## 9.2 The Theory for Realistic Earth Models

The elasticity theory of dislocations presented in the previous section was for a uniform, not self-gravitating, elastic half-space, taken to be a Poisson solid ($\lambda = \mu$). We now extend the theory to more realistic, rotating, self-gravitating, radially inhomogeneous Earth models with a liquid outer core and solid inner core.

Deformations in such realistic Earth models have been considered in Chapter 3. We first consider the equations (3.63) through (3.67) governing spheroidal deformations in the liquid outer core for degree $n \geq 1$. In the static case, the dynamical body force terms vanish and the governing system becomes

$$\frac{dy_1}{dr} = -\frac{2}{r}y_1 + \frac{1}{\lambda}y_2 + \frac{n(n+1)}{r}y_3, \tag{9.26}$$

$$\frac{dy_2}{dr} = -\frac{2\rho_0}{r}\left(2g_0 + r\Omega^2\right)y_1 + \frac{n(n+1)}{r}\rho_0 g_0 y_3 - \rho_0 y_6, \tag{9.27}$$

$$0 = \frac{\rho_0 g_0}{r}y_1 - \frac{1}{r}y_2 - \frac{\rho_0}{r}y_5, \tag{9.28}$$

$$\frac{dy_5}{dr} = 4\pi G\rho_0 y_1 + y_6, \tag{9.29}$$

$$\frac{dy_6}{dr} = -4\pi G\rho_0 \frac{n(n+1)}{r}y_3 + \frac{n(n+1)}{r^2}y_5 - \frac{2}{r}y_6. \tag{9.30}$$

In the liquid outer core, the conditions of hydrostatic equilibrium prevail, even in the deformed state. Thus, the gradient of the pressure is related to gravity by

$$\nabla p = \rho \boldsymbol{g}, \tag{9.31}$$

where $p$ is the pressure, $\rho$ is the density and $\boldsymbol{g}$ is the force of gravity per unit mass. On taking the curl, we find

$$\boldsymbol{g} \times \nabla \rho = 0. \tag{9.32}$$

Hence, equipotential surfaces, isobaric surfaces and surfaces of equal density (isopycnic surfaces) remain parallel after deformation. An individual fluid particle is able to move about force free on such surfaces in the absence of viscosity. Therefore, $y_3$ becomes indeterminate and we can no longer identify individual fluid particles. Eliminating the term in $y_3$ between equations (9.26) and (9.27), we obtain

$$\frac{dy_2}{dr} = \rho_0 g_0 \frac{dy_1}{dr} - \frac{2\rho_0}{r}\left(g_0 + r\Omega^2\right) y_1 - \frac{\rho_0 g_0}{\lambda} y_2 - \rho_0 y_6. \tag{9.33}$$

Substituting for $y_6$ from equation (9.29) and using relation (3.47), we find

$$\frac{dy_2}{dr} = \rho_0 \frac{d}{dr}(g_0 y_1) - \frac{\rho_0 g_0}{\lambda} y_2 - \rho_0 \frac{dy_5}{dr}. \tag{9.34}$$

Multiplying equation (9.28) by $r/\rho_0$ and differentiating gives

$$\rho_0 \frac{d}{dr}\left(g_0 y_1\right) = \rho_0 \frac{d}{dr}\left(\frac{y_2}{\rho_0}\right) + \rho_0 \frac{dy_5}{dr}, \tag{9.35}$$

which, on substitution in (9.34), finally yields

$$\left[\frac{d}{dr}\left(\ln \rho_0\right) + \frac{\rho_0 g_0}{\lambda}\right] y_2 = 0. \tag{9.36}$$

Thus, $y_2 = 0$, unless

$$\frac{d}{dr}\left(\ln \rho_0\right) = -\frac{\rho_0 g_0}{\lambda}, \tag{9.37}$$

which is known as the Adams–Williamson condition and becomes

$$1 + \frac{\lambda}{\rho_0^2 g_0} \frac{d\rho_0}{dr} = 1 + \frac{\alpha^2}{\rho_0 g_0} \frac{d\rho_0}{dr} = 0. \tag{9.38}$$

This is just the condition for the density profile to be perfectly adiabatic (6.166) or neutrally stratified, a condition unlikely to be met everywhere in the liquid outer core. We conclude that $y_2 = 0$. From equation (9.28), this implies that $y_1 = y_5/g_0$ and that the left side of equation (9.27) vanishes, allowing substitution for $y_3$ in equation (9.30). For $n \geq 1$, the equations governing deformation of the liquid outer core degenerate to

$$y_1 = \frac{1}{g_0} y_5, \tag{9.39}$$

$$y_2 = 0, \tag{9.40}$$

$$y_4 = 0, \tag{9.41}$$

$$\frac{dy_5}{dr} = \frac{4\pi G\rho_0}{g_0} y_5 + y_6, \tag{9.42}$$

$$\frac{dy_6}{dr} = \left[ -\frac{16\pi G\rho_0}{g_0 r}\left(1 + \frac{r\Omega^2}{2g_0}\right) + \frac{n\,(n+1)}{r^2} \right] y_5 - \left[ \frac{4\pi G\rho_0}{g_0} + \frac{2}{r} \right] y_6. \tag{9.43}$$

Traditionally, it had been assumed that the variable $y_1$ was continuous at the boundaries of the liquid core, as is the case for dynamical problems. This led Jeffreys and Vincente (1966) to conclude that a solution may be impossible without the assumptions that the Adams–Williamson condition holds and that the density profile in the liquid outer core is perfectly adiabatic. The solution to the dilemma of Jeffreys and Vincente was given by Smylie and Mansinha (1971), who pointed out that in the static case the solid boundaries of the liquid core penetrate the equipotential, isobaric and equal-density surfaces (just as a loaded ship does, with the degree of penetration being an indication of load). Since the equipotential and its normal derivative are continuous, as shown by Israel et al. (1973), equation (9.29) requires a compensating discontinuity in $y_6$ to make $dy_5/dr$ continuous. Of course, an extra hydrostatic pressure is exerted on the penetrating solid boundaries of the liquid core. Taking the radius of the core-mantle boundary to be $r = b$, with $b^+$ the radius just outside the core-mantle boundary and $b^-$ the radius just inside, we have for spheroidal deformations of degree $n \geq 1$,

$$y_1(b^-) = \frac{1}{g_0} y_5, \tag{9.44}$$

$$y_2(b^-) = 0, \tag{9.45}$$

$$y_2(b^+) = -\rho_0(b^-)\, g_0 \left( \frac{y_5}{g_0} - y_1(b^+) \right), \tag{9.46}$$

$$y_6(b^+) = y_6(b^-) - \Delta y_6\,(b)\,, \tag{9.47}$$

where $\Delta y_6(b)$ is the discontinuity in $y_6$ required to make the normal derivative of the equipotential, as expressed by (3.55), continuous. In addition to the discontinuity in $y_1$,

$$\Delta y_1 = y_1(b^-) - y_1(b^+), \tag{9.48}$$

there is a density discontinuity given by

$$\Delta\rho_0(b) = \rho_0(b^-) - \rho_0(b^+). \tag{9.49}$$

The required discontinuity in $y_6$ is then

$$\Delta y_6(b) = 4\pi G\left[\rho_0(b^+)y_1(b^+) - \rho_0(b^-)y_1(b^-)\right]. \tag{9.50}$$

In terms of the discontinuities in $y_1$ and the density, the discontinuity in $y_6$ becomes

$$\Delta y_6(b) = -4\pi G\left[\rho_0(b^-)\Delta y_1(b) + y_1(b^+)\Delta\rho_0(b)\right]. \tag{9.51}$$

Similar conditions prevail at the inner core.

The generalization of the reciprocal theorem of Betti to elasto-gravitational systems whose elastic properties are spatially varying, and which are subject to hydrostatic pre-stress in the equilibrium reference state, yields (3.31)

$$\begin{aligned}
&\int_{\mathcal{S}} (t_i - \rho_0 g_0 u_s \nu_i)\, u_i' d\mathcal{S} + \int_{\mathcal{V}} F_i u_i' d\mathcal{V} \\
&\quad = \int_{\mathcal{S}} \left(t_i' - \rho_0 g_0 u_s' \nu_i\right) u_i d\mathcal{S} + \int_{\mathcal{V}} F_i' u_i d\mathcal{V} \\
&\qquad + \frac{1}{4\pi G}\int_{\mathcal{S}} \left[V_1\left(\frac{\partial V_1'}{\partial x_i} - 4\pi G\rho_0 u_i'\right) - V_1'\left(\frac{\partial V_1}{\partial x_i} - 4\pi G\rho_0 u_i\right)\right]\nu_i d\mathcal{S},
\end{aligned} \tag{9.52}$$

for two systems of surface tractions, body forces and decreases in gravitational potential (primed and unprimed), $t_i$, $F_i$, $V_1$ and $t_i'$, $F_i'$, $V_1'$, acting on material contained in a volume $\mathcal{V}$ by a surface $\mathcal{S}$, producing displacement fields $u_i$ and $u_i'$, respectively. $u_s$ and $u_s'$ are the respective components of displacement in the orthometric direction (opposite to gravity). While this form of the theorem applies to realistic Earth models, it requires generalization to the case where the variables $y_1$ and $y_6$ are discontinuous across the boundaries of the liquid outer core. Considering first the core-mantle boundary, the generalization requires only that we demonstrate that the extra contributions to the surface integrals in (9.52) cancel each other and amount to zero. A similar cancellation is required at the inner core boundary. $y_6$, as defined by equation (3.45), is the radial coefficient of the outward normal component of the gravitational flux vector $\nabla V_1 - 4\pi G\rho_0 \boldsymbol{u}$. Writing

$$f_s = \left(\frac{\partial V_1}{\partial x_i} - 4\pi G\rho_0 u_i\right)\nu_i \tag{9.53}$$

as a shorthand for the outward normal component of the gravitational flux vector, the physical equivalent of equation (9.47) becomes

$$f_s(b^+) = f_s(b^-) + 4\pi G\rho_0(b^-)\Delta u_s, \tag{9.54}$$

where

$$\Delta u_s = u_s(b^-) - u_s(b^+). \tag{9.55}$$

Similarly, writing $t_s$ for the normal traction, the physical equivalent of equation (9.46) becomes

$$t_s(b^+) = -\rho_0(b^-)V_1 + \rho_0(b^-)g_0 u_s(b^+). \tag{9.56}$$

Of course, $t_s(b^-) = 0$ since $y_2(b^-) = 0$. The sum of the surface integrals in (9.52) at the core-mantle boundary is then found to be

$$\int_{\mathcal{S}} (t_s' u_s - t_s u_s')_{r=b^+}\, d\mathcal{S} + \int_{\mathcal{S}} \rho_0(b^-)\left(V_1 \Delta u_s' - V_1' \Delta u_s\right) d\mathcal{S}. \tag{9.57}$$

Substituting for the normal tractions from equation (9.56), the first integral in (9.57) is transformed to

$$\begin{aligned} &-\int_{\mathcal{S}} \rho_0(b^-)\left(V_1' u_s - V_1 u_s'\right)_{r=b^+} d\mathcal{S} + \int_{\mathcal{S}} g_0 \rho_0(b^-)\left(u_s' u_s - u_s u_s'\right)_{r=b^+} d\mathcal{S} \\ &\quad = -\int_{\mathcal{S}} \rho_0(b^-)\left(V_1' u_s - V_1 u_s'\right)_{r=b^+} d\mathcal{S}. \end{aligned} \tag{9.58}$$

The physical equivalent of equation (9.44) is

$$u_S(b^-) = \frac{1}{g_0} V_1. \tag{9.59}$$

This allows the second integral in (9.57) to be written as

$$\begin{aligned} &\int_{\mathcal{S}} \rho_0(b^-)\left(V_1 \frac{1}{g_0} V_1' - V_1' \frac{1}{g_0} V_1\right) d\mathcal{S} - \int_{\mathcal{S}} \rho_0(b^-)\left(V_1 u_S' - V_1' u_S\right)_{r=b^+} d\mathcal{S} \\ &\quad = -\int_{\mathcal{S}} \rho_0(b^-)\left(V_1 u_s' - V_1' u_S\right)_{r=b^+} d\mathcal{S}. \end{aligned} \tag{9.60}$$

Adding expressions (9.58) and (9.60), we find that the extra surface integrals over the core-mantle boundary (9.57) cancel each other. Similar arguments may be constructed for the inner core boundary. Thus, the form (9.52) of Betti's reciprocal theorem is unaltered for discontinuities in the variable $y_6$ given by equation (9.47).

Applied to the mantle and crust (the shell), the surface integrals in Betti's reciprocal theorem remain to be considered for the surface of the Earth and the faces of the dislocation. By arguments analogous to those used at the core-mantle and inner core boundaries, the surface integrals can be shown to vanish over the surface of the Earth. The surface integrals

$$\frac{1}{4\pi G} \int_{\mathcal{S}} \left[ V_1 \left( \frac{\partial V_1'}{\partial x_i} - 4\pi G \rho_0 u_i' \right) - V_1' \left( \frac{\partial V_1}{\partial x_i} - 4\pi G \rho_0 u_i \right) \right] \nu_i d\mathcal{S} \tag{9.61}$$

over the faces of the dislocation cancel each other for slip faults, since for slip faults there is no displacement component normal to the faces of the dislocation and the normal vectors on the two sides of the dislocation are in opposite directions. The surface integrals

$$\int_{\mathcal{S}} (t_i - \rho_0 g_0 u_s \nu_i)\, u_i' d\mathcal{S} \quad \text{and} \quad \int_{\mathcal{S}} \left(t_i' - \rho_0 g_0 u_s' \nu_i\right) u_i d\mathcal{S} \tag{9.62}$$

over the faces of the dislocation cancel each other since, for slip faults, there is no displacement component normal to the faces of the dislocation, and the tractions required to maintain the dislocation are equal and opposite on the two faces. The remaining steps in the derivation of Volterra's formula using Betti's reciprocal theorem follow those in Section 9.1.1. The radius vector to the field point is $\boldsymbol{r} = (x_1, x_2, x_3)$, while the radius vector to a point on the fault surface is $\boldsymbol{r}' = (x_1', x_2', x_3')$ with the fault dip at the angle $\alpha$ to the horizontal. Volterra's formula for a dip-slip fault gives the displacement vector at the field point as

$$\begin{aligned} u_i(\boldsymbol{r}) = \int_{S'} \mu \Delta u \Bigg[ & \left( \frac{\partial u_i^2(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_2'} - \frac{\partial u_i^3(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_3'} \right) \sin 2\alpha \\ & - \left( \frac{\partial u_i^{\prime 2}(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_3'} + \frac{\partial u_i^3(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_2'} \right) \cos 2\alpha \Bigg] dS', \end{aligned} \tag{9.63}$$

with the slip, $\Delta u$, having components $\Delta u_2 = \Delta u \cos \alpha$ and $\Delta u_3 = \Delta u \sin \alpha$. Volterra's formula for a strike-slip fault gives the displacement vector at the field point as

$$\begin{aligned} u_i(\boldsymbol{r}) = \int_{S'} \mu \Delta u_1 \Bigg[ & \left( \frac{\partial u_i^1(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_2'} + \frac{\partial u_i^2(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_1'} \right) \sin \alpha \\ & - \left( \frac{\partial u_i^1(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_3'} + \frac{\partial u_i^3(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_1'} \right) \cos \alpha \Bigg] dS', \end{aligned} \tag{9.64}$$

with the slip having the single component $\Delta u_1$. These forms of Volterra's formula allow the displacement field to be interpreted as being due to the superposition of the displacement fields of a continuous distribution of dipole forces over the fault surface. We take the systems of dipole force distributions shown in Figure 9.6 to be located at $\boldsymbol{r}_0$ with spherical polar co-ordinates $(r_0, \theta_0, 0)$ on the fault surface.

The unit forces entering Volterra's formula are products of unit vectors with the scalar densities

$$\delta = \frac{\delta(r - r_0)\, \delta(\theta - \theta_0)\, \delta(\phi)}{r^2 \sin \theta}, \tag{9.65}$$

since the volume element, $r^2 \sin \theta\, dr\, d\theta\, d\phi$, is a scalar capacity. Taking the local Cartesian focal co-ordinates to be oriented so that $x_1'$, $x_2'$ and $x_3'$ are in the directions

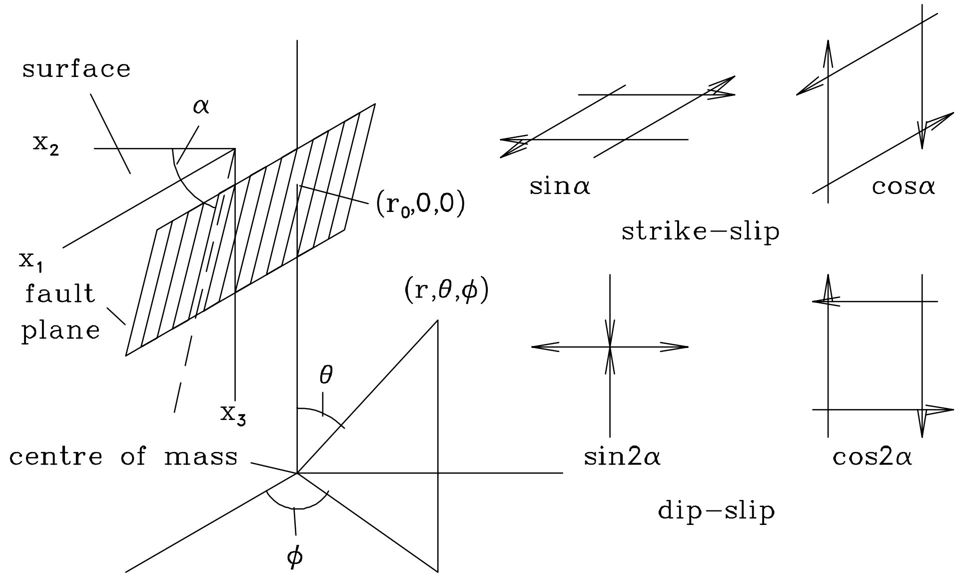


Figure 9.6 Fault geometry and focal force systems.

of increasing $\theta$, decreasing $\phi$ and decreasing $r$, respectively, for spherical polar co-ordinates $(r, \theta, \phi)$ in this *epicentral* system of the field point, we have the dipole force densities for the dip-slip system as

$$\begin{aligned}\frac{1}{r^2 \sin\theta}\Bigg[\hat{\boldsymbol{r}}\Big\{&\frac{1}{r\sin\theta}\delta(r-r_0)\,\delta(\theta-\theta_0)\,\delta'(\phi)\cos 2\alpha \\ &+ \delta'(r-r_0)\,\delta(\theta-\theta_0)\,\delta(\phi)\sin 2\alpha\Big\} \\ + \hat{\boldsymbol{\phi}}\Big\{&-\frac{1}{r\sin\theta}\delta(r-r_0)\,\delta(\theta-\theta_0)\,\delta'(\phi)\sin 2\alpha \\ &+ \delta'(r-r_0)\,\delta(\theta-\theta_0)\,\delta(\phi)\cos 2\alpha\Big\}\Bigg],\end{aligned} \tag{9.66}$$

and the dipole force densities for the strike-slip system as

$$\begin{aligned}\frac{1}{r^2 \sin\theta}\Bigg[&-\frac{\hat{\boldsymbol{r}}}{r}\delta(r-r_0)\,\delta'(\theta-\theta_0)\,\delta(\phi)\cos\alpha \\ &+ \hat{\boldsymbol{\theta}}\Big\{\frac{1}{r\sin\theta}\delta(r-r_0)\,\delta(\theta-\theta_0)\,\delta'(\phi)\sin\alpha \\ &\quad - \delta'(r-r_0)\,\delta(\theta-\theta_0)\,\delta(\phi)\cos\alpha\Big\} \\ &+ \frac{\hat{\boldsymbol{\phi}}}{r}\delta(r-r_0)\,\delta'(\theta-\theta_0)\,\delta(\phi)\sin\alpha\Bigg].\end{aligned} \tag{9.67}$$

While expressions (9.66) and (9.67) take account of the direct effects of the transformation to spherical polar co-ordinates, there are additional terms arising from co-ordinate curvature.

