Solutions

Problem 1.1 (p. 26)

a. (i) A is the ground; B is you. (ii) $\mathbf{F}_{\text{ground on you}}$. (iii) The force of you on the ground: $\mathbf{F}_{\text{you on ground}}$. (iv) downward.

b. (i) A is the earth; B is the stone. (ii) $\mathbf{F}_{\text{earth on stone}}$. (iii) The gravitational force of the stone on the earth: $\mathbf{F}_{\text{stone on earth}}$. (iv) upward.

c. (i) A is the branch; B is the cherry. (ii) $\mathbf{F}_{\text{branch on cherry}}$. (iii) The force of the cherry on the branch: $\mathbf{F}_{\text{stone on earth}}$. (iv) downward.

d. (i) A is the air; B is the hummingbird. (ii) $\mathbf{F}_{\text{air on hummingbird}}$. (iii) The force of the hummingbird on the air: $\mathbf{F}_{\text{hummingbird on air}}$. (iv) downward.

Problem 1.2 (p. 26)

a. passive, electromagnetic, and short range

b. active, gravitational, long range

c. passive, electromagnetic, and short range

d. passive, electromagnetic, and short range

Problem 1.3 (p. 26)

The units are

$$\frac{\mathrm{kg}^{-1}\,\mathrm{m}^3\,\mathrm{s}^{-2} \times \mathrm{kg}}{\mathrm{m}^2}, \tag{S.1}$$

which simplify to $\mathrm{m\,s}^{-2}$.

The powers of ten are

$$\frac{10^{-11} \times 10^{24}}{(10^6)^2}, \tag{S.2}$$

so the exponent of 10 is

$$-11 + 24 - 2 \times 6 = 1. \tag{S.3}$$

So the powers of 10 contribute 10^1.

And the mantissas are

$$\frac{7 \times 6}{6.4^2}. \tag{S.4}$$

The 6.4 in the denominator is almost exactly $\sqrt{40}$, so the mantissas contribute $\approx 42/40$ or roughly 1. Thus, the three-stage calculation gives, approximately,

$$1 \times 10^1\ \mathrm{m\,s}^{-2}. \tag{S.5}$$

Problem 1.4 (p. 27)

With approximate values for the moon's mass and its orbital radius,

$$F_g \approx \frac{\overbrace{7 \times 10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}}^{G} \times \overbrace{7 \times 10^{22}\,\text{kg}}^{m_{\text{moon}}} \times \overbrace{6 \times 10^{24}\,\text{kg}}^{m_{\text{earth}}}}{\big(\underbrace{4 \times 10^{8}\,\text{m}}_{r_{\text{earth–moon}}}\big)^2}. \tag{S.6}$$

A three-stage calculation (units, powers of ten, and mantissas) gives

$$F_g \approx 2 \times 10^{20}\,\text{N}. \tag{S.7}$$

It's roughly 1/200th of the gravitational force of the sun on the earth. Although the moon's mass is a minuscule fraction of the sun's mass, the moon's relative proximity to the earth makes its gravitational force on the earth much greater than a minuscule fraction of the sun's gravitational force on the earth.

Problem 1.5 (p. 27)

The comparison is between

$$\frac{Gm_{\text{earth}}}{R^2_{\text{earth}}} \approx \frac{7 \times 10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2} \times 6 \times 10^{24}\,\text{kg}}{(6.4 \times 10^6\,\text{m})^2} \tag{S.8}$$

and

$$\frac{Gm_{\text{earth}}}{R^2_{\text{earth}}} \approx \frac{7 \times 10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2} \times 6 \times 10^{24}\,\text{kg}}{(6.4 \times 10^6\,\text{m} + 200\,\text{km})^2}. \tag{S.9}$$

In other words, the distance r increased by 200 kilometers, a 3-percent increase on the baseline distance of 6400 kilometers (6.4×10^6 meters). Due to the exponent of -2 on the distance (representing an inverse-square force law), the result is a 6-percent decrease in g.

That fractional-change argument is a bit telegraphic. But a brute-force numerical calculation confirms it. The g at sea level, from (S.8), is 10.25 meters per second squared. The g in low-earth orbit, from (S.9), is 9.64 meters per second squared, which is 5.95 percent smaller than the g at sea level. (The g at sea level from (S.8) is somewhat different from the standard value of 9.8 meters per second squared due to its use of approximate values of the earth's mass and radius.)

Problem 1.6 (p. 27)

In college, I captained our dormitory's Ultimate Frisbee team when we played against a team with Eric Heiden, the Olympic speed skater. Although it was many years after his 1980 Winter Olympics gold-medal performances, he was still an imposing athlete. His arms were thicker than my thighs, his thighs were thicker than my torso, and he could run faster than the wind. I use a faded mental picture of him as my model as I make rough guesses for a speed skater's mass (for calculating dynamic friction), speed, and cross-sectional area: $m \sim 100$ kilograms (probably an overestimate), $v \sim 10$–15 meters per second (skating is probably slightly more efficient than running on land, where a 10-meters-per-second speed was once a world-record pace), and $A \sim 0.5$ square meters.

Then the drag force is

$$F_{\text{drag}} \sim 1\,\text{kg}\,\text{m}^{-3} \times (14\,\text{m}\,\text{s}^{-1})^2 \times 0.5\,\text{m}^2 \sim 100\,\text{N}. \tag{S.10}$$

The dynamic-friction force depends on the normal force. For a skater whose center of mass is moving mostly horizontally (which anyway is more efficient), $N \approx mg$, so

$$F_\mu = \mu N \approx \mu m g \approx 0.001 \times 100\,\text{kg} \times 10\,\text{m}\,\text{s}^{-2} = 1\,\text{N}. \tag{S.11}$$

The drag force is 100 times larger than the friction!

Problem 1.7 (p. 27)

It's the static-friction force of the box on the hill. It points down the slope.

Problem 1.8 (p. 27)

a.
$$F_g \approx \frac{\overbrace{7\times10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}}^{G}\times\overbrace{6\times10^{24}\,\text{kg}}^{m_\text{earth}}\times\overbrace{9\times10^{-31}\,\text{kg}}^{m_\text{electron}}}{\big(\underbrace{6.4\times10^{6}\,\text{m}}_{R_\text{earth}}\big)^2}$$
$$\approx 10^{-29}\,\text{N}$$

b.
$$F_g \approx \frac{\overbrace{7\times10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}}^{G}\times\overbrace{60\,\text{kg}}^{m_1}\times\overbrace{60\,\text{kg}}^{m_2}}{\big(\underbrace{1.3\times10^{7}\,\text{m}}_{2R_\text{earth}}\big)^2}$$
$$\approx 1.5\times10^{-21}\,\text{N}$$

c. $F_\text{drag} \sim \underbrace{0.3\,\text{kg}\,\text{m}^{-3}}_{\rho_\text{high-altitude air}}\times\big(\underbrace{250\,\text{m}\,\text{s}^{-1}}_{v}\big)^2\times\underbrace{40\,\text{m}^2}_{A_\text{cs}}\sim 10^6\,\text{N}.$

d.
$$F_g \approx \frac{\overbrace{7\times10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}}^{G}\times\overbrace{2\times10^{30}\,\text{kg}}^{m_\text{sun}}\times\overbrace{4\times10^{30}\,\text{kg}}^{m_\text{Sirius}}}{\big(\underbrace{8\times10^{16}\,\text{m}}_{d_\text{sun–Sirius}}\big)^2}$$
$$\approx 10^{17}\,\text{N}$$

Problem 2.1 (p. 38)

Ask which kind of fundamental force such a force would be. It cannot be gravitational because any gravitational force points downward (to the center of the earth). It cannot be electromagnetic because the ball isn't in contact with anything. And it's definitely not a nuclear force. So, it's none of the four fundamental types of force – so, it's not a force at all and doesn't belong on the freebody diagram.

Problem 2.2 (p. 38)

In labeling the forces on the track (Figure S.1) and on the earth (Figure S.2), I use "earth" when the earth–ground participates in a gravitational interaction and "ground" when it participates in a contact (electromagnetic) interaction.

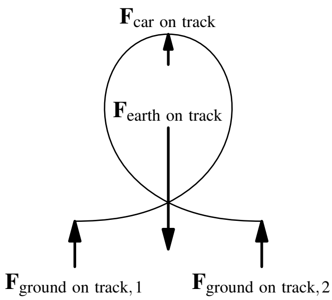

Figure S.1 Freebody diagram of the track. The gravitational force, $\mathbf{F}_{\text{earth on track}}$, is drawn with its tip at the center of mass of the track as reminder that this force is a long-range or body force.

For the freebody diagram of the earth (including the ground as part of the earth):

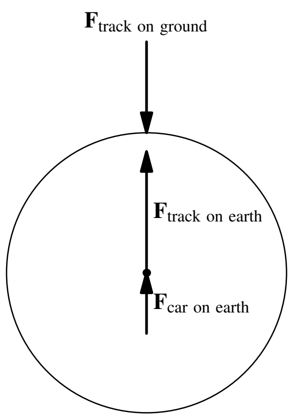

Figure S.2 Freebody diagram of the earth. As a reminder that $\mathbf{F}_{\text{track on earth}}$ and $\mathbf{F}_{\text{car on earth}}$ are long-range or body forces (they are gravitational forces), they are drawn with their tail or tip, respectively, at the earth's center of mass.

Problem 3.1 (p. 46)

Figure S.3 shows the rod lattices drawn for the same two positions of the earth in its orbit as shown in the text.

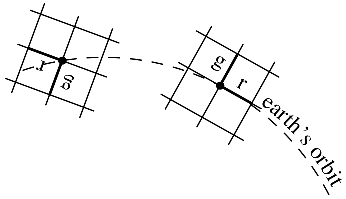

Figure S.3 The rotating and translating frame of the earth in orbit around the sun. These lattices change their location and orientation.

In comparison to the frame moving with the center of the earth but not rotating with the earth, these lattices also change their orientation (which you can see by the changing orientation of the "r" and "g" rods that label the frame's two axes).

Problem 3.2 (p. 46)

As seen from the accelerating train, the rock still has its northward speed of 30 meters per second. But, its backward (westward) speed is now, due to the train's acceleration, not constant at 40 meters per second. Instead, that speed is changing. Thus, what was once a straight path tilted backward at roughly 55 degrees becomes a curved path that curves ever-more backward.

As an illustration, imagine that the train's acceleration is 4 meters per second squared forward (eastward). This acceleration is quite large for a train, but the exaggeration makes the curved nature of the path evident. Figure S.4 shows the resulting path as seen from the train frame.

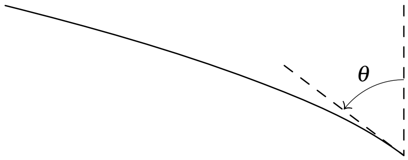

Figure S.4 The path of the rock, as seen in the train frame. As the train travels ever faster eastward, the rock's path approaches due west.

The rock participates in no (horizontal) interactions, yet it moves in a curved line (in the horizontal plane), meaning that its velocity isn't constant. Thus, the accelerating-train frame isn't an inertial frame.

Problem 5.1 (p. 84)

I sometimes find such a triangle hard to analyze because it hangs in midair (Figure S.5a) rather than lying with one leg horizontal and the other vertical. Thus, let's rotate it so that it lies in this more familiar orientation (Figure S.5b).

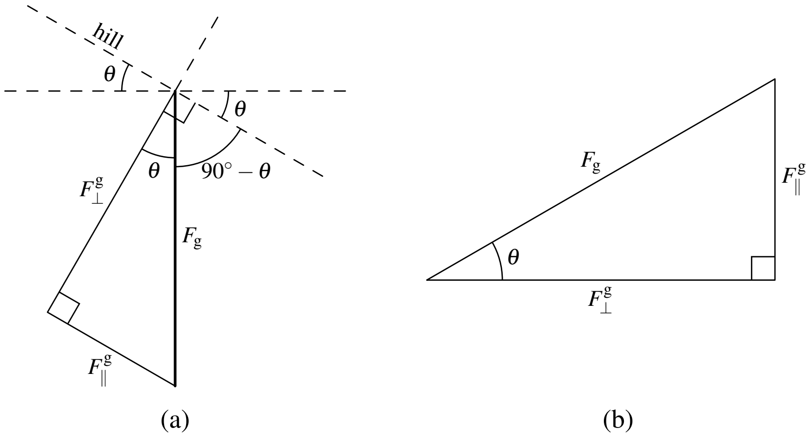

Figure S.5 Right triangles for resolving the gravitational force into parallel and perpendicular portions. (a) This triangle's hypotenuse is (with the addition of a downward arrow) the gravitational force. However, this orientation is unfamiliar, making the trigonometry harder to see. (b) The same triangle has been rotated so that one leg is horizontal, the other leg is vertical, and the hypotenuse is tilted. Because of the changed direction, the hypotenuse is no longer the gravitational force itself, but its length is still F_g.

From trigonometry, the altitude over the hypotenuse is $\sin\theta$, so $F^{g}_{\parallel} = F_g \sin\theta$. Because $F_g = mg$,

$$F^{g}_{\parallel} = mg \sin\theta. \tag{S.12}$$

Similarly, the base over the hypotenuse is $\cos\theta$, so

$$F^{g}_{\perp} = mg \cos\theta. \tag{S.13}$$

Problem 5.2 (p. 84)

First I analyze the new slope and with no dynamic friction. The contact force, calculated in (5.10), becomes

$$F_{\text{contact}} = mg \underbrace{\cos 20^\circ}_{\approx 0.9} \approx 630\,\text{N}. \tag{S.14}$$

The drag force, calculated in (5.11), becomes

$$F_{\text{drag}} = mg \underbrace{\sin 20^\circ}_{\approx 1/3} \approx 230\,\text{N}. \tag{S.15}$$

My terminal speed, calculated in (5.12), becomes

$$v \sim \sqrt{\frac{F_{\text{drag}}}{\rho A_{\text{cs}}}} \approx \sqrt{\frac{230\,\text{N}}{1\,\text{kg}\,\text{m}^{-3} \times 0.25\,\text{m}^2}} = \sqrt{920}\,\text{m}\,\text{s}^{-1} \approx 30\,\text{m}\,\text{s}^{-1} \tag{S.16}$$

(roughly 65 miles or 110 kilometers per hour). It's still fast sledding, but it feels plausible for sledding without air drag down an infinitely long hill (the infinite length lets me reach my terminal speed).

In symbols, for the contact force, (5.10) and (S.14) generalize to

$$F_{\text{contact}} = mg \cos\theta. \tag{S.17}$$

Similarly, for the drag force at the terminal speed, (5.11) and (S.15) generalize to

$$F_{\text{drag}} = mg \sin\theta. \tag{S.18}$$

For the terminal speed, (5.12) and (S.16) generalize to

$$v \sim \sqrt{\frac{F_{\text{drag}}}{\rho A_{\text{cs}}}} = \sqrt{\frac{mg \sin\theta}{\rho A_{\text{cs}}}}. \tag{S.19}$$

Now I include dynamic friction. It doesn't affect the normal (perpendicular) portion **N** of the contact force, whose magnitude is still roughly 630 newtons. It does, however, make the contact force have a parallel portion with magnitude

$$F_\mu = \mu N \approx 0.3 \times 630\,\text{N} \approx 190\,\text{N}. \tag{S.20}$$

The parallel portion $\mathbf{F}_\mu$ and the drag force together balance the parallel portion of the gravitational force. Thus, the drag force (at my terminal speed) is smaller than it was without dynamic friction.

$$F_{\text{drag}} = mg \sin\theta - F_\mu \approx 230\,\text{N} - 190\,\text{N} = 40\,\text{N}. \tag{S.21}$$

The terminal speed, calculated for the 30-degree slope in (5.14), becomes

$$v \sim \sqrt{\frac{40\,\text{N}}{1\,\text{kg}\,\text{m}^{-3} \times 0.25\,\text{m}^2}} = \sqrt{160}\,\text{m}\,\text{s}^{-1} \approx 13\,\text{m}\,\text{s}^{-1} \tag{S.22}$$

(roughly 28 miles or 45 kilometers per hour). That speed matches my experience reasonably sledding down a nearby long hill after a snowstorm.

Problem 5.3 (p. 84)

In this special case, strings 1 and 3 exert only vertical forces on the knot. However, string 2's force on the knot has a horizontal portion. Because neither string 1 nor string 3 can balance this horizontal portion, the knot would have a nonzero net force – very bad! The only way out is for string 2 to have zero tension ($T_2 = 0$), which makes the troublesome horizontal portion also zero. Then string 1 and string 3 act like one longer string, and the tension is mg throughout it ($T_1 = T_3 = mg$).

Now compare these predictions with the calculations.

