Chapter 13

```
> with(plots):
Warning, the name changecoords has been redefined
> with(plottools):
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

Question 1

(i)

Since

$$s - sbar = \left(-\frac{1}{u\,v}\right)(p - pbar)$$

then

$$s = sbar + \frac{1\,[pbar + (p0 - pbar)\,\mathbf{e}^{(-\lambda\,t)} - pbar]}{u\,v}$$

$$= sbar - \frac{(po - pbar)\,\mathbf{e}^{(-\lambda\,t)}}{u\,v}$$

But

$$s0 - sbar = \left(-\frac{1}{u\,v}\right)(p0 - pbar)$$

Therefore

$$s = sbar + (s0 - sbar)\,\mathbf{e}^{(-\lambda\,t)}$$

(ii)

```
> solve(s=105-(1/0.1)*(105-5*exp(-0.011*t)-105),s);
```

$$105. + 50.\,\mathbf{e}^{(-.01100000000\,t)}$$

(iii)

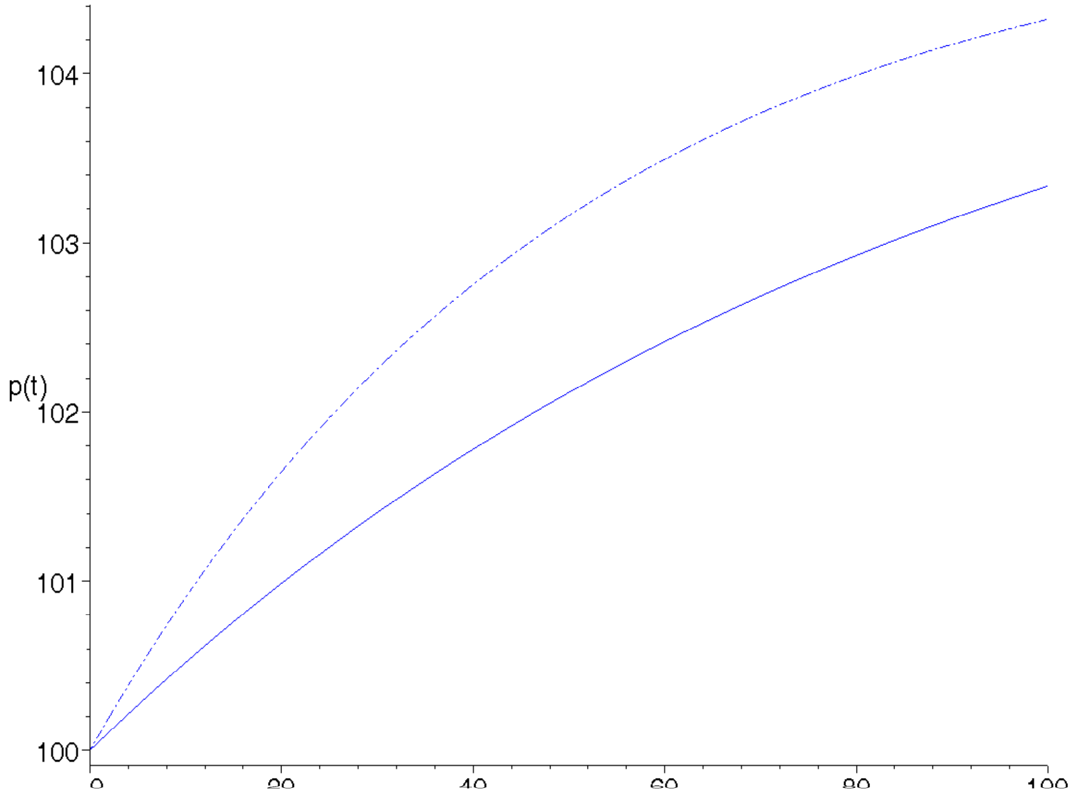
```
> pathp1:=105-5*exp(-0.011*t);
```

$$pathp1 := 105 - 5\,\mathbf{e}^{(-.011\,t)}$$

```
> pathp2:=105.-5*exp(-.02*t);
```

$$pathp2 := 105. - 5\,\mathbf{e}^{(-.02\,t)}$$

```
> plot([pathp1,pathp2],t=0..100,labels=["t","p(t)"],linestyl
e=[1,4],colour=[blue,blue]);
```



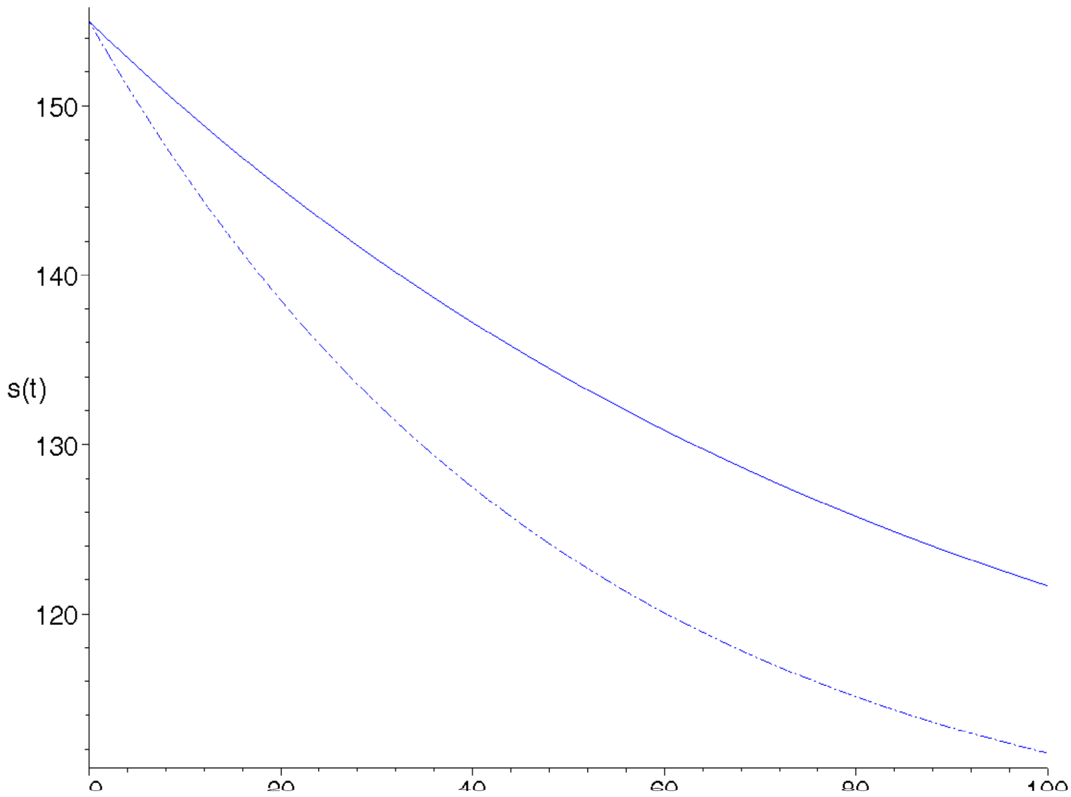

