Chapter 6: Multinode Cooperative Communications

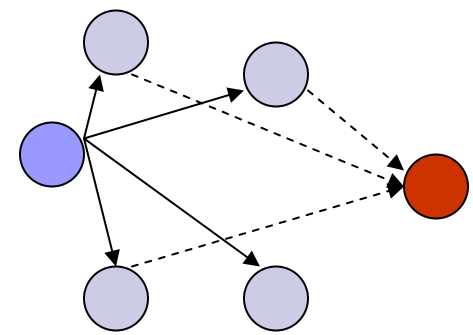
Multinode DF Cooperation

Outline

- Multinode cooperative communications
- System model
- Symbol error rate analysis
  - Approximate symbol error rate analysis
- Optimal power allocation

1. Multi-relay Cooperative Communications

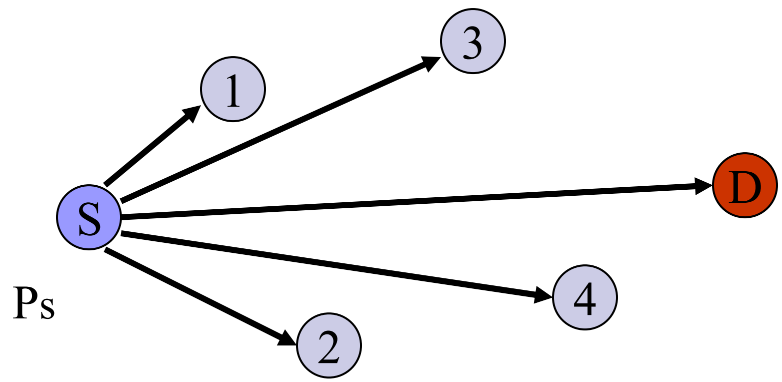
- How to cooperate?
- Characterize the symbol-error-rate to facilitate system design and performance check
- Challenge: channels are not identically distributed
- Can cooperation achieve full diversity gains? (MIMO diversity is the upper bound!)
- What is the optimal power allocation scheme among the source and the relays?



$$d = \lim_{\mathrm{SNR} \to \infty} - \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}}$$

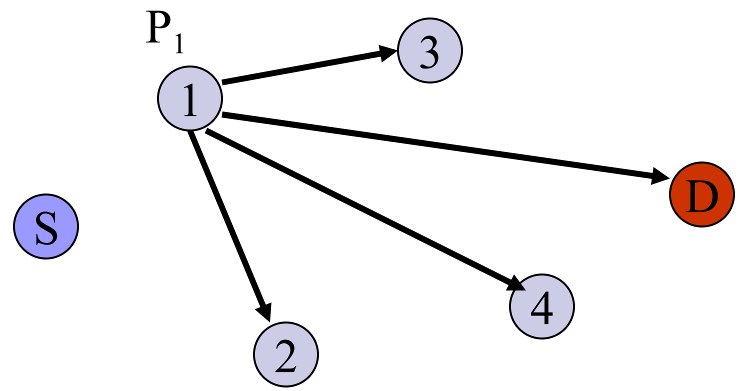
Multi-relay Cooperative Communications

- Arbitrary N-relay networks. D&F relaying
- Relay only forwards if decodes correctly (selective)
- Different possibilities for cooperation



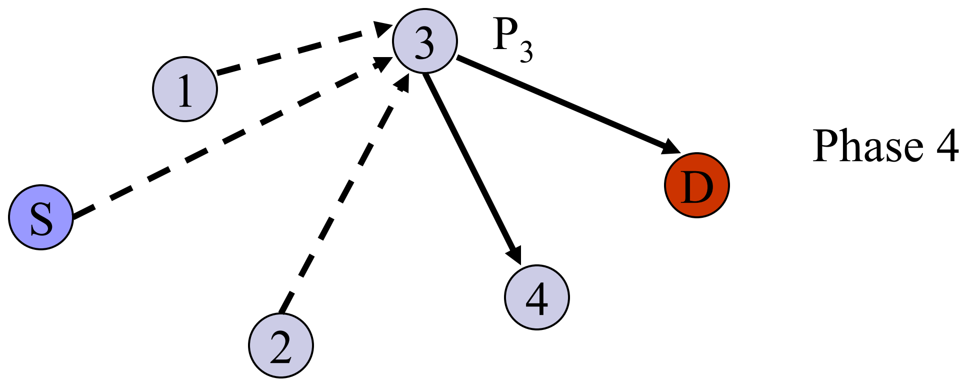
Phase 1

Multi-relay Cooperative Communications



Phase 2

Multi-relay Cooperative Communications



- In general, each relay can combine signals from m previous transmissions (C(m))
- Challenge: channels are not identically distributed
- What is the optimal power allocation scheme?

System Model: *C(m)*

- In Phase 1:

$$y_{s,d} = \sqrt{P_s}\, h_{sd}\, x + n_{sd}\,,$$
$$y_{s,r_i} = \sqrt{P_s}\, h_{sr_i}\, x + n_{sr_i}\,, \qquad 1 \le i \le N$$

where $h_{sd} \sim CN(0, \sigma_{sd}^2)$, $h_{sr_i} \sim CN(0, \sigma_{sr_i}^2)$
and all noise terms are AWGN with $\sim CN(0, N_o)$

- In Phase 2: if relay 1 correctly decodes, it transmits with power P_1 otherwise it remains idle

$$y_{r_1 d} = \sqrt{\tilde{P}_1}\, h_{r_1 d}\, x + n_{r_1 d}\,,$$
$$y_{r_1 r_i} = \sqrt{\tilde{P}_1}\, h_{r_1 r_i}\, x + n_{r_1 r_i}\,, \qquad \tilde{P}_1 \in \{0, P_1\}$$

System Model

- In Phase (i+1): relay i coherently combines all the previous received signals

$$y_{r_i} = \sqrt{P_s} h^*_{sr_i} y_{sr_i} + \sum_{j=\max(1,i-m)}^{i-1} \sqrt{\tilde{P}_j} h^*_{r_j r_i} y_{r_j r_i}$$

where $y_{r_j, r_i} = \sqrt{\tilde{P}_j} h_{r_j r_i} + n_{r_j r_i}$

and $\widetilde{P}_j = P_j$ if relay j decodes correctly, otherwise $\widetilde{P}_j = 0$

- In Phase (N+1): destination coherently combines all the received signals from the previous phases

$$y_d = \sqrt{P_s} h^*_{sd} y_{sd} + \sum_{j=1}^{N} \sqrt{\tilde{P}_j} h^*_{r_j d} y_{r_j d}$$

SER Performance Analysis

- The probability of error at the destination given the CSI can be computed as

Relays State Vector

$$P_{e/CSI_d} = \sum_{i=0}^{2^N-1} \Pr(e \mid \mathbf{S}_N = \mathbf{B}_{i,N}, CSI_d) \cdot \Pr(\mathbf{S}_N = \mathbf{B}_{i,N} \mid CSI_d)$$

