Problems for Chapter 2 of 'Ultra Low Power Bioelectronics'

Problem 2.1

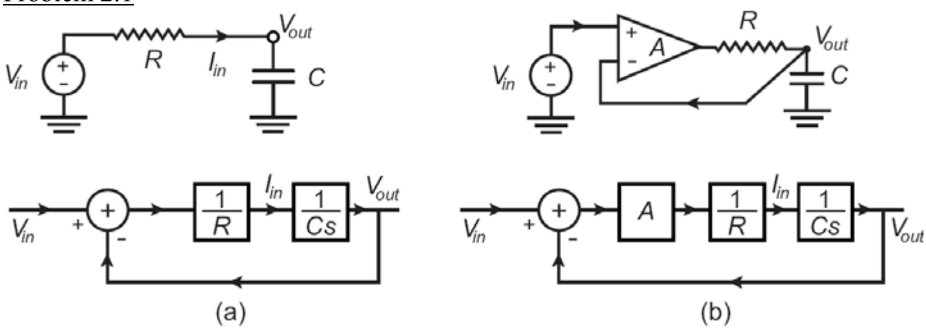

Figure P2.1 Feedback circuits.

If we define $\tau = RC$

a) Find $V_{out}(s)/V_{in}(s)$ in Figure P2.1 (a) in terms of τ

b) Find $V_{out}(s)/V_{in}(s)$ in Figure P2.1 (b) in terms of τ

c) Draw a root-locus plot that illustrates why the circuit of Figure P2.2 (b) speeds up the response of the circuit of Figure P2.1 (a). Mark the location of pole movements in terms of A.

d) If R = 10 kΩ, C = 10 µF, and A = 1000 compute the closed-loop $V_{out}(0)/V_{in}(0)$ dc response of the circuit of Figure P2.2 (b) and the closed-loop time constant of Figure P2.2 (b). Show how your answers are quantitatively consistent with the root-locus plot of part c) using the normalized magnitude rule and Black's formula.

e) By focusing on I_{in} in Figures P2.1 (a) and P2.1 (b) explain physically why the circuit of Figure P2.1 (b) is faster than that of Figure P2.1 (a).

Problem 2.2

A non-inverting amplifier is designed such that its closed-loop gain is 10 at dc when the gain of the operational amplifier used to build it (see Figure 2.3) is assumed to be infinite. If the gain of the operational amplifier is actually 10^5 at dc, what is the actual closed loop gain at dc? How much does this closed-loop gain change at dc if the operational amplifier's gain reduces to 10^4?

Problem 2.3

For each of the four loop transmissions $L(s)$ listed below, sketch the root locus as the gain k varies from 0 to ∞.

a) $$L(s) = \frac{k}{(0.333s+1)(0.5s^2+s+1)}$$

b) $$L(s) = \frac{k}{(s+2)(s+3)(s+100)}$$

c) $L(s)=\dfrac{k(s+1000)(s+2000)}{(s+1)(s+2)(s+3)}$

d) $L(s)=\dfrac{k(s^2+s+1)}{s^2}$

In each case (where applicable), determine centroids and directions of asymptotes for large s and determine points of the root locus where $k=0,\infty$.

Problem 2.4

The quadratic equation $as^2+bs+c=0$ can be converted to a root-locus problem of the form

$$1+\frac{c/a}{s\left(s+\frac{b}{a}\right)}=0$$

with c/a as the root-locus gain parameter and $s=0$ and $s=-\frac{b}{a}$ as the open-loop poles. Draw a root-locus plot that illustrates how the roots of the quadratic equation vary as c/a goes from 0 to $+\infty$. Mark points on the root-locus plot where the closed-loop poles are identical. Show, using the normalized magnitude role of root locus and Pythagoras' theorem, that the roots are located at $s=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Problem 2.5

A feedback loop has a loop transmission $L(s)=\dfrac{k}{s\left(5\times10^{-6}s+1\right)}$. Using a root-locus plot, find the magnitude of k at which the closed-loop poles of the system are

a) Both real and identical
b) Both make angles of 65° with the $j\omega$ frequency axis.

