Problems for Chapter 12 of 'Ultra Low Power Bioelectronics'

Problem 12.1

Derive Equations (12.1) and (12.2). Calculate the transconductance G_m of the circuit in Figure 12.1 for $\kappa = 0.6$, $\phi_t = 26$ mV, and $I_B = 600$ nA assuming that all the transistors are in subthreshold.

Problem 12.2

Calculate the linear range V_L of the transconductor in Figure 12.2 with the bump transistors removed, and with $\kappa = \kappa_p = 0.75$, $\kappa_n = 0.6$ and $\phi_t = 26$ mV. What is the corresponding V_L if the two bump transistors are added and they have a W/L ratio equal to twice the W/L of the diode-connected input transistors?

Problem 12.3

Derive Equation (12.16) for the OTA in Figure 12.2. Explain why, exploiting symmetry via a differential analysis that sets the drain node of the bias transistor (with gate voltage v_B) at ground, yields the same answers as an analysis that does not exploit such symmetry.

Problem 12.4

In Figure P12.4, assume that all p-type transistors have $\kappa_p = 0.75$, that all n-type transistors have $\kappa_n = 0.7$, that all the transistors are operating in subthreshold, that all current mirrors in the circuit have a scaling ratio of 1:1, and that all p-type transistors with missing well connections have their well connected to the supply voltage.

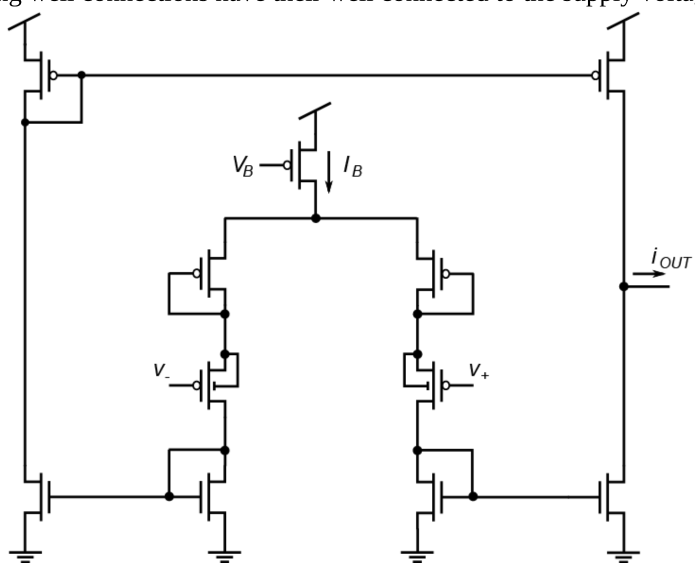

Figure P12.4: An OTA with source-degenerated input transistors.

a) Estimate the minimum power supply voltage needed to keep all transistors in the circuit saturated. Express your answer in terms of V_{DM}, the subthreshold bias voltage dropped across a diode-connected transistor with a drain current of $I_B/2$.

b) Draw a block diagram similar to that in Figure 12.5 and calculate the effective small-signal transconductance, G_m, of the amplifier in terms of κ_n, κ_p, I_B , and ϕ_t.

c) Evaluate G_m and the linear range, V_L, of the OTA of Figure P12.4 for $I_B = 1$ μA.

Problem 12.5

Suppose you want to build a low-offset-voltage G_m-C lowpass filter as in Figure P12.5 (b) using the OTA of Figure P12.4. Unfortunately, due to the low-offset-voltage requirement, some transistors in the circuit have large sizes, thus making their parasitic capacitances significant. Suppose the OTA that you design ends up like the one shown in Figure P12.5 (a) where $C_s = 1$ pF and $C_g = 2$ pF. Assume that $I_B = 1$ μA, $\kappa_p = 0.75$, and $\kappa_n = 0.7$.