- The exact SER for an N-relay network employing C(m) and M-PSK modulation is given by

$$SER = \sum_{i=0}^{2^N-1} F_1\left[\left(1+\frac{b_{psk}P_s\sigma_{s,d}^2}{N_o\sin^2(\theta)}\right)\prod_{j=1}^{N}\left(1+\frac{b_{psk}B_{i,N}(j)P_j\sigma_{r_j,d}^2}{N_o\sin^2(\theta)}\right)\right]\prod_{k=1}^{N}G_k^m\left(B_{i,N}(k)\right)$$

$$G_k^m(0) = F_1\left[\left(1+\frac{b_{psk}P_s\sigma_{s,r_k}^2}{N_o\sin^2(\theta)}\right)\prod_{j=\max(1,k-m)}^{k-1}\left(1+\frac{b_{psk}B_{i,N}(j)P_j\sigma_{r_j,r_k}^2}{N_o\sin^2(\theta)}\right)\right]$$

Asymptotic SER Analysis

- **Theorem:** *At enough high SNR, the SER of an N relay decode-and-forward cooperative diversity network employing cooperation scheme* C(m) *and utilizing M-PSK or M-QAM modulation can be approximated by*

Full Diversity Gain

$$SER \cong \frac{\mathrm{N}_o^{N+1}}{b_q^{N+1}\sigma_{s,d}^2} \sum_{j=1}^{N+1} \frac{g_q(N-j+2)\, g_q^{\,j-1}(1)}{P_s^{\,j} \prod_{i=j}^{N} P_i \sigma_{r_i,d}^2 \prod_{k=1}^{j-1} \sigma_{s,r_k}^2}$$

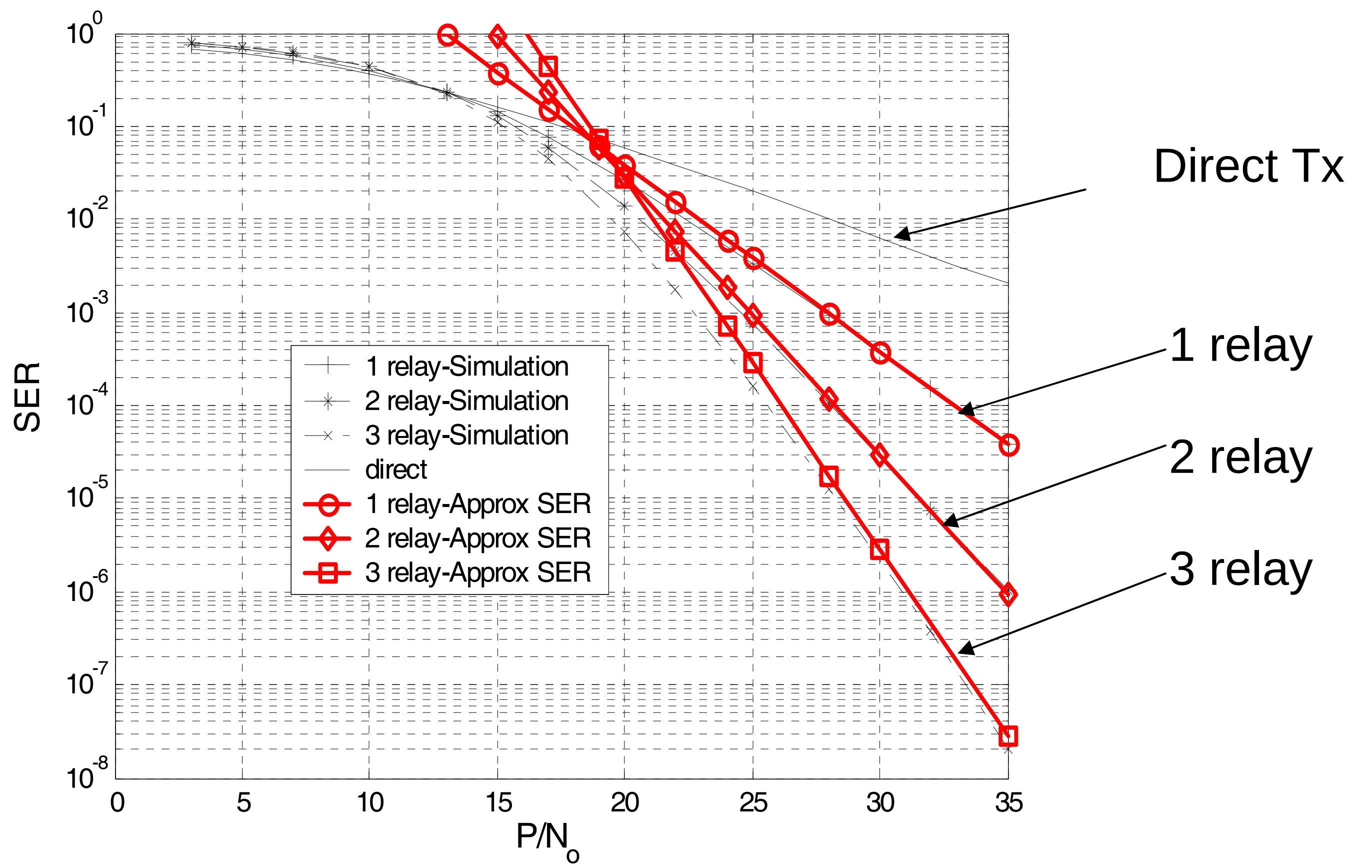
where

$$g_q(x) = \begin{cases} \dfrac{1}{\pi} \displaystyle\int_0^{(M-1)\pi/M} \sin^{2x}(\theta)\, d\theta, & M-PSK, (q=1) \\ \dfrac{4K}{\pi} \left[\displaystyle\int_0^{\pi/2} \sin^{2x}(\theta)\, d\theta - K \int_0^{\pi/4} \sin^{2x}(\theta)\, d\theta \right], & M-QAM, (q=2). \end{cases}$$

Conclusion: At enough high SNR, m does not play a role!

The cooperation protocol utilized is $C(1)$ and the modulation scheme is QPSK.