The calculated string tensions in (5.19) give, when $\theta_3 = 90°$,

$$\begin{aligned} T_1 &= mg; \\ T_2 &= mg\,\frac{0}{\sin(\theta_2 + 90°)}; \\ T_3 &= mg\,\frac{\cos\theta_2}{\sin(\theta_2 + 90°)}. \end{aligned} \tag{S.23}$$

The repeated form $\sin(\theta_2 + 90°)$ can be simplified using a trigonometry identity, slightly massaged. Start with the fundamental symmetry relation between sine and cosine:

$$\sin(90° - \theta) = \cos\theta. \tag{S.24}$$

Now replace θ by $-\theta_2$:

$$\sin(90° + \theta_2) = \cos(-\theta_2). \tag{S.25}$$

But cosine is a symmetry function – namely, $\cos x = \cos(-x)$. So

$$\sin(90° + \theta_2) = \cos\theta_2. \tag{S.26}$$

With this simplification, the special-case tensions in (S.23) become

$$\begin{aligned} T_1 &= mg; \\ T_2 &= mg\,\frac{0}{\cos\theta_2} = 0; \\ T_3 &= mg\,\frac{\cos\theta_2}{\cos\theta_2} = mg. \end{aligned} \tag{S.27}$$

These results agree with the predictions, which increases our confidence in the general formulas (5.19).

Problem 5.4 (p. 84)

It's not possible. The proof is by contradiction. Assume that it *is* possible: that we are moving at the same velocity without accelerating. The parallel portion of the contact force is dynamic friction, which opposes the velocity. Because our velocities are the same, the friction forces on us point in the same direction. If this force balances the string force on one of us, it cannot do so on the other of us. Thus, at least one of us must be accelerating. But that conclusion contradicts the assumption. Therefore, the assumption (that it's possible) is false.

Problem 5.5 (p. 85)

To redo an analysis, use as much of the original analysis as possible, thinking about what in it remains the same and what must change to reflect the new conditions.

The string force on me and the strong force on you still have equal magnitude – otherwise the string would have a nonzero net force on it (impossible for a nonaccelerating body, whether or not it's massless, and impossible for a massless body, whether or not it's accelerating). Thus, the static-friction force on you and on me also have equal magnitudes (because the static-friction force balances the string force).

But the gravitational force on me is now twice as large as on you. The normal force balances the gravitational force, so the normal force on me is twice as large as the normal force on you. The contact force is the normal force plus the static-friction force, so the contact force on me and on you are no longer mirror images of one another.

Figure S.6 shows our freebody diagrams. In this new situation where I have double your mass, I lean less than you do (when our masses were equal, we leaned equally). As I mentioned in the original analysis, the lean cannot be calculated using only Newton's laws; it requires also the ideas of torque and angular momentum. But everyday experience should convince you that my lean would be less than yours and that, if it were the same as yours, I would fall over backward. As an easier and extreme case, imagine playing tug-of-war against an extremely dense slab – imagine neutron-star matter in the shape of a person or of the obelisk in Stanley Kubrick's film "2001: A Space Odyssey." The slab would remain upright: No matter how hard you pull, you couldn't tip it over.

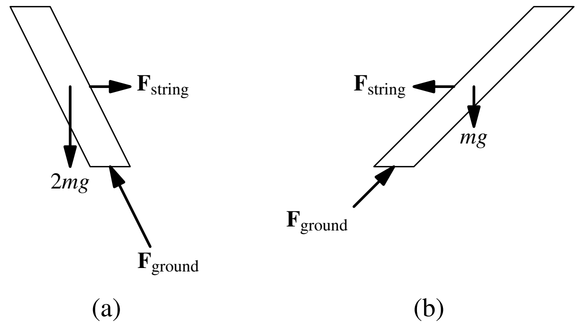

Figure S.6 Freebody diagrams of the contestants. (a) My freebody diagram. My mass has become $2m$, which doubles the gravitational force and changes the contact force in magnitude (it gets stronger) and direction (it gets more vertical). (b) Freebody diagram of you (your mass is still m), which hasn't changed.

The string's freebody diagram doesn't change (Figure S.7a). And the earth's freebody diagram changes to reflect the changes in our freebody diagrams (Figure S.7b).

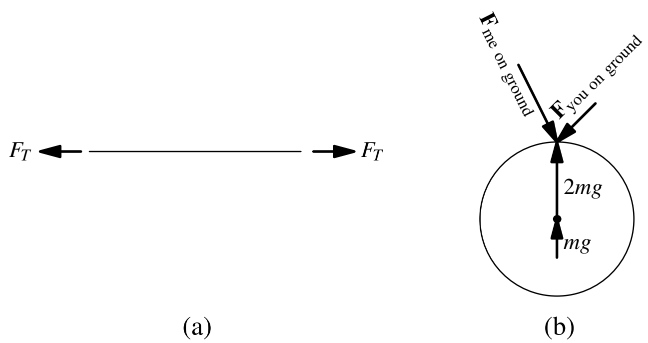

Figure S.7 The remaining freebody diagrams. (a) The freebody diagram of the string (which hasn't changed). (b) The freebody diagram of the earth (which changes to reflect the changes in the contact and gravitational forces on me).

Figure S.8 shows all four freebody diagrams and the interactions, which necessarily cross diagrams.

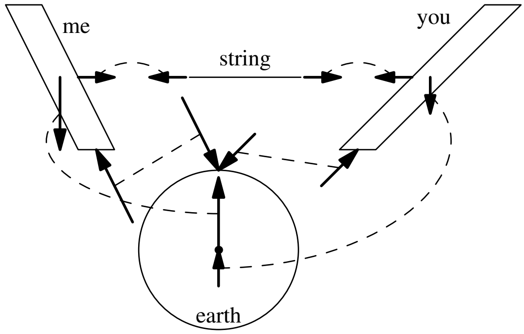

Figure S.8 All four freebody diagrams showing the third-law pairs (connected with dashed lines).

Problem 5.6 (p. 85)

The key word here is "consistent." When the rope gets mass and a gravitational force but the diagrams otherwise remain unchanged, then the rope's freebody diagram shows a nonzero net force: The vertical gravitational force cannot be balanced by either contact force, so the net force on the rope is downward. Thus, the rope would accelerate downward, which contradicts the assumption of no acceleration.

To fix this problem, the contact forces on the rope must have upward portions that, taken together, balance the gravitational force. In other words, the rope sags. Another way of getting to the same conclusion is to discard, for a moment, the assumption of no acceleration. Then, the rope accelerates downward until it sags just the right amount to make the net force zero.

The contact forces on us from the rope are, by Newton's third law, equal and opposite to the contact forces on the rope. Thus, the rope forces (on us) have downward portions.

Besides the change to the orientations of these four forces, the other change is a new third-law pair, from the gravitational interaction between the rope and the earth.

Figures S.9 and S.10 show the resulting freebody diagrams.

The remaining freebody diagrams. (a) Freebody diagram of the sagging rope, with the contact forces on it pointing slightly upward. (b) Freebody diagram of the string.

Figure S.11 shows all four diagrams showing the third-law pairs crossing between diagrams.

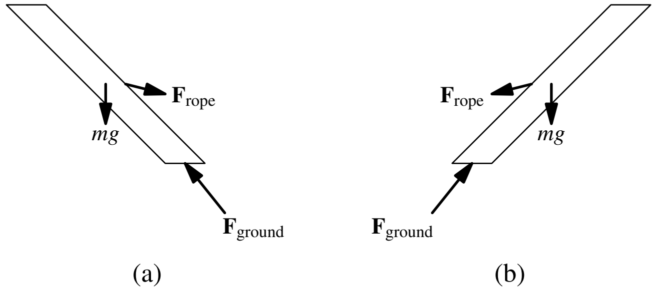

Figure S.9 Freebody diagrams when we pull on a rope (a string with mass). Because the rope sags, the contact force of the rope on either of us points slightly downward (the contact force of either of us on the rope points slightly upward, to balance gravity). (a) My freebody diagram. (b) Your freebody diagram.

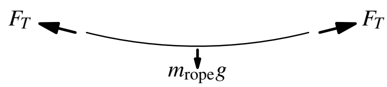

Figure S.10 tug-of-war-rope-correct-fbd-of-earth.pdf

Problem 5.7 (p. 85)

Pulley 2 is the interesting one because it movable: Unlike the pivot of pulley 1 and of Problem 5.10, pulley 2's pivot does not prevent the pulley from moving. Thus, pulley 2 (considered as including its pivot, which moves with it) experiences two forces: from the main pulley string and from the short string holding up the mass.

The short string has tension mg, so it exerts a force mg downward on the pulley.

To find the effect of the main string: In the solution to Problem 5.10, you saw that a string with tension mg exerted a force with magnitude $2mg$ on the pulley around which it was wrapped in the same configuration as the string around this pulley. Thus, the main string exerts a force $2T$ upward.

Because the pulley is massless, no gravitational force acts on it, and the two contact forces are the only forces. Also because the pulley is massless (and because it's not accelerating), the two contact forces balance. Thus,

$$2T = mg \tag{S.28}$$

and

$$T = \frac{mg}{2}. \tag{S.29}$$

Amazingly, T is less than mg. And that result is the reason for this pulley arrangement. It allows you to lift a mass using a force much smaller than the gravitational force on the mass. This factor-of-2 benefit comes with a factor-of-2 cost: To raise the mass by a distance y, you must pull the string by a distance $2y$.

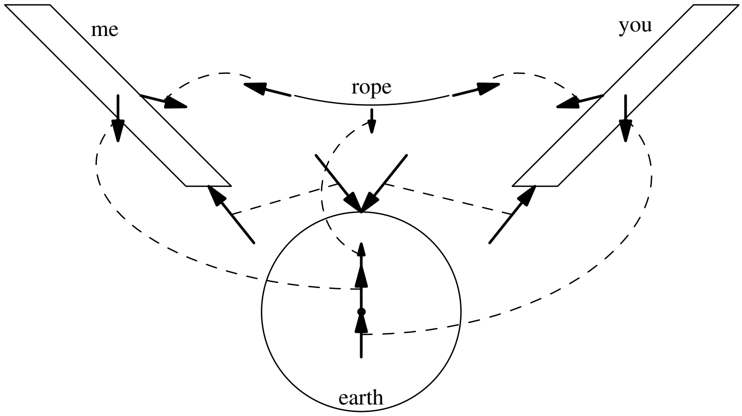

Figure S.11 All four freebody diagrams, with third-law pairs connected by dashed paths.

Problem 5.8 (p. 85)

a. To make the freebody diagrams, start with the top block because you know its freebody diagram already (Figure S.12a): It's the freebody diagram of a body sitting on the ground, with the middle block playing the role of the ground. The top block participates in two interactions: the gravitational interaction with the earth and the contact interaction with the middle block. The result is a two-force diagram, where the contact force balances the gravitational force, leaving the block with no net force and therefore no acceleration. Thus, each force has magnitude $mg/3$.

Now you can make the diagram for the middle block (Figure S.12b). It's similar to the diagram for the top block, but there is a second contact interaction (and therefore an additional force): with the top block. One side of this interaction is the contact force on the top block, which has magnitude $mg/3$. In symbols,

$$\mathbf{F}_{\text{middle on top}} = \frac{mg}{3} \text{ upward.} \tag{S.30}$$

From Newton's third law,

$$\mathbf{F}_{\text{top on middle}} = -\mathbf{F}_{\text{middle on top}} = \frac{mg}{3} \text{ downward.} \tag{S.31}$$

The top block presses the middle block downward.

► *What's the magnitude of the other contact force?*

The three forces, including the contact force from the bottom block, add to zero. Two of these forces are the gravitational force, $mg/3$ downward, and

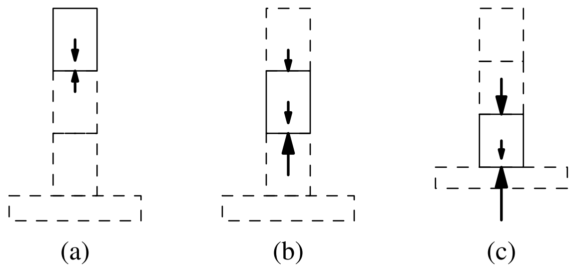

Figure S.12 Freebody diagrams of the blocks. (a) The top block (with the other blocks in ghostly outline, per step 3 of the freebody-diagram procedure in Section 2.1). It's just a body sitting on the ground (with that role played by the middle block), so it experiences the usual two forces that balance. (b) The middle block. Unlike the top block, this block experiences the contact force on its head pushing it down. This force and the gravitational force are balanced by the upward contact force from the bottom block. (c) The bottom block. The gravitational force and the downward contact force, twice as large as the analogous force on the middle block, are balanced by the full-strength upward contact force from the ground.

the contact force from above, also $mg/3$ downward. Thus, the third force must be $2mg/3$ upward. Now the middle block's diagram is complete.

For the bottom block (Figure S.12c), the reasoning follows the same structure as for the middle block. The gravitational force is still $mg/3$ downward. The two contact forces acting on it are from the middle block, providing $2mg/3$ downward, and from the ground, providing mg upward.

Thus, going downward from head to toe, each piece of you feels ever stronger compressive forces acting at its top and bottom surfaces. The rate of increase of these forces depends on m and g. This increasing stress, including its rate of increase, provides the familiar feeling of being earth-bound creatures confined to the earth by gravity. It's what our bones grew to expect. Changes in this distribution of forces – whether by changing g (for example, going to another planet) or by changing the contact forces (for example, by going into orbit) – create strange internal feelings to which astronauts learn slowly to adapt.

The fourth and final diagram is for the earth. The earth participates in three long-range, gravitational interactions, one with each block. The earth – in particular, its surface, the ground – also participates in the contact interaction with the bottom block. Thus, four forces act on the earth (Figure S.13).

The gravitational forces are each $mg/3$ upward – by Newton's third law. The contact force is mg downward – again by Newton's third law. The sum of these forces (the net force) is zero. Thus, the earth does not accelerate. Indeed, it's squeezed between the gravitational forces and the contact forces

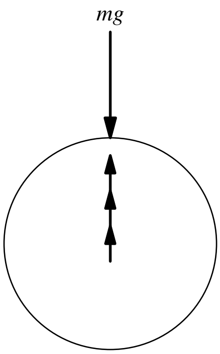

Figure S.13 The freebody diagram of the earth (taken to include the ground). It participates in four interactions – three gravitational and one contact – and therefore experiences four forces. The three gravitational forces, each $mg/3$ upward, together balance the contact force, which is mg upward.

(just as you are, although the one-piece model of the earth does not make this squeezing as clear as does the three-piece model of a person).

b. The next problem is to identify pairs of forces that are "equal and opposite" – that is, have the same magnitude but opposite directions – and decide, for each pair, whether it's so due to the second or the third law. Only the third-law pairs get connected by dashed lines.

The rule for determining which law to apply is simple. If the two forces are two sides of a single interaction, in which case they necessarily act on separate bodies, then they are equal and opposite due to Newton's third law. If the two forces act on the same body and the body isn't accelerating, then they are equal and opposite due to Newton's second law.

I've started the analysis, by pairing up the eight $mg/3$ forces.

a. the contact force on the top block,
b. the $mg/3$ contact force on the middle block,
c. the gravitational force on the top block,
d. the gravitational force of the top block on the earth,
e. the gravitational force on the middle block,
f. the gravitational force of the middle block on the earth,
g. the gravitational force on the bottom block, and
h. the gravitational force of the bottom block on the earth.

In this group, the first two forces are the two sides of the contact interaction between the top and middle blocks, so they are connected by Newton's third law. The first and third forces both act on the same block, and produce the zero net force on it, so they are connected by Newton's second law. The third and fourth forces are the two sides of the gravitational interaction between the top block and the earth, so they are equal and opposite due to Newton's third law. Similarly, the fifth and sixth forces are equal and opposite due to Newton's third law, as are the seventh and eight forces.

Problem 5.9 (p. 86)

Let the tension be T_1 on the left end and T_2 on the right end (where $T_1 \neq T_2$). The string's freebody diagram is show in Figure S.14.

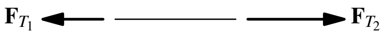

Figure S.14 The freebody diagram of a (straight) string where the tension varies from T_1 to T_2. By assumption, $T_1 \neq T_2$, so the two forces don't balance.

The two contact forces don't balance, giving a nonzero net force on the string. The result is infinite acceleration, which is a contradiction. Thus, in this case of a string with contact forces only at its ends, the two tensions must be equal.

I haven't yet used the assumption that the string is frictionless. It's needed when I try to rescue the preceding situation by placing the string in contact with other objects, in order to prevent it from accelerating. These contacts produce a contact force on the string (which need not be at either end). However, the string is frictionless, so the contact force has no parallel portion and is therefore perpendicular to the string. This perpendicular force cannot balance the net horizontal force. Thus, the string still has an infinite acceleration.

In short, the tensions at the two ends must be equal.

