```
In[78]:= Needs["Graphics`MultipleListPlot`"]

In[79]:= Needs["Graphics`Legend`"]
```

à Question 6

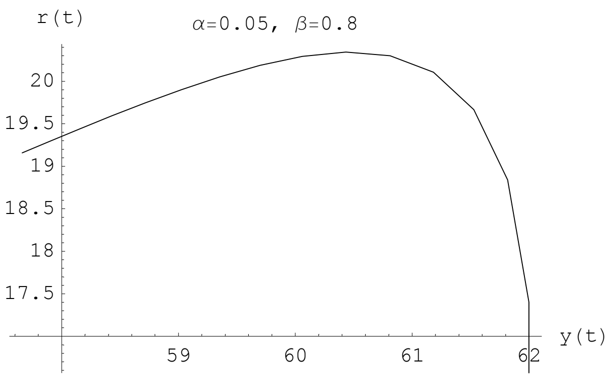
The model is,

$$y_{t+1} - y_t = \alpha[c(1 - \text{trate}) - 1]\, y_t - \alpha\, h\, r_t + \alpha\, a$$
$$r_{t+1} - r_t = \beta\, k\, y_t - \beta\, u\, r_t - \beta\, m_0$$

```
In[80]:= Clear[y, r]

In[81]:= {a = 50, c = 0.75, trate = 0.25, h = 1.525,
          m0 = 5, k = 0.25, u = 0.5, α = 0.05, β = 0.8}

Out[81]= {50, 0.75, 0.25, 1.525, 5, 0.25, 0.5, 0.05, 0.8}

In[82]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[82]= {{y1 → 2.5 - 0.07625 r + 0.978125 y}}

In[83]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[83]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[84]:= Clear[ybar, rbar]

         General::spell1 :
          Possible spelling error: new symbol name "rbar" is similar to existing symbol "ybar".

In[85]:= Solve[{ybar == 2.5 - 0.07625 rbar + 0.978125 ybar,
           rbar == -4 + 0.6 rbar + 0.2 ybar}, {ybar, rbar}]

Out[85]= {{ybar → 54.375, rbar → 17.1875}}

In[86]:= {y[0] = 62, r[0] = 15};
         y[t_] := 2.5 - 0.07625 r[t - 1] + 0.978125 y[t - 1]
         r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[89]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm

Out[89]//TableForm=
```

0	62	15
1	62.	17.4
2	61.817	18.84
3	61.5282	19.6674
4	61.1826	20.1061
5	60.8112	20.3002
6	60.433	20.3423
7	60.06	20.292
8	59.6989	20.1872
9	59.3537	20.0521
10	59.0264	19.902
11	58.7176	19.7465
12	58.4275	19.5914
13	58.1556	19.4404
14	57.9011	19.2953
15	57.6632	19.1574

```
In[90]:= trajdata61 = Table[{y[t], r[t]}, {t, 0, 15}];

In[91]:= ListPlot[trajdata61 , AxesLabel -> {"y(t)", "r(t)"},
           PlotLabel -> "α=0.05, β=0.8", PlotJoined -> True];
```

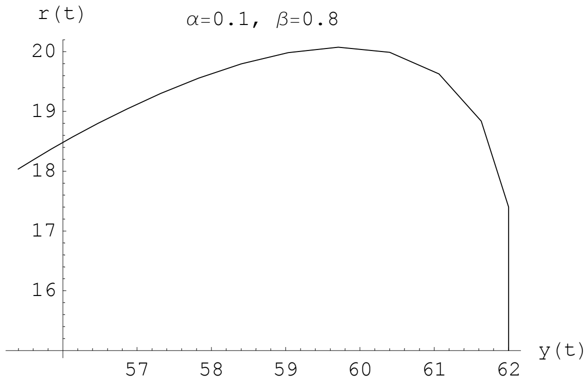



```
In[92]:= Clear[y, r]

In[93]:= {α = 0.1, β = 0.8}

Out[93]= {0.1, 0.8}

In[94]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[94]= {{y1 → 5. - 0.1525 r + 0.95625 y}}

In[95]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[95]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[96]:= {y[0] = 62, r[0] = 15};
         y[t_] := 5 - 0.1525 r[t - 1] + 0.95625 y[t - 1]
         r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[99]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm

Out[99]//TableForm=
```

0	62	15
1	62.	17.4
2	61.634	18.84
3	61.0644	19.6308
4	60.3991	19.9914
5	59.708	20.0746
6	59.0344	19.9864
7	58.4037	19.7987
8	57.8292	19.56
9	57.3163	19.3018
10	56.8652	19.0444
11	56.4731	18.7997
12	56.1354	18.5744
13	55.8469	18.3717
14	55.6019	18.1924
15	55.395	18.0358

```
In[100]:= trajdata62 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[101]:= ListPlot[trajdata62, AxesLabel -> {"y(t)", "r(t)"},
            PlotLabel -> "α=0.1, β=0.8", PlotJoined -> True];
```

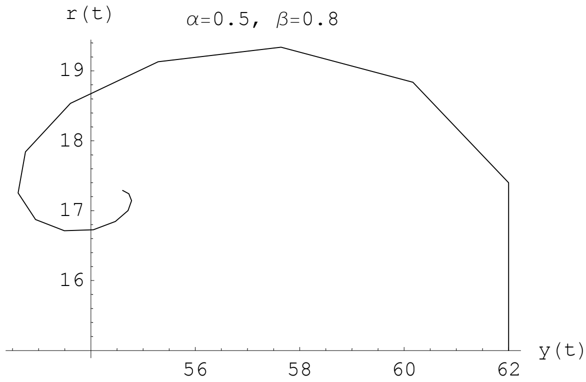



```
In[102]:= Clear[y, r]

In[103]:= {α = 0.5, β = 0.8}

Out[103]= {0.5, 0.8}

In[104]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[104]= {{y1 → 25. - 0.7625 r + 0.78125 y}}

In[105]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[105]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[106]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[109]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm

Out[109]//TableForm=
```

0	62	15
1	62.	17.4
2	60.17	18.84
3	57.6423	19.338
4	55.2878	19.1313
5	53.606	18.5363
6	52.7458	17.843
7	52.6023	17.255
8	52.9387	16.8734
9	53.4923	16.7118
10	54.0481	16.7255
11	54.4719	16.845
12	54.7119	17.0014
13	54.7801	17.1432
14	54.7253	17.2419
15	54.6072	17.2902

```
In[110]:= trajdata63 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[111]:= ListPlot[trajdata63, AxesLabel -> {"y(t)", "r(t)"},
            PlotLabel -> "α=0.5,  β=0.8", PlotJoined -> True];
```



(i) A goods market shock can be considered by changing the value of *a*. (ii) A money-market shock can be considered by changing the value of *m*0.

à Question 7

The system of equations are:

$$\Delta\, y_t = \alpha[c(1-t)-1]\, y_t - \alpha\, h\, r_t + \alpha\, \mathrm{a}\Delta$$
$$\Delta\, r_t = \beta\, h\, y_t - \beta\, u\, r_t - \beta\, m_0$$

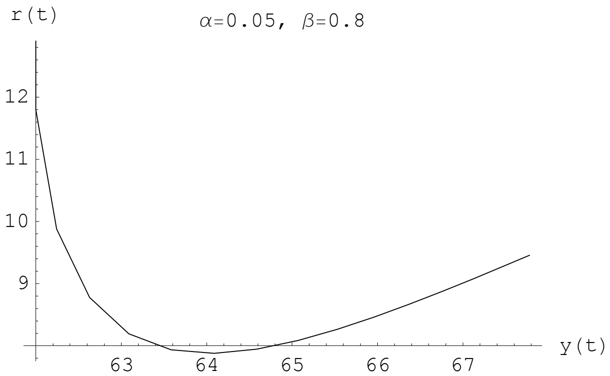
```
In[112]:= {a = 50, c = 0.75, trate = 0.25, h = 1.525, k = 0.25, m0 = 12, u = 0.5}

Out[112]= {50, 0.75, 0.25, 1.525, 0.25, 12, 0.5}

In[113]:= {α = 0.05, β = 0.8}

Out[113]= {0.05, 0.8}

In[114]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[114]= {{y1 → 2.5 - 0.07625 r + 0.978125 y}}

In[115]:= Simplify[Solve[r1 - r == β k y  - β u r  - β m0, r1]]

Out[115]= {{r1 → -9.6 + 0.6 r + 0.2 y}}

In[116]:= Clear[ybar, rbar]

In[117]:= Solve[{ybar == 2.5 - 0.07625 rbar + 0.978125 ybar,
            rbar == -9.6 + 0.6 rbar + 0.2 ybar}, {ybar, rbar}]

Out[117]= {{ybar → 72.1667, rbar → 12.0833}}

In[118]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.07625 r[t - 1] + 0.978125 y[t - 1]
          r[t_] := -9.6 + 0.6 r[t - 1] + 0.2 y[t - 1]
```

```
In[121]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm
```