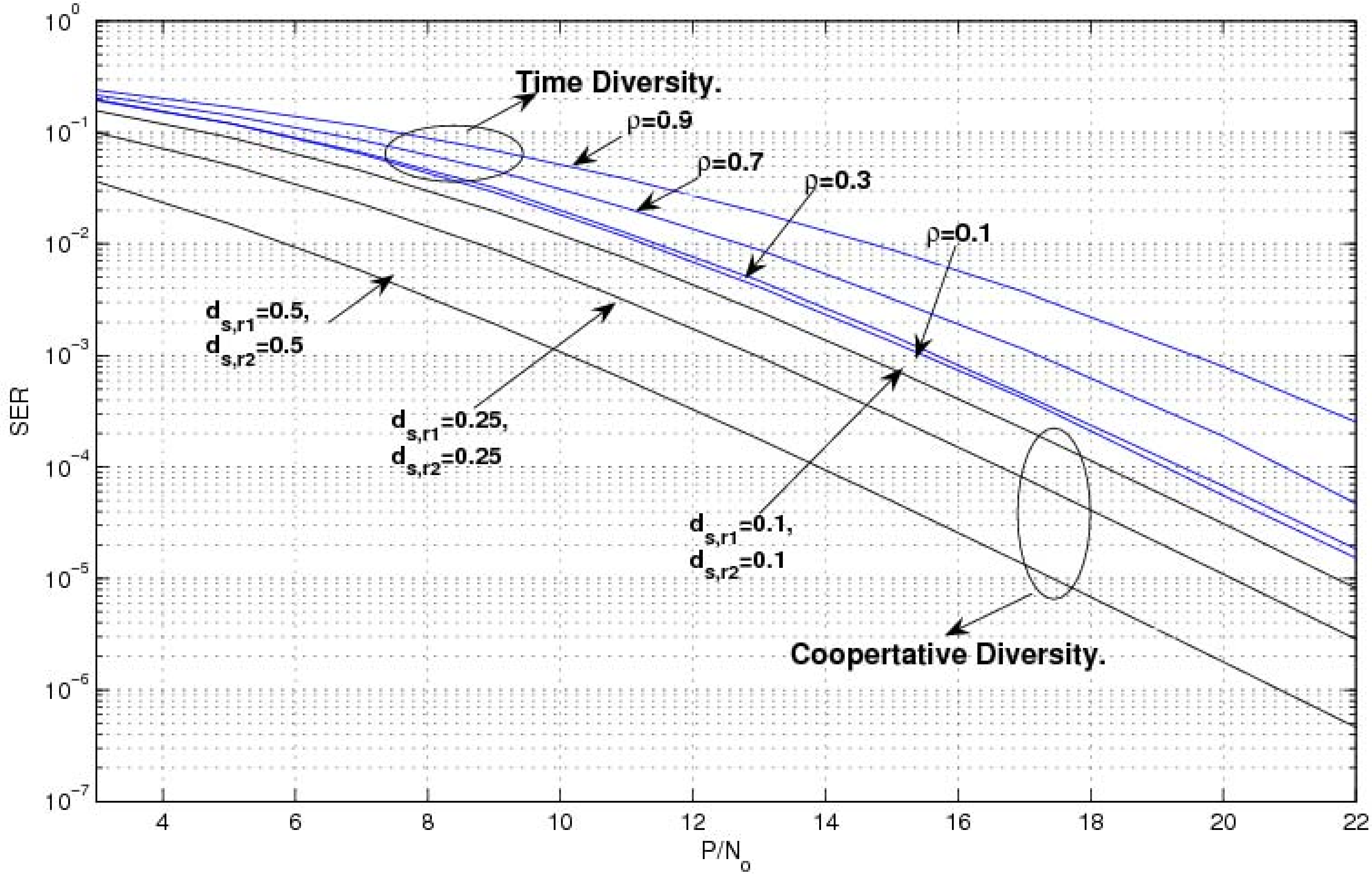
Diversity gain is achieved with increasing # of relays.



Temporal diversity vs. Cooperation

2 relays vs. 3 time slots

1st order Markov model for the temporal correlation



Optimal Power Allocation

- The nonlinear optimization problem can be formulated as follows

$$\mathbf{a}_{\mathbf{opt}} = \arg\min_{\mathrm{a}} \; SER(\mathbf{a})$$

$$\text{subject to } \sum_{i=0}^{N} a_i = 1, \; a_i \geq 0,$$

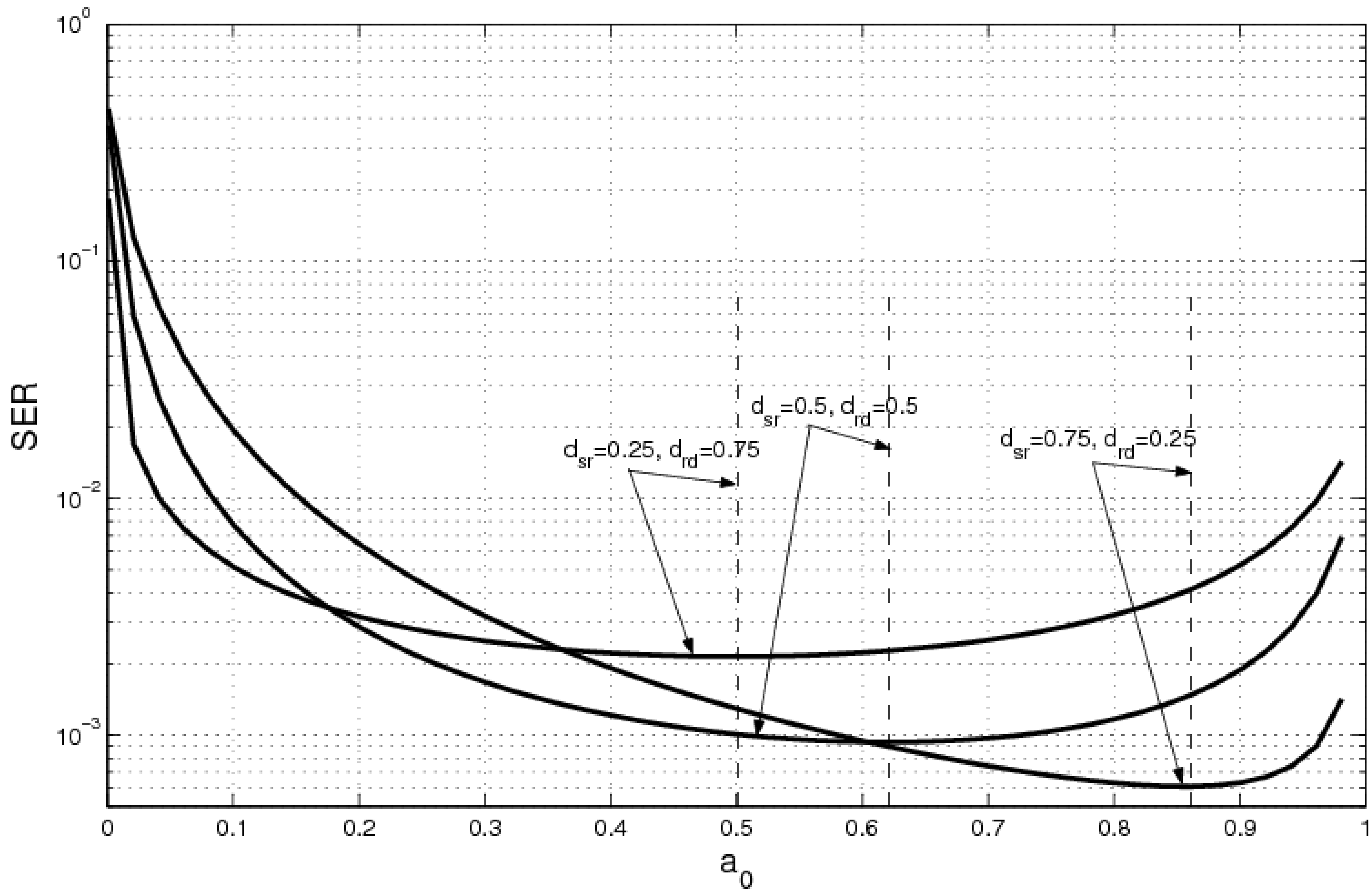
where a_i is the ratio of the total power allocated to the i-th relay.

