Chapter 9

```
In[1]:= Needs["DiscreteMath`RSolve`"]

In[2]:= Needs["Graphics`MultipleListPlot`"]

In[3]:= Needs["Graphics`Graphics3D`"]

In[4]:= Needs["Graphics`PlotField`"]
```

à Question 1

```
In[5]:= Clear[p, Q, q1, q2, pi1, pi2]

In[6]:= Q = q1 + q2
        p = A - B Q
        pi1 = p q1 - a q1
        pi2 = p q2 - a q2

Out[6]= q1 + q2

Out[7]= A - B (q1 + q2)

Out[8]= -a q1 + q1 (A - B (q1 + q2))

Out[9]= -a q2 + q2 (A - B (q1 + q2))

In[10]:= Solve[D[pi1, q1] == 0, q2]
```

Out[10]= $\left\{\left\{q2 \to -\frac{a - A + 2 B q1}{B}\right\}\right\}$

```
In[11]:= Solve[D[pi2, q2] == 0, q2]
```

Out[11]= $\left\{\left\{q2 \to -\frac{a - A + B q1}{2 B}\right\}\right\}$

In[12]:= $\text{solq1} = \text{Simplify}\left[\text{Solve}\left[-\frac{a - A + 2 B q1}{B} == -\frac{a - A + B q1}{2 B}, q1\right]\right]$

Out[12]= $\left\{\left\{q1 \to \frac{-a + A}{3 B}\right\}\right\}$

In[13]:= $\text{solq2} = \text{Simplify}\left[-\frac{a - A + 2 B q1}{B}\right]$ /. solq1

Out[13]= $\left\{\frac{-a + A - \frac{2}{3}(-a + A)}{B}\right\}$

In[14]:= $\text{Simplify}\left[\text{Expand}\left[\frac{-a + A - \frac{2}{3}(-a + A)}{B}\right]\right]$

Out[14]= $\frac{-a + A}{3 B}$

Hence, equilibrium quantities are identical.

à Question 2

This question is best done with paper and pencil. Summations over *j* not equal to *i* are not conveniently dealt with in software packages.

à Question 3

■ (i)

```
In[15]:= Clear[p, Q, q1, q2, pi1, pi2]

In[16]:= Q = q1 + q2
         p = A - B Q
         pi1 = p q1 - a1 q1
         pi2 = p q2 - a2 q2

Out[16]= q1 + q2

Out[17]= A - B (q1 + q2)

Out[18]= -a1 q1 + q1 (A - B (q1 + q2))

Out[19]= -a2 q2 + q2 (A - B (q1 + q2))

In[20]:= Solve[D[pi1, q1] == 0, q2]
```

$$\text{Out[20]=}\ \left\{\left\{q2 \to -\frac{-A + a1 + 2\,B\,q1}{B}\right\}\right\}$$

```
In[21]:= Solve[D[pi2, q2] == 0, q2]
```

$$\text{Out[21]=}\ \left\{\left\{q2 \to -\frac{-A + a2 + B\,q1}{2\,B}\right\}\right\}$$

$$\text{In[22]:=}\ \text{solq1} = \text{Simplify}\left[\text{Solve}\left[-\frac{-A + a1 + 2\,B\,q1}{B} == -\frac{-A + a2 + B\,q1}{2\,B},\ q1\right]\right]$$

$$\text{Out[22]=}\ \left\{\left\{q1 \to \frac{A - 2\,a1 + a2}{3\,B}\right\}\right\}$$

$$\text{In[23]:=}\ \text{solq2} = \text{Simplify}\left[-\frac{-A + a1 + 2\,B\,q1}{B}\right]\ /.\ \text{solq1}$$

$$\text{Out[23]=}\ \left\{\frac{A - a1 - \frac{2}{3}\,(A - 2\,a1 + a2)}{B}\right\}$$

$$\text{In[24]:=}\ \text{Simplify}\left[\text{Expand}\left[\frac{A - a1 - \frac{2}{3}\,(A - 2\,a1 + a2)}{B}\right]\right]$$

$$\text{Out[24]=}\ \frac{A + a1 - 2\,a2}{3\,B}$$

■ (ii)

We already have the following as the reaction functions in the static model:

$q1 = \frac{A-a_1}{2B} - \frac{q_2}{2}$ for firm 1

$q2 = \frac{A-a_2}{2B} - \frac{q_1}{2}$ for firm 2

So the dynamic reaction functions are:

$q_1(t) = \frac{A-a_1}{2B} - \frac{q_2(t)}{2}$

$q_2(t) = \frac{A-a_2}{2B} - \frac{q_1(t)}{2}$

```
In[25]:= RSolve[{q1[t] == (A - a1)/(2 B) - q2[t - 1]/2,
           q2[t] == (A - a2)/(2 B) - q1[t - 1]/2, q1[0] == q10, q2[0] == q20}, {q1[t], q2[t]}, t]
```

```
Out[25]= {{q1[t] → 1/(3 B) (2^(-1-t) (-2 (-1)^t A + 2^(1+t) A + 3 B q10 + 3 (-1)^t B q10 -
              3 B q20 + 3 (-1)^t B q20 + (3 + (-1)^t - 2^(2+t)) a1 + (-3 + (-1)^t + 2^(1+t)) a2)),
          q2[t] → 1/(3 B) (2^(-1-t) (-2 (-1)^t A + 2^(1+t) A - 3 B q10 + 3 (-1)^t B q10 + 3 B q20 +
              3 (-1)^t B q20 + (-3 + (-1)^t + 2^(1+t)) a1 + (3 + (-1)^t - 2^(2+t)) a2))}}
```

This could be simplified but we do not do this here. What matters is that the stability is governed purely by the coefficients of q2(t-1) and q1(t-1) in the reaction functions of firms 1 and 2 respectively. Since these are less than unity, then the system is asymptotically stable. In fact, the coefficients must each take the value of -1/2. Consequently the coefficients of q10 and q20 in the results involve only $(\frac{1}{2})^t$ or $(-\frac{1}{2})^t$ and so tend to zero in the limit regardless of the values of q10 and q20. (Note that the term *B* cancells.)

à Question 4

```
In[26]:= Clear[p, Q, q1, q2, pi1, pi2]
```

■ (i)

```
In[27]:= p = 9 - Q
         Q = q1 + q2
         TC1 = a1 q1
         TC2 = a2 q2
         pi1 = p q1 - TC1
         pi2 = p q2 - TC2

Out[27]= 9 - Q

Out[28]= q1 + q2

Out[29]= a1 q1

Out[30]= a2 q2

Out[31]= -a1 q1 + q1 (9 - q1 - q2)

Out[32]= -a2 q2 + (9 - q1 - q2) q2

In[33]:= Solve[D[pi1, q1] == 0, q2]

Out[33]= {{q2 → 9 - a1 - 2 q1}}

In[34]:= Solve[D[pi2, q2] == 0, q2]
```

$$\text{Out[34]= } \left\{\left\{q2 \to \frac{1}{2}\ (9 - a2 - q1)\right\}\right\}$$

$$\text{In[35]:= } \mathbf{solq1 = Simplify}\left[\mathbf{Solve}\left[9 - a1 - 2\,q1 == \frac{1}{2}\ (9 - a2 - q1),\ q1\right]\right]$$

$$\text{Out[35]= } \left\{\left\{q1 \to \frac{1}{3}\ (9 - 2\,a1 + a2)\right\}\right\}$$

```
In[36]:= solq2 = 9 - a1 - 2 q1 /. solq1
```

$$\text{Out[36]= } \left\{9 - a1 - \frac{2}{3}\ (9 - 2\,a1 + a2)\right\}$$

$$\text{In[37]:= } \mathbf{Simplify}\left[\mathbf{Expand}\left[9 - a1 - \frac{2}{3}\ (9 - 2\,a1 + a2)\right]\right]$$

$$\text{Out[37]= } \frac{1}{3}\ (9 + a1 - 2\,a2)$$

$$\text{In[38]:= } \mathbf{Simplify}\left[\frac{1}{3}\ (9 - 2\,a1 + a2) - \frac{1}{3}\ (9 + a1 - 2\,a2)\right]$$

```
Out[38]= -a1 + a2
```

Hence, equilibrium q_1 is greater than equilibrium q_2 if $a_1 < a_2$.

■ (ii) $a_1 = 3$ and $a_2 = 5$

The model is then

$p = 9 - Q$

$Q = q_1 + q_2$

$TC_1 = 3\,q_1$

$TC_2 = 5\,q_2$

```
In[39]:= pi1 /. a1 → 3

Out[39]= -3 q1 + q1 (9 - q1 - q2)

In[40]:= pi2 /. a2 → 5

Out[40]= -5 q2 + (9 - q1 - q2) q2

In[41]:= Solve[D[-3 q1 + q1 (9 - q1 - q2), q1] == 0, q1]
```

Out[41]= $\left\{\left\{q1 \to \frac{6 - q2}{2}\right\}\right\}$

```
In[42]:= Solve[D[-5 q2 + (9 - q1 - q2) q2, q2] == 0, q2]
```

Out[42]= $\left\{\left\{q2 \to \frac{4 - q1}{2}\right\}\right\}$

Our reaction functions are therefore

$q_1(t) = 3 - \frac{1}{2}\,q_2(t-1)$

$q_2(t) = 2 - \frac{1}{2}\,q_1(t-1)$

The monopoly points are (3, 0) for firm 1 and (0, 2) for firm 2.

