Guided TM Modes

For a TM mode, it is only necessary to find $\mathcal{H}_y$; then the other two nonvanishing field components $\mathcal{E}_x$ and $\mathcal{E}_z$ can be found by using (3.85) and (3.86), respectively. The boundary conditions require that $\mathcal{H}_y$, $\epsilon\mathcal{E}_x$, and $\mathcal{E}_z$ be continuous at the interfaces at $x = \pm d/2$ between layers of different refractive indices. From (3.85) and (3.86), it can be seen that these boundary conditions are equivalent to requiring $\mathcal{H}_y$ and $\epsilon^{-1}\partial\mathcal{H}_y/\partial x$, or $n^{-2}\partial\mathcal{H}_y/\partial x$, be continuous at these interfaces.

For a guided mode, we know that the transverse field patterns in the core, substrate, and cover regions are respectively characterized by the transverse field parameters h_1, γ_2, and γ_3, given in (3.131). A guided TM mode field distribution that satisfies the boundary conditions for the continuity of $\mathcal{H}_y$ at $x = \pm d/2$ has the form:

$$\hat{\mathcal{H}}_y = C_{\text{TM}} \begin{cases} \cos(h_1 d/2 - \psi)\exp[\gamma_3(d/2 - x)], & x > d/2, \\ \cos(h_1 x - \psi), & -d/2 < x < d/2, \\ \cos(h_1 d/2 + \psi)\exp[\gamma_2(d/2 + x)], & x < -d/2. \end{cases} \tag{3.139}$$

Application of the other two boundary conditions for the continuity of $n^{-2}\partial\mathcal{H}_y/\partial x$ at $x = \pm d/2$ yields two eigenvalue equations:

$$\tan h_1 d = \frac{\left(h_1/n_1^2\right)\left(\gamma_2/n_2^2 + \gamma_3/n_3^2\right)}{\left(h_1/n_1^2\right)^2 - \gamma_2\gamma_3/n_2^2 n_3^2} \tag{3.140}$$

and

$$\tan 2\psi = \frac{\left(h_1/n_1^2\right)\left(\gamma_2/n_2^2 - \gamma_3/n_3^2\right)}{\left(h_1/n_1^2\right)^2 + \gamma_2\gamma_3/n_2^2 n_3^2}. \tag{3.141}$$

A guided TM mode can be normalized using the orthonormality relation in (3.22) for

$$C_{\text{TM}} = \sqrt{\frac{\omega\mu_0 n_1^2}{\beta d_{\text{M}}}}, \tag{3.142}$$

where the effective waveguide thickness for a guided TM mode is

$$d_{\text{M}} = d + \frac{1}{\gamma_2 q_2} + \frac{1}{\gamma_3 q_3}, \quad \text{where} \quad q_2 = \frac{\beta^2}{k_1^2} + \frac{\beta^2}{k_2^2} - 1 \quad \text{and} \quad q_3 = \frac{\beta^2}{k_1^2} + \frac{\beta^2}{k_3^2} - 1. \tag{3.143}$$

Modal Dispersion

Guided modes have discrete allowed values of β. They are determined by the allowed values of h_1 because β and h_1 are directly related to each other through (3.131). Because γ_2 and γ_3 are uniquely determined by β through (3.131), they are also uniquely determined by h_1:

$$\gamma_2^2 d^2 = \beta^2 d^2 - k_2^2 d^2 = V^2 - h_1^2 d^2, \tag{3.144}$$

$$\gamma_3^2 d^2 = \beta^2 d^2 - k_3^2 d^2 = (1 + a_{\text{E}})V^2 - h_1^2 d^2. \tag{3.145}$$