- Via the KKT conditions, we get

$$P_o \geq P_N \geq P_{N-1} \geq \cdots \geq P_1.$$

Provide closed form results for some network topologies

Optimal power allocation at the source node for different relay locations



Multinode AF Cooperation

Outline

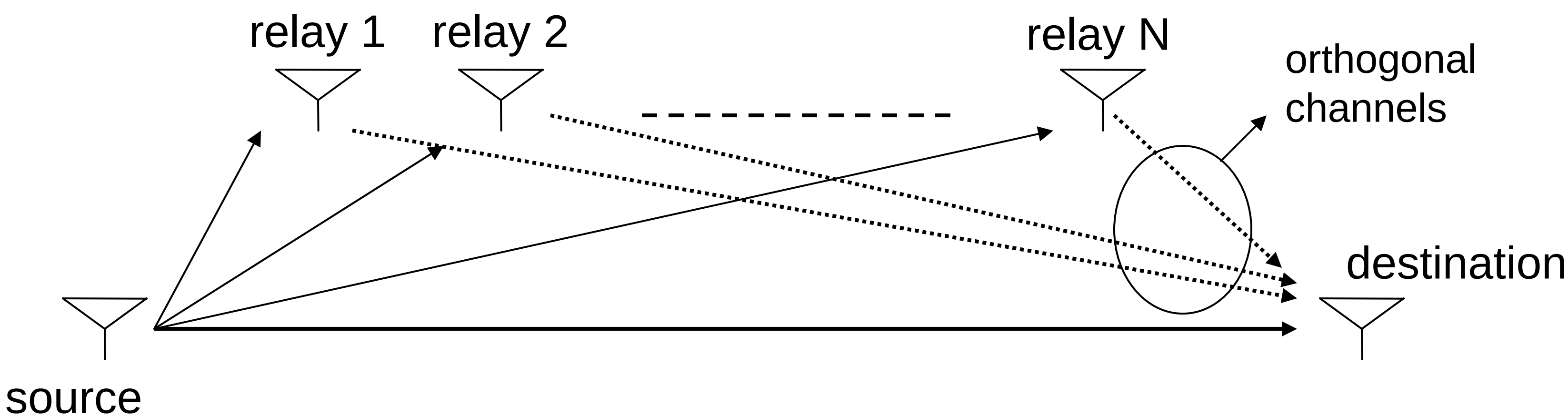
- 2 Phase System Model
- MRC Based System Model
- Performance Analysis of the 2 Phases System
- Problem with the MRC based System
- Conclusion and future work

2 Phases System Model

The system has two phases:

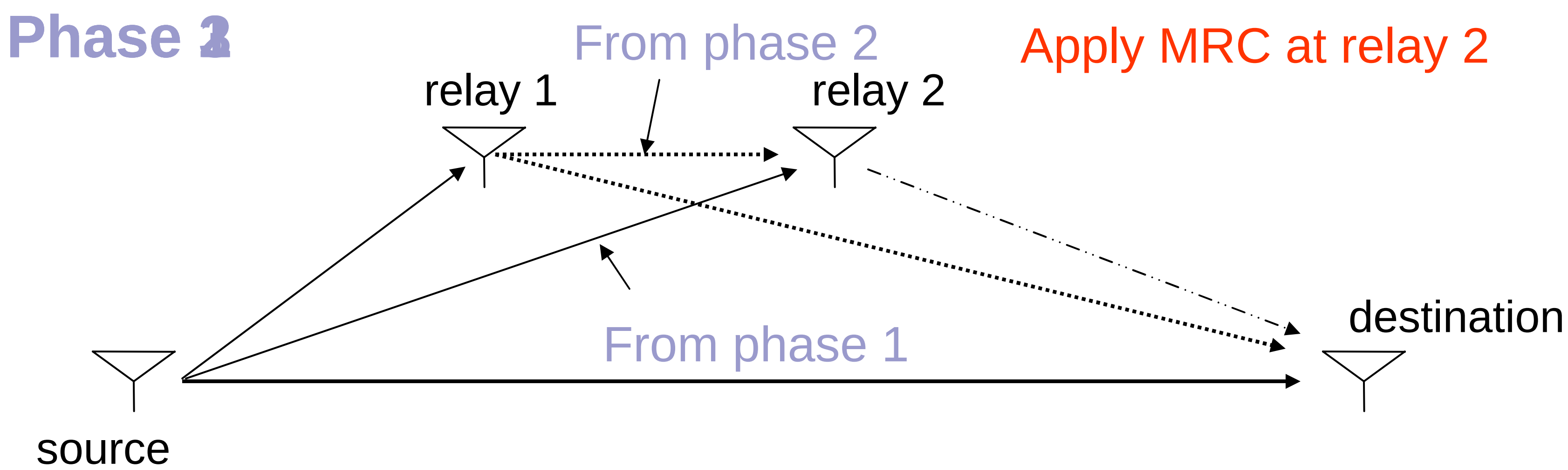
1. Phase 1: The source transmits to all nodes and destination
2. Phase 2: Each node amplifies the source signal only and forwards it to the destination

Phase 2



MRC Based System Model

Consider for simplicity a system of 2 relays



Each relay combines the signals from all the previous relays and source

The system has (N+1) phases

Data Model for the 2 Phases System

Phase 1:

Received data at destination $\longrightarrow$ $y_{s,d} = \sqrt{P_1}\, h_{s,d}\, x + n_{s,d}\,,$

Received data at relay *i* $\longrightarrow$ $y_{s,r_i} = \sqrt{P_1}\, h_{s,r_i}\, x + n_{s,r_i}\,,$

Phase 2:

Received data at destination from relay *i*

$$y_{r_i,d} = \frac{\sqrt{P_{r_i}}}{\sqrt{P_1 \left|h_{s,r_i}\right|^2 + N_0}}\, h_{r_i,d}\, y_{s,r_i} + n_{r_i,d}$$

