Corrections to

Statistical Analysis of Spherical Data

by N.I. Fisher, T. Lewis & B.J.J. Embleton
Cambridge University Press, 1987.

Known errors as at 6th April, 2000. The authors thank all those who have contributed to this list.

Page	**Correction**
8	Add an extra point to the plot, at (5, 0.83)
32^{12}	$-\sin\phi_0 \sin\psi_0,$
32^{15}	Line numbered (3.7): $= \mathbf{A}(\theta_0, \phi_0, \psi_0)$
36^2	$y^* = \rho \sin\psi$
37_5	$\rho = \tan(90 - \frac{1}{2}\theta), \ldots$
38^5	$\rho = 2\sin(90 - \frac{1}{2}\theta), \ldots$
38^{11}	$x^* = \sin\theta\cos\phi,\ y^* = \sin\theta\sin\phi$
48^{16}	Change $\log(\bar{\tau}_3/\bar{\tau}_2)$ to $\log(\bar{\tau}_3/\bar{\tau}_1)$
48_3	"$\bar{\tau}_1$ small; $\bar{\tau}_2, \bar{\tau}_3$ large"
48_1	"rotational symmetry about $\hat{\mathbf{u}}_1$"
51^1	Replace line by: "In this book, we have used slightly modified formulae for $a_1, \ldots, a_n$ given by"
59_{15}	"Colatitude $\Theta = \cos^{-1} S$, longitude $\Phi = 2\pi R_0$."
59_4	"If $V \leq (1 - \kappa S^2)\exp(\kappa S^2)$, go to 6."
66_1	Add final sentence: "Issues related to the accuracy of structural field measurements have been discussed by Woodcock (1976)."
75^{9-10}	*Joint* normality of the random variables is needed if they are not independent.

78^{11} Change reference to Lewis (1988).

89 $\exp(\kappa u^2)\mathrm{d}u$

93^{3-23} The axes are incorrectly labelled. $\boldsymbol{\xi}_3$ is the mean direction or pole, and $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ the major and minor axes respectively.

94_{11} The second term on the right-hand side of (4.46) should be

$$\kappa \sin\alpha[(\mathbf{x}'\boldsymbol{\xi}_2)^2 - (\mathbf{x}'\boldsymbol{\xi}_3)^2]/[1-(\mathbf{x}'\boldsymbol{\xi}_1)^2]^{1/2}.$$

97^{11} Wood (1988)

$98_{9,1}$ Wood (1988)

106 *Example* 5.7:
Line 7: change “set B4” to “set B5”.
Line 8: change “Example 5.4” to “Example 5.5”.
Lines 10–11: change to
“In fact, a test for uniformity (see Examples 5.9 & 5.24) indicates that the Uniform distribution is probably a reasonable model for the data”.

109 Table 5.1:
For Examples 5.2, 5.3, 5.4, 5.5, 5.6 (but not Example 5.1), interchange the values of $\widehat{\gamma}$ and $\widehat{\zeta}$; *e.g.* the row for Example 5.6 should finish

0.98 1.61

111_4 “For $n \geq 25, \ldots$”.

112 Replace Σ on left hand side of equation (5.6) by Σ.

113^8 “Let $(\hat{\gamma}, \hat{\delta})$ …”

113^{16} “ …that the spherical median direction is (γ_0, δ_0)…”

114 **§6.2.1(iv)** should be **§6.3.1(iv)**.