```
> paths1:=105+50*exp(-0.011*t);
```

$$paths1 := 105 + 50\,\mathbf{e}^{(-.011\,t)}$$

```
> paths2:=105+50*exp(-0.02*t);
```

$$paths2 := 105 + 50\,\mathbf{e}^{(-.02\,t)}$$

```
> plot([paths1,paths2],t=0..100,labels=["t","s(t)"],linestyl
e=[1,4],colour=[blue,blue]);
```



Question 2

GM line is $p = s$ and AM line is $p = 110 - .1\, s$

```
> solve({p=s,p=110-0.1*s},{s,p});
```

$$\{s = 100.,\, p = 100.\}$$

```
> am:=m-k*y+u*rstar+u*v*sbar-u*v*s;
```

$$am := m - k\,y + u\,rstar + u\,v\,sbar - u\,v\,s$$

```
> para0:={m=105, k=0.5, y=20, u=0.5, rstar=10, v=0.2,
  sbar=100};
```

$$para0 := \{m = 105,\, k = .5,\, y = 20,\, u = .5,\, rstar = 10,\, v = .2,\, sbar = 100\}$$

```
> am0:=subs(para0,am);
```

$$am0 := 110.00 - .10\, s$$

```
> para1:={m=105, k=0.5, y=20, u=0.5, rstar=12, v=0.2,
  sbar=100};
```

$$para1 := \{m = 105,\, k = .5,\, y = 20,\, u = .5,\, v = .2,\, sbar = 100,\, rstar = 12\}$$

```
> am1:=subs(para1,am);
```

$$am1 := 111.00 - .10\, s$$

```
> solve({p=s, p=111-0.1*s},{s,p});
```

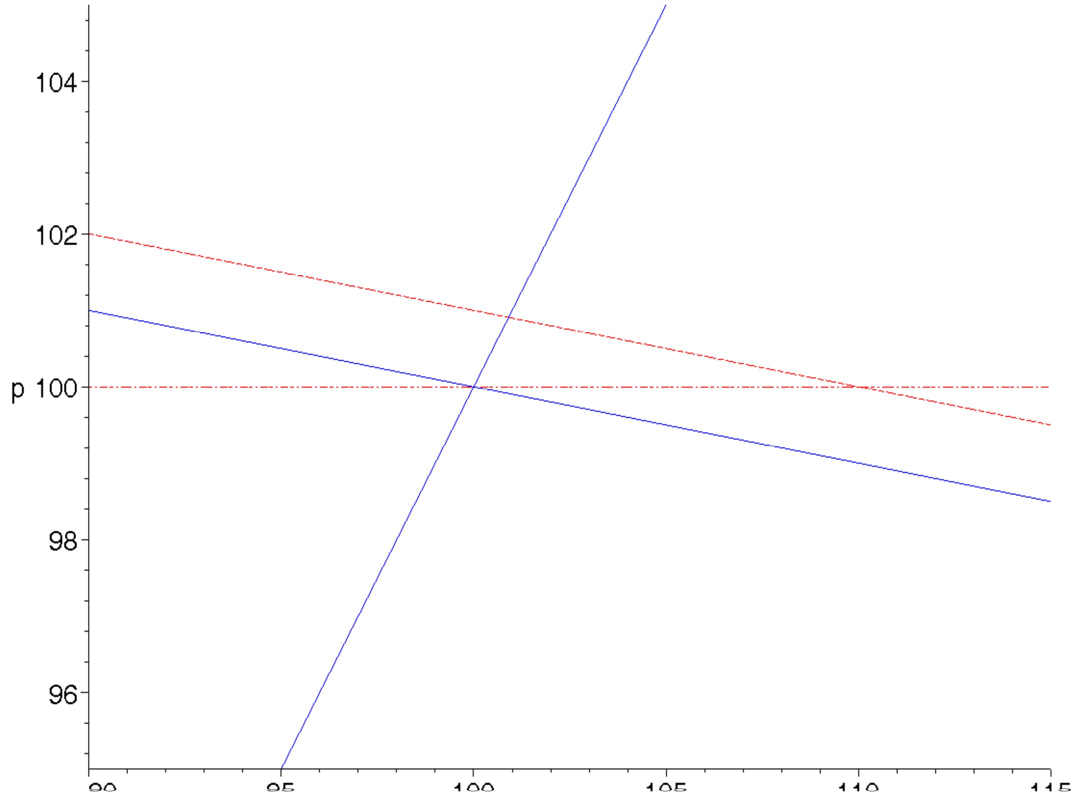
$$\{s = 100.9090909,\, p = 100.9090909\}$$

The value on the new AM line at the original price of $p = 100$ is

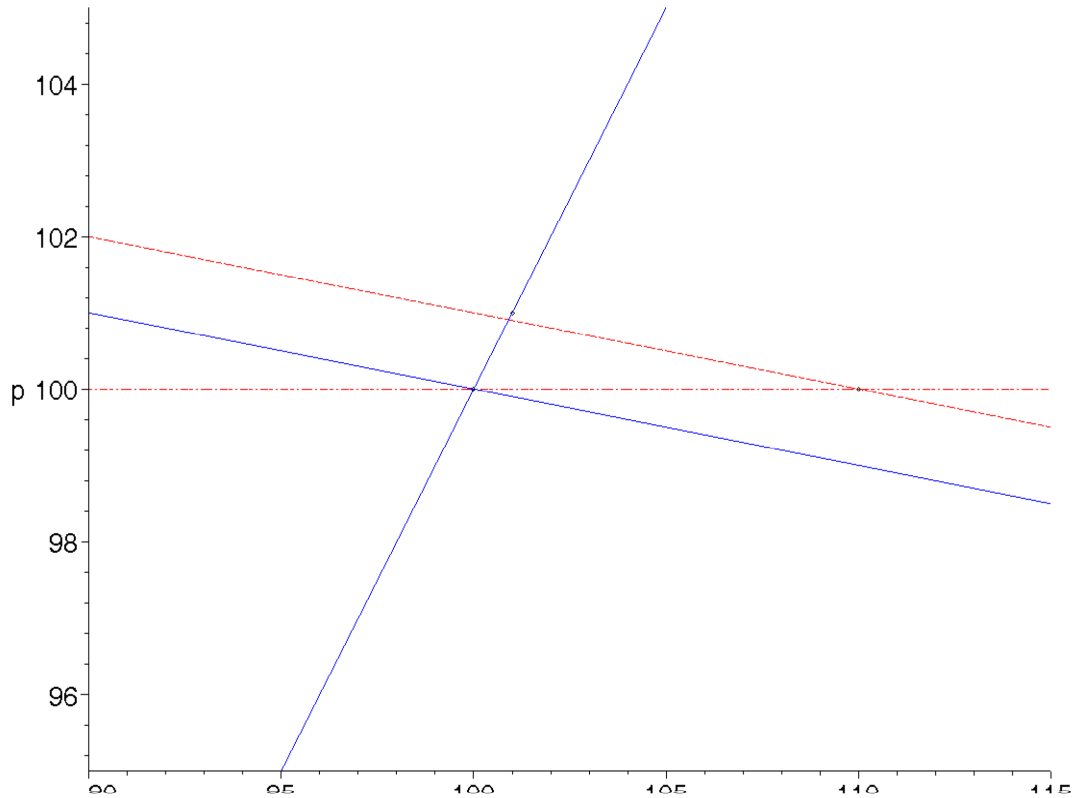
```
> solve(100=111-0.1*s,s);
```

$$110.$$