The spherical polar unit vectors $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}})$ in the epicentral system are related to the geocentric Cartesian unit vectors $(\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3)$ in that system by

$$\hat{\boldsymbol{r}} = \hat{\boldsymbol{e}}_1 \sin\theta \cos\phi - \hat{\boldsymbol{e}}_2 \sin\theta \sin\phi - \hat{\boldsymbol{e}}_3 \cos\theta, \tag{9.68}$$

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{e}}_1 \cos\theta \cos\phi - \hat{\boldsymbol{e}}_2 \cos\theta \sin\phi + \hat{\boldsymbol{e}}_3 \sin\theta, \tag{9.69}$$

$$\hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{e}}_1 \sin\phi - \hat{\boldsymbol{e}}_2 \cos\phi. \tag{9.70}$$

Now consider the double-couple force system arising from the strike-slip expression and illustrated in the upper left of Figure 9.7. Here, unit vectors $\pm\hat{\boldsymbol{\theta}}$ have

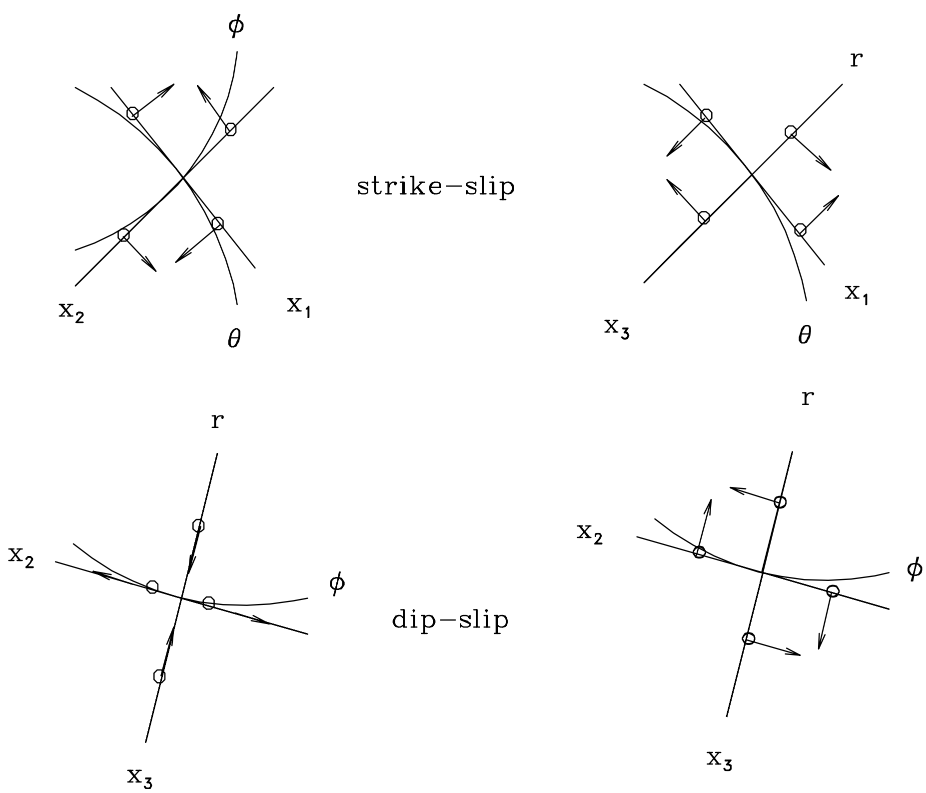


Figure 9.7 Double force and moment transport.

been transported in the $\phi$-direction and unit vectors $\pm\hat{\boldsymbol{\phi}}$ have been transported in the $\theta$-direction. From (9.69) and (9.70), we find

$$\frac{\partial \hat{\boldsymbol{\theta}}}{\partial \phi} = \hat{\boldsymbol{\phi}} \cos\theta \quad \text{and} \quad \frac{\partial \hat{\boldsymbol{\phi}}}{\partial \theta} = 0. \tag{9.71}$$

Thus, only transport in the $\phi$-direction gives a non-null result. In Cartesian geometry the double couple involved is

$$\hat{\boldsymbol{e}}_1' \sin\alpha\, \delta(x_1')\, \delta(x_3') \lim_{\delta x_2' \to 0} \frac{\delta(x_2' + \delta x_2'/2) - \delta(x_2' - \delta x_2'/2)}{\delta x_2'}. \tag{9.72}$$

Each of the two forces gain small extra components

$$\frac{1}{2}\delta\phi \frac{\partial \hat{\boldsymbol{\theta}}}{\partial \phi} \delta(\phi \pm \delta\phi/2)\, \delta(r - r_0)\, \delta(\theta - \theta_0) \sin\alpha \tag{9.73}$$

before division by $\delta x_2'$ and parallel transport into the limit as $\delta x_2' \to 0$. Since $\delta x_2' = r_0 \sin\theta_0\, \delta\phi$, this produces the extra term

$$\frac{1}{r_0 \sin\theta_0\, \delta\phi} \frac{1}{r_0^2 \sin\theta_0} \delta\phi\, \hat{\boldsymbol{\phi}} \cos\theta_0\, \delta(\phi)\, \delta(r - r_0)\, \delta(\theta - \theta_0) \sin\alpha, \tag{9.74}$$

expressed per unit volume. Hence, co-ordinate curvature contributes the term in double-force density

$$\hat{\boldsymbol{\phi}} \frac{\cot\theta_0}{r_0} \delta \sin\alpha, \tag{9.75}$$

arising from the double-couple term for strike-slip faults and illustrated in the top left of Figure 9.7.

In the case of the double-couple force system arising from the strike-slip expression and illustrated in the upper right of Figure 9.7, the unit vectors $\pm\hat{\boldsymbol{r}}$ have been transported in the $\theta$-direction and the unit vectors $\pm\hat{\boldsymbol{\theta}}$ have been transported in the $r$-direction. From (9.68) and (9.69), we find

$$\frac{\partial \hat{\boldsymbol{r}}}{\partial \theta} = \hat{\boldsymbol{\theta}} \quad \text{and} \quad \frac{\partial \hat{\boldsymbol{\theta}}}{\partial r} = 0. \tag{9.76}$$

Thus, only transport in the $\theta$-direction gives a non-null result. With arguments similar to those made before, we find co-ordinate curvature contributes the term in double-force density

$$-\hat{\boldsymbol{\theta}} \frac{\delta}{r_0} \cos\alpha, \tag{9.77}$$

arising from the double-couple term for strike-slip faults and illustrated in the top right of Figure 9.7.

For the double-force system arising from the dip-slip expression and illustrated in the lower left of Figure 9.7, the unit vectors $\pm\hat{\boldsymbol{r}}$ have been transported in the

$r$-direction and the unit vectors $\pm\hat{\boldsymbol{\phi}}$ have been transported in the $\phi$-direction. From (9.68) and (9.70), we find

$$\frac{\partial\hat{\boldsymbol{r}}}{\partial r} = 0 \quad \text{and} \quad \frac{\partial\hat{\boldsymbol{\phi}}}{\partial\phi} = -\hat{\boldsymbol{r}}\sin\theta - \hat{\boldsymbol{\theta}}\cos\theta. \tag{9.78}$$

Thus, only transport in the $\phi$-direction gives a non-null result. With arguments similar to those made before, we find co-ordinate curvature contributes the term in double-force density

$$\hat{\boldsymbol{r}}\frac{\delta}{r_0}\sin 2\alpha + \hat{\boldsymbol{\theta}}\frac{\cot\theta}{r_0}\delta\sin 2\alpha, \tag{9.79}$$

arising from the double-force term for dip-slip faults and illustrated in the lower left of Figure 9.7.

For the double-couple force system arising from the dip-slip expression and illustrated in the lower right of Figure 9.7, the unit vectors $\pm\hat{\boldsymbol{r}}$ have been transported in the $\phi$-direction and the unit vectors $\pm\hat{\boldsymbol{\phi}}$ have been transported in the $r$-direction. From (9.68), (9.69) and (9.70), we find

$$\frac{\partial\hat{\boldsymbol{r}}}{\partial\phi} = \hat{\boldsymbol{\phi}}\sin\theta \quad \text{and} \quad \frac{\partial\hat{\boldsymbol{\phi}}}{\partial r} = 0. \tag{9.80}$$

Thus, only transport in the $\phi$-direction gives a non-null result. With arguments similar to those made before, we find co-ordinate curvature contributes the term in double-force density

$$\hat{\boldsymbol{\phi}}\frac{\delta}{r_0}\cos 2\alpha, \tag{9.81}$$

arising from the double-couple term for dip-slip faults and illustrated in the lower right of Figure 9.7.

In summary, co-ordinate curvature contributes the extra double-force density terms for the dip-slip system,

$$\hat{\boldsymbol{r}}\frac{\delta}{r_0}\sin 2\alpha + \hat{\boldsymbol{\theta}}\frac{\cot\theta_0}{r_0}\delta\sin 2\alpha + \hat{\boldsymbol{\phi}}\frac{\delta}{r_0}\cos 2\alpha, \tag{9.82}$$

and the extra double-force density terms for the strike-slip system,

$$-\hat{\boldsymbol{\theta}}\frac{\delta}{r_0}\cos\alpha + \hat{\boldsymbol{\phi}}\frac{\cot\theta_0}{r_0}\delta\sin\alpha. \tag{9.83}$$

Using the orthogonality relations (1.205) and (1.209), as outlined, the radial spheroidal, transverse spheroidal and torsional radial coefficients can be extracted from the dipole force densities for the dip-slip system (9.66). In the limit as $\theta_0$

approaches zero, we find

$$
\begin{aligned}
u_n^0(r) &= \frac{2n+1}{4\pi r^2}\delta'(r-r_0)\sin 2\alpha,\\
u_n^{-1}(r) &= -\frac{i\,(2n+1)}{8\pi r^3}n\,(n+1)\,\delta(r-r_0)\cos 2\alpha,\\
u_n^{1}(r) &= -\frac{i\,(2n+1)}{8\pi r^3}\delta(r-r_0)\cos 2\alpha,\\
v_n^{-1}(r) &= \frac{i\,(2n+1)}{8\pi r^2}\delta'(r-r_0)\cos 2\alpha,\\
v_n^{1}(r) &= \frac{i\,(2n+1)}{8\pi r^2 n\,(n+1)}\delta'(r-r_0)\cos 2\alpha,\\
v_n^{-2}(r) &= -\frac{2n+1}{8\pi r^3}\,(n-1)\,(n+2)\,\delta(r-r_0)\sin 2\alpha,\\
v_n^{2}(r) &= -\frac{2n+1}{8\pi r^3 n\,(n+1)}\delta(r-r_0)\sin 2\alpha,\\
t_n^{-1}(r) &= -\frac{2n+1}{8\pi r^2}\delta'(r-r_0)\cos 2\alpha,\\
t_n^{1}(r) &= \frac{2n+1}{8\pi r^2 n\,(n+1)}\delta'(r-r_0)\cos 2\alpha,\\
t_n^{-2}(r) &= -\frac{i\,(2n+1)}{8\pi r^3}\,(n-1)\,(n+2)\,\delta(r-r_0)\sin 2\alpha,\\
t_n^{2}(r) &= \frac{i\,(2n+1)}{8\pi r^3 n\,(n+1)}\delta(r-r_0)\sin 2\alpha.
\end{aligned}
\tag{9.84}
$$

Similarly, for the strike-slip system (9.67) we find

$$
\begin{aligned}
u_n^{-1}(r) &= \frac{2n+1}{8\pi r^3}n\,(n+1)\,\delta(r-r_0)\cos\alpha,\\
u_n^{1}(r) &= -\frac{2n+1}{8\pi r^3}\delta(r-r_0)\cos\alpha,\\
v_n^{-1}(r) &= -\frac{2n+1}{8\pi r^2}\delta'(r-r_0)\cos\alpha,\\
v_n^{1}(r) &= \frac{2n+1}{8\pi r^2 n\,(n+1)}\delta'(r-r_0)\cos\alpha,\\
v_n^{-2}(r) &= -\frac{3i\,(2n+1)}{16\pi r^3}\,(n-1)\,(n+2)\,\delta(r-r_0)\sin\alpha,\\
v_n^{2}(r) &= \frac{3i\,(2n+1)}{16\pi r^3 n\,(n+1)}\delta(r-r_0)\sin\alpha,\\
t_n^{0}(r) &= -\frac{2n+1}{8\pi r^3}\delta(r-r_0)\sin\alpha,\\
t_n^{-1}(r) &= -\frac{i\,(2n+1)}{8\pi r^2}\delta'(r-r_0)\cos\alpha,
\end{aligned}
\tag{9.85}
$$

$$t_n^1(r) = -\frac{i\,(2n+1)}{8\pi r^2 n\,(n+1)}\delta'(r-r_0)\cos\alpha,$$
$$t_n^{-2}(r) = \frac{3\,(2n+1)}{16\pi r^3}\,(n-1)\,(n+2)\,\delta(r-r_0)\sin\alpha,$$
$$t_n^2(r) = \frac{3\,(2n+1)}{16\pi r^3 n\,(n+1)}\delta(r-r_0)\sin\alpha.$$

Once again, extracting the radial coefficients of the radial spheroidal, transverse spheroidal and torsional parts of the extra double-force densities (9.82), arising from co-ordinate curvature for the dip-slip system, and taking the limit as $\theta_0$ goes to zero, we find

$$\begin{aligned}
u_n^0(r) &= \frac{2n+1}{4\pi r^3}\delta(r-r_0)\sin 2\alpha,\\
v_n^0(r) &= -\frac{2n+1}{8\pi r^3}\delta(r-r_0)\sin 2\alpha,\\
v_n^{-1}(r) &= \frac{i\,(2n+1)}{8\pi r^3}\delta(r-r_0)\cos 2\alpha,\\
v_n^1(r) &= \frac{i\,(2n+1)}{8\pi n\,(n+1)\,r^3}\delta(r-r_0)\cos 2\alpha,\\
v_n^{-2}(r) &= \frac{(2n+1)\,(n-1)\,(n+2)}{16\pi r^3}\delta(r-r_0)\sin 2\alpha,\\
v_n^2(r) &= \frac{2n+1}{16\pi n\,(n+1)\,r^3}\delta(r-r_0)\sin 2\alpha,\\
t_n^{-1}(r) &= -\frac{2n+1}{8\pi r^3}\delta(r-r_0)\cos 2\alpha,\\
t_n^1(r) &= \frac{2n+1}{8\pi n\,(n+1)\,r^3}\delta(r-r_0)\cos 2\alpha,\\
t_n^{-2}(r) &= \frac{i\,(2n+1)\,(n-1)\,(n-2)}{16\pi r^3}\delta(r-r_0)\sin 2\alpha,\\
t_n^2(r) &= -\frac{i\,(2n+1)}{16\pi n\,(n+1)\,r^3}\delta(r-r_0)\sin 2\alpha.
\end{aligned}\tag{9.86}$$

Similarly, for the extra double-force densities (9.83) arising from co-ordinate curvature for the strike-slip system, we find

$$\begin{aligned}
v_n^{-1}(r) &= -\frac{2n+1}{8\pi r^3}\delta(r-r_0)\cos\alpha,\\
v_n^1(r) &= \frac{2n+1}{8\pi n\,(n+1)\,r^3}\delta(r-r_0)\cos\alpha,\\
v_n^{-2}(r) &= \frac{i\,(2n+1)\,(n-1)\,(n+2)}{16\pi r^3}\delta(r-r_0)\sin\alpha,\\
v_n^2(r) &= -\frac{i\,(2n+1)}{16\pi n\,(n+1)\,r^3}\delta(r-r_0)\sin\alpha,
\end{aligned}$$

$$
\begin{aligned}
t_n^0(r) &= \frac{2n+1}{8\pi r^3}\delta(r-r_0)\sin\alpha, \\
t_n^{-1}(r) &= -\frac{i\,(2n+1)}{8\pi r^3}\delta(r-r_0)\cos\alpha, \\
t_n^1(r) &= -\frac{i\,(2n+1)}{8\pi n\,(n+1)\,r^3}\delta(r-r_0)\cos\alpha, \\
t_n^{-2}(r) &= -\frac{(2n+1)\,(n-1)\,(n+2)}{16\pi r^3}\delta(r-r_0)\sin\alpha, \\
t_n^2(r) &= -\frac{2n+1}{16\pi n\,(n+1)\,r^3}\delta(r-r_0)\sin\alpha.
\end{aligned}
\tag{9.87}
$$

## 9.3 Changes in the Inertia Tensor and the Secular Polar Shift

The effect the displacement fields in realistic Earth models, just described, have on the rotation of the Earth can be found from the changes they produce in the components of the inertia tensor (4.20) (Mansinha and Smylie, 1967), (Smylie and Zuberi, 2009). A displacement field with spherical polar components $(u_r, u_\theta, u_\phi)$, in the epicentral co-ordinate system, contributes changes in the Cartesian co-ordinates, $x_1 = r\sin\theta\cos\phi$, $x_2 = r\sin\theta\sin\phi$, and $x_3 = r\cos\theta$, given by

$$
\begin{aligned}
\Delta x_1 &= u_r\sin\theta\cos\phi + u_\theta\cos\theta\cos\phi - u_\phi\sin\phi, \\
\Delta x_2 &= u_r\sin\theta\sin\phi + u_\theta\cos\theta\sin\phi + u_\phi\cos\phi, \\
\Delta x_3 &= u_r\cos\theta - u_\theta\sin\theta.
\end{aligned}
\tag{9.88}
$$

These changes are small, and we need only work to first order accuracy in the ratios $\Delta x_i/x_j$. By binomial expansion, the components of the inertia tensor, after the displacement field takes effect, can be found, correct to first order in small quantities, to be

$$
\begin{aligned}
I_{11} &= \int\left[(x_2+\Delta x_2)^2 + (x_3+\Delta x_3)^2\right]dm = \int\left[x_2^2 + x_3^2 + 2\,(x_2\Delta x_2 + x_3\Delta x_3)\right]dm, \\
I_{22} &= \int\left[(x_1+\Delta x_1)^2 + (x_3+\Delta x_3)^2\right]dm = \int\left[x_1^2 + x_3^2 + 2\,(x_1\Delta x_1 + x_3\Delta x_3)\right]dm, \\
I_{33} &= \int\left[(x_1+\Delta x_1)^2 + (x_2+\Delta x_2)^2\right]dm = \int\left[x_1^2 + x_2^2 + 2\,(x_1\Delta x_1 + x_2\Delta x_2)\right]dm,
\end{aligned}
\tag{9.89}
$$

$$
\begin{aligned}
I_{12} &= -\int[(x_1+\Delta x_1)\,(x_2+\Delta x_2)]\,dm = -\int[x_1x_2 + (x_1\Delta x_2 + x_2\Delta x_1)]\,dm, \\
I_{13} &= -\int[(x_1+\Delta x_1)\,(x_3+\Delta x_3)]\,dm = -\int[x_1x_3 + (x_1\Delta x_3 + x_3\Delta x_1)]\,dm,
\end{aligned}
$$

$$I_{23} = -\int [(x_2 + \Delta x_2)(x_3 + \Delta x_3)]\, dm = -\int [x_2 x_3 + (x_2 \Delta x_3 + x_3 \Delta x_2)]\, dm.$$

The changes in the components of the inertia tensor, $\Delta I_{ij}$, to first order in small quantities in the epicentral system are then

$$\begin{aligned}
\Delta I_{11} &= 2\int (x_2 \Delta x_2 + x_3 \Delta x_3)\, dm \\
&= 2\int r\left[u_r\left(1 - \sin^2\theta \cos^2\phi\right) - u_\theta \cos\theta \sin\theta \cos^2\phi + u_\phi \sin\theta \cos\phi \sin\phi\right] dm, \\
\Delta I_{22} &= 2\int (x_1 \Delta x_1 + x_3 \Delta x_3)\, dm \\
&= 2\int r\left[u_r\left(1 - \sin^2\theta \sin^2\phi\right) - u_\theta \cos\theta \sin\theta \sin^2\phi - u_\phi \sin\theta \cos\phi \sin\phi\right] dm, \\
\Delta I_{33} &= 2\int (x_1 \Delta x_1 + x_2 \Delta x_2)\, dm \\
&= 2\int r\left[u_r \sin^2\theta + u_\theta \cos\theta \sin\theta\right] dm, \\
\Delta I_{12} &= -\int (x_1 \Delta x_2 + x_2 \Delta x_1)\, dm \\
&= -\int r\Big[2u_r \sin^2\theta \cos\phi \sin\phi + 2u_\theta \cos\theta \sin\theta \cos\phi \sin\phi \qquad (9.90) \\
&\qquad + u_\phi \sin\theta\left(\cos^2\phi - \sin^2\phi\right)\Big] dm, \\
\Delta I_{13} &= -\int (x_1 \Delta x_3 + x_3 \Delta x_1)\, dm \\
&= -\int r\left[2u_r \cos\theta \sin\theta \cos\phi + u_\theta\left(\cos^2\theta - \sin^2\theta\right)\cos\phi - u_\phi \cos\theta \sin\phi\right] dm, \\
\Delta I_{23} &= -\int (x_2 \Delta x_3 + x_3 \Delta x_2)\, dm \\
&= -\int r\left[2u_r \cos\theta \sin\theta \sin\phi + u_\theta\left(\cos^2\theta - \sin^2\theta\right)\sin\phi + u_\phi \cos\theta \cos\phi\right] dm.
\end{aligned}$$

The integrands in (9.90) can all be cast in the form of scalar products of the displacement field with particular spheroidal vectors of degrees zero and two. Thus, the change in the component $\Delta I_{ij}$ can be written

$$\Delta I_{ij} = \int r\boldsymbol{u} \cdot \boldsymbol{S}_{ij} dm, \qquad (9.91)$$

where $\boldsymbol{S}_{ij}$ is a spheroidal vector field of degree zero or two given by the expression (1.195). The radial spheroidal and transverse spheroidal coefficients for each $\Delta I_{ij}$ are given in Table 9.1.

Table 9.1 *Radial spheroidal and transverse spheroidal coefficients of spheroidal vectors of degrees zero and two forming the integrands in (9.90) as scalar products with the displacement field.*

| $\Delta I_{ij}$ | $u_0^0$ | $u_2^0$ | $v_2^0$ | $u_2^1$ | $v_2^1$ | $u_2^{-1}$ | $v_2^{-1}$ | $u_2^2$ | $v_2^2$ | $u_2^{-2}$ | $v_2^{-2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta I_{11}$ | $4/3$ | $2/3$ | $1/3$ | | | | | $-1/6$ | $-1/12$ | $-4$ | $-2$ |
| $\Delta I_{22}$ | $4/3$ | $2/3$ | $1/3$ | | | | | $1/6$ | $1/12$ | $4$ | $2$ |
| $\Delta I_{33}$ | $4/3$ | $4/3$ | $-2/3$ | | | | | | | | |
| $\Delta I_{12}$ | | | | | | | | $i/3$ | $i/6$ | $-8i$ | $-4i$ |
| $\Delta I_{13}$ | | | | $1/3$ | $1/6$ | $-2$ | $-1$ | | | | |
| $\Delta I_{23}$ | | | | $-i/3$ | $-i/6$ | $-2i$ | $-i$ | | | | |

The orthogonality property (1.205) then allows the expressions (9.90) to be reduced to

$$
\begin{aligned}
\Delta I_{11} &= 4\pi \int_0^d r^3 \rho_0(r) \left[ \frac{4}{3} u_0^0(r) + \frac{2}{15} u_2^0(r) - \frac{4}{5} u_2^2(r) - \frac{1}{30} u_2^{-2}(r) \right. \\
&\qquad \left. + \frac{2}{5} v_2^0(r) - \frac{12}{5} v_2^2(r) - \frac{1}{10} v_2^{-2}(r) \right] dr, \\
\Delta I_{22} &= 4\pi \int_0^d r^3 \rho_0(r) \left[ \frac{4}{3} u_0^0(r) + \frac{2}{15} u_2^0(r) + \frac{4}{5} u_2^2(r) + \frac{1}{30} u_2^{-2}(r) \right. \\
&\qquad \left. + \frac{2}{5} v_2^0(r) + \frac{12}{5} v_2^2(r) + \frac{1}{10} v_2^{-2}(r) \right] dr, \\
\Delta I_{33} &= 4\pi \int_0^d r^3 \rho_0(r) \left[ \frac{4}{3} u_0^0(r) - \frac{4}{15} u_2^0(r) - \frac{4}{5} v_2^0(r) \right] dr, \qquad (9.92) \\
\Delta I_{12} &= 4\pi i \int_0^d r^3 \rho_0(r) \left[ -\frac{4}{5} u_2^2(r) + \frac{1}{30} u_2^{-2}(r) - \frac{12}{5} v_2^2(r) + \frac{1}{10} v_2^{-2}(r) \right] dr, \\
\Delta I_{13} &= 4\pi \int_0^d r^3 \rho_0(r) \left[ \frac{2}{5} u_2^1(r) - \frac{1}{15} u_2^{-1}(r) + \frac{6}{5} v_2^1(r) - \frac{1}{5} v_2^{-1}(r) \right] dr, \\
\Delta I_{23} &= 4\pi i \int_0^d r^3 \rho_0(r) \left[ \frac{2}{5} u_2^1(r) + \frac{1}{15} u_2^{-1}(r) + \frac{6}{5} v_2^1(r) + \frac{1}{5} v_2^{-1}(r) \right] dr,
\end{aligned}
$$

with $d$ the radius of the Earth.