■ (a) firm 1 monopolist

```
In[43]:= RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1],
           q2[t] == 2 - (1 / 2) q1[t - 1], q1[0] == 3, q2[0] == 0}, {q1[t], q2[t]}, t]
```

Out[43]= $\left\{\left\{q1[t] \to \frac{1}{3}\,2^{-1-t}\,(3 - (-1)^t + 2^{4+t}),\ q2[t] \to \frac{1}{3}\,2^{-1-t}\,(-3 - (-1)^t + 2^{2+t})\right\}\right\}$

■ (b) firm 2 monopolist

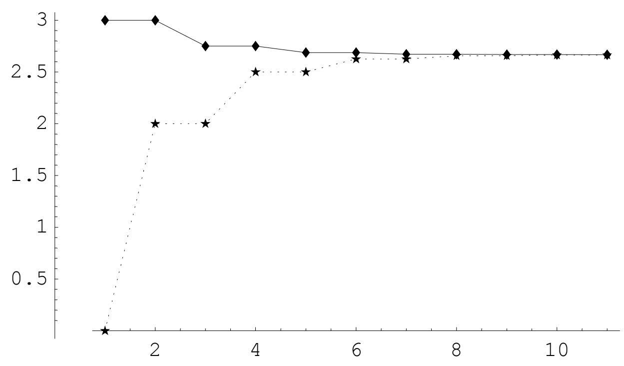
```
In[44]:= RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1],
           q2[t] == 2 - (1 / 2) q1[t - 1], q1[0] == 0, q2[0] == 2}, {q1[t], q2[t]}, t]
```

Out[44]= $\left\{\left\{q1[t] \to \frac{1}{3}\,2^{1-t}\,(-3 - (-1)^t + 2^{2+t}),\ q2[t] \to \frac{1}{3}\,2^{1-t}\,(3 - (-1)^t + 2^t)\right\}\right\}$

In[45]:= q1firm1 = Table$\left[\frac{1}{3}\,2^{-1-t}\,(3 - (-1)^t + 2^{4+t}),\ \{t, 0, 10\}\right]$;

In[46]:= q1firm2 = Table$\left[\frac{1}{3}\,2^{1-t}\,(-3 - (-1)^t + 2^{2+t}),\ \{t, 0, 10\}\right]$;

```
In[47]:= MultipleListPlot[q1firm1, q1firm2,
           PlotJoined → True, PlotRange → All, AxesOrigin → {0, 0}];
```



The equilibrium is reached sooner when firm 1 is the monopolist. This should not be surprising, since firm 1 has the lower unit costs.

à Question 5

```
In[48]:= Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]
```

■ (i)

```
In[49]:= pi1 = (9 - q1 - q2 - q3) q1 - 5 q1
         pi2 = (9 - q1 - q2 - q3) q2 - 5 q2
         pi3 = (9 - q1 - q2 - q3) q3 - 5 q3

Out[49]= -5 q1 + q1 (9 - q1 - q2 - q3)

Out[50]= -5 q2 + q2 (9 - q1 - q2 - q3)

Out[51]= -5 q3 + (9 - q1 - q2 - q3) q3

In[52]:= D[pi1, q1]

Out[52]= 4 - 2 q1 - q2 - q3

In[53]:= D[pi2, q2]

Out[53]= 4 - q1 - 2 q2 - q3

In[54]:= D[pi3, q3]

Out[54]= 4 - q1 - q2 - 2 q3

In[55]:= Solve[{4 - 2 q1 - q2 - q3 == 0, 4 - q1 - 2 q2 - q3 == 0, 4 - q1 - q2 - 2 q3 == 0}, {q1, q2, q3}]

Out[55]= {{q1 → 1, q2 → 1, q3 → 1}}

In[56]:= Solve[4 - 2 q1 - q2 - q3 == 0, q1]
```

Out[56]= $\left\{\left\{q1 \to \frac{1}{2}\ (4 - q2 - q3)\right\}\right\}$

```
In[57]:= Solve[4 - q1 - 2 q2 - q3 == 0, q2]
```

$$\text{Out[57]= } \left\{\left\{q2 \to \frac{1}{2}(4 - q1 - q3)\right\}\right\}$$

```
In[58]:= Solve[4 - q1 - q2 - 2 q3 == 0, q3]
```

$$\text{Out[58]= } \left\{\left\{q3 \to \frac{1}{2}(4 - q1 - q2)\right\}\right\}$$

Hence, the reaction curves are:

$$q_1(t) = 2 - \tfrac{1}{2}\, q_2(t-1) - \tfrac{1}{2}\, q_3(t-1)$$

$$q_2(t) = 2 - \tfrac{1}{2}\, q_1(t-1) - \tfrac{1}{2}\, q_3(t-1)$$

$$q_3(t) = 2 - \tfrac{1}{2}\, q_1(t-1) - \tfrac{1}{2}\, q_2(t-1)$$

■ (iii)

```
In[59]:= RSolve[{q1[t] == 2 - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
           q2[t] == 2 - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
           q3[t] == 2 - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == q10,
           q2[0] == q20, q3[0] == q30}, {q1[t], q2[t], q3[t]}, t]
```

$$\text{Out[59]= } \Big\{\Big\{q1[t] \to \frac{1}{3}\, 2^{-t}\left(-3(-2)^t + 3\, 2^t + (2 + (-2)^t)\, q10 + (-1 + (-2)^t)\, q20 - q30 + (-2)^t\, q30\right),$$

$$q2[t] \to \frac{1}{3}\, 2^{-t}\left(-3(-2)^t + 3\, 2^t + (-1 + (-2)^t)\, q10 + (2 + (-2)^t)\, q20 - q30 + (-2)^t\, q30\right),$$

$$q3[t] \to \frac{1}{3}\, 2^{-t}\left(-3(-2)^t + 3\, 2^t + (-1 + (-2)^t)\, q10 + (-1 + (-2)^t)\, q20 + 2\, q30 + (-2)^t\, q30\right)\Big\}\Big\}$$

Because of the occurrence of the terms $(-1)^t$ then the system also oscillates, eventually oscillating with constant amplitude.

à Question 6

```
In[60]:= Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]
```

■ (i)

```
In[61]:= pi1 = (9 - q1 - q2 - q3) q1 - 3 q1
         pi2 = (9 - q1 - q2 - q3) q2 - 2 q2
         pi3 = (9 - q1 - q2 - q3) q3 - q3
```

```
Out[61]= -3 q1 + q1 (9 - q1 - q2 - q3)

Out[62]= -2 q2 + q2 (9 - q1 - q2 - q3)

Out[63]= -q3 + (9 - q1 - q2 - q3) q3
```

```
In[64]:= D[pi1, q1]
```

```
Out[64]= 6 - 2 q1 - q2 - q3
```

```
In[65]:= D[pi2, q2]

Out[65]= 7 - q1 - 2 q2 - q3

In[66]:= D[pi3, q3]

Out[66]= 8 - q1 - q2 - 2 q3

In[67]:= Solve[{6 - 2 q1 - q2 - q3 == 0, 7 - q1 - 2 q2 - q3 == 0, 8 - q1 - q2 - 2 q3 == 0}, {q1, q2, q3}]
```

Out[67]= $\{\{q1 \to \frac{3}{4}, q2 \to \frac{7}{4}, q3 \to \frac{11}{4}\}\}$

■ (ii)

```
In[68]:= Solve[6 - 2 q1 - q2 - q3 == 0, q1]
```

Out[68]= $\{\{q1 \to \frac{1}{2}(6 - q2 - q3)\}\}$

```
In[69]:= Solve[7 - q1 - 2 q2 - q3 == 0, q2]
```

Out[69]= $\{\{q2 \to \frac{1}{2}(7 - q1 - q3)\}\}$

```
In[70]:= Solve[8 - q1 - q2 - 2 q3 == 0, q3]
```

Out[70]= $\{\{q3 \to \frac{1}{2}(8 - q1 - q2)\}\}$

Hence, the reaction curves are:

$$q_1(t) = 3 - \tfrac{1}{2}\, q_2(t-1) - \tfrac{1}{2}\, q_3(t-1)$$

$$q_2(t) = \tfrac{7}{2} - \tfrac{1}{2}\, q_1(t-1) - \tfrac{1}{2}\, q_3(t-1)$$

$$q_3(t) = 4 - \tfrac{1}{2}\, q_1(t-1) - \tfrac{1}{2}\, q_2(t-1)$$

Monopoly points are (3, 0, 0) for firm 1, (0, 7/2, 0) for firm 2 and (0, 0, 4) for firm 3.