```
> lines2:=plot([s,am0,am1,100],s=90..115,95..105,
  labels=["s","p"],linestyle=[1,1,3,4],colour=[blue,blue,red,re
  d]):
> display(lines2);
```




```
> points2:=[point([100,100]),point([101,101]),point([110,100])]
  :
> display(lines2,points2);
```



System begins with intial equilibrium $(s, p) = (100, 100)$ then moves horizontally across to new AM curve, with value $(s, p) = (110, 100)$. As prices begin to adjust, the system moves up the new AM curve to the new equilibrium at $(s, p) = (101, 101)$.

Question 3

(i)

Since $p = m - k\,y - u\,rstar$ and

$$p = m - k\,y - u\,(rstar + sdote) \qquad sdote \text{ denoting } \frac{ds^e}{dt}$$

$$= m - k\,y - u\,rstar + u\,v\,(sbar - s)$$

$$= m - k\,y - u\,rstar - u\,v\,(s - sbar)$$

Subtracting $pbar$ from p we obtain

$$p - pbar = -u\,v\,(s - sbar)$$

Or

$$s - sbar = -\frac{1\,(p - pbar)}{u\,v}$$

(ii)

We have

$$p = a\,(e - y) = a\,[c\,y - d\,r + g + h\,(s - p) - y]$$

$$= a\,[-(1 - c)\,y - d\,r + g + h\,s - h\,p]$$

```
> pdot:=a*(-(1-c)*y-d*r+g+h*s-h*p);
```

$$pdot := a\,(-(1 - c)\,y - d\,r + g + h\,s - h\,p)$$

But

$$m = k\,y - u\,r + p$$

```
> solve(m=k*y-u*r+p,r);
```

$$\frac{-m + k\,y + p}{u}$$

```
> newpdot:=subs(r=(-m+k*y+p)/u,pdot);
```

$$newpdot := a\left(-(1 - c)\,y - \frac{d\,(-m + k\,y + p)}{u} + g + h\,s - h\,p\right)$$

```
> pdot0:=subs({p=pbar,s=sbar,r=(-m+k*y+pbar)/u},pdot);
```

$$pdot0 := a\left(-(1 - c)\,y - \frac{d\,(-m + k\,y + pbar)}{u} + g + h\,sbar - h\,pbar\right)$$

```
> finalpdot:=simplify(newpdot-pdot0);
```

$$finalpdot := \frac{a\,(-d\,p + h\,s\,u - h\,p\,u + d\,pbar - h\,sbar\,u + h\,pbar\,u)}{u}$$

Which can be written

$$a\,h\,(s - sbar) - a\,h\,(p - pbar) - \frac{a\,d\,(p - pbar)}{u}$$

But from part (i) we know

$$s - sbar = \left(-\frac{1}{u\,v}\right)(p - pbar)$$

Hence,

$$p = \left(-\frac{a\,h}{u\,v}\right)(p - pbar) - a\,h\,(p - pbar) - \frac{a\,d\,(p - pbar)}{u}$$

$$= -a\left(h + \frac{h}{u\,v} + \frac{d}{u}\right)(p - pbar)$$

Let $\lambda = a\left(h + \frac{h}{u\,v} + \frac{d}{u}\right)$, then

```
> dsolve({diff(p(t),t)=-lambda*(p(t)-pbar),p(0)=p0},p(t));
```

$$p(t) = pbar + e^{(-\lambda\,t)}(-pbar + p0)$$

Which can be expressed $p(t) = pbar + (p0 - pbar)\,e^{(-\lambda\,t)}$. So the adjustment coefficient in this model is,

$$\lambda = a\left(h + \frac{h}{u\,v} + \frac{d}{u}\right)$$

Question 4

(i)

```
> e:='e':s:='s':p:='p':r:='r':
```

We suppress the t subscript throughout. To derive the GM line let $p_{t+1} - p_t = 0$.

