WS 15.1 Magnetron anode model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet calculates the properties of a vane-type magnetron anode both without and with straps. The data provided is for the Litton 4J50 magnetron as given in Hull, J. F. (1958). Crossed Field Electron Interaction in Space Charge Limited Beams. New York, Polytechnic Institute of Brooklyn. Doctor of Electrical Engineering.

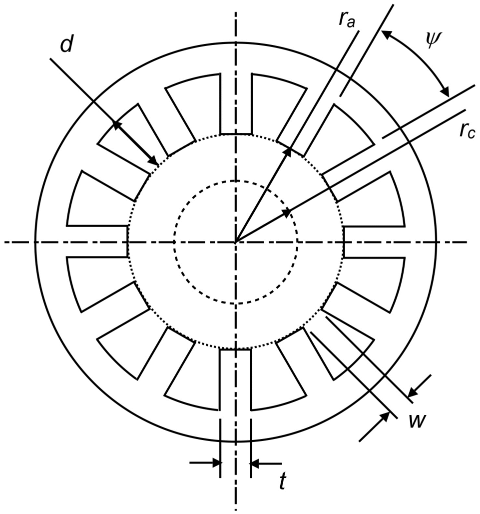
$N_v := 16$ $\psi := \frac{2\cdot\pi}{N_v}$

$r_a := 4.37\cdot mm$ $r_c := 2.84\cdot mm$

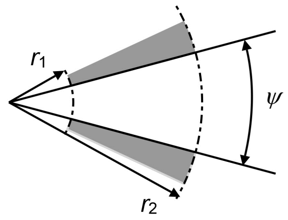
$r_2 := 6.3\cdot mm$ $r_1 := 2.13\cdot mm$

$t := 0.89\cdot mm$ $p := 1.72\cdot mm$

$L_a := 6.35\cdot mm$ $w := 0.83\cdot mm$



(a)



(b)

Input admittance of a cavity

$$Y(k) := j\cdot\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\left(\frac{L_a}{\psi\cdot r_1}\right)\cdot\left(\frac{J0(k\cdot r_1)\cdot Y1(k\cdot r_2) - J1(k\cdot r_2)\cdot Y0(k\cdot r_1)}{J1(k\cdot r_1)\cdot Y1(k\cdot r_2) - J1(k\cdot r_2)\cdot Y1(k\cdot r_1)}\right)$$

Equation 15.57

Find the resonant condition

Guess $k := \frac{\pi}{2\cdot(r_2 - r_1)}$ $k_r := \mathrm{root}(Y(k), k)$ $k_r = 300.92\,\frac{1}{m}$ *CHECK* $Y(k_r) = 0.000\cdot S$

$Y'(k) := \frac{d}{dk}Y(k)$ $Y'(k_r) = 5.996i\times 10^{-5}\cdot m\cdot S$

$C_r := -\frac{j}{2\cdot c}\cdot Y'(k_r)$ Equation 15.62 $C_r = 1.000\times 10^{-13}\,F$

$L_1 := \frac{1}{C_r\cdot {k_r}^2\cdot c^2}$ $L_1 = 1.229\times 10^{-9}\,H$

Calculate the fringing capacitance

$\mu := \frac{N_v\cdot w}{2\cdot\pi\cdot r_a}$ Equation 15.64 $\mu = 0.484$

$C_f := \left(\frac{\varepsilon_0\cdot L_a}{\pi}\right)\cdot\left(1 + \ln\left(\frac{4}{\pi\cdot\mu}\right)\right)$ Equation 15.63 $C_f = 0.035\cdot pF$

$C_1 := C_r + C_f$

$$Y_a(\omega) := j\cdot\left(\omega\cdot C_1 - \frac{1}{\omega\cdot L_1}\right)$$

Equation 15.50

Resonant frequency of a cavity

$$\omega_0 := \frac{1}{\sqrt{L_1\cdot\left(C_r + C_f\right)}} \qquad \omega_0 = 7.758\times 10^{10}\,\frac{1}{s} \qquad f_0 := \frac{\omega_0}{2\cdot\pi} \qquad f_0 = 12.35\cdot GHz$$

