Worksheet 10.2 Multi-element depressed collector model (klystron)

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet calculates the properties of a typical klystron with a multi-element depressed collector

Normalised beam radius $\beta eb := 0.6$

Normalised tunnel radius $\beta ea := 1.0$

Normalised Output gap length $\beta eg := 1.0$

Efficiency at saturation $\eta_{sat} := 0.53$

Number of collector stages N_stages := 5

Normalised electrode voltages

Vc :=

	0
0	0
1	0.2
2	0.4
3	0.6
4	0.8
5	1
6	

Collection probability function

probability :=

- Constant
- Linear
- Sine
- Sine squared

The fraction of the current collected by each stage is estimated by assuming a probability distribution. Current begins to be collected on an electrode when the electron voltage exceeds the stage voltage and rises to 100% when the electron voltage is just less than the voltage of the next stage. Current not collected at the stage voltage is collected at the voltage of the preceding stage. For the final stage the presence of a fictitious electrode is assumed such that the voltage difference between it and the nth voltage is equal to that between the nth and (n-1)th stages. See Section 10.3.2

Define the probability distributions

$$p(x) := \begin{cases} 1 & \text{if probability} = 1 \\ x & \text{if probability} = 2 \\ \sin\left(\frac{\pi \cdot x}{2}\right) & \text{if probability} = 3 \\ \sin\left(\frac{\pi \cdot x}{2}\right)^2 & \text{if probability} = 4 \end{cases}$$

Constant probability

$$\begin{aligned} pc0(Ve) := \; & Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ & \text{for } n \in N\_stages .. 0 \\ & \quad pc_n \leftarrow \begin{cases} 1 & \text{if } Ve \geq Vc_n \wedge Ve < Vc_{n+1} \\ 1 & \text{if } n = N\_stages \wedge Ve \geq Vc_{n+1} \\ 1 & \text{if } n = 0 \wedge Ve < Vc_1 \\ 0 & \text{otherwise} \end{cases} \\ & \text{return } pc \end{aligned}$$

Variable probability

$$\begin{aligned} pc(Ve) := \; & Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ & \text{for } n \in N\_stages .. 0 \\ & \quad pc_n \leftarrow \begin{cases} p\left(\frac{Ve - Vc_n}{Vc_{n+1} - Vc_n}\right) & \text{if } Ve \geq Vc_n \wedge Ve < Vc_{n+1} \\ 1 & \text{if } n = N\_stages \wedge Ve \geq Vc_{n+1} \\ 1 & \text{if } n = 0 \wedge Ve < Vc_1 \\ \left(1 - pc_{n+1}\right) & \text{if } n < N\_stages \wedge \left(Ve \geq Vc_{n+1} \wedge Ve < Vc_{n+2}\right) \\ 0 & \text{otherwise} \end{cases} \\ & \text{return } pc \end{aligned}$$

Plot the probability of collection by each electrode as a function of the electron energy with rectangles showing constant probability for comparison.