As shown in Section 9.2, the solid boundaries of the outer core can penetrate the gravitational equipotentials coincident with the boundaries of the outer core in the undeformed state. For the inner core boundary, the additional contributions to the changes in the inertia tensor are

$$
\Delta J_{11} = \pi a^4 \rho_0(a^+) \left[ \frac{4}{3} \Delta u_0^0(a^+) + \frac{2}{15} \Delta u_2^0(a^+) - \frac{4}{5} \Delta u_2^2(a^+) - \frac{1}{30} \Delta u_2^{-2}(a^+) \right],
$$

$$
\begin{aligned}
\Delta J_{22} &= \pi a^4 \rho_0(a^+) \left[ \frac{4}{3} \Delta u_0^0(a^+) + \frac{2}{15} \Delta u_2^0(a^+) + \frac{4}{5} \Delta u_2^2(a^+) + \frac{1}{30} \Delta u_2^{-2}(a^+) \right], \\
\Delta J_{33} &= \pi a^4 \rho_0(a^+) \left[ \frac{4}{3} \Delta u_0^0(a^+) - \frac{4}{15} \Delta u_2^0(a^+) \right], \\
\Delta J_{12} &= \pi i a^4 \rho_0(a^+) \left[ -\frac{4}{5} \Delta u_2^2(a^+) + \frac{1}{30} \Delta u_2^{-2}(a^+) \right], \\
\Delta J_{13} &= \pi a^4 \rho_0(a^+) \left[ \frac{2}{5} \Delta u_2^1(a^+) - \frac{1}{15} \Delta u_2^{-1}(a^+) \right], \\
\Delta J_{23} &= \pi i a^4 \rho_0(a^+) \left[ \frac{2}{5} \Delta u_2^1(a^+) + \frac{1}{15} \Delta u_2^{-1}(a^+) \right],
\end{aligned}
\tag{9.93}
$$

where $\Delta u_n^m$ is the excess of the radial displacement at the bottom of the outer core over that at the inner core boundary. Similarly, for the outer core boundary, the contributions to the changes in the inertia tensor amount to

$$
\begin{aligned}
\Delta J_{11} &= \pi b^4 \rho_0(b^-) \left[ \frac{4}{3} \Delta u_0^0 + \frac{2}{15} \Delta u_2^0 - \frac{4}{5} \Delta u_2^2 - \frac{1}{30} \Delta u_2^{-2} \right], \\
\Delta J_{22} &= \pi b^4 \rho_0(b^-) \left[ \frac{4}{3} \Delta u_0^0 + \frac{2}{15} \Delta u_2^0 + \frac{4}{5} \Delta u_2^2 + \frac{1}{30} \Delta u_2^{-2} \right], \\
\Delta J_{33} &= \pi b^4 \rho_0(b^-) \left[ \frac{4}{3} \Delta u_0^0 - \frac{4}{15} \Delta u_2^0 \right], \\
\Delta J_{12} &= \pi i b^4 \rho_0(b^-) \left[ -\frac{4}{5} \Delta u_2^2 + \frac{1}{30} \Delta u_2^{-2} \right], \\
\Delta J_{13} &= \pi b^4 \rho_0(b^-) \left[ \frac{2}{5} \Delta u_2^1 - \frac{1}{15} \Delta u_2^{-1} \right], \\
\Delta J_{23} &= \pi i b^4 \rho_0(b^-) \left[ \frac{2}{5} \Delta u_2^1 + \frac{1}{15} \Delta u_2^{-1} \right],
\end{aligned}
\tag{9.94}
$$

where again $\Delta u_n^m$ is the excess of the radial displacement of the base of the mantle over that of the gravitational equipotential coincident with the core-mantle boundary in the undeformed state.

A sudden redistribution of mass within the Earth, such as that accompanying a major earthquake, produces a secular polar shift (Mansinha and Smylie, 1967), (Smylie and Zuberi, 2009),

$$
\frac{\Omega \Delta c}{\sigma_0 A}, \tag{9.95}
$$

where $\Omega$ is the angular velocity of Earth's rotation, $\Delta c = \Delta c_{13} + i\Delta c_{23}$ is the change in the off-diagonal components of the inertia tensor in the geographical system of reference, $\sigma_0$ is the angular frequency of the Chandler wobble and $A$ is the equatorial moment of inertia of the Earth. The real part of the secular polar shift is measured towards Greenwich and the imaginary part is measured in the direction of 90° E. The changes in the off-diagonal components $\Delta c_{13}$ and $\Delta c_{23}$ of the inertia tensor in the geographical system of reference are found by similarity transformation of the changes $\Delta I_{ij} + \Delta J_{ij}$ in the inertia tensor in the epicentral system. A closely equal and opposite contribution to the Chandler wobble occurs, so that only a small instantaneous shift in pole position is realized, with the pole path embarking on a new circular arc about a new centre of rotation.

From expressions (9.92), (9.93) and (9.94), the changes in the inertia tensor in the epicentral system of reference depend on spheroidal displacement fields of zeroth and second degree. The zeroth-degree displacement field makes an equal contribution only to the diagonal components of the inertia tensor. This contribution will, therefore, be the same in all centre of mass co-ordinate systems. Since the excitation of wobble and secular polar shift depend only on two of the off-diagonal components of the inertia tensor, the degree-zero spheroidal displacements can be ignored. In the mantle and crust, the second-degree spheroidal displacement fields obey the system of ordinary differential equations (3.102) through (3.107). Four separate solutions of the non-homogeneous equations are involved with singular sources proportional to

$$u^m_{F\,2} = \frac{5}{8\pi r^3}\mu\,\delta(r - r_0), \tag{9.96}$$

$$u^m_{F\,2} = \frac{5}{8\pi r^2}\mu\,\delta'(r - r_0), \tag{9.97}$$

$$v^m_{F\,2} = \frac{5}{8\pi r^3}\mu\,\delta(r - r_0), \tag{9.98}$$

$$v^m_{F\,2} = \frac{5}{8\pi r^2}\mu\,\delta'(r - r_0). \tag{9.99}$$

Solutions of the homogeneous sixth-order spheroidal system can be represented by the propagator matrix introduced in Section 3.6. It is denoted by $\boldsymbol{Y}(r,\rho)$, which is the solution of the homogeneous differential system (3.299),

$$\frac{d\boldsymbol{Y}(r,\rho)}{dr} = \boldsymbol{A}(r)\cdot\boldsymbol{Y}(r,\rho), \tag{9.100}$$

that propagates the fundamental solutions from radius $\rho$ to radius $r$ with the initial condition

$$\boldsymbol{Y}(\rho,\rho) = \boldsymbol{I}, \tag{9.101}$$

where $\boldsymbol{I}$ is the unit matrix. Each column vector of $\boldsymbol{Y}$ is a linearly independent solution of the differential equation (3.296),

$$\frac{d\boldsymbol{y}(r)}{dr} = \boldsymbol{A}(r)\cdot\boldsymbol{y}(r). \tag{9.102}$$

Some properties of the propagator matrix follow from its definition. Propagation from $r_0$ to $r$ and then back to $r_0$ gives

$$\boldsymbol{Y}(r,r_0)\,\boldsymbol{Y}(r_0,r) = \boldsymbol{I}, \tag{9.103}$$

while propagation from $r$ to $r_0$ and then back to $r$ gives

$$\boldsymbol{Y}(r_0,r)\,\boldsymbol{Y}(r,r_0) = \boldsymbol{I}. \tag{9.104}$$

Differentiating (9.103) with respect to $r_0$ yields

$$\frac{d\boldsymbol{Y}(r,r_0)}{dr_0}\boldsymbol{Y}(r_0,r) = -\boldsymbol{Y}(r,r_0)\frac{d\boldsymbol{Y}(r_0,r)}{dr_0}. \tag{9.105}$$

Substitution from (9.100), with $r_0$ replacing $r$ and $r$ replacing $\rho$, transforms the right-hand side, and we find

$$\frac{d\boldsymbol{Y}(r,r_0)}{dr_0}\boldsymbol{Y}(r_0,r) = -\boldsymbol{Y}(r,r_0)\cdot\boldsymbol{A}(r_0)\cdot\boldsymbol{Y}(r_0,r). \tag{9.106}$$

Right multiplying both sides by $\boldsymbol{Y}(r,r_0)$ and using (9.104) reduces this relation to

$$\frac{d\boldsymbol{Y}(r,r_0)}{dr_0} = -\boldsymbol{Y}(r,r_0)\cdot\boldsymbol{A}(r_0). \tag{9.107}$$

Now consider the solution of the inhomogeneous differential system,

$$\frac{d\boldsymbol{y}(\rho)}{d\rho} = \boldsymbol{A}(\rho)\cdot\boldsymbol{y}(\rho) + \boldsymbol{g}(\rho), \tag{9.108}$$

expressed in terms of the independent variable $\rho$. Multiplying through on the left by $\boldsymbol{Y}(r,\rho)$ as an integrating factor yields

$$\boldsymbol{Y}(r,\rho)\frac{d\boldsymbol{y}(\rho)}{d\rho} - \boldsymbol{Y}(r,\rho)\boldsymbol{A}(\rho)\boldsymbol{y}(\rho) = \boldsymbol{Y}(r,\rho)\boldsymbol{g}(\rho). \tag{9.109}$$

Then, using (9.107), the left-hand side becomes the derivative of the product $\boldsymbol{Y}(r,\rho)\boldsymbol{y}(\rho)$, giving

$$\frac{d}{d\rho}\left[\boldsymbol{Y}(r,\rho)\boldsymbol{y}(\rho)\right] = \boldsymbol{Y}(r,\rho)\boldsymbol{g}(\rho). \tag{9.110}$$

Integrating over $\rho$ from $b$ to $r$ produces

$$\boldsymbol{Y}(r,r)\boldsymbol{y}(r) - \boldsymbol{Y}(r,b)\boldsymbol{y}(b) = \int_b^r \boldsymbol{Y}(r,\rho)\,\boldsymbol{g}(\rho)\,d\rho \tag{9.111}$$

or

$$\boldsymbol{y}(r) = \int_b^r \boldsymbol{Y}(r,\rho)\,\boldsymbol{g}(\rho)\,d\rho + \boldsymbol{Y}(r,b)\,\boldsymbol{y}(b) \tag{9.112}$$

as the solution of the inhomogeneous system (9.108).

If $\boldsymbol{g}(r) = \boldsymbol{G}\delta(r-r_0)$, where $\boldsymbol{G}$ is a constant vector, and using the property (2.260) of the Dirac delta function, the solution is

$$\boldsymbol{y}(r) = \begin{cases} \boldsymbol{Y}(r, r_0)\,\boldsymbol{G} + \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r > r_0 \\ \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r < r_0. \end{cases} \tag{9.113}$$

If $\boldsymbol{g}(r) = \boldsymbol{G}\delta'(r - r_0)$, and using the property (2.262) of the derivative of the Dirac delta function, the solution is

$$\boldsymbol{y}(r) = \begin{cases} -\dfrac{d\boldsymbol{Y}(r, r_0)}{dr_0}\boldsymbol{G} + \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r > r_0 \\ \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r < r_0. \end{cases} \tag{9.114}$$

Equation (9.107) then allows (9.114) to be expressed as

$$\boldsymbol{y}(r) = \begin{cases} \boldsymbol{Y}(r, r_0) \cdot \boldsymbol{A}(r_0) \cdot \boldsymbol{G} + \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r > r_0 \\ \boldsymbol{Y}(r, b)\,\boldsymbol{y}(b), & r < r_0. \end{cases} \tag{9.115}$$

In order to evaluate changes in the inertia tensor in the epicentral co-ordinate system, solutions to the inhomogeneous spheroidal equations for the four singular sources (9.96) through (9.99) are required. This will involve the solutions given by (9.113) and (9.115) as well as the calculation of $\boldsymbol{Y}(r, r_0)$. For this, it is convenient to use the propagator matrix property,

$$\boldsymbol{Y}(r, b) = \boldsymbol{Y}(r, r_0)\boldsymbol{Y}(r_0, b), \tag{9.116}$$

which, on right multiplication by $\boldsymbol{Y}^{-1}(r_0, b)$, produces

$$\boldsymbol{Y}(r, r_0) = \boldsymbol{Y}(r, b)\boldsymbol{Y}^{-1}(r_0, b). \tag{9.117}$$

This allows (9.113) to be written as

$$\boldsymbol{y}(r) = \begin{cases} \boldsymbol{Y}(r, b)\left[\boldsymbol{Y}^{-1}(r_0, b)\boldsymbol{G} + \boldsymbol{y}(b)\right], & r > r_0 \\ \boldsymbol{Y}(r, b)\boldsymbol{y}(b), & r < r_0, \end{cases} \tag{9.118}$$

and (9.115) to be written as

$$\boldsymbol{y}(r) = \begin{cases} \boldsymbol{Y}(r, b)\left[\boldsymbol{Y}^{-1}(r_0, b)\boldsymbol{A}(r_0)\boldsymbol{G} + \boldsymbol{y}(b)\right], & r > r_0 \\ \boldsymbol{Y}(r, b)\boldsymbol{y}(b), & r < r_0. \end{cases} \tag{9.119}$$

Solutions in the inner core are subject to conditions analogous to (9.44) through (9.47) at the inner core boundary with radius $r = a$. Taking $a^-$ to be the radius just inside the boundary and $a^+$ to be the radius just outside the boundary, we have

$$y_1(a^+) = \frac{1}{g_0} y_5, \tag{9.120}$$

$$y_2(a^+) = 0, \tag{9.121}$$

$$y_2(a^-) = -\rho_0(a^+) g_0 \left( \frac{y_5}{g_0} - y_1(a^-) \right), \tag{9.122}$$

$$y_6(a^-) = y_6(a^+) - \Delta y_6(a), \tag{9.123}$$

where $\Delta y_6(a)$ is the discontinuity in $y_6$ required to make the normal derivative of the equipotential, as expressed by (3.55), continuous. In addition to the discontinuity in $y_1$,

$$\Delta y_1(a) = y_1(a^+) - y_1(a^-), \tag{9.124}$$

there is a density discontinuity given by

$$\Delta \rho_0(a) = \rho_0(a^+) - \rho_0(a^-). \tag{9.125}$$

The required discontinuity in $y_6$ is then

$$\Delta y_6(a) = 4\pi G \left[ \rho_0(a^-) y_1(a^-) - \rho_0(a^+) y_1(a^+) \right]. \tag{9.126}$$

In terms of the discontinuities in $y_1$ and the density, the discontinuity in $y_6$ becomes

$$\Delta y_6(a) = -4\pi G \left[ \rho_0(a^+) \Delta y_1(a) + y_1(a^-) \Delta \rho_0(a) \right]. \tag{9.127}$$

For $n \geq 1$, in the inner core, there are only three fundamental, linearly independent solutions regular at the geocentre, as shown in Section 3.5. If they are denoted by $\boldsymbol{y}_1$, $\boldsymbol{y}_2$ and $\boldsymbol{y}_3$, the most general solution in the inner core, regular at the geocentre, is formed by the linear combination

$$\boldsymbol{y} = D_1 \boldsymbol{y}_1 + D_2 \boldsymbol{y}_2 + D_3 \boldsymbol{y}_3, \tag{9.128}$$

where, as before, each $\boldsymbol{y}_j$ is a six-vector with components representing the values of the variables $\left( y_{1j}, y_{2j}, y_{3j}, y_{4j}, y_{5j}, y_{6j} \right)$ describing the spheroidal deformation field, and each $D_j$ is a linear combination coefficient. Applying the condition (9.122) we find

$$\begin{aligned} &D_1 \left[ y_{21}(a) + \rho_0(a^+) y_{51}(a) - \rho_0(a^+) g_0(a) y_{11}(a) \right] \\ &+D_2 \left[ y_{22}(a) + \rho_0(a^+) y_{52}(a) - \rho_0(a^+) g_0(a) y_{12}(a) \right] \\ &+D_3 \left[ y_{23}(a) + \rho_0(a^+) y_{53}(a) - \rho_0(a^+) g_0(a) y_{13}(a) \right] = 0. \end{aligned} \tag{9.129}$$

The shear stress vanishes at the inner core boundary and in the outer core. Thus,

$$D_1 y_{41}(a) + D_2 y_{42}(a) + D_3 y_{43}(a) = 0. \tag{9.130}$$

The two homogeneous equations (9.129) and (9.130) reduce the uncertainty in the inner core solutions to a single free constant. They may be written as

$$D_1 c_{11} + D_2 c_{12} + D_3 c_{13} = 0, \tag{9.131}$$

$$D_1 c_{21} + D_2 c_{22} + D_3 c_{23} = 0, \tag{9.132}$$

where the coefficients are represented by the $c_{ij}$ for brevity. Taking the single free constant $C = D_3$, $D_1 = \delta C$ and $D_2 = \epsilon C$, the equations may be solved to give

$$\delta = \frac{c_{12}c_{23} - c_{13}c_{22}}{c_{11}c_{22} - c_{12}c_{21}}, \;\epsilon = \frac{c_{13}c_{21} - c_{11}c_{23}}{c_{11}c_{22} - c_{12}c_{21}}. \tag{9.133}$$

The gravity potential is continuous across the inner core boundary, giving

$$\delta C y_{51}(a) + \epsilon C y_{52}(a) + C y_{53}(a) = y_5(a^+). \tag{9.134}$$

Applying the condition (9.123), we find

$$\begin{aligned}
&\delta C\left[y_{61}(a) - 4\pi G\left\{\frac{\rho_0(a^+)}{g_0}y_{51}(a) - \rho_0(a^-)y_{11}(a)\right\}\right]\\
&+\epsilon C\left[y_{62}(a) - 4\pi G\left\{\frac{\rho_0(a^+)}{g_0}y_{52}(a) - \rho_0(a^-)y_{12}(a)\right\}\right]\\
&+C\left[y_{63}(a) - 4\pi G\left\{\frac{\rho_0(a^+)}{g_0}y_{53}(a) - \rho_0(a^-)y_{13}(a)\right\}\right] = y_6(a^+).
\end{aligned} \tag{9.135}$$

Equations (9.134) and (9.135) give the values of $y_5(a^+)$ and $y_6(a^+)$ to within the constant factor $C$ for integration through the fluid outer core.

Integration through the fluid outer core involves the integration of the two equations (9.42) and (9.43) in the variables $y_5$ and $y_6$ for $n = 2$. In the fluid outer core, $y_2$ vanishes, implying from equation (9.28) that $y_1 = y_5/g_0$. In addition, the left-hand side of equation (9.27) vanishes, allowing the variable $y_3$ for $n = 2$ to be expressed in terms of $y_5$ and $y_6$ by

$$y_3 = \frac{1}{3g_0}\left[2 + \frac{r\Omega^2}{g_0}\right]y_5 + \frac{r}{6g_0}y_6. \tag{9.136}$$

At the top of the fluid outer core the integration yields

$$y_5(b^-) = C\alpha = y_5(b^+) \text{ and } y_6(b^-) = C\beta, \tag{9.137}$$

since $y_5$ is continuous at the core-mantle boundary.

Integration through the mantle and crust requires values at the base of the mantle at $r = b^+$. To this end, $y_5(b)$ enters the condition (9.46) to give

$$y_2(b^+) = -\rho_0(b^-)g_0\left(\frac{y_5(b)}{g_0} - y_1(b^+)\right). \tag{9.138}$$

Taking the discontinuity in density into account, (9.50) yields

$$y_6(b^+) = y_6(b^-) + 4\pi G\left[\rho_0(b^-)\frac{y_5(b)}{g_0} - \rho_0(b^+)y_1(b^+)\right], \tag{9.139}$$

or, from (9.51),

$$y_6(b^+) = y_6(b^-) + 4\pi G\left[y_1(b^+)\Delta\rho_0(b) + \rho_0(b^-)\Delta y_1(b)\right], \tag{9.140}$$

where $\Delta y_1(b) = y_1(b^-) - y_1(b^+)$ and $\Delta\rho_0(b) = \rho_0(b^-) - \rho_0(b^+)$, as conditions at the base of the mantle at $r = b^+$.