The maximum ratio combining (MRC) $y = \alpha_{s,d}\, y_{s,d} + \sum_{i=1}^{N} \alpha_{r_i,d}\, y_{r_i,d}$

Where,

$\alpha_{s,d} = \sqrt{P_1}\, h^*_{s,d} / N_0$ and $$\alpha_{r_i,d} = \frac{\sqrt{\dfrac{P_1 P_{r_i}}{P_1 \left|h_{s,r_i}\right|^2 + N_0}}\, h^*_{s,ri}\, h^*_{ri,d}}{\left(\dfrac{P_{r_i}\left|h_{r_i,d}\right|^2}{P_1\left|h_{s,r_i}\right|^2 + N_0} + 1\right) N_0}$$

SER Performance Analysis

The SNR at the output of the MRC

$$\gamma = \gamma_{s,d} + \sum_{i=1}^{N} \gamma_{r_i,d}$$

Where at high SNR,

$$\gamma_{s,d} = \frac{P_1 \left| h_{s,d} \right|^2}{N_0} \qquad \text{and} \qquad \gamma_{r_i,d} \approx \frac{1}{N_0} \frac{P_1 \left| h_{s,r_i} \right|^2 \times P_{r_i} \left| h_{r_i,d} \right|^2}{P_1 \left| h_{s,r_i} \right|^2 + P_{r_i} \left| h_{r_i,d} \right|^2}$$

For M-PSK the SER,

$$P_{PSK}^{CSI} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{b_{PSK}\gamma}{\sin^2\theta}\right) \, d\theta$$

Averaging over the Rayleigh fading channel we get,

$$P_{PSK} \approx \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\gamma_{s,d}}\left(\frac{b_{PSK}}{\sin^2\theta}\right) \prod_{i=1}^{N} M_{\gamma_{r_i,d}}\left(\frac{b_{PSK}}{\sin^2\theta}\right) \, d\theta$$

SER Performance Analysis (Cont.)

Where,

$$M_{\gamma_{s,d}}(s) = \frac{1}{1+\frac{sP_1\delta^2_{s,d}}{N_0}}$$

Theorem

At high SNR, the moment generating function of the harmonic mean of two exponential random variables is given by

$$M_{\gamma_{r_i,d}}(s) \approx \frac{\frac{N_0}{P_1\delta^2_{s,r_i}} + \frac{N_0}{P_{ri}\delta^2_{r_i,d}}}{s}$$

The SER for the system is given by,

$$P_{PSK} \approx \frac{N_0^{N+1}C(N)}{b_{PSK}^{N+1}} \frac{1}{P_1\delta^2_{s,d}} \prod_{i=1}^{N} \frac{P_1\delta^2_{s,r_i} + P_{r_i}\delta^2_{r_i,d}}{P_1\delta^2_{s,r_i} P_{r_i}\delta^2_{r_i,d}}$$

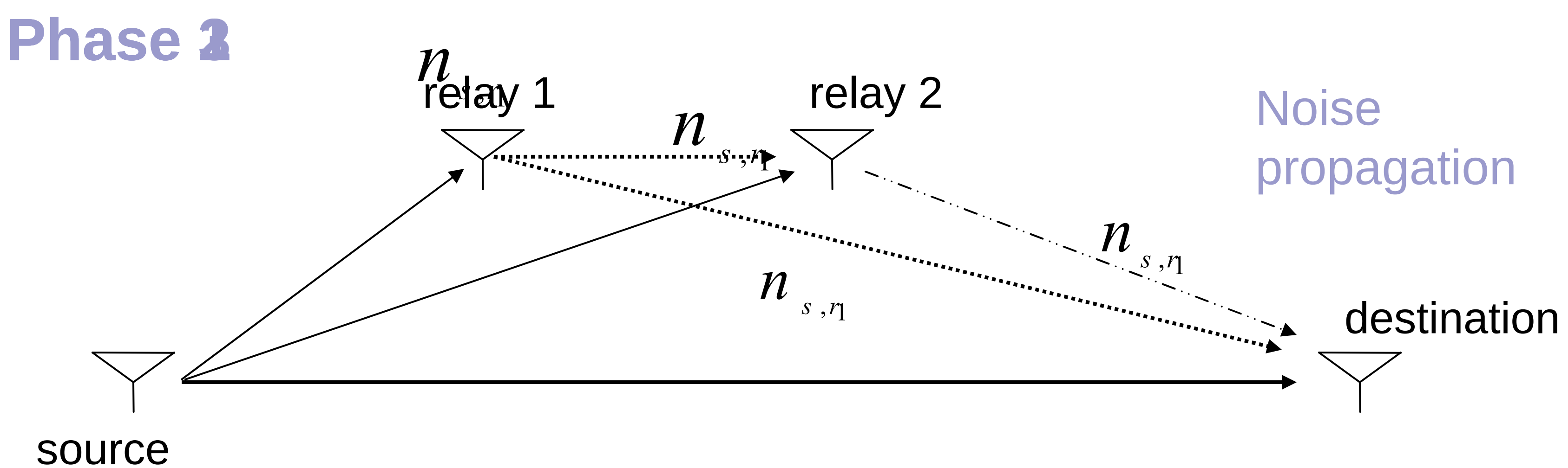
Diversity order N+1

Where,

$$C(N) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \sin^{2(N+1)}\theta \;\; d\theta$$

MRC Based System

The problem with this system is the noise propagation



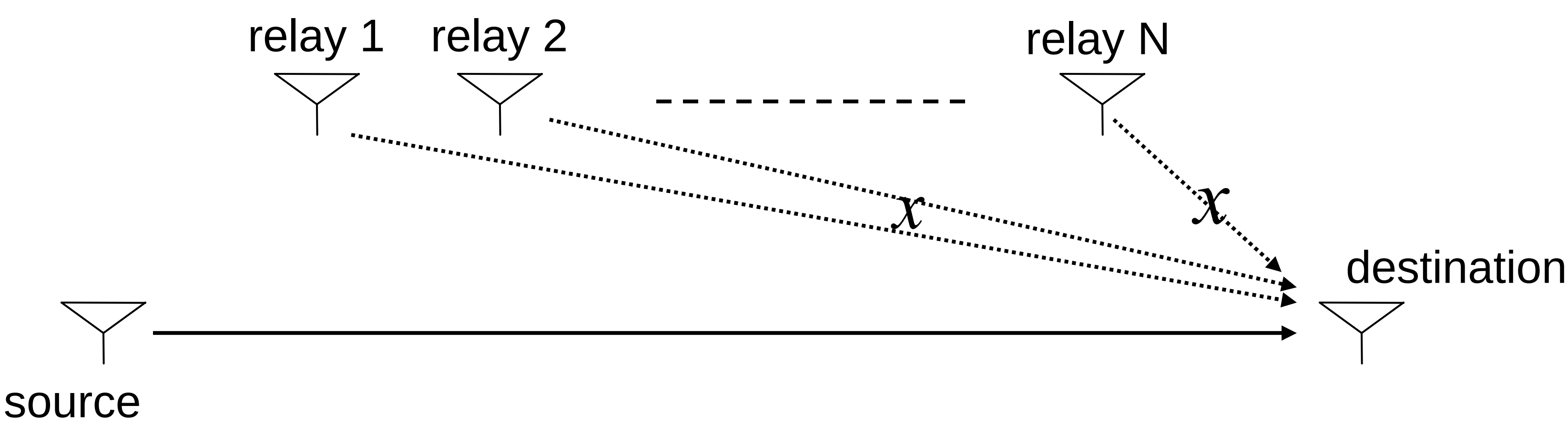
The MRC is no more optimal because of the noise correlation