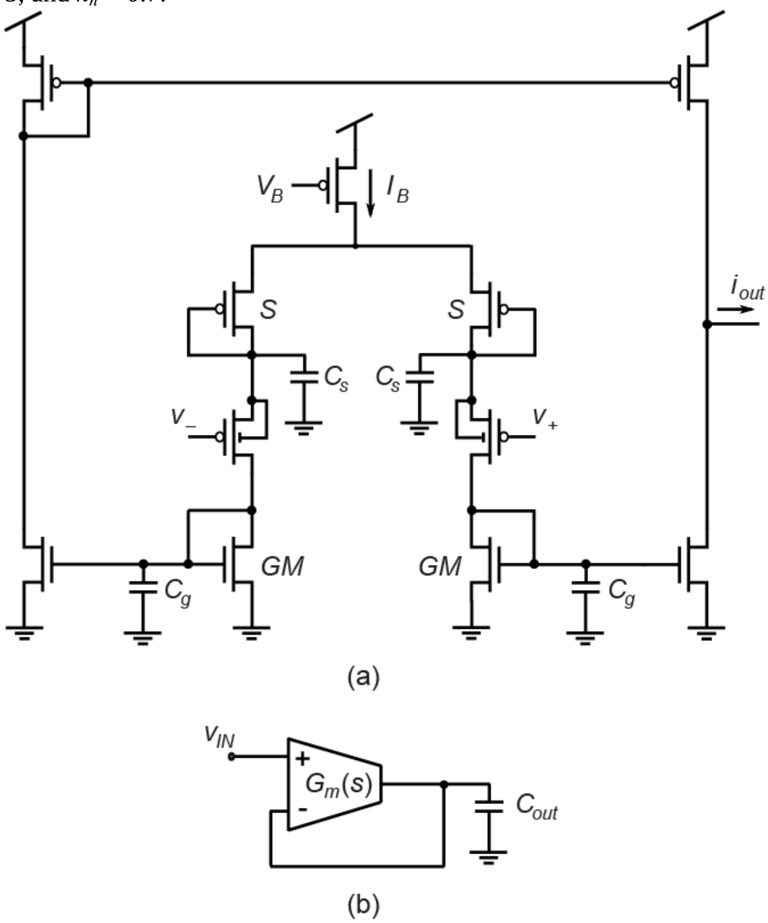

Figure P12.5: OTA circuits for Problem 12.5.

a) Calculate the effective transconductance $G_m(s)$ of this OTA. How many poles does the OTA have and what are their locations?

b) The OTA is now used for designing the low-pass G_m-C filter shown in Figure P12.5 (b). Write down an expression for the loop transmission $L(s)$.

c) Suppose that the only degree of freedom in the design of the filter is the value of C_{out}. Find the value of C_{out} such that $L(s)$ has a phase margin of 45º. Plot the transfer function of the filter in MATLAB. What is the corner frequency of the filter?

<u>Problem 12.6</u>

Consider the OTA of Figure 12.19 (a). The current flowing through the PMOS transistor whose gate is connected to V_B is I_B. Assume that all the NMOS transistors in the circuit are identical except for the two bump transistors, which have $\frac{W}{L} = 2\times\frac{W}{L}$ of other NMOS transistors. Also assume that all the transistors in the circuit are operating in subthreshold and that they all have $\kappa = 0.75$.

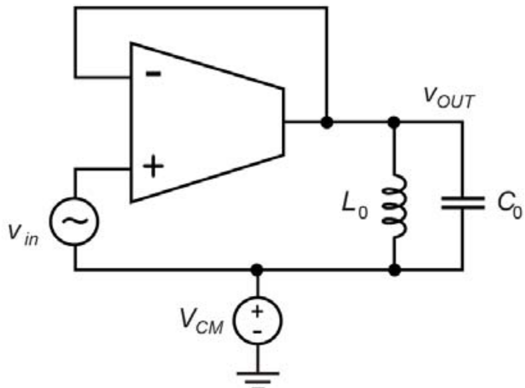

Figure P12.6: A filter circuit.

a) Find the small-signal transconductance G_m of the OTA.

b) Using the expression you find in part a), derive the linear range, V_L, of the OTA.

c) Assume that a dc voltage source of value $V_{DD}/2$ is connected at the v_{OUT} node. Also assume that the flicker noise is negligible. Calculate the noise current power spectral density flowing through this voltage source when $v_+ = v_-$ and all the transistors in the OTA except the bottom bump transistors are in saturation.

d) Calculate the input-referred voltage noise spectral density of the OTA in Figure 12.19 (a).

e) The OTA of Figure 12.19 (a) is used to construct the circuit shown in Figure P12.6 above. For this circuit, derive the transfer function $V_{out}(s)/V_{in}(s)$ in terms of G_m, L_0, and C_0.

