Problems for Chapter 17 of 'Ultra Low Power Bioelectronics'

Problem 17.1

RG-58/U coaxial cable is used in a wide variety of applications. The center conductor of such cable typically consists of solid 20AWG copper wire, with diameter $d = 0.813$ mm. It is surrounded, firstly by a 1.07 mm thick dielectric layer consisting of solid polyethylene, and secondly by a braided copper ground shield. The relative dielectric constant of solid polyethylene is $\epsilon_r \approx 2.25$.

a) Calculate the capacitance and inductance per unit length of the cable. Express your answer in reasonable units, such as pF/m and nH/m.
b) Calculate the characteristic impedance of the cable.
c) Calculate the approximate series resistance per unit length of the cable. The resistivity of copper is approximately $\rho = 1.7 \times 10^{-8}$ Ω-m at room temperature. You may assume that the series resistance is dominated by losses in the center conductor. Do not neglect the skin effect. The skin depth in a conductor at a frequency f is given by

$$\delta = \sqrt{\frac{\rho}{\pi f \mu_0}}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space.

d) Use your results from part c) to estimate the attenuation (in dB/m) suffered by a signal of frequency f as it propagates along the cable. Plot this quantity as a function of f. What properties of the cable would you modify to reduce attenuation?

Problem 17.2

You want to make an antenna out of 20AWG copper wire, which has a diameter of $d = 0.813$ mm, to surreptitiously pick up cell phone signals at 870 MHz present in your neighborhood. Unfortunately the antenna has to fit within an area of 1 cm x 1 cm in order to be inconspicuous.

a) You decide to start with a length of wire that is 1 cm long and make a dipole out of it. What is the radiation impedance and efficiency of this antenna? Explain any assumptions that you make. [Hint: do not neglect the skin effect.]
b) Calculate the maximum power that the antenna can pick up from your neighbor's cell phone. Assume that the phone transmits nearly isotropically with a maximum output of 100 mW and gets no closer than 10 m to your antenna. What voltage amplitude does this received signal generate across the antenna terminals?
c) Being unsatisfied with the performance of your dipole, you decide to create a circular loop of diameter 1 cm out of similar wire. What is the radiation impedance and efficiency of this new antenna? Explain any assumptions that you make.
d) Which antenna would you prefer, and why?
e) How would you increase the radiation efficiency of the dipole antenna while still fitting within an area of 1 cm x 1 cm?

Problem 17.3

Chip packaging adds parasitic capacitance and inductance, which can be significant at high frequencies. The geometry of one pin in a typical wire-bond package is shown in Figure P17.3.

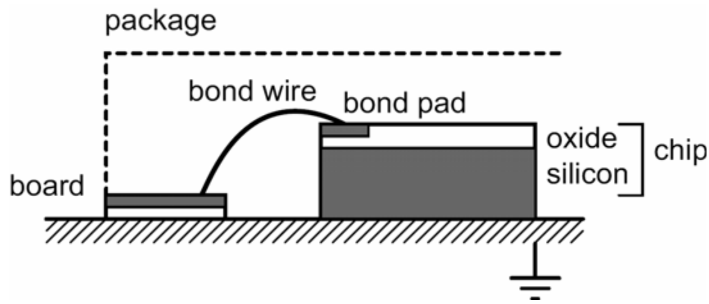

Figure P17.3: The geometry of a typical wire-bond package. Black regions correspond to metallization, while unshaded regions are (approximately) insulating.

a) Typical bond wires are made of gold and have a diameter of 1 mil (25.4 μm). At typical RF frequencies (hundreds of MHz to several GHz) such bond wires can be assumed to have a resistance, inductance and capacitance per mm of approximately 0.15 Ω, 1 nH and 70 fF, respectively. Using these values, draw a lumped equivalent circuit model of a bondwire of length l mm. Justify any assumptions that you make.

b) What circuit components can model the metal regions (bond pads) at either end of the bond wire? Add these components to the model you constructed in part a) to complete your electrical model of a single input/output pin.

c) A typical on-chip bond pad is 75 μm x 75 μm in size and is located on top of a layer of silicon dioxide that is 5 μm thick. Use this information to add numerical values to your pin model. Explain any assumptions that you make.

