

X-Rays and Extreme Ultraviolet Radiation: Principles and Application

Chapter 5. Synchrotron Radiation Homework Problems (August 2018)

5.1 (a) Give a physical explanation as to why synchrotron radiation from relativistic electrons is observed at very short wavelengths and in a narrow forward radiation cone. (b) How do both "scale", that is, how do observed wavelength and cone half angle vary with exponent of the major parameters (excluding numerical coefficients and more slowly varying parameters, if any).

5.2 Give two important technical differences between "third generation" (modern) storage rings and older rings. What is meant by a "fourth generation" synchrotron ring, also known as a "diffraction limited storage ring"? See Chapter 5, the Table 5.1 caption, the footnote on page 189, and comments on page 556. Describe important features of (a) bending magnet radiation, (b) undulator radiation, and (c) wiggler radiation. Each offers very important advantages, even for users at modern storage rings. Be sure to capture these.

5.3 You are to design the radiation source for a biological microscope operating in the water window, between the absorption edges of carbon and oxygen. Assume you are working at a 2.0 GeV storage ring. (a) What are your main considerations? (b) Select major parameters of the radiation source. Consult Chapter 9 regarding microscope options.

5.4 You are to organize an experiment to study the magnetic properties of iron and cobalt. (a) What ring and what radiation source might you choose? (b) What would be your major technical considerations? (c) What major parameters of the ring or magnet structure are relevant to your experiments? (d) What additional equipment or components will you require? (e) Draw a sketch of your planned experiment.

5.5 The Advanced Light Source (ALS) in Berkeley, and the Advanced Photon Source (APS) near Chicago, provide the US with two of its four complimentary state of the art research facilities for materials characterization, biological, chemical, and environmental research, as well as various technological studies. The other two are the Stanford Synchrotron Radiation Facility in Menlo Park, CA, and the National Synchrotron Light Source (NSLS II) in Upton, NY. All four appear in the homework problems that follow. The ALS operates at 1.90 GeV, covering the soft x-ray and extreme ultraviolet regions of the spectrum. The APS operates at 7.00 GeV, covering the multi-keV to hard x-ray spectral regions.

(1) The ALS has 24 ports to collect the spectral continuum of bending magnet radiation. For 1.90 GeV electron beam energy, an average current of 400 mA, and a bending magnet field of 1.27 tesla. (a) What is the critical photon energy and what is its significance. What is the significance of $4E_c$? (b) Graph the photon flux vs. photon energy from 10 eV to 10 keV, assuming a relative spectral bandwidth of 0.1%, a horizontal collection angle of 1 mrad, and a vertical collection angle large enough to collect most of the radiation. Show your results as log-log plot. (c) What vertical collection angle is required to collect 80% of the radiation at photon energies of 0.1, 1, and $3E_c$? (d) What is the power radiated into a 2% spectral bandwidth at 2.5 nm wavelength, as collected by a 2 mrad optic? (e) If the electron beam consists of a large number of "electron bunches," each having a near-Gaussian axial distribution characterized by $\sigma_z = 1.1$ cm, separated from each other by a distance of 60 cm as they move through the storage ring, what is the time structure of the radiation, and what is the ratio of peak to average photon flux?

(2) Repeat calculations (a) through (e) for the APS, which operates at 7.00 GeV and 100 mA, and has a bending magnet field of 0.60 tesla. For Part (2)(b) plot the curve in the region from 70 eV to 70 keV. For Part (2)(d) do the calculation for a wavelength of 1 Å (0.1 nm) and a collection angle of 1 mrad. For

Part (2)(e) assume $\sigma_z = 5.3$ cm and a bunch-to-bunch separation of 85 cm.

5.6 (a) Explain how dipole radiation, with relativistic transformations, is used to explain undulator radiation. (b) How do the two factors of γ enter the undulator equation? (c) Explain the physical significance of each term in the undulator equation. (d) Why is K sometimes called the "deflection parameter"? (e) What is the significance of the central radiation cone? (f) Explain the interdependence between selection of angular acceptance cone and spectral bandpass for undulator radiation. (g) How does the finite number of magnet periods affect the interdependence for acceptance cones narrower than the central radiation cone?

