

## WS 15.2 Rising sun anode model

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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Data given in Collins, G. B. (1948).  
Microwave Magnetrons. New York, McGraw-Hill, p.106

$$N_v := 18$$

$$r_a := 4.602 \cdot \text{mm}$$

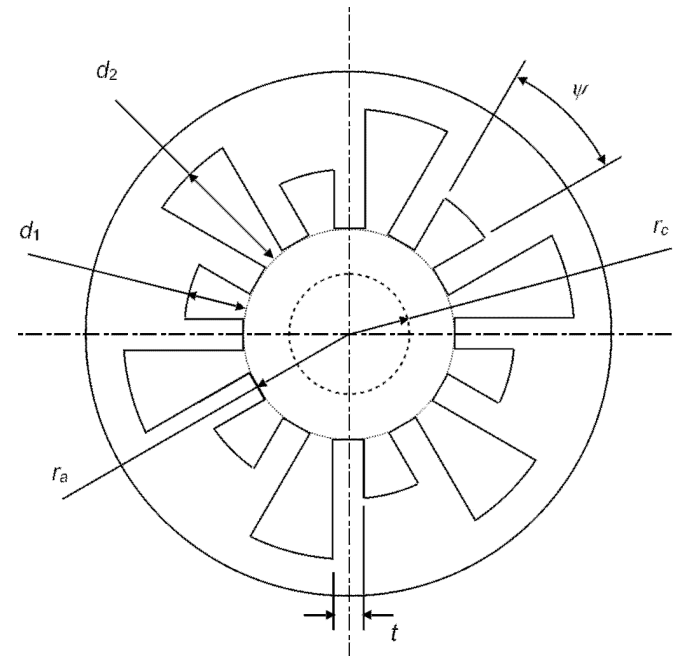
$$r_c := 2.715 \cdot \text{mm}$$

$$d_2 := 6.755 \cdot \text{mm}$$

$$d_1 := 3.806 \cdot \text{mm}$$

$$\theta := 0.068 \quad \text{half angle subtended by each gap}$$

$$\lambda_\pi := \frac{c}{10 \cdot \text{GHz}}$$



Calculate subsidiary dimensions

$$p := \frac{2 \cdot \pi \cdot r_a}{N_v} \quad w := 2 \cdot \theta \cdot r_a \quad t := p - w \quad \psi := \frac{2 \cdot \pi}{N_v} \quad r_1 := r_a - \frac{t}{\psi} \quad h := 0.1 \cdot \lambda \cdot \pi \quad d_{21} := \frac{d_2}{d_1} \quad R_a := \frac{r_a}{r_c}$$

Admittance of a resonator

$$Y(k, r_1, r_2) := j \cdot \sqrt{\frac{\epsilon_0}{\mu_0}} \cdot \left( \frac{h}{\psi \cdot r_1} \right) \cdot \left( \frac{J_0(k \cdot r_1) \cdot Y_1(k \cdot r_2) - J_1(k \cdot r_2) \cdot Y_0(k \cdot r_1)}{J_1(k \cdot r_1) \cdot Y_1(k \cdot r_2) - J_1(k \cdot r_2) \cdot Y_1(k \cdot r_1)} \right) \quad \text{Equation 15.57}$$

$$Y'(k, r_1, r_2) := \frac{d}{dk} Y(k, r_1, r_2)$$

Large resonators

$$k_L := \begin{cases} r_2 \leftarrow r_1 + d_2 \\ k_L \leftarrow \frac{\pi}{4 \cdot d_2} \\ k \leftarrow \text{root}(Y(k_L, r_1, r_2), k_L) \end{cases}$$

$$C_{0L} := \frac{Y'(k_L, r_1, r_1 + d_2)}{2 \cdot j \cdot c} \quad C_{0L} = 0.082 \cdot \text{pF}$$

$$L_{1L} := \frac{1}{C_{0L} \cdot (c \cdot k_L)^2} \quad L_{1L} = 4.744 \cdot \text{nH}$$

