

WS 4.7 Folded waveguide slow-wave structures

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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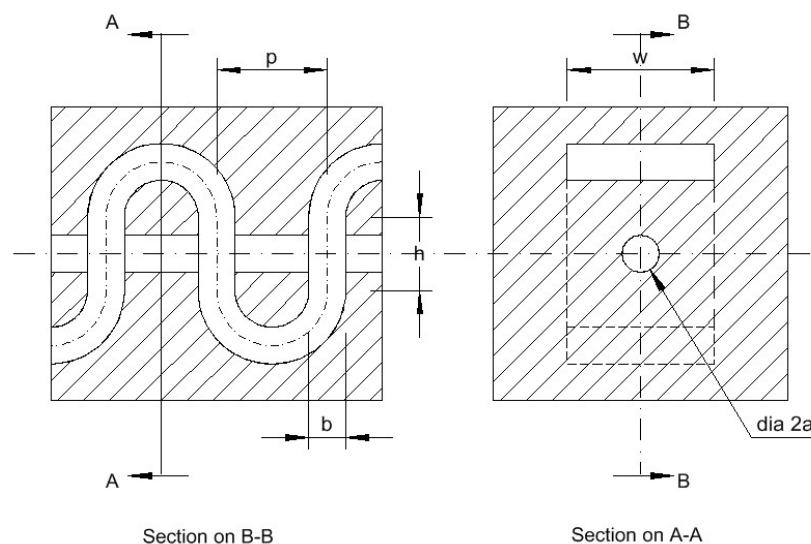
Serpentine waveguide

Derivation of the dispersion equation from Equation 4.108

$$\beta_n p = \left(\frac{\omega^2}{\omega_c^2} - 1 \right)^{\frac{1}{2}} \beta_c (h + \pi p/2) + (2n+1)\pi$$

$$\left(\frac{\beta_n p - (2n+1)\pi}{\beta_c (h + \pi p/2)} \right)^2 + 1 = \left(\frac{\omega^2}{\omega_c^2} \right)$$

$$\frac{\omega}{\omega_c} = \sqrt{\left(\frac{\beta_n p - (2n+1)\pi}{\beta_c (h + \pi p/2)} \right)^2 + 1}$$



Structure dimensions

$p := 10 \cdot \text{mm}$

$h := 10 \cdot \text{mm}$

$w := 20 \cdot \text{mm}$

Cut-off propagation constant and frequency

$\beta_c := \frac{\pi}{w}$

$\omega_c := \beta_c \cdot c$

$\frac{\omega_c}{2 \cdot \pi} = 7.495 \cdot \text{GHz}$

Dispersion equation for the nth space harmonic

$$\omega_{\omega_c}(\theta, n) := \sqrt{\left[\frac{\theta - (2 \cdot n + 1) \cdot \pi}{\beta_c \cdot (h + 0.5 \cdot \pi \cdot p)} \right]^2 + 1}$$

$\theta := 0, 0.01 \dots 4 \cdot \pi$

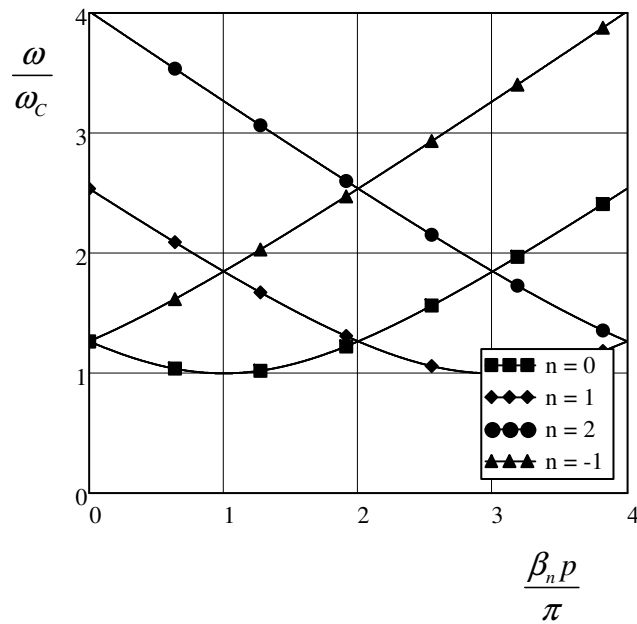


Figure 4.26

Rectangular folded waveguide

Structure dimensions (Note that the variable names are re-used)

$$p := 1 \cdot \text{cm} \quad b_1 := 0.5 \cdot p \quad b_2 := 0.6 \cdot p \quad w := 2 \cdot p \quad h := w - b_2$$

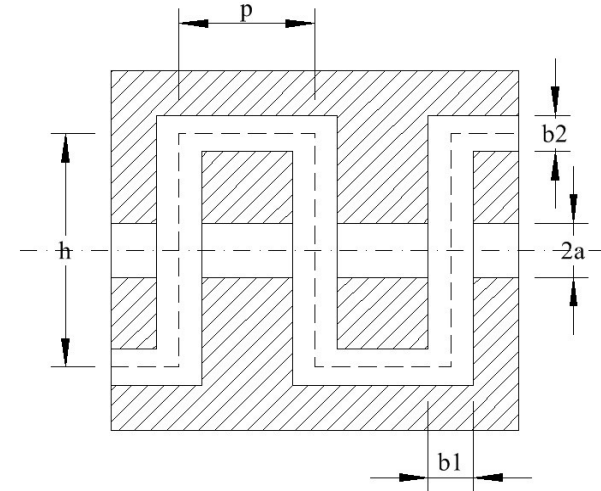
Find the cut-off frequency and the electrical lengths and impedances of the sections.