5.7 Radiated power in the central cone, P_{cen} , is given by Eq. (5.39). For low- K undulator operation the power scales as K^2 . (a) Why is this so? (b) At what K -value does power in the central cone peak? (c) Why does power decrease for large K ? (d) Why is there no dependence on N ? (e) What does the slowly varying K -dependent correction factor, $[JJ]^2$ in Eq. (5.41a), represent? See the online errata regarding the $[JJ]^2$ factor for $n > 1$. (f) What is the usefulness of replacing the K -dependence in Eq. (5.39) by a dependence on $\hbar\omega/\hbar\omega_0$? (g) What does $\hbar\omega_0$ represent? (h) Why is the spectrum of undulator radiation within the central radiation cone, as illustrated in Figure 5.23, generally skewed to lower photon energies? (i) Why is spectrum not symmetric about the central photon energy?

5.8 (a) Use the undulator equation (5.28) to transform the K -dependent expression for power in the central radiation cone, as given in Eq. (5.41a), to a photon energy dependence as in Eq. (5.41c). (b) What is the significance of $\hbar\omega_0$ and what does it represent? What is the power radiated at $\hbar\omega_0$?

5.9 Harmonics are an important feature of undulator radiation, providing access to high brightness, partially coherent radiation at higher photon energies than otherwise available in the fundamental ($n = 1$) at a given facility. (a) What is the physical basis for harmonic wavelengths being shorter by a factor of $1/n$, as indicated by Eq. (5.30)? (b) Why is the relative spectral bandwidth within the central radiation cone equal to $1/nN$? Consider the number of cycles in a Fourier decomposition of the motion. (c) What storage ring or beam parameters might limit the achievement of high harmonics or broaden the anticipated spectral bandwidth within the narrow cone?

5.10 Calculate and graph power in the central cone ($P_{\text{cen},n=1}$ only) as a function of photon energy for two undulators at the Advanced Light Source in Berkeley. (a) For a 5.00 cm period undulator with 89 periods and a magnetic tuning range $0 \leq K \leq 2.5$, and (b) for an 8.00 cm period undulator with 55 periods and a magnetic tuning range $0 \leq K \leq 3.0$. Assume a beam energy of 1.90 GeV and an average current of 400 mA. Present your results in a linear-linear scaled graph. (c) What is the half-angle of the central radiation cone, in each case for $K = 1$? (d) What is the relative spectral bandwidth in each case?

5.11 Repeat problem 5.10 for an undulator at the Advanced Photon Source, near Chicago, Illinois. Do your calculation for the 3.30 cm period undulator, with $N = 72$, a tuning range of $0 \leq K \leq 2.60$, a beam energy of 7.00 GeV, and an average current of 100 mA.

5.12 (a) Calculate central cone power P_{cen} vs. photon energy for the 4.12 cm period undulator at BESSY II in Berlin. Assume 81 periods, a beam energy of 1.70 GeV, an average current of 300 mA, and a magnetic tuning range of $0 \leq K \leq 2.5$. (b) What is the K -value for operation at 500 eV? What is the central cone half-angle at this K value? (c) What is the power in the central radiation cone at 500 eV? (d) What is the photon flux in the central cone? Compare your result with that at the website www.helmholtz-berlin.de, then search 'U41-TXM'. (e) This beamline operates with an SGM monochromator with a relative spectral

resolution as narrow as 1×10^{-4} . What types of science can be pursued at this facility? Consult Chapter 11, Figures 11.20 and 11.22, and Chapter 10, Figure 10.30.