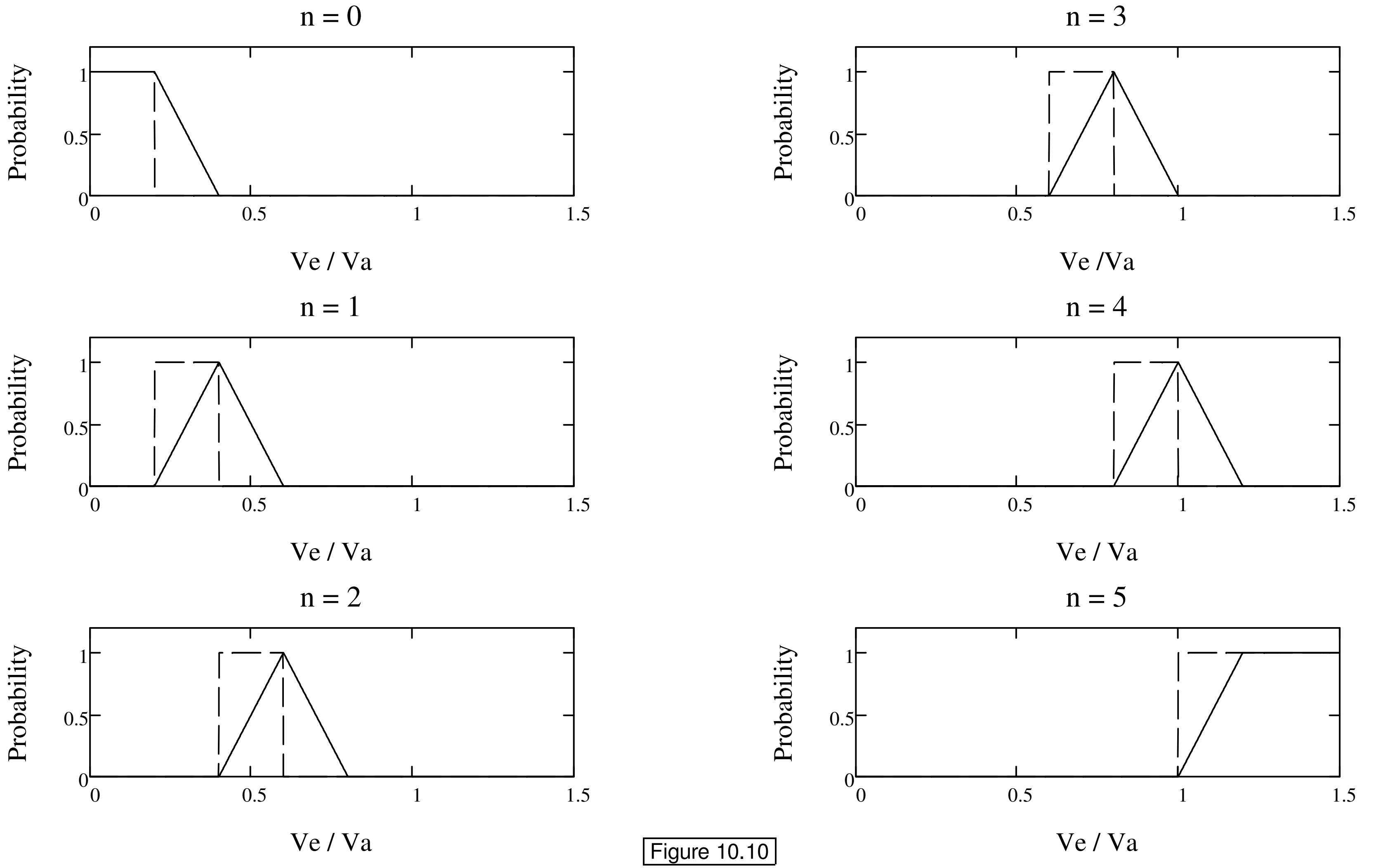


Figure 10.10

Calculate the DC collector efficiency as a function of electron energy

$$\eta_{DC}(Ve) := \frac{1}{Ve} \cdot \sum_{n=0}^{N\_stages} \left(pc(Ve)_n \cdot Vc_n \right)$$ Equation 10.26

Klystron spent beam data as described in Worksheet 13.5

Beam current waveform at the output gap $I(\theta, X) := 1 + 1.33 \cdot X \cdot \cos(\theta) + 0.33 \cdot X^2 \cdot \cos(2\theta)$ Equation 13.52

Normalised fundamental RF beam current $I_1(X) := 1.33 \cdot X$ Equation 13.53

Reciprocal of the mean of the DC and spent electron velocities normalised to the DC velocity as a function of the normalised spent electron energy (Vs / V0).

$$U(V_s) := \frac{2}{1 + \sqrt{V_s}} \qquad \left(U = \frac{2 \cdot u_0}{u_0 + u_s} \right)$$

Effective gap coupling factor

$$M(U) := \frac{2 \cdot I1(\beta eb \cdot U)}{\beta eb \cdot U \cdot I0(\beta ea)} \cdot \mathrm{sinc}(0.5 \cdot U \cdot \beta eg)$$