f) If $I_B = 1\ \mu$A, $L_0 = 1\ \mu$H, and $C_0 = 1$ pF, calculate the total voltage noise present at the output node v_{OUT}. You may find one or more of the following equations useful:

$$\int_0^\infty \frac{dx}{(1-x^2)^2+(ax)^2} = \frac{\pi}{2a}$$
$$\int_0^\infty \frac{x^2 dx}{(1-x^2)^2+(ax)^2} = \frac{\pi}{2a}$$

Problem 12.7

In this problem, we will analyze the low-voltage OTA topology of Figure 12.19 (b).

a) Derive an expression for the transconductance of the OTA in terms of small-signal circuit parameters of the transistors in the figure.
b) Derive an expression for the output impedance of the OTA.
c) If the OTA is used as an open-loop amplifier, derive an expression for the dc voltage gain $(v_{out}^{+} - v_{out}^{-})/(v_{+} - v_{-})$.
d) If the drain-to-source thermal noise current generator of a transistor in strong inversion can be approximated as having a power spectral density of $\frac{8}{3}kTg_m$, derive an expression for the input-referred voltage noise power spectral density of the OTA.

Problem 12.8

One of your co-workers tells you that she has found a way to build a subthreshold OTA with $\kappa = 1$ by using the circuit of Figure P12.8 (a) as an input differential pair in the OTA of Figure P12.8 (b). We will investigate her findings in this problem.