116^8	Insert closing ")" after **§5.3.1(iv)**.
118^{14}	**§3.2.1** should be **§3.2.2**.
123^7	Equation (5.16): 0.467 not 0.567.
123	Second line of Figure caption: Insert opening "(" before 122°.
127	Last text line above figure captions: "... described in §**7.2.3**(**i**)"
131_{6-1}	(Commencing "If $\widehat{\kappa} \geq 5 \ldots$"). Reset lines in accordance with standard left hand margin.
132_8	Replace (n - R) by $(n - R)$
135_{16}	χ^2_{2n-2}
135_{18}	χ^2_{2n-2}
136–140	Corrections to the text in §**5.3.3***(i) Estimation of the parameters of the Kent distribution*):

136 — Equation 5.43: replace first 4 lines of **Step 2** by

"Compute the matrix

$$\mathbf{H} = \mathbf{A}'(\widehat{\theta}, \widehat{\phi}, 0)$$

where $\mathbf{A}$ is defined by (3.9), and then"

137 — Equation 5.45:

$$\widehat{\psi} = \frac{1}{2}\arctan\{2b_{12}/(b_{11} - b_{22})\}$$

137 — Equation 5.46:

$$\mathbf{K} = \begin{pmatrix} \cos\widehat{\psi} & -\sin\widehat{\psi} & 0 \\ \sin\widehat{\psi} & \cos\widehat{\psi} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

137 Equation 5.49:

$$Q = v_{11} - v_{22}, \qquad Q > 0$$

137–138 Example 5.27:

$$\mathbf{B} = \begin{pmatrix} 0.0385 & 0.0042 & 0.0005 \\ 0.0042 & 0.0172 & 0.0009 \\ 0.0005 & 0.0009 & 0.9443 \end{pmatrix}$$

$$\widehat{\mathbf{G}} = \begin{pmatrix} -0.1076 & -0.1170 & -0.9873 \\ -0.1723 & -0.9758 & 0.1345 \\ -0.9791 & 0.1846 & 0.0848 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} 0.0393 & 0.0000 & 0.0007 \\ 0.0000 & 0.0164 & 0.0008 \\ 0.0007 & 0.0008 & 0.9443 \end{pmatrix}$$

$138^{8,9}$ "Let $x_i^* = \sin\theta_i' \cos\phi_i'$, $y_i^* = \sin\theta_i' \sin\phi_i'$, $z_i^* = \cos\theta_i'$, $i = 1, \ldots, n$, ...

138 Equation 5.52:
Replace x by z in the first sum, y by x in the second, and z by y in the third.

138 Equation 5.54:

$$s_1 = \tilde{\sigma}_1 g^{\frac{1}{2}}, \qquad s_2 = \tilde{\sigma}_2 g^{\frac{1}{2}}$$

138_{15} "Then $\arcsin(s_1)$ and $\arcsin(s_2)$ are the major and minor semi–axes ..."

138 Step 3.1:
Let u_0 be any number between $-s_1$ and s_1 (from 5.54).

139 Step 3.2:

$$v_0 = s_2[1-(u_0/s_1)^2]^{\frac{1}{2}}, \qquad w_0 = (1-u_0^2-v_0^2)^{\frac{1}{2}} \tag{5.55}$$

Step 3.3:
... let $x_0 = u_0$, $y_0 = v_0$, $z_0 = w_0$, and $x_0' = u_0$, $y_0' =$

$-v_0,\ z_0' = w_0$
Step 3.4:
The right–hand–side of the right–hand equation in (5.56) should be

$$\hat{\mathbf{G}}\begin{pmatrix} u_0 \\ -v_0 \\ w_0 \end{pmatrix}$$

Replace

$$(v_0^*, w_0^*, u_0^*)$$

by

$$(u_0^*, v_0^*, w_0^*)$$

and

$$(v_0^{**}, w_0^{**}, u_0^{**})$$

by

$$(u_0^{**}, v_0^{**}, w_0^{**})$$

in the two lines immediately following (5.56).

139 Step 3.5:
Repeat Steps 3.1–3.4 for a range of values of u_0 between $-s_1$ and s_1.

139_9 Replace 3.2° by 3.4°.

146_{13-12} $\hat{x} = U/R^*$, $\hat{y} = V/R^*$, and $\hat{z} = W/R^*$, where R^* is defined by (5.67).