Implementing the solutions (9.118) and (9.119) for singular sources at radius $r_0$ in the crust requires the evaluation of the propagator matrix $\boldsymbol{Y}(r, b)$ for all radii in the mantle and crust as well as its inverse $\boldsymbol{Y}^{-1}(r_0, b)$ at the source radius $r_0$. It also requires specification of the components of $\boldsymbol{y}(b)$. The components $y_1(b^+)$ and $y_3(b^+)$ are unknown, while $y_4(b^+)$ vanishes. The components $y_2(b^+)$ and $y_6(b^+)$ are expressed by (9.138) and (9.140) and $y_5(b^+)$ is given by the equation on the left of expression (9.137). Introducing two further unknowns, $A$ and $B$, we have

$$y_1(b^+) = A, \tag{9.141}$$

$$y_2(b^+) = -\rho_0(b^-)g_0\left(\frac{y_5}{g_0} - y_1(b^+)\right) = -\rho_0(b^-)g_0\left(\frac{C\alpha}{g_0} - A\right), \tag{9.142}$$

$$y_3(b^+) = B, \tag{9.143}$$

$$y_4(b^+) = 0, \tag{9.144}$$

$$y_5(b^+) = C\alpha, \tag{9.145}$$

$$\begin{aligned} y_6(b^+) &= y_6(b^-) + 4\pi G\left[\rho_0(b^-)\frac{y_5}{g_0} - \rho_0(b^+)y_1(b^+)\right] \\ &= C\beta + 4\pi G\left(\rho_0(b^-)\frac{C\alpha}{g_0} - \rho_0(b^+)A\right). \end{aligned} \tag{9.146}$$

The vector $\boldsymbol{y}(b)$ can then be represented in terms of the unknowns $A, B, C$ by

$$\boldsymbol{y}(b) = \begin{pmatrix} y_1(b^+) \\ y_2(b^+) \\ y_3(b^+) \\ y_4(b^+) \\ y_5(b^+) \\ y_6(b^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \rho_0(b^-)g_0 & 0 & -\rho_0(b^-)\alpha \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha \\ -4\pi G\rho_0(b^+) & 0 & \beta + 4\pi G\rho_0(b^-)\alpha/g_0 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix}. \tag{9.147}$$

At the free surface of the Earth at $r = d$, both normal and shear stresses vanish, thus $y_2(d) = 0$ and $y_4(d) = 0$. The change in gravity potential is of internal origin and is harmonic outside, giving

$$y_5(d) + \frac{d}{n+1} y_6(d) = 0, \text{ or } y_5(d) + \frac{d}{3} y_6(d) = 0, \tag{9.148}$$

since $n = 2$. The three unknowns $A, B, C$ are determined by the three conditions at the free surface of the Earth through the common term $\boldsymbol{Y}(d, b)$ in expressions (9.118) and (9.119) evaluated at the surface.