Out[121]//TableForm=

0	62	15
1	62.	11.8
2	62.244	9.88
3	62.6291	8.7768
4	63.0898	8.19189
5	63.5851	7.9331
6	64.0893	7.87688
7	64.5867	7.94398
8	65.0681	8.08373
9	65.5284	8.26387
10	65.9648	8.464
11	66.3765	8.67137
12	66.7633	8.87812
13	67.1259	9.07953
14	67.4652	9.2729
15	67.7823	9.45678

```
In[122]:= trajdata71 = Table[{y[t], r[t]}, {t, 0, 15}];

In[123]:= ListPlot[trajdata71, AxesLabel -> {"y(t)", "r(t)"},
            PlotLabel -> "α=0.05, β=0.8", PlotJoined -> True];
```




```
In[124]:= Clear[y, r]

In[125]:= {α = 0.1, β = 0.8}
```

Out[125]= {0.1, 0.8}

```
In[126]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]
```

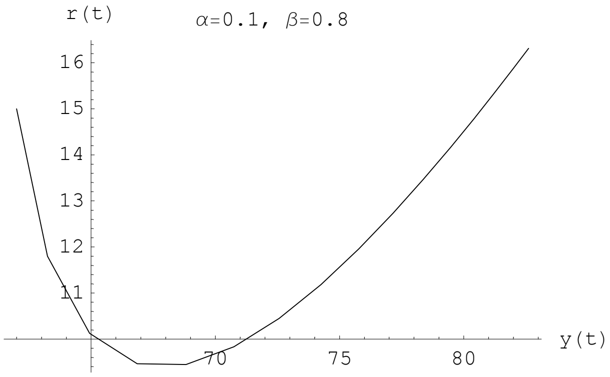
Out[126]= {{y1 → 5. - 0.1525 r + 0.95625 y}}

```
In[127]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]
```

Out[127]= {{r1 → -9.6 + 0.6 r + 0.2 y}}

```
In[128]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.1525 r[t - 1] + 0.97625 y[t - 1]
          r[t_] := -9.6 + 0.6 r[t - 1] + 0.2 y[t - 1]
```

```
In[131]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm

Out[131]//TableForm=
        0       62          15
        1       63.24       11.8
        2       64.9385     10.128
        3       66.8517     9.46451
        4       68.8207     9.44905
        5       70.7452     9.83357
        6       72.5654     10.4492
        7       74.2485     11.1826
        8       75.7797     11.9592
        9       77.1562     12.7315
        10      78.3821     13.4701
        11      79.4664     14.1585
        12      80.4199     14.7884
        13      81.2547     15.357
        14      81.9829     15.8651
        15      82.6164     16.3157

In[132]:= trajdata72 = Table[{y[t], r[t]}, {t, 0, 15}];

In[133]:= ListPlot[trajdata72, AxesLabel -> {"y(t)", "r(t)"},
            PlotLabel -> "α=0.1, β=0.8", PlotJoined -> True];
```



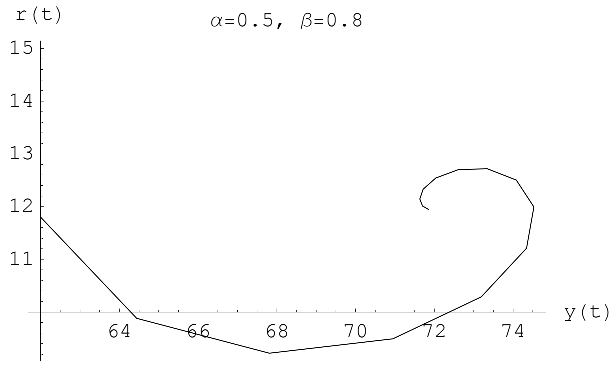

```
In[134]:= Clear[y, r]

In[135]:= {α = 0.5, β = 0.8}

Out[135]= {0.5, 0.8}

In[136]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[136]= {{y1 → 25. - 0.7625 r + 0.78125 y}}

In[137]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[137]= {{r1 → -9.6 + 0.6 r + 0.2 y}}

In[138]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -9.6 + 0.6 r[t - 1] + 0.2 y[t - 1]
```

```
In[141]:= Table[{t, y[t], r[t]}, {t, 0, 15}] // TableForm
```

Out[141]//TableForm=

0	62	15
1	62.	11.8
2	64.44	9.88
3	67.8103	9.216
4	70.9496	9.49165
5	73.192	10.2849
6	74.339	11.2093
7	74.5302	11.9934
8	74.0818	12.5021
9	73.3435	12.7176
10	72.6025	12.6993
11	72.0375	12.5401
12	71.7175	12.3315
13	71.6265	12.1424
14	71.6996	12.0108
15	71.8571	11.9464

```
In[142]:= trajdata73 = Table[{y[t], r[t]}, {t, 0, 15}];

In[143]:= ListPlot[trajdata73, AxesLabel -> {"y(t)", "r(t)"},
            PlotLabel -> "α=0.5, β=0.8", PlotJoined -> True];
```



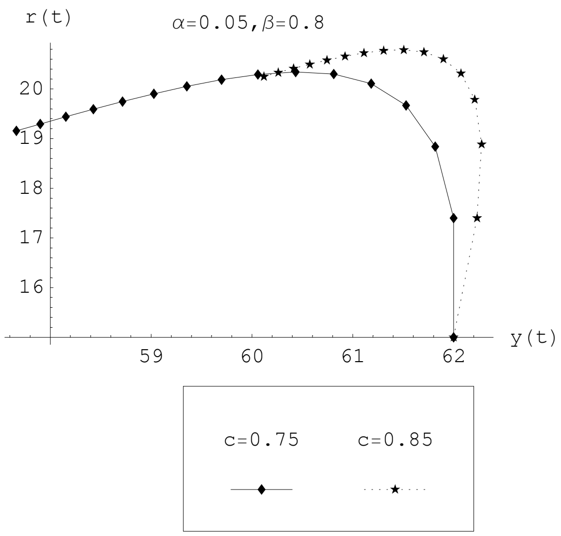
à Question 8

(i) **A rise in the marginal propensity to consume**

We shall increase the marginal propensity to consume from c = 0.75 to c = 0.85.

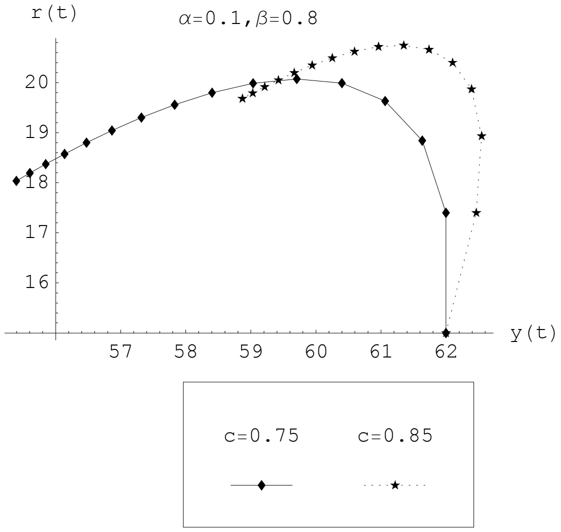
```
In[144]:= Clear[y, r]

In[145]:= {a = 50, c = 0.85, trate = 0.25, h = 1.525,
            m0 = 5, k = 0.25, u = 0.5, α = 0.05, β = 0.8}

Out[145]= {50, 0.85, 0.25, 1.525, 5, 0.25, 0.5, 0.05, 0.8}

In[146]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[146]= {{y1 → 2.5 - 0.07625 r + 0.981875 y}}
```

```
In[147]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[147]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[148]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.07625 r[t - 1] + 0.981875 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[151]:= trajdata81 = Table[{y[t], r[t]}, {t, 0, 15}];

In[152]:= MultipleListPlot[trajdata61, trajdata81, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"c=0.75", "c=0.85"},
            PlotLabel -> "α=0.05,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



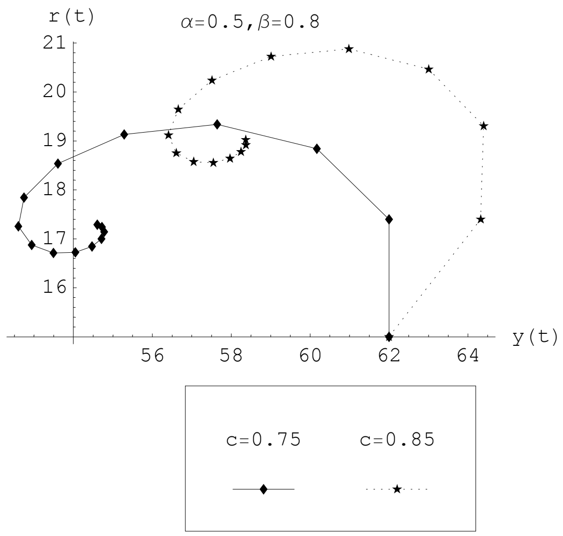