5.13 The Taiwan Photon Source (TPS) in Hsinchu has a beam energy of 3 GeV, a circumference of 518 m, a repetition rate of 500 Mhz, and operates with an average current of 500 mA. Its electron beam parameters are $\sigma_x = 121 \mu\text{m}$, $\sigma_y = 5.1 \mu\text{m}$, $\sigma'_x = 17 \mu\text{rad}$, and $\sigma'_y = 3.1 \mu\text{rad}$, and an axial bunch length of $\sigma_z = 2.8 \text{ mm}$.

(a) Calculate power in the central radiation cone P_{cen} for a 2.2 cm period, $N = 273$, linear undulator, and graph for the K range 0 to 1.54 and graph your results on two-cycle by two-cycle log-log plot. (b) Calculate the spectral brightness for this undulator over the same spectral range and plot your results on a multicycle log-log plot. Compare your result with that at the facility website www.nsrcc.org.tw/english/tps_phase.aspx. Why is the calculated brightness similar to that at the website while the website power, or photon flux, is smaller by a significant factor? See slides 36-39, 54-56, and 57-59. Slides 60 & 61 have further comments relevant to the definition of θ_{cen} and its effect on calculated values of photon flux, brightness, power and coherent power, which are surprisingly different. (c) What is the temporal duration of each pulse (FWHM) and what is the time separation between pulses? See Figure 11.23 and accompanying text for an example application.

5.14 Calculate power in the central radiation cone, P_{cen} vs. photon energy for the 42.0 mm undulator at the ESRF in Grenoble, France. Assume $N = 38$, a beam energy of 6.04 GeV, a beam current of 200 mA, and a magnetic tuning range of $0 \leq K \leq 2.1$. (b) What is the half-angle of the central radiation cone, photon energy, relative spectral bandwidth and power within the central ($n = 1$ only) at $K = 1$? (c) Calculate and graph the spectral brightness on a log-log scale for this undulator, assuming horizontal and vertical rms beam sizes (σ) of 412 μm and 6.2 μm respectively, and rms horizontal and vertical beam divergences (σ') of 12 μrad and 5.1 μrad , respectively. Note that at ERSF undulators are also available with three times this length. The facility is presently (2015-2020) involved in a major emittance upgrade which will result in significantly smaller electron beam parameters. With anticipated rms beam parameters of 28 μm , 6.1 μm , 7.2 μrad and 5.1 μrad , respectively, by what factor do you estimate the spectral brightness will be increased compared to the present facility? See the paper by J. Susini et al., "Challenges in Beamline Instrumentation for the ESRF Upgrade Programme Phase II", *J.Synchrotron Rad.* **21**, 986 (2014). For updated beam parameters consult the website www.esrf.fr/machine/support/ids/Public/Sizes/sizes.html.

5.15 (a) Calculate power in the central cone P_{cen} vs. photon energy for a 3.20 cm, 140 period undulator at SPring-8 in Harima, Japan. The beam energy is 8.00 GeV and average beam current is 100 mA. Plot P_{cen} vs. photon energy on a log-log graph for a magnetic tuning range of $0 < K < 2.46$. (b) At $K = 1$, what is the power in the central radiation cone ($n = 1$ only), its half-angle, center photon energy, and relative spectral bandwidth within this cone? An additional homework problem regarding a proposed Spring-8 upgrade is considered in problem 5.24.

5.16 Calculate the on-axis spectral brightness (0.1 % bw, $n = 1$) for (a) an 8.00 cm period, $N = 55$ undulator, and (b) a 5.00 cm period, $N = 89$, undulator at the ALS in Berkeley, assuming 1.90 GeV, 500 mA operation, and σ , σ' values given in Table 5.1, p. 154. Assume typical K -tuning ranges of zero to 3. Use K -dependent values for the central radiation cone. Express your answer in units of photons/s \cdot mm² \cdot mrad² (0.1% bandwidth). Graph your results on three cycle log-log scales.