Four separate solutions of the inhomogeneous sixth-order spheroidal system with singular sources given by expressions (9.96) through (9.99) are obtained, using the methods outlined. The programme SOURCES.FOR finds these solutions throughout the Earth's interior by integrating through the inner core, fluid outer core, mantle and crust. Input to the programme is the Earth model file, the name of which is typed in at the request of the programme. The input Earth model is written out to the screen for verification. Integration begins at the geocentre by power series expansion of the three regular solutions there. The first three terms in the power series expansions are written out to the screen. All three free solutions in the inner core are found by Runge-Kutta integration throughout the inner core and are written to output files. The propagator matrix is used as already described to complete the four source solutions, as output, in the files source1.dat, source2.dat, source3.dat and source4.dat. In order to find the changes in the inertia tensor in the epicentral co-ordinate system as given by (9.89), the radial and transverse coefficients $u$ and $v$ are integrated over radius for the four source solutions. The integrations over the inner core, mantle and crust are performed using the trapezoidal rule because of the large number of points, taking discontinuities at the focus into account. For the fluid outer core, the extended Simpson's rule is used. The results, for the integrals of $u$ and $v$ for the four source solutions are written to the screen, broken down into individual contributions from the inner core, outer core, mantle and crust as well as the summation over the whole Earth. As well, the results are output in the file results.dat. The focal depth is the only free parameter input to SOURCES.FOR.

```
C              PROGRAMME SOURCES.FOR
C SOURCES.FOR solves the sixth-order spheroidal system for the four singular sources.
C For a given input Earth model, the solutions are obtained by integrating through
C the inner core, fluid outer core, mantle and crust for singular sources located at
C the focal depth in either the mantle or crust. The four source solutions are given,
C as output, in the files source1.dat, source2.dat, source3.dat and source4.dat.
C As required for the calculation of changes in the inertia tensor in epicentral
C co-ordinates, the radial and transverse coefficients u and v are integrated
C over radius for the four source solutions. The results of these integrations are
C output in the file results.dat.
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION R(100),RHO(100),GZERO(100),RI(300),RHOI(300),
     1 GZEROI(300),ENAME(10),NM(4),NI(4),NK(4),B(98,198),C(100,100),
     1 Y(6,6),YSCAL(6,3),YSCAL2(6,3),YA(6),A(6,6),AS(6,6),CAUG(6,13),
```

```
     1 RM1(6,3),RM2(6,3),YP(6,6),YIC(6,3),Y1(6,6),Y2(6,6),DELTA(6,3),
     2 YINFD(6,6),YINFDS(6,6),RAD(1000),FS(6,3,1000),G1(6),G2(6),G3(6),
     2 G4(6),ABC(6,3),PROD(6,3),AA(3,3),AAS(3,3),RHOIC(725),
     2 B1(3),B2(3),B3(3),B4(3),ROC(15),RHOOC(15),YOC(6,15),RM(725),
     3 RHOM(725),YYM(6,6,725),RC(75),RHOC(75),YYC(6,6,65),YF1(6),
     3 YF2(6),YF3(6),YF4(6),UV(6),UV1(6),UV2(6),UV3(6),UV4(6),
     3 CAUG2(3,7),SIC(8,725),SOC(8,15),SM(8,725),SC(8,75),VOC(8,15),
     4 WOC(15),SX(8),RES(8,5),DISC(4,2)
      DOUBLE PRECISION MU(100),LAMBDA(100),MUI(300),LAMBDAI(300),
     1 K1(6,6),MUFD
      CHARACTER*7 ENAME
      CHARACTER*20 EMODEL
C Enter Earth model file name.
      WRITE(6,10)
 10   FORMAT(1X,'Type in Earth model file name.')
      READ(5,11) EMODEL
 11   FORMAT(A20)
C Open Earth model file.
      OPEN(UNIT=1,FILE=EMODEL,STATUS='OLD')
C Open free solutions files.
      OPEN(UNIT=3,FILE='fs1.dat',STATUS='UNKNOWN')
      OPEN(UNIT=4,FILE='fs2.dat',STATUS='UNKNOWN')
      OPEN(UNIT=7,FILE='fs3.dat',STATUS='UNKNOWN')
C Open source solution files.
      OPEN(UNIT=8,FILE='source1.dat',STATUS='UNKNOWN')
      OPEN(UNIT=9,FILE='source2.dat',STATUS='UNKNOWN')
      OPEN(UNIT=10,FILE='source3.dat',STATUS='UNKNOWN')
      OPEN(UNIT=11,FILE='source4.dat',STATUS='UNKNOWN')
      OPEN(UNIT=12,FILE='results.dat',STATUS='UNKNOWN')
C Set angular frequency of Earth's rotation (WGS 84).
      WE=7.292115D-5
C Calculate square of angular frequency of Earth's rotation.
      WES=WE*WE
C Set value of pi.
      PI=3.141592653589793D0
C Set value of universal constant of gravitation (CODATA 2006).
      G=6.67428D-11
C Set maximum relative error tolerance for integrations.
      EPS=1.D-5
C Set minimum starting radius for variable Earth properties.
      RMIN=1200.D0
C Set maximum dimensions for interpolation.
      M1=100
      M2=198
      M3=98
C Read in and write out Earth model.
C Read in and write out headers.
C Earth model name.
      READ(1,12)(ENAME(I),I=1,10)
 12   FORMAT(10A7)
      WRITE(6,13)(ENAME(I),I=1,10)
 13   FORMAT(11X,10A7)
C Read in next line of header.
      READ(1,14)NN
 14   FORMAT(I10)
C Read in number of model points and number of integration steps.
      READ(1,15)(NM(I),NI(I),I=1,4)
 15   FORMAT(8I10)
      WRITE(6,16)
 16   FORMAT(//16X,'Number of model points and number of integration',
     1 ' steps.'/)
```

```
      WRITE(6,15)(NM(I),NI(I),I=1,4)
      WRITE(6,17)
 17   FORMAT(//5X,'Radius',6X,'Rho',4X,'Lambda',7X,'Mu',6X,'Gzero')
      WRITE(6,18)
 18   FORMAT(6X,'(km)',5X,'(gm/cc)',2X,'(kbars)',4X,'(kbars)',2X,
     1 '(cm/sec/sec)'/)
C Read in Earth model, inner core, outer core, mantle and crust.
      K=0
      DO 19 M=1,4
        N1=NM(M)
        READ(1,20)(R(I),RHO(I),LAMBDA(I),MU(I),GZERO(I),I=1,N1)
 20     FORMAT(1X,F10.1,F10.2,F10.1,F10.1,F10.1)
        DO 21 I=1,N1
C Scale Earth model to SI values and store.
          J=K+I
          RI(J)=R(I)*1.D3
          RHOI(J)=RHO(I)*1.D3
          LAMBDAI(J)=LAMBDA(I)*1.D8
          MUI(J)=MU(I)*1.D8
          GZEROI(J)=GZERO(I)*1.D-2
C Write Earth model out to screen.
          WRITE(6,20)R(I),RHO(I),LAMBDA(I),MU(I),GZERO(I)
 21     CONTINUE
        NK(M)=K
        K=K+N1
 19   CONTINUE
C Store surface radius.
      D=RI(NK(4)+NM(4))
C Begin integration through the inner core.
C Set up interpolation for the inner core.
      N1=NM(1)
      N2=2*N1-2
      N3=N1-2
C Put inner core values in active locations.
      DO 22 I=1,N1
        J=NK(1)+I
        R(I)=RI(J)
        RHO(I)=RHOI(J)
        MU(I)=MUI(J)
        LAMBDA(I)=LAMBDAI(J)
        GZERO(I)=GZEROI(J)
 22   CONTINUE
C Store density just inside inner core.
      RHOAM=RHO(N1)
C Calculate gravity gradient at geocentre.
      GAMMA=(4.D0*PI*G*RHO(1)-2.D0*WES)/3.D0
C Set gravity at minimum radius.
      GZERO(1)=GAMMA*RMIN
C Construct interpolation matrix.
      CALL SPMAT(N1,N2,N3,C,R,B,M1,M2,M3)
C Set degree to 2.
      N=2
      AN=DFLOAT(N)
C Set ANGS and COR to zero for dc solutions.
      ANGS=0.D0
      COR=0.D0
C Begin power series expansions for fundamental solutions for N=2.
C Set initial values of fundamental solutions.
C Set initial values of first fundamental solution.
      Y(1,1)=1.D0
      Y(2,1)=2.D0*(AN-1.D0)*MU(1)
```

```
      Y(3,1)=1.D0/AN
      Y(4,1)=Y(2,1)/AN
      Y(5,1)=4.D0*PI*G*RHO(1)/AN
      Y(6,1)=0.D0
C Set initial values of second fundamental solution.
      Y(1,2)=0.D0
      Y(2,2)=0.D0
      Y(3,2)=0.D0
      Y(4,2)=0.D0
      Y(5,2)=1.D0/AN
      Y(6,2)=1.D0
C Set initial values of third fundamental solution.
      Y(3,3)=P2(AN,LAMBDA(1),MU(1))/P1(AN,LAMBDA(1),MU(1))
      Y(2,3)=-Q1(AN,LAMBDA(1),MU(1))*Y(3,3)+Q2(AN,LAMBDA(1),MU(1))
      Y(1,3)=-AN*Y(3,3)+1.D0/MU(1)
      Y(4,3)=1.D0
      Y(5,3)=2.D0*PI*G*RHO(1)*((AN+3.D0)*Y(1,3)-AN*(AN+1.D0)*Y(3,3))/
     1 (2.D0*AN+3.D0)
      Y(6,3)=(AN+2.D0)*Y(5,3)-4.D0*PI*G*RHO(1)*Y(1,3)
C Calculate coefficients of second terms in series expansions.
C Calculate coefficients of second terms
C for first fundamental solution.
      YSCAL(3,1)=RHO(1)*((3.D0-AN)*GAMMA+ANGS+2.D0*WES-COR/AN)/
     1 P1(AN,LAMBDA(1),MU(1))
      YSCAL(2,1)=-Q1(AN,LAMBDA(1),MU(1))*YSCAL(3,1)
      YSCAL(1,1)=-AN*YSCAL(3,1)
      YSCAL(4,1)=0.D0
      YSCAL(5,1)=2.D0*PI*G*RHO(1)*((AN+3.D0)*YSCAL(1,1)
     1 -AN*(AN+1.D0)*YSCAL(3,1))/(2.D0*AN+3.D0)
      YSCAL(6,1)=(AN+2.D0)*YSCAL(5,1)-4.D0*PI*G*RHO(1)*YSCAL(1,1)
C Calculate coefficients of second terms
C for second fundamental solution.
      YSCAL(3,2)=RHO(1)/P1(AN,LAMBDA(1),MU(1))
      YSCAL(2,2)=-Q1(AN,LAMBDA(1),MU(1))*YSCAL(3,2)
      YSCAL(1,2)=-AN*YSCAL(3,2)
      YSCAL(4,2)=0.D0
      YSCAL(5,2)=2.D0*PI*G*RHO(1)*((AN+3.D0)*YSCAL(1,2)
     1 -AN*(AN+1.D0)*YSCAL(3,2))/(2.D0*AN+3.D0)
      YSCAL(6,2)=(AN+2.D0)*YSCAL(5,2)-4.D0*PI*G*RHO(1)*YSCAL(1,2)
C Generate matrix for evaluation of second terms in power series
C expansion of third fundamental solution.
      CALL MATRIX(A,RHO,LAMBDA,MU,PI,G,AN,WE)
C Set right-hand side for calculation of second terms
C in third solution.
      YA(1)=0.D0
      YA(2)=RHO(1)*(-(4.D0*GAMMA+ANGS+2.D0*WES)*Y(1,3)
     1 +(AN*(AN+1.D0)*GAMMA+COR)*Y(3,3)-Y(6,3))
      YA(3)=0.D0
      YA(4)=RHO(1)*((GAMMA+COR/(AN*(AN+1.D0)))*Y(1,3)
     1 -(ANGS-COR/(AN*(AN+1.D0)))*Y(3,3)-Y(5,3))
      YA(5)=0.D0
      YA(6)=0.D0
C Store matrix A in AS.
      DO 23 I=1,6
        DO 24 J=1,6
          AS(I,J)=A(I,J)
 24     CONTINUE
 23   CONTINUE
C Solve for second terms in third solution.
      CALL LINSOL(A,YA,6,CAUG,DET,6,13)
      YSCAL(1,3)=YA(1)
```

```
      YSCAL(2,3)=YA(2)
      YSCAL(3,3)=YA(3)
      YSCAL(4,3)=YA(4)
      YSCAL(5,3)=YA(5)
      YSCAL(6,3)=YA(6)
C Set right-hand side for calculation of third terms
C in first solution.
      YA(1)=0.D0
      YA(2)=RHO(1)*(-(4.D0*GAMMA+ANGS+2.D0*WES)*YSCAL(1,1)
     1 +(AN*(AN+1.D0)*GAMMA+COR)*YSCAL(3,1)-YSCAL(6,1))
      YA(3)=0.D0
      YA(4)=RHO(1)*((GAMMA+COR/(AN*(AN+1.D0)))*YSCAL(1,1)
     1 -(ANGS-COR/(AN*(AN+1.D0)))*YSCAL(3,1)-YSCAL(5,1))
      YA(5)=0.D0
      YA(6)=0.D0
C Copy stored matrix AS into matrix A.
      DO 25 I=1,6
        DO 26 J=1,6
          A(I,J)=AS(I,J)
 26     CONTINUE
 25   CONTINUE
C Solve for third terms in first solution.
      CALL LINSOL(A,YA,6,CAUG,DET,6,13)
      YSCAL2(1,1)=YA(1)
      YSCAL2(2,1)=YA(2)
      YSCAL2(3,1)=YA(3)
      YSCAL2(4,1)=YA(4)
      YSCAL2(5,1)=YA(5)
      YSCAL2(6,1)=YA(6)
C Set right-hand side for calculation of third terms
C in second solution.
      YA(1)=0.D0
      YA(2)=RHO(1)*(-(4.D0*GAMMA+ANGS+2.D0*WES)*YSCAL(1,2)
     1 +(AN*(AN+1.D0)*GAMMA+COR)*YSCAL(3,2)-YSCAL(6,2))
      YA(3)=0.D0
      YA(4)=RHO(1)*((GAMMA+COR/(AN*(AN+1.D0)))*YSCAL(1,2)
     1 -(ANGS-COR/(AN*(AN+1.D0)))*YSCAL(3,2)-YSCAL(5,2))
      YA(5)=0.D0
      YA(6)=0.D0
C Copy stored matrix AS into matrix A.
      DO 27 I=1,6
        DO 28 J=1,6
          A(I,J)=AS(I,J)
 28     CONTINUE
 27   CONTINUE
C Solve for third terms in second solution.
      CALL LINSOL(A,YA,6,CAUG,DET,6,13)
      YSCAL2(1,2)=YA(1)
      YSCAL2(2,2)=YA(2)
      YSCAL2(3,2)=YA(3)
      YSCAL2(4,2)=YA(4)
      YSCAL2(5,2)=YA(5)
      YSCAL2(6,2)=YA(6)
C Set right-hand side for calculation of third terms
C in third solution.
      YA(1)=0.D0
      YA(2)=RHO(1)*(-(4.D0*GAMMA+ANGS+2.D0*WES)*YSCAL(1,3)
     1 +(AN*(AN+1.D0)*GAMMA+COR)*YSCAL(3,3)-YSCAL(6,3))
      YA(3)=0.D0
      YA(4)=RHO(1)*((GAMMA+COR/(AN*(AN+1.D0)))*YSCAL(1,3)
     1 -(ANGS-COR/(AN*(AN+1.D0)))*YSCAL(3,3)-YSCAL(5,3))
```

```
      YA(5)=0.D0
      YA(6)=0.D0
C Copy stored matrix AS into matrix A and add 2.0 to diagonal of A.
      DO 29 I=1,6
        DO 30 J=1,6
          A(I,J)=AS(I,J)
          IF(I.EQ.J)A(I,J)=A(I,J)+2.D0
 30     CONTINUE
 29   CONTINUE
C Solve for third terms in third solution.
      CALL LINSOL(A,YA,6,CAUG,DET,6,13)
      YSCAL2(1,3)=YA(1)
      YSCAL2(2,3)=YA(2)
      YSCAL2(3,3)=YA(3)
      YSCAL2(4,3)=YA(4)
      YSCAL2(5,3)=YA(5)
      YSCAL2(6,3)=YA(6)
C Calculate terms in power series expansions at minimum radius RMIN.
      XV=RMIN
      XVS=XV*XV
      XV3=XV*XVS
      XV4=XVS*XVS
      DO 31 I=1,6
        DO 32 J=1,3
          RM1(I,J)=XVS*YSCAL(I,J)
          RM2(I,J)=XV4*YSCAL2(I,J)
 32     CONTINUE
 31   CONTINUE
C Write out terms in power series expansions.
      DO 33 J=1,3
        WRITE(6,34)J
 34     FORMAT(/5X,'Fundamental solution number ',I3)
        WRITE(6,35)
 35     FORMAT(/6X,'first term',10X,'second term',9X,'third term')
        WRITE(6,36)(Y(I,J),RM1(I,J),RM2(I,J),I=1,6)
 36     FORMAT(5X,D15.8,5X,D15.8,5X,D15.8)
 33   CONTINUE
C Set last three columns of arrays Y(6,6) and YP(6,6) to zero
C for inner core.
      DO 37 I=1,6
        DO 38 J=4,6
          Y(I,J)=0.D0
          YP(I,J)=0.D0
 38     CONTINUE
 37   CONTINUE
C Put inner core solutions at minimum radius in array YIC(6,3) and
C initialize free solutions array in y-variables, FS(6,3,1), at the geocentre.
      DO 39 I=1,6
        DO 40 J=1,3
          YIC(I,J)=Y(I,J)
          FS(I,J,1)=0.D0
          IF(J.EQ.1.AND.(I.EQ.2.OR.I.EQ.4)) FS(I,J,1)=Y(I,J)
 40     CONTINUE
 39   CONTINUE
C Write out starting values of free solutions in y-variables at the geocentre.
      RAD(1)=0.D0
      RHOIC(1)=RHO(1)
      WRITE(3,42)RAD(1),(FS(I,1,1),I=1,6)
      WRITE(4,42)RAD(1),(FS(I,2,1),I=1,6)
      WRITE(7,42)RAD(1),(FS(I,3,1),I=1,6)
C Store minimum radius and density at minimum radius.
```

```
      RAD(2)=XV*1.D-3
      CALL INTPL(XV,RHOR,N1,C,R,RHO,M1)
      RHOIC(2)=RHOR
C Convert solutions back to y-variables.
      ANM2=AN-2.D0
      DO 41 J=1,3
        IF(J.EQ.3) ANM2=AN
        CONST=XV**ANM2
        FS(2,J,2)=CONST*(Y(2,J)+RM1(2,J)+RM2(2,J))
        FS(4,J,2)=CONST*(Y(4,J)+RM1(4,J)+RM2(4,J))
        CONST=CONST*XV
        FS(1,J,2)=CONST*(Y(1,J)+RM1(1,J)+RM2(1,J))
        FS(3,J,2)=CONST*(Y(3,J)+RM1(3,J)+RM2(3,J))
        FS(6,J,2)=CONST*(Y(6,J)+RM1(6,J)+RM2(6,J))
        CONST=CONST*XV
        FS(5,J,2)=CONST*(Y(5,J)+RM1(5,J)+RM2(5,J))
 41   CONTINUE
C Write out free solutions at minimum radius, RMIN.
      WRITE(3,42)RAD(2),(FS(I,1,2),I=1,6)
      WRITE(4,42)RAD(2),(FS(I,2,2),I=1,6)
      WRITE(7,42)RAD(2),(FS(I,3,2),I=1,6)
 42   FORMAT(F7.2,6D14.6)
      IRAD=2
C Begin Runge-Kutta integration for the inner core.
C Find maximum ratio of second coefficient to first term
C in power series expansions.
      CALL REL(ERRMAX,YSCAL,YIC,N)
C Set initial stepsize so that fourth-order method would have relative error
C bound EPS.
      H=(EPS/ERRMAX)**(0.2D0)
C Find solutions and derivatives at minimum radius RMIN
C by power series expansions.
      DO 43 I=1,6
        DO 44 J=1,3
C Find solutions.
          Y(I,J)=Y(I,J)+XVS*YSCAL(I,J)+XV4*YSCAL2(I,J)
C Find derivatives.
          YP(I,J)=2.D0*XV*YSCAL(I,J)+4.D0*XV3*YSCAL2(I,J)
 44     CONTINUE
 43   CONTINUE
C Set inner core index.
      IR=1
 45   CONTINUE
C Calculate derivatives at beginning of current step.
      CALL YPRIME(XV,Y,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,
     1 IR,RMIN,ANGS,COR,WES)
C Initialize function values at beginning of integration step.
      DO 46 I=1,6
        DO 47 J=1,6
          Y1(I,J)=Y(I,J)
          Y2(I,J)=Y(I,J)
 47     CONTINUE
 46   CONTINUE
C Integrate solution forward one full step.
      CALL RK4(XV,Y1,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,H,IR,
     1 RMIN,ANGS,COR,WES)
C Integrate solution forward two half-steps.
C Halve step.
      HH=0.5D0*H
C Reset radius.
      XV=XV-H
```

```
      CALL RK4(XV,Y2,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,HH,IR,
     1 RMIN,ANGS,COR,WES)
      CALL YPRIME(XV,Y2,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,IR,
     1 RMIN,ANGS,COR,WES)
      CALL RK4(XV,Y2,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,HH,IR,
     1 RMIN,ANGS,COR,WES)
C Find error matrix and maximum relative error.
      DO 48 I=1,6
        DO 49 J=1,3
          DELTA(I,J)=Y1(I,J)-Y2(I,J)
 49     CONTINUE
 48   CONTINUE
C Copy Y2(6,6) into inner core solution array YIC(6,3).
      DO 50 I=1,6
        DO 51 J=1,3
          YIC(I,J)=Y2(I,J)
 51     CONTINUE
 50   CONTINUE
C Find maximum relative error.
      CALL REL(ERRMAX,DELTA,YIC,N)
C Scale maximum relative error by specified relative error tolerance.
      ERRMAX=ERRMAX/EPS
C Test whether step was successful.
      IF(ERRMAX.LT.1.D0) GO TO 52
C Step too large, reduce stepsize and repeat.
      XV=XV-H
      H=0.9D0*H*(ERRMAX**(-0.25D0))
      GO TO 45
 52   CONTINUE
C Step successful, accept solution and improve truncation error.
      DO 53 I=1,6
        DO 54 J=1,3
          Y(I,J)=Y2(I,J)+DELTA(I,J)/15.D0
 54     CONTINUE
 53   CONTINUE
C Prepare to write out free solutions.
      IRAD=IRAD+1
      RAD(IRAD)=XV*1.D-3
C Write out inner core density.
      CALL INTPL(XV,RHOR,N1,C,R,RHO,M1)
      RHOIC(IRAD)=RHOR
C Convert solutions back to y-variables.
      ANM2=AN-2.D0
      DO 55 J=1,3
        IF(J.EQ.3) ANM2=AN
        CONST=XV**ANM2
        FS(2,J,IRAD)=CONST*Y(2,J)
        FS(4,J,IRAD)=CONST*Y(4,J)
        CONST=CONST*XV
        FS(1,J,IRAD)=CONST*Y(1,J)
        FS(3,J,IRAD)=CONST*Y(3,J)
        FS(6,J,IRAD)=CONST*Y(6,J)
        CONST=CONST*XV
        FS(5,J,IRAD)=CONST*Y(5,J)
 55   CONTINUE
      WRITE(3,42)RAD(IRAD),(FS(I,1,IRAD),I=1,6)
      WRITE(4,42)RAD(IRAD),(FS(I,2,IRAD),I=1,6)
      WRITE(7,42)RAD(IRAD),(FS(I,3,IRAD),I=1,6)
C Increase stepsize.
      H=0.9D0*H*(ERRMAX**(-0.20D0))
C Test if solution is finished.
```

```
      XVT=XV+H
      IF(XVT.LT.R(N1)) GO TO 45
C Find remaining distance to the ICB.
      H=R(N1)-XV
C Compute fraction of radius.
      RATIO=H/R(N1)
C Continue integration only if remaining distance is more than
C 1.D-9 of radius.
      IF(RATIO.GT.1.D-9) GO TO 45
C Begin integration through the fluid outer core.
C Set up interpolation for the fluid outer core.
      N1=NM(2)
      N2=2*N1-2
      N3=N1-2
C Put fluid outer core values in active locations.
      DO 56 I=1,N1
        J=NK(2)+I
        R(I)=RI(J)
        RHO(I)=RHOI(J)
        MU(I)=MUI(J)
        LAMBDA(I)=LAMBDAI(J)
        GZERO(I)=GZEROI(J)
 56   CONTINUE
C Store density and gravity at bottom of the outer core.
      RHOAP=RHO(1)
      GZEROA=GZERO(1)
C Store density and gravity at top of the outer core.
      RHOBM=RHO(N1)
      GZEROB=GZERO(N1)
C Construct interpolation matrix.
      CALL SPMAT(N1,N2,N3,C,R,B,M1,M2,M3)
C Find starting values of y5 and y6 to within constant factor C.
C Find coefficients cij.
      C11=FS(2,1,IRAD)+RHOAP*FS(5,1,IRAD)-RHOAP*GZEROA*FS(1,1,IRAD)
      C12=FS(2,2,IRAD)+RHOAP*FS(5,2,IRAD)-RHOAP*GZEROA*FS(1,2,IRAD)
      C13=FS(2,3,IRAD)+RHOAP*FS(5,3,IRAD)-RHOAP*GZEROA*FS(1,3,IRAD)
      C21=FS(4,1,IRAD)
      C22=FS(4,2,IRAD)
      C23=FS(4,3,IRAD)
C Find delta (DEL) and epsilon (EPS).
      DEL=(C12*C23-C13*C22)/(C11*C22-C12*C21)
      EPS=(C13*C21-C11*C23)/(C11*C22-C12*C21)
C Find starting values of y5 and y6.
      Y5S=DEL*FS(5,1,IRAD)+EPS*FS(5,2,IRAD)+FS(5,3,IRAD)
      Y6S=DEL*(FS(6,1,IRAD)-4.D0*PI*G*(RHOAP*FS(5,1,IRAD)/GZEROA
     1 -RHOAM*FS(1,1,IRAD)))+EPS*(FS(6,2,IRAD)-4.D0*PI*G*(RHOAP
     2 *FS(5,2,IRAD)/GZEROA-RHOAM*FS(1,2,IRAD)))
     3 +FS(6,3,IRAD)-4.D0*PI*G*(RHOAP*FS(5,3,IRAD)/GZEROA
     4 -RHOAM*FS(1,3,IRAD))
C Initialize propagator matrix for the fluid outer core.
      DO 57 I=1,2
        Y(I,I)=1.D0
        DO 58 J=1,2
          IF(I.NE.J) Y(I,J)=0.D0
 58     CONTINUE
 57   CONTINUE
C Begin Runge-Kutta integration for the fluid outer core.
      M=NI(2)
      AM=DFLOAT(M)
C Find stepsize.
      H=(R(N1)-R(1))/AM
```

```
C Set initial radius and density.
      XV=R(1)
      ROC(1)=XV*1.D-3
      RHOOC(1)=RHO(1)
C Set initial value of solution vector.
      YOC(1,1)=Y5S/GZERO(1)
      YOC(2,1)=0.D0
      YOC(3,1)=(2.D0+XV*WES/GZERO(1))*Y5S/(3.D0*GZERO(1))+
     1 XV*Y6S/(6.D0*GZERO(1))
      YOC(4,1)=0.D0
      YOC(5,1)=Y5S
      YOC(6,1)=Y6S
      DO 59 I=1,M
        CALL YPRIMEOC(XV,Y,A,K1,N1,C,R,RHO,GZERO,PI,G,WES)
        CALL RK4OC(XV,Y,A,K1,N1,C,R,RHO,GZERO,PI,G,H,WES)
C Find value GXV of GZERO at radius XV.
        CALL INTPL(XV,GXV,N1,C,R,GZERO,M1)
        IP1=I+1
C Store radius.
        ROC(IP1)=XV*1.D-3
C Write out outer core density.
        CALL INTPL(XV,RHOR,N1,C,R,RHO,M1)
        RHOOC(IP1)=RHOR
C Update solution vector.
        ALPHA=Y(1,1)*Y5S+Y(1,2)*Y6S
        BETA=Y(2,1)*Y5S+Y(2,2)*Y6S
        YOC(1,IP1)=ALPHA/GXV
        YOC(2,IP1)=0.D0
        YOC(3,IP1)=(2.D0+XV*WES/GXV)*ALPHA/(3.D0*GXV)+
     1  XV*BETA/(6.D0*GXV)
        YOC(4,IP1)=0.D0
        YOC(5,IP1)=ALPHA
        YOC(6,IP1)=BETA
 59   CONTINUE
C Input focal depth.
      WRITE(6,60)
 60   FORMAT(1X,'Enter focal depth in kilometres.')
      READ(5,*) FD
C Find radius of focus in kilometres.
      RFD=1.D-3*D-FD
C Find radius at the bottom of the crust.
      J=NK(4)+1
      RC(1)=1.D-3*RI(J)
C Set switch for focus in the mantle.
      IFSW=1
C Set switch for focus in the crust.
      IF(RFD.GT.RC(1)) IFSW=-1
C Begin integration of propagator matrix through the mantle.
C Set up interpolation for the mantle.
      N1=NM(3)
      N2=2*N1-2
      N3=N1-2
C Put mantle values in active locations.
      DO 61 I=1,N1
        J=NK(3)+I
        R(I)=RI(J)
        RHO(I)=RHOI(J)
        MU(I)=MUI(J)
        LAMBDA(I)=LAMBDAI(J)
        GZERO(I)=GZEROI(J)
 61   CONTINUE
```

```
C Store density at bottom of the mantle.
      RHOBP=RHO(1)
C Construct interpolation matrix.
      CALL SPMAT(N1,N2,N3,C,R,B,M1,M2,M3)
C Initialize propagator matrix for the mantle and store initial value.
      DO 62 I=1,6
        Y(I,I)=1.D0
        YYM(I,I,1)=1.D0
        DO 63 J=1,6
          IF(I.NE.J) Y(I,J)=0.D0
          IF(I.NE.J) YYM(I,J,1)=0.D0
 63     CONTINUE
 62   CONTINUE
C Begin Runge-Kutta integration for the mantle.
      M=NI(3)
      AM=DFLOAT(M)
C Find stepsize.
      H=(R(N1)-R(1))/AM
C Set initial radius and density.
      XV=R(1)
      RM(1)=XV*1.D-3
      RHOM(1)=RHO(1)
C Set integration region counter.
      IR=3
      DO 64 I=1,M
        CALL YPRIME(XV,Y,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,
     1  IR,RMIN,ANGS,COR,WES)
        CALL RK4(XV,Y,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,H,IR,
     1  RMIN,ANGS,COR,WES)
        IP1=I+1
C Store radius and density.
        RM(IP1)=XV*1.D-3
        CALL INTPL(XV,RHOR,N1,C,R,RHO,M1)
        RHOM(IP1)=RHOR
C Store propagator matrix.
        DO 65 J=1,6
          DO 66 K=1,6
            YYM(J,K,IP1)=Y(J,K)
 66       CONTINUE
 65     CONTINUE
 64   CONTINUE
C Set exact radius at the top of the mantle.
      RM(IP1)=R(N1)*1.D-3
C For focus in the mantle, calculate coefficient matrix AS(6,6).
      XV=RFD*1.D3
      IF(IFSW.EQ.1) CALL
     1 YPRIME(XV,Y,AS,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,
     2 IR,RMIN,ANGS,COR,WES)
C If focus is in the mantle, find shear modulus MUFD at focal depth.
      IF(IFSW.EQ.1) CALL
     1 INTPL(XV,MUFD,N1,C,R,MU,M1)
C If focus is in the mantle, find density at focal depth.
      IF(IFSW.EQ.1) CALL
     1 INTPL(XV,RHOFD,N1,C,R,RHO,M1)
C Begin integration of propagator matrix through the crust.
C Set integration region counter.
      IR=4
C Set number of steps in integration.
      M=NI(4)
      AM=DFLOAT(M)
C Set up interpolation for the crust.
```

```
      N1=NM(4)
      N2=2*N1-2
      N3=N1-2
C Put crust values in active locations.
      DO 67 I=1,N1
        J=NK(4)+I
        R(I)=RI(J)
        RHO(I)=RHOI(J)
        MU(I)=MUI(J)
        LAMBDA(I)=LAMBDAI(J)
        GZERO(I)=GZEROI(J)
 67   CONTINUE
C Construct interpolation matrix.
      CALL SPMAT(N1,N2,N3,C,R,B,M1,M2,M3)
C Initialize propagator matrix for the crust.
      DO 68 I=1,6
        DO 69 J=1,6
          Y(I,J)=YYM(I,J,IP1)
          YYC(I,J,1)=Y(I,J)
 69     CONTINUE
 68   CONTINUE
C Find stepsize.
      H=(R(N1)-R(1))/AM
C Set initial radius and density.
      XV=R(1)
      RC(1)=XV*1.D-3
      RHOC(1)=RHO(1)
      DO 70 I=1,M
        CALL YPRIME(XV,Y,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,
     1  IR,RMIN,ANGS,COR,WES)
        CALL RK4(XV,Y,A,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,H,IR,
     1  RMIN,ANGS,COR,WES)
        IP1=I+1
C Store radius and density.
        RC(IP1)=XV*1.D-3
        CALL INTPL(XV,RHOR,N1,C,R,RHO,M1)
        RHOC(IP1)=RHOR
C Store propagator matrix.
        DO 71 J=1,6
          DO 72 K=1,6
            YYC(J,K,IP1)=Y(J,K)
 72       CONTINUE
 71     CONTINUE
 70   CONTINUE
C If focus is in the crust, calculate coefficient matrix AS(6,6).
      XV=RFD*1.D3
      IF(IFSW.EQ.-1) CALL
     1 YPRIME(XV,Y,AS,K1,N1,C,R,RHO,MU,LAMBDA,GZERO,N,PI,G,
     2 IR,RMIN,ANGS,COR,WES)
C If focus is in the crust, find shear modulus MUFD at focal depth.
      IF(IFSW.EQ.-1) CALL
     1 INTPL(XV,MUFD,N1,C,R,MU,M1)
C If focus is in the crust, find density at focal depth.
      IF(IFSW.EQ.-1) CALL
     1 INTPL(XV,RHOFD,N1,C,R,RHO,M1)
C Determine whether focus is in the mantle or the crust.
      IF(IFSW.EQ.-1) GO TO 73
C Focus is in the mantle.
C Find location of focus in the mantle and select propagator matrix to be
C inverted.
      M=NI(3)
```

```
      MP1=M+1
      DO 74 I=2,MP1
        IM1=I-1
        IF(RFD.GT.RM(IM1).AND.RFD.LT.RM(I)) GO TO 75
        IF(RFD.EQ.RM(I)) GO TO 76
        GO TO 74
 75     IEQ=-1
        IL1=I-1
        DO 77 J=1,6
          DO 78 K=1,6
C Take average propagator matrix.
            YINFD(J,K)=(YYM(J,K,I)*(RFD-RM(IM1))+
     1      YYM(J,K,IM1)*(RM(I)-RFD))/(RM(I)-RM(IM1))
 78       CONTINUE
          UV(J)=1.D0
 77     CONTINUE
        GO TO 74
 76     IEQ=1
        IL1=I-1
        DO 79 J=1,6
          DO 80 K=1,6
C Use propagator matrix at this point.
            YINFD(J,K)=YYM(J,K,I)
 80       CONTINUE
          UV(J)=1.D0
 79     CONTINUE
 74   CONTINUE
      GO TO 81
C Focus is in the crust.
C Find location of focus in the crust and select propagator matrix to be
C inverted.
 73   M=NI(4)
      MP1=M+1
      DO 82 I=2,MP1
        IM1=I-1
        IF(RFD.GT.RC(IM1).AND.RFD.LT.RC(I)) GO TO 83
        IF(RFD.EQ.RC(I)) GO TO 84
        GO TO 82
 83     IEQ=-1
        IL1=I-1
        DO 85 J=1,6
          DO 86 K=1,6
C Take average propagator matrix.
            YINFD(J,K)=(YYC(J,K,I)*(RFD-RC(IM1))+
     1      YYC(J,K,IM1)*(RC(I)-RFD))/(RC(I)-RC(IM1))
 86       CONTINUE
          UV(J)=1.D0
 85     CONTINUE
        GO TO 82
 84     IEQ=1
        IL1=I-1
        DO 87 J=1,6
          DO 88 K=1,6
C Use propagator matrix at this point.
            YINFD(J,K)=YYC(J,K,I)
 88       CONTINUE
          UV(J)=1.D0
 87     CONTINUE
 82   CONTINUE
C Store propagator matrix at the focus.
 81   DO 89 I=1,6
```

```
        DO 90 J=1,6
          YINFDS(I,J)=YINFD(I,J)
 90     CONTINUE
 89   CONTINUE
C Invert propagator matrix at the focus.
      CALL LINSOL(YINFD,UV,6,CAUG,DET,6,13)
C Begin solutions for four sources.
C Initialize four source vectors G1,G2,G3,G4.
      XV=RFD*1.D3
C Construct G1.
      FACT=-5.D0*MUFD/(8.D0*PI*XV*XV*XV)
      DO 91 I=1,6
        G1(I)=FACT*YINFD(I,2)
 91   CONTINUE
C Construct G2.
      FACT=FACT*XV
      DO 92 I=1,6
        UV(I)=FACT*AS(I,2)
 92   CONTINUE
      DO 93 I=1,6
        HOLD=0.D0
        DO 94 J=1,6
          HOLD=HOLD+YINFD(I,J)*UV(J)
 94     CONTINUE
        G2(I)=HOLD
 93   CONTINUE
C Construct G3.
      FACT=FACT/XV
      DO 95 I=1,6
        G3(I)=FACT*YINFD(I,4)
 95   CONTINUE
C Construct G4.
      FACT=FACT*XV
      DO 96 I=1,6
        UV(I)=FACT*AS(I,4)
 96   CONTINUE
      DO 97 I=1,6
        HOLD=0.D0
        DO 98 J=1,6
          HOLD=HOLD+YINFD(I,J)*UV(J)
 98     CONTINUE
        G4(I)=HOLD
 97   CONTINUE
C Initialize matrix ABC(6,3) connecting unknowns to the vector y(b).
      ABC(1,1)=1.D0
      ABC(1,2)=0.D0
      ABC(1,3)=0.D0
      ABC(2,1)=RHOBM*GZEROB
      ABC(2,2)=0.D0
      ABC(2,3)=-RHOBM*ALPHA
      ABC(3,1)=0.D0
      ABC(3,2)=1.D0
      ABC(3,3)=0.D0
      ABC(4,1)=0.D0
      ABC(4,2)=0.D0
      ABC(4,3)=0.D0
      ABC(5,1)=0.D0
      ABC(5,2)=0.D0
      ABC(5,3)=ALPHA
      ABC(6,1)=-4.D0*PI*G*RHOBP
      ABC(6,2)=0.D0
```

```
      ABC(6,3)=BETA+4.D0*PI*G*RHOBM*ALPHA/GZEROB
C Multiply propagator matrix at Earth's surface into matrix ABC(6,3)
C to get PROD(6,3).
      DO 99 I=1,6
        DO 100 K=1,3
          HOLD=0.D0
          DO 101 J=1,6
            HOLD=HOLD+Y(I,J)*ABC(J,K)
 101      CONTINUE
          PROD(I,K)=HOLD
 100    CONTINUE
 99   CONTINUE
C Set ABC equal to PROD.
      DO 102 I=1,6
        DO 103 J=1,3
          ABC(I,J)=PROD(I,J)
 103    CONTINUE
 102  CONTINUE
C Multiply four source vectors by propagator matrix at Earth's surface.
      DO 104 I=1,6
        HOLD1=0.D0
        HOLD2=0.D0
        HOLD3=0.D0
        HOLD4=0.D0
        DO 105 J=1,6
          HOLD1=HOLD1+Y(I,J)*G1(J)
          HOLD2=HOLD2+Y(I,J)*G2(J)
          HOLD3=HOLD3+Y(I,J)*G3(J)
          HOLD4=HOLD4+Y(I,J)*G4(J)
 105    CONTINUE
        UV1(I)=HOLD1
        UV2(I)=HOLD2
        UV3(I)=HOLD3
        UV4(I)=HOLD4
 104  CONTINUE
C Apply boundary conditions at Earth's surface.
C Construct coefficient matrix AA(3,3).
C Normal stress vanishes.
      DO 106 I=1,3
        AA(1,I)=ABC(2,I)
 106  CONTINUE
C Shear stress vanishes.
      DO 107 I=1,3
        AA(2,I)=ABC(4,I)
 107  CONTINUE
C Gravity potential is harmonic outside the Earth.
      DO 108 I=1,3
        AA(3,I)=ABC(5,I)+D*ABC(6,I)/3.D0
 108  CONTINUE
C Store coefficient matrix AA(3,3) as AAS(3,3).
      DO 109 I=1,3
        DO 110 J=1,3
          AAS(I,J)=AA(I,J)
 110    CONTINUE
 109  CONTINUE
C Construct constant vectors B1(3), B2(3), B3(3), B4(3) for four sources.
      B1(1)=-UV1(2)
      B1(2)=-UV1(4)
      B1(3)=-UV1(5)-D*UV1(6)/3.D0
C Solve for first source coefficients.
      CALL LINSOL(AA,B1,3,CAUG2,DET,3,7)
```

```
      B2(1)=-UV2(2)
      B2(2)=-UV2(4)
      B2(3)=-UV2(5)-D*UV2(6)/3.D0
C Restore matrix AA(3,3).
      DO 111 I=1,3
        DO 112 J=1,3
          AA(I,J)=AAS(I,J)
 112    CONTINUE
 111  CONTINUE
C Solve for second source coefficients.
      CALL LINSOL(AA,B2,3,CAUG2,DET,3,7)
      B3(1)=-UV3(2)
      B3(2)=-UV3(4)
      B3(3)=-UV3(5)-D*UV3(6)/3.D0
C Restore matrix AA(3,3).
      DO 113 I=1,3
        DO 114 J=1,3
          AA(I,J)=AAS(I,J)
 114    CONTINUE
 113  CONTINUE
C Solve for third source coefficients.
      CALL LINSOL(AA,B3,3,CAUG2,DET,3,7)
      B4(1)=-UV4(2)
      B4(2)=-UV4(4)
      B4(3)=-UV4(5)-D*UV4(6)/3.D0
C Restore matrix AA(3,3).
      DO 115 I=1,3
        DO 116 J=1,3
          AA(I,J)=AAS(I,J)
 116    CONTINUE
 115  CONTINUE
C Solve for fourth source coefficients.
      CALL LINSOL(AA,B4,3,CAUG2,DET,3,7)
C Find four source solutions in the inner core.
      DO 117 I=1,IRAD
        DO 118 J=1,6
          UV1(J)=B1(3)*(DEL*FS(J,1,I)+EPS*FS(J,2,I)+FS(J,3,I))
          UV2(J)=B2(3)*(DEL*FS(J,1,I)+EPS*FS(J,2,I)+FS(J,3,I))
          UV3(J)=B3(3)*(DEL*FS(J,1,I)+EPS*FS(J,2,I)+FS(J,3,I))
          UV4(J)=B4(3)*(DEL*FS(J,1,I)+EPS*FS(J,2,I)+FS(J,3,I))
 118    CONTINUE
        SIC(1,I)=UV1(1)
        SIC(2,I)=UV1(3)
        SIC(3,I)=UV2(1)
        SIC(4,I)=UV2(3)
        SIC(5,I)=UV3(1)
        SIC(6,I)=UV3(3)
        SIC(7,I)=UV4(1)
        SIC(8,I)=UV4(3)
        WRITE(8,42)RAD(I),(UV1(K),K=1,6)
        WRITE(9,42)RAD(I),(UV2(K),K=1,6)
        WRITE(10,42)RAD(I),(UV3(K),K=1,6)
        WRITE(11,42)RAD(I),(UV4(K),K=1,6)
 117  CONTINUE
C Initialize radial displacements at the inner core boundary.
      DISC(1,1)=-UV1(1)
      DISC(2,1)=-UV2(1)
      DISC(3,1)=-UV3(1)
      DISC(4,1)=-UV4(1)
C Find four source solutions in the outer core.
      M=NI(2)
```

```
      MP1=M+1
      DO 119 I=1,MP1
        DO 120 J=1,6
          UV1(J)=B1(3)*YOC(J,I)
          UV2(J)=B2(3)*YOC(J,I)
          UV3(J)=B3(3)*YOC(J,I)
          UV4(J)=B4(3)*YOC(J,I)
 120    CONTINUE
        SOC(1,I)=UV1(1)
        SOC(2,I)=UV1(3)
        SOC(3,I)=UV2(1)
        SOC(4,I)=UV2(3)
        SOC(5,I)=UV3(1)
        SOC(6,I)=UV3(3)
        SOC(7,I)=UV4(1)
        SOC(8,I)=UV4(3)
C Record discontinuities in radial displacement at the inner core boundary.
        IF(I.EQ.1)DISC(1,1)=DISC(1,1)+UV1(1)
        IF(I.EQ.1)DISC(2,1)=DISC(2,1)+UV2(1)
        IF(I.EQ.1)DISC(3,1)=DISC(3,1)+UV3(1)
        IF(I.EQ.1)DISC(4,1)=DISC(4,1)+UV4(1)
        WRITE(8,42)ROC(I),(UV1(K),K=1,6)
        WRITE(9,42)ROC(I),(UV2(K),K=1,6)
        WRITE(10,42)ROC(I),(UV3(K),K=1,6)
        WRITE(11,42)ROC(I),(UV4(K),K=1,6)
 119  CONTINUE
C Initialize displacements at the core-mantle boundary.
      DISC(1,2)=-UV1(1)
      DISC(2,2)=-UV2(1)
      DISC(3,2)=-UV3(1)
      DISC(4,2)=-UV4(1)
C Find four source solutions in the mantle.
C Initialize solution vectors at the base of the mantle.
      UV1(1)=B1(1)
      UV2(1)=B2(1)
      UV3(1)=B3(1)
      UV4(1)=B4(1)
      UV1(2)=-RHOBM*(B1(3)*ALPHA-GZEROB*B1(1))
      UV2(2)=-RHOBM*(B2(3)*ALPHA-GZEROB*B2(1))
      UV3(2)=-RHOBM*(B3(3)*ALPHA-GZEROB*B3(1))
      UV4(2)=-RHOBM*(B4(3)*ALPHA-GZEROB*B4(1))
      UV1(3)=B1(2)
      UV2(3)=B2(2)
      UV3(3)=B3(2)
      UV4(3)=B4(2)
      UV1(4)=0.D0
      UV2(4)=0.D0
      UV3(4)=0.D0
      UV4(4)=0.D0
      UV1(5)=B1(3)*ALPHA
      UV2(5)=B2(3)*ALPHA
      UV3(5)=B3(3)*ALPHA
      UV4(5)=B4(3)*ALPHA
      UV1(6)=B1(3)*BETA+4.D0*PI*G*(RHOBM*B1(3)*ALPHA/GZEROB
     1 -RHOBP*B1(1))
      UV2(6)=B2(3)*BETA+4.D0*PI*G*(RHOBM*B2(3)*ALPHA/GZEROB
     2 -RHOBP*B2(1))
      UV3(6)=B3(3)*BETA+4.D0*PI*G*(RHOBM*B3(3)*ALPHA/GZEROB
     3 -RHOBP*B3(1))
      UV4(6)=B4(3)*BETA+4.D0*PI*G*(RHOBM*B4(3)*ALPHA/GZEROB
     4 -RHOBP*B4(1))
```

```
C Solve for four source solutions in the mantle.
      M=NI(3)
      MP1=M+1
C Switch for focus in the mantle.
      IF(IFSW.EQ.1) GO TO 121
      DO 122 I=1,MP1
        DO 123 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 124 K=1,6
            HOLD1=HOLD1+YYM(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYM(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYM(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYM(J,K,I)*UV4(K)
 124      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 123    CONTINUE
        SM(1,I)=YF1(1)
        SM(2,I)=YF1(3)
        SM(3,I)=YF2(1)
        SM(4,I)=YF2(3)
        SM(5,I)=YF3(1)
        SM(6,I)=YF3(3)
        SM(7,I)=YF4(1)
        SM(8,I)=YF4(3)
C Record radial displacement discontinuities at the core-mantle boundary.
        IF(I.EQ.1)DISC(1,2)=DISC(1,2)+YF1(1)
        IF(I.EQ.1)DISC(2,2)=DISC(2,2)+YF2(1)
        IF(I.EQ.1)DISC(3,2)=DISC(3,2)+YF3(1)
        IF(I.EQ.1)DISC(4,2)=DISC(4,2)+YF4(1)
        WRITE(8,42)RM(I),(YF1(J),J=1,6)
        WRITE(9,42)RM(I),(YF2(J),J=1,6)
        WRITE(10,42)RM(I),(YF3(J),J=1,6)
        WRITE(11,42)RM(I),(YF4(J),J=1,6)
 122  CONTINUE
      GO TO 125
C Focus is in the mantle.
C Output solutions below focal depth.
 121  DO 126 I=1,IL1
        DO 127 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 128 K=1,6
            HOLD1=HOLD1+YYM(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYM(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYM(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYM(J,K,I)*UV4(K)
 128      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 127    CONTINUE
        SM(1,I)=YF1(1)
```

```
        SM(2,I)=YF1(3)
        SM(3,I)=YF2(1)
        SM(4,I)=YF2(3)
        SM(5,I)=YF3(1)
        SM(6,I)=YF3(3)
        SM(7,I)=YF4(1)
        SM(8,I)=YF4(3)
C Record radial displacement discontinuities at the core-mantle boundary.
        IF(I.EQ.1)DISC(1,2)=DISC(1,2)+YF1(1)
        IF(I.EQ.1)DISC(2,2)=DISC(2,2)+YF2(1)
        IF(I.EQ.1)DISC(3,2)=DISC(3,2)+YF3(1)
        IF(I.EQ.1)DISC(4,2)=DISC(4,2)+YF4(1)
        WRITE(8,42)RM(I), (YF1(J), J=1,6)
        WRITE(9,42)RM(I), (YF2(J), J=1,6)
        WRITE(10,42)RM(I),(YF3(J),J=1,6)
        WRITE(11,42)RM(I),(YF4(J),J=1,6)
 126  CONTINUE
C Augment vectors UV1, UV2, UV3, UV4.
      DO 129 I=1,6
        UV1(I)=UV1(I)+G1(I)
        UV2(I)=UV2(I)+G2(I)
        UV3(I)=UV3(I)+G3(I)
        UV4(I)=UV4(I)+G4(I)
 129  CONTINUE
C Output solutions at and above focal depth.
      IF(IEQ.EQ.1) GO TO 130
C Find solutions at focus.
      DO 131 I=1,6
        HOLD1=0.D0
        HOLD2=0.D0
        HOLD3=0.D0
        HOLD4=0.D0
        DO 133 J=1,6
          HOLD1=HOLD1+YINFDS(I,J)*UV1(J)
          HOLD2=HOLD2+YINFDS(I,J)*UV2(J)
          HOLD3=HOLD3+YINFDS(I,J)*UV3(J)
          HOLD4=HOLD4+YINFDS(I,J)*UV4(J)
 133    CONTINUE
        YF1(I)=HOLD1
        YF2(I)=HOLD2
        YF3(I)=HOLD3
        YF4(I)=HOLD4
 131  CONTINUE
      IST=IL1+1
      SM(1,IST)=YF1(1)
      SM(2,IST)=YF1(3)
      SM(3,IST)=YF2(1)
      SM(4,IST)=YF2(3)
      SM(5,IST)=YF3(1)
      SM(6,IST)=YF3(3)
      SM(7,IST)=YF4(1)
      SM(8,IST)=YF4(3)
      WRITE(8,42)RFD, (YF1(I), I=1,6)
      WRITE(9,42)RFD, (YF2(I), I=1,6)
      WRITE(10,42)RFD, (YF3(I),I=1,6)
      WRITE(11,42)RFD, (YF4(I),I=1,6)
C Modify vector RM to include RFD, modify vector RHOM to include RHOFD.
      DO 132 I=IST,MP1
        J=MP1+IST-I+1
        RM(J)=RM(J-1)
        RHOM(J)=RHOM(J-1)
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 132  CONTINUE
      RM(IST)=RFD
      RHOM(IST)=RHOFD
 130  IST=IL1+1
      DO 134 I=IST, MP1
        IP=I
        IF(IEQ.EQ.-1)IP=I+1
        DO 135 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 136 K=1,6
            HOLD1=HOLD1+YYM(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYM(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYM(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYM(J,K,I)*UV4(K)
 136      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 135    CONTINUE
        SM(1,IP)=YF1(1)
        SM(2,IP)=YF1(3)
        SM(3,IP)=YF2(1)
        SM(4,IP)=YF2(3)
        SM(5,IP)=YF3(1)
        SM(6,IP)=YF3(3)
        SM(7,IP)=YF4(1)
        SM(8,IP)=YF4(3)
        WRITE(8,42)RM(IP), (YF1(J),J=1,6)
        WRITE(9,42)RM(IP), (YF2(J),J=1,6)
        WRITE(10,42)RM(IP), (YF3(J),J=1,6)
        WRITE(11,42)RM(IP), (YF4(J),J=1,6)
 134  CONTINUE
 125  CONTINUE
C Find four source solutions in the crust.
      M=NI(4)
      MP1=M+1
C Switch for focus in the crust.
      IF(IFSW.EQ.-1) GO TO 137
      DO 138 I=1,MP1
        DO 139 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 140 K=1,6
            HOLD1=HOLD1+YYC(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYC(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYC(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYC(J,K,I)*UV4(K)
 140      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 139    CONTINUE
        SC(1,I)=YF1(1)
        SC(2,I)=YF1(3)
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      SC(3,I)=YF2(1)
      SC(4,I)=YF2(3)
      SC(5,I)=YF3(1)
      SC(6,I)=YF3(3)
      SC(7,I)=YF4(1)
      SC(8,I)=YF4(3)
      WRITE(8,42)RC(I), (YF1(J), J=1,6)
      WRITE(9,42)RC(I), (YF2(J), J=1,6)
      WRITE(10,42)RC(I),(YF3(J), J=1,6)
      WRITE(11,42)RC(I),(YF4(J), J=1,6)
 138  CONTINUE
      GO TO 141
C Focus is in the crust.
C Output solutions below focal depth.
 137  DO 142 I=1,IL1
        DO 143 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 144 K=1,6
            HOLD1=HOLD1+YYC(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYC(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYC(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYC(J,K,I)*UV4(K)
 144      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 143    CONTINUE
        SC(1,I)=YF1(1)
        SC(2,I)=YF1(3)
        SC(3,I)=YF2(1)
        SC(4,I)=YF2(3)
        SC(5,I)=YF3(1)
        SC(6,I)=YF3(3)
        SC(7,I)=YF4(1)
        SC(8,I)=YF4(3)
        WRITE(8,42) RC(I), (YF1(J), J=1,6)
        WRITE(9,42) RC(I), (YF2(J), J=1,6)
        WRITE(10,42)RC(I),(YF3(J), J=1,6)
        WRITE(11,42)RC(I),(YF4(J), J=1,6)
 142  CONTINUE
C Augment vectors UV1, UV2, UV3, UV4.
      DO 145 I=1,6
        UV1(I)=UV1(I)+G1(I)
        UV2(I)=UV2(I)+G2(I)
        UV3(I)=UV3(I)+G3(I)
        UV4(I)=UV4(I)+G4(I)
 145  CONTINUE
C Output solutions at and above focal depth.
      IF(IEQ.EQ.1) GO TO 146
C Find solutions at the focus.
      DO 147 I=1,6
        HOLD1=0.D0
        HOLD2=0.D0
        HOLD3=0.D0
        HOLD4=0.D0
        DO 149 J=1,6
          HOLD1=HOLD1+YINFDS(I,J)*UV1(J)
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          HOLD2=HOLD2+YINFDS(I,J)*UV2(J)
          HOLD3=HOLD3+YINFDS(I,J)*UV3(J)
          HOLD4=HOLD4+YINFDS(I,J)*UV4(J)
 149    CONTINUE
        YF1(I)=HOLD1
        YF2(I)=HOLD2
        YF3(I)=HOLD3
        YF4(I)=HOLD4
 147  CONTINUE
      IST=IL1+1
      SC(1,IST)=YF1(1)
      SC(2,IST)=YF1(3)
      SC(3,IST)=YF2(1)
      SC(4,IST)=YF2(3)
      SC(5,IST)=YF3(1)
      SC(6,IST)=YF3(3)
      SC(7,IST)=YF4(1)
      SC(8,IST)=YF4(3)
      WRITE(8,42)RFD, (YF1(I), I=1,6)
      WRITE(9,42)RFD, (YF2(I), I=1,6)
      WRITE(10,42)RFD,(YF3(I), I=1,6)
      WRITE(11,42)RFD,(YF4(I), I=1,6)
C Modify vector RC to include RFD, modify vector RHOC to include RHOFD.
      DO 148 I=IST,MP1
        J=MP1+IST-I+1
        RC(J)=RC(J-1)
        RHOC(J)=RHOC(J-1)
 148  CONTINUE
      RC(IST)=RFD
      RHOC(IST)=RHOFD
 146  IST=IL1+1
      DO 150 I=IST,MP1
        IP=I
        IF(IEQ.EQ.-1)IP=I+1
        DO 151 J=1,6
          HOLD1=0.D0
          HOLD2=0.D0
          HOLD3=0.D0
          HOLD4=0.D0
          DO 152 K=1,6
            HOLD1=HOLD1+YYC(J,K,I)*UV1(K)
            HOLD2=HOLD2+YYC(J,K,I)*UV2(K)
            HOLD3=HOLD3+YYC(J,K,I)*UV3(K)
            HOLD4=HOLD4+YYC(J,K,I)*UV4(K)
 152      CONTINUE
          YF1(J)=HOLD1
          YF2(J)=HOLD2
          YF3(J)=HOLD3
          YF4(J)=HOLD4
 151    CONTINUE
        SC(1,IP)=YF1(1)
        SC(2,IP)=YF1(3)
        SC(3,IP)=YF2(1)
        SC(4,IP)=YF2(3)
        SC(5,IP)=YF3(1)
        SC(6,IP)=YF3(3)
        SC(7,IP)=YF4(1)
        SC(8,IP)=YF4(3)
        WRITE(8,42)RC(IP), (YF1(J), J=1,6)
        WRITE(9,42)RC(IP), (YF2(J), J=1,6)
        WRITE(10,42)RC(IP),(YF3(J), J=1,6)
```