■ (a) firm 1 monopolist

```
In[71]:= Clear[q1, q2, q3]

In[72]:= RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
            q2[t] == (7 / 2) - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
            q3[t] == 4 - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == 3,
            q2[0] == 0, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]
```

Out[72]= $\{\{q1[t] \to -3\left(-\frac{1}{4} - 2^{-t} + \frac{1}{4}\, e^{i\pi t}\right), q2[t] \to \frac{7}{4} - 2^{-t} - \frac{3}{4}\, e^{i\pi t}, q3[t] \to \frac{11}{4} - \frac{3\,(-1)^t}{4} - 2^{1-t}\}\}$

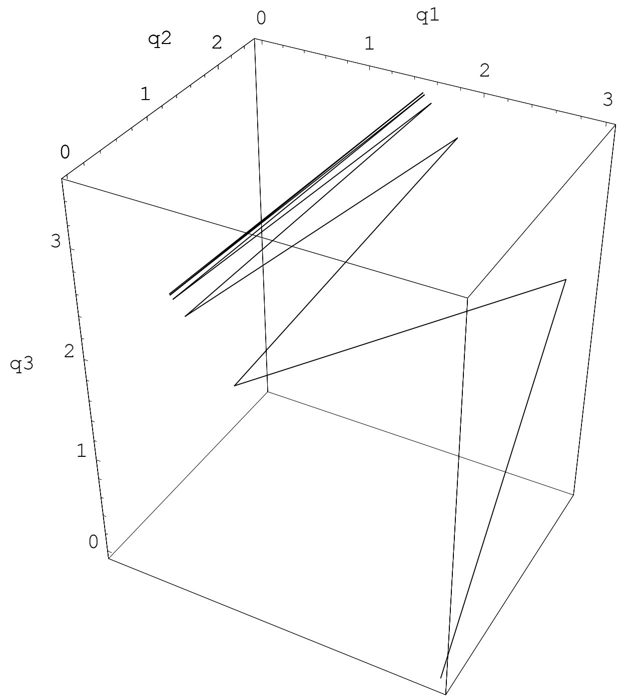
In[73]:= $q1[t\_] := -3\left(-\frac{1}{4} - 2^{-t} + \frac{1}{4}\, e^{i\pi t}\right)$

In[74]:= $q2[t\_] := \frac{7}{4} - 2^{-t} - \frac{3}{4}\, e^{i\pi t}$

In[75]:= **q3[t_] := $\frac{11}{4} - \frac{3\,(-1)^{t}}{4} - 2^{1-t}$**

In[76]:= **points6a = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];**

In[77]:= **ScatterPlot3D[points6a, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];**



■ (b) firm 2 as monopolist

In[78]:= **Clear[q1, q2, q3]**

In[79]:= **RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
q2[t] == (7 / 2) - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
q3[t] == 4 - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == 0,
q2[0] == 7 / 2, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]**

Out[79]= $\left\{\left\{q1[t] \to \frac{1}{12}\,(9 - 7\,(-1)^{t} - 2^{1-t}),\; q2[t] \to -\frac{7}{12}\,(-3 - 2^{2-t} + e^{i\pi t}),\; q3[t] \to \frac{1}{3}\,2^{-2-t}\,(-26 - 7\,(-2)^{t} + 33\,2^{t})\right\}\right\}$

In[80]:= **q1[t_] := $\frac{1}{12}\,(9 - 7\,(-1)^{t} - 2^{1-t})$**

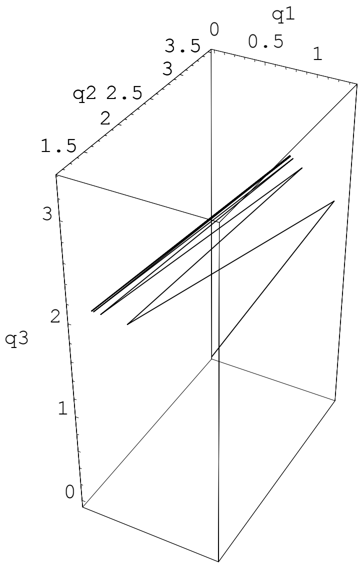
In[81]:= **q2[t_] := $-\frac{7}{12}\,(-3 - 2^{2-t} + e^{i\pi t})$**

In[82]:= **q3[t_] := $\frac{1}{3}\,2^{-2-t}\,(-26 - 7\,(-2)^{t} + 33\,2^{t})$**

In[83]:= **points6b = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];**

```
General::spell1 :
 Possible spelling error: new symbol name "points6b" is similar to existing symbol "points6a".
```

```
In[84]:= ScatterPlot3D[points6b, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];
```



■ (c) firm 3 monopolist

```
In[85]:= Clear[q1, q2, q3]

In[86]:= RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
            q2[t] == (7 / 2) - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
            q3[t] == 4 - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == 0,
            q2[0] == 0, q3[0] == 4}, {q1[t], q2[t], q3[t]}, t]
```

Out[86]= $\left\{\left\{q1[t] \to \frac{3}{4} - \frac{2^{-t}}{3} - \frac{5}{12} e^{i \pi t},\right.\right.$
$\left.\left. q2[t] \to \frac{1}{12} (21 - 5 (-1)^t - 2^{4-t}),\ q3[t] \to \frac{11}{4} + \frac{5\ 2^{-t}}{3} - \frac{5}{12} e^{i \pi t}\right\}\right\}$

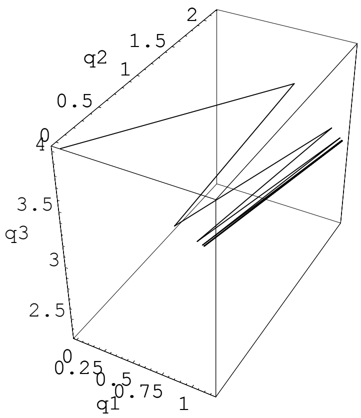
In[87]:= $q1[t\_] := \frac{3}{4} - \frac{2^{-t}}{3} - \frac{5}{12} e^{i \pi t}$

In[88]:= $q2[t\_] := \frac{1}{12} (21 - 5 (-1)^t - 2^{4-t})$

In[89]:= $q3[t\_] := \frac{11}{4} + \frac{5\ 2^{-t}}{3} - \frac{5}{12} e^{i \pi t}$

```
In[90]:= points6c = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];

    General::spell :
     Possible spelling error: new symbol name "points6c" is similar to existing symbols {points6a, points6b}.
```

In[91]:= `ScatterPlot3D[points6c, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];`



■ **(iii)**

In[92]:= `Clear[q1, q2, q3]`

In[93]:=
```
RSolve[{q1[t] == 3 - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
  q2[t] == (7 / 2) - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
  q3[t] == 4 - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == q10,
  q2[0] == q20, q3[0] == q30}, {q1[t], q2[t], q3[t]}, t]
```

Out[93]= $\left\{\left\{q1[t] \to \frac{1}{3}\, 2^{-2-t}\, (12 + 9\, 2^t + 8\, q10 - 4\, q20 - 4\, q30 + 2^t\, e^{i \pi t}\, (-21 + 4\, q10 + 4\, q20 + 4\, q30)),\right.\right.$

$q2[t] \to \frac{1}{3}\, 2^{-2-t}\, (-21\, (-2)^t + 21\, 2^t + 4\, (-1 + (-2)^t)\, q10 + 4\, (2 + (-2)^t)\, q20 - 4\, q30 + (-1)^t\, 2^{2+t}\, q30),$

$\left.\left. q3[t] \to \frac{1}{3}\, 2^{-2-t}\, (-12 - 21\, (-2)^t + 33\, 2^t + 4\, (-1 + (-2)^t)\, q10 + 4\, (-1 + (-2)^t)\, q20 + 8\, q30 + (-1)^t\, 2^{2+t}\, q30)\right\}\right\}$

à Question 7

In[94]:= `Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]`

■ **(i)**

In[95]:=
```
pi1 = (15 - 2 q1 - 2 q2 - 2 q3) q1 - 5 q1
pi2 = (15 - 2 q1 - 2 q2 - 2 q3) q2 - 3 q2
pi3 = (15 - 2 q1 - 2 q2 - 2 q3) q3 - 2 q3
```

Out[95]= −5 q1 + q1 (15 − 2 q1 − 2 q2 − 2 q3)

Out[96]= −3 q2 + q2 (15 − 2 q1 − 2 q2 − 2 q3)

Out[97]= −2 q3 + (15 − 2 q1 − 2 q2 − 2 q3) q3

In[98]:= `D[pi1, q1]`

Out[98]= 10 − 4 q1 − 2 q2 − 2 q3

```
In[99]:= D[pi2, q2]

Out[99]= 12 - 2 q1 - 4 q2 - 2 q3

In[100]:= D[pi3, q3]

Out[100]= 13 - 2 q1 - 2 q2 - 4 q3

In[101]:= Solve[{10 - 4 q1 - 2 q2 - 2 q3 == 0,
            12 - 2 q1 - 4 q2 - 2 q3 == 0, 13 - 2 q1 - 2 q2 - 4 q3 == 0}, {q1, q2, q3}]
```