$$\beta_0(\omega) := \frac{\omega}{c} \quad Z_0 := \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\beta_g := \frac{\pi}{w} \quad \omega_g := c \cdot \beta_c \quad \beta_g(\omega) := \sqrt{\beta_0(\omega)^2 - \beta_c^2} \quad \text{Equation 2.11}$$

$$\psi_1(\omega) := 0.5 \cdot h \cdot \beta_g(\omega) \quad Z_{g1}(\omega) := \frac{(2 \cdot b_1 \cdot \beta_g(\omega) \cdot Z_0)}{w \cdot \beta_0(\omega)} \quad \text{Equation 2.65}$$

$$\psi_2(\omega) := p \cdot \beta_g(\omega) \quad Z_{g2}(\omega) := \frac{(2 \cdot b_2 \cdot \beta_g(\omega) \cdot Z_0)}{w \cdot \beta_0(\omega)}$$



Compute the transfer matrices of the sections of waveguide and of one period of the structure neglecting the reactances of the bends

$$T_1(\omega) := \begin{pmatrix} \cos(\psi_1(\omega)) & j \cdot \frac{Z_{g1}(\omega)}{\Omega} \cdot \sin(\psi_1(\omega)) \\ j \cdot \frac{\Omega}{Z_{g1}(\omega)} \cdot \sin(\psi_1(\omega)) & \cos(\psi_1(\omega)) \end{pmatrix} \quad T_2(\omega) := \begin{pmatrix} \cos(\psi_2(\omega)) & j \cdot \frac{Z_{g2}(\omega)}{\Omega} \cdot \sin(\psi_2(\omega)) \\ j \cdot \frac{\Omega}{Z_{g2}(\omega)} \cdot \sin(\psi_2(\omega)) & \cos(\psi_2(\omega)) \end{pmatrix} \quad \text{Equation 4.111}$$

$$T_p(\omega) := T_1(\omega) \cdot T_2(\omega) \cdot T_1(\omega) \quad \text{Equation 4.112}$$

Find the phase shift per section and the total impedance

$$\psi(\omega, n) := \text{acos}\left(T_p(\omega)_{0,0}\right) + (2 \cdot n + 1) \cdot \pi$$

Equation 4.113

$$Z_T(\omega) := \sqrt{\frac{T_p(\omega)_{0,1}}{T_p(\omega)_{1,0}}}$$

Equation 4.114

$$\frac{\omega_c}{2 \cdot \pi} = 7.495 \text{ GHz}$$

$$\frac{b_2}{b_1} = 1.2$$

Frequency range for the graphs

$$\omega := \omega_c, 1.001 \cdot \omega_c \dots 3 \cdot \omega_c$$

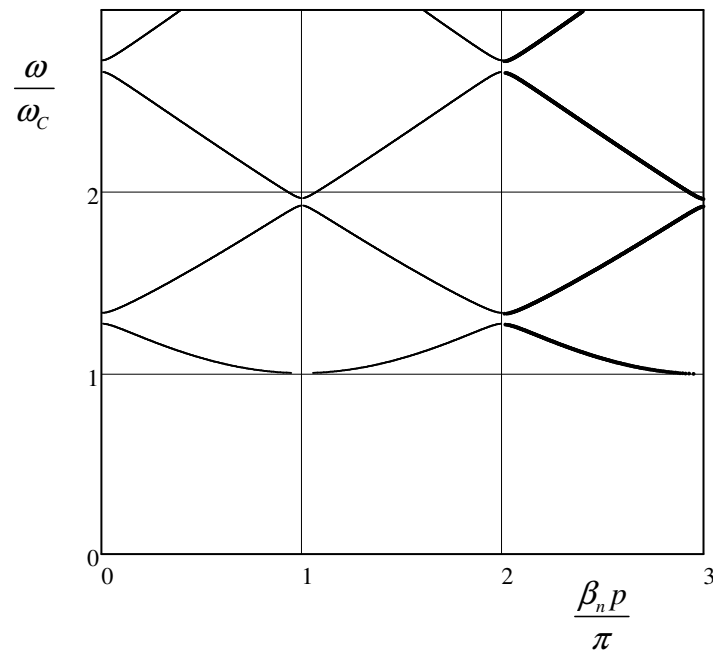


Figure 4.28a

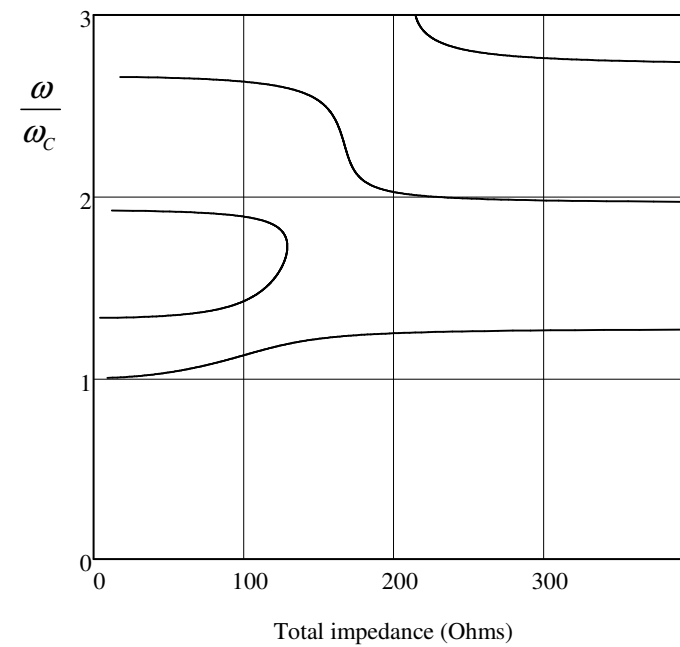


Figure 4.28b