```
In[153]:= Clear[y, r]

In[154]:= {α = 0.1, β = 0.8}

Out[154]= {0.1, 0.8}

In[155]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[155]= {{y1 → 5. - 0.1525 r + 0.96375 y}}

In[156]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[156]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[157]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.1525 r[t - 1] + 0.96375 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[160]:= trajdata82 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[161]:= MultipleListPlot[trajdata62, trajdata82, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"c=0.75", "c=0.85"},
            PlotLabel -> "α=0.1,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```




```
In[162]:= Clear[y, r]

In[163]:= {α = 0.5, β = 0.8}

Out[163]= {0.5, 0.8}

In[164]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[164]= {{y1 → 25. - 0.7625 r + 0.81875 y}}

In[165]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[165]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[166]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.81875 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[169]:= trajdata83 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[170]:= MultipleListPlot[trajdata63, trajdata83, PlotJoined -> True,
             AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"c=0.75", "c=0.85"},
             PlotLabel -> "α=0.5,β=0.8", LegendPosition -> {-0.35, -1.2},
             LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```

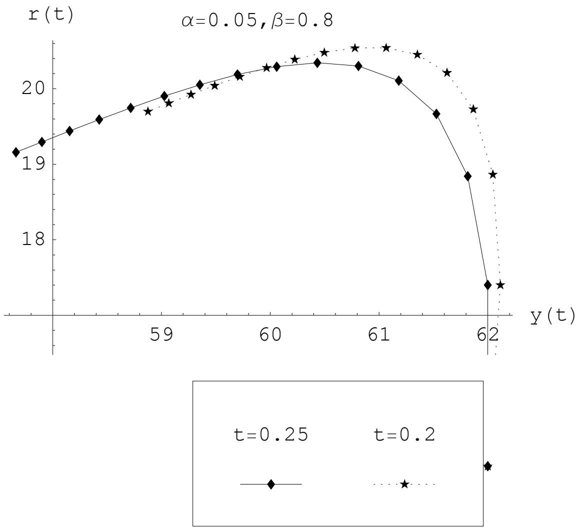


(ii) **A reduction in the marginal rate of tax**

A reduction in the marginal rate of tax, from $t = 0.25$ to $t = 0.2$. The marginal propensity to consume is returned to its initial value of $c = 0.75$.

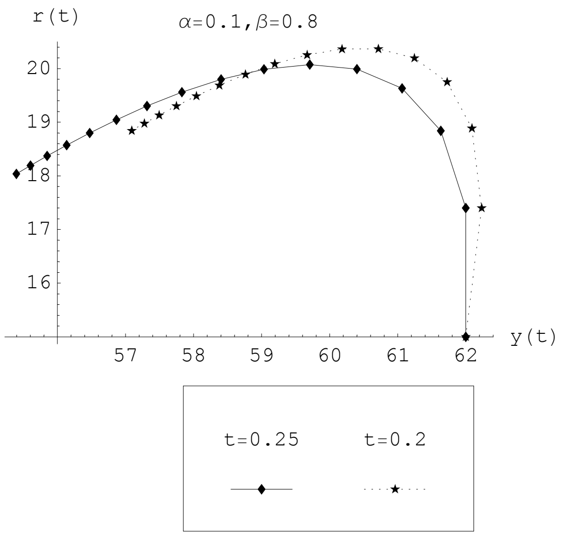
```
In[171]:= Clear[y, r]

In[172]:= {c = 0.75, trate = 0.2}

Out[172]= {0.75, 0.2}

In[173]:= {α = 0.05, β = 0.8}

Out[173]= {0.05, 0.8}

In[174]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[174]= {{y1 → 2.5 - 0.07625 r + 0.98 y}}

In[175]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[175]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[176]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.07625 r[t - 1] + 0.98 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[179]:= trajdata821 = Table[{y[t], r[t]}, {t, 0, 15}];

     General::spell1 : Possible spelling error: new
        symbol name "trajdata821" is similar to existing symbol "trajdata81".
```

```
In[180]:= MultipleListPlot[trajdata61, trajdata821, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"t=0.25", "t=0.2"},
            PlotLabel -> "α=0.05,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



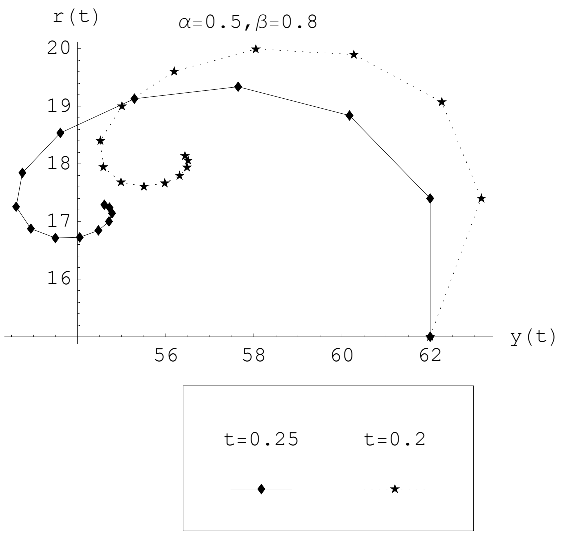

```
In[181]:= Clear[y, r]

In[182]:= {α = 0.1, β = 0.8}

Out[182]= {0.1, 0.8}

In[183]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[183]= {{y1 → 5. - 0.1525 r + 0.96 y}}

In[184]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[184]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[185]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.1525 r[t - 1] + 0.96 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[188]:= trajdata822 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[189]:= MultipleListPlot[trajdata62, trajdata822, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"t=0.25", "t=0.2"},
            PlotLabel -> "α=0.1,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```




```
In[190]:= Clear[y, r]

In[191]:= {α = 0.5, β = 0.8}

Out[191]= {0.5, 0.8}

In[192]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[192]= {{y1 → 25. - 0.7625 r + 0.8 y}}

In[193]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[193]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[194]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.8 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[197]:= trajdata823 = Table[{y[t], r[t]}, {t, 0, 15}];

      General::spell1 : Possible spelling error: new
         symbol name "trajdata823" is similar to existing symbol "trajdata83".
```

```
In[198]:= MultipleListPlot[trajdata63, trajdata823, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"t=0.25", "t=0.2"},
            PlotLabel -> "α=0.5,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```

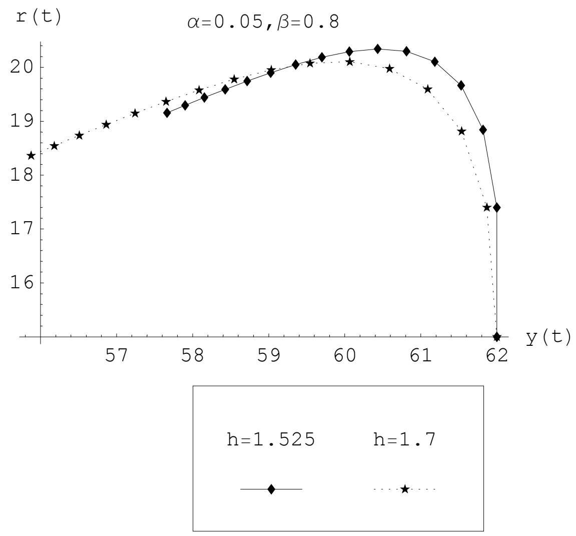


(iii) **A rise in the interest-rate sensitivity of investment**

A rise in h from h = 1.525 to h = 1.7, all other parameters returning to their original values.

