ANALYTICAL AND COMPUTER EXERCISES

MACROECONOMIC MODELS

Solow growth model
Ramsey-Cass-Koopmans model
Diamond model

for use with Excel and iDMC software
iDMC available http://code.google.com/p/idmc/

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THE SOLOW GROWTH MODEL

EXERCISE SET 1

PART A.
Exercises with Excel

I. Calculate the evolution of the population, beginning with an initial population of 100, for different values of the proportional growth rate: n=0.01, 0.005, 0, -0.005, -0.01, from 0 to 20 periods, for period 100, for period 1000. Comment on the effects of changing the proportional growth rate. Example:

POPULATION GROWTH under hypotheses for n

N(t) = N(0)e^nt = N(0)*EXP(n*t)

	N(0)=100				
time	growthrate				
t	n=0.01	n=0.005	n=0	n=-0.005	n=-0.01
0	100				
1	101.005				
2					

II. For a given set of parameter values, calculate the evolution over 20 periods of the per-capita variables k, y, c, i, $(\delta+n)k$, Δk. Comment on this evolution. Example:

EVOLUTION OF MACROVARIABLES (per capita terms)

y=f(k)=k^alpha, c=MPC*y, i=s*f(k), k(0)=4, y(0)=2

t	k	y	c	i	(delta+n)k	change k
0	4	2	1.4	0.6	0.44	0.16
1	4.16					
2						
3						

PART B.
Exercises with iDMC using parameter and initial values of Exercise A.II . The model is called Solow. See appendix for instructions on getting started with iDMC.

File - Open model – choose solow from economic directory
File - New plot - trajectory
Inital values k(o)=4
Parameter values, choose one set from the following

1. α=0.5 s=0.2 d= 0.04 n=0.01 g=0.01; with α=0.25
2. α=0.5 s=0.3 d= 0.04 n=0.01 g=0.01; with α=0.25

3. α=0.5 s=0.1 d= 0.04 n=0.01 g=0.01 ; with α=0.25
4. α=0.5 s=0.2 d= 0.03 n=0.01 g=0.01; with α=0.25
5. α=0.5 s=0.2 d= 0.02 n=0.01 g=0.01 ; with α=0.25
6. α=0.5 s=0.2 d= 0.0 n=0.01 g=0.01 ; with α=0.25
7. α=0.5 s=0.2 d= 0.04 n=0.005 g=0.01 ; with α=0.25
8. α=0.5 s=0.2 d= 0.04 n=0.0 g=0.01 ; with α=0.25
9. α=0.5 s=0.2 d= 0.04 n=-0.005 g=0.01 ; with α=0.25
10. α=0.5 s=0.2 d= 0.04 n=0.01 g=0.02; with α=0.25
11. α=0.5 s=0.2 d= 0.04 n=0.01 g=0.005; with α=0.25
12. α=0.5 s=0.2 d= 0.04 n=0.01 g=0.0; with α=0.25

Algorithm stepsize 0.2
transients 0
iterations 15000 (3000 periods)
Auto ranges 500

Click on Start. Simulate a trajectory for your parameter set first with alpha 0.5 and then with alpha 0.25.

To see that the steady state is independent of the initial value we use the Variation routine to simultaneously plot trajectories from different initial values. Click Reset. Under Plot options, click on Variation. Only the initial value will be changed. In the box next to k(0) put 1 (for example), which means at every run the initial value will be increased by 1 (giving initial values of 4, 5, etc.). Put 0 in the variation boxes for all other parameters and set Variation at 10. Print the plot and repeat the exercise for the other value of alpha. Comment on these plots.

