Answers to Exercises
Chapter 6

Exercise 6.1

(a) The terminal velocity, u_b, of spherical solid particles within an infinite viscous Newtonian fluid can be estimated using Stokes' law,

$$u_b = \frac{2g(\rho - \rho_b)r_b^2}{9\eta}$$

where g is the acceleration due to gravity, ρ and η are the fluid density and viscosity respectively, and ρ_b and r_b are the bubble density and radius. Assuming that the bubbles act similarly to solid spheres, the bubble density is negligible with respect to that of the magma, and that the magma can be represented as a Newtonian fluid with a viscosity of 100 Pa s and density of 2600 kg m^{-3}, for diameters of 10^{-4}, 10^{-3}, 0.01 and 0.1 m, Stokes' law gives rise velocities of 1.4×10^{-7}, 10^{-5}, 10^{-3} and 10^{-1} m s^{-1} respectively. Within this we are also assuming that the conduit is much wider than the diameter of the bubbles, so that there is negligible wall interaction and the fluid can be considered as infinite in extent. Note that, since u_b is inversely proportional to the fluid viscosity, increasing viscosity by a factor of ten (to 1 kPa s) would reduce estimated rise speeds by an order of magnitude.

(b) Calculating Reynolds numbers for the bubble rise velocities shows that all bubbles in (a) rise at Re < 0.3 with the exception of the 10 cm bubble in a 100 Pa s melt (Re = 1.0). So, the laminar flow assumptions used above are supported, with the exception of this one case, for which Stokes' law should not be used to estimate rise velocity.

(c) Increasing Reynolds numbers reflect increasing inertial control over viscous effects, and, consequently, increasing turbulence. For Re > 0.3, the influence of turbulence in reducing terminal velocities can no longer be neglected, thus velocities are smaller that would be anticipated using the laminar-flow Stokes' model. Note that also, bubbles outside of the Stokes regime are not necessarily spherical but can become oval or cap-shaped, possibly with trailing skirts.

Exercise 6.2

(a) For a straightforward estimate, assume that the vapor acts as an ideal gas under isothermal conditions, in which case, the product of pressure, P, and volume, V is constant, so, $PV = P_0V_0$, where P_0 and V_0 are the initial starting pressure and volume respectively. During ascent, P can be given by the sum of the atmospheric pressure and the static pressure of the magma overlying the bubble. At the surface, pressure is atmospheric (~10^5 Pa) and, if calculations are carried out using a magma density of 2600 kg m^{-3} then the gas volume at the surface is 256 m^3.

(b) Converting this volume to a cylindrical form with radius 1.2 m requires a cylinder length of 56.6 m.

(c) These calculations have assumed that pressure in the bubble can adjust to magma-static during ascent. However, a real gas slug retains pressure due to the dynamic effects resulting from having to accelerate the fluid above it as the gas expands. The evidence for this is that at the surface, the slug bursts violently, suddenly releasing the retained overpressure. If the gas in the slug was at a greater-than-atmospheric pressure, then the slug volume (and hence length) would be less than if one assumed atmospheric pressure. In order to account for this, a dynamic model that includes liquid accelerations is required. However, we have also been assuming that the gas acts as an ideal gas, but at 1000 m, or pressures of ~25 MPa, H_2O will be a supercritical fluid.

(d) Recalculating for the extraterrestrial conditions demonstrates that, at depths below ~10 m or so, the results appear to be very similar. This is because at depth the fractional pressure change is dominated by the change in height. As the surface is approached, the surface pressure becomes increasingly important and, under low atmospheric pressure conditions, very large expansions occur. This is demonstrated by the results which suggest surface gas volumes of 5.2×10^6 m^3 and slug lengths of 1150 km; clearly invalid results. Thus, low atmospheric pressures strongly favor more violent eruptions and would be expected to produce strombolian activity from magmas with only very small volatile concentrations and explosive fountaining from most others.

Exercise 6.3

(a) In order to fragment the magma (i.e., to generate the pyroclasts) and then to eject the pyroclasts, the pressure inside the slug must have been greater than that of the atmosphere. The slug must have been overpressured at burst.

(b) Immediately prior to "burst", a curved membrane of magma will have likely separated the slug gas from the atmosphere. When this membrane ruptured, material from it will be ejected as pyroclasts by the rapidly accelerating (and decompressing) slug gas. Further pyroclasts may well be formed from magma in the falling film around the slug, by being ripped off by the expanding gas jet. Some pyroclasts may also be generated at the base of the slug as any bubbles in the trailing wake expand suddenly during the rapid decompression of the slug at burst.

(c) The pyroclast tracks show near-parabolic paths, indicating that inertial and gravitational forces control their dispersal. They will also be influenced by gas drag. Near the vent, the gas drag will be from the decompressing slug gas, and is responsible for most of the pyroclast acceleration. Further on in their trajectories, as the influence of the slug gas decreases, drag will begin to act to decelerate the pyroclasts as they enter near-stationary atmosphere.

(d) Strombolian deposits are significantly less dispersed than plinian deposits because the finer and more vesicular nature of plinian pyroclasts (smaller and less dense particles) make them much more capable of transferring thermal energy to the atmosphere and more susceptible to gas drag. Futhermore, plinian eruptions produce a more sustained supply of pyroclasts, and hence greater thermal energy.. These factors combine to ensure a wide dispersal of plinian deposits; transfer of thermal energy into the atmosphere creates an effective convection column, the gas velocities in which produce drag forces on the small and relatively low density particles that exceed their gravitational and inertial forces. Hence, particles are lofted into high (tens of km) plumes and are dispersed by the wind. The larger and denser strombolian clasts do not transfer heat to the atmosphere efficiently and produce only weakly convecting columns (see Fig. 6.8). The dominance of gravitational forces means that most particles are little affected by the gas drag of this convection.

Exercise 6.4

(a) From the relatively similar dispersion of their eruption products, the intensities of the 2003 paroxysm at Stromboli and the 1959 hawaiian fountaining event of Kilauea are comparable, with the Stromboli event being slightly weaker. However, the Hawaiian eruption produced deposits an order of magnitude thicker, so was significantly more voluminous than Stromboli.

(b) In this case, the contrast reflects the sustained nature of hawaiian fountains (lasting days to weeks), compared to the relatively short (minutes) Stromboli paroxysm.

(c) Although ‘normal’ strombolian eruptions are short in duration (seconds to tens of seconds) they usually occur much more frequently than hawaiian fountaining events. Consequently, sampling strombolian eruption deposits often entails sampling superimposed and intermingled products of numerous eruptions, and the results reflect a wide averaging of the eruption and dispersal parameters for all the source events.