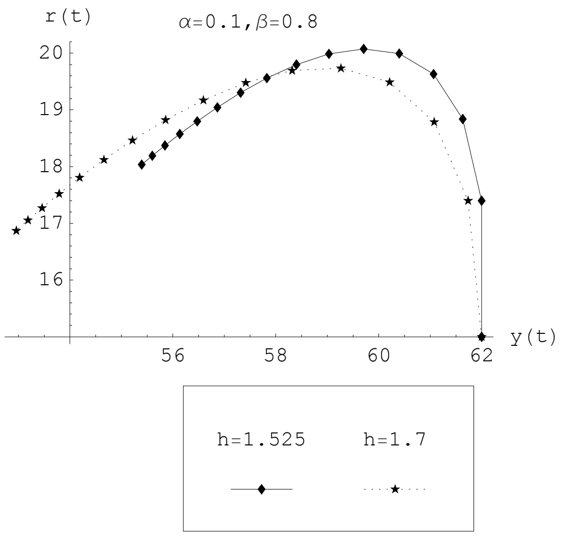
```
In[199]:= Clear[y, r]

In[200]:= {trate = 0.25, h = 1.7}

Out[200]= {0.25, 1.7}

In[201]:= {α = 0.05, β = 0.8}

Out[201]= {0.05, 0.8}

In[202]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[202]= {{y1 → 2.5 - 0.085 r + 0.978125 y}}

In[203]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[203]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[204]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.085 r[t - 1] + 0.978125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[207]:= trajdata831 = Table[{y[t], r[t]}, {t, 0, 15}];

    General::spell1 : Possible spelling error: new
       symbol name "trajdata831" is similar to existing symbol "trajdata81".
```

```
In[208]:= MultipleListPlot[trajdata61, trajdata831, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"h=1.525", "h=1.7"},
            PlotLabel -> "α=0.05,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



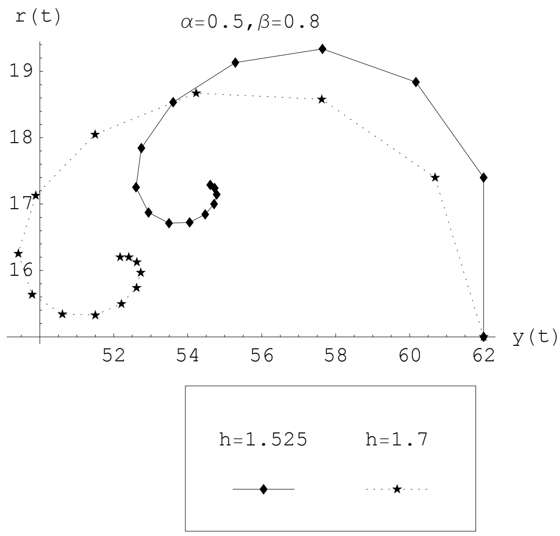

```
In[209]:= Clear[y, r]

In[210]:= {α = 0.1, β = 0.8}

Out[210]= {0.1, 0.8}

In[211]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[211]= {{y1 → 5. - 0.17 r + 0.95625 y}}

In[212]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[212]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[213]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.17 r[t - 1] + 0.95625 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[216]:= trajdata832 = Table[{y[t], r[t]}, {t, 0, 15}];

     General::spell : Possible spelling error: new symbol
        name "trajdata832" is similar to existing symbols {trajdata82, trajdata823}.
```

```
In[217]:= MultipleListPlot[trajdata62, trajdata832, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"h=1.525", "h=1.7"},
            PlotLabel -> "α=0.1,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```




```
In[218]:= Clear[y, r]

In[219]:= {α = 0.5, β = 0.8}

Out[219]= {0.5, 0.8}

In[220]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[220]= {{y1 → 25. - 0.85 r + 0.78125 y}}

In[221]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[221]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[222]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.85 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[225]:= trajdata833 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[226]:= MultipleListPlot[trajdata63, trajdata833, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"h=1.525", "h=1.7"},
            PlotLabel -> "α=0.5,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```

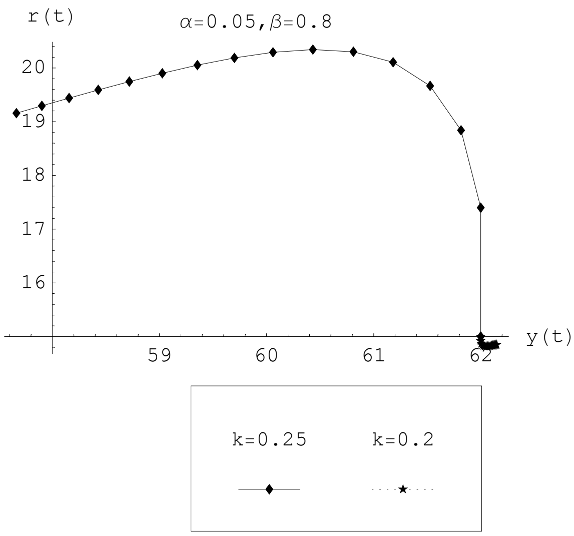


(iv) **Higher income velocity of circulation of money (lower *k*)**

A fall in k from $k = 0.25$ to $k = 0.2$, all other parameters returning to their original values.

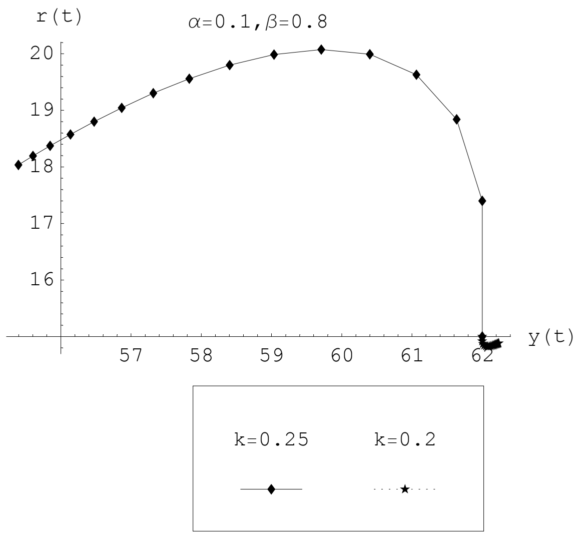
```
In[227]:= Clear[y, r]

In[228]:= {h = 1.525, k = 0.2}

Out[228]= {1.525, 0.2}

In[229]:= {α = 0.05, β = 0.8}

Out[229]= {0.05, 0.8}

In[230]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[230]= {{y1 → 2.5 - 0.07625 r + 0.978125 y}}

In[231]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[231]= {{r1 → -4. + 0.6 r + 0.16 y}}

In[232]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.07625 r[t - 1] + 0.978125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.16 y[t - 1]

In[235]:= trajdata841 = Table[{y[t], r[t]}, {t, 0, 15}];

     General::spell1 : Possible spelling error: new
        symbol name "trajdata841" is similar to existing symbol "trajdata81".
```

```
In[236]:= MultipleListPlot[trajdata61, trajdata841, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"k=0.25", "k=0.2"},
            PlotLabel -> "α=0.05,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



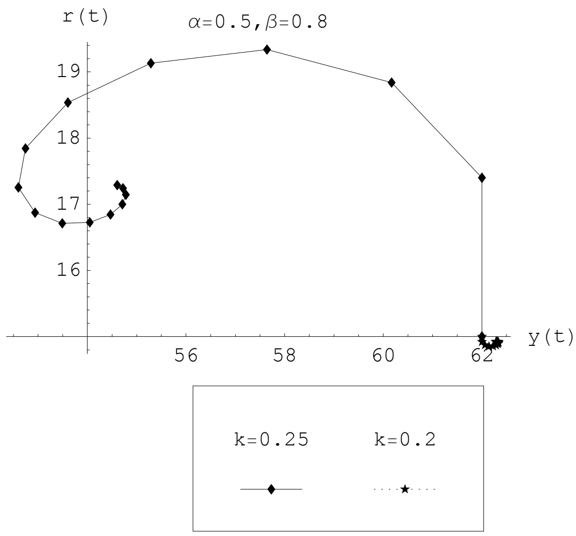

```
In[237]:= Clear[y, r]

In[238]:= {α = 0.1, β = 0.8}

Out[238]= {0.1, 0.8}

In[239]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[239]= {{y1 → 5. - 0.1525 r + 0.95625 y}}

In[240]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[240]= {{r1 → -4. + 0.6 r + 0.16 y}}

In[241]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.1525 r[t - 1] + 0.95625 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.16 y[t - 1]

In[244]:= trajdata842 = Table[{y[t], r[t]}, {t, 0, 15}];

     General::spell1 : Possible spelling error: new
        symbol name "trajdata842" is similar to existing symbol "trajdata82".
```

```
In[245]:= MultipleListPlot[trajdata62, trajdata842, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"k=0.25", "k=0.2"},
            PlotLabel -> "α=0.1,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```




```
In[246]:= Clear[y, r]

In[247]:= {α = 0.5, β = 0.8}

Out[247]= {0.5, 0.8}

In[248]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[248]= {{y1 → 25. - 0.7625 r + 0.78125 y}}

In[249]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[249]= {{r1 → -4. + 0.6 r + 0.16 y}}

In[250]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.16 y[t - 1]

In[253]:= trajdata843 = Table[{y[t], r[t]}, {t, 0, 15}];

      General::spell1 : Possible spelling error: new
         symbol name "trajdata843" is similar to existing symbol "trajdata83".
```