Small resonators

$$\text{nH} := \text{H} \cdot 10^{-9}$$

$$k_S(d_{21}) := \begin{cases} r_2 \leftarrow r_1 + d_2 \cdot d_{21}^{-1} \\ k_S \leftarrow \frac{\pi}{4 \cdot d_1} \\ k \leftarrow \text{root}(Y(k_S, r_1, r_2), k_S) \end{cases}$$

$$C_{0S}(d_{21}) := \frac{Y'(k_S(d_{21}), r_1, r_1 + d_2 \cdot d_{21}^{-1})}{2 \cdot j \cdot c} \quad C_{0S}(d_{21}) = 0.056 \cdot \text{pF}$$

$$L_{1S}(d_{21}) := \frac{1}{C_{0S}(d_{21}) \cdot (c \cdot k_S(d_{21}))^2} \quad L_{1S}(d_{21}) = 1.859 \cdot \text{nH}$$

Calculate the fringing capacitance

$$\mu := \frac{N_V \cdot w}{2 \cdot \pi \cdot r_a} \quad \text{Equation 15.64} \quad C_f := \left( \frac{h \cdot \epsilon_0}{\pi} \right) \cdot \left( 1 + \ln \left( \frac{4}{\pi \cdot \mu} \right) \right) \quad \text{Equation 15.63} \quad C_f = 0.018 \cdot \text{pF}$$

$$C_{1L} := C_{0L} + C_f \quad C_{1L} = 0.101 \text{ pF}$$

$$C_{1S}(d_{21}) := C_{0S}(d_{21}) + C_f \quad C_{1S}(d_{21}) = 0.075 \text{ pF}$$

Resonator frequencies

$$\omega_{1L} := \frac{1}{\sqrt{L_{1L} \cdot C_{1L}}}$$

$$\omega_{1S}(d_{21}) := \frac{1}{\sqrt{L_{1S}(d_{21}) \cdot (C_{1S}(d_{21}))}}$$

$$f_{1L} := \frac{\omega_{1L}}{2 \cdot \pi \cdot \text{GHz}} \quad f_{1L} = 7.278$$

$$f_{1S}(d_{21}) := \frac{\omega_{1S}(d_{21})}{2 \cdot \pi \cdot \text{GHz}} \quad f_{1S}(d_{21}) = 13.501$$

**Estimate anode/cathode capacitance per vane**

Upper bound  $C_U := \frac{p}{r_a} \cdot \epsilon_0 \cdot h \cdot \frac{1}{\ln(R_a)} \quad C_U = 0.018 \cdot \text{pF}$

Lower bound  $C_L := \frac{t}{r_a} \cdot \epsilon_0 \cdot h \cdot \frac{1}{\ln(R_a)} \quad C_L = 0.011 \cdot \text{pF}$

Best estimate  $C_0 := \sqrt{C_U \cdot C_L} \quad C_0 = 0.014 \text{ pF}$

**Dispersion diagrams for the uncoupled cavities****Equation 15.56**

$$\omega_S(\phi, d_{21}) := \begin{cases} \omega \leftarrow 0 & \text{if } \phi = 0 \vee \phi = 2 \cdot \pi \\ \omega \leftarrow \omega_{1S}(d_{21}) \cdot \left[ 1 + \frac{C_0}{2 \cdot C_{1S}(d_{21}) \cdot (1 - \cos(\phi))} \right]^{-0.5} & \text{otherwise} \\ \text{return } \omega \end{cases}$$

$$\omega_L(\phi) := \begin{cases} \omega \leftarrow 0 & \text{if } \phi = 0 \vee \phi = 2 \cdot \pi \\ \omega \leftarrow \omega_{1L} \cdot \left[ 1 + \frac{C_0}{2 \cdot C_{1L} \cdot (1 - \cos(\phi))} \right]^{-0.5} & \text{otherwise} \\ \text{return } \omega \end{cases}$$

$$f_S(\phi, d_{21}) := \frac{\omega_S(\phi, d_{21})}{\omega_{1L}}$$

$$\frac{C_0}{2 \cdot C_{1S}(d_{21})} = 0.092$$

$$f_L(\phi) := \frac{\omega_L(\phi)}{\omega_{1L}}$$

$$\frac{C_0}{2 \cdot C_{1L}} = 0.068$$

**Dispersion diagram for the rising sun anode**

Define  $\alpha_1(d_{21}) := \frac{2 \cdot C_{1S}(d_{21})}{C_0}$        $\alpha_2 := \frac{2 \cdot C_{1L}}{C_0}$