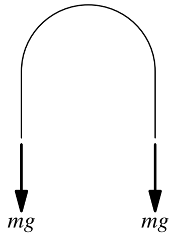

Figure S.15 An incomplete freebody diagram of the string. The string experiences a nonzero net force, which gives it an infinite acceleration. Thus, the diagram lacks at least one force.

Problem 5.10 (p. 86)

Following my own advice, I first make a freebody diagram of the string. The tension in the string is mg, and the contact interaction at each end produces a downward force mg at each end (Figure S.15).

An easy trap to fall into, having landing there myself, is to think that this diagram is complete. However, it cannot be, as it implies a nonzero net force on the string, which (by definition) is massless and therefore would get an infinite acceleration. Very bad!

To find the missing force that prevents the infinite acceleration, I let the world reason for me. I imagine that the string does accelerate as directed by the two downward tension forces. In other words, the string starts accelerating downward. Then I ask myself, "What body prevents this downward acceleration?" In other words, why doesn't it just jet ever faster downward? Answer: The pulley!

Thus, the missing force is the contact force of the pulley (Figure S.16). In order to balance the two tension forces, it must be $2mg$ upward. Now the string's freebody diagram is complete: Each of its three contact interactions, with the two masses and with the pulley, has contributed one of the three forces acting on it.

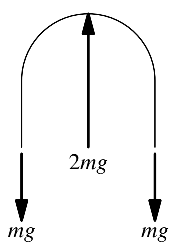

Figure S.16 The complete freebody diagram of the string. The contact force of the pulley makes the net force zero and prevents the string from accelerating.

Now I can make the freebody diagram of the pulley (Figure S.17). It experiences a gravitational force $m_{\text{pulley}}g$ downward. It also experiences the other side of the string–pulley interaction, which produces a contact force $2mg$ downward. But the story cannot end there, or the pulley would have only downward forces on it. Although it has mass and therefore wouldn't have an infinite acceleration, it still would accelerate downward.

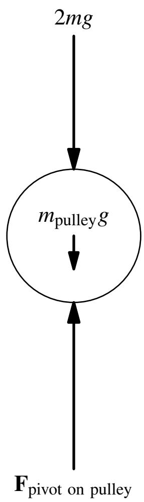

Figure S.17 The freebody diagram of the pulley. The force of the pivot on the pulley actually acts at the center of the wheel (at the pivot). However, I have placed it with its tip at the edge of the wheel for two reasons. First, it doesn't overlap the gravitational force. Second, its new location reminds us that it's a contact force (based on the convention that a contact force has its tip or tail at the body's surface).

► *What body prevents this acceleration (and resulting motion)?*

The pivot! Thus, I forgot the contact force of the pivot on the pulley; in other words, I forgot the pivot–pulley interaction. Because the net force on the pulley is zero, this contact force must be $(2m + m_{\text{pulley}})g$ upward.

$$F_{\text{pivot on pulley}} = \left(2m + m_{\text{pulley}}\right)g. \tag{S.32}$$

My drawing is slightly misleading about the point of application of $\mathbf{F}_{\text{pivot on pulley}}$. It actually acts at the middle of the pulley (rather than at the bottom of the pulley wheel, as shown in the diagram). But if I had drawn it there, it would have looked like an upward, long-range force (antigravity).

Problem 5.11 (p. 86)

From (5.23), the pressure p versus "depth" h below the top of the atmosphere is

$$p = \rho_0 g h, \tag{S.33}$$

where ρ_0 is the sea-level density and where p_0, the pressure at the top of the pool, disappeared because it's zero. When $h = H$, the pressure p is the sea-level pressure. Thus,

$$H = \frac{p_{\text{sea level}}}{\rho_0 g}, \tag{S.34}$$

so

$$H \approx \frac{\overbrace{10^5\,\text{Pa}}^{p_{\text{sea level}}}}{\underbrace{1.2\,\text{kg}\,\text{m}^{-3}}_{\rho_0} \times \underbrace{10\,\text{m}\,\text{s}^{-2}}_{g}} = 8 \times 10^3\,\text{m}, \tag{S.35}$$

or 8 kilometers (a little lower than Mount Everest). For the sea-level density, I've used the more accurate value of 1.2 kilograms per cubic meter, rather than the convenient but rough value of 1 kilogram per cubic meter, in order to get an estimate of H accurate enough to use in the more realistic model of Problem 5.15.

In that more realistic model, the air density decreases with height. The atmosphere then doesn't abruptly disappear at 8 kilometers. Rather, it fades slowly with height and, by 8 kilometers, has decreased significantly in density and pressure (roughly by a factor of 3).

Problem 5.12 (p. 86)

a. As with ropes, make a freebody diagram of a segment of the cable. In particular, take a segment of length z starting at the bottom of the cable. It participates in two interactions: the long-range, gravitational interaction with the earth and the short-range, contact interaction with the rest of the cable.

   Thus, it experiences two forces (Figure S.18). The gravitational force is mg downward, where m is the segment's mass. The segment has volume Az, so it has mass ρAz. Thus, the gravitational force is ρAzg downward. Meanwhile, the contact force points upward and has magnitude $T(z)$.

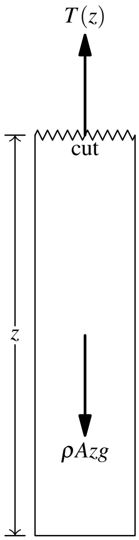

Figure S.18 The freebody diagram of a segment of the cable. The segment, which starts at the bottom of the cable, has length z and mass ρAz. It experiences two forces: the gravitational force and the contact force from the cable above it.

The cable segment isn't accelerating, so the two forces balance:

$$T(z) = \rho Azg. \tag{S.36}$$

This formula is similar to the pressure in the lake given by (5.23), as it should be. Newton's laws don't care whether the substance in question is a solid, such as the steel in this cable, or a liquid, such as the water in the lake. The main difference between tension and pressure is that the tension includes a factor for area whereas pressure is akin to force per area. Thus, the tension (S.36) contains a factor of A absent from the pressure (5.23).

b. Based on the tension in (S.36), the maximum tension is when z is a maximum, at the top of the cable where $z = l$. Thus,

$$T_{\max} = \rho Alg. \tag{S.37}$$

The maximum tensile stress $\sigma_{\max}$ is $T_{\max}/A$, so

$$\sigma_{\max} = \rho lg. \tag{S.38}$$

Solving for l in terms of $\sigma_{\max}$,

$$l = \frac{\sigma_{\max}}{\rho g}. \tag{S.39}$$

For steel, the breaking strength is roughly 10^9 pascals, and ρ is roughly 7×10^3 kilograms per cubic meter (7 times denser than water). With those values, the cable's maximum length is

$$l \approx \frac{10^9\,\mathrm{Pa}}{7\times 10^3\,\mathrm{kg\,m^{-3}} \times 10\,\mathrm{m\,s^{-2}}} \approx 1.4\times 10^4\,\mathrm{m}, \qquad \text{(S.40)}$$

or about 14 kilometers.

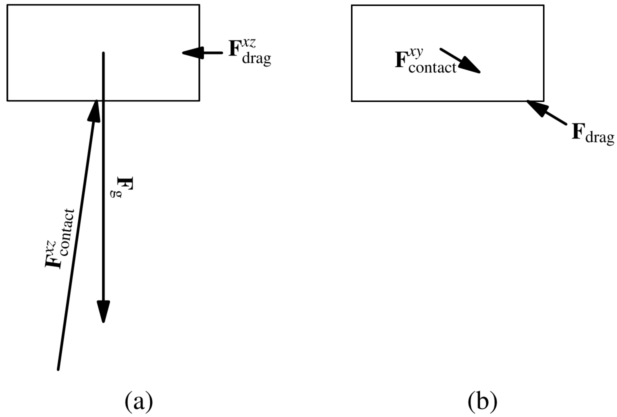

Figure S.19 The two views of the freebody diagram for bicycling with wind. (a) The side view, showing the forces' xz portions. (b) The top view, showing the forces' xy portions.

Problem 5.13 (p. 87)

The wind has no effect on the gravitational force, whose two views therefore do not change: The side view shows the entire gravitational force, and the top view shows no force.

In contrast, wind does affect the other two forces. As usual, it's easiest to consider the passive forces – here, the contact force of the ground – after considering the other forces. Thus, first consider the effect of the wind on the drag force.

Drag is the contact force of the air on the body, and it opposes the body's motion *relative to the air*. The wind means that the air is moving (to the north). Thus, the bicycle's motion relative to the air is east and south, and the drag force will point west and north (whereas without wind it pointed purely west).

The third force, the contact force, balances the other two forces (the bicycle isn't accelerating). Thus, its downward portion balances the gravitational force, and its planar portion (its portion in the xy plane) balances the drag force. So, the contact force points mostly downward, somewhat east, and slightly south.

Its side view shows only the downward portion. Its top view shows only the planar portion.

Putting the preceding points into the freebody diagram gives the two views (Figure S.19).

An unusual feature of the arrows is worth noting: The drag force (on the side view) and the contact force (on the top view) do not have their tips at the edge of the body, even though these forces are short range. The reason is that the surface is two-dimensional.

The drag force, at least with the direction and placement for it that I've chosen, acts on the body's side surface, at a point slightly behind the front of the body. (In the top view, you can see where that point is.) Thus, on the side view, the drag force's tip looks like it's within the body.

Similarly, the contact force also acts at the body's surface, in particular at its bottom surface. Thus, in the top view, the contact force has its tip inside the body (and its tail, because its *xy* portion isn't large).

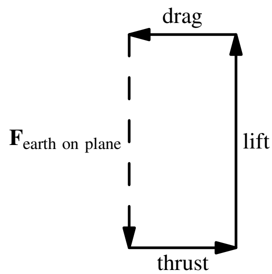

Figure S.21 The vector sum of the gravitational force and the three portions of the contact force (lift, drag, and thrust).

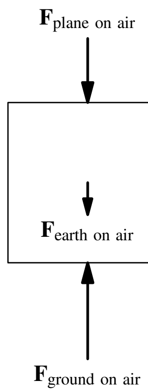

Figure S.22 The freebody diagram of the air. The air interacts with three other bodies: the earth (gravity), the plane (contact), and the ground (contact). Thus, it experiences three forces. The gravitational force and the downward contact force (from the plane) balance the upward contact force (from the ground).

Problem 5.14 (p. 87)

a. the plane interacts with only two other bodies: the earth and the air. Thus, the new force $\mathbf{F}_{\text{air on plane}}$, contributed by the plane's interaction with the air, is the second and only other force (Figure S.20). Because the plane isn't accelerating, the two forces balance.

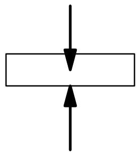

Figure S.20 The complete freebody diagram for the plane. The missing force was due to its interaction with the air.

If the plane were climbing at a steady rate, it would still be moving at constant velocity (the velocity just would have an upward portion). Thus, the two forces on the plane still would balance and the freebody diagram would look the same.

b. Lift points upward, and thrust and drag point horizontally (Figure S.21). Because the sum of these three forces is upward (to balance gravity), thrust and drag must balance.

c. The air interacts with three bodies: (1) the earth (in a long-range, gravitational interaction), (2) the plane (in a short-range, contact interaction), and (3) the ground (in the other short-range, contact interaction). Thus, three forces act on the air (Figure S.22).

   a. $\mathbf{F}_{\text{earth on air}}$. This force, because the air is assumed to have one-half the mass of the plane, is one-half of $\mathbf{F}_{\text{earth on plane}}$. This force easy to forget because air is invisible (a belief that I held in abeyance while I lived in Los Angeles). How can gravity act on an invisible body? But it does! Gravity sees everything.

   b. $\mathbf{F}_{\text{plane on air}}$. This force isn't so easy to forget, but its direction isn't obvious, especially if one thinks of the air as the whole atmosphere or even just as a giant box of air that extends above the plane. But this force is the third-law counterpart of $\mathbf{F}_{\text{air on plane}}$. So, the two forces are equal and opposite, meaning that $\mathbf{F}_{\text{plane on air}}$ points downward.

   c. $\mathbf{F}_{\text{ground on air}}$. This force, which I find even easier to forget than I do $\mathbf{F}_{\text{earth on air}}$, is the third-law counterpart of $\mathbf{F}_{\text{air on ground}}$, which itself is the pressure force of the air on the ground. Thus, neither force is zero! In particular, $\mathbf{F}_{\text{ground on air}}$ points upward and, because the air isn't accelerating, must balance the other two downward forces on the air.

Now for the freebody diagram of the earth (which includes the ground). It participates in three interactions: (1) the long-range, gravitational interaction with the plane; (2) the long-range, gravitational interaction with the air; and (3) the short-range, contact interaction with the air. Thus, it experiences three forces: $\mathbf{F}_{\text{plane on earth}}$, $\mathbf{F}_{\text{air on earth}}$, $\mathbf{F}_{\text{air on ground}}$ (Figure S.23). Each force is a third-law counterpart, and therefore equal and opposite, to a force on one of the two preceding freebody diagrams.

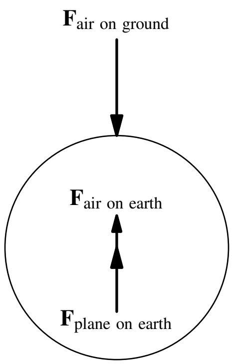

Figure S.23 The freebody diagram of the earth. It interacts with three other bodies: the plane (gravity), the air (gravity), and the air again (contact with the ground). Thus, it experiences three forces.

d. Figure S.24 shows the three freebody diagrams with the third-law pairs connected by labeled dashed paths.

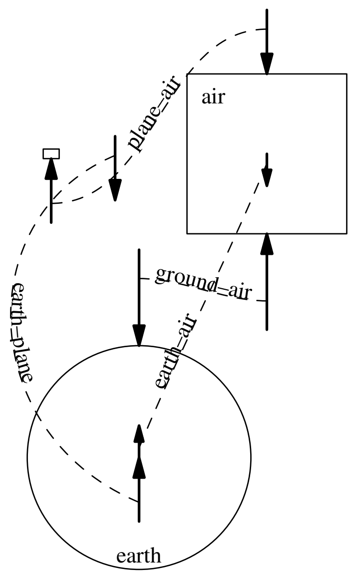

Figure S.24 All the freebody diagrams together with third-law pairs connected by dashed lines.

Problem 5.15 (p. 88)

a. Like the tube of lake water, this tube of air experiences two contact forces, an upward one on its bottom surface and a downward one on its top surface, and one long-range force, the gravitational force. Also like the tube of lake water, the contact force on its side is zero by symmetry.

The upward contact force is a pressure force with magnitude $p(z)A$. The downward contact force is a pressure force with magnitude $p(z+\Delta z)A$ (the $+\Delta z$ accounts for the pressure being different at the top of the slab).

The gravitational force is $g\Delta m$ downward, where Δm is the slab's mass. Its mass is the its volume $A\Delta z$ times the air density ρ.

$$\Delta m = A\Delta z. \qquad \text{(S.41)}$$

This calculation is why the slab has to be thin. If the slab were thick, the density would vary too much from top to bottom, and we wouldn't know what density to use. (For the tube of lake water, where I assumed that water was incompressible, the density was fixed and the slab could be as thick as needed.)

With the mass given in (S.41), the gravitational force has magnitude

$$F_g = g\Delta m = \rho A g \Delta z. \qquad \text{(S.42)}$$

Like the tube of lake water, the slab has zero acceleration, so the net force on it must be zero. Thus, sum of the two contact forces balances the gravitational force. As an equation for vertical components (with upward as the positive direction),

$$p(z)A - p(z+\Delta z)A - \rho A \Delta z = 0. \qquad \text{(S.43)}$$

The common factor of A divides out and leaves

$$p(z) - p(z+\Delta z) = \rho g \Delta z. \qquad \text{(S.44)}$$

Because Δp is $p(z+\Delta z) - p(z)$,

$$\Delta p = -\rho g \Delta z. \qquad \text{(S.45)}$$

Finally, we can eliminate the interloper quantity ρ by using the ideal-gas law (5.26). Then,

$$\Delta p = -\frac{m_{\text{molar}} g}{RT} p \Delta z. \qquad \text{(S.46)}$$

b. With the thin-slab approximation

$$\Delta p \approx \frac{dp}{dz}\Delta z, \qquad \text{(S.47)}$$

the relation (S.46) between Δp and Δz becomes

$$\frac{dp}{dz} = -\frac{m_{\text{molar}} g}{RT} p. \qquad \text{(S.48)}$$

c. This first-order, linear differential equation is the equation of exponential decay. Its solution is

$$p = p_0 e^{-\frac{m_{\text{molar}} g z}{RT}}, \tag{S.49}$$

where p_0 is the pressure at sea level ($z = 0$).