d) How would you estimate numerical values for the remaining circuit components in your pin model?

e) Sketch the transfer function of your pin model without using a circuit simulator. What range of frequencies will pass through the pin without significant attenuation for l = 1, 3, and 7 mm?

f) A friend tells you that the effective bandwidth of the pin can be increased by simultaneously sending a signal through several adjacent pins (which are shorted together). Would you believe her? Explain your reasoning and any assumptions that you make.

g) Assume that you are using a single pin and that the on-chip bond pad is connected to a circuit with an input impedance of 2 kΩ. Your goal is to design a lossless impedance-matching network to connect this circuit to a 50 Ω microstrip transmission line on the printed circuit board. What is the maximum fractional bandwidth (around a center frequency of 2.4 GHz) that can be obtained with infinite-order, second-order, and first-order matching networks when no more than 1% of the incident power can be reflected? Explain your reasoning and any assumptions that you make.

h) Repeat part g), but now assume that up to 10% of the incident power can be reflected.

Problem 17.4

This problem requires the use of an antenna simulation program. Many such programs are available. A significant fraction of them are based on NEC (Numerical Electromagnetics Code), a public-domain program for solving Maxwell's equations in sinusoidal steady-state conditions. It uses the boundary element method, also known as the method of moments. A free NEC-based program that we have found useful is 4nec2. It can be downloaded from http://home.ict.nl/~arivoors/.

a) Implement a square loop antenna of total length $l = 30$ cm in free space. Assume that the antenna uses copper wire of diameter $d = l/50$.
b) What is the lowest frequency f_0 at which you expect the antenna to be resonant, i.e., have an input impedance that is purely real?
c) Use the simulator to find the input impedance of the antenna as a function of frequency, and plot the real and imaginary parts for frequencies between $0.1f_0$ and $5f_0$. Explain your results.
d) Estimate the first resonant frequency of the antenna from the results obtained in part c), and compare it with the theoretical estimate from part b). What is the fractional difference between these two quantities?
e) Plot the current distribution on the surface of the antenna at two frequencies: $0.1f_0$ and f_0. Explain your results.
f) What is the maximum gain of the antenna at f_0, and in which direction does it occur? Express your answer in dBi.
g) Repeat parts a) through e), but now assume that $d = l/10$. What are the main changes that you observe in the behavior of the antenna?

Problem 17.5

a) Show that the block labeled 'antennas' in Figure 17.02 is an equivalent circuit representation of any linear, passive, two-port network that has a Z-parameter matrix given by

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

b) Derive Equations (17.13) and (17.14) by directly solving for the impedances seen across the terminals of the voltage source and load, respectively, in Figure 17.2.

Problem 17.6

Consider the coaxial transmission line shown in Figure P17.6 in sinusoidal steady state. The outer conductor (which has radius r_2) is grounded, while the amplitude of the voltage on the inner conductor (which has radius r_1) is V_0. The current along the line is I_0. Both conductors and the medium between them are assumed to be perfectly lossless. The figure also shows the cylindrical co-ordinate system that we will use for this problem, with r, θ and z being the radial, azimuthal, and axial co-ordinates, respectively.

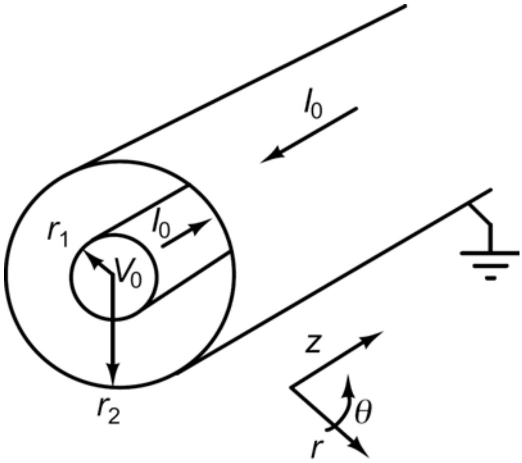

Figure P17.6: A coaxial transmission line.

a) Assume that the electric and magnetic fields within the transmission line have no components in the z-direction for all frequencies of interest (this statement is true from DC up to a frequency f_{max}, which you will estimate later). This is known as the transverse electro-magnetic (TEM) mode of wave propagation. Sketch both electric and magnetic fields within the line in this case [Hint: remember that, in addition to having no axial component, the electric and magnetic fields must also be perpendicular to each other.].