```
In[254]:= MultipleListPlot[trajdata63, trajdata843, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"k=0.25", "k=0.2"},
            PlotLabel -> "α=0.5,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```

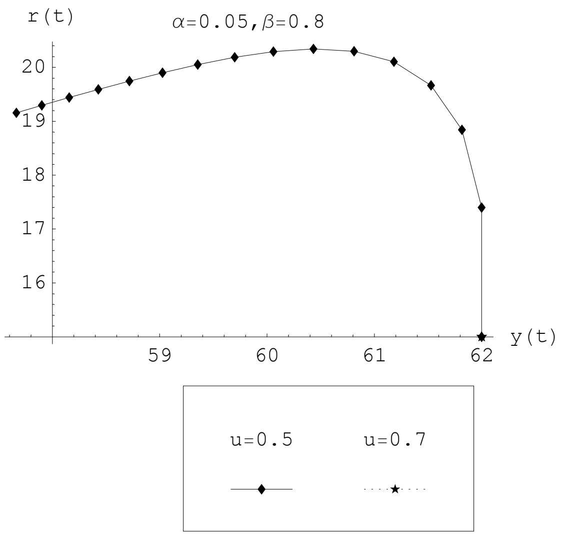


(v) **Rise in interest-sensitivity of demand for money (rise in *u*)**

A rise in *u* from *u* = 0.5 to *u* = 0.7, all other parameters returning to their original values.

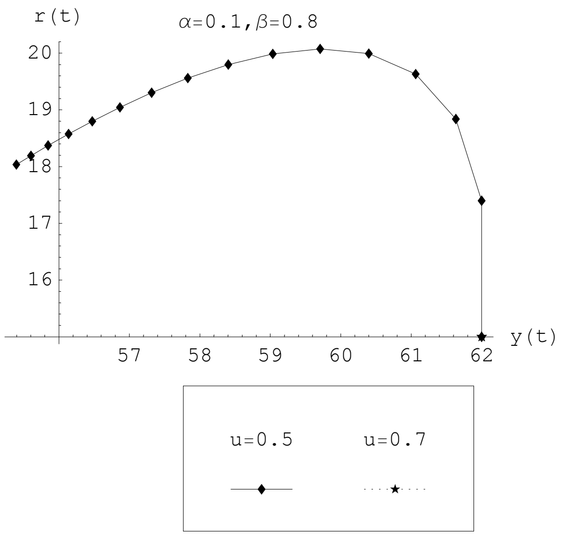
```
In[255]:= Clear[y, r]

In[256]:= {k = 0.25, u = 0.7}

Out[256]= {0.25, 0.7}

In[257]:= {α = 0.05, β = 0.8}

Out[257]= {0.05, 0.8}

In[258]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[258]= {{y1 → 2.5 - 0.07625 r + 0.978125 y}}

In[259]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[259]= {{r1 → -4. + 0.44 r + 0.2 y}}

In[260]:= {y[0] = 62, r[0] = 15};
          y[t_] := 2.5 - 0.07625 r[t - 1] + 0.978125 y[t - 1]
          r[t_] := -4 + 0.44 r[t - 1] + 0.2 y[t - 1]

In[263]:= trajdata851 = Table[{y[t], r[t]}, {t, 0, 15}];

        General::spell1 : Possible spelling error: new
           symbol name "trajdata851" is similar to existing symbol "trajdata81".
```

```
In[264]:= MultipleListPlot[trajdata61, trajdata851, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"u=0.5", "u=0.7"},
            PlotLabel -> "α=0.05,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



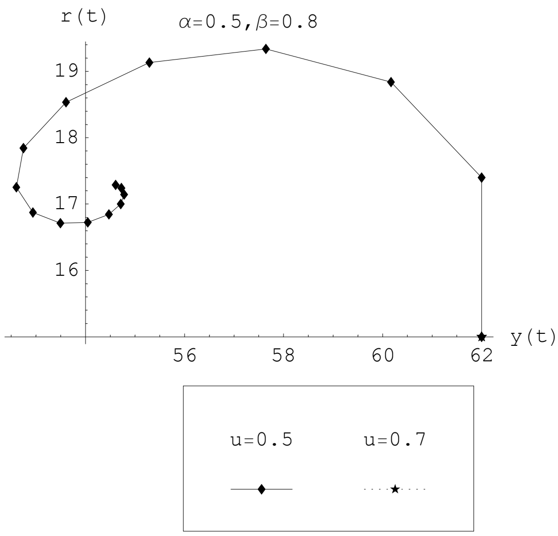

```
In[265]:= Clear[y, r]

In[266]:= {α = 0.1, β = 0.8}

Out[266]= {0.1, 0.8}

In[267]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[267]= {{y1 → 5. - 0.1525 r + 0.95625 y}}

In[268]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[268]= {{r1 → -4. + 0.44 r + 0.2 y}}

In[269]:= {y[0] = 62, r[0] = 15};
          y[t_] := 5 - 0.1525 r[t - 1] + 0.95625 y[t - 1]
          r[t_] := -4 + 0.44 r[t - 1] + 0.2 y[t - 1]

In[272]:= trajdata852 = Table[{y[t], r[t]}, {t, 0, 15}];

     General::spell1 : Possible spelling error: new
        symbol name "trajdata852" is similar to existing symbol "trajdata82".
```

```
In[273]:= MultipleListPlot[trajdata62, trajdata852, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"u=0.5", "u=0.7"},
            PlotLabel -> "α=0.1,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```




```
In[274]:= Clear[y, r]

In[275]:= {α = 0.5, β = 0.8}

Out[275]= {0.5, 0.8}

In[276]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[276]= {{y1 → 25. - 0.7625 r + 0.78125 y}}

In[277]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[277]= {{r1 → -4. + 0.44 r + 0.2 y}}

In[278]:= {y[0] = 62, r[0] = 15};
          y[t_] := 25 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.44 r[t - 1] + 0.2 y[t - 1]

In[281]:= trajdata853 = Table[{y[t], r[t]}, {t, 0, 15}];

    General::spell1 : Possible spelling error: new
       symbol name "trajdata853" is similar to existing symbol "trajdata83".
```

```
In[282]:= MultipleListPlot[trajdata63, trajdata853, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLegend -> {"u=0.5", "u=0.7"},
            PlotLabel -> "α=0.5,β=0.8", LegendPosition -> {-0.35, -1.2},
            LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



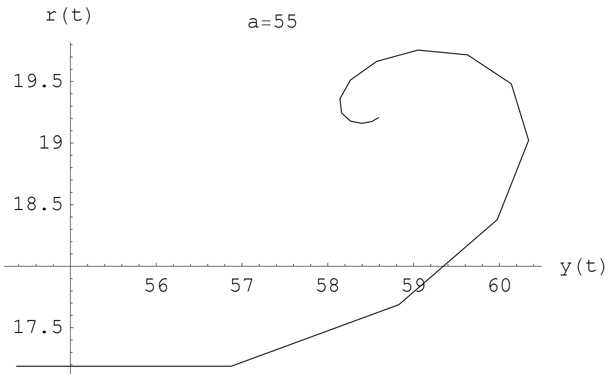
à Question 9

The model is,

$$y_{t+1} - y_t = \alpha[c(1 - \text{trate}) - 1]\, y_t - \alpha\, h\, r_t + \alpha\, a$$
$$r_{t+1} - r_t = \beta\, k\, y_t - \beta\, u\, r_t - \beta\, m_0$$

```
In[283]:= Clear[y, r]

In[284]:= {a = 55, c = 0.75, trate = 0.25, h = 1.525,
           m0 = 5, k = 0.25, u = 0.5, α = 0.5, β = 0.8}

Out[284]= {55, 0.75, 0.25, 1.525, 5, 0.25, 0.5, 0.5, 0.8}

In[285]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[285]= {{y1 → 27.5 - 0.7625 r + 0.78125 y}}

In[286]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[286]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[287]:= Clear[ybar, rbar]

In[288]:= Solve[{ybar == 27.5 - 0.7625 rbar + 0.78125 ybar,
            rbar == -4 + 0.6 rbar + 0.2 ybar}, {ybar, rbar}]