To confirm this solution, substitute it into the differential equation (S.48). The left side, which is the z derivative of p, becomes

$$-p_0 \frac{m_{\text{molar}} g}{RT} e^{-\frac{m_{\text{molar}} g z}{RT}}. \tag{S.50}$$

The solution (S.49) greatly simplifies this derivative, which becomes

$$\frac{m_{\text{molar}} g}{RT} p. \tag{S.51}$$

This result is just the right side of the differential equation (S.48). So, its two sides are equal, which confirms the proposed solution.

d. The main idea here for the ideal-gas law (5.26) here is that the density and pressure are proportional. Thus, they both decay exponentially with the same dependence on height.

$$\rho = \rho_0 e^{-\frac{m_{\text{molar}} g z}{RT}}. \tag{S.52}$$

e. The ratio ρ/ρ_0 is determined by the value of the quantity

$$\frac{m_{\text{molar}} g z}{RT} \tag{S.53}$$

in the exponent of (S.52). Putting in the numerical values, including that $z = 10$ kilometers,

$$\frac{m_{\text{molar}} g z}{RT} \approx \frac{3 \times 10^{-2}\,\text{kg}\,\text{mol}^{-1} \times 10\,\text{m}\,\text{s}^{-2} \times 10^4\,\text{m}}{8\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1} \times 300\,\text{K}} = 1.25. \tag{S.54}$$

Thus,

$$\frac{\rho}{\rho_0} \approx e^{-1.25} \approx 0.3. \tag{S.55}$$

Thus, at 10 kilometers, the air is about 30 percent as dense as at sea level.

At the scale height of $H \approx 8$ kilometers, the ratio in the exponent is even easier to estimate:

$$\frac{m_{\text{molar}} g z}{RT} \approx \frac{3 \times 10^{-2}\,\text{kg}\,\text{mol}^{-1} \times 10\,\text{m}\,\text{s}^{-2} \times 8 \times 10^3\,\text{m}}{8\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1} \times 300\,\text{K}} = 1. \tag{S.56}$$

Despite all the approximate values that I used in this estimate, the result, that the ratio is 1, is exact: The scale height is defined as the height at which this ratio becomes 1. With this ratio equal to 1, the density ratio is $1/e$ or about 36 percent.

Figure S.25 shows the graphs of density versus height in three models of the atmosphere. The constant-density model of Problem 5.11 is the blue curve; the density is its sea-level value until H, where it drops abruptly to zero. The isothermal model of this problem, whose prediction is given in (S.52), is the red curve. And the actual atmosphere (actually, the so-called US Standard Atmosphere, which models the actual atmosphere closely) is the black, dashed curve. The isothermal model is remarkably accurate. (The actual density is slightly higher than predicted by the isothermal model because the temperature falls with altitude, and colder air is denser.)

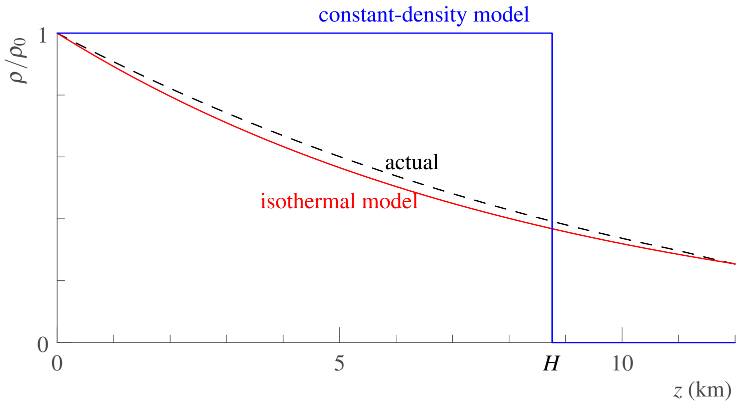

Figure S.25 A comparison of the constant-density and isothermal models against the actual atmosphere. The curves plot density versus height above sea level (with density measured relative to the sea-level density). The isothermal model is close to the actual model.

Problem 6.1 (p. 114)

a. The particle spends much time backtracking (the second straight segment undoes the first). Thus, its travels farther than the distance between points A and B. Thus, v_{avg}, which has distance traveled in the numerator, is going to be greater than $|\mathbf{v}_{\text{avg}}|$, which has the A–B distance in the numerator. For this reason,

$$v_{\text{avg}} \geq |\mathbf{v}_{\text{avg}}| \tag{S.57}$$

(with equality only when the path is a straight line and the particle never reverses).

b. I calculate $\mathbf{v}_{\text{avg}}$ first (from which $|\mathbf{v}_{\text{avg}}|$ follows). The numerator, the displacement, is the same as in the example. The duration is increased by the time required to travel the two straight segments. For the first segment, the extra time is

$$\frac{3\,\text{m}}{10\,\text{m}\,\text{s}^{-1}} = 0.3\,\text{s}. \tag{S.58}$$

For the second segment, it's

$$\frac{3\,\text{m}}{12\,\text{m}\,\text{s}^{-1}} = 0.25\,\text{s}. \tag{S.59}$$

Thus,

$$\mathbf{v}_{\text{avg}} = \frac{2\,\text{m downward}}{\frac{2}{7}\,\text{s} + 0.3\,\text{s} + 0.25\,\text{s}} \approx 2.4\,\text{m}\,\text{s}^{-1}\text{ downward}. \tag{S.60}$$

Thus,

$$|\mathbf{v}_{\text{avg}}| \approx 2.4\,\text{m}\,\text{s}^{-1}. \tag{S.61}$$

For the average speed, the duration also increases as calculated above, but the numerator also increases (to include the length of the two segments):

$$v_{\text{avg}} \approx \frac{\overbrace{6\,\text{m}}^{\text{extra}} + \pi\,\text{m}}{\frac{2}{7}\,\text{s} + 0.3\,\text{s} + 0.25\,\text{s}} \approx 10.9\,\text{m}\,\text{s}^{-1}. \tag{S.62}$$

As it should be, the average speed lies between the lowest and highest speed, 10 and 12 meters per second, respectively. The average is slightly lower than the midpoint of 11 meters per second because the particle spends slightly less time on the second segment (where it moves faster) than on the first segment.

Problem 6.2 (p. 115)

For all the parts of this problem, the average and instantaneous accelerations are the same, so you can calculate an average acceleration for any convenient time interval.

a. Taking the quantities in the order $\mathbf{a}$, a, and a_x:

   i. The acceleration vector $\mathbf{a}$ has magnitude 1 meter per second squared. But its direction requires a bit of care. The train starts at 100 meters per second to the left. As mentioned in the problem, 5 seconds later, the train is moving 105 meters per second, also to the left. Thus,

   $$\Delta\mathbf{v} = 5\,\mathrm{m\,s^{-1}}\ \text{to the left.} \tag{S.63}$$

   Using the $\Delta\mathbf{v}$ form of the average acceleration (6.22),

   $$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{5\,\mathrm{m\,s^{-1}}\ \text{to the left}}{5\,\mathrm{s}} = 1\,\mathrm{m\,s^{-2}}\ \text{to the left.} \tag{S.64}$$

   Because the acceleration is constant, the average acceleration is the (instantaneous) acceleration. Thus,

   $$\mathbf{a} = 1\,\mathrm{m\,s^{-2}}\ \text{to the left.} \tag{S.65}$$

   ii. As predicted, $\mathbf{a}$ has magnitude

   $$a = 1\,\mathrm{m\,s^{-2}}. \tag{S.66}$$

   iii. a_x is the component of $\mathbf{a}$, which points to the left. The positive x direction is to the right, so a_x is negative. Thus,

   $$a_x = -1\,\mathrm{m\,s^{-2}}. \tag{S.67}$$

b. Again, compute the average acceleration $\mathbf{a}_{\text{avg}}$, and the other quantities follow from it. I imagine waiting 5 seconds. Then the train is moving 90 meters per second to the left. Thus,

   $$\begin{aligned}\Delta\mathbf{v} &= \mathbf{v}_{\text{after}} - \mathbf{v}_{\text{before}}\\ &= 90\,\mathrm{m\,s^{-1}}\ \text{to the left} - 100\,\mathrm{m\,s^{-1}}\ \text{to the left}\\ &= -10\,\mathrm{m\,s^{-1}}\ \text{to the left.}\end{aligned} \tag{S.68}$$

   Alternatively, and with one fewer minus sign,

   $$\Delta\mathbf{v} = 10\,\mathrm{m\,s^{-1}}\ \text{to the right.} \tag{S.69}$$

   Thus,

   $$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{10\,\mathrm{m\,s^{-1}}\ \text{to the right}}{5\,\mathrm{s}} = 2\,\mathrm{m\,s^{-2}}\ \text{to the right.} \tag{S.70}$$

   i. $\mathbf{a}$ is, because the acceleration is constant, equal to $\mathbf{a}_{\text{avg}}$.

   $$\mathbf{a} = 2\,\mathrm{m\,s^{-2}}\ \text{to the right.} \tag{S.71}$$

ii. a is the magnitude of **a**.

$$a = 2\,\mathrm{m\,s^{-2}}. \tag{S.72}$$

iii. a_x is the x component of **a**. Because **a** points in the positive x direction, a_x is positive.

$$a_x = +2\,\mathrm{m\,s^{-2}}. \tag{S.73}$$

c. This situation has no minus-sign traps. However, for completeness and just to be sure, I follow the full procedure that starts with finding $\mathbf{a}_{\text{avg}}$. In 5 seconds, the car is moving 65 meters per second to the right. Thus,

$$\begin{aligned}\Delta\mathbf{v} &= \mathbf{v}_{\text{after}} - \mathbf{v}_{\text{before}}\\ &= 65\,\mathrm{m\,s^{-1}}\text{ to the right} - 50\,\mathrm{m\,s^{-1}}\text{ to the right}\\ &= 15\,\mathrm{m\,s^{-1}}\text{ to the right.}\end{aligned} \tag{S.74}$$

Thus,

$$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{15\,\mathrm{m\,s^{-1}}\text{ to the right}}{5\,\mathrm{s}} = 3\,\mathrm{m\,s^{-2}}\text{ to the right.} \tag{S.75}$$

i. **a** is, because the acceleration is constant, equal to $\mathbf{a}_{\text{avg}}$.

$$\mathbf{a} = 3\,\mathrm{m\,s^{-2}}\text{ to the right.} \tag{S.76}$$

ii. a is the magnitude of **a**.

$$a = 3\,\mathrm{m\,s^{-2}}. \tag{S.77}$$

iii. a_x is the x component of **a**. Because **a** points in the positive x direction, a_x is positive.

$$a_x = +3\,\mathrm{m\,s^{-2}}. \tag{S.78}$$

d. This car is, wisely, losing speed: After 5 seconds, it's moving 35 meters per second to the right (a much safer speed). Thus,

$$\begin{aligned}\Delta\mathbf{v} &= \mathbf{v}_{\text{after}} - \mathbf{v}_{\text{before}}\\ &= 35\,\mathrm{m\,s^{-1}}\text{ to the right} - 50\,\mathrm{m\,s^{-1}}\text{ to the right}\\ &= -15\,\mathrm{m\,s^{-1}}\text{ to the right.}\end{aligned} \tag{S.79}$$

With one fewer minus sign,

$$\Delta\mathbf{v} = 15\,\mathrm{m\,s^{-1}}\text{ to the left.} \tag{S.80}$$

Then,

$$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{15\,\mathrm{m\,s^{-1}}\text{ to the left}}{5\,\mathrm{s}} = 3\,\mathrm{m\,s^{-2}}\text{ to the left.} \tag{S.81}$$

i. **a** is, because the acceleration is constant, equal to $\mathbf{a}_{\text{avg}}$.

$$\mathbf{a} = 3\,\mathrm{m\,s^{-2}}\text{ to the left.} \tag{S.82}$$

ii. a is the magnitude of $\mathbf{a}$.

$$a = 3\,\mathrm{m\,s^{-2}}. \tag{S.83}$$

iii. a_x is the x component of $\mathbf{a}$. Because $\mathbf{a}$ points in the *negative* x direction, a_x is negative.

$$a_x = -3\,\mathrm{m\,s^{-2}}. \tag{S.84}$$

e. For this situation, I wait only 1 second (to avoid the minus sign that would result if I waited for 5 seconds). Then the ball is moving 10 meters per second upward. The $\Delta\mathbf{v}$ is Thus,

$$\begin{aligned}\Delta\mathbf{v} &= \mathbf{v}_{\text{after}} - \mathbf{v}_{\text{before}}\\ &= 10\,\mathrm{m\,s^{-1}}\text{ upward} - 20\,\mathrm{m\,s^{-1}}\text{ upward}\\ &= -10\,\mathrm{m\,s^{-1}}\text{ upward.}\end{aligned} \tag{S.85}$$

With one fewer minus sign,

$$\Delta\mathbf{v} = 10\,\mathrm{m\,s^{-1}}\text{ downward.} \tag{S.86}$$

Then,

$$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{10\,\mathrm{m\,s^{-1}}\text{ downward}}{1\text{ s}} = 10\,\mathrm{m\,s^{-2}}\text{ downward.} \tag{S.87}$$

i. $\mathbf{a}$ is, because the acceleration is constant, equal to $\mathbf{a}_{\text{avg}}$.

$$\mathbf{a} = 10\,\mathrm{m\,s^{-2}}\text{ downward.} \tag{S.88}$$

ii. a is the magnitude of $\mathbf{a}$.

$$a = 10\,\mathrm{m\,s^{-2}}. \tag{S.89}$$

iii. a_x is the x component of $\mathbf{a}$. Because $\mathbf{a}$ points perpendicular to the x direction,

$$a_x = 0. \tag{S.90}$$

Problem 6.3 (p. 115)

Because the earth moves at constant speed, its acceleration points inward and is perpendicular to its motion (to its velocity). The magnitude of the perpendicular, and here the entire, acceleration is v^2/r, where v is the earth's orbital speed and r is its orbital radius. The orbital speed is 30 kilometers per second, from (6.14), and is also worth memorizing because it's such a useful quantity and such a round number (in metric units). Using

$$r = 1\,\mathrm{AU} \approx 1.5 \times 10^{11}\,\mathrm{m}, \tag{S.91}$$

$$a = \frac{v^2}{r} \approx \frac{(3 \times 10^4\,\mathrm{m\,s^{-1}})^2}{1.5 \times 10^{11}\,\mathrm{m}} = 6 \times 10^{-3}\,\mathrm{m\,s^{-2}}. \tag{S.92}$$

It's a tiny acceleration (comparing it, for example, to g)!

You can check this result by calculating it using the gravitational force of the sun on the earth (Section 1.4) along with the earth's mass.

Problem 6.4 (p. 115)

Imagine a *right*-moving bob just before and just after point C, with the two imagined locations symmetric around point C. The bob's location just before point C is to the left and slight above point C. Its velocity, always tangent to the circle, points mostly to the right and slightly downward (Figure S.26). Just after point C, the velocity points again mostly to the right but now slightly upward.

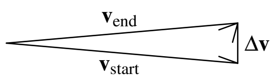

Figure S.26 Calculating $\Delta\mathbf{v}$ around point C (for a right-moving bob). In contrast to Figure 6.12, for calculating $\Delta\mathbf{v}$ for a left-moving bob, these velocities point mostly to the right. Similarly Figure 6.12, however, the starting velocity points slightly downward, and the ending velocity points slightly upward. Thus, the difference $\Delta\mathbf{v}$ is the same as for the left-moving bob!

Because of symmetry, these before-and-after speeds are identical. So, the two velocity vectors making up $\Delta\mathbf{v}$, namely $\mathbf{v}_{\text{start}}$ and $\mathbf{v}_{\text{end}}$, have the same length. Their difference,

$$\Delta\mathbf{v} \equiv \mathbf{v}_{\text{end}} - \mathbf{v}_{\text{start}}, \tag{S.93}$$

points upward – just as it did for the left-moving bob. Thus, the acceleration points upward or inward, as it should for circular motion at constant speed (which describes the motion around point C).

Problem 6.5 (p. 115)

This right-moving bob, just before point B, is climbing the hill toward point B. Its velocity, always tangent to the circle, points upward and to the right. Its velocity just after point B points slightly more upward. If this change of direction in $\mathbf{v}$, a counterclockwise rotation, were the whole cause of $\Delta\mathbf{v}$, then the acceleration would point directly inward along the string (befitting circular motion at constant speed).

However, as you know from the corresponding analysis of the left-moving bob (redrawn in Figure S.27a), the story isn't complete. For this bob slows down as it travels uphill along the circular path. Thus, it moves slower after point B than it does before point B. So, the velocity not only rotates (counterclockwise), it also shrinks.