Out[101]= $\left\{\left\{q1 \to \frac{5}{8}, q2 \to \frac{13}{8}, q3 \to \frac{17}{8}\right\}\right\}$

```
In[102]:= Solve[10 - 4 q1 - 2 q2 - 2 q3 == 0, q1]
```

Out[102]= $\left\{\left\{q1 \to \frac{1}{2} (5 - q2 - q3)\right\}\right\}$

```
In[103]:= Solve[12 - 2 q1 - 4 q2 - 2 q3 == 0, q2]
```

Out[103]= $\left\{\left\{q2 \to \frac{1}{2} (6 - q1 - q3)\right\}\right\}$

```
In[104]:= Solve[13 - 2 q1 - 2 q2 - 4 q3 == 0, q3]
```

Out[104]= $\left\{\left\{q3 \to \frac{1}{4} (13 - 2 q1 - 2 q2)\right\}\right\}$

Hence, the reaction functions are:

$$q_1(t) = \frac{5}{2} - \frac{1}{2}\, q_2(t-1) - \frac{1}{2}\, q_3(t-1)$$

$$q_2(t) = 3 - \frac{1}{2}\, q_1(t-1) - \frac{1}{2}\, q_3(t-1)$$

$$q_3(t) = \frac{13}{4} - \frac{1}{2}\, q_1(t-1) - \frac{1}{2}\, q_2(t-1)$$

Monopoly points are ($\frac{5}{2}$, 0, 0) for firm 1, (0, 3, 0) for firm 2 and (0, 0, $\frac{13}{4}$) for firm 3.

■ (a) firm 1 monopolist

```
In[105]:= RSolve[{q1[t] == (5 / 2) - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
            q2[t] == 3 - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
            q3[t] == (13 / 4) - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1],
            q1[0] == 5 / 2, q2[0] == 0, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]
```

Out[105]= $\left\{\left\{q1[t] \to -5\, 2^{-3-t} \left(-4 - 2^t + 2^t e^{i \pi t}\right), q2[t] \to \frac{13}{8} - \frac{5 (-1)^t}{8} - 2^{-t}, q3[t] \to 2^{-3-t} \left(-12 - 5 (-2)^t + 17\, 2^t\right)\right\}\right\}$

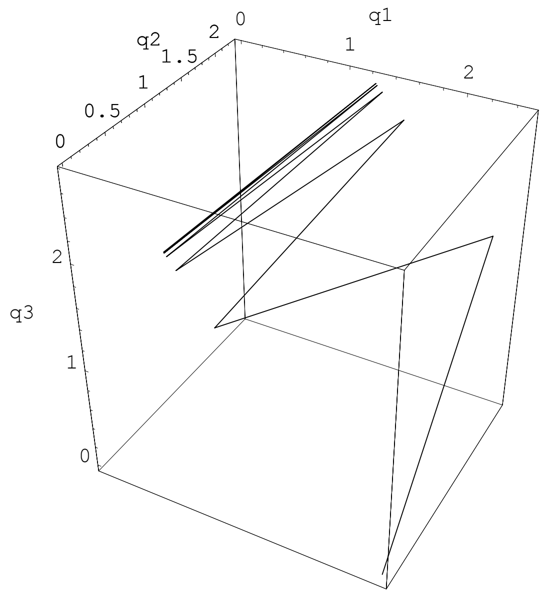
In[106]:= **q1[t_] := $-5\, 2^{-3-t} \left(-4 - 2^t + 2^t e^{i \pi t}\right)$**

In[107]:= **q2[t_] := $\frac{13}{8} - \frac{5 (-1)^t}{8} - 2^{-t}$**

In[108]:= **q3[t_] := $2^{-3-t} \left(-12 - 5 (-2)^t + 17\, 2^t\right)$**

```
In[109]:= points7a = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];
```

```
In[110]:= ScatterPlot3D[points7a, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];
```



■ (b) firm 2 as monopolist

```
In[111]:= Clear[q1, q2, q3]

In[112]:= RSolve[{q1[t] == (5 / 2) - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
            q2[t] == 3 - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
            q3[t] == (13 / 4) - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1],
            q1[0] == 0, q2[0] == 3, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]
```

Out[112]= $\left\{\left\{q1[t] \to \frac{1}{3}\, 2^{-3-t}\, (-4 - 11\, (-2)^t + 15\, 2^t),\; q2[t] \to \frac{1}{3}\, 2^{-3-t}\, (44 - 11\, (-2)^t + 39\, 2^t),\; q3[t] \to \frac{1}{24}\, (51 - 11\, (-1)^t - 5\, 2^{3-t})\right\}\right\}$

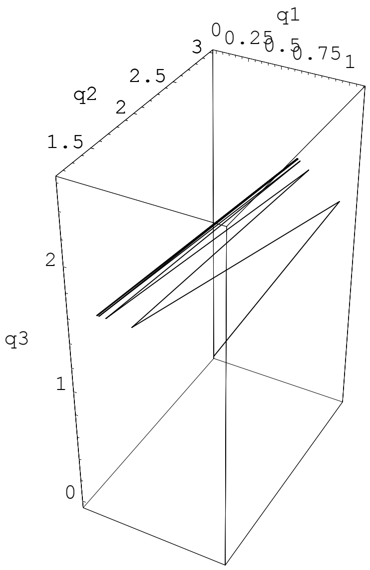
In[113]:= q1[t_] := $\frac{1}{3}\, 2^{-3-t}\, (-4 - 11\, (-2)^t + 15\, 2^t)$

In[114]:= q2[t_] := $\frac{1}{3}\, 2^{-3-t}\, (44 - 11\, (-2)^t + 39\, 2^t)$

In[115]:= q3[t_] := $\frac{1}{24}\, (51 - 11\, (-1)^t - 5\, 2^{3-t})$

```
In[116]:= points7b = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];

General::spell1 :
 Possible spelling error: new symbol name "points7b" is similar to existing symbol "points7a".
```

```
In[117]:= ScatterPlot3D[points7b, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];
```



■ (c) firm 3 monopolist

```
In[118]:= Clear[q1, q2, q3]

In[119]:= RSolve[{q1[t] == (5 / 2) - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
            q2[t] == 3 - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
            q3[t] == (13 / 4) - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == 0,
            q2[0] == 0, q3[0] == 13 / 4}, {q1[t], q2[t], q3[t]}, t]
```

Out[119]= $\left\{\left\{q1[t] \to 2^{-3-t} (-2 - 3 (-2)^t + 5\, 2^t),\ q2[t] \to 2^{-3-t} (-10 - 3 (-2)^t + 13\, 2^t),\ q3[t] \to \frac{1}{8} (17 + 3\, 2^{2-t} - 3\, e^{i \pi t})\right\}\right\}$

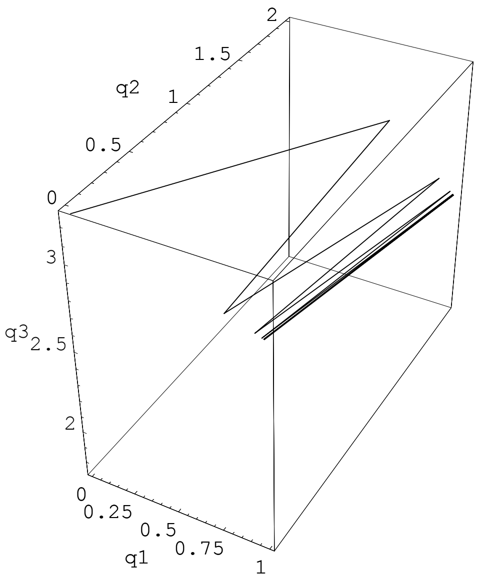
In[120]:= **q1[t_] := $2^{-3-t} (-2 - 3 (-2)^t + 5\, 2^t)$**

In[121]:= **q2[t_] := $2^{-3-t} (-10 - 3 (-2)^t + 13\, 2^t)$**

In[122]:= **q3[t_] := $\frac{1}{8} (17 + 3\, 2^{2-t} - 3\, e^{i \pi t})$**

```
In[123]:= points7c = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];

General::spell :
 Possible spelling error: new symbol name "points7c" is similar to existing symbols {points7a, points7b}.
```

In[124]:= **ScatterPlot3D[points7c, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];**



■ **(iii)**

In[125]:= **Clear[q1, q2, q3]**

In[126]:= **RSolve[{q1[t] == (5 / 2) - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
q2[t] == 3 - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
q3[t] == (13 / 4) - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1], q1[0] == q10,
q2[0] == q20, q3[0] == q30}, {q1[t], q2[t], q3[t]}, t]**

Out[126]= $\{\{q1[t] \to \frac{1}{3} 2^{-3-t} (20 - 35 (-2)^t + 15\, 2^t + 8 (2 + 2^t e^{i\pi t})\, q10 + 8 (-1 + (-2)^t)\, q20 - 8\, q30 + (-1)^t 2^{3+t} q30),$
$q2[t] \to \frac{1}{3} 2^{-3-t} (-4 - 35 (-2)^t + 39\, 2^t + 8 (-1 + (-2)^t)\, q10 + 8 (2 + (-2)^t)\, q20 - 8\, q30 + (-1)^t 2^{3+t} q30),$
$q3[t] \to \frac{1}{3} 2^{-3-t} (-16 - 35 (-2)^t + 51\, 2^t + 8 (-1 + (-2)^t)\, q10 + 8 (-1 + (-2)^t)\, q20 + 16\, q30 + (-1)^t 2^{3+t} q30)\}\}$

No. Because of the presence of $(-1)^t$, the system will eventually oscillate.