b) Can the electric and magnetic fields vary with azimuthal angle θ? Explain.

c) Imagine a cylinder of radius r, where $r_1 < r < r_2$, and infinitesimal axial length dz. Apply Gauss' law to this cylinder to find the electric field as a function of r, i.e., $\vec{E}(r)$. Remember that Gauss' law in integral form can be written as

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon}$$

where Q is the total charge enclosed by the closed surface S, and ϵ is the permittivity of the medium between the conductors.

d) Integrate the electric field you obtained in part c) from r_1 to r_2 to get the voltage difference between the two conductors.

e) Use the definition of capacitance, i.e., $C = Q/V$, to prove that the capacitance per unit length of the line is given by the expression in Equation (17.12).

f) Imagine a circle of radius r, where $r_1 < r < r_2$. Apply Ampere's law to this circle to find the magnetic field as a function of r, i.e., $\vec{B}(r)$. Remember that Ampere's law in integral form can be written as

$$\oint_C \vec{B} \cdot d\vec{l} = \mu I_0$$

where μ is the permeability of the medium, and the current I_0 flows normal to the surface enclosed by the closed curve C.

g) Imagine a plane extending from r_1 to r_2 in the radial direction and with an infinitesimal length dz in the axial direction. Integrate $\vec{B}(r)$ over this plane to find the magnetic flux Φ flowing through it.
h) Explain why the plane of integration in part g) can be limited to the region $r_1 < r < r_2$.
i) Use the definition of inductance, i.e., $L = \Phi / I$, to prove that the inductance per unit length of the line is given by the expression in Equation (17.12).
j) Calculate the characteristic impedance of the transmission line and its wave propagation velocity.
k) Show that the maximum frequency f_{max} at which only the TEM mode propagates is approximately

$$f_{max} \approx \frac{v}{\pi(r_1 + r_2)}$$

where v is the wave propagation velocity [Hint: in non-TEM modes the electric and magnetic fields can vary with azimuthal angle.].
l) Estimate f_{max} for the RG-58/U cable discussed in Problem 17.1.

<u>Problem 17.7</u>
This problem requires the use of a circuit-simulation program such as SPICE.
Consider the H-bridge rectifier circuit shown in Figure 17.12 (a).

a) Intuitively explain the operation of this circuit [Hint: it is much easier to understand if you assume that v_P and $\overline{v_P}$ are driven by square waves.].
b) Implement the circuit in a circuit-simulation program. Connect V_L and V_H to ground and the output load, respectively. The load consists of a resistor R_L in parallel with a capacitor C_L. Drive v_P and $\overline{v_P}$ with anti-phase square waves of amplitude V_{in} and period T_{in} = 10 μs.
c) Perform transient simulations to estimate the steady-state output voltage of the rectifier as a function of R_L and V_{in}. Vary R_L between 1 kΩ and 1 MΩ, and V_{in} between 0.2 V and 2 V. Use at least 10 values for each parameter [consider automating the simulation and data-extraction process as much as possible to save time.].
d) From the data obtained in part c), estimate the voltage drop V_{drp} and output impedance R_{out} of the rectifier as a function of R_L and V_{in}. Plot these quantities, and qualitatively explain their behavior.
e) What changes would you make to the rectifier to reduce V_{drp} for a given value of R_L? Would these changes have undesirable side-effects? Explain. <u>Note</u>: do not change the circuit topology itself. Only adjust parameter values.
f) Repeat parts b), c), and d), but with the square wave inputs replaced by sinusoids that have the same amplitude and period.
g) Compare the plots obtained in parts d) and f), and qualitatively explain any differences between them. What are the relative advantages of square versus sinusoidal inputs?

<u>Problem 17.8</u>

a) Under "best-case" conditions, how much power does WEEI (Boston sports radio) have to broadcast in order to be picked up by a receiver located on the other side of the earth, i.e., at the point on the earth's surface diametrically opposite Boston (which, by the way, happens to be in the Indian Ocean, southwest of Australia)? Explain any assumptions that you make.

b) Repeat part a), but now assume that the receiver is a communication satellite in geosynchronous orbit. You may assume that the satellite is always overhead at the point on the equator that shares the longitude of Boston (71.1°W).