The optimal is to use a whitening filter before the MRC

Lower Bound on System SER Performance

The problem with the analysis begins from relay 2

The best any system can do is

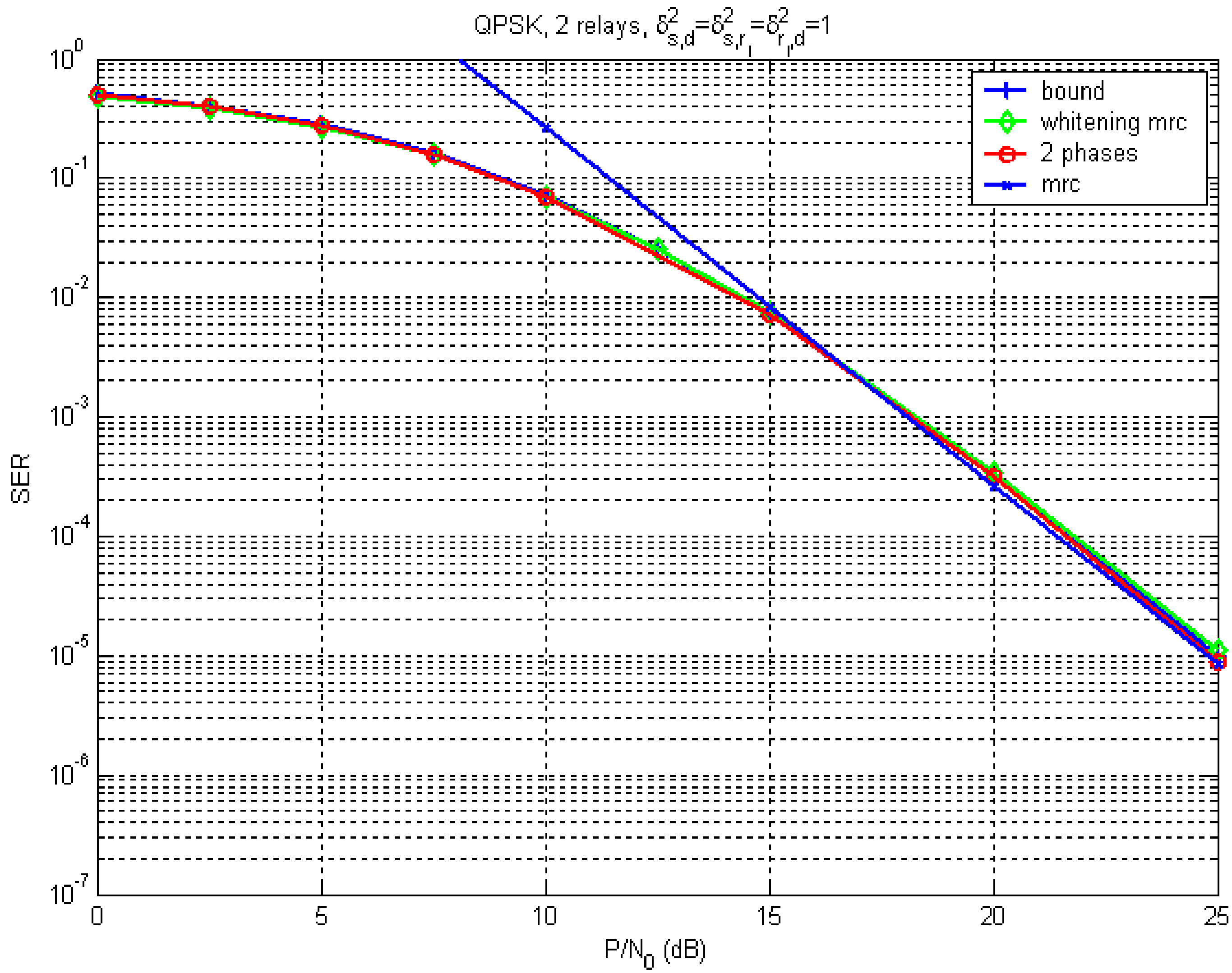


The bound of the SER in this case is given by

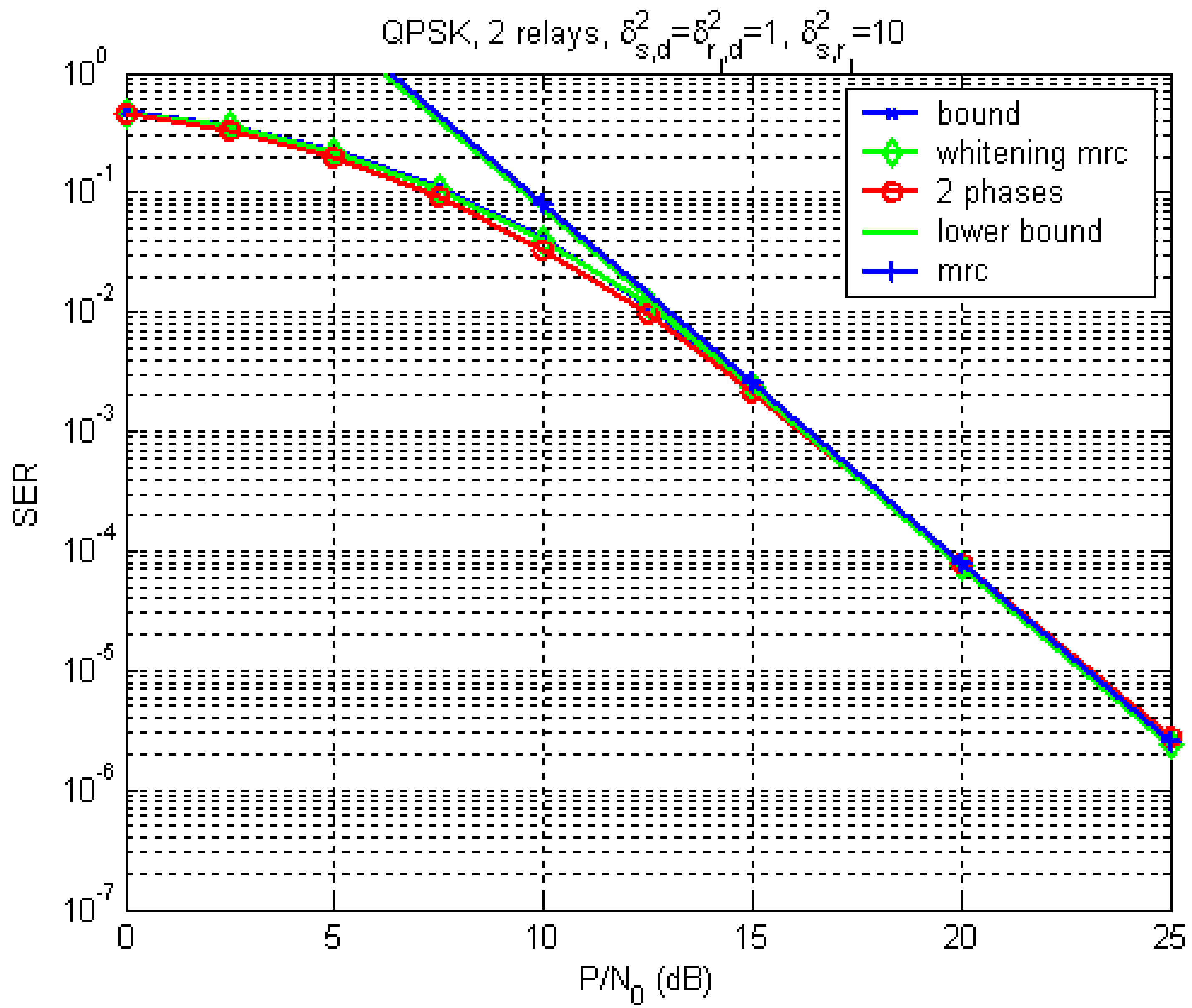
