WS 18.1 Two-surface multipactor model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet can be used to explore the properties of multipactor discharges between plane parallel electrons in the absence of a magnetic field (see Section 18.8.1).

Define the frequency and the separation between the plates

$f := 1 \cdot GHz$ $\qquad$ $d := 10 \cdot mm$

Define the secondary emission constants using the The Furman and Pivi model for oxidised aluminium with data from Lin (2005)

$\delta_m := 3.5$ $\qquad$ $E_{pm} := 400 \cdot V$ $\qquad$ $ss := 1.65$ $\qquad$ Secondary emission energy $\quad V_0 := 2 \cdot V$

Define the charge/mass ratio of the electron $\qquad \eta := 1.759 \cdot 10^{11} \cdot \frac{C}{kg}$

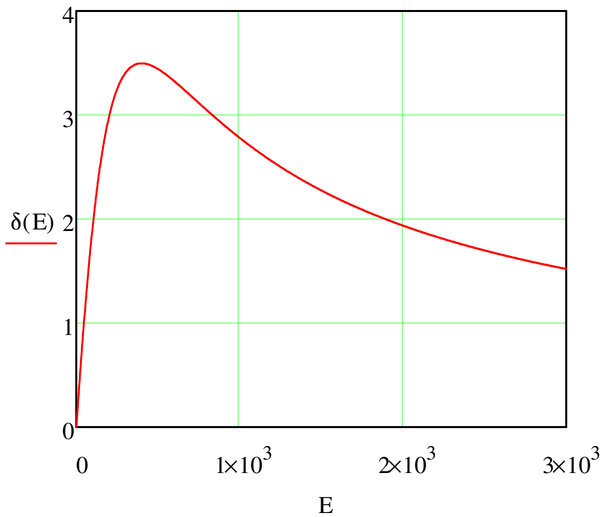
Secondary electron emission model

$$D(x) := \frac{ss \cdot x}{ss - 1 + x^{ss}} \qquad \delta(E) := \delta_m \cdot D\left(\frac{E}{E_{pm}}\right) \qquad \boxed{\text{Equation 18.19}}$$

Find the electron impact energies at which δ = 1

$E1_s := 0 \cdot \text{volt}$ $\qquad V_{min} := \text{root}\left[\left(\delta\left(E1_s\right) - 1\right), E1_s\right]$ $\qquad V_{min} = 47\ \text{V}$

$E2_s := 1.2 \cdot E_{pm}$ $\qquad V_{max} := \text{root}\left[\left(\delta\left(E2_s\right) - 1\right), E2_s\right]$ $\qquad V_{max} = 5869\ \text{V}$



RF voltage for resonance

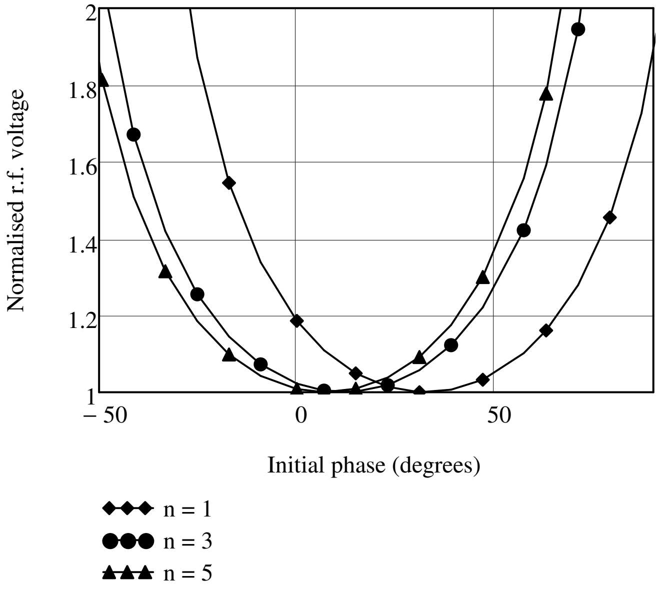
$\omega := 2\cdot\pi\cdot f$ $\qquad\qquad \omega d := \omega\cdot d$

$$V_1(n, V_0, \omega d, \theta_1) := \frac{\omega d\cdot\left(\omega d - n\cdot\pi\cdot\sqrt{2\cdot\eta\cdot V_0}\right)}{\eta\cdot\left(n\cdot\pi\cdot\cos(\theta_1) + 2\cdot\sin(\theta_1)\right)}$$ Equation 18.56

$$\theta_{max}(n) := \operatorname{atan}\left(\frac{2}{n\cdot\pi}\right)$$ Equation 18.57