Out[288]= {{ybar → 58.5417, rbar → 19.2708}}

In[289]:= {y[0] = 54.375, r[0] = 17.1875};
          y[t_] := 27.5 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[292]:= trajdata9 = Table[{y[t], r[t]}, {t, 0, 15}];
```

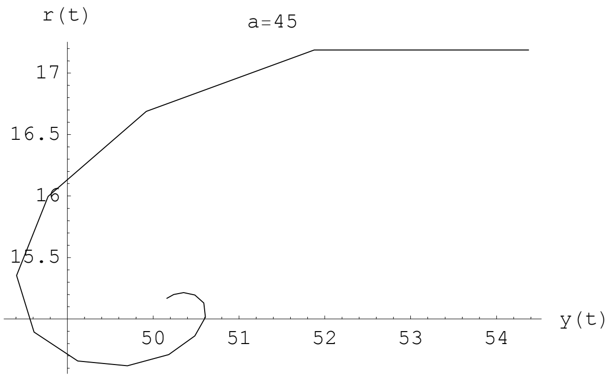
```
In[293]:= ListPlot[trajdata9, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLabel -> "a=55"];
```



à Question 10

```
In[294]:= Clear[y, r]

In[295]:= {a = 45}

Out[295]= {45}

In[296]:= Simplify[Solve[y1 - y == α (c (1 - trate) - 1) y - α h r + α a, y1]]

Out[296]= {{y1 → 22.5 - 0.7625 r + 0.78125 y}}

In[297]:= Simplify[Solve[r1 - r == β k y - β u r - β m0, r1]]

Out[297]= {{r1 → -4. + 0.6 r + 0.2 y}}

In[298]:= Clear[ybar, rbar]

In[299]:= Solve[{ybar == 22.5 - 0.7625 rbar + 0.78125 ybar,
            rbar == -4 + 0.6 rbar + 0.2 ybar}, {ybar, rbar}]

Out[299]= {{ybar → 50.2083, rbar → 15.1042}}

In[300]:= {y[0] = 54.375, r[0] = 17.1875};
          y[t_] := 22.5 - 0.7625 r[t - 1] + 0.78125 y[t - 1]
          r[t_] := -4 + 0.6 r[t - 1] + 0.2 y[t - 1]

In[303]:= trajdata10 = Table[{y[t], r[t]}, {t, 0, 15}];
```

```
In[304]:= ListPlot[trajdata10, PlotJoined -> True,
            AxesLabel -> {"y(t)", "r(t)"}, PlotLabel -> "a=45"];
```



à Question 11

(i) Example 10.5

The model is,

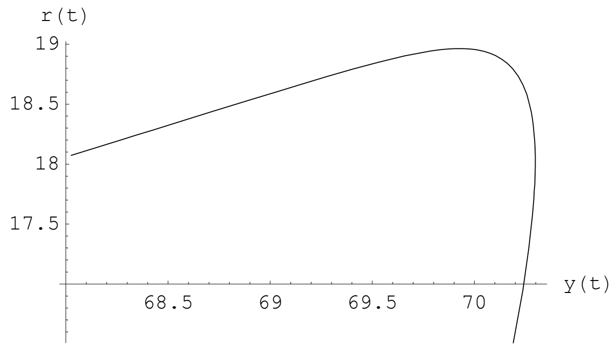
$$\dot{y} = \alpha[c(1 - \text{trate}) + j - 1]\, y - \alpha\, h\, r + \alpha\, a$$
$$\dot{r} = \beta\, k\, y - \beta\, u\, r - \beta\, m_0$$

```
In[305]:= {a = 2, c = 0.75, trate = 0.25, h = 1.525,
           j = 0.8, m0 = 8, k = 0.25, u = 0.5, α = 0.05, β = 0.8}

Out[305]= {2, 0.75, 0.25, 1.525, 0.8, 8, 0.25, 0.5, 0.05, 0.8}

In[306]:= Simplify[Solve[ydot == α (c (1 - trate) + j - 1) y - α h r + α a, ydot]]

Out[306]= {{ydot → 0.1 - 0.07625 r + 0.018125 y}}

In[307]:= Simplify[Solve[rdot == β k y - β u r - β m0, rdot]]

        General::spell1 :
         Possible spelling error: new symbol name "rdot" is similar to existing symbol "ydot".

Out[307]= {{rdot → -6.4 - 0.4 r + 0.2 y}}

In[308]:= Clear[ybar, rbar]

In[309]:= Solve[{0 == 0.1 - 0.07625 rbar + 0.018125 ybar,
            0 == -6.4 - 0.4 rbar + 0.2 ybar}, {ybar, rbar}]

Out[309]= {{ybar → 66., rbar → 17.}}

In[310]:= iscurve = Simplify[Solve[0 == α (c (1 - trate) + j - 1) y - α h r + α a, r]]

Out[310]= {{r → 1.31148 + 0.237705 y}}

In[311]:= lmcurve = Simplify[Solve[0 == β k y - β u r - β m0, r]]

Out[311]= {{r → -16. + 0.5 y}}
```

We shall take the initial point to be $(y, r) = (70, 15)$.

```
In[312]:= Clear[y, r]
```

```
In[313]:= sol11 = NDSolve[{y'[t] == 0.1 - 0.07625 r[t] + 0.018125 y[t], r'[t] ==
              -6.4 - 0.4 r[t] + 0.2 y[t], y[0] == 70, r[0] == 15}, {y, r}, {t, 0, 40}]

Out[313]= {{y → InterpolatingFunction[{{0., 40.}}, <>],
            r → InterpolatingFunction[{{0., 40.}}, <>]}}

In[314]:= tr11 = ParametricPlot[{y[t], r[t]} /. sol11,
             {t, 0, 40}, AxesLabel -> {"y(t)", "r(t)"}];

          ParametricPlot::ppcom : Function {y[t], r[t]} /. sol11
             cannot be compiled; plotting will proceed with the uncompiled function.
```



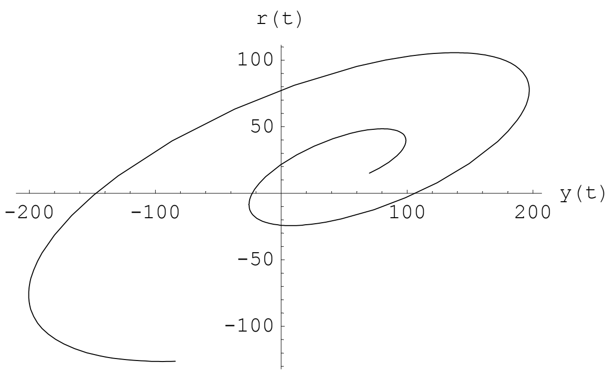
Which illustrates the statement in the text that we have a stable path which appears to traverse a straight line path after a certain time period.

(ii) Example 10.6

```
In[315]:= {a = 2, c = 0.8, trate = 0.2, h = 1.525,
           j = 0.95, m0 = 8, k = 0.25, u = 0.25, α = 0.2, β = 0.3}

Out[315]= {2, 0.8, 0.2, 1.525, 0.95, 8, 0.25, 0.25, 0.2, 0.3}

In[316]:= Simplify[Solve[ydot == α (c (1 - trate) + j - 1) y - α h r + α a, ydot]]

Out[316]= {{ydot → 0.4 - 0.305 r + 0.118 y}}

In[317]:= Simplify[Solve[rdot == β k y - β u r - β m0, rdot]]

Out[317]= {{rdot → -2.4 - 0.075 r + 0.075 y}}

In[318]:= Clear[ybar, rbar]

In[319]:= Solve[{0 == 0.4 - 0.305 rbar + 0.118 ybar,
            0 == -2.4 - 0.075 rbar + 0.075 ybar}, {ybar, rbar}]

Out[319]= {{ybar → 54.3316, rbar → 22.3316}}

In[320]:= iscurve = Simplify[Solve[0 == α (c (1 - trate) + j - 1) y - α h r + α a, r]]

Out[320]= {{r → 1.31148 + 0.386885 y}}

In[321]:= lmcurve = Simplify[Solve[0 == β k y - β u r - β m0, r]]

Out[321]= {{r → -32. + 1. y}}
```

We shall take the initial point to be (y, r) = (70, 15).

