Problems for Chapter 13 of 'Ultra Low Power Bioelectronics'

Problem 13.1

The circuit in Figure P13.1 yields a second-order filter. Assume all transconductance amplifiers have a linear range of V_L, zero offset, and infinite dc gain (i.e., r_o can be neglected). Each amplifier has N devices worth of white noise.

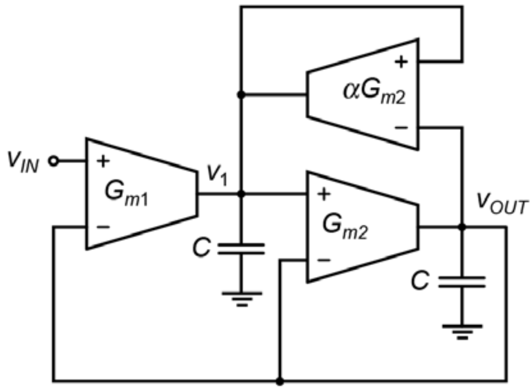


Figure P13.1: A second-order filter circuit

a) Draw a block diagram including the input-referred noise PSD of each amplifier.
b) Find the transfer function $H(s)$ from v_{IN} to v_{OUT}. What are the τ and Q of the filter? When does the filter go unstable?
c) Find the transfer function from each noise source to the output in terms of $H(s)$.
d) Find the output voltage PSD of the filter, $\langle v_o^2\rangle/\Delta f$. Simplify your algebraic expression as much as possible. Try to express all results (especially any G_m ratios) in terms of Q, α and, $|H(s)|^2$.
e) Find the total output noise of the filter, $\langle v_o^2\rangle$, and show that it reduces to the following expression: $\frac{NqV_L}{4C}\frac{1}{1-\alpha}\left[2+\frac{\alpha(1+\alpha)}{Q^2(1-\alpha)^2}\right]$
Now write down the minimum detectable signal, a_{min}, at f_{90} (the frequency at which the phase shift between the input and output of the filter is -90°).
f) Explain what causes the total output noise of this filter to vary with Q while that of the second-order filter shown in Figure 13.8 (c) does not. Given a constant Q, how does $\langle v_o^2\rangle$ vary with α (for values of α between 0 and 1)? If $\alpha = 0.5$, for what values of Q is the noise of this filter less than that of the filter in the text? If your result is non-physical, say so.
g) Find the transfer function from v_{IN} to the differential input voltage of each OTA in terms of τ, Q, and $H(s)$.
h) Using these transfer functions, find the maximum acceptable input amplitude, a_{max}, at f_{90} as a function of V_L and Q. Here a_{max} is the largest input amplitude for which the differential input voltage across each transconductor is less than V_L. For $\alpha = 0.5$, which OTA limits a_{max} as Q varies?

i) **MATLAB:** Find a_{max}/a_{min} at f_{90} as a function of Q for two filters: the one shown in Figure P13.1 and the topology shown in Figure 13.8 (c). Plot your results using MATLAB over a range of Q's from 0.1 to 100. Which filter has more dynamic range?

Problem 13.2

Consider the gyrator circuit shown in Figure 13.3 (b).

a) Assume that the dc current through the transistor is 10 μA, and that it is wide enough to remain in subthreshold. What is its small-signal transconductance g_m, assuming that $\kappa = 0.7$?

b) We want the gyrator to behave as an inductor of value $L = 100\ \mu$H. What is the required value of the time constant $\tau = RC$?

c) What is the minimum frequency at which the gyrator circuit behaves as an acceptable inductor? For the purposes of this problem, an acceptable inductor must have a quality factor $Q \geq 1$.

d) We want the circuit to behave as an acceptable inductor over at least two decades, i.e., a frequency range of 100:1. What values of R and C would you pick to meet this requirement?

Problem 13.3

An important characteristic of any filter is *group delay*. It is a measure of the time taken by a signal to propagate through the filter. Frequency-dependent group delay results in phase distortion (think of what happens to a pulse if different frequency components of the pulse are subjected to different amounts of delay). Group delay is defined as

$$\tau_g = -\left(\frac{\partial \phi}{\partial \omega}\right),$$

where ϕ is the phase of the frequency response.