Estimate anode/cathode capacitance per vane

Upper bound

Lower bound

Best estimate

$$C_U := \frac{p}{r_a}\cdot\varepsilon_0\cdot L_a\cdot\frac{1}{\ln\left(\frac{r_a}{r_c}\right)} \qquad C_L := \frac{t}{r_a}\cdot\varepsilon_0\cdot L_a\cdot\frac{1}{\ln\left(\frac{r_a}{r_c}\right)} \qquad C_0 := \sqrt{C_U\cdot C_L}$$

$$C_U = 0.051\cdot pF \qquad C_L = 0.027\cdot pF \qquad C_0 = 0.037\cdot pF$$

$$Y_k(\omega,\phi) := \frac{j\cdot\omega\cdot C_0}{2\cdot(1-\cos(\phi))}$$

$$Yr(\omega,\phi) := Y_a(\omega) + Y_k(\omega,\phi)$$

Equation 15.52

Corrected π mode frequency

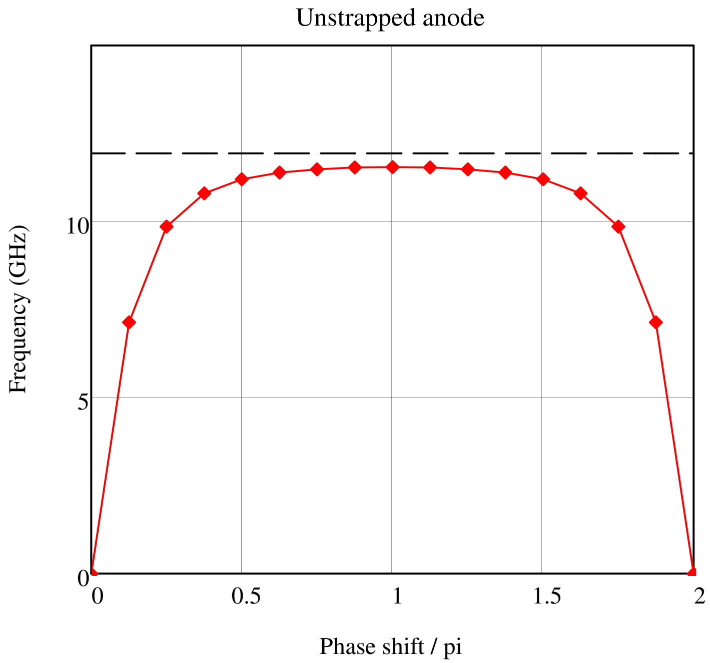
$$\omega_1 := \frac{1}{\sqrt{L_1\cdot\left(C_r + C_f + 0.25\cdot C_0\right)}} \qquad \omega_1 = 7.506\times 10^{10}\,\frac{1}{s} \qquad f_1 := \frac{\omega_1}{2\cdot\pi} \qquad f_1 = 11.95\cdot GHz$$

Dispersion diagram for the unstrapped anode

$$f_u(\phi) := \left| \begin{array}{l} \omega \leftarrow f_1 \cdot \left(1 + \dfrac{C_0}{2 \cdot C_1} \cdot \dfrac{1}{1 - \cos(\phi)}\right)^{-0.5} \quad \text{if } \cos(\phi) \neq 1 \\ \omega \leftarrow 0 \quad \text{otherwise} \end{array} \right.$$

Equation 15.56

$$\phi := 0, \frac{2 \cdot \pi}{N_v} .. 2 \cdot \pi$$



Strapped Anode

The strap dimensions are the strap radius (rs) and the equivalent wire radius (as). The characteristic admittance of the straps is estimated from their dimensions using the formula for the self-inductance of a wire loop given by Ramo et al. with a correction for the mutual inductance

Number of straps $N_s := 4$ Two straps on each end of the anode

Strap mean radii $r_{s1} := 5.145\ \text{mm}$ $r_{s2} := 5.785\ \text{mm}$

Strap width $w_s := 0.33 \cdot \text{mm}$ Strap thickness $h_s := 0.305 \cdot \text{mm}$

Mean strap radius $r_s := \dfrac{r_{s1} + r_{s2}}{2}$ This assumes that the properties of the straps are all the same.