At saturation the effective normalised gap voltage is

$$V_{eff\_sat} := \frac{2 \cdot \eta_{sat}}{I_1(1)}$$ Equation 13.54

The normalised energy of the slowest electrons is

$$V_{min\_sat} := 1 - V_{eff\_sat}$$

The normalised load resistance is

$$R_L := \frac{V_{eff\_sat}}{M\left(U\left(V_{min\_sat}\right)\right)^2 \cdot I_1(1)}$$ Equation 13.55

Now use an iterative calculation to find self-consistent values of the spent-beam voltage and the effective gap voltage as a function of X

$$Vs(X) := \left| \begin{array}{l} Vs_0 \leftarrow 1.0 \\ \text{for } n \in 1..10 \\ \quad \left| \begin{array}{l} U \leftarrow \dfrac{2}{1+\sqrt{Vs_{(n-1)}}} \\ Vs_n \leftarrow 1 - M(U)^2 \cdot I_1(X) \cdot R_L \end{array} \right. \\ Vs_{10} \end{array} \right.$$

$$V_{geff}(X) := 1 - Vs(X)$$

$$V_s(\theta, X) := 1 - V_{geff}(X) \cdot \cos(\theta)$$

$\boxed{\text{Equation 13.56}}$

$$\eta_e(X) := \frac{1}{2} \cdot I_1(X) \cdot V_{geff}(X)$$

$$V_g(X) := \frac{V_{geff}(X)}{M(U(Vs(X)))}$$

$$PdB(X) := 10 \cdot \log\left(\frac{\eta_e(X)}{\eta_e(1)}\right)$$

Output backoff in dB

Check the total current and the total power entering the collector

$$I_0(X) := \frac{1}{\pi} \cdot \int_0^{\pi} I(\theta, X)\, d\theta$$

$$I_0(1) = 1$$

$$P_{ent}(X) := \frac{1}{\pi} \cdot \int_0^{\pi} I(\theta, X) \cdot V_s(\theta, X)\, d\theta$$

$$P_{ent}(1) = 0.47$$

Check $\quad 1 - P_{ent}(1) = 0.53$

$$\eta_e(1) = 0.53$$

Calculate the rf efficiency of the collector as a function of the drive parameter X. (X = 1 at saturation)

$$\eta_c(X) := \frac{1}{\pi \cdot P_{ent}(X)} \cdot \int_0^{\pi} I(\theta, X) \cdot V_s(\theta, X) \cdot \eta_{DC}\left(\left|V_s(\theta, X)\right|\right) d\theta$$

Compute the RF efficiency as a function of drive level

$$\eta_{rf}(X) := \frac{\eta_e(X)}{1 - \eta_c(X)\cdot\left(1 - \eta_e(X)\right)}$$

Equation 10.13

Current collected

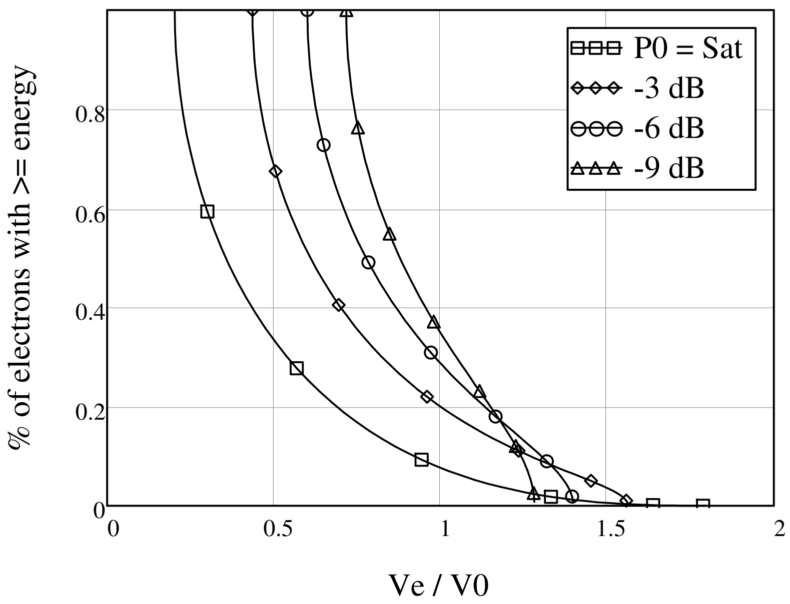
$$Ic(\theta, X) := \frac{1}{\pi}\cdot\int_{\theta}^{\pi} I(\theta, X)\, d\theta$$