à Question 8

In[127]:= **Clear[p, Q, q1, q2, pi1, pi2]**

■ (i)

■ (a)

```
In[128]:= Q = q1 + q2
          p = 20 - 3 Q
          pi1 = p q1 - 4 q1
          pi2 = p q2 - 4 q2

Out[128]= q1 + q2

Out[129]= 20 - 3 (q1 + q2)

Out[130]= -4 q1 + q1 (20 - 3 (q1 + q2))

Out[131]= -4 q2 + q2 (20 - 3 (q1 + q2))

In[132]:= D[pi1, q1]

Out[132]= 16 - 3 q1 - 3 (q1 + q2)

In[133]:= D[pi2, q2]

Out[133]= 16 - 3 q2 - 3 (q1 + q2)

In[134]:= Solve[{16 - 3 q1 - 3 (q1 + q2) == 0, 16 - 3 q2 - 3 (q1 + q2) == 0}, {q1, q2}]
```

Out[134]= $\left\{\left\{q1 \to \frac{16}{9}, q2 \to \frac{16}{9}\right\}\right\}$

■ (b)

```
In[135]:= Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]

In[136]:= Q = q1 + q2 + q3
          p = 20 - 3 Q
          pi1 = p q1 - 4 q1
          pi2 = p q2 - 4 q2
          pi3 = p q3 - 4 q3

Out[136]= q1 + q2 + q3

Out[137]= 20 - 3 (q1 + q2 + q3)

Out[138]= -4 q1 + q1 (20 - 3 (q1 + q2 + q3))

Out[139]= -4 q2 + q2 (20 - 3 (q1 + q2 + q3))

Out[140]= -4 q3 + q3 (20 - 3 (q1 + q2 + q3))

In[141]:= D[pi1, q1]

Out[141]= 16 - 3 q1 - 3 (q1 + q2 + q3)

In[142]:= D[pi2, q2]

Out[142]= 16 - 3 q2 - 3 (q1 + q2 + q3)
```

```
In[143]:= D[pi3, q3]

Out[143]= 16 - 3 q3 - 3 (q1 + q2 + q3)

In[144]:= Solve[{16 - 3 q1 - 3 (q1 + q2 + q3) == 0,
            16 - 3 q2 - 3 (q1 + q2 + q3) == 0, 16 - 3 q3 - 3 (q1 + q2 + q3) == 0}, {q1, q2, q3}]
```

Out[144]= $\{\{q1 \to \frac{4}{3}, q2 \to \frac{4}{3}, q3 \to \frac{4}{3}\}\}$

■ (c)

```
In[145]:= Clear[p, Q, q1, q2, pi1, pi2]

In[146]:= Q = q1 + q2
          p = 20 - 3 Q
          pi1 = p q1 - 4 q1^2
          pi2 = p q2 - 4 q2^2

Out[146]= q1 + q2

Out[147]= 20 - 3 (q1 + q2)

Out[148]= -4 q1^2 + q1 (20 - 3 (q1 + q2))

Out[149]= -4 q2^2 + q2 (20 - 3 (q1 + q2))

In[150]:= D[pi1, q1]

Out[150]= 20 - 11 q1 - 3 (q1 + q2)

In[151]:= D[pi2, q2]

Out[151]= 20 - 11 q2 - 3 (q1 + q2)

In[152]:= Solve[{20 - 11 q1 - 3 (q1 + q2) == 0, 20 - 11 q2 - 3 (q1 + q2) == 0}, {q1, q2}]
```

Out[152]= $\{\{q1 \to \frac{20}{17}, q2 \to \frac{20}{17}\}\}$

■ (d)

```
In[153]:= Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]

In[154]:= Q = q1 + q2 + q3
          p = 20 - 3 Q
          pi1 = p q1 - 4 q1^2
          pi2 = p q2 - 4 q2^2
          pi3 = p q3 - 4 q3^2

Out[154]= q1 + q2 + q3

Out[155]= 20 - 3 (q1 + q2 + q3)

Out[156]= -4 q1^2 + q1 (20 - 3 (q1 + q2 + q3))

Out[157]= -4 q2^2 + q2 (20 - 3 (q1 + q2 + q3))

Out[158]= -4 q3^2 + q3 (20 - 3 (q1 + q2 + q3))
```

```
In[159]:= D[pi1, q1]

Out[159]= 20 - 11 q1 - 3 (q1 + q2 + q3)

In[160]:= D[pi2, q2]

Out[160]= 20 - 11 q2 - 3 (q1 + q2 + q3)

In[161]:= D[pi3, q3]

Out[161]= 20 - 11 q3 - 3 (q1 + q2 + q3)

In[162]:= Solve[{20 - 11 q1 - 3 (q1 + q2 + q3) == 0,
            20 - 11 q2 - 3 (q1 + q2 + q3) == 0, 20 - 11 q3 - 3 (q1 + q2 + q3) == 0}, {q1, q2, q3}]

Out[162]= {{q1 → 1, q2 → 1, q3 → 1}}
```

■ (ii)

Here we are comparing models (a) and (b).

```
In[163]:= Clear[p, Q, q1, q2, pi1, pi2]

In[164]:= Solve[16 - 3 q1 - 3 (q1 + q2) == 0, q1]
```

Out[164]= $\{\{q1 \to \frac{1}{6}(16 - 3\,q2)\}\}$

```
In[165]:= Solve[16 - 3 q2 - 3 (q1 + q2) == 0, q2]
```

Out[165]= $\{\{q2 \to \frac{1}{6}(16 - 3\,q1)\}\}$

The two reaction functions are therefore:

$$q_1(t) = \frac{8}{3} - \frac{1}{2}\,q_2(t-1)$$

$$q_2(t) = \frac{8}{3} - \frac{1}{2}\,q_1(t-1)$$

If firm 1 is the monopolist then the initial point is (8 / 3, 0).

```
In[166]:= RSolve[{q1[t] == (8 / 3) - (1 / 2) q2[t - 1],
            q2[t] == (8 / 3) - (1 / 2) q1[t - 1], q1[0] == 8 / 3, q2[0] == 0}, {q1[t], q2[t]}, t]
```

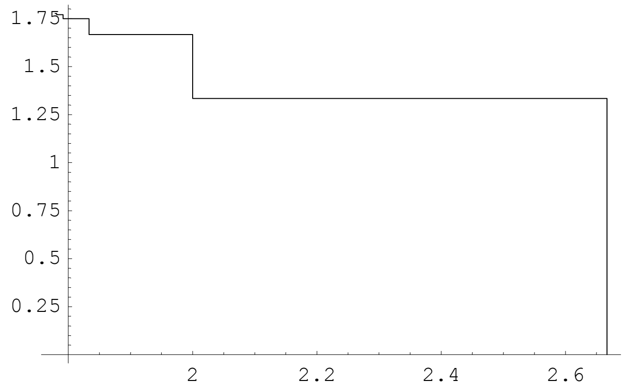
Out[166]= $\{\{q1[t] \to \frac{1}{9}\,2^{2-t}\,(3 - (-1)^t + 2^{2+t}),\ q2[t] \to \frac{1}{9}\,2^{2-t}\,(-3 - (-1)^t + 2^{2+t})\}\}$

In[167]:= $q1[t\_] := \frac{1}{9}\,2^{2-t}\,(3 - (-1)^t + 2^{2+t})$

In[168]:= $q2[t\_] := \frac{1}{9}\,2^{2-t}\,(-3 - (-1)^t + 2^{2+t})$

```
In[169]:= points8a = Table[{q1[t], q2[t]}, {t, 0, 10}];
```

In[170]:= **ListPlot[points8a, PlotJoined → True, AxesOrigin → {1.8, 0}];**



In[171]:= **Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]**

In[172]:= **Solve[16 - 3 q1 - 3 (q1 + q2 + q3) == 0, q1]**

Out[172]= $\{\{q1 \to \frac{1}{6}(16 - 3\,q2 - 3\,q3)\}\}$

In[173]:= **Solve[16 - 3 q2 - 3 (q1 + q2 + q3) == 0, q2]**

Out[173]= $\{\{q2 \to \frac{1}{6}(16 - 3\,q1 - 3\,q3)\}\}$

In[174]:= **Solve[16 - 3 q3 - 3 (q1 + q2 + q3) == 0, q3]**

Out[174]= $\{\{q3 \to \frac{1}{6}(16 - 3\,q1 - 3\,q2)\}\}$

The three reaction functions are therefore:

$$q_1(t) = \frac{8}{3} - \frac{1}{2}\,q_2(t-1) - \frac{1}{2}\,q_3(t-1)$$

$$q_2(t) = \frac{8}{3} - \frac{1}{2}\,q_1(t-1) - \frac{1}{2}\,q_3(t-1)$$

$$q_3(t) = \frac{8}{3} - \frac{1}{2}\,q_1(t-1) - \frac{1}{2}\,q_2(t-1)$$

If firm 1 is the monopolist then the initial point is (8/3, 0, 0).