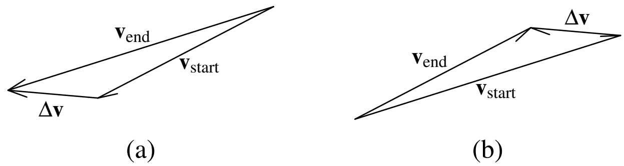

Figure S.27 Calculating $\Delta\mathbf{v}$ at point B. (a) Left-moving bob. This diagram is a copy of Figure 6.13a from the text. (b) Right-moving bob. This bob is moving to the right (and slightly upward). The resulting $\Delta\mathbf{v}$ is identical to the $\Delta\mathbf{v}$ for the left-moving bob.

The resulting difference $\Delta\mathbf{v}$ is the same as it was for the left-moving bob (Figure S.27b). This equality, which is suggested by the graphical calculation, has the following proof. Changing from a left-moving to a right-moving bob changes two parts of the $\Delta\mathbf{v}$ calculation.

First, the change swaps the meaning of start and end: For the left-moving bob, the starting location was just to the right of point B, which is the ending location for the right-moving bob. Similarly, for the left-moving bob, the ending location was just to the left of point B, which is the starting location for the right-moving bob.

Second, the change negates the velocities: at a given location, the left-moving bob has the opposite velocity to the right-moving bob (both are tangent to the circle but along different directions).

To summarize these relations symbolically,

$$\begin{aligned}\mathbf{v}_{\text{start}}^{\text{left-moving}} &= -\mathbf{v}_{\text{end}}^{\text{right-moving}};\\ \mathbf{v}_{\text{end}}^{\text{left-moving}} &= -\mathbf{v}_{\text{start}}^{\text{right-moving}}.\end{aligned} \tag{S.94}$$

Thus, the calculation of $\Delta\mathbf{v}$, as

$$\Delta\mathbf{v} \equiv \mathbf{v}_{\text{end}} - \mathbf{v}_{\text{start}}, \tag{S.95}$$

gets changed in two ways. First, the two terms switch places; this change introduces a factor of -1. Second, each velocity gets negated; this change introduces another factor of -1. The result is a factor of $(-1)^2$ or 1: Thus, $\Delta\mathbf{v}$ doesn't change.

Having $\Delta\mathbf{v}$, we can split it into parallel and perpendicular portions (Figure S.28). Because $\Delta\mathbf{v}$ hasn't changed and because the velocity for the left-moving bob and the velocity for the right-moving bob lie along the same line (though in opposite directions), the splitting produces identical portions and therefore identical $\mathbf{a}_{\parallel}$ and $\mathbf{a}_{\perp}$ (which are just the respective portions divided by the time interval Δt).

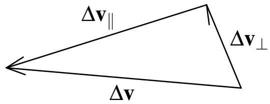

Figure S.28 Splitting $\Delta\mathbf{v}$ at point B into parallel and perpendicular components. The split is identical for left- and right-moving bobs. Thus, this figure is just a copy of Figure 6.13b.

However, for the right-moving bob, $\Delta\mathbf{v}_\parallel$ and $\mathbf{a}_\parallel$ point opposite to the velocity: $\mathbf{a}_\parallel$ points downhill and tangent to the circle, whereas the velocity points uphill and tangent to the circle. According to (6.59), the relation between $\mathbf{a}_\parallel$ and $\mathbf{v}$ is

$$\mathbf{a}_\parallel = \frac{dv}{dt} \text{ along } \mathbf{v}. \tag{S.96}$$

Therefore, dv/dt must be negative. And it's: At point B, the right-moving bob is climbing the hill and is therefore slowing down.

Problem 6.6 (p. 115)

The claim, here made explicit but not simplified by giving the car's velocity a known direction, talks about the change in the car's speed. And changing speed appears in the *parallel* component of acceleration:

$$a_\parallel = \frac{dv}{dt}. \tag{S.97}$$

Going backward from instantaneous rate of change (dv/dt) to average rate of change (undoing the limit $\Delta t \to 0$),

$$(a_\parallel)_{\text{avg}} = \frac{\Delta v}{\Delta t}. \tag{S.98}$$

The claim specifies Δv (60 mph) and a Δt (4 seconds), so

$$(a_\parallel)_{\text{avg}} = \frac{60\,\text{mph}}{4\,\text{s}} = 15\,\text{mph}\,\text{s}^{-1}. \tag{S.99}$$

If the salesman knew he were speaking to a physics student, he might have said to her, "This car's parallel component of acceleration, averaged over the first 4 seconds after it races off, is 15 miles per hour every second (or roughly 7 meters per second squared)." But that salesman wouldn't sell many cars.

Problem 7.1 (p. 165)

The toast falls for a time given by (7.3):

$$t_{\text{fall}} = \sqrt{\frac{2h}{g}}. \tag{S.100}$$

Because its acceleration is constant and has magnitude g, its speed at impact is gt_{fall}:

$$v_{\text{impact}} = gt_{\text{fall}} = g \times \sqrt{\frac{2h}{g}} = \sqrt{2gh}. \tag{S.101}$$

For $h \approx 0.6$ meters (standard table height),

$$v_{\text{impact}} \approx \sqrt{2 \times 10\,\text{m}\,\text{s}^{-2} \times 0.6\,\text{m}} \approx 3.5\,\text{m}\,\text{s}^{-1}. \tag{S.102}$$

This result is consistent with the free-fall time of 0.35 seconds (calculated in (7.3) for the same height). With an acceleration of 10 meters per second squared downward, the velocity will change by

$$10\,\text{m}\,\text{s}^{-2}\ \text{downward} \times 0.35\,\text{s} = 3.5\,\text{m}\,\text{s}^{-1}\ \text{downward}. \tag{S.103}$$

Thus, after starting from rest, the toast hits with an impact velocity of 3.5 meters per second downward.

Problem 7.2 (p. 165)

This distance is the plane's average speed times the time spent taxiing:

$$d = v_{\text{avg}}t. \tag{S.104}$$

Like the freely falling stone, the 747 had a constant acceleration and started from rest. Thus, its average speed is just one-half of its final speed, which I had estimated as 80 meters per second. Meanwhile, it spent roughly 40 seconds taxiing. So,

$$d \approx \underbrace{\frac{1}{2} \times 80\,\text{m}\,\text{s}^{-1}}_{v_{\text{avg}}} \times 40\,\text{s} = 1600\,\text{m}. \tag{S.105}$$

It's 1.6 kilometers or almost exactly 1 mile: Runways that handle long-distance flights need to be long.

Problem 7.3 (p. 166)

a. The stone loses 10 meters per second of speed every second, so its speed is zero after 2 seconds.

b. No! The only force on the stone, whether rising or falling, is the gravitational force. During the stone's upward journey, this force points opposite to the stone's direction of motion. This situation is another reminder that force is connected not to velocity but rather to acceleration. Similarly, at the end of the upward journey, when the stone is at its maximum height, its velocity is zero (zero motion) yet the only force on it points downward. Force and velocity have no necessary connection.

c. Its speed decreases steadily from 20 to 0 meters per second, so its average speed is 10 meters per second. Its average velocity is 10 meters per second upward. Because its acceleration is constant, its average acceleration is its acceleration: 10 meters per second squared downward.

d. On its upward journey, its average speed is 10 meters per second, and it travels for 2 seconds. Thus, it travels 20 meters.

e. Its downward journey is the mirror image of the upward journey: Its speed starts at zero and increases steadily, at 10 meters per second per second, for 2 seconds. Then it has traveled 20 meters downward. Thus, 4 seconds after its launch the stone has returned to its launch height.

f. Because the upward and downward journeys have the same duration, the round-trip average speed is the average of the downward and upward average speeds. Each was 10 meters per second, so the round-trip average speed is also 10 meters per second.

   The average velocity is the displacement divided by the journey time. Because the displacement is zero for a round trip, the average velocity is also zero. This result agrees with a direct calculation. On the upward journey, the average velocity is 10 meters per second upward. On the downward journey, it's 10 meters per second downward. The average of the two average velocities is zero. (Don't forget that, in computing their average, their sum is a vector sum.)

   The stone's acceleration is constant, so its average acceleration is its acceleration: 10 meters per second squared downward. This result agrees with a direct calculation using the definition of $\mathbf{a}_{\text{avg}}$ in (6.22):

$$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta \mathbf{v}}{\Delta t}. \tag{S.106}$$

Here, the velocity started at 20 meters per second upward and ended at 20 meters per second downward. Thus,

$$\begin{aligned}\Delta\mathbf{v} &= 20\,\mathrm{m\,s^{-1}}\text{ downward} - 20\,\mathrm{m\,s^{-1}}\text{ upward}\\ &= 40\,\mathrm{m\,s^{-1}}\text{ downward.}\end{aligned} \tag{S.107}$$

With $\Delta t = 4$ seconds,

$$\mathbf{a}_{\text{avg}} \equiv \frac{\Delta\mathbf{v}}{\Delta t} = \frac{40\,\mathrm{m\,s^{-1}}\text{ downward}}{4\,\mathrm{s}} = 10\,\mathrm{m\,s^{-2}}\text{ downward.} \tag{S.108}$$

Problem 7.4 (p. 166)

a. Each mass's freebody diagram has an upward, contact force $\mathbf{F}_T$ (Figure S.29). (These two tension forces are the same on each mass because the tension is constant throughout the string.) Meanwhile, each mass also experiences a gravitational force.

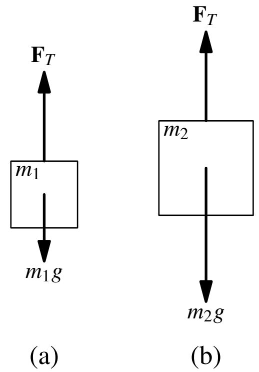

Figure S.29 Freebody diagrams for (a) m_1 and (b) m_2.

b. We know something, but not everything, about the forces: We know the gravitational forces, but not the tension force. We also know something, but not everything, about the motion. We know that the upward acceleration of m_1 is the same as the downward acceleration of m_2 (otherwise the string would change its length, which it doesn't). We don't know what this acceleration is. So, this problem is a mixture of type C (calculating) and type I (inferring).

With a_z as the upward component of m_1's acceleration and applying Newton's second law to m_1 to infer a_z:

$$a_z = \frac{T - m_1 g}{m_1}. \tag{S.109}$$

In applying the second law to m_2, careful with the signs. Now a_z is the downward component of its acceleration. Thus,

$$a_z = \frac{m_2 g - T}{m_2}. \tag{S.110}$$

Now comes the algebra: Equate the two routes to getting a_z, via (S.109) and (S.110), to get an equation for T.

$$\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}. \tag{S.111}$$

To solve for T, cross-multiply.

$$m_2 T - m_1 m_2 g = m_1 m_2 g - m_1 T. \tag{S.112}$$

Its solution is

$$T = \frac{2m_1 m_2}{m_1 + m_2} g. \tag{S.113}$$

Once T is known, then either equation for a_z, (S.110) or (S.109), determines a_z. Using (S.109), one route through the algebra is as follows.

$$\begin{aligned}
a_z &= \frac{T - m_1 g}{m_1} \\
\text{(dividing by } m_1 \text{ term by term)} \quad &= \frac{T}{m_1} - g \\
\text{(substituting for } T \text{ from (S.113))} \quad &= \frac{2m_2}{m_1 + m_2} g - g \\
\text{(factoring out } g\text{)} \quad &= \left[\frac{2m_2}{m_1 + m_2} - 1\right] g \\
\text{(making a common denominator)} \quad &= \left[\frac{2m_2}{m_1 + m_2} - \frac{m_1 + m_2}{m_1 + m_2}\right] g \\
\text{(subtracting)} \quad &= \left[\frac{2m_2 - (m_1 + m_2)}{m_1 + m_2}\right] g \\
\text{(simplifying)} \quad &= \frac{m_2 - m_1}{m_1 + m_2} g.
\end{aligned} \tag{S.114}$$

As a check that T is correct: If we substitute T from (S.113) into the a_z equation due to the m_2 freebody diagram, namely into (S.110), we should get this same a_z.

$$\begin{aligned}
a_z &= \frac{m_2 g - T}{m_2} \\
\text{(after dividing by } m_2 \text{ term by term)} &= g - \frac{T}{m_2} \\
\text{(after substituting for } T \text{ from (S.113))} &= g - \frac{2m_1}{m_1 + m_2} g \\
\text{(after factoring out } g\text{)} &= \left[1 - \frac{2m_1}{m_1 + m_2}\right] g \\
\text{(after making a common denominator)} &= \left[\frac{m_1 + m_2}{m_1 + m_2} - \frac{2m_1}{m_1 + m_2}\right] g \\
\text{(after simplifying)} &= \frac{m_2 - m_1}{m_1 + m_2} g,
\end{aligned} \tag{S.115}$$

so a_z is the same by either route.

c. It's easy to do algebra on autopilot and end up with nonsense without realizing it. Thus, now it's time to check that the results for T and a_z make physical sense.

i. Another check on T and a_z is whether they work in the easy case $m_1 = m_2$. For T, in (S.113), the easy case gives, using m for either m_1 or m_2 (because they are equal),

$$T = \frac{2m^2}{2m} g = mg, \tag{S.116}$$

which agrees with the analysis in Section 5.6.3.

For a_z, in (S.114), the easy case gives

$$a_z = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{0}{2m} g = 0. \tag{S.117}$$

This result also agrees with the analysis in Section 5.6.3, where we found that, in an equal-mass Atwood machine, the masses could move only at constant speed (which need not be zero!).

ii. Finally, we check whether the general result for a_z has the correct sign when $m_1 > m_2$ and when $m_1 < m_2$. When $m_1 > m_2$, then m_2 cannot fully hold m_1 back, so m_1 wins the tug-of-war and accelerates downward. Thus, a_z, which is the *upward* component of m_1's acceleration, should be negative. And it's: In (S.114), the numerator $m_2 - m_1$ is negative, which makes a_z negative.

Similarly, when $m_1 < m_2$, then m_1 cannot fully hold m_2 back, so m_2 wins the tug-of-war and accelerates downward. Thus, m_1 accelerates upward, meaning that a_z should be positive. And it is: In (S.114), the numerator $m_2 - m_1$ is positive, which makes a_z positive.

(Passing the preceding tests doesn't guarantee that T and a_z are correct. However, it does guarantee that, if they are wrong, at least they are not nonsense.)

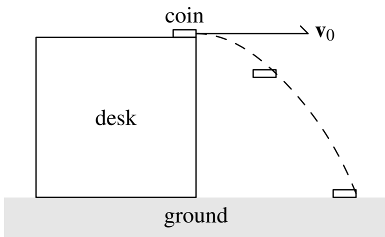

Figure S.30 A coin launched horizontally from a desk.

Problem 7.5 (p. 166)

a. To illustrate the first caveat, launch a coin of mass m horizontally with speed v_0 from a desk and ignore air resistance (Figure S.30). Once the coin no longer touches the desk, it experiences only one force, the gravitational force mg downward. Thus, its acceleration is simply g downward – a constant.

However, its speed, which starts at v_0, does not change steadily – as the following calculation shows. Its speed v is the Pythagorean sum of the x component v_x and the y component v_y.

$$v = \sqrt{v_x^2 + v_y^2}. \quad \text{(S.118)}$$

v_x is always v_0, the initial speed (the coin's initial velocity was purely horizontal). Meanwhile, v_y is gt. So,

$$v = \sqrt{v_0^2 + (gt)^2}. \quad \text{(S.119)}$$

Although the speed increases, it does not increase steadily (at a constant rate). Figure S.31 shows a graph of v, calculated for a coin launched from a standard-height desk (0.75 meters) with the correct initial speed (roughly 1.9 meters per second) so that it lands 0.75 meters beyond the desk.

After impact, at around 0.4 seconds after launch, the v curve becomes dashed because of its subjunctive, or hypothetical, nature: If there were no ground (for example, launching the coin off a cliff), then v would keep increasing beyond the impact speed of roughly 4.3 meters per second.

b. An example of *one*-dimensional motion where **a** is constant yet the speed does not change steadily: the stone of Problem 7.3. Its acceleration is always g downward, so it's constant. However, its speed, which decreases during the upward journey, then increases during the downward journey. In each half of the journey, whether upward or downward, the speed changes steadily.

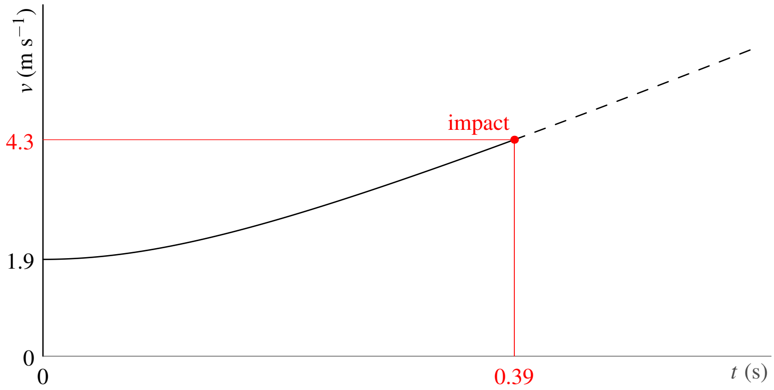

Figure S.31 The coin's speed versus time. The coin speeds up (the curve is increasing), though not steadily (the curve isn't a straight line). If the ground were not there, the coin's speed would keep increasing beyond its speed at the impact time.