Expand equation 15.83 as a quadratic equation in  $\omega^2$ . The coefficients are in ascending powers of  $\omega^2$

$$V_{rs}(\phi, d_{21}) := \begin{bmatrix} \alpha_1(d_{21}) \cdot \alpha_2 \cdot \omega_{1S}(d_{21})^2 \cdot \omega_{1L}^2 \cdot (1 - \cos(\phi))^2 \cdot s^4 \\ -\alpha_1(d_{21}) \cdot \alpha_2 \cdot (\omega_{1S}(d_{21})^2 + \omega_{1L}^2) \cdot (1 - \cos(\phi))^2 \cdot s^2 - \alpha_1(d_{21}) \cdot \omega_{1S}(d_{21})^2 \cdot s^2 - \alpha_2 \cdot \omega_{1L}^2 \cdot s^2 \\ \alpha_1(d_{21}) \cdot \alpha_2 \cdot (1 - \cos(\phi))^2 + \alpha_1(d_{21}) + \alpha_2 + 1 \end{bmatrix}$$

The roots of the equation are

$$\omega(\phi, d_{21}) := \frac{1}{s} \cdot \sqrt{\text{polyroots}(V_{rs}(\phi, d_{21}))}$$

$$f_{rs1}(\phi, d_{21}) := \frac{\omega(\phi, d_{21})_0}{\omega_{1L}}$$

$$f_{rs2}(\phi, d_{21}) := \frac{\omega(\phi, d_{21})_1}{\omega_{1L}}$$

# Experimental data

$$\lambda_e := \begin{pmatrix} 1.944 \\ 1.456 \\ 1.381 \\ 1.360 \\ 0.754 \\ 0.766 \\ 0.800 \\ 0.920 \\ 1.000 \end{pmatrix} \cdot \lambda_\pi$$

$$f_e := \frac{c}{\text{GHz} \cdot \lambda_e}$$

$$f_e = \begin{pmatrix} 5.144 \\ 6.868 \\ 7.241 \\ 7.353 \\ 13.263 \\ 13.055 \\ 12.5 \\ 10.87 \\ 10 \end{pmatrix}$$

Lower mode

$$f_{1e} := \begin{pmatrix} 0 \\ 5.144 \\ 6.868 \\ 7.241 \\ 7.353 \\ 7.353 \\ 7.241 \\ 6.868 \\ 5.144 \\ 0 \end{pmatrix}$$

Upper mode

$$f_{2e} := \begin{pmatrix} 10 \\ 10.87 \\ 12.5 \\ 13.055 \\ 13.263 \\ 13.263 \\ 13.055 \\ 12.5 \\ 10.87 \\ 10 \end{pmatrix}$$