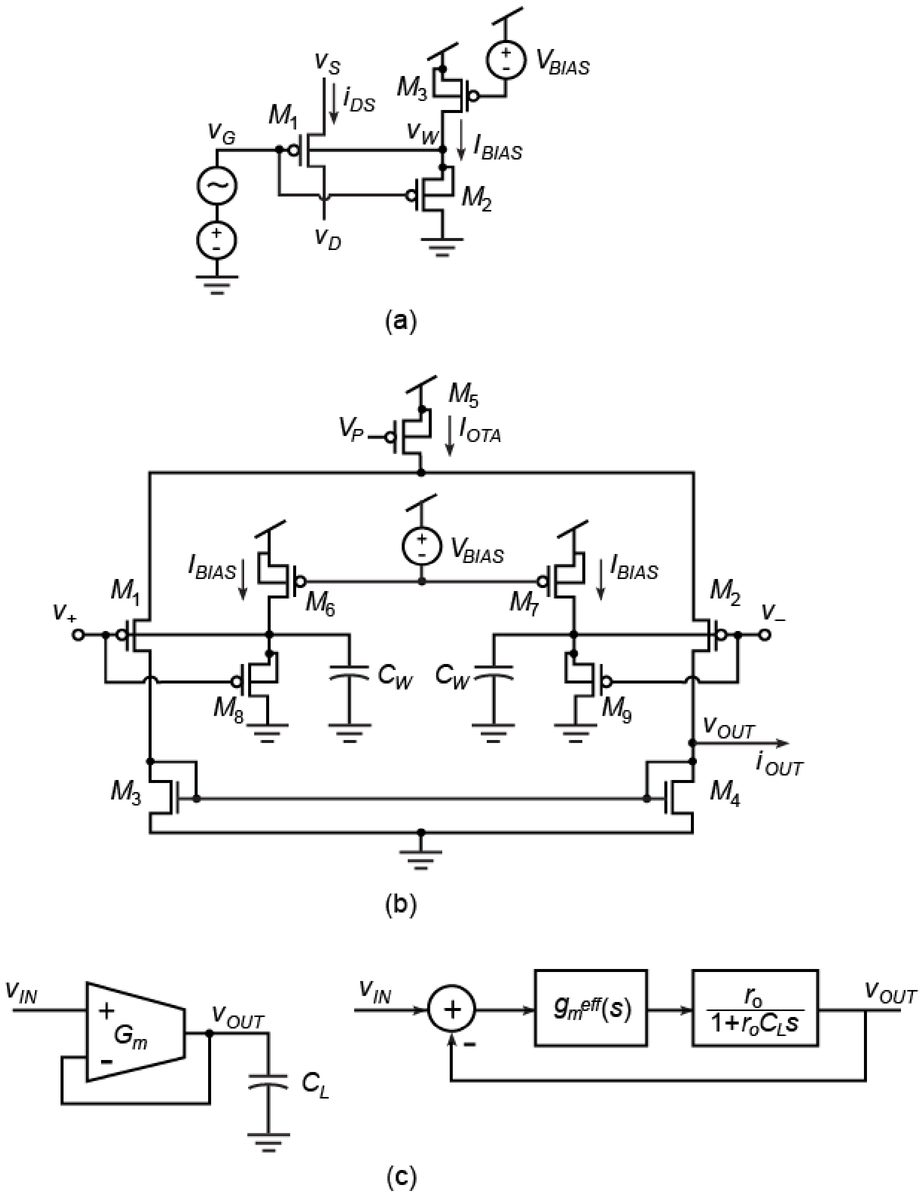

Figure P12.8: An OTA with $\kappa = 1$.

a) In Figure P12.8 (a), assume that all transistors are identical and that $I_{BIAS} > I_{DS}$. Show that the small-signal transconductance of M_1 is $\dfrac{\partial i_{DS}}{\partial v_G} = \dfrac{I_{DS}}{\phi_t}$ and is independent of κ.

b) Calculate the noise power spectral density of the circuit in Figure P12.8 (a) referred to the input v_G.

c) Figure P12.8 (c) shows a G_m-C first order low-pass filter with the transconductance amplifier built from $\kappa = 1$ transistors as shown in Figure P12.8 (b). The parasitic capacitance C_W represents the nwell-to-substrate capacitance and cannot be ignored. Assume that all the transistors have an Early voltage = V_A. Show that the lowpass filter can be represented by the block diagram show at the right of Figure P12.8(c). Derive $g_m^{eff}(s)$ and r_o in terms of small-signal circuit parameters.

d) Compute the loop transmission $L(s)$ for the block diagram of Figure P12.8 (c). Also identify the locations for poles and zeros for I_{OTA} = 10 nA, I_{BIAS} = 50 nA, C_L = 1 pF, C_W = 100 fF, and V_A = 100 V. Draw Bode gain and phase plots of the loop transmission.

e) What is the total noise of this G_m-C filter using circuit parameters that you computed in part d).

Problem 12.9

Figure P12.9 shows a circuit constructed from the wide-linear range transconductance amplifier of Figure P12.4. Assume that $\kappa = 0.7$ for all transistors and that I_{B1} and I_{B2} are the bias currents flowing through the transistors whose gates are connected to V_B for the G_{m1} and G_{m2} OTA's respectively.

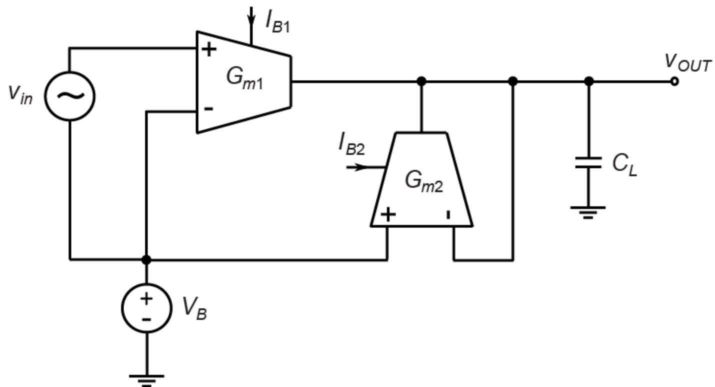

Figure P12.9: An OTA circuit for problem 12.9.

a) Derive the transfer function $V_{out}(s)/V_{in}(s)$. What function does this circuit topology implement? What is the corner frequency of the circuit in terms of small-signal circuit parameters?

b) If I_{B1} = 500 nA and I_{B2} = 100 nA, what is the corner frequency? What is the maximum input amplitude at the corner frequency at which the circuit remains linear? What is the corresponding maximum input amplitude at one-tenth of the corner frequency?

c) Calculate the total input-referred noise at the input for the bias setting in part b).

d) Based on your answers in parts b) and c), calculate the maximum signal-to-noise ratio of this circuit at its center frequency.

Problem 12.10

In this problem, we will study the settling behavior of a first-order lowpass filter when its input voltage undergoes a large step change.