However, because of the change of direction at the top of the stone's trajectory, the stone's speed for the whole journey does not change steadily (the graph of v versus t has a kink at the top of the journey).

Problem 7.6 (p. 166)

The contact force's magnitude is the Pythagorean sum of mg and ma (the magnitudes of its two perpendicular portions):

$$F_{\text{contact}} = \sqrt{(mg)^2 + (ma)^2} = m\sqrt{g^2 + a^2}. \qquad \text{(S.120)}$$

Problem 7.7 (p. 166)

a. In free fall themselves, the three internal blocks accelerate downward, toward the earth, with acceleration magnitude g (Figure S.32). Each block also experiences a gravitational force mg downward. Thus, by itself, the gravitational force on each block causes, and explains, each block's acceleration. So, any other forces on the block must add to zero (otherwise they would make the acceleration different from free fall).

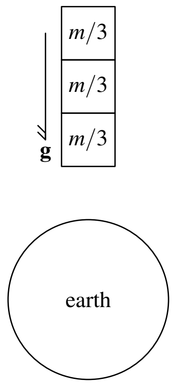

Figure S.32 A person freely falling toward the earth. The person is modeled as three blocks in a stack.

To find out whether any other forces even exist, start with the bottom block. Besides its gravitational interaction with the earth, it participates in only one other interaction: with the middle block. Thus, the only other possible force on the bottom block is the contact force of the middle block – which must therefore be zero. In summary, the bottom block must experience only a gravitational force (Figure S.33a).

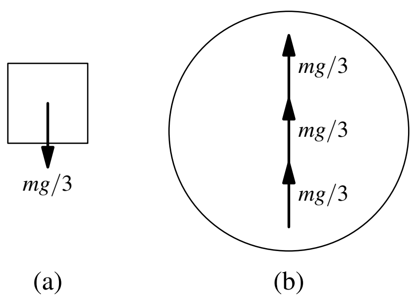

Figure S.33 The freebody diagrams of (a) any of the three blocks and (b) the earth.

By almost the same argument, the top block must also experience only a gravitational force.

Finally, consider the middle block. It participates in two contact interactions: with the bottom block and with the top block. Thus, it experiences two contact forces, one from the bottom block and one from the top block. But, by the third law, the contact force of the bottom block is zero: It's the third-law counterpart of the contact force of the middle block on the bottom block, which, we just showed, must be zero. Therefore, with no possible force to balance it, the contact force of the top block must also be zero. Thus, the middle block also experiences only a gravitational force.

To summarize the freebody diagrams of the three blocks: They are identical and are the freebody diagram of a freely falling stone.

To make the earth's freebody diagram, either (1) just draw the third-law counterpart of each force on the three blocks, or (2) start from first principles by considering the interactions in which the earth participates.

For practice, let's start from first principles. The earth, even including the ground, touches no other bodies. Thus, it participates in only long-range, gravitational interactions: the gravitational interactions with the three blocks. Thus, it experiences three forces (Figure S.33b): the gravitational forces of the three blocks. Each force points upward and, like its third-law counterpart the force on the block, has magnitude $mg/3$.

b. Now that the freebody diagrams are complete, we come to the most important step: interpreting them. The blocks' freebody diagrams, which show no contact forces between the blocks, tell you that the internal forces in you are zero. In contrast, when you stand on the ground, the internal forces increase in magnitude from zero at your head to mg at your toes. Thus, when you step off that diving board and go into free fall – or, more accurately described, into free gravitational motion – these internal forces vanish. Thus, you feel funny (a more extreme version of the feeling when zooming downhill on a roller-coaster ride).

If these forces are absent for a long time, your bones don't experience the forces that signal them to repair and grow as normal. You may wonder how the forces could be absent for a long time; after all, the interval between jumping off a dividing board and reaching the water lasts, at most, a matter of seconds.

Don't forget orbits! An astronaut in orbit is in free gravitational motion. To help me remember this situation, I try to say "free gravitational motion" rather than "free fall." For it's hard to remember that an astronaut, a satellite, and the moon are even falling, let alone falling freely – even though they are, which was Newton's great insight. He realized that the motion of an apple in its fall and the motion of the moon in its orbit are described by one and the same law of gravitation, called therefore the law of *universal* gravitation.

c. The earth, in contrast to when you stand on the ground, now has a net upward force on it (with magnitude mg). Thus, the earth accelerates upward, albeit slowly! Just as you are in free gravitational motion toward the earth, the earth is in free gravitational motion toward you.

Problem 7.8 (p. 167)

Look at the pendulum motion of Section 6.5. There, except at the extremes of the motion, $\mathbf{a}_\perp$ is nonzero. In particular, at point C, the acceleration is purely perpendicular. Although the pendulum bob moved in a circle because of the pendulum string, the particular cause is irrelevant. All that matters for $\mathbf{a}$ and $\mathbf{a}_\perp$ is the motion itself. Thus, if we can construct a hill that produces the same motion, then $\mathbf{a}_\perp$ will also be nonzero. And we can: Make the hill a large skateboarding park or, more mundanely, a hemispherical bowl. A body sliding down the bowl ("downhill") will have a nonzero $\mathbf{a}_\perp$.

In particular, from (6.61),

$$\mathbf{a}_\perp = \frac{v^2}{r_{\text{curvature}}} \text{ inward,} \tag{S.121}$$

where v is the body's speed and $r_{\text{curvature}}$ is the hill's radius of curvature. Thus, keeping $\mathbf{a}_\perp$ zero requires either the trivial case $v = 0$ (no motion at all) or, more interestingly, making $r_{\text{curvature}}$ infinity – which is the description of a straight line.

Problem 7.9 (p. 167)

Such problems can be tricky for an important reason: It's not obvious how to represent the string's angle in the language of Newton's laws and forces. Let's start, therefore, with the effect of the string seen through Newton's laws. The string, through the contact interaction between the string and the mass, contributes a tension, or contact, force on the mass. This force points along the string: The direction of this force is the direction of the string.

Thus, the goal of the problem is to find the direction of the tension force. The problem is then of type I, inferring force from motion. (It may also of type C, inferring motion from force, if we don't know everything about the motion. Let's see.) Thus, we know what to do: Make a freebody diagram of the mass and include on it what we know of the mass's motion.

The mass, yoked to your motion, accelerates downhill with acceleration $g \sin \theta$. Thus, we know everything that we need to know about the mass's motion (making the problem purely of type I). First, Newton's laws concern themselves with a body's acceleration, which here we know. Second, although particular forces could depend on a body's velocity or position, here they don't. (The most common such force is air resistance, which depends on the body's velocity, but here air resistance is assumed to be zero.) So, even though we don't know the mass's velocity, we don't need to know it.

As for the forces, the mass experiences two: the gravitational force and the tension force.

Its freebody diagram, including the motion description, is then identical to the freebody diagram of you in Section 7.1.4 as you slid down a frictionless hill. There, you accelerated downhill with an acceleration magnitude $g \sin \theta$. You also experienced two forces, a gravitational force and a contact force. Thus, the tension force here must be the same as the contact force there ($\mathbf{N}$) – which had magnitude $mg \cos \theta$ and pointed perpendicularly to the hill.

Thus, the tension force, and therefore the string, also points perpendicularly to the hill (Figure S.34). This 90-degree angle with the hill means an angle of θ with the vertical.

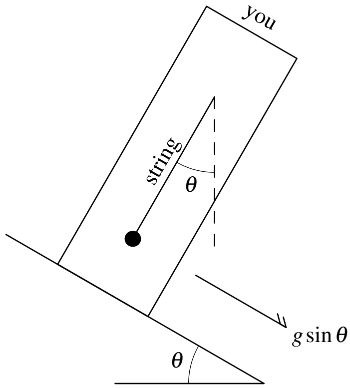

Figure S.34 A pendulum accelerating downhill. The string hangs perpendicularly to the hill.

Problem 7.10 (p. 167)

a. At point A, the satellite's distance from the planet is an extremum (a maximum or a minimum). Equivalently, the satellite's velocity is perpendicular to the line pointing toward the planet. For if the velocity weren't perpendicular to that line, the satellite's distance from the planet would be changing (either increasing or decreasing), meaning that the distance would *not* be an extremum – a contradiction.

Thus, the satellite's velocity and its acceleration, which points toward the planet (along the gravitational force), are perpendicular (Figure S.35a). Therefore, $\mathbf{a}_{\parallel}$ is zero, and so, therefore, is dv/dt. In other words, at point A, the satellite's speed is neither increasing nor decreasing – it's an extremum, either a maximum or a minimum. (It actually is a minimum.)

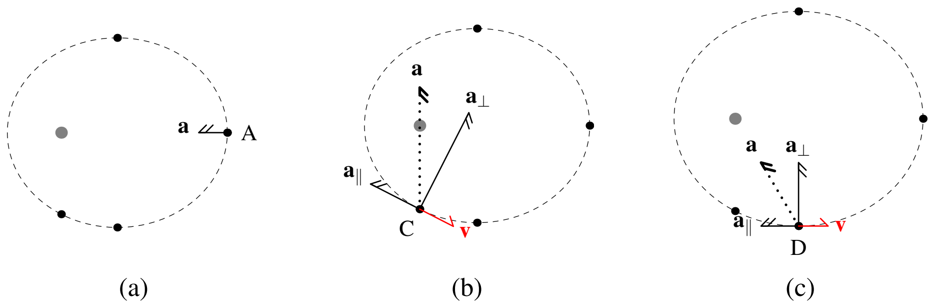

Figure S.35 The acceleration and velocity of the satellite orbiting counterclockwise. (a) At point A. (b) At point C. (c) At point D.

b. At point C, the satellite's acceleration points, as always, toward the planet – directly upward on the diagram (Figure S.35b). The velocity points, as always, straight ahead – on the diagram, to the left and slightly downward. Thus, $\mathbf{a}_{\parallel}$, the parallel portion of the acceleration, points backward (opposite to the velocity). In other words, dv/dt is negative, and satellite's speed is decreasing. And it should be: The satellite is getting farther away from the sun, so, as it fights the sun's gravity, it loses speed.

   At point D (Figure S.35c), the situation is similar to point C. The satellite, as it recedes from the planet, is slowing down. Correspondingly, $\mathbf{a}_{\parallel}$ points backward (opposite to the satellite's velocity).

If the satellite orbits clockwise rather than counterclockwise, that change affects **v** but leaves **a** alone. The acceleration is determined by the gravitational force, which depends only on the satellite's position (not on its velocity). Thus, $\mathbf{a}_{\parallel}$ is unaffected. However, because **v** gets flipped by the change of orbital direction, so does whether $\mathbf{a}_{\parallel}$ is along or opposite to **v**.

Thus, at point A, where $\mathbf{a}_{\parallel}$ was zero (in the counterclockwise orbit), it's still zero, and the speed is an extremum. Meanwhile, at points C and D, $\mathbf{a}_{\parallel}$ and **v** are now aligned, and the satellite is speeding up.

Problem 7.11 (p. 167)

In thinking about both situations, I find it helpful to describe the forces and acceleration using their forward (straight ahead of the car), inward (toward the center), and outward (away from the center) portions.

a. The car's acceleration, which before was purely inward (no $\mathbf{a}_{\parallel}$) and reflected its constant-speed motion, now has a forward, parallel portion reflecting its increasing speed. Thus, the net force, the cause of this acceleration, must acquire a forward portion (Figure S.36a). (The acceleration's inward, perpendicular portion doesn't change because its magnitude is v^2/R and v hasn't changed.)

   One source of this forward portion could be a reduction in F_{drag} (as the drag force points backward). However, the car's speed hasn't (yet) changed, so neither has F_{drag}. The only other source of the forward portion is the frictional force **f**. Thus, its forward portion also grows.

   Once you know the forces roughly in magnitude and direction, you can place them onto the freebody diagram (Figure S.36b)

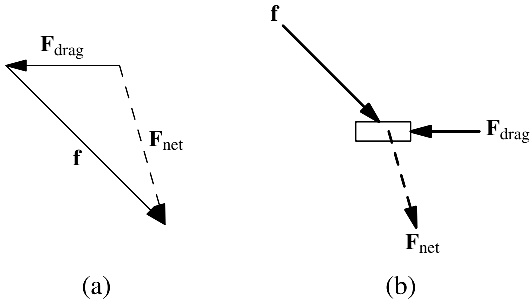

Figure S.36 (a) The vector sum modified so that the net force has a forward portion (which speeds up the car). (b) The modified freebody diagram.

For the extra curious. When you look closely at the freebody diagram, you'll see that $\mathbf{F}_{net}$'s tail lies slightly ahead of the car's center of mass. Also, **f**'s line of application, which before passed through the car's center of mass, now passes slightly ahead of the car's center of mass (and through $\mathbf{F}_{net}$'s tail). These shifts relative to the original situation do not affect the car's acceleration and are not determined by Newton's laws. Thus, you can pretend that you never noticed them.

However, in case you are curious about their origin: These shifts do affect the car's rate of rotation (about its center of mass) – a topic touched upon in Section 8.2. Before, when the car moved at constant speed around the circle, its rate of rotation was constant, which is why **f**'s line of application passed through the center of mass. Such forces are said to produce no torque and don't change a body's rate of rotation. Now, however, with the car increasing its speed, its rate of rotation is also increasing. This increase requires a torque, which is produced by force whose line of application is offset from the center of mass. (This explanation will help you with Problem 8.1.)

b. In this situation, where the car's speed has finished doubling and is back to being constant, the net force is purely inward. However, compared to the original situation, it's four times larger: Its magnitude, $mv^2/r_{\text{curvature}}$, is proportional to v^2 and v has doubled. Meanwhile, the drag force still points backward, but it too is proportional to v^2, so it's four times larger. The frictional force f, being the difference of the net force and the drag force, must therefore also be four times larger (than in the original situation).

Thus, the freebody diagram and the vector sum look identical in shape to the original freebody diagram and vector sum, except that all the force arrows become four times longer.

Problem 7.12 (p. 167)

a. To avoid being too negative, let's start with what's correct. First, the gravitational force is present and points downward. Second, the two (allegedly) actual forces, the gravitational and the centrifugal forces, add up to the net force. Finally, the ground is drawn with a dashed line (rather than a solid line), which is a reminder that the cyclist is, for this diagram, to be freed from the ground.

b. However, the diagram has many problems. First, as discussed in Section 1.5.1, the centrifugal force does not exist (Section 1.5.1). Therefore, it shouldn't appear on the diagram.

   Second, the net force has downward and outward portions. Neither portion is correct. According to Newton's second law, a downward portion means that the acceleration has a downward portion. The cyclist would be accelerating toward, and therefore into, the ground! An outward portion is also impossible. For it implies an outward acceleration portion – which is mathematically impossible. Such a portion would be a perpendicular portion of the acceleration, but the perpendicular portion, if it's nonzero, points inward – as summarized in point 6 of Section 6.6. Said another way, the cyclist's velocity is changing from coming directly at you, as in the diagram, to coming mostly at you but also slightly to the left. Thus, its change in velocity is to the left – that is, inward.

c. With such fundamental errors in the diagram, just make a new one from scratch, while keeping in mind the diagram's two correct features: that the gravitational force points down and that the actual forces must add up to the net force.

   The cyclist participates in two interactions: the gravitational interaction with the earth and a contact interaction with the ground. Thus, the cyclist experiences two forces: the downward, gravitational force (already in mind and applied at the center of mass) and the contact force of the ground (applied at the point of contact).

   These two forces, as the only forces, must add to the net force. To find the direction of the net force, find the direction of the acceleration. For a body in circular motion at constant speed, such as this cyclist, the acceleration points inward. Thus, the net force also points inward (Figure S.37).

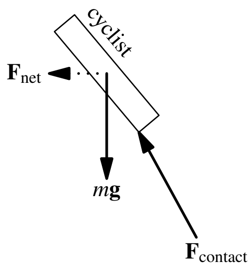

Figure S.37 The corrected freebody diagram.

Problem 7.13 (p. 168)

Perhaps the first step in making these freebody diagrams correctly is to get the net force correct. From the second law, the net force is proportional to the car's acceleration (shown in Figure 6.10).

Its acceleration at each point was the subject of Section 6.4, on constant-speed motion around an ellipse. In summary, because the car's speed is constant, its acceleration is purely perpendicular ($\mathbf{a}_{\parallel} = 0$). Furthermore, the farther away the point from the center, the smaller the path's radius of curvature there and therefore the greater the acceleration magnitude.

