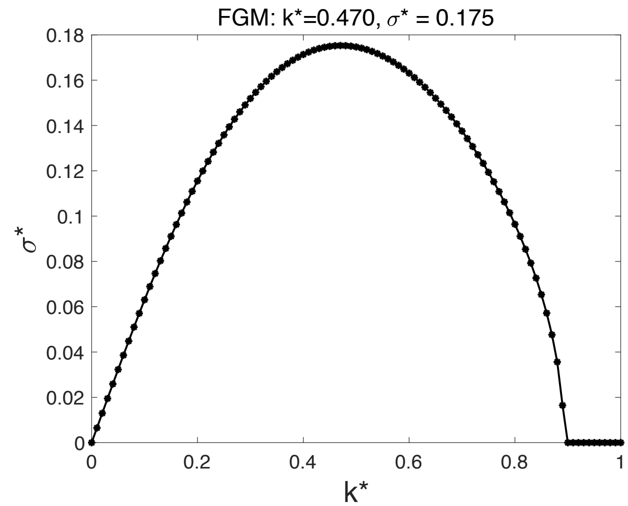

Figure 1: Dispersion relation for the hyperbolic tangent shear layer, showing the fastest growing mode near $k* = 0.45$.

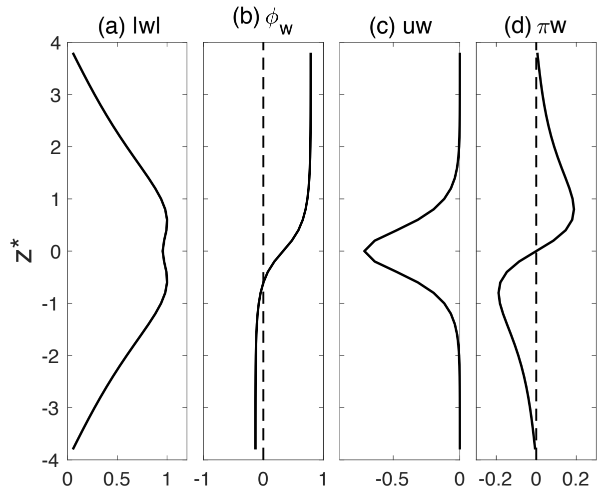

Figure 2: Eigenfunction, momentum flux and energy flux for the fastest growing mode.

10: Energy analysis for a shear layer

The growth rate $\sigma^\star(k^\star)$ for the hyperbolic tangent shear layer $U^\star = \tanh(z^\star)$ is shown in figure 1. The maximum growth rate $\sigma^\star(k^\star) = 0.175$ is found at $k^\star = 0.470$ as found in problem 7.

(a) Profiles of $\hat{w}$ (amplitude and phase), $\overline{u'w'}$ and $\overline{\pi'w'}$ are shown on figure 2. The vertical velocity eigenfunction is is even about z=0. The phase of the eigenfunction tilts with the mean shear. (This is consistent with the fact that phase lines tilt opposite to the shear.) The momentum flux $u'w'$ carries positive (rightward) momentum downward through the shear layer and negative momentum upward, so the flux is negative everywhere. The energy flux carries energy vertically away from the shear layer.

(b) Profiles of the kinetic energy $K'(z)$, the shear production rate $SP(z)$ and the flux convergence $FC(z)$ are shown on figure 3, plus the energy flux again for good measure.