5.17 MAX IV in Lund, Sweden, has recently become the first synchrotron built with a significantly reduced

horizontal emittance based on the use of a 7-bend multi-bend achromat (MBA), an electron beam optical system (lattice) with an increased number of more gentle bending magnets. The horizontal emittance scales roughly as γ^2 / N_b^3 , where N_b is the number of bends and γ is the Lorentz factor. (a) Calculate the spatially coherent power within a $1/N$ relative spectral power for the 18 mm period, $N = 111$, $0 \leq K \leq 2.0$ undulator at MAX IV. Assume beam parameters of 3.00 GeV, 500 mA, rms horizontal and vertical beam sizes (σ) of 49 μm and 6.2 μm , and rms horizontal and vertical beam divergences (σ') of 5.4 μrad and 1.3 μrad , respectively. (b) Calculate the spatially coherent power within a relative spectral bandwidth of $\Delta\lambda/\lambda = 2.5 \times 10^{-4}$, assuming a hard x-ray monochromator insertion loss of 0.8, i.e., an efficiency of 80% (for photon energies near 5 keV, see Fig.10.40 in the text). Note that beamline/monochromator efficiencies are much lower for softer x-rays. (c) Consult the website [<http://www.maxlab.lu.se>] to confirm electron beam parameters. What type of experiments would benefit from spatially filtered radiation such as this? For a discussion of the new MBA designs, known as “Diffraction Limited Storage Rings (DLDRs)”, see the paper by M. Eriksson, J. Fresco van der Veen and C. Quitmann, *J. Synchr. Rad.* **21**, 837 (September 2014).

5.18 (a) Calculate P_{cen} and P_{coh} in both the fundamental ($n = 1$) and third harmonic ($n = 3$) for a 5 cm period undulator at the Advanced Light Source (ALS) in Berkeley. Assume $N = 89$, $E = 1.9$ GeV, $I = 500$ mA, and beam parameters (σ_x , σ_y , σ'_x , σ'_y) from Chapter 5, Table 5.1, p.154. Display your results on linear-linear scaled graphs (x - photon energy, y - power) for magnetic deflection parameters $0 \leq K \leq 3.0$. (b) Furthermore, calculate $P_{\text{coh}, \lambda/\Delta\lambda}$ following a soft x-ray monochromator set for $\Delta\lambda/\lambda = 2.5 \times 10^{-4}$, with an insertion efficiency of $\eta = 10\%$. (c) By what factor is the single pulse peak coherent power greater than the time averaged coherent power if all buckets are full and the RF is set to provide 60 ps FWHM pulses every 2 ns? For calculations of radiated power in the fundamental and harmonics see slides 37-39. For spatially coherent power see slides 56-57. Slides 62 & 63 have comments relative to the definition of θ_{cen} .

5.19 The ALS has recently received approval for an emittance upgrade based on a transition from the present triple bend achromat (TBA) electron beam lattice, to a nine beam multi-bend achromat (MBA), part of the international move known as DLSRs (See reference at the end of problem 5.17). As horizontal electron beam emittance scales roughly as γ^2 / N_b^3 a factor of about 1/30 horizontal emittance reduction is expected. New beam parameters are estimated to be approximately circular with $\sigma_x = 10$ μm , $\sigma_y = 10$ μm , $\sigma'_x = 5$ μrad , and $\sigma'_y = 5$ μrad , an average electron beam current of 500 mA and an electron beam energy increased slightly to 2.0 GeV ($\gamma = 3914$). A candidate undulator would have a period of 2.67 cm, 150 periods (N), and a magnetic tuning range of $0.4 \leq K \leq 2.98$. (a) What do you estimate will be the increase in spectral brightness and coherent power, or coherent flux, with these upgraded electron beam parameters and this shorter period undulator? Be careful to use θ_{total} for the photons rather than the electron's σ' . (b) This upgrade will have a dramatic effect on all imaging experiments. What do you estimate will be the reduction in exposure time for all nanoprobe, zone plate microscope, and coherent imaging techniques? (c) Calculate the spatially coherent power within a $1/nN$ relative spectral bandwidth for the fundamental ($n = 1$) and third harmonic radiation ($n = 3$) with the proposed ALS upgrade using the candidate 2.67 cm period undulator over its operational (K) range. (d) Convert the results in (c) to spatially coherent photon flux within a 0.1 % relative spectral bandwidth. Present your results without beamline/monochromator consideration (100% efficient), and separately assuming a soft x-ray beamline/monochromator insertion efficiency of 10%. See slides 46-48 regarding harmonic power calculations, 56 regarding DLSRs, 68-73 regarding spatially coherent power calculations and the ALS-U upgrade. Slides 74 & 75 have further comments relevant to the definition of θ_{cen} and its effect on calculated values of photon flux, brightness, power and coherent power, which are surprisingly different. Slide 76 discusses the effects of electron beam energy spread.