Problem 2.6

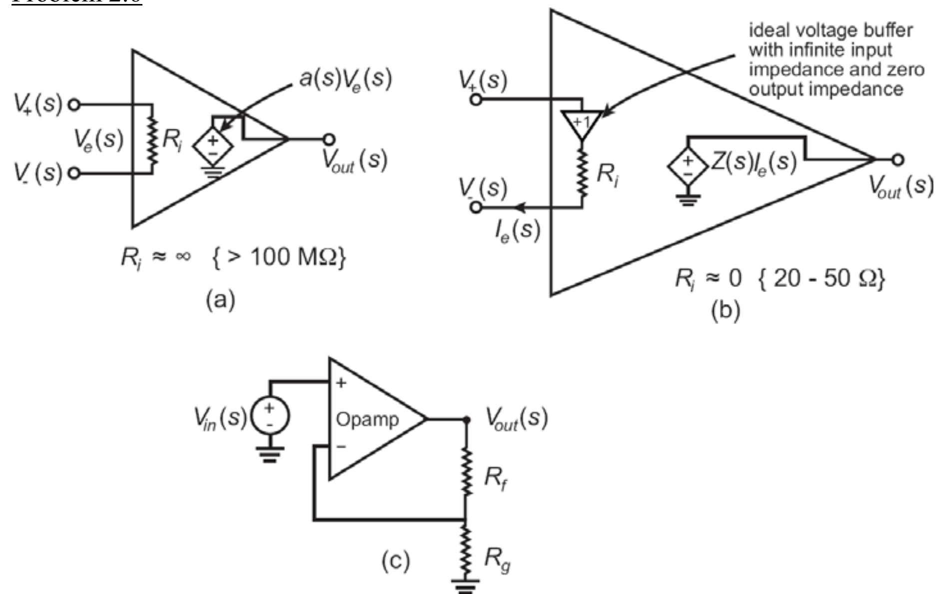

Figure P2.6 (a) shows a model for a traditional 'voltage-feedback' amplifier while (b) shows a model for a 'current-feedback' operational amplifier. Both op amps are used in the non-inverting amplifier circuit of (c) to provide voltage gain.

a) Draw a feedback block diagram that illustrates, why in both circuits, if either $|a(s)|$ or $|Z(s)|$ are sufficiently large, $V_{out}(s)/V_{in}(s) = \dfrac{R_f + R_g}{R_g}$ for all s.

   Assume $R_i = \infty$ in Figure P2.6 (a) and that $R_i = 0$ in Figure P2.6 (b) for all the following parts.

b) If R_f is fixed and R_g varies such that the closed-loop gain varies, show that $L(s)$ is invariant with R_g if the op amp of Figure P2.6 (b) is used in Figure P2.6(c) but not if the op amp of Figure P2.6 (a) is used in Figure P2.6 (c).

c) If $R_i = 0$ in Figure P2.6 (b), what is the 'R_g-invariant' loop transmission of Figure P2.6 (c) when such an ideal current-feedback operational amplifier is used to construct it?

d) Explain intuitively why current-feedback operation results in loop transmission that is invariant with R_g but voltage-feedback operation does not.

Problem 2.7

The piezoelectric electromechanical amplification system in the biological inner ear or cochlea can be modeled by the negative-feedback loop transmission

$$L(s) = \frac{k}{\left(10^{-3}s+1\right)\left(2.56\times10^{-10}s^2+2.24\times10^{-5}s+1\right)}$$

in a certain range of frequencies. Draw a root-locus plot that illustrates how the closed-loop amplification system's poles change with increasing piezoelectric gain k. From this plot explain why increasing k provides both speedup and increased amplification of frequencies near 10 kHz in the system. Estimate the value of k at which the system is just unstable.

Problem 2.8

For two scalars, $\frac{z_1 z_2}{z_1 + z_2} \simeq z_2$ if $|z_1| \gg |z_2|$. The insensitivity of a negative feedback loop to the forward path gain may be viewed as the vector generalization of this concept:

$$\frac{a(s)}{1+a(s)f(s)} = \frac{a(s)\frac{1}{f(s)}}{a(s)+\frac{1}{f(s)}} = \frac{1}{f(s)} \quad \text{if } |a(s)| \gg \frac{1}{|f(s)|},$$

where $a(s)$ and $\frac{1}{f(s)}$ are complex numbers represented as vectors on the complex plane.

a) If z_1 and z_2 are two complex numbers represented as vectors, then prove that the following construction yields $\measuredangle\left(\frac{z_1 z_2}{z_1 + z_2}\right)$.