(c) Kinetic energy is transferred from the mean flow to the perturbation via $SP(z)$ in a layer surrounding

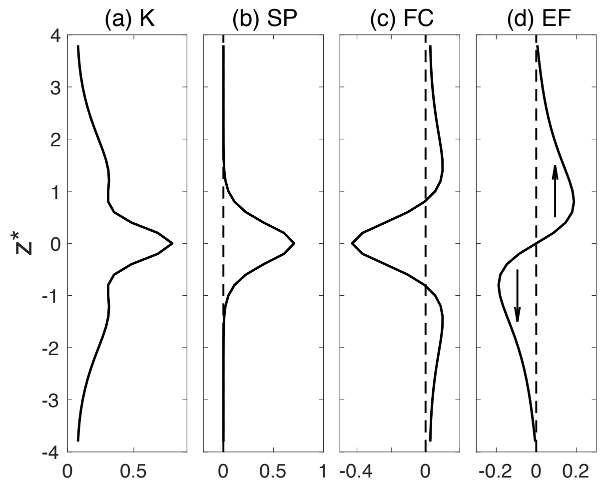

Figure 3: Energy budget terms for the fastest growing mode.

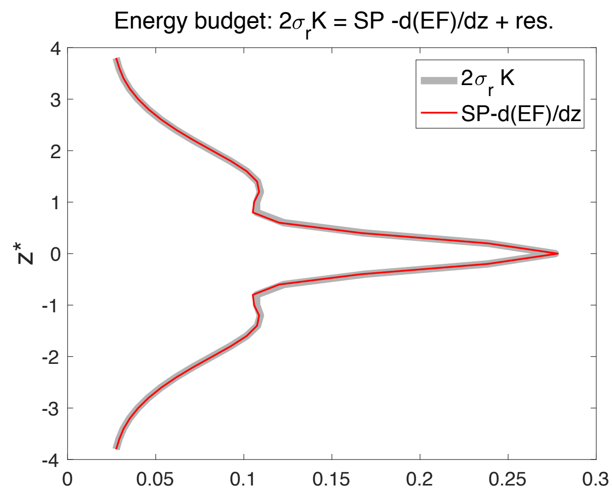

Figure 4: Balancing the energy budget.

$z = 0$, peaking at $z = 0$ (figure 3b). Part of this energy is fluxed outward (figure 3c,d), so that the energy flux diverges near $z = 0$ (approximately $|z| < 1$ and converges outside that layer. As a result, the kinetic energy profile (figure 3a) shows energetic regions extending above and below the shear layer.

(d) Profiles of $2\sigma_r K'(z)$ and $SP(z) + FC(z)$ are combined on figure 4, and show that the kinetic energy budget balances to within a small tolerance. The error decreases as Δ is reduced. This indicates not only that the code is correct but also that the solution of the finite difference equation is a good approximation to that of the differential equation.

(e) Figure 5 shows $w'(x,z)$ for an arbitrary time. We used the Matlab function contourf for this. The tilt is consistent with positive shear production.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Project 10: Energy analysis for a shear layer
clear
```

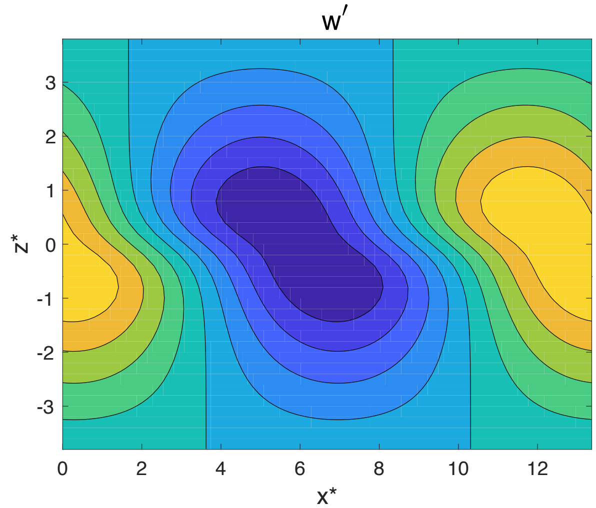

Figure 5: The vertical velocity perturbation for the fastest-growing mode.