Normalised RF voltage

$$V1(n, \theta) := \frac{V_1(n, V_0, \omega d, \theta)}{V_1(n, V_0, \omega d, \theta_{max}(n))}$$



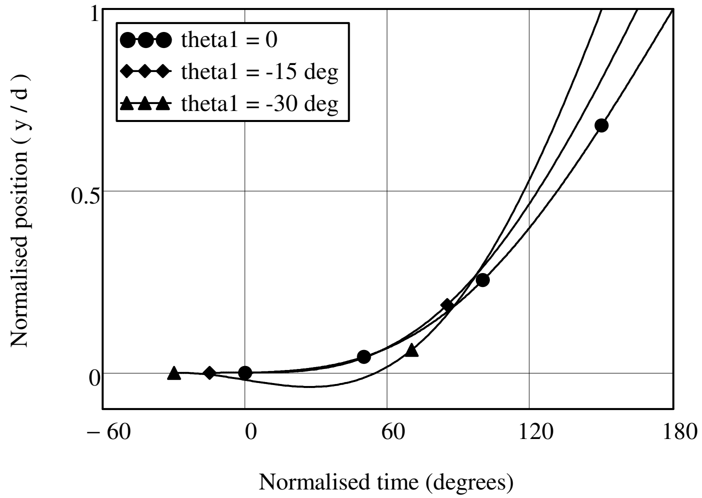
Normalised electron position as a function of normalised time

$$Y(\theta, n, V_0, \omega d, \theta_1) := \frac{\eta \cdot V_1(n, V_0, \omega d, \theta_1)}{\omega d^2} \cdot \left[(\theta - \theta_1) \cdot \cos(\theta_1) - \sin(\theta) + \sin(\theta_1) \right] + \frac{\sqrt{2 \cdot \eta \cdot V_0}}{\omega d} \cdot (\theta - \theta_1)$$

Equation 18.53

Plotting ranges

$\theta a := 0, 0.5 \cdot deg .. 180 \cdot deg$ $\theta b := -15 \cdot deg, -14 \cdot deg .. 180 \cdot deg$ $\theta c := -30 \cdot deg, -29 \cdot deg .. 180 \cdot deg$



$V_0 = 2\ V$

Find the minimum phase for which an electron can escape from the first plate

$$\theta_{esc}(n, V_0, \omega d) := \begin{array}{l} \text{for } i \in 0, 0.1 .. 60 \\ \quad \theta_i \leftarrow -i \cdot 1 \cdot deg \\ \quad \text{for } \theta \in 0, 0.1deg .. 90deg \\ \qquad Y1 \leftarrow Y(\theta + \theta_i, n, V_0, \omega d, \theta_i) \\ \qquad \text{return } \theta_i + 0.1 \cdot deg \text{ if } Y1 < 0 \end{array}$$

$\theta_{esc}(1, V_0, \omega d) = -17.2 \cdot deg$

Find the two limits of the initial phase which satisfy the phase focusing condition

$$\theta 1(n) := \theta_{max}(n)$$

$$\theta 0 := 0 \qquad \theta 2(n, V_0, \omega d) := \text{atan}\left[-\frac{4n\,\pi \cdot \sqrt{2 \cdot \eta \cdot V_0} + 2 \cdot \omega d}{4 \cdot \sqrt{2 \cdot \eta \cdot V_0} + n \cdot \pi \cdot \omega d - (n\,\pi)^2 \cdot \sqrt{2 \cdot \eta \cdot V_0}}\right]$$