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        WRITE(11,42)RC(IP),(YF4(J), J=1,6)
 150  CONTINUE
 141  CONTINUE
C Begin integrations of the radial and transverse components of the
C four fundamental solutions over radius.
C Initialize integrals.
      S1IU=0.D0
      S1IV=0.D0
      S2IU=0.D0
      S2IV=0.D0
      S3IU=0.D0
      S3IV=0.D0
      S4IU=0.D0
      S4IV=0.D0
C Begin integration through the inner core using the trapezoidal rule.
C Set number of segments.
      IRADM1=IRAD-1
      DO 153 I=1,IRADM1
        IP1=I+1
C Change radii to metres.
        RAD(I)=RAD(I)*1.D3
        RAD(IP1)=RAD(IP1)*1.D3
C Find cubes of radii.
        RC1=RAD(I)*RAD(I)*RAD(I)
        RC2=RAD(IP1)*RAD(IP1)*RAD(IP1)
C Find increment in radius for segment.
        DR=RAD(IP1)-RAD(I)
C Find densities at segment boundaries.
        RHO1=RHOIC(I)
        RHO2=RHOIC(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RC1*RHO1*SIC(1,I)+RC2*RHO2*SIC(1,IP1))/2.D0
        S1IV=S1IV+DR*(RC1*RHO1*SIC(2,I)+RC2*RHO2*SIC(2,IP1))/2.D0
        S2IU=S2IU+DR*(RC1*RHO1*SIC(3,I)+RC2*RHO2*SIC(3,IP1))/2.D0
        S2IV=S2IV+DR*(RC1*RHO1*SIC(4,I)+RC2*RHO2*SIC(4,IP1))/2.D0
        S3IU=S3IU+DR*(RC1*RHO1*SIC(5,I)+RC2*RHO2*SIC(5,IP1))/2.D0
        S3IV=S3IV+DR*(RC1*RHO1*SIC(6,I)+RC2*RHO2*SIC(6,IP1))/2.D0
        S4IU=S4IU+DR*(RC1*RHO1*SIC(7,I)+RC2*RHO2*SIC(7,IP1))/2.D0
        S4IV=S4IV+DR*(RC1*RHO1*SIC(8,I)+RC2*RHO2*SIC(8,IP1))/2.D0
 153  CONTINUE
C Store results for inner core.
      RES(1,1)=S1IU
      RES(2,1)=S1IV
      RES(3,1)=S2IU
      RES(4,1)=S2IV
      RES(5,1)=S3IU
      RES(6,1)=S3IV
      RES(7,1)=S4IU
      RES(8,1)=S4IV
C Begin integration through the outer core using extended Simpson's rule.
C Find number of model points.
      N1=NM(2)
C Find stepsize.
      M=NI(2)
      AM=DFLOAT(M)
      H=(R(N1)-R(1))/AM
      MP1=M+1
C Establish vector of functions, VOC, at each segment border, and vector,
C WOC, of weights.
      DO 154 I=1,MP1
C Convert radius to metres.
```

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        CUR=ROC(I)*1.D3
C Cube radius.
        CUR=CUR*CUR*CUR
C Construct VOC.
        VOC(1,I)=H*(CUR*RHOOC(I)*SOC(1,I))/3.D0
        VOC(2,I)=H*(CUR*RHOOC(I)*SOC(2,I))/3.D0
        VOC(3,I)=H*(CUR*RHOOC(I)*SOC(3,I))/3.D0
        VOC(4,I)=H*(CUR*RHOOC(I)*SOC(4,I))/3.D0
        VOC(5,I)=H*(CUR*RHOOC(I)*SOC(5,I))/3.D0
        VOC(6,I)=H*(CUR*RHOOC(I)*SOC(6,I))/3.D0
        VOC(7,I)=H*(CUR*RHOOC(I)*SOC(7,I))/3.D0
        VOC(8,I)=H*(CUR*RHOOC(I)*SOC(8,I))/3.D0
 154  CONTINUE
C Construct WOC.
      WOC(1)=1.D0
      WOC(2)=4.D0
      WOC(3)=2.D0
      WOC(4)=4.D0
      WOC(5)=2.D0
      WOC(6)=4.D0
      WOC(7)=2.D0
      WOC(8)=4.D0
      WOC(9)=2.D0
      WOC(10)=4.D0
      WOC(11)=1.D0
C Find integrals as the scalar products of the vectors WOC and VOC.
      HOLD1=0.D0
      HOLD2=0.D0
      HOLD3=0.D0
      HOLD4=0.D0
      HOLD5=0.D0
      HOLD6=0.D0
      HOLD7=0.D0
      HOLD8=0.D0
      DO 155 I=1,MP1
        HOLD1=HOLD1+WOC(I)*VOC(1,I)
        HOLD2=HOLD2+WOC(I)*VOC(2,I)
        HOLD3=HOLD3+WOC(I)*VOC(3,I)
        HOLD4=HOLD4+WOC(I)*VOC(4,I)
        HOLD5=HOLD5+WOC(I)*VOC(5,I)
        HOLD6=HOLD6+WOC(I)*VOC(6,I)
        HOLD7=HOLD7+WOC(I)*VOC(7,I)
        HOLD8=HOLD8+WOC(I)*VOC(8,I)
 155  CONTINUE
C Update integrals.
      S1IU=S1IU+HOLD1
      S1IV=S1IV+HOLD2
      S2IU=S2IU+HOLD3
      S2IV=S2IV+HOLD4
      S3IU=S3IU+HOLD5
      S3IV=S3IV+HOLD6
      S4IU=S4IU+HOLD7
      S4IV=S4IV+HOLD8
C Store results for outer core.
      RES(1,2)=S1IU-RES(1,1)
      RES(2,2)=S1IV-RES(2,1)
      RES(3,2)=S2IU-RES(3,1)
      RES(4,2)=S2IV-RES(4,1)
      RES(5,2)=S3IU-RES(5,1)
      RES(6,2)=S3IV-RES(6,1)
      RES(7,2)=S4IU-RES(7,1)
```

```
      RES(8,2)=S4IV-RES(8,1)
C Begin integration through the mantle using the trapezoidal rule.
C Find number of segments M.
      M=NI(3)
C Switch for focus in the mantle.
      IF(IFSW.EQ.1) GO TO 156
      DO 157 I=1,M
        IP1=I+1
C Change radii to metres.
        RM(I)=RM(I)*1.D3
        RM(IP1)=RM(IP1)*1.D3
C Find cubes of radii.
        RCM1=RM(I)*RM(I)*RM(I)
        RCM2=RM(IP1)*RM(IP1)*RM(IP1)
C Find increment in radius for segment.
        DR=RM(IP1)-RM(I)
C Find densities at segment boundaries.
        RHO1=RHOM(I)
        RHO2=RHOM(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCM1*RHO1*SM(1,I)+RCM2*RHO2*SM(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCM1*RHO1*SM(2,I)+RCM2*RHO2*SM(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCM1*RHO1*SM(3,I)+RCM2*RHO2*SM(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCM1*RHO1*SM(4,I)+RCM2*RHO2*SM(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCM1*RHO1*SM(5,I)+RCM2*RHO2*SM(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCM1*RHO1*SM(6,I)+RCM2*RHO2*SM(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCM1*RHO1*SM(7,I)+RCM2*RHO2*SM(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCM1*RHO1*SM(8,I)+RCM2*RHO2*SM(8,IP1))/2.D0
 157  CONTINUE
      GO TO 158
C Focus is in the mantle.
C Add contributions of segments to two below focal depth to integrals.
 156  IL2=IL1-1
      DO 159 I=1,IL2
        IP1=I+1
C Change radii to metres.
        RM(I)=RM(I)*1.D3
        RM(IP1)=RM(IP1)*1.D3
C Find cubes of radii.
        RCM1=RM(I)*RM(I)*RM(I)
        RCM2=RM(IP1)*RM(IP1)*RM(IP1)
C Find increment in radius for segment.
        DR=RM(IP1)-RM(I)
C Find densities at segment boundaries.
        RHO1=RHOM(I)
        RHO2=RHOM(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCM1*RHO1*SM(1,I)+RCM2*RHO2*SM(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCM1*RHO1*SM(2,I)+RCM2*RHO2*SM(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCM1*RHO1*SM(3,I)+RCM2*RHO2*SM(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCM1*RHO1*SM(4,I)+RCM2*RHO2*SM(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCM1*RHO1*SM(5,I)+RCM2*RHO2*SM(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCM1*RHO1*SM(6,I)+RCM2*RHO2*SM(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCM1*RHO1*SM(7,I)+RCM2*RHO2*SM(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCM1*RHO1*SM(8,I)+RCM2*RHO2*SM(8,IP1))/2.D0
 159  CONTINUE
C Add contributions at and above focal depth.
C Do linear extrapolation of sources to focus.
      IST=IL1+1
      DO 161 I=1,8
        SX(I)=SM(I,IL2)+(SM(I,IL1)-SM(I,IL2))*(RM(IST)-RM(IL2))/
```

```
     1 (RM(IL1)-RM(IL2))
 161  CONTINUE
C Add contribution of segment before focus.
C Change radii to metres.
      RM(IL1)=RM(IL1)*1.D3
      RM(IST)=RM(IST)*1.D3
C Find cubes of radii.
      RCM1=RM(IL1)*RM(IL1)*RM(IL1)
      RCM2=RM(IST)*RM(IST)*RM(IST)
C Find increment in radius for segment.
      DR=RM(IST)-RM(IL1)
C Find densities at segment boundaries.
      RHO1=RHOM(IL1)
      RHO2=RHOM(IST)
C Update contributions.
      S1IU=S1IU+DR*(RCM1*RHO1*SM(1,IL1)+RCM2*RHO2*SX(1))/2.D0
      S1IV=S1IV+DR*(RCM1*RHO1*SM(2,IL1)+RCM2*RHO2*SX(2))/2.D0
      S2IU=S2IU+DR*(RCM1*RHO1*SM(3,IL1)+RCM2*RHO2*SX(3))/2.D0
      S2IV=S2IV+DR*(RCM1*RHO1*SM(4,IL1)+RCM2*RHO2*SX(4))/2.D0
      S3IU=S3IU+DR*(RCM1*RHO1*SM(5,IL1)+RCM2*RHO2*SX(5))/2.D0
      S3IV=S3IV+DR*(RCM1*RHO1*SM(6,IL1)+RCM2*RHO2*SX(6))/2.D0
      S4IU=S4IV+DR*(RCM1*RHO1*SM(7,IL1)+RCM2*RHO2*SX(7))/2.D0
      S4IV=S4IV+DR*(RCM1*RHO1*SM(8,IL1)+RCM2*RHO2*SX(8))/2.D0
C Add contributions at focus and above.
      MP1=M+1
      IFIN=MP1
      IF(IEQ.EQ.1) IFIN=M
      DO 160 I=IST,IFIN
        IP1=I+1
C Change radii to metres.
        RM(I)=RM(I)*1.D3
        RM(IP1)=RM(IP1)*1.D3
C Find cubes of radii.
        RCM1=RM(I)*RM(I)*RM(I)
        RCM2=RM(IP1)*RM(IP1)*RM(IP1)
C Find increment in radius for segment.
        DR=RM(IP1)-RM(I)
C Find densities at segment boundaries.
        RHO1=RHOM(I)
        RHO2=RHOM(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCM1*RHO1*SM(1,I)+RCM2*RHO2*SM(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCM1*RHO1*SM(2,I)+RCM2*RHO2*SM(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCM1*RHO1*SM(3,I)+RCM2*RHO2*SM(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCM1*RHO1*SM(4,I)+RCM2*RHO2*SM(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCM1*RHO1*SM(5,I)+RCM2*RHO2*SM(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCM1*RHO1*SM(6,I)+RCM2*RHO2*SM(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCM1*RHO1*SM(7,I)+RCM2*RHO2*SM(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCM1*RHO1*SM(8,I)+RCM2*RHO2*SM(8,IP1))/2.D0
 160  CONTINUE
 158  CONTINUE
C Store results for mantle.
      RES(1,3)=S1IU-RES(1,2)-RES(1,1)
      RES(2,3)=S1IV-RES(2,2)-RES(2,1)
      RES(3,3)=S2IU-RES(3,2)-RES(3,1)
      RES(4,3)=S2IV-RES(4,2)-RES(4,1)
      RES(5,3)=S3IU-RES(5,2)-RES(5,1)
      RES(6,3)=S3IV-RES(6,2)-RES(6,1)
      RES(7,3)=S4IU-RES(7,2)-RES(7,1)
      RES(8,3)=S4IV-RES(8,2)-RES(8,1)
C Begin integration through the crust using the trapezoidal rule.
```

```
C Find number of segments M.
      M=NI(4)
C Switch for focus in the crust.
      IF(IFSW.EQ.-1) GO TO 162
      DO 163 I=1,M
        IP1=I+1
C Change radii to metres.
        RC(I)=RC(I)*1.D3
        RC(IP1)=RC(IP1)*1.D3
C Find cubes of radii.
        RCC1=RC(I)*RC(I)*RC(I)
        RCC2=RC(IP1)*RC(IP1)*RC(IP1)
C Find increment in radius for segment.
        DR=RC(IP1)-RC(I)
C Find densities at segment boundaries.
        RHO1=RHOC(I)
        RHO2=RHOC(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCC1*RHO1*SC(1,I)+RCC2*RHO2*SC(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCC1*RHO1*SC(2,I)+RCC2*RHO2*SC(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCC1*RHO1*SC(3,I)+RCC2*RHO2*SC(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCC1*RHO1*SC(4,I)+RCC2*RHO2*SC(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCC1*RHO1*SC(5,I)+RCC2*RHO2*SC(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCC1*RHO1*SC(6,I)+RCC2*RHO2*SC(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCC1*RHO1*SC(7,I)+RCC2*RHO2*SC(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCC1*RHO1*SC(8,I)+RCC2*RHO2*SC(8,IP1))/2.D0
 163  CONTINUE
      GO TO 164
C Focus is in the crust.
C Add contributions of segments to two below focal depth to integrals.
 162  IL2=IL1-1
      DO 165 I=1,IL2
        IP1=I+1
C Change radii to metres.
        RC(I)=RC(I)*1.D3
        RC(IP1)=RC(IP1)*1.D3
C Find cubes of radii.
        RCC1=RC(I)*RC(I)*RC(I)
        RCC2=RC(IP1)*RC(IP1)*RC(IP1)
C Find increment in radius for segment.
        DR=RC(IP1)-RC(I)
C Find densities at segment boundaries.
        RHO1=RHOC(I)
        RHO2=RHOC(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCC1*RHO1*SC(1,I)+RCC2*RHO2*SC(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCC1*RHO1*SC(2,I)+RCC2*RHO2*SC(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCC1*RHO1*SC(3,I)+RCC2*RHO2*SC(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCC1*RHO1*SC(4,I)+RCC2*RHO2*SC(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCC1*RHO1*SC(5,I)+RCC2*RHO2*SC(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCC1*RHO1*SC(6,I)+RCC2*RHO2*SC(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCC1*RHO1*SC(7,I)+RCC2*RHO2*SC(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCC1*RHO1*SC(8,I)+RCC2*RHO2*SC(8,IP1))/2.D0
 165  CONTINUE
C Add contributions at and above focal depth.
C Do linear extrapolation of sources to focus.
      IST=IL1+1
      DO 166 I=1,8
        SX(I)=SC(I,IL2)+(SC(I,IL1)-SC(I,IL2))*(RC(IST)-RC(IL2))/
     1 (RC(IL1)-RC(IL2))
 166  CONTINUE
```

```
C Add contribution of segment before focus.
C Change radii to metres.
      RC(IL1)=RC(IL1)*1.D3
      RC(IST)=RC(IST)*1.D3
C Find cubes of radii.
      RCC1=RC(IL1)*RC(IL1)*RC(IL1)
      RCC2=RC(IST)*RC(IST)*RC(IST)
C Find increment in radius for segment.
      DR=RC(IST)-RC(IL1)
C Find densities at segment boundaries.
      RHO1=RHOC(IL1)
      RHO1=RHOC(IST)
C Update contributions.
      S1IU=S1IU+DR*(RCC1*RHO1*SC(1,IL1)+RCC2*RHO2*SX(1))/2.D0
      S1IV=S1IV+DR*(RCC1*RHO1*SC(2,IL1)+RCC2*RHO2*SX(2))/2.D0
      S2IU=S2IU+DR*(RCC1*RHO1*SC(3,IL1)+RCC2*RHO2*SX(3))/2.D0
      S2IV=S2IV+DR*(RCC1*RHO1*SC(4,IL1)+RCC2*RHO2*SX(4))/2.D0
      S3IU=S3IU+DR*(RCC1*RHO1*SC(5,IL1)+RCC2*RHO2*SX(5))/2.D0
      S3IV=S3IV+DR*(RCC1*RHO1*SC(6,IL1)+RCC2*RHO2*SX(6))/2.D0
      S4IU=S4IU+DR*(RCC1*RHO1*SC(7,IL1)+RCC2*RHO2*SX(7))/2.D0
      S4IV=S4IV+DR*(RCC1*RHO1*SC(8,IL1)+RCC2*RHO2*SX(8))/2.D0
C Add contributions at focus and above.
      MP1=M+1
      IFIN=MP1
      IF(IEQ.EQ.1) IFIN=M
      DO 167 I=IST,IFIN
        IP1=I+1
C Change radii to metres.
        RC(I)=RC(I)*1.D3
        RC(IP1)=RC(IP1)*1.D3
C Find cubes of radii.
        RCC1=RC(I)*RC(I)*RC(I)
        RCC2=RC(IP1)*RC(IP1)*RC(IP1)
C Find increment in radius for segment.
        DR=RC(IP1)-RC(I)
C Find densities at segment boundaries.
        RHO1=RHOC(I)
        RHO2=RHOC(IP1)
C Add contributions of segment to integrals.
        S1IU=S1IU+DR*(RCC1*RHO1*SC(1,I)+RCC2*RHO2*SC(1,IP1))/2.D0
        S1IV=S1IV+DR*(RCC1*RHO1*SC(2,I)+RCC2*RHO2*SC(2,IP1))/2.D0
        S2IU=S2IU+DR*(RCC1*RHO1*SC(3,I)+RCC2*RHO2*SC(3,IP1))/2.D0
        S2IV=S2IV+DR*(RCC1*RHO1*SC(4,I)+RCC2*RHO2*SC(4,IP1))/2.D0
        S3IU=S3IU+DR*(RCC1*RHO1*SC(5,I)+RCC2*RHO2*SC(5,IP1))/2.D0
        S3IV=S3IV+DR*(RCC1*RHO1*SC(6,I)+RCC2*RHO2*SC(6,IP1))/2.D0
        S4IU=S4IU+DR*(RCC1*RHO1*SC(7,I)+RCC2*RHO2*SC(7,IP1))/2.D0
        S4IV=S4IV+DR*(RCC1*RHO1*SC(8,I)+RCC2*RHO2*SC(8,IP1))/2.D0
 167  CONTINUE
 164  CONTINUE
C Store results for crust.
      RES(1,4)=S1IU-RES(1,3)-RES(1,2)-RES(1,1)
      RES(2,4)=S1IV-RES(2,3)-RES(2,2)-RES(2,1)
      RES(3,4)=S2IU-RES(3,3)-RES(3,2)-RES(3,1)
      RES(4,4)=S2IV-RES(4,3)-RES(4,2)-RES(4,1)
      RES(5,4)=S3IU-RES(5,3)-RES(5,2)-RES(5,1)
      RES(6,4)=S3IV-RES(6,3)-RES(6,2)-RES(6,1)
      RES(7,4)=S4IU-RES(7,3)-RES(7,2)-RES(7,1)
      RES(8,4)=S4IV-RES(8,3)-RES(8,2)-RES(8,1)
C Store results for whole Earth.
      RES(1,5)=S1IU
      RES(2,5)=S1IV
```

```
      RES(3,5)=S2IU
      RES(4,5)=S2IV
      RES(5,5)=S3IU
      RES(6,5)=S3IV
      RES(7,5)=S4IU
      RES(8,5)=S4IV
C Write out results to screen.
      WRITE(6,168) FD
 168  FORMAT(/5X,'Results for focal depth =',1X,F5.1,1X,'km.')
      WRITE(6,169)
 169  FORMAT(/11X,'INTEGRALS OVER RADIUS')
      WRITE(6,170)
 170  FORMAT(/1X,'source',5X,'integral over u',5X,'integral over v',5X,
     1 'region')
      WRITE(6,171)RES(1,1),RES(2,1)
 171  FORMAT(3X,'1'/14X,D11.4,9X,D11.4,7X,'inner core')
      WRITE(6,172)RES(1,2),RES(2,2)
 172  FORMAT(14X,D11.4,9X,D11.4,7X,'outer core')
      WRITE(6,173)RES(1,3),RES(2,3)
 173  FORMAT(14X,D11.4,9X,D11.4,7X,'mantle')
      WRITE(6,174)RES(1,4),RES(2,4)
 174  FORMAT(14X,D11.4,9X,D11.4,7X,'crust')
      WRITE(6,175)RES(1,5),RES(2,5)
 175  FORMAT(14X,D11.4,9X,D11.4,7X,'whole Earth')
      WRITE(6,176)RES(3,1),RES(4,1)
 176  FORMAT(3X,'2'/14X,D11.4,9X,D11.4,7X,'inner core')
      WRITE(6,177)RES(3,2),RES(4,2)
 177  FORMAT(14X,D11.4,9X,D11.4,7X,'outer core')
      WRITE(6,178)RES(3,3),RES(4,3)
 178  FORMAT(14X,D11.4,9X,D11.4,7X,'mantle')
      WRITE(6,179)RES(3,4),RES(4,4)
 179  FORMAT(14X,D11.4,9X,D11.4,7X,'crust')
      WRITE(6,180)RES(3,5),RES(4,5)
 180  FORMAT(14X,D11.4,9X,D11.4,7X,'whole Earth')
      WRITE(6,181)RES(5,1),RES(6,1)
 181  FORMAT(3X,'3'/14X,D11.4,9X,D11.4,7X,'inner core')
      WRITE(6,182)RES(5,2),RES(6,2)
 182  FORMAT(14X,D11.4,9X,D11.4,7X,'outer core')
      WRITE(6,183)RES(5,3),RES(6,3)
 183  FORMAT(14X,D11.4,9X,D11.4,7X,'mantle')
      WRITE(6,184)RES(5,4),RES(6,4)
 184  FORMAT(14X,D11.4,9X,D11.4,7X,'crust')
      WRITE(6,185)RES(5,5),RES(6,5)
 185  FORMAT(14X,D11.4,9X,D11.4,7X,'whole Earth')
      WRITE(6,186)RES(7,1),RES(8,1)
 186  FORMAT(3X,'4'/14X,D11.4,9X,D11.4,7X,'inner core')
      WRITE(6,187)RES(7,2),RES(8,2)
 187  FORMAT(14X,D11.4,9X,D11.4,7X,'outer core')
      WRITE(6,188)RES(7,3),RES(8,3)
 188  FORMAT(14X,D11.4,9X,D11.4,7X,'mantle')
      WRITE(6,189)RES(7,4),RES(8,4)
 189  FORMAT(14X,D11.4,9X,D11.4,7X,'crust')
      WRITE(6,190)RES(7,5),RES(8,5)
 190  FORMAT(14X,D11.4,9X,D11.4,7X,'whole Earth')
C Write out discontinuities in radial displacements.
      WRITE(6,191)
 191  FORMAT(/11X,'DISCONTINUITIES IN RADIAL DISPLACEMENTS')
      WRITE(6,192)
 192  FORMAT(/1X,'source',5X,'discontinuity at ICB',5X,'discontinuity at
     1 CMB')
      WRITE(6,193)DISC(1,1),DISC(1,2)
```

```
 193  FORMAT(3X,'1',/14X,D11.4,14X,D11.4)
      WRITE(6,194)DISC(2,1),DISC(2,2)
 194  FORMAT(3X,'2',/14X,D11.4,14X,D11.4)
      WRITE(6,195)DISC(3,1),DISC(3,2)
 195  FORMAT(3X,'3',/14X,D11.4,14X,D11.4)
      WRITE(6,196)DISC(4,1),DISC(4,2)
 196  FORMAT(3X,'4',/14X,D11.4,14X,D11.4)
C Write out results to file.
      WRITE(12,197) FD,MUFD
 197  FORMAT(1X,2D15.8)
C Write out integrals over radius.
      DO 198 I=1,4
        I2=2*I
        I2M1=I2-1
        WRITE(12,199)(RES(I2M1,J),RES(I2,J),J=1,5)
 199    FORMAT(1X,2D20.8)
 198  CONTINUE
C Write out discontinuities in radial displacements at the ICB and CMB.
      DO 200 I=1,4
        WRITE(12,199)DISC(I,1),DISC(I,2)
 200  CONTINUE
      END
```