The following facts may help you solve this problem:

1. WEEI broadcasts at a frequency of 850 kHz.
2. AM channels in North America have a bandwidth of 10 kHz.
3. The radius of the earth is approximately 6378.1 km.
4. The dielectric constant of air is approximately 1.
5. The input-referred noise power spectral density of both receivers is equal to that of a 50 Ω resistor at 300 K.
6. Geosynchronous orbits are located approximately 35,800 km above the surface of the earth.
7. The latitude of Boston is 42.4° N.

Problem 17.9

Consider the multi-stage rectifier circuit, known as a Dickson charge pump, shown in Figure P17.9. The input terminals v_P and $\overline{v_P}$ are connected to anti-phase square-wave or sinusoidal signals. Assume that each input has a peak-to-peak swing of V_P.

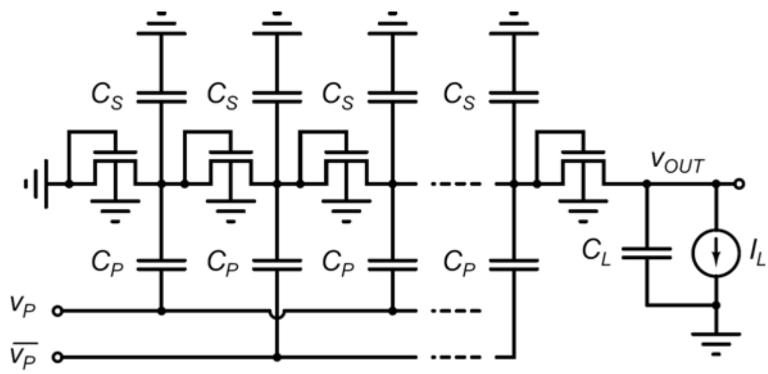

Figure P17.9: A rectifier circuit.

a) Qualitatively explain the behavior of this circuit.

b) Assume v_P and $\overline{v_P}$ are square waves. Do they need to be non-overlapping? Explain.

c) Show that the DC output voltage is approximately given by

$$V_{OUT} \approx NV_P\left(\frac{C_P}{C_P + C_S}\right) - \sum_{n=1}^{N+1} V_{drop}(n)$$

Where $V_{drop}(n)$ is the DC voltage drop across the n-th NMOS transistor, and there are a total of N input capacitors (each denoted by C_P).

d) What factors determine the value of $V_{drop}(n)$? Derive an approximate expression for $V_{drop}(n)$. Justify any assumptions that you make.
e) Derive an approximate expression for R_{out}, the output resistance of the rectifier. Justify any assumptions that you make.
f) How do $V_{drop}(n)$ and R_{out} vary as N increases?
g) Sketch the expected behavior of V_{OUT} as a function of N. Qualitatively explain the important characteristics of your sketch.

Problem 17.10

Very low frequency (VLF) radio signals between 3 kHz and 30 kHz are used to communicate with underwater vehicles. This problem examines why such low frequencies are necessary.

a) Calculate the skin depth of RF propagation in sea water as a function of frequency. Plot your result over the range between 1 kHz and 1 MHz. The average conductivity of sea water is 4.8 S/m.
b) Assume that the maximum acceptable attenuation through the ocean is S (measured in dB). Calculate the maximum depth at which a submarine can communicate with the surface as a function of frequency.
c) Plot your result from part b) in the following cases: $S = 20$, 40, 60 and 80.
d) An additional source of attenuation is impedance mismatch between air and sea water. What fraction of RF power broadcast from above the surface do you expect to propagate into the ocean? Assume that the wave is incident normal to the ocean surface, and express your result in dB. The relative permittivity of sea water is approximately 78 at 300K [Hint: calculate the characteristic impedances of the two media, and then apply Equation (17.5).].
e) What is the length of a half-wavelength dipole antenna tuned to 10 kHz (a typical VLF frequency) when it is located in air?
f) Repeat part e), but now assume that the antenna is located below the ocean surface. Comment on the practical problems that might arise in VLF communication systems.