```
close all

fs=18;
ms=16;
lw=2;

ks=[0:.01:1.0];
l=0;

dz=.2;
ztop=4;
zbot=-ztop;
z=[zbot+dz:dz:ztop-dz]';
iBC=[1 1];
U=tanh(z);
for i=1:length(ks)
    k=ks(i);
    [sig1(i),w1]=Ray(z,U,k,l,iBC);
end

figure
plot(ks,real(sig1),'k-*');hold on
yl=ylim;ylim([0 yl(2)]);
xlabel('k*')
ylabel('\sigma*')
%

sFGM=max(real(sig1));
kFGM=ks(real(sig1)==sFGM);
titl=sprintf('FGM: k*=%.3f, \\sigma* = %.3f',kFGM,sFGM)
title(titl,'fontweight','normal')
```

```
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

%% Part A) eigfn, fluxes

k=kFGM; % pick FGM
[sig1,w1,uw,SP,pw,PKE]=Ray(z,U,k,0,iBC); % recalculate with budget terms
FC=-ddz(z)*pw;


figure

subplot(1,8,1:2)
plot(abs(w1),z,'k','linewidth',lw)
title('(a) |w|','fontsize',fs,'fontweight','normal')
ylabel('z*','fontsize',fs)
xlim([0 1.2])
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,3:4)
plot(phase(w1),z,'k','linewidth',lw)
title('(b) \phi_w','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
%xlim(1.0*[-.2 1])
hold on
plot([0 0],ylim,'k--')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,5:6)
plot(uw,z,'k','linewidth',lw)
title('(c) uw','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
xlim([-0.9 0])
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,7:8)
plot(pw,z,'k','linewidth',lw)
title('(d) \piw','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
hold on
xlim(.3*[-1 1])
plot([0 0],ylim,'k--')
xarrow=max(pw)/2;
% arrow6([xarrow .5],[xarrow 1.5],.04)
% arrow6([-xarrow -.5],[-xarrow -1.5],.04)
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)


%% Part B) PKE budget
```

```
figure
subplot(1,8,1:2)
plot(PKE,z,'k','linewidth',lw)
title('(a) K','fontsize',fs,'fontweight','normal')
ylabel('z*','fontsize',fs)
xlim([0 0.9])
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,3:4)
plot(SP,z,'k','linewidth',lw)
title('(b) SP','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
xlim(1.0*[-.2 1])
hold on
plot([0 0],ylim,'k--')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,5:6)
plot(FC,z,'k','linewidth',lw)
title('(c) FC','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
hold on
xlim([-.5 .2])
plot([0 0],ylim,'k--')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

subplot(1,8,7:8)
plot(pw,z,'k','linewidth',lw)
title('(d) EF','fontsize',fs,'fontweight','normal')
set(gca,'yticklabel','')
hold on
xlim(.3*[-1 1])
plot([0 0],ylim,'k--')
xarrow=max(pw)/2;
arrow6([xarrow .5],[xarrow 1.5],.04)
arrow6([-xarrow -.5],[-xarrow -1.5],.04)

set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.5,'titlefontsizemultiplier',1.2)

%% Part D) check KE budget
figure
l(1)=plot(2*real(sig1)*PKE,z,'k','linewidth',lw*3);
set(l(1),'color',.7*[1 1 1])
hold on
l(2)=plot(SP+FC,z,'r','linewidth',lw*.7)
h=legend(l,'2\sigma_r K','SP-d(EF)/dz')
set(h,'fontsize',fs)
```

```
title('Energy budget: 2\sigma_rK = SP -d(EF)/dz + res.','fontweight','normal')
ylabel('z*','fontsize',fs)

set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.4,'titlefontsizemultiplier',1.2)

%% Part E) plot FGM
x=[0:.01:1]*2*pi/k;
[X, Z]=meshgrid(x,z);
ii=complex(0.,1.);
ePhi=exp(ii*k*X);
clear w
for i=1:length(z)
    w(i,:)=real(w1(i)*ePhi(i,:));
end

figure
contourf(X,Z,w);
title('  w^\prime','fontweight','normal')
xlabel('x*')
ylabel('z*')
set(gca,'fontsize',fs-2,'labelfontsizemultiplier',1.2,'titlefontsizemultiplier',1.4)
```