The net force is the sum of the two actual forces in the horizontal plane: the frictional force from the road ($\mathbf{f}$) and the drag force ($\mathbf{F}_{drag}$). (The gravitational force and the normal force are vertical, so they won't appear on the diagram. They also balance; considered together, they wouldn't contribute to the net force anyway.) Because the car's speed is constant, so is the drag-force magnitude. Thus, the drag force cannot account for the changes in the net force. Rather, those changes are the result of changes in $\mathbf{f}$ (Figure S.38). As a passive force, it adjusts itself to whatever it needs to be in order to keep the car on the track.

If the car drives clockwise (rather than counterclockwise) at the same speed as in the preceding discussion, the net force doesn't change: $\mathbf{a}_{\parallel}$ is still zero; and $|\mathbf{a}_{\perp}|$, which is $v^2/r_{curvature}$, also doesn't change because neither v nor $r_{curvature}$ change. Meanwhile, the drag force, whose magnitude depends on v, keeps its magnitude but flips its direction (Figure S.39). And the frictional force from the road, as a passive force, adjusts accordingly in order to keep the car on the track.

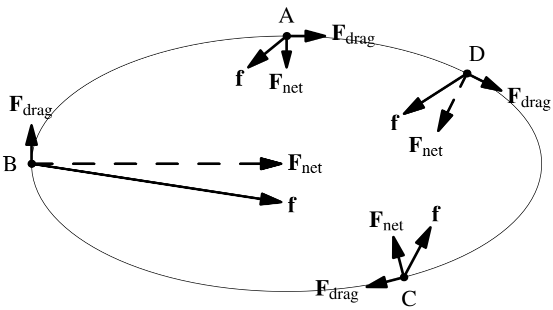

Figure S.38 Freebody diagrams of the car at various points (for counterclockwise motion).

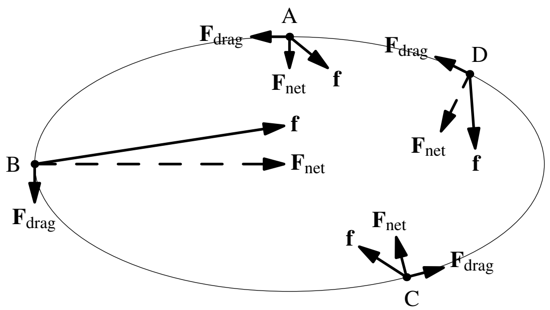

Figure S.39 Freebody diagrams of the car at various points (for clockwise motion).

Problem 7.14 (p. 168)

a. This pendulum bob and the bob in the pendulum accelerometer experience the same two forces: a gravitational force and a contact force (the tension force). They also have the same acceleration, purely horizontal. Thus, they share the same relation between the tilt angle θ and the horizontal acceleration magnitude a:

$$\tan\theta = \frac{a}{g}. \tag{S.122}$$

Here, with uniform circular motion, $a = v^2/R$, where R is the radius of the circle. From right-angle trigonometry, $R = l\sin\theta$ (Figure S.40).

Thus,

$$\tan\theta = \frac{v^2}{gl\sin\theta}. \tag{S.123}$$

Solving for v and replacing $\tan\theta$ with $\sin\theta/\cos\theta$,

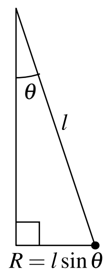

Figure S.40 The right triangle formed by the vertical, by the radius of the circle, and by the string.

$$v = \sqrt{gl\frac{\sin^2\theta}{\cos\theta}} = \sqrt{\frac{gl}{\cos\theta}}\sin\theta. \quad \text{(S.124)}$$

b. Because the bob moves at constant speed, the period is easy:

$$T = \frac{\text{circumference}}{v} == \frac{2\pi R}{v}. \quad \text{(S.125)}$$

With $R = l\sin\theta$ and using v from (S.124),

$$T = \frac{2\pi l\sin\theta}{\sqrt{gl/\cos\theta}\,\sin\theta} = 2\pi\sqrt{\frac{l}{g}}\sqrt{\cos\theta}. \quad \text{(S.126)}$$

c. When θ is small, $\cos\theta \approx 1$. So, the period (S.126) becomes

$$T \approx 2\pi\sqrt{\frac{l}{g}}. \quad \text{(S.127)}$$

And this period is indeed what you find for a (small-amplitude) pendulum after solving difficult differential equations!

Problem 7.15 (p. 169)

Air resistance has two related effects. First, reflecting that I am slowing down at the bottom of the arc, my acceleration gets a horizontal (parallel) portion. Second, air resistance results in two new forces appearing on my freebody diagram: air resistance itself and a passive, static-friction force at my feet.

However, these two effects change neither my vertical (perpendicular) acceleration nor the two vertical forces acting on me. The gravitational force is still mg downward, and the normal force, which adjusts itself to produce the inward acceleration required to keep me moving along the arc, is still given by (7.53),

$$\underbrace{N - mg}_{F_y^{\text{net}}} \to m a_y^{\text{CM}}. \tag{S.128}$$

I have added a subscript y to a_{CM} in order to clarify that only its vertical component matters. (Without air resistance, this clarification was not strictly necessary because $\mathbf{a}$ was purely vertical.)

With the assumption that my speed at the bottom of the arc is unaffected by adding the air resistance (perhaps I start higher or get an extra push at the start), a_y^{CM} now is the same as a_{CM} before. Thus, N, which is my weight, is still $1.4mg$.

Problem 7.16 (p. 169)

a. The perpendicular (vertical) portion of your acceleration points downward and has magnitude $v^2/r_{\text{curvature}}$. As with the analysis of my weight while swinging, this acceleration is produced by the two vertical forces: the gravitational force and the normal force. However, your perpendicular acceleration points downward rather than upward. Thus, with upward as the positive y direction and being careful with the signs:

$$a_y = -\frac{v^2}{r_{\text{curvature}}}, \tag{S.129}$$

and

$$\underbrace{N - mg}_{F_y^{\text{net}}} \to ma_y = -\frac{mv^2}{r_{\text{curvature}}}. \tag{S.130}$$

Thus, your weight N is

$$N = m\left(g - \frac{v^2}{r_{\text{curvature}}}\right). \tag{S.131}$$

With $v = 5$ meters per second and $r_{\text{curvature}} = 5$ meters (the track height of 10 meters is irrelevant!),

$$\frac{v^2}{r_{\text{curvature}}} = \frac{5\,\text{m}\,\text{s}^{-1} \times 5\,\text{m}\,\text{s}^{-1}}{5\,\text{m}} = 5\,\text{m}\,\text{s}^{-2}, \tag{S.132}$$

which is approximately $0.5g$. Thus,

$$N \approx 0.5mg. \tag{S.133}$$

This reduction in your weight is the physics description of the roller-coaster feeling.

b. A seat belt provides an attractive normal force (particularly if you represent it mentally as velcro or glue clasping you to the seat). Thus, a seat belt makes **N** point downward. At the threshold speed where you start needing a seat belt, **N** is zero. Setting $N = 0$ in (S.131) gives

$$g = \frac{v^2}{r_{\text{curvature}}}, \tag{S.134}$$

or

$$v = \sqrt{g r_{\text{curvature}}} \approx \sqrt{10\,\text{m}\,\text{s}^{-2} \times 5\,\text{m}} \approx 7\,\text{m}\,\text{s}^{-1} \tag{S.135}$$

(about 25 kilometers or 15 miles per hour).

Problem 7.17 (p. 169)

Don't make this question harder than it needs to be! You're standing peacefully on a kind of ground (admittedly, a slightly shaky ground). Thus, the normal force on you is mg upward, and your weight is mg.

Problem 7.18 (p. 169)

This skier and scale are zooming down a frictionless hill with acceleration $g \sin\theta$ down the hill. The skier's freebody diagram is therefore the same as yours when you slid down the frictionless ramp (Section 7.1.4). Thus, two forces act on you: a gravitational force mg downward and a contact force $mg \cos\theta$ perpendicular to the hill. (Their sum, $mg \sin\theta$ down the hill, causes your acceleration.)

Because the scale lies parallel to the hill, the contact force is also perpendicular to the scale. Thus, the scale displays $mg \cos\theta$.

When the skier is skiing at his or her terminal speed, it's once again the situation of constant-speed sledding down a frictionless slope (Section 5.5.1). Now the skier's acceleration is zero. Air resistance balances the parallel portion of the gravitational force. Because the hill is frictionless, the contact force is still normal to the hill, and it still balances the perpendicular portion of gravity. Thus, it still has magnitude $mg \cos\theta$ – which is the skier's weight.

Problem 7.19 (p. 170)

a. The rider's perpendicular acceleration is $v^2/r_{\text{curvature}}$ downward. This acceleration is caused by two downward forces: the gravitational force and the normal force of the scale. Thus, from the second law (for the vertical components of force and acceleration)

$$-N - mg = -\frac{mv^2}{r_{\text{curvature}}}. \tag{S.136}$$

For my sanity, I get rid of all those awful minus signs (equivalently, I use a coordinate system in which downward is the positive y direction).

$$N + mg = \frac{mv^2}{r_{\text{curvature}}}. \tag{S.137}$$

Thus,

$$N = m\left(\frac{v^2}{r_{\text{curvature}}} - g\right). \tag{S.138}$$

With the given values,

$$\frac{v^2}{r_{\text{curvature}}} = \frac{15\,\text{m}\,\text{s}^{-1} \times 15\,\text{m}\,\text{s}^{-1}}{15\,\text{m}} = 15\,\text{m}\,\text{s}^{-2}, \tag{S.139}$$

which is approximately $1.5g$. Thus,

$$N \approx m(1.5g - -g) = 0.5mg \approx 0.5 \times 60\,\text{kg} \times 10\,\text{m}\,\text{s}^{-2} = 300\,\text{N}. \tag{S.140}$$

i. A positive scale reading means that the scale pushes on the rider and, by third law, that the rider pushes equally hard on the scale. Thus, the rider is "stuck" to the scale and needs no seat belt to stay there. Even after solving many such problems, I still find this physical consequence hard to believe!

ii. The rider stays in the car because the car, which follows the path of the track, "falls" faster than the rider would when subject only to gravity. That is, the track is more sharply curved than the rider's path would be in free gravitational motion (at 15 meters per second horizontally). Thus, the track pushes the rider downward (and no seat belt is needed).

b. At the bottom of the loop, the rider's perpendicular acceleration is upward, as is the normal force. But the gravitational force is still downward. Thus, the vertical-component equation is

$$+N - mg = +\frac{mv^2}{r_{\text{curvature}}}. \tag{S.141}$$

(It has the same form as the equation describing my scale reading while swinging.) With the given values,

$$\frac{v^2}{r_{\text{curvature}}} = \frac{25\,\text{m}\,\text{s}^{-1} \times 25\,\text{m}\,\text{s}^{-1}}{15\,\text{m}\,\text{s}^{-1}} \approx 42\,\text{m}\,\text{s}^{-2} \approx 4.2g. \tag{S.142}$$

Thus,

$$N = m\left(\frac{v^2}{r_{\text{curvature}}} + g\right) \approx 5.2mg. \tag{S.143}$$

With $m = 60\,\text{kg}$,

$$N \approx 5.2 \times 60\,\text{kg} \times 10\,\text{m}\,\text{s}^{-2} \approx 3100\,\text{N}. \tag{S.144}$$

Such a high normal force – in colloquial terms, a high g force – is exciting for a short time but debilitating if it lasts too long (as it nearly did on the Gemini 8 mission, where the spacecraft malfunctioned and was spinning rapidly, but Neil Armstrong managed to pilot it to safety).

Problem 7.20 (p. 170)

This horizontal portion points forward and causes your forward acceleration – necessary for you to remain on the accelerating wedge (which itself has a forward acceleration). To see the effect another way, imagine reducing this portion to zero by oiling the scale. As it moves ever faster down the hill, it would slide out from under your feet.

Problem 7.21 (p. 170)

In order to understand, we need to go back to the drawing board! Here, the drawing board is what we know about force: The net external force is mass times acceleration (Newton's second law). To find the external forces, we use our recipe of starting with the ball's interactions. The ball, while it's contact with the table, participates in two interactions: a gravitational one with the earth and an electromagnetic one with the table. Thus, there are two forces, the gravitational force with magnitude mg and pointing downward, and a contact (or normal) force pointing upward with (unknown) magnitude N.

Thus, $N - mg$ is the vertical component of the acceleration. That leads to the next question: What's the acceleration? Here is one possible analysis. Acceleration is defined as the derivative of the velocity,

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}. \tag{S.145}$$

Because we are studying the moment, or instant, when the ball is just stationary on the ground, the velocity is zero. Because the derivative of a number, including zero, is zero, the acceleration is also zero.

► *What do you think of this analysis?*

It's rubbish! If generalized, it would lead to nonsense everywhere. At any instant, for any object, the velocity is just a number (and units, such as meters per second). Thus, acceleration would always be zero. That nonsense conclusion should encourage us to find the flaw.

It's in confusing a momentarily zero velocity with a velocity that is a constant zero. If you set the ball on the table, and it sits there, then its velocity is a constant zero, and its derivative is also zero. In that case, the acceleration is zero, so the two external forces add to zero. And they do: The normal force balances the gravitational force.

In that case! In the case of the bouncing ball, the story is different. To see what happens, let's approximate the instantaneous acceleration with the average acceleration. We'll need only an approximate value anyway in order to decide on the contact force.

Thus,

$$\mathbf{a} \sim \frac{\Delta\mathbf{v}}{\Delta t}. \tag{S.146}$$

For the change in velocity, we need to know the velocity at two instants. We know it approximately when the ball first touches the table, and also when the ball first leaves contact with the table. With the idea of energy conservation, touched on in the chapter on what comes next, Chapter 8, you can predict it. But here we need just an approximate value, and home experiments should convince you that it's about 1 meter per second, give or take a factor of 2. It thus starts out at 1 meter per second downward, and ends up at 1 meter per second upward. The difference (end minus start) is 2 meters per second up.

The denominator is trickier. It's the time interval between these two velocity instants. Thus, it's the time in which the ball remains in contact with the table. There are several ways to get an idea of this time. The first, the crudest but simplest, is to note that it's not perceptible. That is, the bounce seems to happen instantly. Thus, it's well below the typical human perception threshold of, say, 100 milliseconds. Let's guess that it's below 10 milliseconds. Then the acceleration is about 200 meters per seconds squared, upward. That is about $20g$. So, the net force is $20mg$ (upward), making the normal force roughly the same (it's $21mg$, but having made such crude estimates, it's dangerous to give the results spurious precision).

A slightly more accurate estimate of the contact time comes from the following idea. At the first time instant, the bottom of the ball has just reached the table. The top of the ball, however, has no knowledge of this event, and keeps falling – thereby compressing the ball. Eventually, however, the bottom of the ball tells the top, "Stop! We've hit bottom here, and there's no more room." Then the top stops, turns around, and starts moving upward. The time required for the top to receive this message is roughly the contact time.

But what kind of message is it? It's a sound wave! And sound travels at about 5 kilometers per second in steel. If the ball has a diameter of, say 1 or 2 centimeters (the upper end of this range is a big ball bearing), the contact time is roughly

$$\Delta t \sim \frac{2\,\text{cm}}{5\,\text{km/s}} \approx 5\ \mu\text{s}. \tag{S.147}$$

Let's round this time to the nearest power of 10 and call it 10 microseconds.

Now we can estimate the acceleration:

$$\mathbf{a} \sim \frac{\Delta \mathbf{v}}{\Delta t}. \tag{S.148}$$

With Δv about 2 meters per second or, rounded to the closest power of 10, about 1 meter per second, and $\Delta\mathbf{v}$ therefore 1 meter per second upward, $\mathbf{a}$ becomes 10^5 meters per second squared. This acceleration is 10^4 times the gravitational acceleration (and in the opposite direction). Thus, the net force is roughly $10^4 mg$ upward, as is the contact force and the weight of the ball during the bounce.

It turns out that this simple estimate is mostly right. Because the ball is spherical, rather than a rod (whose cross-section does not change as it compresses during contact), the acceleration and net force are slightly smaller. The correction factor is roughly

$$\left(\frac{\text{speed of sound in steel}}{\text{impact speed}}\right)^{1/5}. \tag{S.149}$$

In this example, that ratio is

$$\left(\frac{5\,\mathrm{km\,s^{-1}}}{1\,\mathrm{m\,s^{-1}}}\right)^{1/5} \sim 5. \tag{S.150}$$

Thus, the net force is roughly a few thousand *mg* upward.

Problem 7.22 (p. 170)

At all three points, the ball participates in one long-range interaction, the gravitational interaction with the earth. Thus, it experiences a gravitational force mg downward. At all three points, the ball also participates in a short-range, contact interaction with the air. Thus, it experiences a drag force opposite to its velocity. Finally, at point C, the ball also participates in a second short-range, contact interaction – with the ground. Thus, at point C, it experiences a third force, the contact force of the ground. Because the ball–ground contact is frictionless, this contact force has no horizontal portion (no static or dynamic friction). Thus, the contact force points (directly) upward.