In[175]:=
```
RSolve[{q1[t] == (8 / 3) - (1 / 2) q2[t - 1] - (1 / 2) q3[t - 1],
   q2[t] == (8 / 3) - (1 / 2) q1[t - 1] - (1 / 2) q3[t - 1],
   q3[t] == (8 / 3) - (1 / 2) q1[t - 1] - (1 / 2) q2[t - 1],
   q1[0] == 8 / 3, q2[0] == 0, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]
```

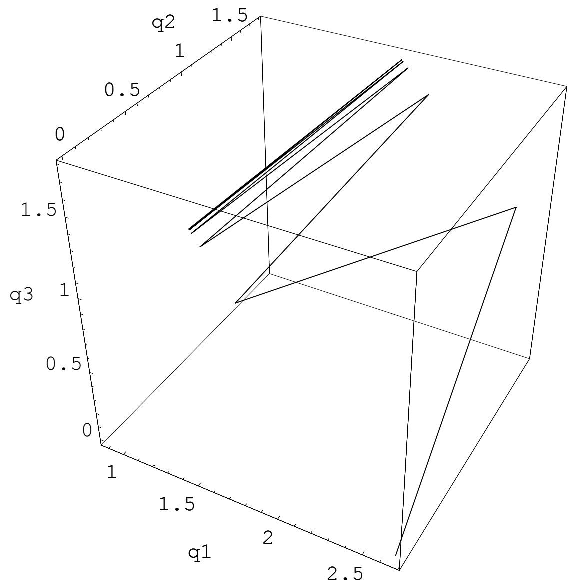
Out[175]= $\{\{q1[t] \to -\frac{1}{9}\,2^{2-t}\,(-4 + (-2)^t - 3\,2^t),$
$q2[t] \to \frac{4}{9}\,(3 - 2^{-t}\,(2 + (-2)^t)),\ q3[t] \to \frac{4}{9}\,(3 - 2^{-t}\,(2 + (-2)^t))\}\}$

In[176]:= **q1[t_] :=** $-\frac{1}{9}\,2^{2-t}\,(-4 + (-2)^t - 3\,2^t)$

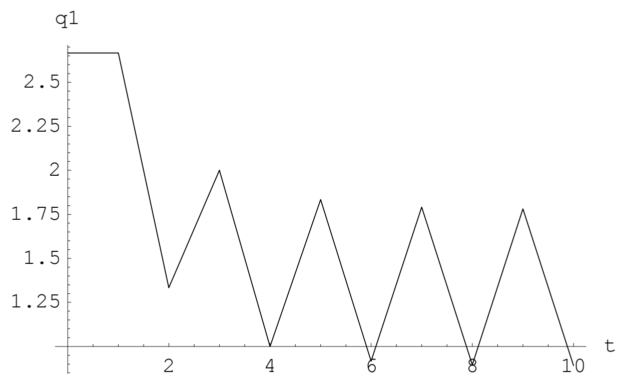
In[177]:= **q2[t_] :=** $\frac{4}{9}\,(3 - 2^{-t}\,(2 + (-2)^t))$

In[178]:= **q3[t_] :=** $\frac{4}{9}\,(3 - 2^{-t}\,(2 + (-2)^t))$

```
In[179]:= points8b = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];

    General::spell1 :
     Possible spelling error: new symbol name "points8b" is similar to existing symbol "points8a".

In[180]:= ScatterPlot3D[points8b, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];
```




```
In[181]:= points8b1 = Table[{t, q1[t]}, {t, 0, 10}];

In[182]:= ListPlot[points8b1, PlotJoined → True, AxesLabel → {"t", "q1"}];
```



When $n = 2$ the system was stable. However, for $n = 3$ the system soon begins to oscillate. Here we show it only for q1 but it applies equally to q2 and q3, which is apparent from the presence of $(-1)^t$ in the solutions.

■ (iii)

Part (ii) already compares (a) and (b) and so here we consider just (c) and (d).

```
In[183]:= Clear[p, Q, q1, q2, pi1, pi2]

In[184]:= Solve[20 - 11 q1 - 3 (q1 + q2) == 0, q1]
```

Out[184]= $\left\{\left\{q1 \to \frac{1}{14}\ (20 - 3\ q2)\right\}\right\}$

```
In[185]:= Solve[20 - 11 q2 - 3 (q1 + q2) == 0, q2]
```

Out[185]= $\left\{\left\{q2 \to \frac{1}{14}\ (20 - 3\, q1)\right\}\right\}$

Hence the reaction curves are:

$$q_1(t) = \tfrac{10}{7} - \tfrac{3}{14}\, q_2(t-1)$$

$$q_2(t) = \tfrac{10}{7} - \tfrac{3}{14}\, q_1(t-1)$$

If firm 1 is a monopoly then the initial point is (10 / 7, 0).

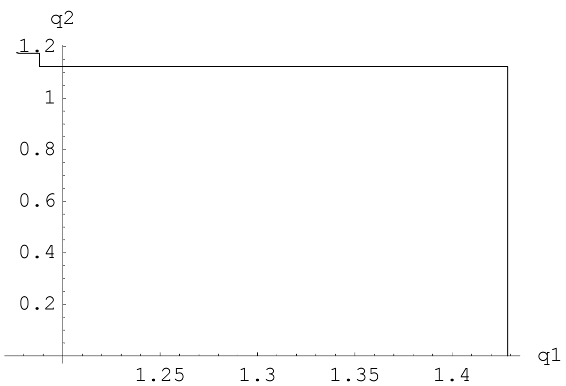
```
In[186]:= RSolve[{q1[t] == (10 / 7) - (3 / 14) q2[t - 1],
            q2[t] == (10 / 7) - (3 / 14) q1[t - 1], q1[0] == 10 / 7, q2[0] == 0}, {q1[t], q2[t]}, t]
```

Out[186]= $\left\{\left\{q1[t] \to \frac{5}{17}\, 2^{-t}\, 7^{-1-t}\, (17\, 3^t + 2^{2+t}\, 7^{1+t} - 11\, 3^t\, e^{i \pi t}),\right.\right.$
$\left.\left. q2[t] \to \frac{5}{119}\left(28 - 11\left(-\frac{3}{14}\right)^t - 17\left(\frac{3}{14}\right)^t\right)\right\}\right\}$

In[187]:= $q1[t\_] := \frac{5}{17}\, 2^{-t}\, 7^{-1-t}\, (17\, 3^t + 2^{2+t}\, 7^{1+t} - 11\, 3^t\, e^{i \pi t})$

In[188]:= $q2[t\_] := \frac{5}{119}\left(28 - 11\left(-\frac{3}{14}\right)^t - 17\left(\frac{3}{14}\right)^t\right)$

```
In[189]:= points8c = Table[{q1[t], q2[t]}, {t, 0, 10}];

      General::spell :
       Possible spelling error: new symbol name "points8c" is similar to existing symbols {points8a, points8b}.

In[190]:= ListPlot[points8c, PlotJoined → True, AxesLabel → {"q1", "q2"}];
```




```
In[191]:= Clear[p, Q, q1, q2, q3, pi1, pi2, pi3]

In[192]:= Solve[20 - 11 q1 - 3 (q1 + q2 + q3) == 0, q1]
```

Out[192]= $\left\{\left\{q1 \to \frac{1}{14}\ (20 - 3\, q2 - 3\, q3)\right\}\right\}$

```
In[193]:= Solve[20 - 11 q2 - 3 (q1 + q2 + q3) == 0, q2]
```

Out[193]= $\left\{\left\{q2 \to \frac{1}{14}\ (20 - 3\, q1 - 3\, q3)\right\}\right\}$

```
In[194]:= Solve[20 - 11 q3 - 3 (q1 + q2 + q3) == 0, q3]
```

Out[194]= $\left\{\left\{q3 \to \frac{1}{14}\ (20 - 3\, q1 - 3\, q2)\right\}\right\}$

