Worksheet 7.1 Calculation of beam parameters

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Define physical constants $c = 2.998 \times 10^{8} \frac{m}{s}$ $\varepsilon_0 = 8.854 \times 10^{-12} \cdot \frac{F}{m}$

Rest mass of the electron $m_0 := 9.10938291 \cdot 10^{-31} \cdot kg$

Elementary charge $q_e := 1.602176565 \cdot 10^{-19} \cdot C$

Charge/mass ratio of the electron $\eta := \frac{q_e}{m_0}$ $\eta = 1.759 \times 10^{11}\, C \cdot kg^{-1}$

Rest energy of the electron (eV) $V_R := \frac{m_0 \cdot c^2}{q_e}$ $V_R = 510.999\, kV$

Define micro-perveance $\mu perv := \mu A \cdot V^{-1.5}$

When an electron is accelerated from rest through a potential difference V_0 (in kV) its velocity can be computed either using the full relativistic formula or using an approximation which is valid at low voltages.

$$u_0(V) := c \cdot \sqrt{1 - \frac{1}{\left(1 + \frac{V}{V_R}\right)^2}}$$

Equation 1.4

$$\gamma_R(V) := 1 + \frac{V}{V_R}$$

Equation 7.12

$$u_{nonrel}(V) := c \cdot \sqrt{\frac{2 \cdot V}{V_R}}$$

Equation 1.5

$V0 := 1 \cdot kV, 2 \cdot kV .. 1000 \cdot kV$

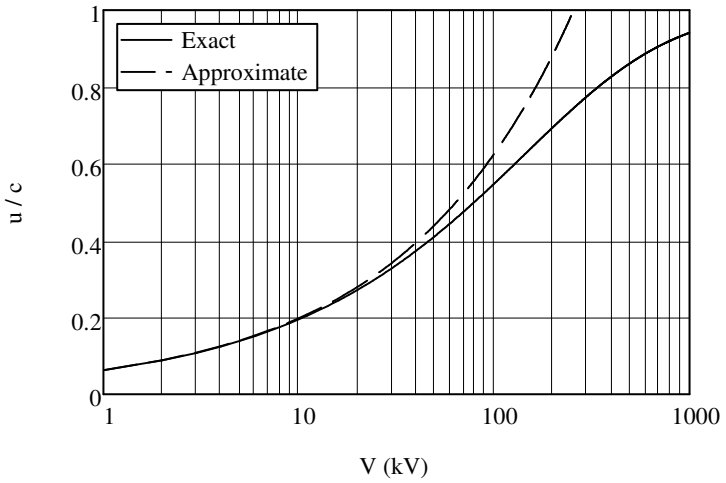

Figure 1.3

Computation of the parameters of an electron beam

Define the anode voltage and current and calculate the perveance, DC beam power and electron velocity

$V_a := 30 \cdot kV$ $I_0 := 5 \cdot A$

$\frac{I_0}{V_a^{1.5}} = 0.962 \cdot \mu perv$ $I_0 \cdot V_a = 150 \cdot kW$ $u_0(V_a) = 9.844 \times 10^7 \frac{m}{s}$

Define the tunnel and mean beam radii

$a := 5 \cdot mm$ $b := 3 \cdot mm$

Calculate the beam voltage and velocity allowing for space-charge potential depression and relativity

$V_0 :=$ $V_0 \leftarrow V_a$ $V_0 = 29.1 \cdot kV$

for $n \in 0..3$

$u_n \leftarrow c \cdot \left[1 - \frac{1}{\left(1 + \frac{V_n}{V_R}\right)^2}\right]^{0.5}$ Equation 1.4

$V_{n+1} \leftarrow V_0 - \frac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left(\frac{1}{2} - \ln\left(\frac{b}{a}\right)\right)$ Equation 7.8

return V_{n+1}

Calculate the plasma frequency and the Brillouin field

$$\omega_p := \left[\frac{\eta}{\varepsilon_0}\cdot\frac{I_0}{\left(\pi\cdot b^2\cdot u_0(V_0)\right)}\right]^{0.5}$$

Equation 7.47

$\omega_p = 6.017\times 10^{9}\cdot s^{-1}$

$\frac{\omega_p}{2\cdot\pi} = 0.958\cdot GHz$

$$B_B := \sqrt{\frac{2}{\gamma_R(V_0)}}\cdot\frac{\omega_p}{\eta}$$

Equation 7.55

$B_B = 0.047\,T$

Solenoid focusing

Specify the ratio of the solenoid field to the Brillouin field and calculate the focusing field

$m_B := 2$

$B_0 := m_B\cdot B_B$

$B_0 = 0.094\,T$

Calculate the cathode flux

$$K_c := \sqrt{1-\frac{1}{m_B^{\,2}}}$$

Equation 7.57

$$\Phi_k := K_c\cdot\pi\cdot b^2\cdot B_0$$

Equation 7.51

$\Phi_k = 2.3\times 10^{-6}\,Wb$

The flux density required at the cathode is then calculated using the radius of the cathode.

P.P.M. focusing

Specify the p.p.m. ripple and calculate the p.p.m. period and the peak magnetic field

$\delta r := 0.1$ $\quad B_{pk} := \sqrt{2} \cdot B_0$ $\quad B_{pk} = 0.133\,T$

Calculate the value of the parameters in terms of the mean radius for the same stiffness

$$\alpha_m := \frac{2 \cdot \delta r \cdot (1 - \delta r)}{\left[1 - K_c \cdot \left[1 - \frac{1}{(1 - \delta r)^2}\right]\right]}$$ Equation 7.98 $\quad \alpha_m = 0.15$ $\quad \alpha := 2 \cdot \alpha_m$

$\omega_L := \frac{1}{2} \cdot \eta \cdot B_{pk}$ Larmor frequency: Equation 7.39 $\quad \beta_L := \frac{\omega_L}{u_0(V_0)}$ $\quad \omega_L = 1.171 \times 10^{10} \cdot s^{-1}$ $\quad \frac{\omega_L}{2 \cdot \pi} = 1.863 \cdot GHz$

$\beta_0 := \frac{\beta_L}{\sqrt{2 \cdot \alpha}}$ Equation 7.89 $\quad PPM\_Period := \frac{2 \cdot \pi}{\beta_0}$ $\quad PPM\_Period = 40.3 \cdot mm$