The programme SOURCES uses the subroutines SPMAT and INTPL to interpolate the Earth model, as described in Section 1.6. Power series expansions of the free solutions, regular at the geocentre, are performed with the assistance of the double precision function subprogrammes, P1, P2, Q1 and Q2, as well as the subroutine MATRIX, and the subroutine LINSOL, described in Section 1.5. The subroutine REL tracks the relative error in both the power series expansions and the Runge-Kutta integrations. Derivatives of the propagator matrix, required for the Runge-Kutta integrations in the inner core, mantle and crust, are calculated by the subroutine YPRIME, described in Section 3.6, and the Runge-Kutta integration there itself is performed by the subroutine RK4.

In addition, new subroutines YPRIMEOC and RK4OC, are used for integrations in the fluid outer core. They are presented below.

```
      SUBROUTINE YPRIMEOC(R,Y,A,YP,N1,C,RM,RHOM,GZEROM,PI,G,WES)
C This subroutine finds the derivatives, YP, of the two fundamental solutions in
C the fluid outer core.
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION RM(100),RHOM(100),GZEROM(100),C(100,100),
     1 Y(6,6),YP(6,6),A(6,6)
C Zero fill derivative matrix YP(I,J) and coefficient matrix A(I,J).
      DO 10 I=1,6
        DO 11 J=1,6
          YP(I,J)=0.D0
          A(I,J)=0.D0
 11     CONTINUE
 10   CONTINUE
C Set maximum number of points for interpolation.
      M1=100
C Interpolate Earth properties at radius R.
      CALL INTPL(R,RHO,N1,C,RM,RHOM,M1)
      CALL INTPL(R,GZERO,N1,C,RM,GZEROM,M1)
```

```
C Construct matrix A.
      A(1,1)=4.D0*PI*G*RHO/GZERO
      A(1,2)=1.D0
      A(2,1)=-16.D0*PI*G*RHO*(1.D0+R*WES/(2.D0*GZERO))/(GZERO*R)
     1 +6.D0/(R*R)
      A(2,2)=-4.D0*PI*G*RHO/GZERO-2.D0/R
C Multiply A into Y to find derivative matrix YP.
      DO 12 I=1,2
        DO 13 J=1,2
          DO 14 K=1,2
            YP(I,J)=YP(I,J)+A(I,K)*Y(K,J)
 14       CONTINUE
 13     CONTINUE
 12   CONTINUE
      RETURN
      END


      SUBROUTINE RK4OC(R,Y,A,K1,N1,C,RM,RHOM,GZEROM,PI,G,H,WES)
C This subroutine completes the last three steps of a fourth-order
C Runge-Kutta integration, given the derivatives K1(6,6) at the starting
C radius R.
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION RM(100),RHOM(100),GZEROM(100),C(100,100),
     1 Y(6,6),YY(6,6),A(6,6)
      DOUBLE PRECISION K1(6,6),K2(6,6),K3(6,6),K4(6,6)
C Set half stepsize, one-sixth stepsize.
      HH=0.5D0*H
      H6=H/6.D0
C Complete remaining steps of Runge-Kutta integration.
      DO 10 I=1,2
        DO 11 J=1,2
          YY(I,J)=Y(I,J)+HH*K1(I,J)
 11     CONTINUE
 10   CONTINUE
C Increment radius and solve for new derivatives.
      R=R+HH
      CALL YPRIMEOC(R,YY,A,K2,N1,C,RM,RHOM,GZEROM,PI,G,WES)
      DO 12 I=1,2
        DO 13 J=1,2
          YY(I,J)=Y(I,J)+HH*K2(I,J)
 13     CONTINUE
 12   CONTINUE
C Find new derivatives.
      CALL YPRIMEOC(R,YY,A,K3,N1,C,RM,RHOM,GZEROM,PI,G,WES)
      DO 14 I=1,2
        DO 15 J=1,2
          YY(I,J)=Y(I,J)+H*K3(I,J)
 15     CONTINUE
 14   CONTINUE
C Increment radius and solve for new derivatives.
      R=R+HH
      CALL YPRIMEOC(R,YY,A,K4,N1,C,RM,RHOM,GZEROM,PI,G,WES)
      DO 16 I=1,2
        DO 17 J=1,2
          Y(I,J)=Y(I,J)+H6*(K1(I,J)+2.D0*(K2(I,J)+K3(I,J))+K4(I,J))
 17     CONTINUE
 16   CONTINUE
      RETURN
      END
```

The four separate solutions of the inhomogeneous sixth-order spheroidal system with singular sources at the focus are given by expressions (9.96) through (9.99) and are illustrated in Figures (9.8) through (9.11) for focal depth 0.1*d*, with *d* the Earth's radius.