```
> e:=0.01*(s-p)+0.8*(20)-0.1*r+5;
```

$$e := .01\,s - .01\,p + 21.0 - .1\,r$$

```
> solve(0=0.2*(e-20),p);
```

$$100. + s - 10.\,r$$

We need to eliminate r using equilibrium in the money market.

```
> solve(105=p+0.5*(20)-0.25*r,r);
```

$$-380. + 4.\,p$$

Furthermore, in equilibrium $r = rstar = 10$, so we can solve for equilibrium p from

```
> solve(105=p+0.5*(20)-0.25*(10),p);
```

$$97.50000000$$

Then the GM line is

```
> solve(p=100.-10*(-380+4*p)+s,p);
```

$$95.12195122 + .02439024390\,s$$

Given equilibrium $p = 97.5$, then equilibrium s is given by

```
> solve(97.5=95.12195122+.2439024390e-1*s,s);
```

$$97.49999999$$

Which is, to one decimal place, the same value.

We can now derive the equation for the AM line as follows.

```
> solve(105=p+0.5*(20)-0.25*(10+0.25*(97.5-s)),p);
```

$$103.5937500 - .06250000000\,s$$

To summarise:

GM line: $p_t = 95.12195122 + .02439024390\,s_t$

AM line: $p_t = 103.5937500 - .06250000000\,s_t$

To check these equations

```
> solve({p=95.12195122+.2439024390e-1*s,p=103.5937500-.62500
00000e-1*s},{s,p});
```

$$\{p = 97.50000000, s = 97.50000000\}$$

(ii)

We have already established for this model that

$$p_{t+1} - pbar = -a\left(h + \frac{h}{u\,v} + \frac{d}{u}\right)(p_t - pbar)$$

$$p_{t+1} = pbar - \lambda\,(p_t - pbar)$$

with $ppbar = 97.5$.

```
> lambda:=a*(h+(h/(u*v))+(d/u));
```

$$\lambda := a\left(h + \frac{h}{u\,v} + \frac{d}{u}\right)$$

```
> para4:={a=0.2, h=0.01, u=0.25, v=0.25, d=0.1};
```

$$para4 := \{a = .2, h = .01, u = .25, v = .25, d = .1\}$$

```
> valuel:=subs(para4,lambda);
```

$$valuel := .1140000000$$

```
> subs(para4,solve(s0-97.5=-(1/(u*v))*(100-97.5),s0));
```

$$57.50000000$$

```
> pathp:=97.5+(100-97.5)*exp(-valuel*t);
```

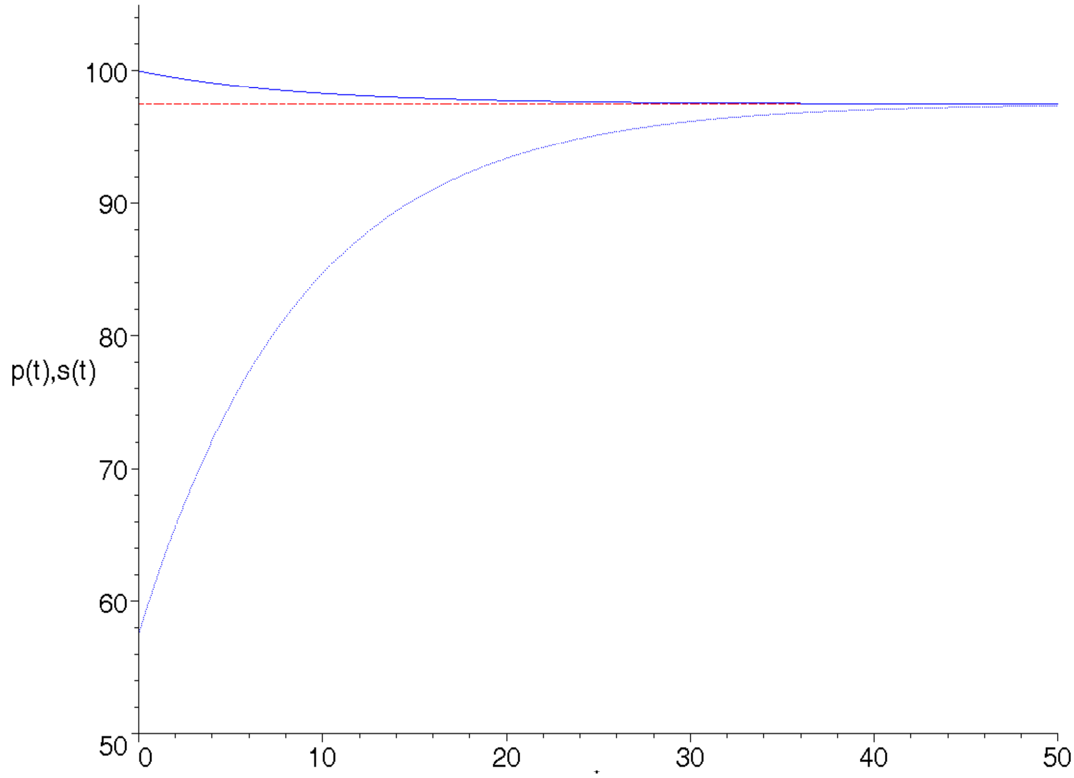
$$pathp := 97.5 + 2.5\,\mathbf{e}^{(-.1140000000\,t)}$$