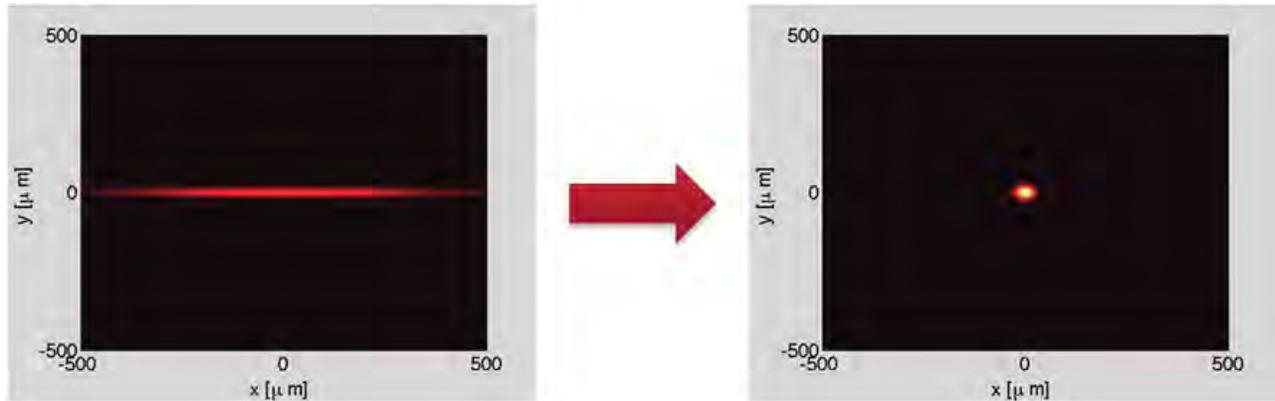


Figure courtesy of C. Steier, ALS.