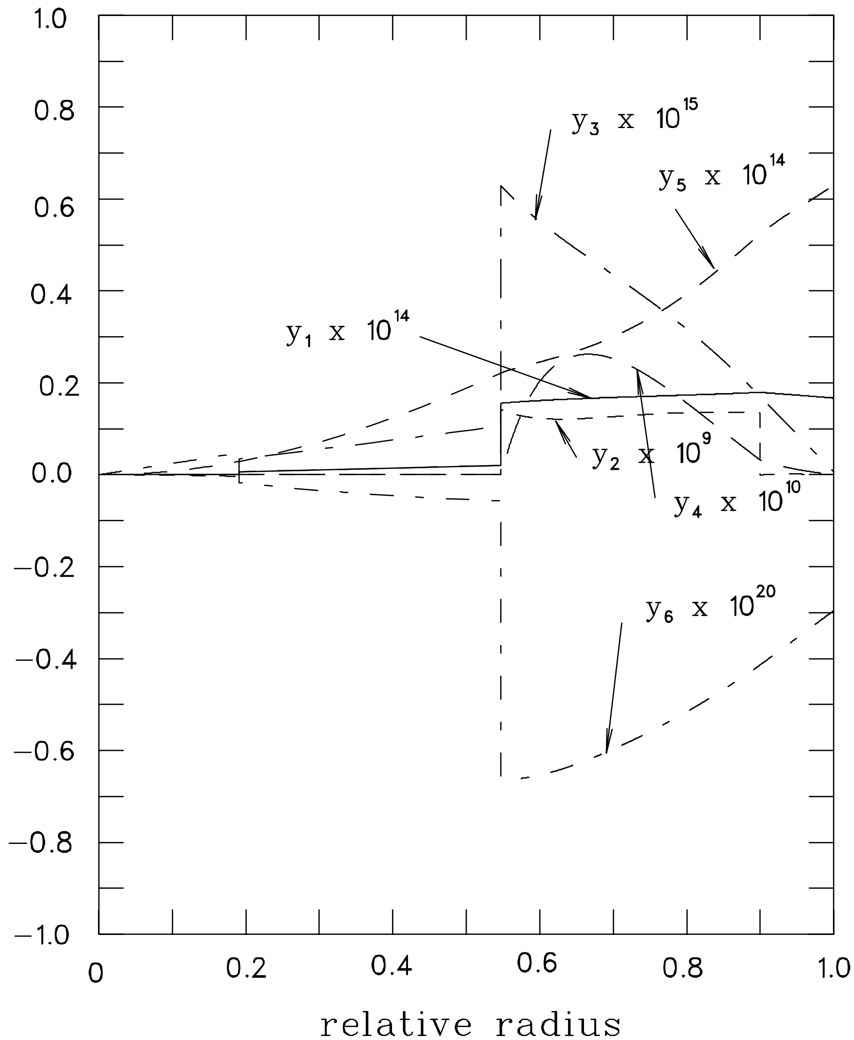


Figure 9.8 Source1 solution for singular source (9.96) at the focus.

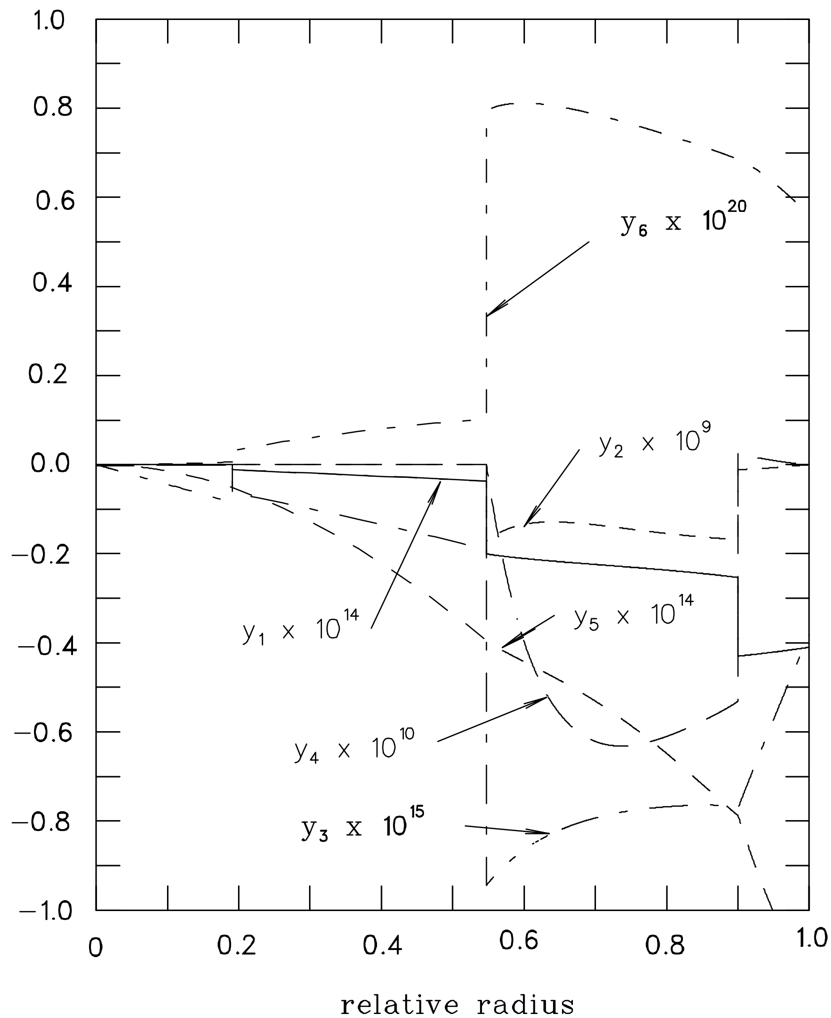


Figure 9.9 Source2 solution for singular source (9.97) at the focus.

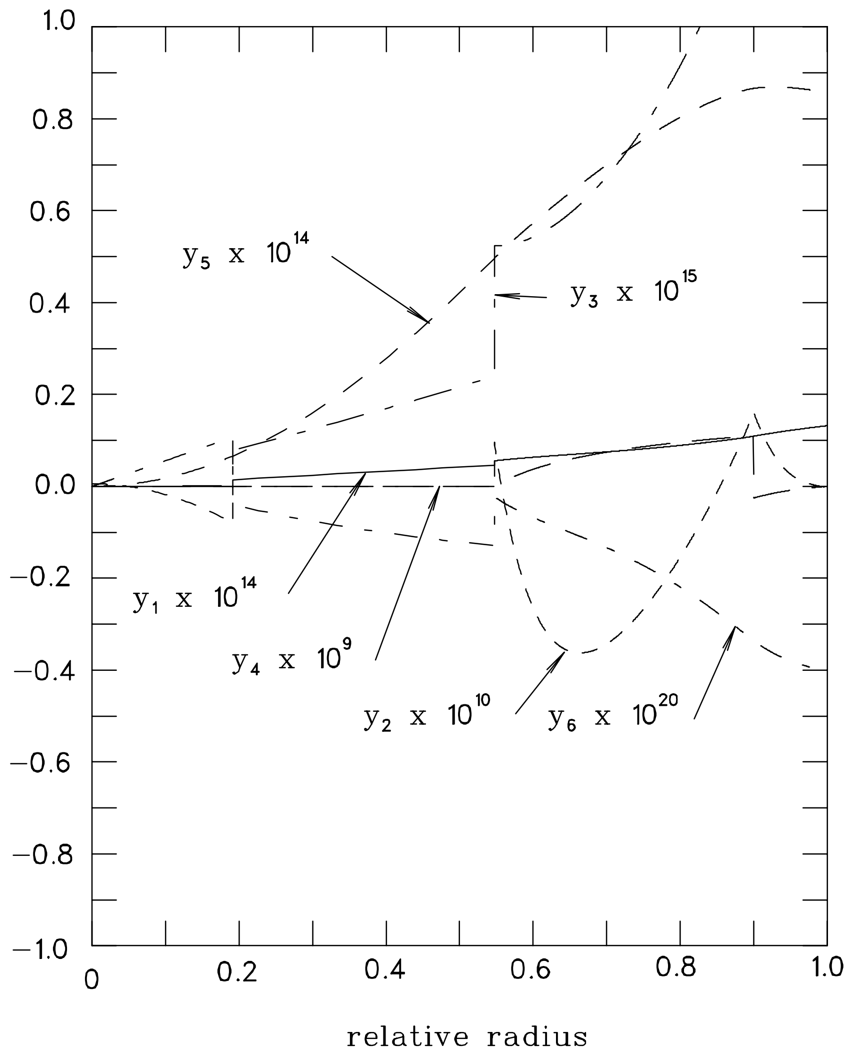


Figure 9.10 Source3 solution for singular source (9.98) at the focus.

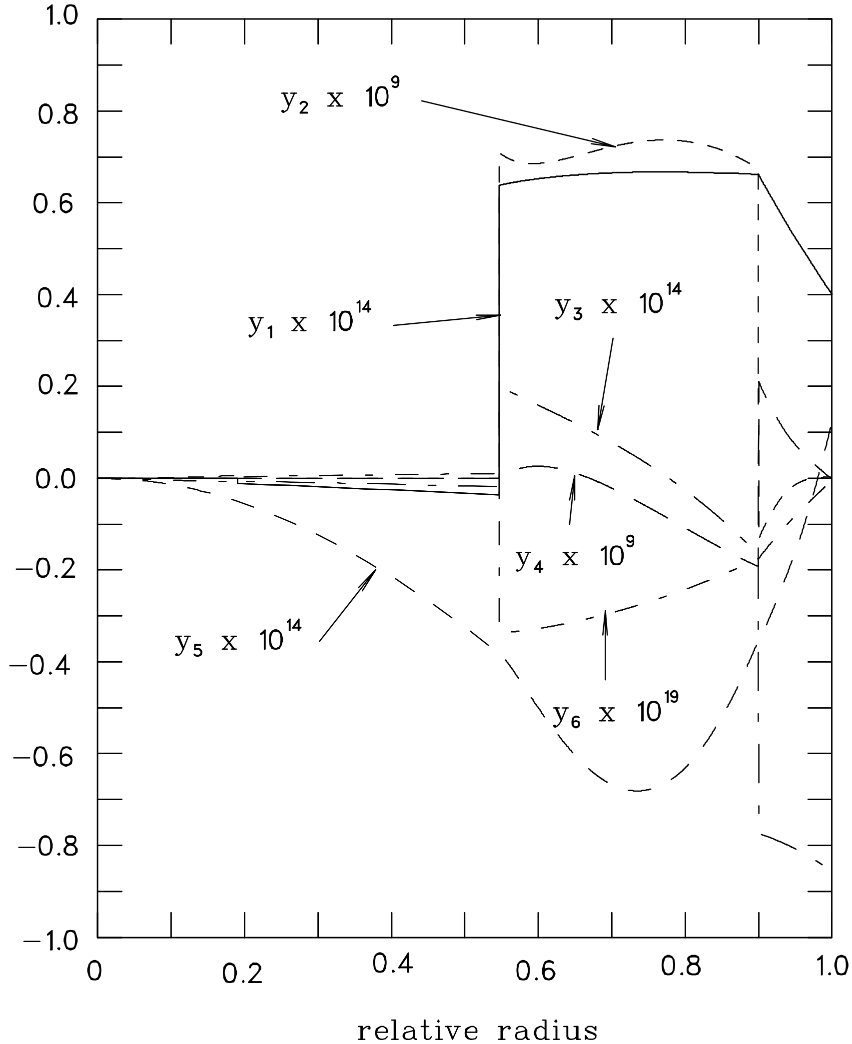


Figure 9.11 Source4 solution for singular source (9.99) at the focus.

The displacement fields are given by Volterra's formula as the integral over the fault surface of the product of the double force densities, the slip and the local modulus of rigidity. With focus at radius $r_0$, the four singular sources at the focus may be labelled as

$$s_1 = u_2^m = \frac{5}{8\pi r^3}\mu\,\delta(r - r_0), \tag{9.149}$$

$$s_2 = u_2^m = \frac{5}{8\pi r^2}\mu\,\delta'(r - r_0), \tag{9.150}$$

$$s_3 = v_2^m = \frac{5}{8\pi r^3}\mu\,\delta(r - r_0), \tag{9.151}$$

$$s_4 = v_2^m = \frac{5}{8\pi r^2}\mu\,\delta'(r - r_0). \tag{9.152}$$

Including the modulus of rigidity, the sources for the dip-slip system given by (9.84) yield

$$\begin{aligned}
u_2^0 &= \frac{5\mu}{4\pi r^2}\delta'(r - r_0)\sin 2\alpha = 2s_2 \sin 2\alpha,\\
u_2^{-1} &= -i\frac{30\mu}{8\pi r^3}\delta(r - r_0)\cos 2\alpha = -6is_1\cos 2\alpha,\\
u_2^1 &= -i\frac{5\mu}{8\pi r^3}\delta(r - r_0)\cos 2\alpha = -is_1\cos 2\alpha,\\
v_2^{-1} &= i\frac{5\mu}{8\pi r^2}\delta'(r - r_0)\cos 2\alpha = is_4\cos 2\alpha,\\
v_2^1 &= i\frac{5\mu}{48\pi r^2}\delta'(r - r_0)\cos 2\alpha = \frac{i}{6}s_4\cos 2\alpha,\\
v_2^{-2} &= -\frac{20\mu}{8\pi r^3}\delta(r - r_0)\sin 2\alpha = -4s_3\sin 2\alpha,\\
v_2^2 &= -\frac{5\mu}{48\pi r^3}\delta(r - r_0)\sin 2\alpha = -\frac{1}{6}s_3\sin 2\alpha.
\end{aligned} \tag{9.153}$$

In addition, the sources arising from co-ordinate curvature for the dip-slip system given by (9.86) contribute

$$\begin{aligned}
u_2^0 &= \frac{5\mu}{4\pi r^3}\delta(r - r_0)\sin 2\alpha = 2s_1\sin 2\alpha,\\
v_2^0 &= -\frac{5\mu}{8\pi r^3}\delta(r - r_0)\sin 2\alpha = -s_3\sin 2\alpha,\\
v_2^{-1} &= i\frac{5\mu}{8\pi r^3}\delta(r - r_0)\cos 2\alpha = is_3\cos 2\alpha,\\
v_2^1 &= i\frac{5\mu}{48r^3}\delta(r - r_0)\cos 2\alpha = \frac{i}{6}s_3\cos 2\alpha,\\
v_2^{-2} &= \frac{5\mu}{4\pi r^3}\delta(r - r_0)\sin 2\alpha = 2s_3\sin 2\alpha,\\
v_2^2 &= \frac{5\mu}{96\pi r^3}\delta(r - r_0)\sin 2\alpha = \frac{1}{12}s_3\sin 2\alpha.
\end{aligned} \tag{9.154}$$

Again including the modulus of rigidity, the sources for the strike-slip system given by (9.85) contribute

$$
\begin{aligned}
u_2^{-1} &= \frac{15\mu}{4\pi r^3}\delta(r-r_0)\cos\alpha = 6s_1\cos\alpha,\\
u_2^{1} &= -\frac{5\mu}{8\pi r^3}\delta(r-r_0)\cos\alpha = -s_1\cos\alpha,\\
v_2^{-1} &= -\frac{5\mu}{8\pi r^2}\delta'(r-r_0)\cos\alpha = -s_4\cos\alpha,\\
v_2^{1} &= \frac{5\mu}{48\pi r^2}\delta'(r-r_0)\cos\alpha = \frac{s_4}{6}\cos\alpha,\\
v_2^{-2} &= -i\frac{15\mu}{4\pi r^3}\delta(r-r_0)\sin\alpha = -6is_3\sin\alpha,\\
v_2^{2} &= i\frac{15\mu}{96\pi r^3}\delta(r-r_0)\sin\alpha = i\frac{s_3}{4}\sin\alpha.
\end{aligned}
\tag{9.155}
$$

In addition, the sources arising from co-ordinate curvature for the strike-slip system given by (9.87) contribute

$$
\begin{aligned}
v_2^{-1} &= -\frac{5\mu}{8\pi r^3}\delta(r-r_0)\cos\alpha = -s_3\cos\alpha,\\
v_2^{1} &= \frac{5\mu}{48\pi r^3}\delta(r-r_0)\cos\alpha = \frac{s_3}{6}\cos\alpha,\\
v_2^{-2} &= i\frac{5\mu}{4\pi r^3}\delta(r-r_0)\sin\alpha = i2s_3\sin\alpha,\\
v_2^{2} &= -i\frac{5\mu}{96\pi r^3}\delta(r-r_0)\sin\alpha = -i\frac{s_3}{12}\sin\alpha.
\end{aligned}
\tag{9.156}
$$

Once the four source solutions have been used to evaluate the changes in the components of the inertia tensor in the epicentral co-ordinate system as given by expression (9.89), it is necessary to transform the inertia tensor to the geographic system of co-ordinates. We follow the methods of classical mechanics including the use of Euler angles (Goldstein, 1956, Ch. 4).

We first examine the process of going from the geographic co-ordinates $(x_1, x_2, x_3)$ to the epicental system. Given an epicentre at east longitude $\phi$, co-latitude $\theta$, the $x_1$ and $x_2$ co-ordinate axes are first rotated around the $x_3$ axis by the angle $\phi$ to produce new co-ordinates $x_1', x_2'$, related to $x_1, x_2$ by

$$
\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{9.157}
$$

The three new co-ordinates are related to the original ones by

$$
\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \tag{9.158}
$$

Of course, the co-ordinate $x_3$ is unaffected and $x'_3 = x_3$.

The transformation (9.158) is orthogonal, so that if it is multiplied on the left by its transpose, we obtain

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}. \tag{9.159}$$

Then, the $x'_1$ and $x'_3$ axes are rotated around the $x'_2$ axis by the angle $\theta$. The new co-ordinates $x''_3, x''_1$ are related to $x'_3, x'_1$ by

$$\begin{pmatrix} x''_3 \\ x''_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x'_3 \\ x'_1 \end{pmatrix}. \tag{9.160}$$

The three new co-ordinates are related to the three old co-ordinates by

$$\begin{pmatrix} x''_1 \\ x''_2 \\ x''_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}. \tag{9.161}$$

Again, the co-ordinate $x'_2$ is unaffected and $x''_2 = x'_2$.

Making use of the orthogonality of (9.161), we are led to

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} x''_1 \\ x''_2 \\ x''_3 \end{pmatrix}. \tag{9.162}$$

The final step in going to epicentral co-ordinates is to allow for the fault azimuth $\psi$ measured clockwise from the north. The axes $x''_1$ and $x''_2$ are rotated about the $-x''_3$ axis in order to maintain a right-hand co-ordinate system. The new co-ordinates $x'''_1, x'''_2$ are related to the old ones by

$$\begin{pmatrix} x'''_1 \\ x'''_2 \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} x''_1 \\ x''_2 \end{pmatrix}. \tag{9.163}$$

In this final rotation, the new co-ordinates are related to the old ones by

$$\begin{pmatrix} x'''_1 \\ x'''_2 \\ x'''_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x''_1 \\ x''_2 \\ x''_3 \end{pmatrix}. \tag{9.164}$$

In this case, the new co-ordinate $x'''_3 = -x''_3$.

Once again, making use of the orthogonality of (9.164), we find

$$\begin{pmatrix} x''_1 \\ x''_2 \\ x''_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x'''_1 \\ x'''_2 \\ x'''_3 \end{pmatrix}. \tag{9.165}$$

The transformation from epicentral to geographic co-ordinates is found by first substituting from (9.164) into (9.161), and then from (9.158), to get

$$\begin{pmatrix} x_1''' \\ x_2''' \\ x_3''' \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

Carrying out the matrix multiplications, we find the transformation from geographic to epicentral co-ordinates to be

$$\begin{pmatrix} x_1''' \\ x_2''' \\ x_3''' \end{pmatrix} = \boldsymbol{T}^T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \tag{9.166}$$

with the transpose of the transformation matrix being

$$\boldsymbol{T}^T = \begin{pmatrix} \cos\phi\cos\theta\cos\psi + \sin\phi\sin\psi & \sin\phi\cos\theta\cos\psi - \cos\phi\sin\psi & -\sin\theta\cos\psi \\ \cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & -\sin\theta\cos\psi \\ -\cos\phi\sin\theta & -\sin\phi\sin\theta & -\cos\theta \end{pmatrix}.$$

Similarly, the transformation from epicentral to geographic co-ordinates is obtained by first substituting from (9.165) into (9.162) and then into (9.159) to obtain

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1''' \\ x_2''' \\ x_3''' \end{pmatrix}.$$

Again, carrying out the matrix multiplications, the transformation from epicentral to geographic co-ordinates is found to be

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \boldsymbol{T} \begin{pmatrix} x_1''' \\ x_2''' \\ x_3''' \end{pmatrix}, \tag{9.167}$$

with transformation matrix $\boldsymbol{T}$ given by

$$\boldsymbol{T} = \begin{pmatrix} \cos\phi\cos\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & -\cos\phi\sin\theta \\ \sin\phi\cos\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & -\sin\phi\sin\theta \\ -\sin\theta\cos\psi & -\sin\theta\sin\psi & -\cos\theta \end{pmatrix}.$$