We now use Variation to simulate how the values of the balanced growth path change as one parameter is varied. For this exercise simulate and print the plot obtained by varying alpha (set variation to 0 for initial value and to 0.05 for alpha, do 6 variations using Autoranges 15000) and then simulate and print the plot obtained by varying one of the other parameters s, d or n. Remain within typical parameter ranges for consideration
$0<\alpha<0.5$, $0<s<0.3$, $0<d<0.04$, $-0.01<n<0.01$, $0<g<0.02$

APPENDIX

Idmc - interactive Dynamical Model Calculator
Software description*

The software program is a research tool for studying nonlinear dynamic systems and is available free at http://www.dss.uniud.it/nonlinear . To download the program click on the current version of iDMC in your preferred operating system.

Installation
To install iDMC you need to download Java2 Standard Edition (J2SE), version 1.4.2 or later from the java site or //www.java.com/en/download/manual.jsp

then run the installer, double clicking on the program. More instructions are on the nonlinear site.

Running iDMC
The first step is to select a model from the directory models. Next click on Plot and, for example, Trajectory. You will be presented with an interface that must be completed with values for initial conditions, parameters, number of transients, number of iterations after the transients. Clicking start the trajectory should appear. A number of graphical options are available, including plotting points (dots, big dots, connected dots), manual bounds for the variables, continue the trajectory, zoom (right click and select desired area of plot), either time evolution or state space. If you want to save a particular parameter set use the sample option. The print and save options are under File, the file format is .png. Click New Model to open a session with a new model. Exit to leave the program.

THE RAMSEY-CASS-KOOPMANS MODEL
EXERCISE SET 2

PART A.
Using the model equations (2.23) and (2.24) find the non-trivial steady state values for k and c, in terms of the parameters, under the assumption of a Cobb-Douglas production function. Hint: one of the steady state couples is (0,0). Use (2.23) to first find the positive valued steady state k, then (2.24) to find the positive valued steady state c.

PART B.
Exercise with Excel

Calculate the utility, marginal utility, for four different values of theta, 0, 0.5, 1, 1.5 and calculate the relative risk. Comment on these values. Example.

UTILITY, MARGINAL UTILITY, RRA and THETA

C(t)	Utility (C(t))	MARGINAL UTLITY (C(t))				RRA			
	θ=0	θ=0.5	θ=1	θ=1.5	θ=0	θ=0.5	θ=1	θ=1.5	γ
0.25	0.25	1	-1.38629	-4					
0.5	0.5								
0.75									

PART C.
I Exercise with iDMC.

Choose one set of parameter values from the list below and produce and print a graph showing trajectories with different initial values of capital (use the Variation routine described in iDMC exercise set 1). An example run follows. Draw in approximate curves for the lines with dc/dt=0 and dk/dt=0. Comment on the figure.

File - Open model – ramsey
File - New plot - trajectory
Inital values c 0.5 k 3
Parameters choose ONE set from following list

1. α=0.25 ρ=0.03 θ=0.01 n=0.01 g=0.01
2. α=0.25 ρ=0.03 θ=0.5 n=0.01 g=0.01
3. α=0.25 ρ=0.03 θ=1.0 n=0.01 g=0.01
4. α=0.25 ρ=0.03 θ=1.5 n=0.01 g=0.01
5. α=0.25 ρ=0.03 θ=1.0 n=0.01 g=0.01
6. α=0.25 ρ=0.03 θ=1.0 n=0.005 g=0.01
7. α=0.25 ρ=0.03 θ=1.0 n=0.0 g=0.01

8. α=0.25 ρ=0.03 θ=1.0 n=-0.005 g=0.01
9. α=0.25 ρ=0.05 θ=1.0 n=0.01 g=0.01
10. α=0.25 ρ=0.10 θ=1.0 n=0.01 g=0.01
11. α=0.25 ρ=0.15 θ=1.0 n=0.01 g=0.01
12. α=0.25 ρ=0.20 θ=1.0 n=0.01 g=0.01
13. α=0.25 ρ=0.03 θ= 0.04 n=0.01 g=0.0
14. α=0.25 ρ=0.03 θ= 0.04 n=0.01 g=0.015
15. α=0.25 ρ=0.03 θ= 0.04 n=0.01 g=0.02
16. α=0.25 ρ=0.03 θ= 0.04 n=0.01 g=0.025

Algorithm
stepsize 0.02 transients 0 iterations 5000
Auto ranges 5000

Axes
domain k
range c

Plot – click on Variation
c in second box put 0.1
all others put 0 in second box
Variation
10

Plot – click Manual Bounds
Horizontal min 0 max 30
Vertical min 0 max 4

II. Insert your parameter values into your results for Part A to calculate steady state values of c and k for your parameter set. Are they close to what you find in your plot?

