WS 6.3 Electrostatic solution for a tetrode

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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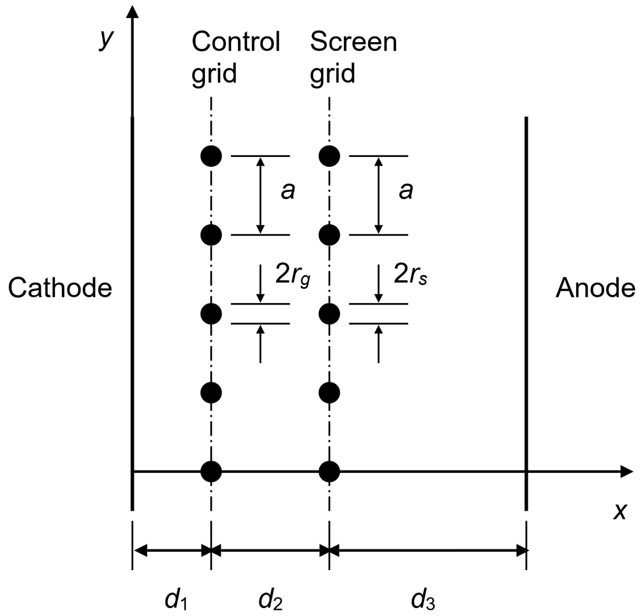


Dimensions in millimetres

$a := 1$ $\quad r := 0.05$

$d_1 := 0.442$ $\quad d_2 := 0.620$ $\quad d_3 := 9.983$

Electrode potentials

$V_g := 20 \cdot V$ $\quad V_s := 900 \cdot V$ $\quad V_a := 6000 \cdot V$

The working equations are

$$f_s(x,y) := \frac{a}{4\cdot\pi}\cdot\ln\left[\frac{\cosh\left[\frac{2\cdot\pi\cdot(x+d_1+d_2)}{a}\right]-\cos\left(\frac{2\cdot\pi\cdot y}{a}\right)}{\cosh\left[\frac{2\cdot\pi\cdot(x-d_1-d_2)}{a}\right]-\cos\left(\frac{2\cdot\pi\cdot y}{a}\right)}\right]$$

Equation 6.76

$$D_s := -\frac{a}{2\cdot\pi\cdot d_3}\cdot\ln\left(2\cdot\sin\left(\frac{\pi\cdot r}{a}\right)\right)$$

Equation 6.77 $D_s = 0.019$ $\frac{1}{D_s} = 54$

$$f_g(x,y) := \frac{a}{4\cdot\pi}\cdot\ln\left[\frac{\cosh\left[\frac{2\cdot\pi\cdot(x+d_1)}{a}\right]-\cos\left(\frac{2\cdot\pi\cdot y}{a}\right)}{\cosh\left[\frac{2\cdot\pi\cdot(x-d_1)}{a}\right]-\cos\left(\frac{2\cdot\pi\cdot y}{a}\right)}\right]$$

Equation 6.24

$$D_g := -\frac{a}{2\cdot\pi\cdot d_2}\cdot\ln\left(2\cdot\sin\left(\frac{\pi\cdot r}{a}\right)\right)$$

Equation 6.81 $D_g = 0.298$ $\frac{1}{D_g} = 3.4$

$$A1 := \begin{pmatrix} d_1 + d_2\cdot D_g & d_1 & d_1 \\ d_1 & d_1 + d_2 + d_3\cdot D_s & d_1 + d_2 \\ d_1 & d_1 + d_2 & d_1 + d_2 + d_3 \end{pmatrix}$$

Equation 6.82 $\frac{1}{D_s\cdot D_g} = 181$

$$V_t(x,y) := \begin{pmatrix} f_g(x,y) & f_s(x,y) & x \end{pmatrix}\cdot(A1)^{-1}\cdot\begin{pmatrix} V_g \\ V_s \\ V_a \end{pmatrix}$$

Equation 6.75

Generate a matrix for a potential plot

$n := 50$

$$
Vt\_plot := \left|
\begin{array}{l}
\text{for } i \in 0 .. \left(\dfrac{d_1 + d_2 + d_3}{a}\right)\cdot n \\
\quad \text{for } j \in 0 .. n \\
\quad\quad \left|
\begin{array}{l}
x \leftarrow \dfrac{i}{n}\cdot a \\
y \leftarrow \dfrac{j}{n}\cdot a \\
V_{j,i} \leftarrow \left|
\begin{array}{ll}
V_g & \text{if } \left[\left(x - d_1\right)^2 + y^2\right] \le r^2 \vee \left[\left(x - d_1\right)^2 + (a - y)^2\right] \le r^2 \\
V_s & \text{if } \left[\left(x - d_1 - d_2\right)^2 + y^2\right] \le r^2 \vee \left[\left(x - d_1 - d_2\right)^2 + (a - y)^2\right] \le r^2 \\
V_t(x, y) & \text{otherwise}
\end{array}
\right.
\end{array}
\right. \\
\text{return } V
\end{array}
\right.
$$