```
> paths:=97.5+(57.5-97.5)*exp(-valuel*t);
```

$$paths := 97.5 - 40.0\,\mathbf{e}^{(-.1140000000\,t)}$$

t	p(t)	s(t)
0	100.0000	57.5000
1	99.7306	61.8096
2	99.4903	65.6550
3	99.2758	69.0860
4	99.0845	72.1474
5	98.9138	74.8789
6	98.7614	77.3162
7	98.6255	79.4908
8	98.5042	81.4312
9	98.3960	83.1624
10	98.2995	84.7072
11	98.2134	86.0855
12	98.1365	87.3153
13	98.0679	88.4126
14	98.0067	89.3917
15	97.9521	90.2653
16	97.9034	91.0448
17	97.8599	91.7403
18	97.8211	92.3608
19	97.7865	92.9145
20	97.7557	93.4086
21	97.7281	93.8494
22	97.7035	94.2427
23	97.6816	94.5937
24	97.6620	94.9068
25	97.6446	95.1862
26	97.6290	95.4355
27	97.6151	95.6579
28	97.6027	95.8564

```
> plot([pathp,paths,97.5],t=0..50,50..105,linestyle=[1,2,3],colour=[blue,blue,red],labels=["t","p(t),s(t)"]);
```



(iii)

The expenditure function in terms of s,p and r remains unaffected. However, the value of r in terms of p now differs.

```
> solve(110=p+0.5*(20)-0.25*r,r);
```

$$-400. + 4.\, p$$

Equilibrium price is

```
> solve(110=p+0.5*(20)-0.25*(10),p);
```

$$102.5000000$$

The GM line is then,

```
> solve(p=100.-10*(-400+4*p)+s,p);
```

$$100. + .02439024390\, s$$

```
> solve(102.5=100.+.2439024390e-1*s,s);
```

$$102.5000000$$

The AM line is then,

```
> solve(110=p+0.5*(20)-0.25*(10+0.25*(102.5-s)),p);
```

$$108.9062500 - .06250000000\, s$$

```
> solve({p=100.+.2439024390e-1*s,p=108.9062500-.6250000000e-1*s},{s,p});
```

$$\{p = 102.5000000, s = 102.5000000\}$$

Hence,

$$dm_t = ds_t = dp_t = 5$$

Question 5

(i)

The goods market is solved as before.

```
> e:='e':s:='s':p:='p':
> e:=0.8*(20)+4+0.01*(s-p);
```

$$e := 20.0 + .01\, s - .01\, p$$

```
> solve(0=0.1*(e-20),p);
```

$$s$$

No solution is given here because $p = s$
the origin. The major difference in this model lies in the asset market, the AM line.

In equilibrium $s = sbar$, so $sdote = 0$, $bp = 0$ and $r = rstar$.

```
> pbar:='pbar':
> solve(105=pbar+0.5*(20)-0.5*(10),pbar);
```

$$100.$$

```
> solve(0=0.01*(s-p)+0.0045*(r-10-0.2*(100-s)),r);
```

$$30. - 2.422222222\, s + 2.222222222\, p$$

```
> solve(105=p+0.5*(20)-0.5*(30.-2.422222222*s+2.222222222*p)
  ,p);
```

$$-990.0000010 + 10.90000001\, s$$

To check

```
> solve({p=s,p=-990.0000010+10.90000001*s},{s,p});
```

$$\{p = 100.,\, s = 100.\}$$

```
>
```

(ii) Rise in ms from 105 to 110

The goods line remains as the 45-degree line though the origin. But

```
> solve(110=pbar+0.5*(20)-0.5*(10),pbar);
```

$$105.$$

```
> solve(0=0.01*(s-p)+0.0045*(r-10-0.2*(105-s)),r);
```

$$31. - 2.422222222\, s + 2.222222222\, p$$

```
> solve(110=p+0.5*(20)-0.5*(31.-2.422222222*s+2.222222222*p)
  ,p);
```

$$-1039.500001 + 10.90000001\, s$$

```
> solve({p=s,p=-1039.500001+10.90000001*s},{s,p});
```

$$\{p = 105.0000000,\, s = 105.0000000\}$$

Point C arises on the new AM line at a price of 100.

```
> solve(100=-1039.500001+10.90000001*s,s);
```

$$104.5412844$$

Hence, coordinates of point C are $(s, p) = (104.5412844, 100)$.

(iii)

In the new equilibrium $p = s = 105$ and hence $dp = ds = 5 = dm$.

Question 6

(i) Model 13.4

```
> solr:=solve(0=h*(s-p)+b*(r-rstar-v*(sbar-s)),r);
```

$$solr := \frac{-h\,s + h\,p + b\,rstar + b\,v\,sbar - b\,v\,s}{b}$$

```
> valuep:=m-k*y+u*solr;
```

$$valuep := m - k\,y + \frac{u\,(-h\,s + h\,p + b\,rstar + b\,v\,sbar - b\,v\,s)}{b}$$

```
> valuepbar:=m-k*y-u*(h*sbar-h*pbar-b*rstar-b*v*sbar+b*v*sba
r)/b;
```

$$valuepbar := m - k\,y - \frac{u\,(h\,sbar - h\,pbar - b\,rstar)}{b}$$

```
> diffp:=simplify(valuep-valuepbar);
```

$$diffp := \frac{u\,(-h\,s + h\,p + b\,v\,sbar - b\,v\,s + h\,sbar - h\,pbar)}{b}$$

```
> collect(diffp,{s,sbar,p,pbar});
```

$$-\frac{u\,h\,pbar}{b} + \frac{u\,(b\,v + h)\,sbar}{b} + \frac{u\,(-h - b\,v)\,s}{b} + \frac{h\,p\,u}{b}$$

Which can be written,

$$-\left(u\,v + \frac{v\,h}{b}\right)(s - sbar) + \frac{u\,h\,(p - pbar)}{b}$$

Hence,

$$p - pbar = -\left(u\,v + \frac{u\,h}{b}\right)(s - sbar) + \frac{u\,h\,(p - pbar)}{b}$$

$$\left(1 - \frac{u\,h}{b}\right)(p - pbar) = -u\,v + \frac{u\,h\,(s - sbar)}{b}$$

$$s - sbar = -\frac{\left(1 - \frac{u\,h}{b}\right)(p - pbar)}{u\,v + \frac{u\,h}{b}}$$

Since in equilibrium $r = rstar + v\,(sbar - s) = rstar - v\,(s - sbar)$ then