The preceding discussion specifies the forces and their directions. The next task before drawing the diagrams is to order the forces by magnitude. Three of the forces, the gravitational forces, have magnitude mg (where m is the ball's mass).

► *Where do the other forces lie relative to this magnitude?*

On a solid rubber ball, which is quite massive for its size, the drag force is weaker than the gravitational force. (This ordering could be reversed for a hollow plastic ball, as estimated in Section 1.3.3.)

But how do the three drag forces rank among themselves? Drag's magnitude increases with speed. The ball moves the fastest at point A, where the ball has fallen far but not yet lost speed in the first bounce nor in the battle against drag from point A to point C. It moves slowest at point B, the peak of the trajectory, having lost all its vertical velocity fighting gravity. At point C, its speed is in between. Thus, drag is the largest at point A, is next at point C, and the smallest at point B (and all are smaller than mg).

The final force is the normal force $\mathbf{N}$ acting on the ball at point C. Because rubber is far less stiff than steel, N is not so large as $10^4 mg$, which was the normal-force magnitude estimated in (7.41) for a steel ball bouncing from a table. Even so, N is far larger than mg. (If you want to reduce N to only a few times larger than mg, imagine dropping a giant, squishy, and light foam ball from not very high.)

Thus, the forces ordered from weakest to strongest are $\mathbf{F}^{\mathrm{B}}_{\mathrm{drag}}$, $\mathbf{F}^{\mathrm{C}}_{\mathrm{drag}}$, $\mathbf{F}^{\mathrm{A}}_{\mathrm{drag}}$, the three gravitational forces, and $\mathbf{N}$. Now you can draw the freebody diagrams (Figure S.41).

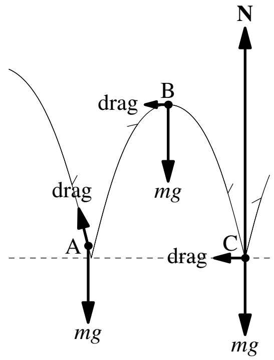

Figure S.41 Freebody diagrams of the ball at various points.

Problem 7.23 (p. 171)

a. In region 1 of Figure 7.35, the elevator is sitting waiting patiently for me to decide where I want to go.

In region 2, the weight dropped: from 134 "grams," meaning from

$$\underbrace{0.134\,\mathrm{kg}}_{134\,\mathrm{g}} \times 10\,\mathrm{m\,s^{-2}} = 1.34\,\mathrm{N}, \qquad \text{(S.151)}$$

to 101 "grams" (1.01 newtons). To determine what kind of elevator motion resulted in this reduction, imagine the extreme case of an elevator in free gravitational motion (in free fall). As in an orbiting spacecraft, the food would be weightless. In reality, the food didn't become weightless, but its weight did first drop. Thus, the accelerator was partly in free gravitational motion (partly in free fall) – what you'd expect from an elevator starting its descent from the sixth floor and picking up speed (accelerating downward).

Thus, in region 2, $v_z < 0$ and $a_z < 0$.

In region 3, the weight is back to normal (134 "grams"). Thus, $a_z = 0$ and, therefore, v_z isn't changing. So, v_z is still negative.

In region 4, as the elevator approaches the first floor, the weight rises, indicating an upward acceleration: $a_z > 0$. However, the elevator is still descending: $v_z < 0$.

As an analogy, imagine jumping from a table (if you try it at home, please use a low table for safety). As you land and bend your knees, you push hard against the ground, harder than you do when you hold yourself upright against your normal weight. By the third law, the ground also pushes hard on you (the other side of the interaction), giving you an upward acceleration and slowing you down. The elevator, the scale, and the food on the scale are going through the same process of slowing down. During it, they weigh more than their normal weight.

Only in region 5 has the elevator arrived at the first floor. The weight has returned to normal, meaning that a_z is zero and the elevator has finished slowing itself down. Thus, it has reached its final speed of zero ($v_z = 0$).

b. The maximum upward acceleration occurs in region 4, when the weight reaches its peak of 167 "grams" or, in force units, 1.67 newtons. Symbolically, Newton's second law for the food, applied to upward components, says (with N as the normal force or weight):

$$N - mg \rightarrow ma_z. \qquad \text{(S.152)}$$

Thus,

$$a_z = \left(\frac{N}{mg} - 1\right) g = \left(\frac{N - mg}{mg}\right) g. \tag{S.153}$$

The product mg is the normal weight. Doing the calculation of the parenthesized quantity using fake grams,

$$\frac{N - mg}{mg} = \frac{(167 - 134)\text{ “grams”}}{134\text{ “grams”}} \approx 0.25. \tag{S.154}$$

Thus, the upward acceleration is about $0.25g$.

c. The downward acceleration reaches its maximum magnitude in the trough in region 2. Using (S.153) there,

$$a_z \approx \frac{(101 - 134)\text{ “grams”}}{133\text{ “grams”}} g \approx -0.25g. \tag{S.155}$$

(The negative sign of a_z is a welcome check on the labeling of region 2 in item a.)

In thinking about why the weight dropped in region 2, I found myself thinking about forces and causation. For I first reasoned that the elevator accelerating downward caused the drop in the normal force (the weight). But that statement seems to contradict the fundamental point about Newton's laws that I have stated often, maybe too often, in this book: that force causes acceleration and not vice versa.

The seeming contradiction is reconciled through a slow-motion replay of the causal sequence. It begins at the end of region 1, with the elevator, scale, and food all at rest.

1. The tension force, from the steel cable holding up the elevator, drops (an effect itself the result of a causal chain started by the elevator's motor).

2. This upward force then can no longer balance downward gravitational force on the elevator. Illustrating that (net) force causes acceleration, the elevator starts accelerating downward.

3. Once the elevator has moved downward slightly, the spring interaction between the elevator floor the bottom of the scale drops in magnitude.

4. The upward, spring force on the scale no longer balances the two downward forces on the scale: the gravitational force on the scale and the contact force of the food. Illustrating again that (net) force causes acceleration, the scale starts accelerating downward.

5. Rinse and repeat, as they say on shampoo bottles (so that you buy more shampoo). Once the scale has moved downward slightly, the spring interaction

between it and the food drops in magnitude. In other words, the weight decreases.

In this sequence, steps 2 and 4 illustrate the Newtonian direction of causation: a downward net force caused a downward acceleration.

However, steps 3 and 5 look like examples of the opposite direction of causation, whereby (downward) acceleration decreased a spring force's magnitude. In both cases, intervening steps were motion and, implicitly, a new position. For example, in step 3, the scale's downward acceleration led to its downward motion, which led to its new position, slightly farther away from the food. And this new position decompressed the chemical bonds in the food and the scale at the contacting surface.

As formalized in (1.18), an (ideal) spring interaction's strength is proportional to the spring's compression or extension. Thus, the strength drops, as does the magnitude of the normal force on the food (because the normal force is one side of the interaction).

Thus, this seemingly inverted causation results from a repulsive spring interaction and the relation between its (relative) position and interaction strength. And the repulsive spring interaction, for a material-to-material contact, is the result of Coulomb's law of electrostatics. As the outer-shell electrons in the food molecules at the contact and in the scale molecules at the contact increase their separation, the repulsive force, being an inverse-square force, decreases in magnitude.

Thus, when you think you've found an inverted causation where acceleration seems to cause force, chercher la spring force (if you'll pardon my French) and, at its root, an electrostatic repulsion.

Problem 7.24 (p. 172)

Before trying to demonstrate any result, make sure that the result is at least plausible! This advice generalizes Wheeler's First Moral Principle, introduced in Section 6.3: "Never make a calculation until you already know the answer" [22, p. 20].

Here, and often in other problems, the first and best way to check plausibility is to check just the effect's sign. If a quantity decreases, should it? If it increases, should it? Or, if it remains fixed, should it?

Here, the claimed scale reading Mg is less than the usual weight of $(M+m)g$. Thus, the check question becomes: Should sending the bees into free fall reduce the scale reading (the weight)?

It should! To see why, make a sign-only comparison of the motion and forces with their respective values in the original situation (when the bees had zero acceleration).

For the motion: The composite body of box and bees, which before had zero acceleration, now accelerates downward. For the composite body's acceleration – defined as its center-of-mass acceleration – is the weighted (sorry!) average of its constituents' accelerations. Although the box does not accelerate, the bees accelerate downward – and, therefore, so does the composite body.

For the forces: Before, where the composite body had zero acceleration, the two forces on it – the upward, normal force and the downward, gravitational force – balanced. Thus, N was equal to F_g. Now, where the composite body has a downward acceleration, the upward force must be not quite strong enough to balance the downward force. In other words, N must be less than F_g. Because $F_g = (M+m)g$ (the usual weight), N is now less than $(M+m)g$ – making the claim that $N = Mg$ plausible.

Determining whether it's also correct requires the full analysis. But now you are ready for it. For in making the sign-only analysis, you determined the structure of the full analysis. All that remains is to make it exact. Thus, there are two questions to answer. What's the composite body's exact acceleration? What's the exact resulting N?

For the composite body's center-of-mass acceleration, the weighted average is

$$\mathbf{a}_{\mathrm{CM}} = \frac{M\mathbf{a}_{\mathrm{box}} + m\mathbf{a}_{\mathrm{bees}}}{M+m}. \tag{S.156}$$

The box is stationary, so $\mathbf{a}_{\mathrm{box}} = 0$; and $\mathbf{a}_{\mathrm{bees}} = g$ downward. Thus,

$$\mathbf{a}_{\mathrm{CM}} = \frac{m}{M+m} g \text{ downward.} \tag{S.157}$$

Determining N uses Newton's second-a-half law. Only the vertical (z) portions or components matter here. Taking upward as the positive z direction,

$$\frac{1}{M+m}\left(N-(M+m)g\right) \to -\frac{m}{M+m}g. \qquad \text{(S.158)}$$

Conveniently, the $M + m$ denominator is in the left and right sides, so it cancels. The resulting equation is

$$N-(M+m)g=-mg. \qquad \text{(S.159)}$$

Its solution is

$$N = Mg. \qquad \text{(S.160)}$$

The bees, who are now in free gravitational motion (in free fall) and are themselves weightless, now do not show up in the overall weight.

Figure S.42 shows the freebody diagram including the motion. 1

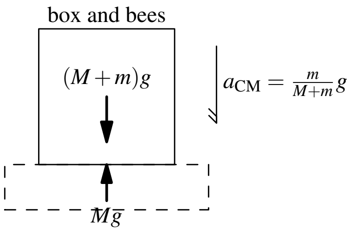

Figure S.42 The freebody diagram of the box and bees when the bees are in free fall. The gravitational force is slightly stronger than the normal force. Thus, the composite body's center of mass has a downward acceleration. Although the box itself has zero acceleration, the bees have a downward acceleration.

An interesting question arises from this freebody diagram. It shows that the normal force, which is the passive force provided by the scale, adjusts itself based on the acceleration of the bees. When the bees were hover ($\mathbf{a}_{\text{bees}} = 0$), N was $(M + m)g$. Now, when the bees are in free gravitational motion ($\mathbf{a}_{\text{bees}} = g$ downward), N has magically become Mg.

How does the scale know the bees' acceleration? The only body touching the scale is the box, whose motion is independent of the bees' acceleration (the box stays motionless even as the bees' change their acceleration). The scale and the bees participate in some kind of an action-at-distance (long-range) interaction.

The explanation and mechanism is the air. To see how, follow the forces. When the bees hover, the air exerts on them an upward force mg (balancing the gravitational force on them). By Newton's third law, the bees exert on the air a downward force mg. Despite this force, the air has zero acceleration – because the box exerts on the air a balancing force mg upward.

Warning! Even though this force is equal in magnitude and opposite in direction to the downward mg force that the bees exert on the air, these two forces are not a third-law pair. Rather, their common magnitude and opposite direction are a consequence of the second law and the requirement that the air not accelerate.

The third-law counterpart of the upward force experienced by the air is the downward force of the air on the box. The normal force on the box (from the scale) must compensate for this downward force, which has magnitude mg; otherwise the box would accelerate. Thus, N isn't merely Mg, which is enough only to support the empty box, but also includes an mg term. Thus,

$$N = Mg + mg = (M + m)g. \tag{S.161}$$

This story changes after the bees start their free gravitational motion (their free fall). Now the air exerts no upward force on the bees and, therefore, no downward force on the box. The mg term in (S.161) vanishes, making $N = Mg$.

The air carries the action-at-a-distance interaction between the bees and the box.

Microscopically, the box isn't completely motionless. Rather, when the bees hover, the box floor is slightly compressed by the extra pressure forces from the air. When the bees start their free gravitational motion, these extra forces disappear, and the box floor slightly uncompresses. These tiny changes, which result in changes in the internal forces within the box floor and therefore in the contact force on the scale, tell the scale what the normal force should be.

Problem 7.25 (p. 172)

a. For finding T_3, the right composite body joins all three sleds together and has mass $m_{123} = m_1 + m_2 + m_3$ and acceleration a to the right. (Two strings are part of the composite body, but they are massless.)

   From Newton's second-and-a-half law (7.44), the net *external* force on the composite body must be $m_{123}a$. The external forces are three: the normal force (without friction, the contact force is perpendicular to the ice); the gravitational force, which balances the normal force; and tension force with magnitude T_3 and pointing to the right. Thus, the net external force, which is T_3 to the right, must equal $m_{123}a$.

$$T_3 = m_{123}a = (m_1 + m_2 + m_3)a. \tag{S.162}$$

b. For finding T_2, the right composite body joins the first two sleds and has mass $m_{12} = m_1 + m_2$. Now just follow the argument in part (a) for finding T_3. The net external force on this composite body is T_2 to the right: The string force (on the m_2 part) is the composite body's only unbalanced external force. From (7.44),

$$T_2 = (m_1 + m_2)a. \tag{S.163}$$

c. For finding T_1, the right body isn't composite: It's just the first sled, which has mass m_1. The net external force on this body is T_1 to the right (the string force is, again, the body's only unbalanced external force). From (7.44),

$$T_1 = m_1 a. \tag{S.164}$$

The moral: It's better to be the first string than the last string, which, in order to accelerate the entire sled train, has the highest tension.

Problem 8.1 (p. 191)

a. The line of action's placement can be deduced from the effect of **f** on the car's rotation. The first step is to determine what's happening to the rotation – in particular, to the rotation rate.

Imagine looking at the car from above, keeping yourself directly above the car's center of mass. But don't rotate your head. In other words, the origin of your reference frame (say, the bridge of your nose) accelerates – like the car, it's in uniform circular motion – but your reference frame does not rotate about its origin (your eyes stay aligned left to right, say).

► *In your reference frame, how does the car's motion look?*

A body's motion, fully described, includes two aspects: the motion of its center of mass (translation) and its motion about its center of mass (rotation).

In your reference frame, the translation vanishes by design: The reference frame tracks the car's center of mass. Thus, the only possible remaining motion is the car's rotation (about its center of mass).

And you do see the car rotate. For example, when it's at the 12 o'clock position of the circle, it's heading right (in the 3-o'clock direction). And when it's at the 3 o'clock position, it's heading down (in the 6-o'clock direction). For every full rotation around the circular track, the car makes one full rotation about the vertical axis through its center of mass.

Because the car is moving at constant speed, its rate of rotation around the track and, therefore, its rate of rotation around this axis are constant. A constant rate of rotation, meaning zero angular acceleration, implies zero net torque (about the vertical axis): Net torque causes angular acceleration.

Thus, all the forces acting on the car must combine to produce zero net torque about the vertical axis. Without air resistance, three forces act on the car: the gravitational force, the normal force, and **f**. The gravitational force acts at the center of mass, so it produces no torque at all about any axis through the center of mass, including the vertical axis. The normal force might not act underneath the center of mass; however, as a vertical force, it doesn't affect the car's rotation about the vertical axis (it doesn't turn the car).

Because neither the normal force nor the gravitational force affects the car's rotation around the vertical axis, **f** is the only source of torque about this axis. To make this torque zero, **f** must have zero lever arm: It's line of action must pass through the axis. This requirement is reflected in choice (ii).

b. When the car speeds up (accelerates), its rotation speed (about the same vertical axis) increases. This increase is caused by a torque provided by **f**. Its line of action should lie such that **f** tries to spin up the car clockwise, to augment the clockwise rotation rate. Thus, the line of action should lie ahead of the center of mass: choice (iii).

c. When the car slows down (decelerates), its rotation speed decreases. This decrease is also caused by a torque from **f**. Its line of action should lie such that **f** tries to spin up the car counterclockwise (to reduce the clockwise rotation rate). Thus, the line of action should lie behind the center of mass: choice (i).