149 Giné's

150 Figure caption should refer to Example 5.35.

154 Caption to Figure 6.5, line 3:
"(Colatitude, Longitude). See Example 6.5."

154 Replace Figure 6.5 by new figure (supplied).

158 Replace plots in Figure 6.11 by two new figures (supplied).

160_8	$\bar{\tau}_3$
165^{14}	$(1-2\bar{\tau}_3+\Gamma)$
176^1	$D(\widehat{\kappa})=\bar{\tau}_3$
176^{6-9}	Replace by

$$\widehat{\kappa}=\begin{cases} 3.75(3\bar{\tau}_3-1) & \frac{1}{3}\le\bar{\tau}_3\le 0.34 \\ -5.95+14.9\bar{\tau}_3+1.48(1-\bar{\tau}_3)^{-1}-11.05\bar{\tau}_3^{-2}, & 0.34<\bar{\tau}_3\le 0.64 \\ -7.96+21.5\bar{\tau}_3+(1-\bar{\tau}_3)^{-1}-13.25\bar{\tau}_3^2 & \bar{\tau}_3>0.64 \end{cases}$$

176^{23}	Insert $1-$ at beginning of right hand side of equation (6.24):

$$\bar{\tau}_3^*=1-\ldots$$

177^{19}	"Best & Fisher (1986)".
177_{15}	Change line to: "Best & Fisher (1986); note the correction, 21.5 instead of 2.15, for $\bar{\tau}_3>0.32$."
189	Replace equation (6.38) by

$$\widehat{\kappa}=\begin{cases} (2\bar{\tau}_1)^{-1} & 0\le\bar{\tau}_1\le 0.06 \\ 0.961-7.08\bar{\tau}_1+0.466/\bar{\tau}_1 & 0.06<\bar{\tau}_1\le 0.32 \\ 3.75(1-3\bar{\tau}_1), & 0.32<\bar{\tau}_1\le\frac{1}{3} \end{cases}$$

200	In equation (7.4), replace $\mathbf{\Sigma}$ by $\mathbf{\Sigma}^{-1}$
200_2	"…using the method in **(iia)** …".
200_1	"…(colat. 72.3°, long. 338.0° …".
201^1	Replace the number 326.9 by 263.8.
202	In equations (7.5) and (7.6) replace $\mathbf{W}_i^{-1}$ by $\mathbf{W}_i$

$203_{13,12}$ Change to:

... and the $\mathbf{V}$–matrix in (7.7) is then given by

$$\mathbf{V}^{-1} = \sum_{i=1}^{r} n_i \ \mathbf{W}_i^{-1}/N^2 \qquad (7.9)$$

203_1 "and minor semi–axes 9.9° and 4.3°."

204_{8-4} Replace these lines by:

"For formal testing, define the quantities

$$\widehat{x}_* = \sum_{i=1}^{r} \widehat{x}_i/\widehat{\sigma}_i^2, \qquad \widehat{y}_* = \sum_{i=1}^{r} \widehat{y}_i/\widehat{\sigma}_i^2, \qquad \widehat{z}_* = \sum_{i=1}^{r} \widehat{z}_i/\widehat{\sigma}_i^2,$$

$$R_* = (\widehat{x}_*^2 + \widehat{y}_*^2 + \widehat{z}_*^2)^{\frac{1}{2}}, \qquad \rho_* = \sum_{i=1}^{r} 1/\widehat{\sigma}_i^2$$

The test statistic is

$$G_r = 4(\widehat{\rho}_* - R_*) \qquad (7.11)$$

206 Last four lines of Example 7.7. Change sentences to:

"For the pooled data, we get $G_4 = 10.8$, corresponding to a $P-$value of about 0.1 for a χ_6^2 variate. We conclude that the four samples can reasonably be taken to be drawn from distributions with a common mean direction."

206_9 $\sin\widehat{\alpha}_w \cos\widehat{\beta}_w = \cdots$

207_{10} "If $n_1^{\frac{1}{2}}\widehat{\sigma}_1, \ldots$"

207_4 Equation number should be (7.23).