```
> pdot:=a*(c*y-d*(rstar-v*(s-sbar))+g+h*(s-p)-y);
```

$$pdot := a\,(c\,y - d\,(rstar - v\,(s - sbar)) + g + h\,(s - p) - y)$$

```
> pdot0:=a*(c*y-d*rstar+g+h*(sbar-pbar)-y);
```

$$pdot0 := a\,(c\,y - d\,rstar + g + h\,(sbar - pbar) - y)$$

```
> diffpdot:=simplify(pdot-pdot0);
```

$$diffpdot := -a\,d\,v\,sbar + a\,d\,v\,s + a\,h\,s - a\,h\,p - a\,h\,sbar + a\,h\,pbar$$

Which can be written,

$$a\,(d\,v\,(s - sbar) + h\,(s - sbar) - h\,(p - pbar))$$

and substituting the value for $s - sbar$, we get for pdot

$$pdot = a\left(-(h+d\,v)\left[\frac{1-\frac{u\,h}{b}}{u\,v+\frac{u\,h}{b}}\right](p-pbar)-h\,(p-pbar)\right)$$

$$-a\left((h+d\,v)\left[\frac{1-\frac{u\,h}{b}}{u\,v+\frac{u\,h}{b}}+h\right](p-pbar)\right)$$

and for λ we have

$$\lambda = a\left(\frac{(h+d\,v)\left(1-\frac{u\,h}{b}\right)}{u\,v+\frac{u\,h}{b}}+h\right)$$

```
> newlambda:=a*((h+d*v)*(1-u*h/b)/(u*v+u*h/b)+h);
```

$$newlambda := a\left(\frac{(h+d\,v)\left(1-\frac{u\,h}{b}\right)}{u\,v+\frac{u\,h}{b}}+h\right)$$

(ii) Limit for d->0

```
> limit(newlambda,d=0);
```

$$\frac{a\,h\,b\,(1+u\,v)}{u\,(b\,v+h)}$$

Which is the same as

$$a\,h\left(\frac{1-\frac{u\,h}{b}}{u\,v+\frac{u\,h}{b}}+1\right)$$

which we can verify by

```
> simplify(a*h*(((1-u*h/b)/(u*v+u*h/b))+1));
```

$$\frac{a\,h\,b\,(1+u\,v)}{u\,(b\,v+h)}$$

(iii) Limit for b->∞

```
> limit(newlambda,b=infinity);
```

$$\frac{a\,(h+d\,v+u\,h\,v)}{u\,v}$$

Which is the same as

$$a\left(\frac{h}{u\,v}+\frac{d}{u}+h\right)$$

(iv) Limit for d->0 and b->∞

```
> limit(limit(newlambda,d=0),b=infinity);
```

$$\frac{a\,h\,(1+u\,v)}{u\,v}$$

Which is the same as

$$a\,h\left(\frac{1}{u\,v}+1\right)$$

Question 7

(i)

(a) Solve for equilibrium

```
> solve(105=p+0.5*(20)-0.5*(10),p);
```

$$100.$$

(b) GM line

To derive the GM line we first solve for r and then substitute this into the equation for *pdot* p.

```
> solve(m=p+k*y-u*r,r);
```

$$\frac{-m+k\,y+p}{u}$$

```
> solGM7:=solve(0=-(1-c)*y-d*((-m+k*y+p)/u)+g+h*(s-p),p);
```

$$solGM7 := \frac{-y\,u+y\,u\,c+d\,m-d\,k\,y+g\,u+h\,s\,u}{d+u\,h}$$

```
> collect(solGM7,{y,m,s});
```

$$\frac{(-d\,k-u+u\,c)\,y}{d+u\,h}+\frac{u\,h\,s}{d+u\,h}+\frac{d\,m}{d+u\,h}+\frac{g\,u}{d+u\,h}$$

Which can be expressed

$$p=-\frac{\left(1-c+\frac{d\,k}{u}\right)y}{h+\frac{d}{u}}+\frac{g+\frac{d\,m}{u}}{h+\frac{d}{u}}+\frac{h\,s}{h+\frac{d}{u}}$$

```
> para7:={c=0.8,d=0.1,k=0.5,u=0.5,y=20,g=5,m=105,h=0.01,v=0.2,b=0.004};
```

$$para7 := \{m=105,\ k=.5,\ y=20,\ u=.5,\ v=.2,\ c=.8,\ g=5,\ b=.004,\ h=.01,\ d=.1\}$$

```
> GMline:=subs(para7,solGM7);
```

$$GMline := 95.23809524+.04761904762\,s$$

Hence the equation for the GM line is

$$p=95.23809524+.04761904762\,s$$

(c) AM line

```
> solve(0=h*(s-p)+b*(r-rstar-v*(sbar-s)),r);
```

$$\frac{-h\,s + h\,p + b\,rstar + b\,v\,sbar - b\,v\,s}{b}$$

```
> solLM7:=solve(m=p+k*y-u*(-(h*s-h*p-b*rstar-b*v*sbar+b*v*s)/b),p);
```

$$solLM7 := -\frac{-m\,b + k\,y\,b + h\,s\,u - u\,b\,rstar - u\,b\,v\,sbar + u\,b\,v\,s}{b - u\,h}$$

Which can be expressed

$$p = \frac{m - k\,y + u\,rstar}{1 - \frac{u\,h}{b}} + \frac{u\,v\,sbar}{1 - \frac{u\,h}{b}} - \frac{\left(u\,v + \frac{u\,h}{b}\right)s}{1 - \frac{u\,h}{b}}$$