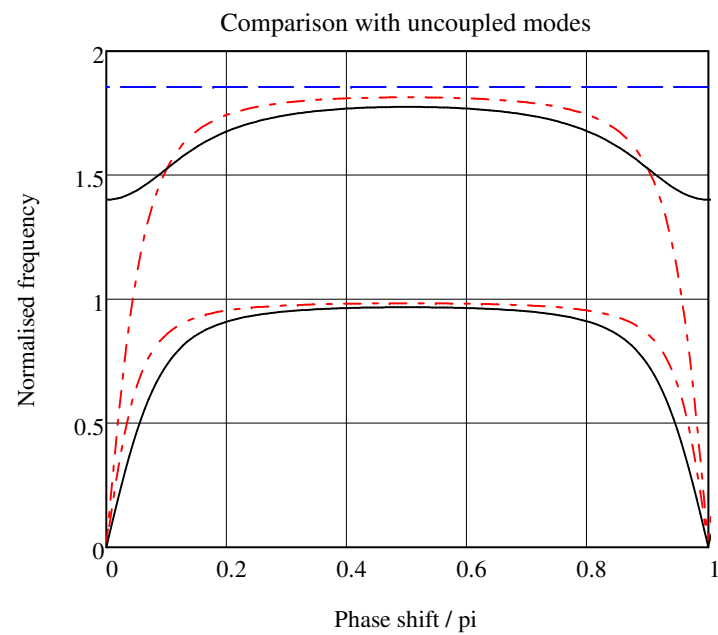
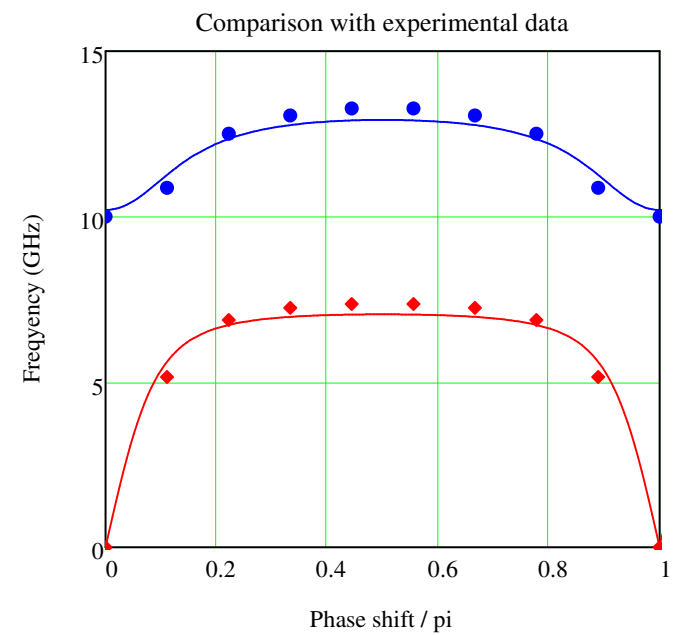


Figure 15.23



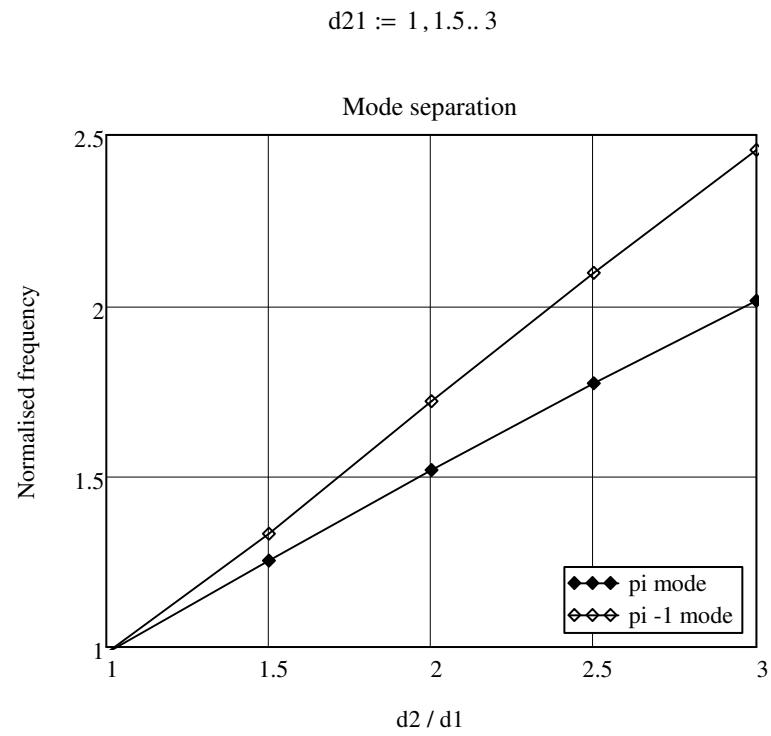


Figure 15.24