210_{8-6} Change sentence:

"Note that when these two samples were two of the four compared jointly, in Example 7.7, there was effectively no evidence of overall difference."

215^2 Change "Example 5.26" to "Example 5.27".

218^{13} "Let $(\theta_1', \phi_1'), \ldots$"

233_{10} "$(\mathbf{X}_i, \ \mathbf{X}_i^*)$ omitted, …"

$235^{14,15}$ Replace by

$$\left.\begin{array}{lcl} \widehat{\boldsymbol{\Sigma}}_{11} & = & \sum_{i=1}^n (\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})' \\ \widehat{\boldsymbol{\Sigma}}_{12} & = & \sum_{i=1}^n (\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i^* - \bar{\mathbf{X}}^*)' \\ \widehat{\boldsymbol{\Sigma}}_{22} & = & \sum_{i=1}^n (\mathbf{X}_i^* - \bar{\mathbf{X}}^*)(\mathbf{X}_i^* - \bar{\mathbf{X}}^*)' \end{array}\right\} \qquad (8.11)$$

where

$$\bar{\mathbf{X}} = (1/n)\sum_{i=1}^n \mathbf{X}_i, \qquad \bar{\mathbf{X}}^* = (1/n)\sum_{i=1}^n \mathbf{X}_i^*$$

236^{12} "(cf. §**3.7(i)**)"

237 Replace last paragraph of *Example 8.8* beginning "From (8.14), …" by:
"From (8.14), we obtain the estimated correlation as $\widehat{\rho}_A = 0.042$, so that $3n\widehat{\rho}_A \simeq 12.73$. This corresponds to a P–value of about 0.18 for a χ_9^2–variate. We conclude that there is little evidence of correlation between the orientations of the shortest and longest axes."

238^2 Replace $\mathbf{X}_i \mathbf{X}_i'$ by $(\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})'$

238^3 Replace $\mathbf{X}_i \mathbf{Y}_i'$ by $(\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{Y}_i - \bar{\mathbf{Y}})'$

238^4 Replace $\mathbf{Y}_i \mathbf{Y}_i'$ by $(\mathbf{Y}_i - \bar{\mathbf{Y}})(\mathbf{Y}_i - \bar{\mathbf{Y}})'$
Add line:
(where $\bar{\mathbf{X}} = (1/n)\sum_{i=1}^n \mathbf{X}_i$, $\bar{\mathbf{Y}} = (1/n)\sum_{i=1}^n \mathbf{Y}_i$)

239 In (8.25) the second equation should be

$$\tan\hat{\alpha}_1 = \mathrm{trace}(\mathbf{S}_{vx}\ \hat{\mathbf{U}}_1')/\bar{\mathbf{X}}'\ \hat{\mathbf{U}}_1\mathbf{w}$$

264–265 The values of $A^{-1}(x)$ in Table A10 are accurate; however, those of $D^{-1}(x)$ are inaccurate. A corrected table of values $D^{-1}(x)$ is attached.

295^3 “*Coordinates* Colatitude, Longitude.”

313_8 “Diggle, P.J. & Fisher, N.I. ...”

316^{17-18} Change to:
Lewis, T. (1988). A simple improved–accuracy normal approximation for χ^2. *Austral. J. Statist.* **30A**, 160–167.

320 Delete last five lines.

322 Insert new entry:
“colatitude plot 118, 168”.

322 Existing entry “colatitude test” :
Insert ”122, 169”, and delete “see Fisher distribution, Watson distribution”

325 Existing entry “longitude plot”:
Insert “118, 168” before “to assess ... ”, and delete “see also Fisher distribution, Watson distribution”.

325 Insert new entry:
“longitude test 123, 170”.

328 Insert new entry:
“two–variable plot 118”.

328 Insert new entry:
“two–variable test 125”.

320 Change reference to Wood (1986) to:
Wood, A. T. A. (1988). Some notes on the Fisher–Bingham family on the sphere. *Commun. Statist.– Theor. Meth.* **17**, 3881–3897.