Hence the reaction curves are:

$$q_1(t) = \tfrac{10}{7} - \tfrac{3}{14}\, q_2(t-1) - \tfrac{3}{14}\, q_3(t-1)$$

$$q_2(t) = \tfrac{10}{7} - \tfrac{3}{14}\, q_{21}(t-1) - \tfrac{3}{14}\, q_3(t-1)$$

$$q_3(t) = \tfrac{10}{7} - \tfrac{3}{14}\, q_1(t-1) - \tfrac{3}{14}\, q_2(t-1)$$

With firm 1 the monopolist, then the initial point is (10/7, 0, 0).

```
In[195]:= RSolve[{q1[t] == (10 / 7) - (3 / 14) q2[t - 1] - (3 / 14) q3[t - 1],
            q2[t] == (10 / 7) - (3 / 14) q1[t - 1] - (3 / 14) q3[t - 1],
            q3[t] == (10 / 7) - (3 / 14) q1[t - 1] - (3 / 14) q2[t - 1],
            q1[0] == 10 / 7, q2[0] == 0, q3[0] == 0}, {q1[t], q2[t], q3[t]}, t]
```

Out[195]= $\left\{\left\{q1[t] \to -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t - 20\, 3^t - 3\, 2^t\, 7^{1+t}\right),\right.\right.$
$q2[t] \to -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t + 10\, 3^t - 3\, 2^t\, 7^{1+t}\right),$
$\left.\left. q3[t] \to -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t + 10\, 3^t - 3\, 2^t\, 7^{1+t}\right)\right\}\right\}$

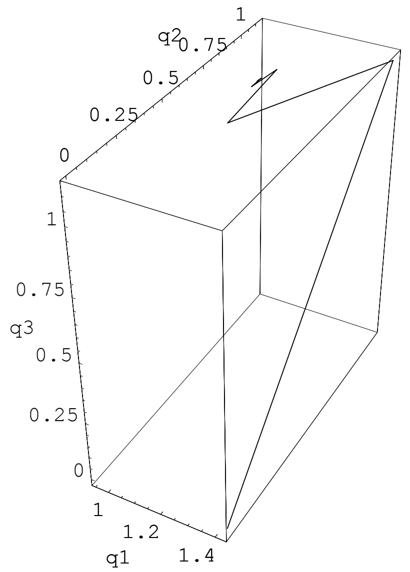
In[196]:= $q1[t\_] := -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t - 20\, 3^t - 3\, 2^t\, 7^{1+t}\right)$

In[197]:= $q2[t\_] := -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t + 10\, 3^t - 3\, 2^t\, 7^{1+t}\right)$

In[198]:= $q3[t\_] := -\frac{1}{3}\, 2^{-t}\, 7^{-1-t}\, \left(11\, (-6)^t + 10\, 3^t - 3\, 2^t\, 7^{1+t}\right)$

```
In[199]:= points8d = Table[{q1[t], q2[t], q3[t]}, {t, 0, 10}];

        General::spell : Possible spelling error: new symbol
           name "points8d" is similar to existing symbols {points8a, points8b, points8c}.
```

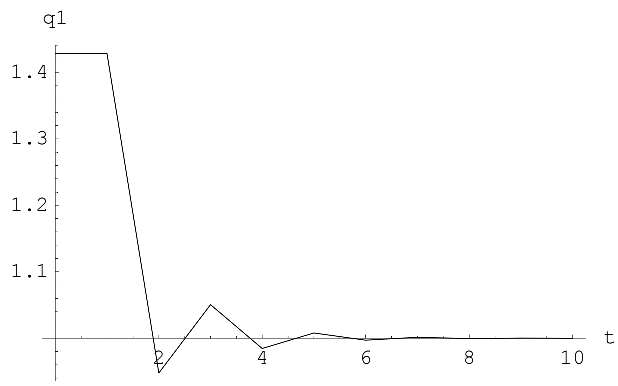
In[200]:= **ScatterPlot3D[points8d, PlotJoined → True, AxesLabel → {"q1", "q2", "q3"}];**



In[201]:= **points8d1 = Table[{t, q1[t]}, {t, 0, 10}];**

```
General::spell1 :
 Possible spelling error: new symbol name "points8d1" is similar to existing symbol "points8b1".
```

In[202]:= **ListPlot[points8d1, PlotJoined → True, AxesLabel → {"t", "q1"}];**



When marginal costs are not constant then the system is more stable and equilibrium is reached sooner. The oscillatory pattern observed for $n = 3$ under constant marginal costs now disappears and the system is stable.

à Question 9

In[203]:= **Clear[p, Q, q1, q2, pi1, pi2]**

■ (i)

The Cournot solution is the same as for model 8(i)(a), namely $(\frac{16}{9}, \frac{16}{9})$.

■ (ii)

In question 8(ii) we derived the reaction functions, which in their continuous dynamic form are:

$x_1(t) = \frac{8}{3} - \frac{1}{2} q_2(t)$

$x_2(t) = \frac{8}{3} - \frac{1}{2} q_1(t)$

Hence

$$q_1'(t) = 0.2\,(\tfrac{8}{3} - \tfrac{1}{2} q_2(t) - q_1(t))$$

$$q_2'(t) = 0.2\,(\tfrac{8}{3} - \tfrac{1}{2} q_1(t) - q_2(t))$$

```
In[204]:= Simplify[0.2 ((8 / 3) - (1 / 2) q2[t] - q1[t])]

Out[204]= 0.533333 - 0.2 q1[t] - 0.1 q2[t]

In[205]:= Simplify[0.2 ((8 / 3) - (1 / 2) q1[t] - q2[t])]

Out[205]= 0.533333 - 0.1 q1[t] - 0.2 q2[t]
```

The matrix of the system is therefore

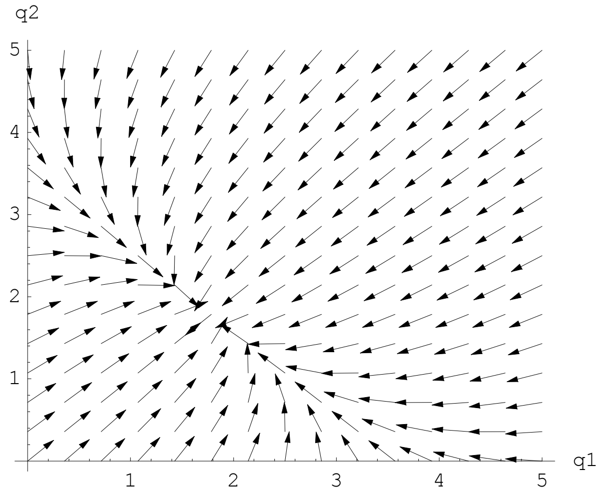
```
In[206]:= mA = {{-0.2, -0.1}, {-0.1, -0.2}}

Out[206]= {{-0.2, -0.1}, {-0.1, -0.2}}

In[207]:= Tr[mA]

Out[207]= -0.4

In[208]:= Det[mA]

Out[208]= 0.03

In[209]:= Tr[mA]^2 - 4 Det[mA]

Out[209]= 0.04
```

Hence the equilibrium is dynamically stable.

■ (iii)

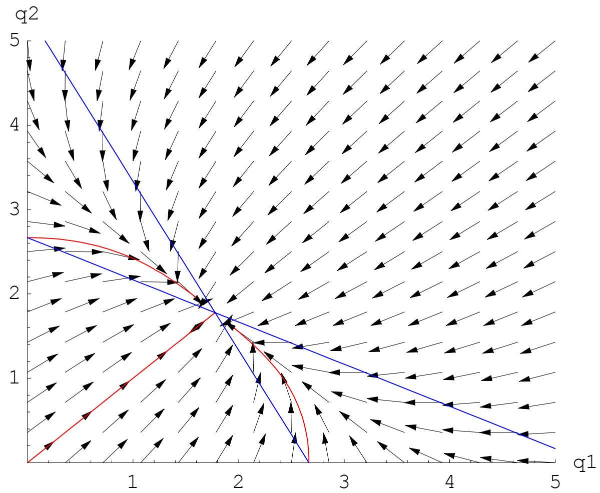
The initial points are (8/3, 0) for firm 1 a monopolist and (0, 8/3) for firm 2 being a monopolist and (0, 0) indicating both firms are to enter the industry.

```
In[210]:= arrows9 = PlotVectorField[{0.2 ((8 / 3) - (1 / 2) q2 - q1), 0.2 ((8 / 3) - (1 / 2) q1 - q2)},
             {q1, 0, 5}, {q2, 0, 5}, Axes → True, AxesLabel → {"q1", "q2"},
             ScaleFunction → (1 &), AspectRatio → 0.8];
```




```
In[211]:= path91 = NDSolve[{q1'[t] == 0.2 ((8 / 3) - (1 / 2) q2[t] - q1[t]), q2'[t] ==
              0.2 ((8 / 3) - (1 / 2) q1[t] - q2[t]), q1[0] == 8 / 3, q2[0] == 0}, {q1, q2}, {t, 0, 20}]

Out[211]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}

In[212]:= path92 = NDSolve[{q1'[t] == 0.2 ((8 / 3) - (1 / 2) q2[t] - q1[t]), q2'[t] ==
              0.2 ((8 / 3) - (1 / 2) q1[t] - q2[t]), q1[0] == 0, q2[0] == 8 / 3}, {q1, q2}, {t, 0, 20}]

Out[212]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}

In[213]:= path93 = NDSolve[{q1'[t] == 0.2 ((8 / 3) - (1 / 2) q2[t] - q1[t]), q2'[t] ==
              0.2 ((8 / 3) - (1 / 2) q1[t] - q2[t]), q1[0] == 0, q2[0] == 0}, {q1, q2}, {t, 0, 20}]

Out[213]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}

In[214]:= traj91 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path91], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[215]:= traj92 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path92], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[216]:= traj93 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path93], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[217]:= lines9 = Plot[{16 / 3 - 2 q1, 8 / 3 - (1 / 2) q1}, {q1, 0, 5}, PlotRange → {0, 5},
             PlotStyle → {{RGBColor[0, 0, 1]}, {RGBColor[0, 0, 1]}, {Thickness[.02]}},
             DisplayFunction → Identity];
```

```
In[218]:= Show[arrows9, traj91, traj92, traj93, lines9, PlotRange → {{0, 5}, {0, 5}}];
```



The diagram reveals the stability of the Cournot solution.