Consider the canonical second-order band-pass filter. Its transfer function is given by

$$H(s) = \frac{s\tau / Q}{\tau^2 s^2 + s\tau / Q + 1}.$$

a) Show that the group delay of this filter is given by

$$\tau_g(\omega) = \frac{\tau}{Q}\left[\frac{1+\omega^2\tau^2}{\left(1-\omega^2\tau^2\right)^2 + \left(\omega\tau / Q\right)^2}\right]$$

b) Plot the group delay function for $\tau = 1$ and $Q = 1$, 2 and 4. Will the amount of phase distortion increase or decrease as Q increases?

c) What is the group delay at the center frequency $\omega_0 = 1/\tau$? Is there an intuitive physical explanation for your result?

d) What is the group delay at very low and very high frequencies ($\omega \to 0$ and $\omega \to \infty$, respectively)?

Now consider the canonical second-order resonant low-pass filter, which has a transfer function given by

$$H(s) = \frac{1}{\tau^2 s^2 + s\tau / Q + 1}$$

e) Find the group delay of the low-pass filter as a function of frequency. [Hint: you do not need to do any math. Use existing knowledge to solve the problem.]

Problem 13.4

Consider the resonant low-pass filter shown in Figure 13.8 (c).

a) Write down the transfer function $V_{out}(s)/ V_{in}(s)$ in canonical form. Show that while the transfer function between $V_{in}(s)$ and $V_{out}(s)$ is a resonant low-pass transfer function, the transfer function between $V_{in}(s)$ and $V_1(s)$ is band-pass.

b) Based on your findings in part a), what modifications would you make to this circuit in order to use it as a band-pass filter? [Hint: you don't need to modify the topology of the circuit.]

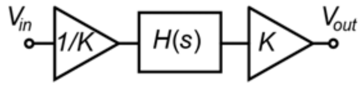
c) Find the bias current of both transconductors as a function of the center frequency and quality factor of the filter. You may assume that the linear range of both transconductors is V_L, a constant.

d) How would the linear range, noise, and dynamic range of the filter be changed by the modifications you made in part b)?

Problem 13.5

The linear range of active filters limits the largest signal swings that they can handle. A strategy for increasing the linear range of any active filter is shown in Figure P13.5 (a). The signal is passed through an attenuator of value $1/K$, where $K > 1$, before being sent through the filter, and gained up by a value K afterwards. If the attenuator and gain blocks are frequency-independent it should be clear that the overall transfer function between V_{in} and V_{out} is unchanged, but that the filter itself sees signals that are K times smaller than before. This problem considers a second-order band-pass filter that uses this linear-range extension technique. The circuit is shown in Figure P13.5 (b). It is based on the band-pass filter that you developed in Problem 13.4 (you are advised to complete Problem 13.4 before attempting this problem), but also implements the linear-range extension technique. Assume that A is a constant.

a) Explain intuitively which parts of the circuit of Figure P13.5 (b) carry out the functions of the attenuator and the amplifier shown in Figure P13.5 (a). What is the value of K?



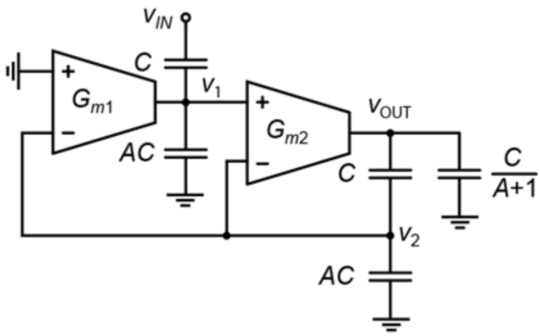


Figure P13.5: (a) A generic idea for increasing the linear range of any active filter, and (b) a second-order band-pass filter that uses this idea.

b) Will this circuit work in real life? If not, how would you modify it to make it work? [Hint: think about the dc voltage of node v_2.]

c) Let us assume that you have solved the dc-voltage problem described in part b). Draw a small-signal block diagram that describes the filter.

d) Using your block diagram, show that the transfer function $H(s) = V_{out}(s)/V_{in}(s)$ is that of a canonical second-order band-pass filter. Find the center frequency and quality factor ($1/\tau$ and Q, respectively) of this filter as a function of G_{m1}, G_{m2}, A and C.