```
> LMline:=subs({rstar=10,sbar=100},subs(para7,solLM7));
```

$$LMline := -440.00 + 5.40\,s$$

Hence, equation for the LM curve is

$p = -440.00 + 5.40\,s$

We can check the equilibrium results by solving

```
> solve({p=95.23809524+.4761904762e-1*s,p=-440.00+5.40*s},{s,p});
```

$$\{p = 100.0000000, s = 100.0000000\}$$

```
>
```

(ii) Rise in money supply from 105 to 110

```
> para72:={c=0.8,d=0.1,k=0.5,u=0.5,y=20,g=5,m=110,h=0.01,v=0.2,b=0.004};
```

$$para72 := \{k = .5, y = 20, u = .5, v = .2, c = .8, g = 5, b = .004, h = .01, d = .1, m = 110\}$$

```
> solve(110=p+0.5*(20)-0.5*(10),p);
```

$$105.$$

```
> GMline72:=subs(para72,solGM7);
```

$$GMline72 := 100.0000000 + .04761904762\,s$$

```
> LMline72:=subs({rstar=10,sbar=105},subs(para72,solLM7));
```

$$LMline72 := -462.00 + 5.40\,s$$

Hence, the two new equations are

$p = 100.0000000 + .04761904762\,s$

$p = -462.00 + 5.40\,s$

The new equilibrium is

```
> solve({p=100.0000000+.4761904762e-1*s,p=-462.00+5.40*s},{s,p});
```

$$\{p = 105.0000000, s = 105.0000000\}$$

Point C on the new AM line is at a price of $p = 100$.

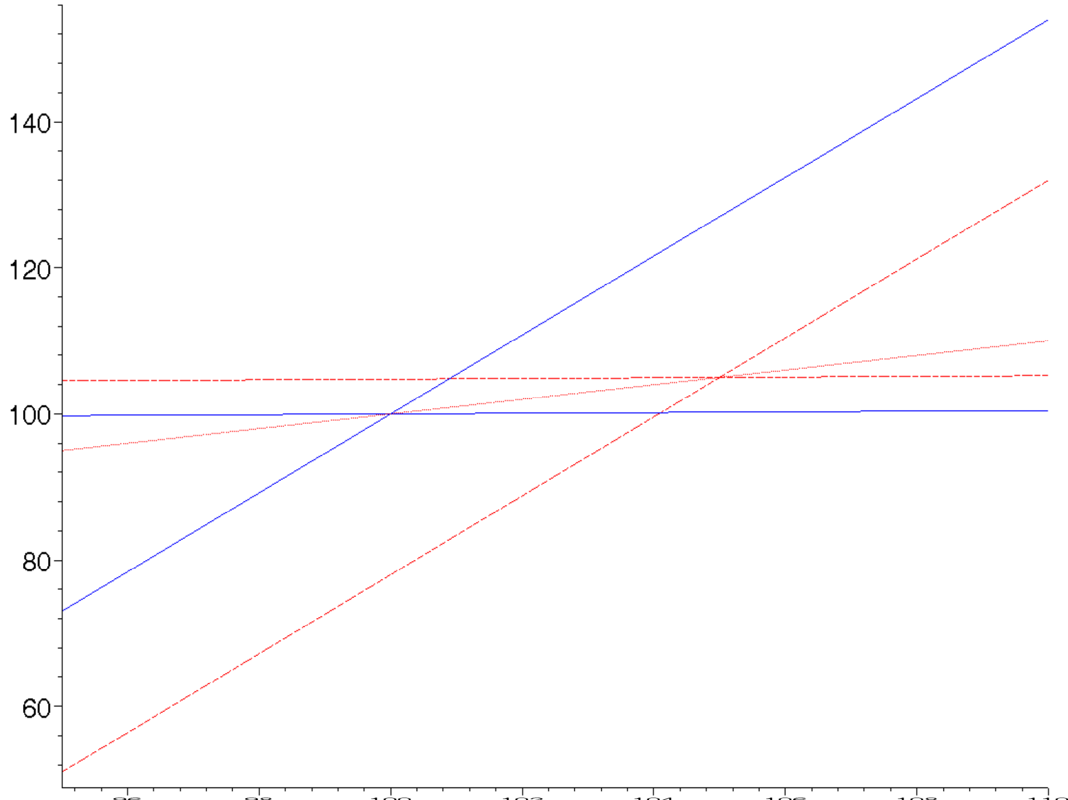
```
> solve(100=-462.00+5.40*s,s);
```

$$104.0740741$$

Hence, the coordinates of point C are $(s, p) = (104.0740741, 100)$.

Also, we have that $dp = ds = 5 = dm$.

```
> lines7:=plot([s,GMline,LMline,GMline72,LMline72],s=95..110
,linestyle=[2,1,1,3,3],colour=[red,blue,blue,red,red]):
> display(lines7);
```




```
> points7:=[point([100,100]),point([105,105]),point([104.074
,100])]:
> display(lines7,points7, labels=["s","p"],title="Rise in
money supply");
```

Question 8

(i) Equilibrium

```
> pdot:='pdot': sdot:='sdot':
> pbar:=105-0.5*(20)+0.5*(18);
```

$$pbar := 104.0$$

Hence, *pbar* = *sbar* = 104.

(ii) Saddle path solutions

```
> pdot:=0.01*(0.1*(s-p)+0.8*(20)+4-y);
```

$$pdot := .001\ s - .001\ p + .200 - .01\ y$$

```
> newpdot:=-0.001*(p-pbar)+0.001*(s-sbar);
```

$$newpdot := -.001\ p + .1040 + .001\ s - .001\ sbar$$

```
> sdot:=(1/0.5)*(s-sbar);
```

$$sdot := 2.000000000\ s - 2.000000000\ sbar$$

```
> A:=matrix([[-0.001, 0.001], [2, 0]]);
```

$$A := \begin{bmatrix} -.001 & .001 \\ 2 & 0 \end{bmatrix}$$

```
> eigenvalues(A);
```

$$-.04522415455, .04422415455$$

(iii) Saddle-path equations

(a)