à Question 10

```
In[219]:= Clear[p, Q, q1, q2, pi1, pi2, mA]
```

■ (i)

The Cournot solution is the same as for model 8(i)(c), namely $(\frac{20}{17}, \frac{20}{17})$.

■ (ii)

In question 8(iii) we derived the reaction functions, which in their continuous dynamic form are:

$$x_1(t) = \frac{10}{7} - \frac{3}{14}\, q_2(t)$$

$$x_2(t) = \frac{10}{7} - \frac{3}{14}\, q_1(t)$$

Hence

$$q_1'(t) = 0.2\,(\frac{10}{7} - \frac{3}{14}\, q_2(t) - q_1(t))$$

$$q_2'(t) = 0.2\,(\frac{10}{7} - \frac{3}{14}\, q_1(t) - q_2(t))$$

```
In[220]:= Simplify[0.2 ((10 / 7) - (3 / 14) q2[t] - q1[t])]

Out[220]= 0.285714 - 0.2 q1[t] - 0.0428571 q2[t]

In[221]:= Simplify[0.2 ((10 / 7) - (3 / 14) q1[t] - q2[t])]

Out[221]= 0.285714 - 0.0428571 q1[t] - 0.2 q2[t]
```

The matrix of the system is therefore

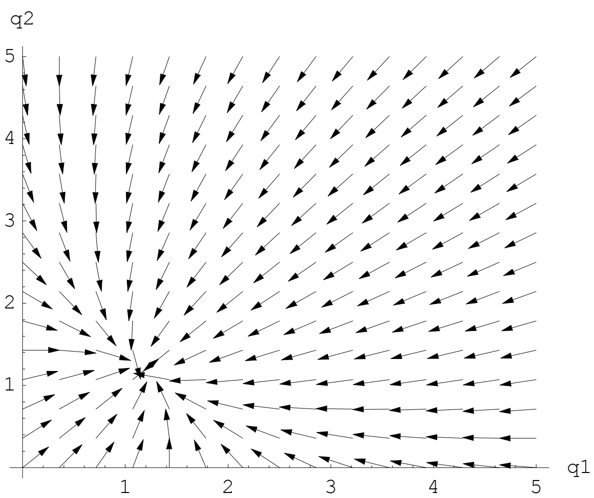
```
In[222]:= mA = {{-0.2, -0.0428571}, {-0.0428571, -0.2}}

Out[222]= {{-0.2, -0.0428571}, {-0.0428571, -0.2}}

In[223]:= Tr[mA]

Out[223]= -0.4

In[224]:= Det[mA]

Out[224]= 0.0381633

In[225]:= Tr[mA]^2 - 4 Det[mA]

Out[225]= 0.00734692
```

Hence the equilibrium is dynamically stable.

■ (iii)

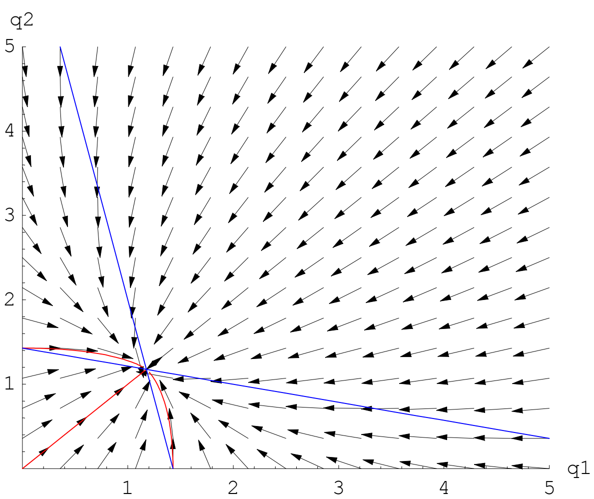
The initial points are ($\frac{10}{7}$, 0) for firm 1 a monopolist and (0, $\frac{10}{7}$) for firm 2 being a monopolist and (0, 0) indicating both firms are to enter the industry.

```
In[226]:= arrows10 =
            PlotVectorField[{0.2 ((10 / 7) - (3 / 14) q2 - q1), 0.2 ((10 / 7) - (3 / 14) q1 - q2)},
             {q1, 0, 5}, {q2, 0, 5}, Axes → True, AxesLabel → {"q1", "q2"},
             ScaleFunction → (1 &), AspectRatio → 0.8];
```




```
In[227]:= path101 = NDSolve[{q1'[t] == 0.2 ((10 / 7) - (3 / 14) q2[t] - q1[t]),
              q2'[t] == 0.2 ((10 / 7) - (3 / 14) q1[t] - q2[t]),
              q1[0] == 10 / 7, q2[0] == 0}, {q1, q2}, {t, 0, 20}]

Out[227]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}
```

```
In[228]:= path102 = NDSolve[{q1'[t] == 0.2 ((10 / 7) - (3 / 14) q2[t] - q1[t]),
             q2'[t] == 0.2 ((10 / 7) - (3 / 14) q1[t] - q2[t]),
             q1[0] == 0, q2[0] == 10 / 7}, {q1, q2}, {t, 0, 20}]

Out[228]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}

In[229]:= path103 = NDSolve[{q1'[t] == 0.2 ((10 / 7) - (3 / 14) q2[t] - q1[t]), q2'[t] ==
              0.2 ((10 / 7) - (3 / 14) q1[t] - q2[t]), q1[0] == 0, q2[0] == 0}, {q1, q2}, {t, 0, 20}]

Out[229]= {{q1 → InterpolatingFunction[{{0., 20.}}, <>],
            q2 → InterpolatingFunction[{{0., 20.}}, <>]}}

In[230]:= traj101 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path101], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[231]:= traj102 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path102], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[232]:= traj103 = ParametricPlot[
             Evaluate[{q1[t], q2[t]} /. path103], {t, 0, 20}, DisplayFunction → Identity,
             PlotStyle → {{RGBColor[1, 0, 0]}, {Thickness[.02]}}];

In[233]:= lines10 = Plot[{20 / 3 - (14 / 3) q1, 10 / 7 - (3 / 14) q1}, {q1, 0, 5}, PlotRange → {0, 5},
             PlotStyle → {{RGBColor[0, 0, 1]}, {RGBColor[0, 0, 1]}, {Thickness[.02]}},
             DisplayFunction → Identity];

In[234]:= Show[arrows10, traj101, traj102, traj103, lines10, PlotRange → {{0, 5}, {0, 5}}];
```



The diagram reveals the stability of the Cournot solution for the chosen initial points.

```
In[235]:= DSolve[{q1'[t] == 0.2 ((10 / 7) - (3 / 14) q2[t] - q1[t]),
            q2'[t] == 0.2 ((10 / 7) - (3 / 14) q1[t] - q2[t]),
            q1[0] == q10, q2[0] == q20}, {q1[t], q2[t]}, t]
```

Out[235]= $\{\{q1[t] \to 0.0294118\, e^{-0.242857\, t}$
$(-40. + 40.\, e^{0.242857\, t} + 17.\, q10 + 17.\, e^{0.0857143\, t}\, q10 + 17.\, q20 - 17.\, e^{0.0857143\, t}\, q20),$
$q2[t] \to 0.0294118\, e^{-0.242857\, t}$
$(-40. + 40.\, e^{0.242857\, t} + 17.\, q10 - 17.\, e^{0.0857143\, t}\, q10 + 17.\, q20 + 17.\, e^{0.0857143\, t}\, q20)\}\}$

Since all coefficients of q10 and q20 involve $e^{-0.242857\,t}$, $e^{0.242857\,t}$, or $e^{0.0857143\,t}$ then the system converges on the Cournot equilibrium regardless of the initial values.

à Questions 11-15

These questions are more easily done with a spreadsheet and are therefore not undertaken here with *Mathematica*.