Equation 13.57

Value of X for output backoff D dB

$X1 := 1$ $\quad$ $X(D) := \mathrm{root}(PdB(X1) - D, X1)$

$\theta := 0, 0.05 .. \pi$



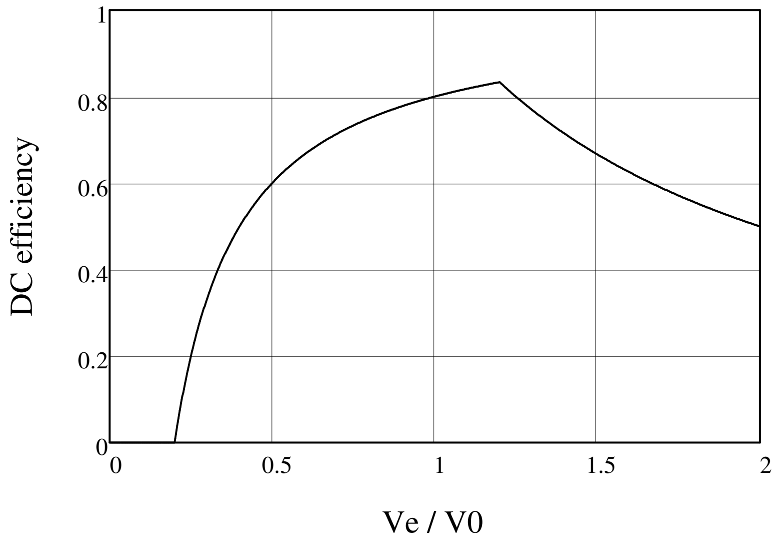
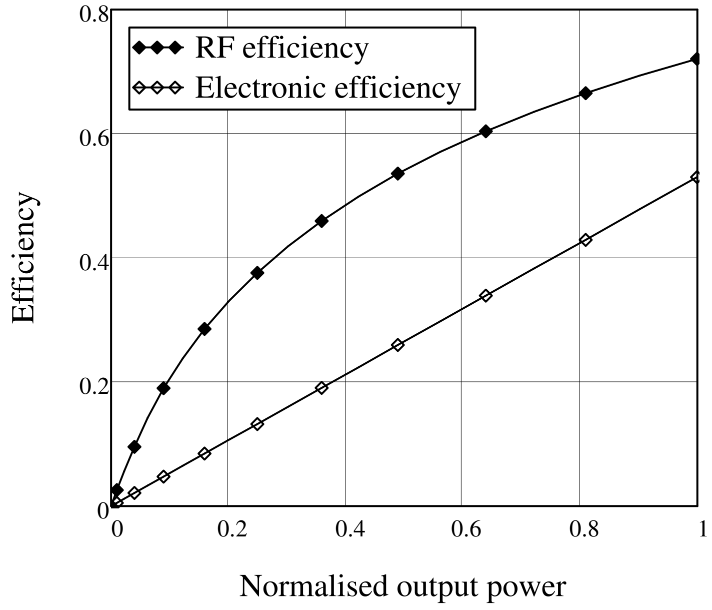


Figure 10.11

The shape of this curve is very insensitive to different assumptions about the current waveform: linear, sine or sine squared.

$X2 := 0, 0.05 .. 1$



Normalised load resistance $R_L = 0.953$

Normalised effective gap voltage $V_{geff}(1) = 0.797$

Normalised effective gap voltage $V_g(1) = 1.005$