We have taken the topics of closed-loop log modulus an Nichols chart out of the text. One can apply the concept of closed-loop modulus in controller design easily with a computer. Nichols chart, on the other hand, takes some thinking and requires reading a chart even if that may be prepared by a computer (MATLAB can do that). This method has fallen out of favor, and we don't see much of it in more recent texts. We provide here only the important equations associated with the closed-loop modulus and phase angle.

Closed-loop Log Modulus

A Bode plot uses the open-loop function. There is no reason why we cannot plot the closed-loop transfer function versus frequency on a log-log scale. The idea is that we may want to see the actual closed-loop frequency response since we are designing the closed-loop system. Of course, we lose the ability to use the Nyquist Stability Criterion. We only use the closed-loop modulus after application of either a Bode plot or Nyquist plot for stability analysis.

All we have here is an empirical criterion. Again, using the idea that we are designing a slightly underdamped system, we expect to see a peak magnitude and we put a limit on it so that it will not be too oscillatory. This peak magnitude is the **maximum closed-loop log modulus**, M_p. We try to limit M_p in the range of 1.25 to 1.3 (which corresponds to a damping ratio ζ of roughly 0.5 to 0.4).

The frequency at M_p is the peak frequency ω_p. We also would like to have a large ω_p. A large ω_p means $C/R \approx 1$ over a larger frequency range, i.e., the response is fast enough that the controlled variable can track the set point change closely over a wide range of operating conditions.

Nichols Chart

We rewrite the open-loop function in polar form

$$G_c(j\omega)G_p(j\omega) = \left|G_{OL}\right| e^{j\phi_{OL}} \tag{1}$$

where the subscript OL stands for open-loop. For a servo problem with a unity feedback loop (*i.e.*, $G_m = 1$)

$$\frac{C}{R} = \frac{G_c(j\omega)G_p(j\omega)}{1 + G_c(j\omega)G_p(j\omega)} = \frac{\left|G_{OL}\right| e^{j\phi_{OL}}}{1 + \left|G_{OL}\right| e^{j\phi_{OL}}} \tag{2}$$

We want to find a polar form for the closed-loop function,

$$\frac{C}{R} = \left|G_C\right| e^{j\phi_C}$$

Equate the last two equations, and we can find [1]

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[1] It takes *some* algebraic work to arrive at the results (3) and (4). Along the way, you need the help of Euler's identity and the trigonometry identity $\sin^2\phi + \cos^2\phi = 1$.

$$|G_C| = \frac{|G_{OL}|}{\sqrt{|G_{OL}|^2 + 2|G_{OL}|\cos\phi_{OL} + 1}} \tag{3}$$

and

$$\phi_C = \tan^{-1}\left(\frac{\sin\phi_{OL}}{|G_{OL}| + \cos\phi_{OL}}\right) \tag{4}$$

These conversions can be done easily with a small program. The Nichols chart is prepared for any general system with a unity feedback loop. It plots the magnitude of the open loop function log $|G_{OL}|$ versus the open-loop phase lag, ϕ_{OL}, with contours of constant $|G_C|$ and ϕ_C. Thus for any given pairs of open-loop magnitude and phase lag, we can read off the corresponding closed-loop values.