Strap separation $d_s := r_{s2} - r_{s1}$

Strap wire radius $a_s := \dfrac{w_s + h_s}{\pi}$ This assumes that the inductance can be calculated for an equivalent round wire having the same perimeter as a strap.

Self inductance per unit length of a strap based on a round loop of wire

$$L_s := \frac{\mu_0}{2 \cdot \pi} \cdot \left(\ln\left(\frac{8 \cdot r_s}{a_s} \right) - 2 \right)$$

Ramo, S., J. R. Whinnery, et al. (1965). Fields and Waves in Communication Electronics. New York, Wiley, Equation (2) on p.311.

Characteristic admittance of a strap

$$Y_c := \frac{1}{c \cdot L_s}$$

Admittance presented to the cavity by each strap

$$Y_s(\omega,\phi) := j\cdot Y_c\cdot\frac{\cos(2\cdot\phi)-\cos\left(\frac{2\cdot\omega\cdot p}{c}\right)}{\sin\left(\frac{2\omega\cdot p}{c}\right)\cdot(1-\cos(\phi))}$$

Equation 15.81

Admittance of strapped resonators with adjustable parameter F_s

$$F_s := 1.5$$

$$Ys(\omega,\phi) := Yr(\omega,\phi) + F_s\cdot N_s\cdot Y_s(\omega,\phi)$$

Equation 15.82

Resonant frequencies of a strapped anode

$$n := 0..N_v$$

$$\text{fs} := \left|\begin{array}{l} \omega \leftarrow \omega_1 \\ \text{for } i \in 0..N_v \\ \quad \left|\begin{array}{l} \phi \leftarrow \frac{2\cdot\pi\cdot i}{N_v} \\ \omega u_i \leftarrow 0 \quad \text{if } i = 0 \vee i = N_v \\ \omega u_i \leftarrow \text{root}(Ys(\omega,\phi),\omega) \quad \text{otherwise} \end{array}\right. \\ \frac{\omega u}{2\cdot\pi} \end{array}\right.$$

Pi mode frequency $fs_8 = 9.409\,\text{GHz}$

Pi - 1 mode frequency $fs_7 = 11.547\,\text{GHz}$

Vaughan, J. R. M. (1973). "A model for calculation of magnetron performance." IEEE Transactions on Electron Devices **20**(9): 818-826 gives 9.396 GHz and 11.600 GHz for these two modes.

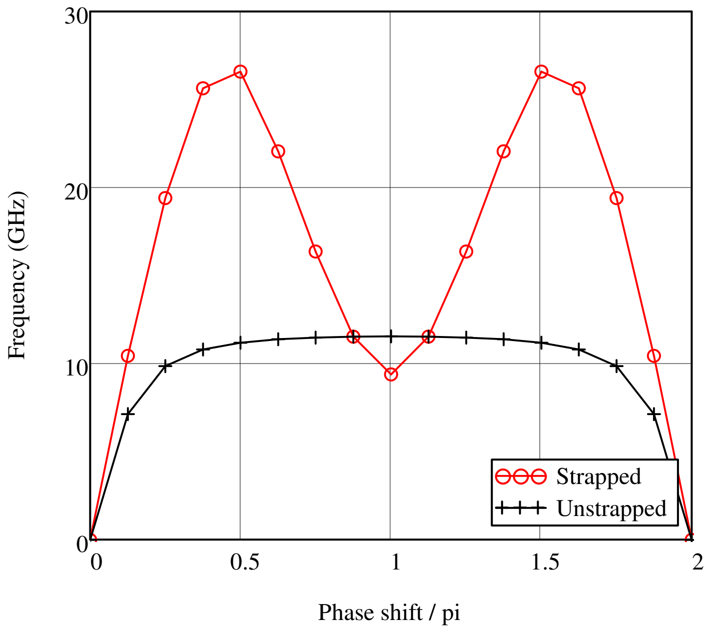


Figure 15.19