5.20 The Advanced Photon Source (APS) near Chicago is presently in the design stage for a significant upgrade to the present 7 GeV, 100 mA, ring, double bend achromat lattice, to a 6 GeV, 200 mA, seven bend, multi-bend achromat (MBA). As the horizontal emittance scales roughly as γ^2 / N_b^3 , where N_b is the number of bends, the horizontal emittance is expected to decrease by a factor of about 60. With the factor of two increase in current, this implies an increase in spectral brightness and coherent power by a factor of order 100. Undulators with an increased number of periods will further increase this factor. The upgrade will reduce data collection time for all imaging experiments at the APS by similar factors, enabling new experimental techniques. For example, see Chapter 11, pp. 555-556, and Figure 11.28 where increases of several hundred would permit the extension to three-dimensional (tomographic) ptychographic imaging. (a) Calculate the anticipated on-axis spectral brightness and spatially coherent flux within a 0.1% relative spectral bandwidth for a possible 1.80 cm period, $N = 205$, undulator operating in 3rd harmonic at 20 keV. Assume anticipated beam parameters of $\sigma_x = 17.4 \mu\text{m}$, $\sigma_y = 10.2 \mu\text{m}$, $\sigma'_x = 2.5 \mu\text{rad}$, and $\sigma'_y = 4.2 \mu\text{rad}$, and assume a beamline efficiency of 50% (DCM and two mirrors). (b) What is the K-value required to radiate a 3rd harmonic at 20 keV. What are the fundamental and 3rd harmonic wavelengths? What would be the total photon tuning range for this undulator, including both fundamental and third harmonic, assuming a maximum K-value of 2.5? (c) What is the central radiation cone half angle for the 3rd harmonic? Is the undulator condition well satisfied in this case? If not, how might the calculation of spectral brightness according to Eq.(5.65) be affected? What is the correction factor [see the comments below Eq.(5.65) and use Fig.5.23 to estimate a correction factor]. What is your corrected value for on-axis spectral brightness at 20 keV utilizing this correction factor? (d) Does the narrowed third harmonic spectrum require a correction to the calculated spectral brightness and coherent flux? Give an estimate. Would this correction be stronger at the 5th or higher harmonics? For calculations of radiated power in the fundamental and harmonics see slides 37-39. For spatially coherent power see slides 56-57. Slides 48 & 59 discuss DLSRs, slides 62 & 63 discuss definitions of θ_{cen} and the effects on calculations of brightness, coherent power and coherent flux. Slide 64 discusses the effect of electron beam energy spread on spectral width and spectral brightness. Confirm updated sigma values, estimated spectral brightness and coherent flux, at the APS website www.aps.anl.gov/Beamline-Selection/Technical-Information/Storage-Ring-Parameters

5.21 Calculate the spatially coherent power within a relative spectral bandwidth $\Delta\lambda/\lambda = 2 \times 10^{-4}$ for the U20 undulator at Brookhaven National Laboratory's NSLS II. The undulator has a 2.0 cm period, $N = 148$, and operation in the range $0 \leq K \leq 1.83$. Assume a beamline efficiency of 80%. The x-ray ring operates at 3.0

GeV with a maximum current of 500 mA. Assume beam parameters of $\sigma_x = 28 \mu\text{m}$, $\sigma_y = 2.6 \mu\text{m}$, $\sigma'_x = 19 \mu\text{rad}$, $\sigma'_y = 3.2 \mu\text{rad}$. Present your results in a multicycle log-log graph. Consult current beam parameters at the website [http://www.nsls.bnl.gov/nsls2/docs/PDF/summary_of_NSLSII_Source_Parameters.pdf]. You might also consult the NSLS website section "Accelerator Physics" for formulae regarding beam parameters in terms of emittance and betatron function.

5.22 (a) Calculate spatially coherent power within a relative spectral bandwidth of $1/N$, and also 1×10^{-4} , for the $N = 81$, 41.2 mm period undulator at BESSY II, operating at a beam energy of 1.7 GeV and a current of 300 mA. (b) Graph your results versus photon energy, for K -values from 0 to 2.5. Assume beam parameters of $\sigma_x = 314 \mu\text{m}$, $\sigma_y = 24 \mu\text{m}$, $\sigma'_x = 18 \mu\text{rad}$, $\sigma'_y = 2 \mu\text{rad}$. (c) What experiments at BESSY II would benefit from the use of spatially coherent radiation? (d) What type of experiments would require this spectral resolution?

5.23 (a) Calculate the anticipated spatially coherent power within a relative spectral bandwidth of $1/nN$ for the fundamental ($n = 1$) and 3rd harmonic ($n = 3$) generated with a 35 mm period undulator at the upgraded (2015-2020) European Synchrotron Radiation Facility (ESRF) in Grenoble, France, operating at a beam energy of 6.00 GeV and a current of 200 mA. Assume $N = 137$, $0 \leq K \leq 2.1$. Use rms beam parameters $\sigma_x = 28 \mu\text{m}$, $\sigma_y = 6.1 \mu\text{m}$, $\sigma'_x = 7.2 \mu\text{rad}$, $\sigma'_y = 5.1 \mu\text{rad}$, or consult the website [<http://www.esrf.fr>] for updated parameter values. Plot your results on a multicycle log-log graph. Compare your results to those in the paper J. Susini et al., "Challenges in Beamline Instrumentation for the ESRF Upgrade Programme Phase II", *J.Synchrotron Rad.* **21**, 986 (2014), their figure 2. (b) Repeat this calculation for a relative spectral bandwidth of 2×10^{-4} and a hard x-ray beamline efficiency of 80%. Again plot your results on a log-log graph. (c) What type of experiments would benefit from use of spatially coherent power in this spectral region? For calculations of radiated power in the fundamental and harmonics see slides 37-39. For spatially coherent power see slides 54-56. Slides 60 & 61 have comments relative to the definition of θ_{cen} .