$$P_{PSK} \approx \frac{N_0^{N+1} C(N)}{b_{PSK}^{N+1}} \times \frac{1}{P_1 \delta_{s,d}^2} \times \frac{P_1 \delta_{s,r_1}^2 + P_{r_1} \delta_{r_1,d}^2}{P_1 \delta_{s,r_1}^2 P_{r_1} \delta_{r_1,d}^2} \prod_{i=2}^{N} \frac{1}{P_{r_i} \delta_{r_i,d}^2}$$

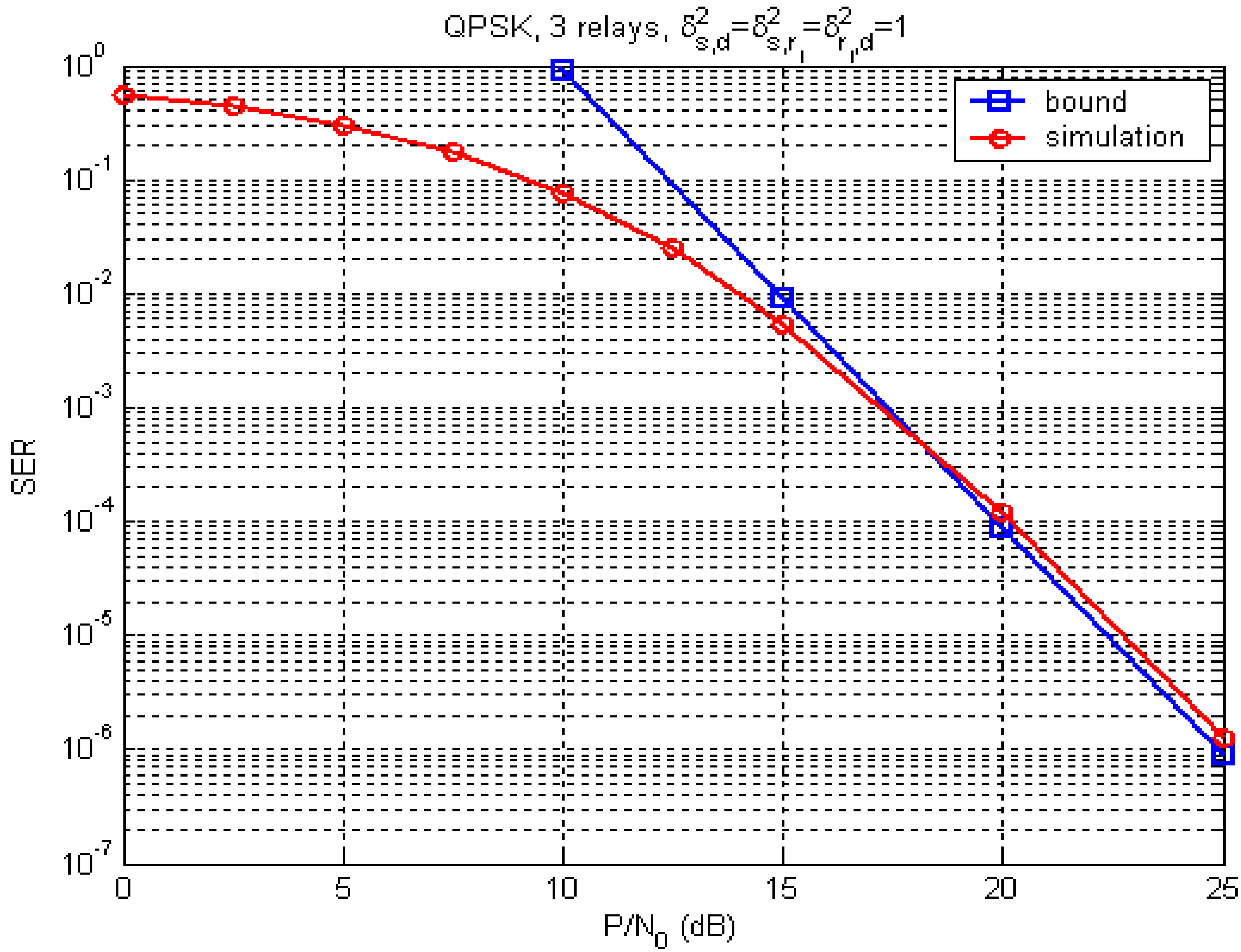
This performance can not be achieved by any system (bench mark)