```
In[322]:= Clear[y, r]

In[323]:= sol112 = NDSolve[
            {y'[t] == 0.4 - 0.305 r[t] + 0.118 y[t], r'[t] == -2.4 - 0.075 r[t] + 0.075 y[t],
             y[0] == 70, r[0] == 15}, {y, r}, {t, 0, 100}]

Out[323]= {{y → InterpolatingFunction[{{0., 100.}}, <>],
            r → InterpolatingFunction[{{0., 100.}}, <>]}}

In[324]:= tr112 = ParametricPlot[{y[t], r[t]} /. sol112,
             {t, 0, 100}, AxesLabel -> {"y(t)", "r(t)"}];

      ParametricPlot::ppcom : Function {y[t], r[t]} /. sol112
         cannot be compiled; plotting will proceed with the uncompiled function.
```



Which illustrates the statement in the text that we have an unstable spiral.

à Question 12

```
In[325]:= {a = -25, c = 0.75, trate = 0.25, h = 1,
           j = 0.95, m0 = 8, k = 0.22, u = 0.75, α = 0.1, β = 0.8}

Out[325]= {-25, 0.75, 0.25, 1, 0.95, 8, 0.22, 0.75, 0.1, 0.8}

In[326]:= Simplify[Solve[ydot == α (c (1 - trate) + j - 1) y - α h r + α a, ydot]]

Out[326]= {{ydot → -2.5 - 0.1 r + 0.05125 y}}

In[327]:= Simplify[Solve[rdot == β k y - β u r - β m0, rdot]]

Out[327]= {{rdot → -6.4 - 0.6 r + 0.176 y}}

In[328]:= Clear[ybar, rbar]

In[329]:= Solve[{0 == -2.5 - 0.1 rbar + 0.05125 ybar,
             0 == -6.4 - 0.6 rbar + 0.176 ybar}, {ybar, rbar}]

Out[329]= {{ybar → 65.3992, rbar → 8.51711}}

In[330]:= iscurve = Simplify[Solve[0 == α (c (1 - trate) + j - 1) y - α h r + α a, r]]

Out[330]= {{r → -25. + 0.5125 y}}

In[331]:= lmcurve = Simplify[Solve[0 == β k y - β u r - β m0, r]]

Out[331]= {{r → -10.6667 + 0.293333 y}}
```

Writing the system in matrix form we have,

$$\begin{pmatrix} \dot{y} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} 0.05125 & -0.1 \\ 0.176 & -0.6 \end{pmatrix} \begin{pmatrix} y - y^* \\ r - r^* \end{pmatrix}$$

```
In[332]:= matrixA = ( 0.05125  -0.1 )
                    ( 0.176    -0.6 )

Out[332]= {{0.05125, -0.1}, {0.176, -0.6}}

In[333]:= Det[matrixA]

Out[333]= -0.01315

In[334]:= traceA = Sum[matrixA[[i, i]], {i, 2}]

Out[334]= -0.54875

In[335]:= Eigenvalues[matrixA]

Out[335]= {-0.57175, 0.0229996}

In[336]:= Eigenvectors[matrixA]

Out[336]= {{0.158485, 0.987361}, {0.962336, 0.271864}}
```

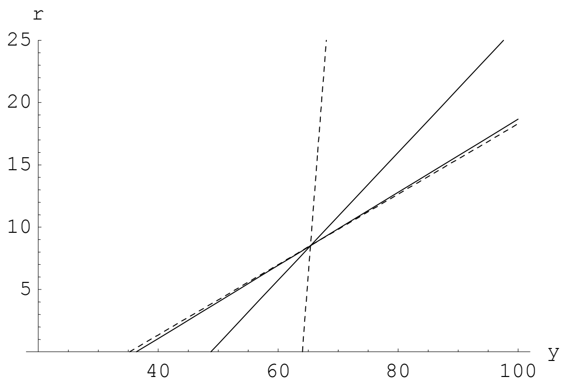
Let $u = y - y^*$ and $v = r - r^*$, then for the first eigenvalue we can solve,

```
In[337]:= Clear[u, v]

In[338]:= Simplify[Solve[0.05125 u - 0.1 v == -0.57175 u, v]]

Out[338]= {{v → 6.23 u}}
```

Using the second eigenvalue we obtain,

```
In[339]:= Clear[u, v]

In[340]:= Simplify[Solve[0.05125 u - 0.1 v == 0.0229996 u, v]]

Out[340]= {{v → 0.282504 u}}

In[341]:= islm = Plot[{-25 + 0.5125 y, -10.6667 + 0.29333 y}, {y, 20, 100}, PlotRange ->
               {0, 25}, AxesLabel -> {"y", "r"}, DisplayFunction -> Identity];

In[342]:= Clear[y, r]

In[343]:= Simplify[Solve[r - 8.51711 == 6.23 (y - 65.3992), r]]

Out[343]= {{r → -398.92 + 6.23 y}}

In[344]:= Clear[y, r]

In[345]:= Simplify[Solve[r - 8.51711 == 0.282504 (y - 65.3992), r]]

Out[345]= {{r → -9.95843 + 0.282504 y}}

In[346]:= saddles = Plot[{-398.92 + 6.23 y, -9.95843 + 0.282504 y},
             {y, 20, 100}, PlotRange -> {0, 25},
             PlotStyle -> Dashing[{0.01}], DisplayFunction -> Identity];
```

```
In[347]:= Show[islm, saddles, DisplayFunction -> $DisplayFunction];
```



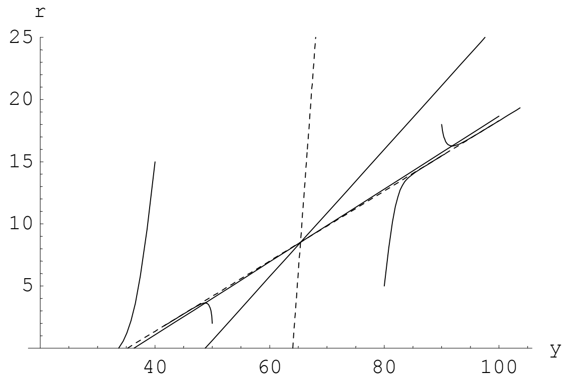
Taking the following four initial points, one from each sector,

$(y, r) = (40, 15)$ $(y, r) = (90, 18)$

$(y, r) = (80, 5)$ $(y, r) = (50, 2)$

```
In[348]:= sol121 = NDSolve[
            {y'[t] == -2.5 - 0.1 r[t] + 0.05125 y[t], r'[t] == -6.4 - 0.6 r[t] + 0.176 y[t],
             y[0] == 40, r[0] == 15}, {y, r}, {t, 0, 20}]

          General::spell : Possible spelling error: new
             symbol name "sol121" is similar to existing symbols {sol11, sol112}.

Out[348]= {{y → InterpolatingFunction[{{0., 20.}}, <>],
            r → InterpolatingFunction[{{0., 20.}}, <>]}}

In[349]:= sol122 = NDSolve[
            {y'[t] == -2.5 - 0.1 r[t] + 0.05125 y[t], r'[t] == -6.4 - 0.6 r[t] + 0.176 y[t],
             y[0] == 90, r[0] == 18}, {y, r}, {t, 0, 20}]

Out[349]= {{y → InterpolatingFunction[{{0., 20.}}, <>],
            r → InterpolatingFunction[{{0., 20.}}, <>]}}

In[350]:= sol123 = NDSolve[
            {y'[t] == -2.5 - 0.1 r[t] + 0.05125 y[t], r'[t] == -6.4 - 0.6 r[t] + 0.176 y[t],
             y[0] == 80, r[0] == 5}, {y, r}, {t, 0, 20}]

Out[350]= {{y → InterpolatingFunction[{{0., 20.}}, <>],
            r → InterpolatingFunction[{{0., 20.}}, <>]}}

In[351]:= sol124 = NDSolve[
            {y'[t] == -2.5 - 0.1 r[t] + 0.05125 y[t], r'[t] == -6.4 - 0.6 r[t] + 0.176 y[t],
             y[0] == 50, r[0] == 2}, {y, r}, {t, 0, 20}]

Out[351]= {{y → InterpolatingFunction[{{0., 20.}}, <>],
            r → InterpolatingFunction[{{0., 20.}}, <>]}}

In[352]:= trj121 = ParametricPlot[{y[t], r[t]} /. sol121,
             {t, 0, 20}, DisplayFunction -> Identity];

          ParametricPlot::ppcom : Function {y[t], r[t]} /. sol121
             cannot be compiled; plotting will proceed with the uncompiled function.

In[353]:= trj122 = ParametricPlot[{y[t], r[t]} /. sol122,
             {t, 0, 20}, DisplayFunction -> Identity];

          ParametricPlot::ppcom : Function {y[t], r[t]} /. sol122
             cannot be compiled; plotting will proceed with the uncompiled function.
```

```
In[354]:= trj123 = ParametricPlot[{y[t], r[t]} /. sol123,
             {t, 0, 20}, DisplayFunction -> Identity];

          ParametricPlot::ppcom : Function {y[t], r[t]} /. sol123
             cannot be compiled; plotting will proceed with the uncompiled function.

In[355]:= trj124 = ParametricPlot[{y[t], r[t]} /. sol124,
             {t, 0, 20}, DisplayFunction -> Identity];

          ParametricPlot::ppcom : Function {y[t], r[t]} /. sol124
             cannot be compiled; plotting will proceed with the uncompiled function.