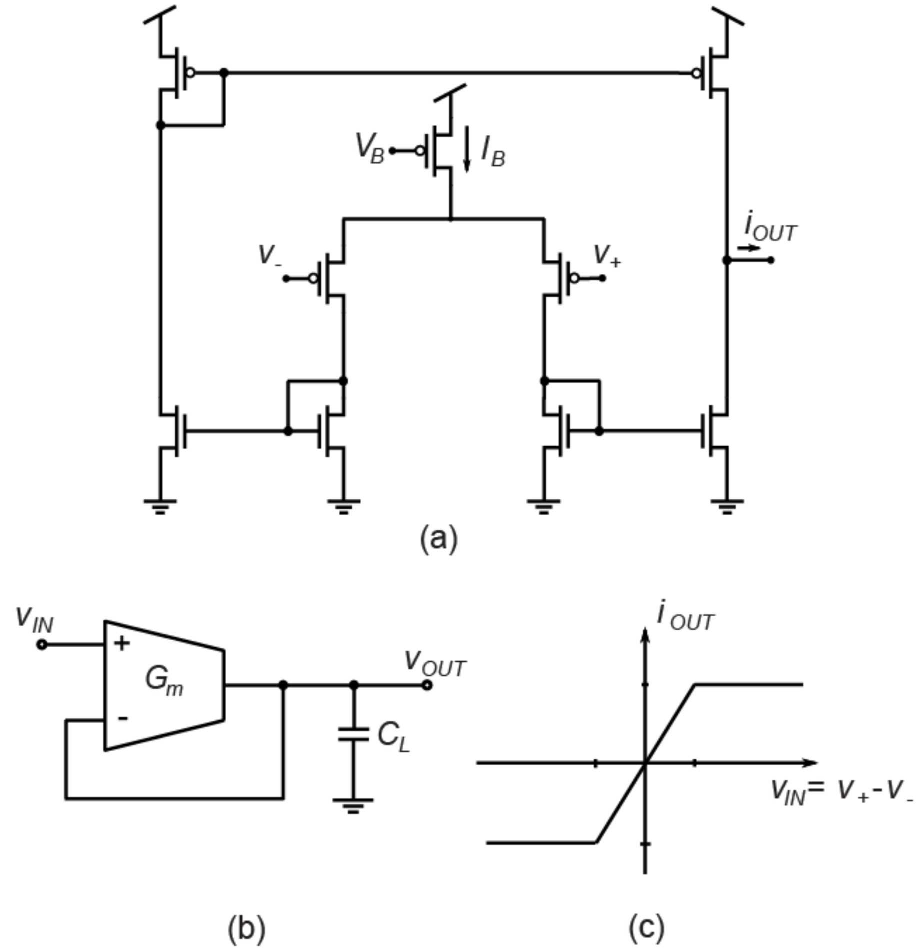

Figure P12.10: A first-order G_m-C lowpass filter.

Figure P12.10 (a) shows a standard transconductance amplifier. Assume that all the PMOS transistors are identical and that all the NMOS transistors are identical. Also assume that all transistors are operating in the subthreshold regime with the same exponential coefficient κ. Figure P12.10 (c) shows a piecewise-linear approximation of the DC transfer characteristic of the OTA when its differential input $v_{IN} = v_+ - v_-$ is swept from $-V_{DD}$ to V_{DD} and the output is fixed at the middle of the rail with an ideal voltage source.

a) Label the axes in Figure P12.10 (c) in terms of circuit's and physical parameters (κ, I_B, and ϕ_t).
b) Referring to Figure P12.10 (b), suppose during the time $t < 0$, v_{IN} has been held at 300 mV (with respect to a reference) for a long time such that v_{OUT} has settled to its final value. At $t = 0$, v_{IN} is abruptly changed to 325 mV.

Assuming that $\kappa = 0.7$, $I_B = 500$ nA, $\phi_t = 26$ mV, and $C_L = 10$ pF, sketch the response $v_{OUT}(t)$ of the filter. Label all important aspects of the curve (initial slope, settling time, initial value, final value).

c) For the scenario of part b), find the time at which $v_{OUT}(t)$ reaches 99% of its asymptotic final value.

d) Repeat parts b) and c) when v_{IN} is abruptly changed from 300 mV to 2.3 V at $t = 0$.