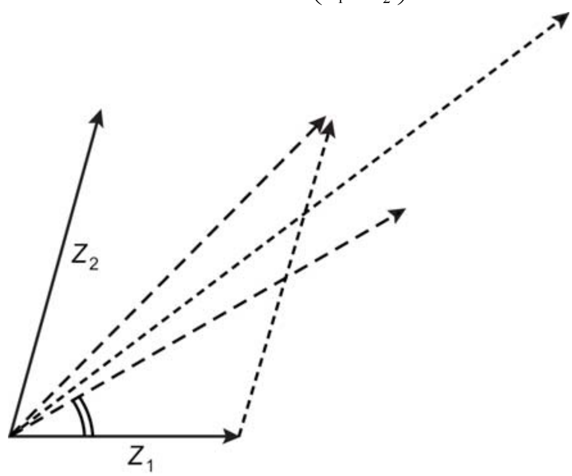

Figure P2.8 (a) z_1 and z_2 vectors.

i.e., the angular direction of $\frac{z_1 z_2}{z_1 + z_2}$ is obtained by reflecting $z_1 + z_2$ across the line that is the angular bisector of z_1 and z_2. (Hints: 1) set $z_1 = r_1 e^{j\theta_1}$ and

$z_2 = r_2 e^{j\theta_2}$; 2) You can rotate both vectors by θ_2 and rotate the answer by $-\theta_2$ without loss of generality.)

b) Prove that the magnitude of $\frac{z_1 z_2}{z_1 + z_2}$ can be constructed by:

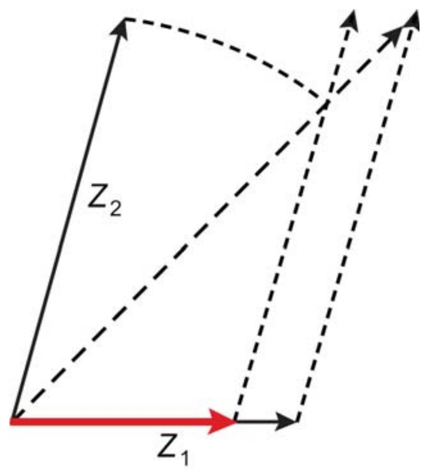

Figure P2.8 (b) z_1 and z_2 vectors.

i.e. Draw $z_1 + z_2$. Then draw an arc from z_2 to $z_1 + z_2$. Translate a copy of z_2 so that it passes through the intersection point of the arc and $z_1 + z_2$. The magnitude of the vector from z_1 to the intersection of the translated z_2 and the ray along z_1 has the same length as $\frac{z_1 z_2}{z_1 + z_2}$. (Hint: Look for ratios of similar triangles.)

c) Illustrate geometrically why

$$\frac{a(s)\frac{1}{f(s)}}{a(s)+\frac{1}{f(s)}}$$ is always slightly less than $\frac{1}{f(s)}$ in magnitude if

$\measuredangle \frac{1}{f(s)} = 0^\circ$, $\measuredangle a(s) = -90^\circ$, and $|a(s)| \gg \left|\frac{1}{f(s)}\right|$. This scenario is the typical state of affairs for most frequency-independent feedback networks used with an op-amp whose frequency response approximates that of an integrator.

Problem 2.9

Figure 24.3 illustrates the feedback loops involved in enzyme-substrate binding chemical reactions in biology. If the substrate concentration $[S]$ is fixed, find the transfer function between $E_b(s)$, the Laplace transform of $[ES]$ in Figure 24.3, and $E_t(s)$, the Laplace transform of $[E_t]$ in Figure 24.3 in terms of the k_f, k_r and $[S]$ parameters. Draw a root-locus plot that illustrates how the dynamics of the chemical reaction speeds up if $[S]$ and k_r are fixed but k_f varies.

<u>Problem 2.10</u>

In positive-feedback systems, K in Equation (2.18) is negative. Show that the new magnitude and angle conditions for the roots of a closed-loop positive-feedback system are given by an altered form of Equation (2.20)

$$\frac{1}{a(s')f(s')} = |K| \leftarrow \text{magnitude condition}$$

$$\measuredangle a(s')f(s') = 2n\pi \leftarrow \text{angle condition}$$

With these conditions, derive eight new root-locus rules that are relevant for positive-feedback systems: That is, how does s' change as $|K|$ goes from $0 \rightarrow \infty$ with the magnitude and angle conditions listed above.