In[356]:= Show[islm, saddles, trj121, trj122,
            trj123, trj124, DisplayFunction -> $DisplayFunction];
```



à Question 13

■ (i)

```
In[357]:= Clear[b1, k, u, y, m0, q]
```

Given

$$q = f(y) = \frac{b_1\, y}{(k\, y - \text{m0})/u}$$

with linear approximation

$$q = q^* + f'(y^*)\,(y - y^*)$$

then

```
In[358]:= Simplify[D[b1 y / ((k y - m0) / u), y]]
```

$$\text{Out[358]= } -\frac{\text{b1 m0 u}}{(\text{m0} - \text{k y})^2}$$

Hence,

$$q = q^* - \left[\frac{b_1\, m_0\, u}{(k\, y^* - m_0)^2}\right](y - y^*)$$

which is the same as the expression in the text on p.458, since $\frac{\text{dq}}{\text{dy}}$ can be simplified as shown here.

■ (ii)

First we need to solve for equilibrium y and q.

```
In[359]:= Solve[{0 == 14 - 0.4 y + 0.4 q, 0 == 1.25 q y - 0.1 y - 40 q}, {y, q}]

Out[359]= {{y → 31.3193, q → -3.68072}, {y → 35.7607, q → 0.760721}}
```

Ignoring the negative values, then

```
In[360]:= Simplify[Solve[q == 13.7604 - (0.1 (8) (0.2))/(((0.25) (35.7607) - 8)^2) (y - 35.7607), q]]

Out[360]= {{q → 20.2334 - 0.18101 y}}
```

To obtain the stable arm of the saddle point equilibrium, we note that

$$\dot{q} = \phi(q, y) = 1.25\,q\,y - 0.1\,y - 40\,q$$

with the linear approximation,

$$\dot{q} = \phi_y(y - y^*) + \phi_q(q - q^*)$$

and where ϕ_y and ϕ_q are both evaluated at the equilibrium point (y^*, q^*).

```
In[361]:= eq = 1.25 q y - 0.1 y - 40 q

Out[361]= -40 q - 0.1 y + 1.25 q y

In[362]:= D[eq, y] /. {y -> 35.7607, q → 0.7607}

Out[362]= 0.850875

In[363]:= D[eq, q] /. {y -> 35.7607, q → 0.7607}

Out[363]= 4.70087
```

The linear approximation of the system is, therefore,

$$\dot{y} = -0.4\,(y - y^*) + 0.4\,(q - q^*)$$
$$\dot{q} = 0.850875\,(y - y^*) + 4.70087\,(q - q^*)$$

whose matrix of the system is,

$$\mathbf{A} = \begin{pmatrix} -0.4 & 0.4 \\ 0.850875 & 4.70087 \end{pmatrix}$$

In order to derive the equation of the stable arm we need to obtain the eigenvectors of the stable arm.

Let us use *Mathematica* to obtain both the eigenvalues and the eigenvectors of the system.

```
In[364]:= matrixA = ( -0.4      0.4
                      0.850875  4.70087 )

Out[364]= {{-0.4, 0.4}, {0.850875, 4.70087}}

In[365]:= Eigenvalues[matrixA]

Out[365]= {4.76674, -0.465873}

In[366]:= Eigenvectors[matrixA]

Out[366]= {{-0.0771872, -0.997017}, {-0.986709, 0.162494}}
```

To see that these are very similar to those given on p.298 of the text, we readily establish

```
In[367]:= {{1, (-0.997017) / (-0.0771872)}, {1, 0.162494 / (-0.986709)}}

Out[367]= {{1, 12.9169}, {1, -0.164683}}
```

As indicated in the text, the stable arm is the eigenvector associated with the eigenvalue -0.465873. Hence,

$$q - q^* = -0.164683\,(y - y^*)$$

```
In[368]:= Simplify[Solve[q == 0.7607 - 0.164683 (y - 35.7607), q]]

Out[368]= {{q → 6.64988 - 0.164683 y}}

In[369]:= Simplify[Solve[0 == 0.850875 (y - 35.7607) + 4.70087 (q - 0.7607), q]]

Out[369]= {{q → 7.23352 - 0.181004 y}}
```

à Question 14

■ (i)

```
In[370]:= Clear[y, r]

In[371]:= Solve[{0 == 5 + 0.75 (1 - 0.25) y - 0.3 r + 0.4 y - y, 0 == 0.5 y - 0.3 r - 10}, {y, r}]

Out[371]= {{y → 27.907, r → 13.1783}}
```

■ (ii)

```
In[372]:= Simplify[Solve[0 == 5 + 0.75 (1 - 0.25) y - 0.3 r + 0.4 y - y, r]]

Out[372]= {{r → 16.6667 - 0.125 y}}

In[373]:= Simplify[Solve[0 == 0.5 y - 0.3 r - 10, r]]

Out[373]= {{r → -33.3333 + 1.66667 y}}
```

■ (iii)

```
In[374]:= Simplify[0.25 (5 + 0.75 (1 - 0.25) y - 0.3 r + 0.4 y - y)]

Out[374]= 1.25 - 0.075 r - 0.009375 y

In[375]:= Simplify[0.4 (0.5 y - 0.3 r - 10)]

Out[375]= -4. - 0.12 r + 0.2 y

In[376]:= mA = {{-0.009375, -0.075}, {0.2, -0.12}}

Out[376]= {{-0.009375, -0.075}, {0.2, -0.12}}

In[377]:= Tr[mA]

Out[377]= -0.129375

In[378]:= Det[mA]

Out[378]= 0.016125
```

```
In[379]:= Tr[mA]^2 - 4 Det[mA]

Out[379]= -0.0477621
```

Hence we have a stable sprial.

à Question 15

■ (i)

```
In[380]:= Clear[y, q]
```

First we need to solve for the equilibrium y and q.

```
In[381]:= Solve[{0 == 14 - 0.4 y + 0.4 q, 0 == 2 q y - 0.15 y - 64 q}, {y, q}]

Out[381]= {{y → 31.3549, q → -3.64514}, {y → 35.7201, q → 0.720137}}
```

Ignoring the negative values, then

In[382]:= $\text{Simplify}\left[\text{Solve}\left[q == 0.720137 - \frac{0.15\ (16)\ (0.25)\ (y - 35.7201)}{(0.5\ (35.7201) - 16)^2}, q\right]\right]$

```
Out[382]= {{q → 6.91476 - 0.173421 y}}
```

i.e.

$$q = 6.91476 - 0.173421\, y$$

To obtain the saddle arm of the saddle point equilibrium, we note that

$$\dot{q} = \phi(q,\, y) = 2\, q\, y - 0.15\, y - 64\, q$$

with linear approximation,

$$\dot{q} = \phi_y (y - \text{ye}) + \phi_q (q - \text{qe})$$

where ϕ_y and ϕ_q are both evaluated at the equilibrium point point (ye, qe).

```
In[383]:= eq = 2 q y - 0.15 y - 64 q

Out[383]= -64 q - 0.15 y + 2 q y

In[384]:= D[eq, y] /. q → 0.720137

Out[384]= 1.29027

In[385]:= D[eq, q] /. {q → 0.720137, y → 35.7201}

Out[385]= 7.4402
```

The linear approximation of the system is, therefore,

$$\dot{y} = (-0.4)\,(y - \text{ye}) + 0.4\,(q - \text{qe})$$
$$\dot{q} = 1.29027\,(y - \text{ye}) + 7.4402\,(q - \text{qe})$$

whose matrix of the system is,

$$\mathbf{A} = \begin{pmatrix} -0.4 & 0.4 \\ 1.29027 & 7.4402 \end{pmatrix}$$

In order to derive the equation of the stable arm we need to obtain the eigenvectors of the stable arm.

Let us use *Mathematica* to obtain both the eigenvalues and the eigenvectors of the system.

```
In[386]:= A = {{-0.4, 0.4}, {1.29027, 7.4402}}

Out[386]= {{-0.4, 0.4}, {1.29027, 7.4402}}

In[387]:= Eigenvalues[A]

Out[387]= {7.50548, -0.465285}

In[388]:= Eigenvectors[A]

Out[388]= {{-0.0505331, -0.998722}, {-0.986941, 0.161081}}
```

These can be expressed

```
In[389]:= {{1, -0.998722 / (-0.0505331)}, {1, 0.161081 / (-0.986941)}}

Out[389]= {{1, 19.7637}, {1, -0.163212}}
```

As indicated in the text, the stable arm is the eigenvector associated with the eigenvalue -0.465285. Hence
$q - \text{qe} = -0.163212\,(y - \text{ye})$

```
In[390]:= Simplify[Solve[q == 0.720137 - 0.163212 (y - 35.7201), q]]

Out[390]= {{q → 6.55009 - 0.163212 y}}
```

Solving for the linear approximation $\partial_t\, q = 0$

```
In[391]:= Simplify[Solve[0 == 1.29027 (y - 35.7201) + 7.4402 (q - 0.720137), q]]

Out[391]= {{q → 6.91467 - 0.173419 y}}
```