# Answers to Exercises
# Chapter 4

**Exercise 4.1**

Assume a cylindrical conduit so that velocity is in the $z$ direction and velocity in the radial ($r$, cross-conduit) direction is 0. Neglect the possible effects of inertia, and assume a Newtonian magma. Equation (4.43) implies that

$$\frac{\partial u_z}{\partial r} = \frac{r}{2\eta}\frac{\partial p}{\partial z} \quad .$$

Integrating once with respect to $r$, and using the boundary condition $u_z = 0$ and $r = a$, gives

$$u_z = \frac{1}{4\eta}\frac{\partial p}{\partial z}\left(a^2 - r^2\right) \quad ,$$

where $a$ is the conduit radius. Setting $r = 0$ yields the maximum velocity along the centerline of the flow, i.e., 125 m $s^{-1}$ and 0.0125 m $s^{-1}$ for basalt and rhyolite, respectively.

**Exercise 4.2**

The strain rate at the conduit walls is $du_z/dr$ at $r = a$, i.e.,

$$\frac{a}{2\eta}\frac{\partial p}{\partial z}$$

For the basalt and rhyolite magmas, we obtain strain rates of $2.5 \times 10^2$ $s^{-1}$ and $2.5 \times 10^{-3}$ $s^{-1}$, respectively.

The viscous relaxation times (section 4.5.2) of our basalt and rhyolite are $\tau_r = \eta/G_\infty = 10^{-10}$ s and $10^{-4}$ s, respectively.

To become shear thinning, following Eq. (4.18), we need strain rates > $10^6$ and 1 $s^{-1}$, respectively. This is not the case. Structural failure requires strain rates that are 100 times higher.

**Exercise 4.3**

We assume a surface tension $\gamma = 0.1$ N $m^{-1}$. The Capillary numbers (section 4.3.4) are 25 and 250 for the basalt and rhyolite, respectively. As this is >> 1 we would expect that centimeter-sized bubbles will become highly deformed near conduit walls.

**Exercise 4.4**

Assuming that Re for bubble motion is small and the gas inside the bubble has negligible density and viscosity, the speed is given by $\Delta\rho g R/3\eta$, where $R$ is the bubble radius, and $\Delta\rho$ is the difference between the magma and vapor densities. With respect to the surrounding melt, the rise speeds are 0.93 m $s^{-1}$ and $0.83 \times 10^{-4}$ m $s^{-1}$ for basalt and rhyolite, respectively. For comparison, the magma ascent rates from the solution to 4.1 are 125 m $s^{-1}$ and 0.0125 m $s^{-1}$ for basalt and rhyolite, respectively. In both cases the magma is rising much faster that the speed at which bubbles move with respect to the surrounding melt so the bubbly flow regime of Figure 4.4 should be relevant. We can, after the fact, calculate Re for the bubble in basalt, and find a value of ~25, large enough that the Stokes flow solution will not be accurate. Nevertheless, the bubble rise speed will still remain much less than the magma ascent speed.