Simulation Results



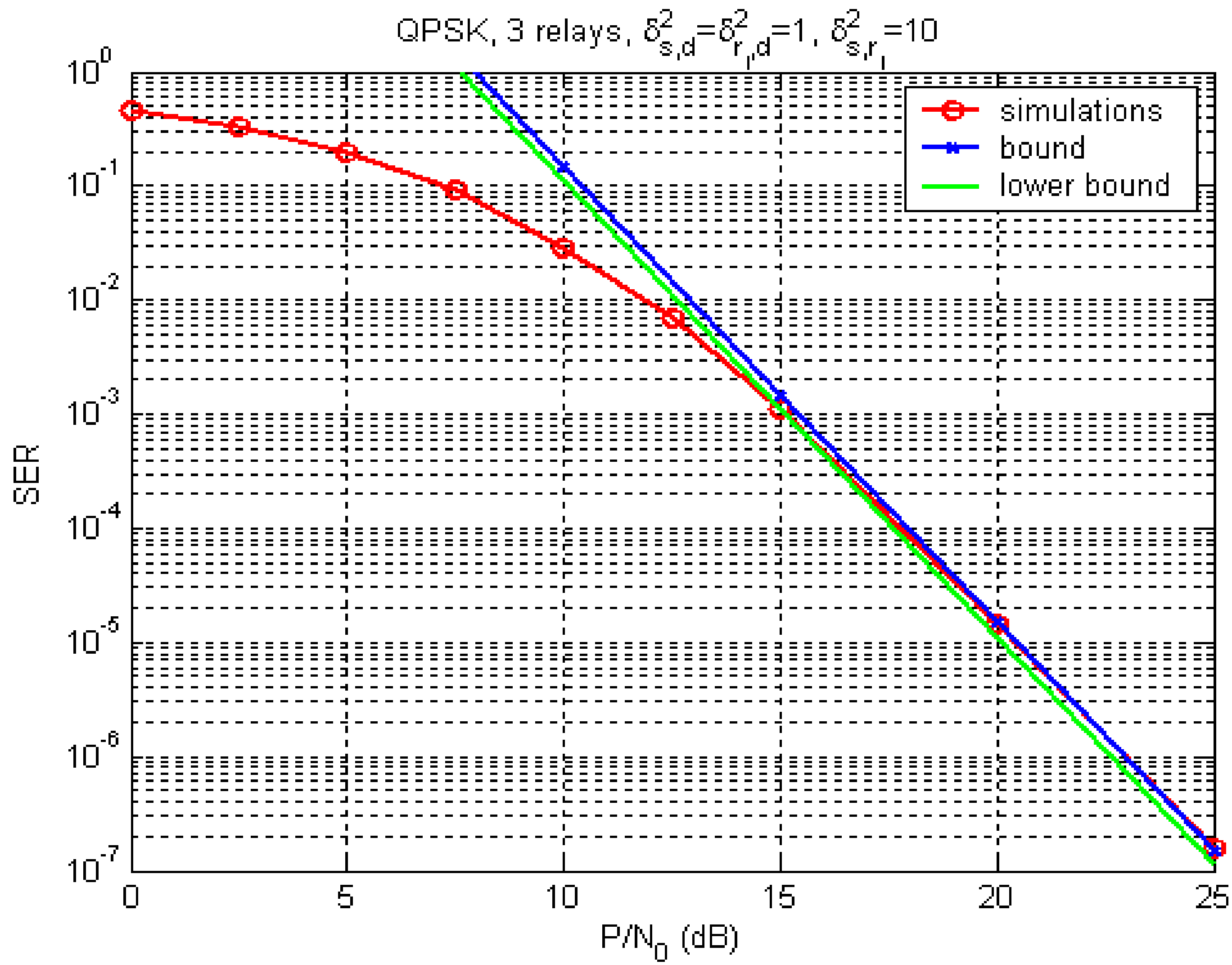
2 Relays close to the source





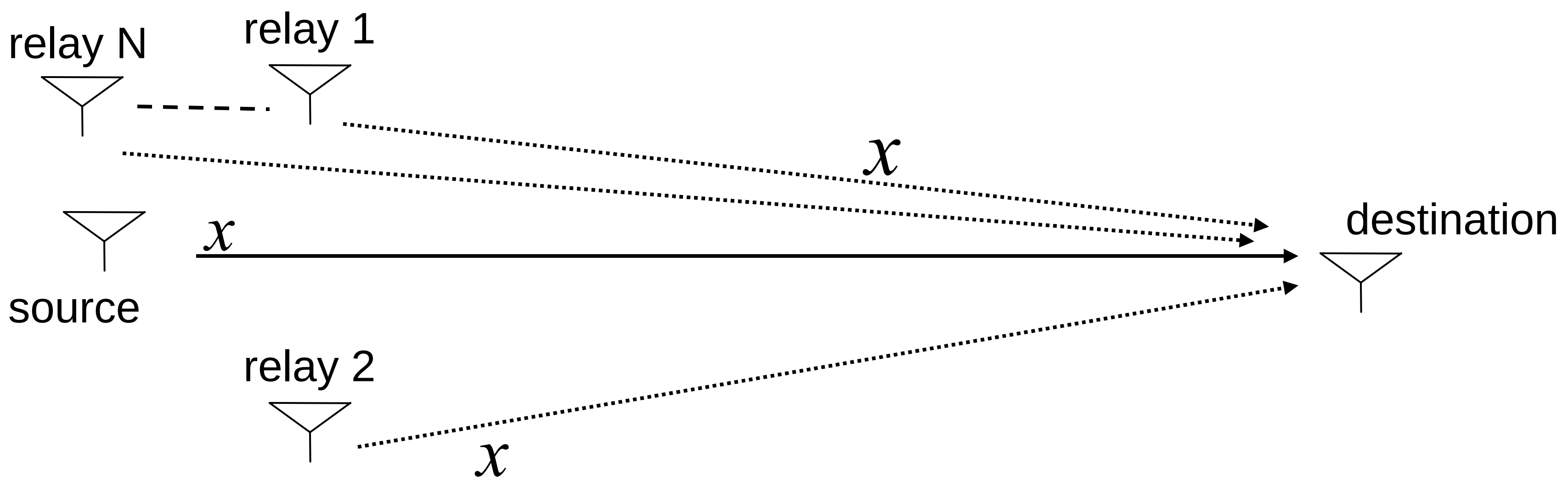
QPSK, 3 relays, $\delta^2_{s,d}=\delta^2_{s,r_i}=\delta^2_{r_i,d}=1$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
SER
0
5
10
15
20
25
P/N_0 (dB)
bound
simulation

3 Relays close to the source



Explanation

If the relays are close to source



Or from the SER bound

$$\frac{P_1\delta^2_{s,r_i} + P_{r_i}\delta^2_{r_i,d}}{P_1\delta^2_{s,r_i} P_{r_i}\delta^2_{r_i,d}} \xrightarrow{\delta^2_{s,r_i}\uparrow} \frac{1}{P_{r_i}\delta^2_{r_i,d}}$$

So the 2 phase system approach the bench mark performance as the relays becomes closer to the source

Summary

- Multinode cooperation can achieve full diversity gain as in a MIMO system
- Both decode and forward relaying and amplify and forward relaying are considered
- Approximate SER expressions are derived
- Optimal power allocation at the source and relay nodes is characterized