THE DIAMOND MODEL

EXERCISE SET 3

PART A.
Assuming logarithmic utility and Cobb-Douglas production the basic equation of the model is (2.59). Select one of the parameter sets from those below and find the non-trivial steady state values for k, and discuss their stability.

PART B.
Exercises with iDMC.

I. Use the same set of parameter values as in Part A. Compare the simulated steady state value to your calculation in Part A.

model: Diamond
file - new plot -trajectory:
time evolution

parameters: choose one and use as standard set
try an initial value k near 0 and one larger than the steady state
transients 0, iterations 20

PARAMETER SET CHOICES, CHOOSE ONE

α=0.25 ρ=1 θ=1.0 n=0.01 g=0.01	α=0.30 ρ=1 θ=1.0 n=0.01 g=0.01
α=0.35 ρ=1 θ=1.0 n=0.01 g=0.01	α=0.35 ρ=10 θ=1.0 n=0.01 g=0.01
α=0.35 ρ=0 θ=1.0 n=0.01 g=0.01	α=0.35 ρ=-0.5 θ=1.0 n=0.01 g=0.01
α=0.35 ρ=1 θ=1.0 n=0.0 g=0.01	α=0.35 ρ=1 θ=1.0 n=-0.01 g=0.01
α=0.35 ρ=1 θ=1.0 n=-0.1 g=0.01	α=0.35 ρ=1 θ=1.0 n=0.1 g=0.01
α=0.30 ρ=10 θ=1.0 n=0.01 g=0.01	α=0.30 ρ=0 θ=1.0 n=0.01 g=0.01
α=0.35 ρ=1 θ=1.0 n=0.01 g=0.0	α=0.35 ρ=1 θ=1.0 n=0.01 g=0.05
α=0.35 ρ=1 θ=1.0 n=0.01 g=0.1	α=0.35 ρ=1 θ=1.0 n=0.01 g=0.2

II. Shifted and cobweb
These two routines are available for one-dimensional maps, like the Diamond model.
file - new plot - Shifted & Cobweb

Shifted. This plots the values of the variable, shifted forward k periods, on the ordinate axis against the current values on the abscissa.

order 1, transients 0, ranges, for example (0,0.3) (0,0.3)

Cobweb animation. From the Plot menu select Cobweb animation. This routine draws the kth iterate of the map, the bisector and the forward trajectory using the bisector to reflect back to the abscissa at each pass in order to determine the next iterate value from the curve.
file – new plot - cobweb and animation

order 1, transients 0, ranges, for example (0,0.3) (0,0.3)

Notice that the motion is slowed down and the speed can be adjusted by dragging on the arrow above the plot. Use the Stop button to stop the trajectory plotting.

III. Bifurcation. This routine represents the limit sets of a dynamical system as a parameter is varied.

File – new plot – bifurcation
The default setting it to plot the asymptotic behavior of the system as a single parameter value is varied over a defined range of values. Select the parameter to be varied and the minimum and maximum values to be used for that parameter. The Vertical range refers to the axis values for the limit sets of the variable which will be plotted on the ordinate axis. In order to be sure that the trajectory has reached the attractor a sufficiently large number of transients must be excluded, otherwise the plot will contain points that are not really part of the limit set. Further, to avoid misleading plots, the number of iterations should be sufficiently large so as to cover the entire limit set.

Choose a parameter and study the effects of changing its values over a reasonable range
try using transients 200, iterations 500