e) Find the total bias current of both transconductors as a function of τ and Q. You may assume that the linear range of both transconductors is V_L, a constant.

f) Assume that each transconductor contributes N devices of noise, i.e., that the output shot noise PSD is $2NqI_B$, where I_B is the steady-state bias current through each noisy transistor. Show that the total output noise of the filter when flicker noise is negligible is given by

$$\overline{v_{no}^2} = \frac{NqV_L(A+1)}{2C}$$

g) What is the maximum input signal amplitude that can be handled by the filter? Explain any assumptions that you make.

h) What is the maximum signal-to-noise ratio (SNR) at the output of the filter?

i) Assume that this circuit and the simpler one analyzed in Problem 13.4 have the same transfer function and maximum allowable input amplitude. Now compare their performance based on the following metrics: output noise, maximum output SNR, layout area, and power consumption.

j) Based on the analysis in part i), what are the advantages and disadvantages of the linear-range extension technique proposed in this problem?

Problem 13.6

Consider the filter circuit shown in Figure P13.6.

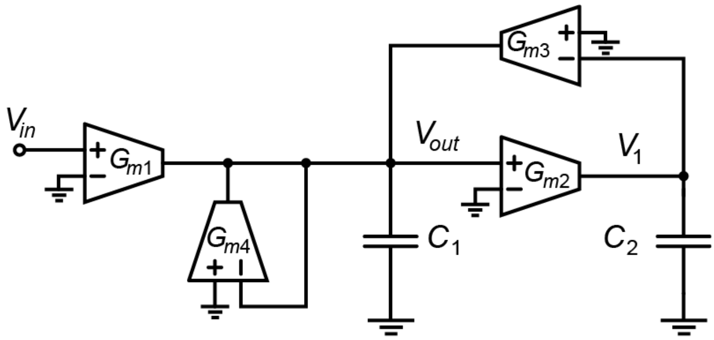


Figure P13.6: A filter circuit.

a) Compute the transfer function of the filter from $V_{in}(s)$ to $V_{out}(s)$ as a function of the transconductances and capacitances in the circuit.
b) Rewrite the transfer function found in part a) in canonical form and find the peak gain A, time constant τ, and quality factor Q in terms of circuit parameters. What type of filter is implemented by this circuit?
c) Map this circuit to a passive RLC prototype by element replacement. Use this prototype to intuitively explain the function of each transconductor.
d) Repeat parts a) and b) for the transfer function between $V_{in}(s)$ and $V_1(s)$.
e) Draw a small-signal block diagram of the filter.
f) Assume that each transconductor contributes N devices of noise, i.e., that the output shot noise PSD is $2NqI_B$, where I_B is the steady-state bias current through each noisy transistor. Also assume that the linear range of all transconductors is V_L, a constant. Now use the block diagram from part e) to show that the total output noise of the filter when flicker noise is negligible is given by

$$\overline{v_{no}^2} = \frac{NqV_L}{4\beta C}\left[1 + A + \frac{Q}{\alpha}\left(\beta + \frac{1}{\beta}\right)\right]$$

Use the following definitions while deriving this formula: $\tau = C/G_\tau$, $C_1 = C\beta$, $C_2 = C/\beta$, $G_{m2} = G_\tau\alpha$ and $G_{m3} = G_\tau/\alpha$.
g) What is the amplitude of the smallest input signal that can be detected by this filter?
h) What is the amplitude of the largest input signal that can be handled by this filter? Explain any assumptions that you make.
i) Find the total bias current consumed by the filter as a function of G_τ, V_L and dimensionless parameters.
j) **MATLAB**: Imagine that you are designing this filter for an analog circuit design company and that the required values of τ and Q are fixed by management. Now assume A, α, and β are constants and pick values that result in reasonable trade-offs between dynamic range, layout area and power consumption.

k) Repeat part j), but now allow one dimensionless circuit parameter to vary as a function of signal amplitude. Which parameter would you vary, if any? Explain how your scheme would improve performance.