Calculate electron trajectories

$E_x(x, y) := -\dfrac{d}{dx}V_t(x, y)\cdot\dfrac{1}{V}$ $\quad$ $E_y(x, y) := -\dfrac{d}{dy}V_t(x, y)\cdot\dfrac{1}{V}$ $\quad$ $\eta := -1.759\cdot 10^{11}$

Estimate the maximum flight time

$d_{max} := d_1 + d_2 + d_3$ $\quad$ $s = \dfrac{1}{2}\cdot a\cdot t^2$ $\quad$ $t = \sqrt{\dfrac{2\cdot s}{a}}$ $\quad$ $t_{max} := 1.5\cdot\sqrt{\dfrac{2\cdot d_{max}{}^2\cdot V}{|\eta|\cdot V_a}}$ $\quad$ $t_{max} = 7.212\times 10^{-7}$

Set up the simultaneous first order differential equations for the electron motion

$$\frac{d}{dt}v_x = \eta\cdot E_x$$

$$\frac{d}{dt}x = v_x$$

$$\frac{d}{dt}v_y = \eta\cdot E_x$$

$$\frac{d}{dt}y = v_y$$

$$Dt(t,z) := \begin{pmatrix} \eta\cdot E_x(z_1, z_3) \\ z_0 \\ \eta\cdot E_y(z_1, z_3) \\ z_2 \end{pmatrix}$$

Number of time steps and number of trajectories

$nsteps := 1000$ $\quad$ $ntraj := 25$

$$Z := \left| \begin{array}{l} \text{for } n \in 0..ntraj \\ \quad \left| \begin{array}{l} init \leftarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ \dfrac{a\cdot n}{2\cdot ntraj} \end{pmatrix} \\ Z_n \leftarrow AdamsBDF(init, 0, t_{max}, nsteps, Dt) \end{array} \right. \\ \text{return } Z \end{array} \right.$$

The matrix Z contains the time in the first column then, in groups of four columns v_x, x, v_y and y for each trajectory.

Unpack the solution matrix

$$xn := \left| \begin{array}{l} nmax \leftarrow 2 \cdot ntraj \\ \text{for } n \in 0..nmax \\ \quad \text{for } i \in 0..nsteps \\ \quad\quad xn_{i,n} \leftarrow \left| \begin{array}{ll} \left[\left(Z_n\right)^{\langle 2 \rangle}\right]_i & \text{if } n \le ntraj \\ \left[\left(Z_{nmax-n}\right)^{\langle 2 \rangle}\right]_i & \text{otherwise} \end{array} \right. \\ \text{return } xn \end{array} \right.$$

$$yn := \left| \begin{array}{l} nmax \leftarrow 2 \cdot ntraj \\ \text{for } n \in 0..nmax \\ \quad \text{for } i \in 0..nsteps \\ \quad\quad yn_{i,n} \leftarrow \left| \begin{array}{ll} \left[\left(Z_n\right)^{\langle 4 \rangle}\right]_i & \text{if } n \le ntraj \\ a - \left[\left(Z_{nmax-n}\right)^{\langle 4 \rangle}\right]_i & \text{otherwise} \end{array} \right. \\ \text{return } yn \end{array} \right.$$

Stop trajectories at x = d1 if they are intercepted by the control grid

$$xm := \left| \begin{array}{l} xm \leftarrow xn \\ \text{for } n \in 0..20 \\ \quad \left| \begin{array}{l} intflag \leftarrow 0 \\ \text{for } i \in 0..nsteps \\ \quad \left| \begin{array}{l} intflag \leftarrow 1 \quad \text{if } \left(xn_{i,n} - d_1\right)^2 + \left(yn_{i,n}\right)^2 \le r^2 \vee \left(xn_{i,n} - d_1\right)^2 + \left(a - yn_{i,n}\right)^2 \le r^2 \\ xm_{i,n} \leftarrow d_1 \quad \text{if } intflag = 1 \end{array} \right. \end{array} \right. \\ \text{return } xm \end{array} \right.$$

Calculate the cathode current density

$$K_1 := \frac{4\cdot\varepsilon_0}{9}\cdot\sqrt{2\cdot 1.759\cdot 10^{11}\cdot C\cdot kg^{-1}}$$

Parameters of the equivalent triode

$$V_e := \frac{V_s + D_s\cdot V_a}{1 + D_s\cdot\left(1 + \frac{d_3}{d_2}\right)}$$ $\boxed{\text{Equation 6.91}}$ $V_e = 767.9\,V$

$$d_e := \frac{d_1 + d_2 + D_s\cdot(d_1 + d_2 + d_3)}{1 + D_s\cdot\left(1 + \frac{d_3}{d_2}\right)}$$ $\boxed{\text{Equation 6.92}}$ $d_e = 0.962$

$$J_0(V_g) := \frac{K_1}{mm^2}\cdot\sqrt{1 + D_g}\cdot\frac{(V_g + D_g\cdot V_e)^{1.5}}{(d_1 + D_g\cdot d_e)^2}$$ $\boxed{\text{Equation 6.93}}$ $J_0(V_g) = 1.97\,A\cdot cm^{-2}$

