Valve Characteristics and Steady State Gains

In process engineering, one of the most important actuating devices is the regulating valve. There are many different kinds of valves, but they all are modeled with the same basic equation that is based on the use of the Bernoulli equation across an orifice. The key point in this note is to show how flow rate, and hence the value of the steady state gain of a valve, may vary with the valve opening.

The liquid flow rate q through a valve generally can be described with the equation

$$q = C_v(\ell) \sqrt{\frac{\Delta P_v}{\rho_f}} \tag{1}$$

From introductory fluid mechanics, we should be familiar with the functional form involving the pressure drop across the valve ΔP_v, and the fluid density ρ_f. In the process industry, the units of the flow are commonly in gpm, pressure drop in psi, and the density in specific gravity.

The flow through the valve is also governed by the valve coefficient (also called the capacity factor) C_v that describes the valve characteristics. The coefficient is defined such that it is the flow rate in gpm of water at a pressure drop of 1 psi across a given valve. The valve coefficient is a function of the percent valve opening, also called the *lift* $\boldsymbol{\ell}$, where $0 \le \boldsymbol{\ell} \le 1$. The manufacturer generally provides the characteristics curve.

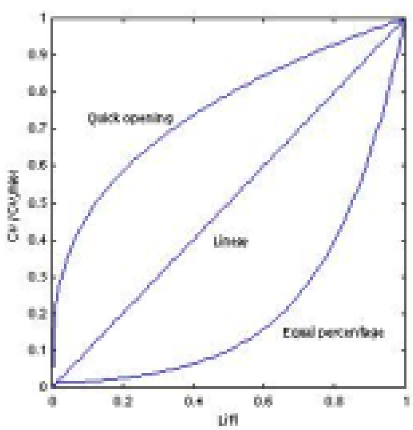

Figure 1. Valve characteristic curves. The quick opening curve is plotted with $\boldsymbol{\ell}^{1/3}$. The equal percentage curve is plotted with R = 100.

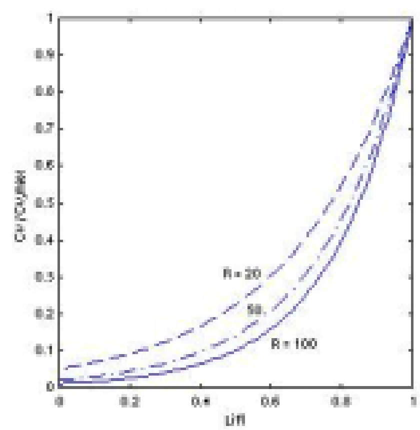

Figure 2. The effect of the rangeability parameter on the equal percentage valve characteristics. The curves are based on Eq. (4) and not adjusted to have zero flow at zero lift.

There are three common valve characteristics, and they are:

Linear $$C_v / C_{v,max} = \boldsymbol{\ell} \tag{2}$$

Quick opening: $$C_v / C_{v,max} = \boldsymbol{\ell}^{1/\alpha} \quad , \alpha > 0 \tag{3}$$

Equal percentage: $$C_v / C_{v,max} = R^{\boldsymbol{\ell}-1} \tag{4}$$

Their general shapes are illustrated in Fig. 1. With the quick opening valve, most of the flow variation is introduced in the lower end of the valve stem travel (*i.e.*, lift). This kind of valve is good for use as on-off or relief valves that we want to deliver a large flow rate as quickly as possible.

To regulate flow rates, we prefer either the linear or equal percentage valves. The flow through a linear valve is simply proportional to the valve opening. The steady state gain of the valve will also be a constant.

With an equal percentage valve, the flow rate increases more gradually with the valve position. In Eq. (4), the quantity R is the *rangeability* parameter. In manufactured products, a rangeability of 50 is common. Fig. 2 shows how the valve characteristic function varies with the lift for different values of R. With the "power law" definition in (4), C_v and hence q, will not be zero when ℓ is zero. So in reality, actual valve characteristic curves will deviate from Eq. (4) in the lowest 5% or so of the lift such that the flow will indeed be zero when the valve is closed.

You may now ask why the term "equal percentage"? If we substitute (4) in (1) and take the derivative of q with respect to ℓ, we should find:

$$\frac{dq}{d\ell} = C_{v,max} \ln R \; R^{\ell-1} \sqrt{\frac{\Delta P_v}{\rho_f}}$$

Here, we should recognize that most of the terms make up q. Thus we really have

$$\frac{dq}{d\ell} = \ln R \; q \qquad \text{or} \qquad \frac{1}{q}\frac{dq}{d\ell} = \ln R \tag{5}$$

The relative (percent) change in flow rate with respect to a given change in the lift is a constant (hence the term "equal"). Because of this feature, with a larger valve opening (or flow rate), the same percent change in the lift will lead to a larger increase in the actual flow rate.[1]

The result is that the steady state gain of an equal percentage valve increases with the operating flow rate. How do we calculate the valve gain? We need first the controller output p so we can define the gain as the change in flow as a result of the change in controller output:

$$K_v = \frac{\Delta q}{\Delta p} = \frac{\Delta q}{\Delta \ell}\frac{\Delta \ell}{\Delta p} \tag{6}$$

We already have $\Delta q/\Delta \ell$ from (5), while $\Delta \ell/\Delta p$ depends on how we handle (theoretically) the controller output. We can use 4-20 mA, or for simplicity, we take the controller output as 0-100 %C, so $\Delta \ell/\Delta p$ is 1/100 fractional opening/%C.[2]

Hence, we can write (6) as

$$K_v = \pm \frac{1}{100} \ln R \; q^s \qquad \left[\frac{\text{flow units}}{\%\text{C}}\right] \tag{7}$$

where we have added the ± sign to indicate that the valve may be either air-to-open or air-to-close. We also have added the superscript s to flow rate q with the understanding that when we linearize a function, it has to be with respect to some steady state value. With (7), it may finally be obvious that with equal percentage valves,

$$K_v \propto q^s$$

When should we choose the equal percentage valve? With some nonlinear processes, their steady state gain K_p may decrease with throughput, *i.e.*, the processing flow rate. If we implement an equal percentage valve in the control system, we can now keep the system characteristic in terms of the product K_vK_p fairly constant. In other words, we would not have to worry about retuning the controller as much when the flow rate changes.

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[1] Say, a 1% increase of 100 gpm is 1 gpm, while a 1% increase of 10 gpm is only 0.1 gpm.

[2] See our supplementary notes of percent steady state gains.