5.24 (a) For the SPring-8 synchrotron facility in Harima, Japan, operating at 8 GeV and 100 mA, calculate the spatially coherent power for both the fundamental ($n = 1$) and the 3rd harmonic ($n = 3$) in a relative spectral bandwidth of 2×10^{-4} for the 32 mm period, $N = 140$ undulator. Do the calculations and graph your results for $0 \leq K \leq 2.46$. Plot your results on a three cycle log-log graph. Assume a hard x-ray beamline efficiency of 80%. For beam parameters assume $\sigma_x = 316 \mu\text{m}$, $\sigma_y = 4.9 \mu\text{m}$, $\sigma'_x = 8.8 \mu\text{rad}$, $\sigma'_y = 1.0 \mu\text{rad}$, or consult the updated beam parameters at the website [<http://www.spring8.or.jp>]. (b) What applications might take advantage of coherent radiation at these short wavelengths? (c) Spring-8 is presently planning a DLSR upgrade to Spring-8-II based on a five bend achromat. Calculate the coherent power $P_{\text{coh},\lambda/\Delta\lambda}$ within a relative spectral bandwidth of 2×10^{-4} , and assume a hard x-ray beamline efficiency of 80%. The new ring parameters are expected to include a reduced beam energy of 6 GeV, an average current of 100 mA, and horizontal and vertical rms beam size and divergence parameters of $\sigma_x = 23 \mu\text{m}$, $\sigma_y = 5.4 \mu\text{m}$, $\sigma'_x = 4.2 \mu\text{rad}$, and $\sigma'_y = 1.8 \mu\text{rad}$. Assume a linear undulator with a 1.6 cm period, 225 periods, and a tuning range $0 < K < 1.61$. See the paper by H. Tanaka et al., "Spring-8 Upgrade Project", IPAC 2016, Busan, Korea, and the 2014 conceptual design report at rsc.Riken.jp/pdf/Spring-8-II.pdf. See slides 36-38, 53-54, 57-58. Slides 59 & 60 have comments relative to the definition of θ_{cen} . For calculations of radiated power in the fundamental and harmonics see slides 37-39. For spatially coherent power see slides 54-56. Slides 60 & 61 have comments relative to the definition of θ_{cen} .

5.25 (a) For the Taiwan Photon Source (TPS) described in problem 5.13, calculate the spatially coherent power within a relative spectral bandwidth of $1/nN$ for the fundamental ($n = 1$) and third harmonic ($n = 3$),

and (b), within a relative spectral bandwidth of 0.1%. For part (b) assume a hard x-ray beamline efficiency of 50%. For calculations of radiated power in the fundamental and harmonics see slides 37-39. For spatially coherent power see slides 56-57. Slides 62 & 63 have comments relative to the definition of θ_{cen} and the different ways these definitions affect central cone power and flux ($\sim 1/4$) compared to calculated spectral brightness and coherent power (no effect). Compare your calculated values to those posted at the facility website.