Calculation of mean cathode current density when there is island formation

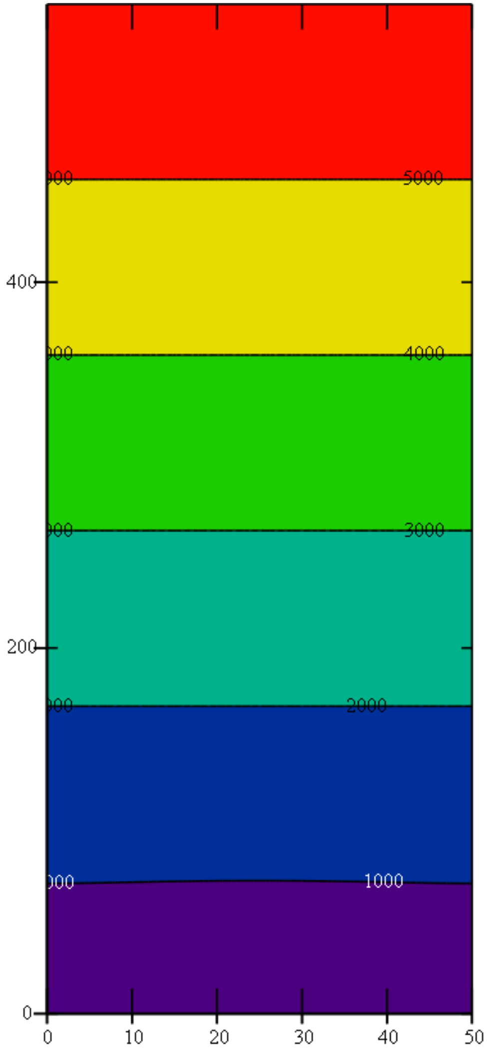
$$f1_g(x,y) := \frac{d}{dx} f_g(x,y)$$

$$D_E(y) := \frac{d_1 \cdot \left(1 - f1_g(0,y)\right) + d_e \cdot D_g}{\left(d_1 + d_e\right) \cdot f1_g(0,y) - d_1}$$ $\boxed{\text{Equation 6.94}}$

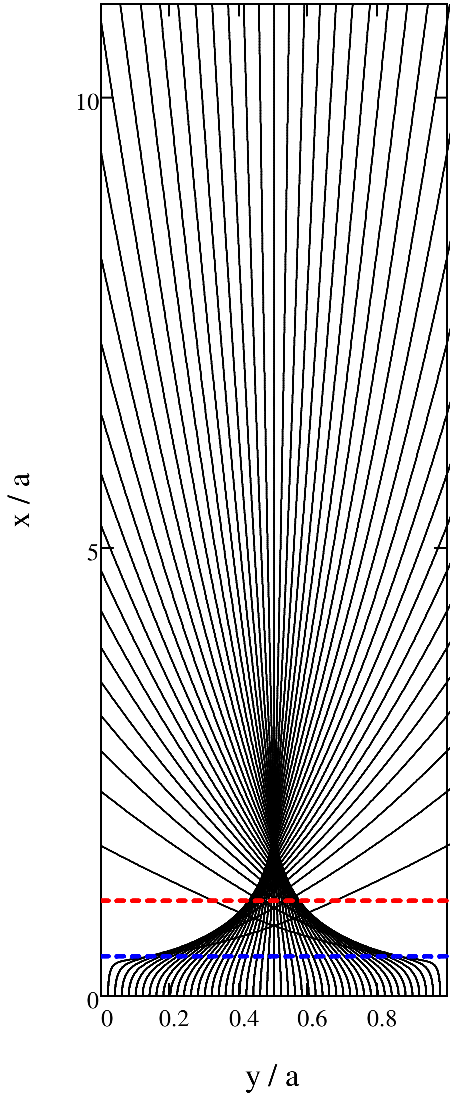
$$F_1\left(V_g, y\right) := \begin{cases} K_1 \cdot \dfrac{\sqrt{1 + D_E(y)} \cdot \left(V_g + D_E(y) \cdot V_e\right)^{1.5}}{\left(d_1 + D_E(y) \cdot d_e\right)^2} & \text{if } V_g + D_E(y) \cdot V_a \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$J_1\left(V_g\right) := \frac{1}{a \cdot mm^2} \cdot \int_0^a F_1\left(V_g, y\right) dy$$ $\boxed{\text{Equation 6.96}}$

$$J_1\left(V_g\right) = 1.98\ A \cdot cm^{-2}$$



Vt_plot



Note: The horizontal and vertical scales of these figures are not normally the same as each other.