Problem 13.7

This problem requires the use of a circuit simulator such as SPICE. Consider the simple first-order low-pass filter circuit shown in Figure 13.2 (c).

a) Assume that the transconductor contributes N devices of noise, i.e., that the output shot noise PSD is $2qNI_B$, where I_B is the steady-state bias current through each noisy transistor. Also assume that the linear range of the transconductor is V_L, a constant. Prove that the total output noise of the filter when flicker noise is negligible is given by

$$\overline{v_{no}^2} = \frac{NqV_L}{4C}$$

b) Implement the transconductor in two ways: using a five-transistor OTA, and a wide-linear-range (WLR) OTA (as in Figure 12.2 of Chapter 12). Bias each OTA in subthreshold, and perform a dc I-V simulation to find its linear range.

c) Use the same bias current as in part b), and set C = 1 pF. Now find the total output noise of the filter in two cases: using the five-transistor OTA, and using the WLR OTA. Verify that the formula you derived in part a) correctly predicts the noise of both filters. [Note: noise simulations are usually very accurate. If you end up with a significant discrepancy between theory and simulation, you are probably doing something wrong.]

d) Feed a sinusoidal input signals of frequency $0.2\omega_c$ into each low-pass filter circuit analyzed in part c), where ω_c is the cutoff frequency of the filter in question. Plot the total harmonic distortion (THD) at the output of both filters as the input amplitude increases between 0 and 2 V.

e) Repeat part d) for input frequencies equal to ω_c and $5\omega_c$, and explain your results.

Problem 13.8

This problem requires the use of a circuit simulator such as SPICE. Real transconductors have finite dc gain, i.e., they are not ideal current sources but have a finite output impedance R_o. One effect of finite dc gain is to lower the quality factors of practical high-Q filters relative to those obtained with ideal transconductors.

a) Consider the resonant low-pass filter shown in Figure 13.8 (c). Implement each transconductor using your favorite subthreshold OTA, and hook them up to create the filter. Assume $C_1 = C_2$ = 1 pF, and use Equation (13.18) to set the two OTA bias currents such that the predicted values of τ and Q are 0.1 ms and 1, respectively. Note that we expect the DC gain of this filter to be A_{DC} = 1 (independent of all other parameters).

b) Perform small-signal ac simulations on your filter and find the actual values of A_{DC}, τ and Q. Repeat your simulations for various values of Q between 1 and 10. Remember that you must re-bias your filter every time!

c) Plot the theoretically-expected values of A_{DC}, τ, and Q versus what you actually obtained from your simulations. Explain your results. Do the simulated values of Q and A_{DC} fit Equation (13.33)?

d) Repeat parts a), b) and c), but for the low-pass filter shown in Figure 13.12 (b).
e) Based on the results of your simulations, which circuit would you use in applications where high values of Q are required?

Problem 13.9

This problem requires the use of a circuit simulator such as SPICE. The fact that OTA I-V curves have saturating nonlinearities means that the effective transconductance decreases as the amplitude of the differential input signal increases. As a result, the center frequency and other parameters of G_m-C filters can change with signal amplitude. This problem investigates such effects.

a) Consider the band-pass filter that you analyzed in Problem 13.4. Implement each transconductor using your favorite subthreshold OTA, and hook them up to create the filter. Assume $C_1 = C_2 = 1$ pF, and use Equation (13.18) to set the two OTA bias currents such that the predicted values of τ and Q are 0.1 ms and 5, respectively. Note that we expect the peak gain of this filter to be $A_0 = 1$ (independent of all other parameters).
b) Feed sinusoidal input signals of amplitudes varying between 0 and $2V_L / Q$ into your filter, where V_L is the linear range of the OTAs. Perform a transient simulation at each amplitude level to find the actual values of A_0 and τ as a function of input amplitude. [Hint: the easiest way to obtain A_0 and τ from transient simulations is to use 'chirp' inputs with slowly changing frequencies. Keep the input amplitude constant with time, but gradually sweep the frequency up (rising chirp) or down (falling chirp).]
c) Plot the theoretically-expected values of A_0 and τ versus what you actually obtained from simulations using rising chirps. Is there an intuitive explanation for your results?
d) Repeat part c) by simulating with falling chirps instead. Are there significant differences between the two sets of simulation results?

Problem 13.10

For the $-s^2$ –plane geometry of Figure 13.14 (c), prove all results in the last column of Table 13.2.