Steady state gains with percent units

In more applied textbooks, we may find that signals are reported on a percent basis. This practice arises from how some commercial instrumentation panels are actually implemented. Under such circumstances, the physical or engineering units are transparent to a user. So instead of using, for example, 4 to 20 mA, or 3 to 15 psig, the signal is reported as in the range 0 to 100%. The consequence is that we also have to work with steady state gains with percent units.

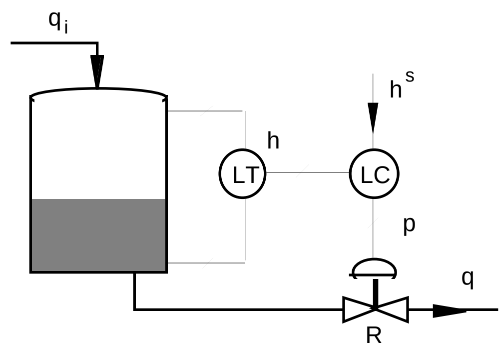

Figure 1. Liquid level control system taken from Chapter 5, Fig. 5.1.

We will use the liquid level system in Fig. 1 to illustrate how the units may appear. Its block diagram is in Fig. 2. Since we are trying to decipher the units, the block diagram contains only the notations of different gains. We also use American engineering units since this is a matter of industrial implementation.

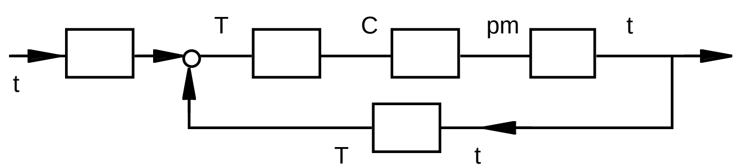

Figure 2. Block diagram of the liquid level control system illustrating the use of percent transmitter and percent controller output units.

We start with the process function, or rather now, the process steady state gain K_p. It still has the engineering units of ft/gpm, representing how changes in the flow rate may lead to changes in the liquid level. Now, the transmitter gain K_T has the units of %T/ft, where %T stands for percent transmitter output. Suppose the level transmitter is capable of measuring a calibrated range of 0 to 10 ft, the transmitter gain would be [1]

$$K_T = \frac{100\ \%T}{(10 - 0)\ \text{ft}} = 10\ \frac{\%T}{\text{ft}}$$

How may this percent-based transmitter gain relate to our more familiar engineering units? For lack of better notations, we consider K_T a product of K_t and K_m (Fig. 3). The measurement gain with units of mA/ft is K_m, while the conversion between physical and percent units is handled by K_t.

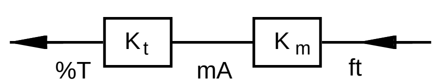

Figure 3. How the percent transmitter gain can be related to our more familiar measurement gain: $K_T = K_t K_m$.

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[1] There is no significance in choosing the two terms transmitter and measurement gains. They are arbitrarily chosen just to have different units. Strictly speaking, a transducer is made of a sensor which produces a measurable physical or chemical phenomenon, and a transmitter then converts this phenomenon into a signal.

If we are using 4-20 mA as the signal, then K_t must be

$$K_t = \frac{100\ \%T}{(20-4)\ mA} = 6.25\ \frac{\%T}{mA}$$

And thus working backward, K_m in this example should be

$$K_m = \frac{10}{6.25} = 1.6\ \frac{mA}{ft}$$

Of course, in any given problem, we may have the information for K_m and use that to calculate K_T instead. But most often, if a problem is posed with the use of percent outputs, we usually just need to work with K_T without having to know K_t and K_m.

The unit of the controller gain K_c remains dimensionless, but now we would write it as %/% (or more precisely as percent controller output to percent transmitter output, %C/%T). The steady state gain of the valve (actuator) would now be based on the percent output of the controller. In this example, the steady state gain of the valve will take on the units:

$$K_v\ [=]\ \frac{gpm}{\%C}$$

We should see that the overall unit of all the gains around the loop remains dimensionless, *i.e.*, they are consistent:

$$K_c\, K_v\, K_p\, K_T\ \ [=]\ \ \frac{\%C}{\%T}\,\frac{gpm}{\%C}\,\frac{ft}{gpm}\,\frac{\%T}{ft} = 1$$

It is not uncommon that a user can measure easily only the lumped $K_vK_pK_T$ with a unit of %/%, or more precisely %T/%C.