```
> eigenvectors(A);
```

$$[-.04522415455, 1, \{[-.009145577155, .4044554176]\}],$$
$$[.04422415455, 1, \{[-.01222409585, -.5528244001]\}]$$

```
> Imatrix:=array(identity,1..2,1..2);print(Imatrix);
```

$$Imatrix := \text{array}(identity, 1..2, 1..2, [\ ])$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
> evalm(A-evalm(.4422415455e-1*Imatrix));
```

$$\begin{bmatrix} -.04522415455 & .001 \\ 2. & -.04422415455 \end{bmatrix}$$

```
> solve(-.4522415455e-1*(p-104)+.1e-2*(s-104)=0,p);
```

$$101.7003440 + .02211207727\, s$$

(b)

```
> evalm(A+evalm(.4522415455e-1*Imatrix));
```

$$\begin{bmatrix} .04422415455 & .001 \\ 2. & .04522415455 \end{bmatrix}$$

```
> solve(.4422415455e-1*(p-104)+.1e-2*(s-104)=0,p);
```

$$106.3516560 - .02261207727\, s$$

(iv) Rise in the money supply to 110

The change in the money supply has no affect on the dynamics of the system, i.e., the matrix of the system remains unaffected. However, the equilibrium value changes to

```
> newpbar=110-0.5*(20)+0.5*(18);
```

$$newpbar = 109.0$$

Hence, *pbar* = *sbar* = 109.

```
> solve(-.4522415455e-1*(p-109)+.1e-2*(s-109)=0,p);
```

$$106.5897836 + .02211207727\, s$$

```
> solve(.4422415455e-1*(p-109)+.1e-2*(s-109)=0,p);
```

$$111.4647164 - .02261207727\, s$$

Question 9

(i) Initial situation

```
> pdot:='pdot': sdot:='sdot': pbar:='pbar':
> pbar0:=105-0.5*(20)+0.5*(10);
```

$$pbar0 := 100.0$$

Hence, *pbar* = *sbar* = 100.

(ii) Saddlepaths

```
> pdot=-0.1*(0.01)*(p-pbar)+0.1*(0.01)*(s-sbar);
```

$$pdot = -.001\, p + .001\, pbar + .001\, s - .001\, sbar$$

```
> sdot:=(0.8/0.5)*(p-pbar)+(0.2/0.5)*(s-sbar);
```

$$sdot := 1.600000000\, p - 1.600000000\, pbar + .4000000000\, s - .4000000000\, sbar$$

```
> A9:=matrix([[-0.001, 0.001], [1.6, 0.4]]);
```

$$A9 := \begin{bmatrix} -.001 & .001 \\ 1.6 & .4 \end{bmatrix}$$

```
> eigenvalues(A9);
```

$$-.004951094397, .4039510944$$

```
> eigenvectors(A9);
```

$$[-.0049510944, 1, \{[-.009992203554, .03948013947]\}], [.4039510944, 1, \{[-.002447480793, -.9911100253]\}]$$

```
> evalm(A9-evalm(.4039510944*Imatrix));
```

$$\begin{bmatrix} -.4049510944 & .001 \\ 1.6 & -.0039510944 \end{bmatrix}$$

```
> solve(-.4049510944*(p-100)+.1e-2*(s-100),p);
```

$$99.75305660 + .002469433998\, s$$

```
> evalm(A9+evalm(.4951094397e-2*Imatrix));
```

$$\begin{bmatrix} .003951094397 & .001 \\ 1.6 & .4049510944 \end{bmatrix}$$

```
> solve(.3951094397e-2*(p-100)+.1e-2*(s-100)=0,p);
```

$$125.3094434 - .2530944340\, s$$

The saddle paths are, then,

$p = 99.75305660 + .002469433998\, s$

$p = 125.3094434 - .2530944340\, s$

(iii) Resource discovery

```
> solve({p=s+(4-(1-0.8)*(20)+2*(0.5))/0.01,p=((105-0.5*(20)-
0.3+0.5*(10))/0.8)-(0.2/0.8)*s},{s,p});
```

$$\{s = 19.70000000, p = 119.7000000\}$$

The matrix of the system remains unaffected and so we have the same eigenvalues

```
> evalm(A9-evalm(.4039510944*Imatrix));
```

$$\begin{bmatrix} -.4049510944 & .001 \\ 1.6 & -.0039510944 \end{bmatrix}$$

```
> solve(-.4049510944*(p-119.7)+.1e-2*(s-19.7),p);
```

$$119.6513522 + .002469433998\, s$$

```
> evalm(A9+evalm(.4951094397e-2*Imatrix));
```

$$\begin{bmatrix} .003951094397 & .001 \\ 1.6 & .4049510944 \end{bmatrix}$$

```
> solve(.3951094397e-2*(p-119.7)+.1e-2*(s-19.7),p);
```

$$124.6859603 - .2530944340\, s$$

Hence, the saddle path equations are

$p = 119.6513522 + .002469433998\, s$

$p = 124.6859603 - .2530944340\, s$

Question 11

(i)

```
> s:='s':sdot:='sdot':
> solve(106-s-1=0.5*20-0.5*(10+sdot),sdot);
```

$$-200. + 2.\, s$$

Hence the differential equation is

$$\frac{\partial}{\partial t} s = -200 + 2\, s$$

(ii)

```
> solve(0=-200.+2.*s,s);
```

$$100.$$

Question 12

```
> p:='p':s:='s':sdot:='sdot':pstar:='pstar':lambda:='lambda':
```

$m - s - pstar = k\, y - u\,(rstar + sdot)$

Since we have defined a stationary equilibrium where real money balances are constant, then

$mdot - sdot - pdotstar = 0$

or

$\lambda - sdot - pistar = 0$

```
> subs(sdot=lambda-pistar,m-s-pstar-k*y+u*(rstar+sdot));
```

$$m - s - pstar - k\, y + u\,(rstar + \lambda - pistar)$$

Hence,

$sbar = m - pstar - k\, y + u\,(rstar + \lambda - pistar)$