320 Add reference:
Woodcock, N. H. (1976). The accuracy of structural field measurements. *Journal of Geology* **84**, 350–355. (66)

Table A10

x	$D^{-1}(x)$	x	$D^{-1}(x)$	x	$D^{-1}(x)$
0.0005	–1000.0	0.240	–1.202	0.620	2.930
0.001	–500.0	0.245	–1.127	0.630	3.044
0.002	–250.0	0.250	–1.053	0.640	3.160
0.003	–166.7	0.255	–0.982	0.650	3.280
0.004	–125.0	0.260	–0.911	0.660	3.402
0.005	–100.0	0.265	–0.842	0.670	3.529
0.006	–83.33	0.270	–0.774	0.680	3.659
0.007	–71.43	0.275	–0.707	0.690	3.794
0.008	–62.50	0.280	–0.642	0.700	3.933
0.009	–55.56	0.285	–0.578	0.710	4.079
0.010	–50.00	0.290	–0.514	0.720	4.231
0.015	–33.33	0.295	–0.452	0.730	4.389
0.020	–25.00	0.300	–0.390	0.740	4.556
0.025	–20.00	0.305	–0.329	0.750	4.731
0.030	–16.67	0.310	–0.270	0.760	4.917
0.035	–14.29	0.315	–0.211	0.770	5.115
0.040	–12.50	0.320	–0.152	0.780	5.326
0.045	–11.11	0.325	–0.095	0.790	5.552
0.050	–10.00	0.330	–0.038	0.800	5.797
0.055	–9.087	0.331	–0.026	0.810	6.063
0.060	–8.327	0.332	–0.015	0.820	6.354
0.065	–7.681	0.333	–0.004	0.830	6.676
0.070	–7.126	0.334	0.008	0.840	7.035
0.075	–6.641	0.335	0.019	0.850	7.438
0.080	–6.215	0.336	0.030	0.860	7.897
0.085	–5.835	0.337	0.041	0.870	8.426
0.090	–5.495	0.338	0.052	0.880	9.043
0.095	–5.188	0.339	0.063	0.890	9.776
0.100	–4.907	**0.340**	0.075	**0.900**	10.66
0.105	–4.651	0.350	0.184	0.905	11.17
0.110	–4.415	0.360	0.292	0.910	11.75
0.115	–4.196	0.370	0.398	0.915	12.39
0.120	–3.992	0.380	0.503	0.920	13.11
0.125	–3.802	0.390	0.606	0.925	13.94
0.130	–3.624	0.400	0.708	0.930	14.88
0.135	–3.457	0.410	0.809	0.935	15.97
0.140	–3.298	0.420	0.908	0.940	17.24
0.145	–3.148	0.430	1.008	0.945	18.75
0.150	–3.006	0.440	1.106	0.950	20.56
0.155	–2.870	0.450	1.204	0.955	22.77
0.160	–2.741	0.460	1.302	0.960	25.55
0.165	–2.617	0.470	1.399	0.965	29.11
0.170	–2.499	0.480	1.497	0.970	33.87
0.175	–2.385	0.490	1.594	0.975	40.53
0.180	–2.275	0.500	1.692	0.980	50.52
0.185	–2.170	0.510	1.790	0.985	67.18
0.190	–2.068	0.520	1.888	**0.990**	100.5
0.195	–1.970	0.530	1.987	0.991	111.6
0.200	–1.874	0.540	2.087	0.992	125.5
0.205	–1.782	0.550	2.188	0.993	143.4
0.210	–1.692	0.560	2.289	0.994	167.2
0.215	–1.605	0.570	2.392	0.995	200.5
0.220	–1.520	0.580	2.496	0.996	250.5
0.225	–1.438	0.590	2.602	0.997	333.8
0.230	–1.357	0.600	2.709	0.998	500.5
0.235	–1.279	0.610	2.818	0.999	1000.5