5.26 Calculate $P_{\text{coh},\lambda/\Delta\lambda}$ as a function of photon energy for ALS bending magnet beamline 6.3, using present beam parameters, a monochromator set for $\Delta\lambda/\lambda = 5 \times 10^{-4}$, and a soft x-ray beamline efficiency of 10%. For bending magnet beam parameters assume $\sigma_x = 33 \mu\text{m}$, $\sigma_y = 33 \mu\text{m}$, $\sigma'_x = 110 \mu\text{rad}$, $\sigma'_y = 11 \mu\text{rad}$. (Hint: Start with beam size, then choose angles for spatially coherent radiation, then calculate photon flux and power radiated within these angles. Present your results in a graph of coherent power versus photon energy on a multicycle log-log scale). How would these values be enhanced with completion of the recently approved ALS upgrade described in problem 5.19?

5.27 (a) Explain the difference between a wiggler and an undulator. (b) What are the relevant K values? (c) Why is this the important parameter? (d) Which attributes are most important for each? (e) What features are less attractive? (f) How does each compare to bending magnet radiation? (g) Which could be retrofitted to an older storage ring? (h) Which would not reach its potential on an older storage ring, and why not? (i) Describe the evolution from undulator spectrum to wiggler spectrum as the magnet gap is closed. (j) Explain the concept of the "critical harmonic". (k) What is the significance of n_c ? (l) Why do wigglers and bending magnet spectra have the same general spectral shape for high K ? (m) How do the two differ?

5.28 (a) Calculate the total radiated power for a 16.0 cm period wiggler at the ALS, $N = 19$, with a 2.1 Tesla maximum field on axis, a beam energy of 1.90 GeV and a current of 500 mA. (b) What is the K -value and the critical photon energy? (c) What is the significance of the critical photon energy? (d) Graph the on-axis photon flux in a log-log plot extending from 10 eV to 10 keV, in units of photons/s·mrad²·(0.1% BW). (e) Also plot the photon flux per unit horizontal acceptance angle, over the same photon energy range, in units of photons/s·mrad·(0.1% BW). (f) What would the photon flux at the critical photon energy be within a 5 mrad acceptance angle and a 2% relative spectral bandwidth?

5.29 (a) Calculate the total radiated power for a 16.0 cm period wiggler at the ALS ($N = 19$) with a 2.1 tesla maximum field on axis, a beam energy of 1.90 GeV and a current of 400 mA. (b) What is the K -value and the critical photon energy? (c) What is the significance of the critical photon energy? (d) Graph the on-axis photon flux in a log-log plot extending from 10 eV to 10 keV, in units of photons/s·mrad²·(0.1% BW). (e) Also plot the photon flux per unit horizontal acceptance angle, over the same photon energy range, in units of photons/s·mrad·(0.1% BW). (f) What would be the photon flux at the critical photon energy within a 5 mrad acceptance angle and a 2% relative spectral bandwidth?

5.30 (a) Calculate the total power radiated, and the power per unit solid angle (kW/mrad²) for the 8.5 cm, $N = 28$, APS wiggler when operating at 7 GeV and 100 mA, at its peak field on axis of 1.60 Tesla. (b) What is the critical photon energy? (c) What is the K value? (d) Describe the angular radiation pattern. (e) What is the photon energy of the first harmonic? (f) What is n_c ?

5.31 (a) Calculate the total power radiated by a 20 pole wiggler ($N = 10$), with 23 cm period and 2.04 Tesla peak field) at the Stanford Synchrotron Radiation Laboratory (SSRL). The electron beam energy is 3.0 GeV and the average current is 500 mA. (b) What is the critical photon energy E_c ? (c) What is K ? (d) What is the angular extent of the radiation fan in the horizontal and vertical planes? (e) Would this be suitable

for use with an x-ray microprobe? To what photon energy?

5.32 (a) Calculate total radiated power and radiated power per unit solid angle, in kW and kW/mrad², respectively, for the superconducting wiggler (SCW) on the 3.0 GeV, 500 mA ring at Brookhaven National Laboratory's National Synchrotron Light Source II (NSLS II). The wiggler has a magnet period of 6.0 cm, 17 periods, and $K = 19.6$. (b) What is the critical photon energy E_c ? What is $4E_c$? Why is $4E_c$ important?