Equation 18.64

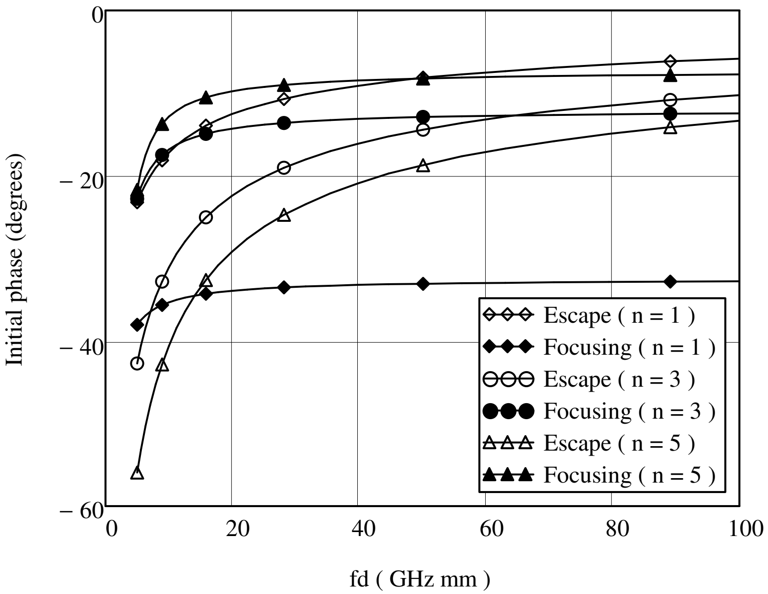
$$N1 := 1, 3 .. 9$$

$\theta 1(N1) =$ (·deg)	$\theta 2(N1, V_0, 0.1 \cdot \omega d) =$ (·deg)	$\theta 2(N1, V_0, \omega d) =$ (·deg)	$\theta 2(N1, V_0, 10\omega d) =$ (·deg)
32.5	-57.3	-35.286	-32.764
12.0	74.9	-16.798	-12.418
7.3	31.9	-12.827	-7.714
5.2	18.1	-11.510	-5.664
4.0	12.4	-11.244	-4.520

The upper limit of θ is independent of ωd and equal to the phase corresponding to the minimum of V_1. The lower limit of θ varies with ωd

Comparison between the lower phase limits set by electron escape and by phase focusing

$ny := 0.7, 0.75 .. 2 \qquad \Omega mm := 2 \cdot \pi \cdot GHz \cdot mm$



For all V_0 the escape phase gets more negative as the order of the multipactor increases. The phase focusing limit shows the opposite behaviour. When V_0 = 0 the escape limit is theta = 0. When V_0 > 0 the limits are electron escape for n = 1.For V_0 = 1V and n = 3 the limit is phase focusing up to fd = 50 GHz mm and escape thereafter. The crossover value of fd increases as V_0 increases. For higher values of n the phase focusing limit dominates

Find the limiting RF voltages for which the secondary electron emission coefficient is unity

Lower impact limit

$$V_L(n, V_0, \omega d) := V_1(n, V_0, \omega d, \theta_{max}(n))$$

Equation 18.56

Select the greater of the two phases for the upper impact limit

$$\theta_{min}(n, V_0, \omega d) := \left| \begin{array}{l} \theta a \leftarrow \theta 2(n, V_0, \omega d) \\ \theta b \leftarrow \theta_{esc}(n, V_0, \omega d) \\ \theta \leftarrow \left| \begin{array}{ll} \theta a & \text{if } \theta a > \theta b \\ \theta b & \text{otherwise} \end{array} \right. \\ \text{return } \theta \end{array} \right.$$

$$V_U(n, V_0, \omega d) := \frac{\omega d \cdot \left(\omega d - n \cdot \pi \cdot \sqrt{2 \cdot \eta \cdot V_0}\right)}{\eta \cdot \left(n \cdot \pi \cdot \cos\left(\theta_{min}(n, V_0, \omega d)\right) + 2 \cdot \sin\left(\theta_{min}(n, V_0, \omega d)\right)\right)}$$

Dependence of r.f. voltage on the impact velocity

$$Vi_1(n, V_0, V_i, \omega d) := \left| \begin{array}{l} v_0 \leftarrow \sqrt{2 \cdot \eta \cdot V_0} \\ v_i \leftarrow \sqrt{2 \cdot \eta \cdot V_i} \\ V1 \leftarrow \dfrac{\omega d}{2 \cdot \eta} \cdot \sqrt{\left[\omega d - \dfrac{n \cdot \pi}{2} \cdot (v_i + v_0)\right]^2 + (v_i - v_0)^2} \\ \text{return } V1 \end{array} \right.$$

Equation 18.66

Find the line which is approximately tangential to these curves

$$fd(n) := \left| \begin{array}{l} v_0 \leftarrow \sqrt{2\cdot\eta\cdot V_0} \\ v_i \leftarrow \sqrt{2\cdot\eta\cdot V_{min}} \\ (v_i + v_0)\cdot\frac{n}{4} + \frac{(v_i - v_0)^2}{v_i + v_0}\cdot\frac{1}{n\cdot\pi^2} \end{array} \right.$$

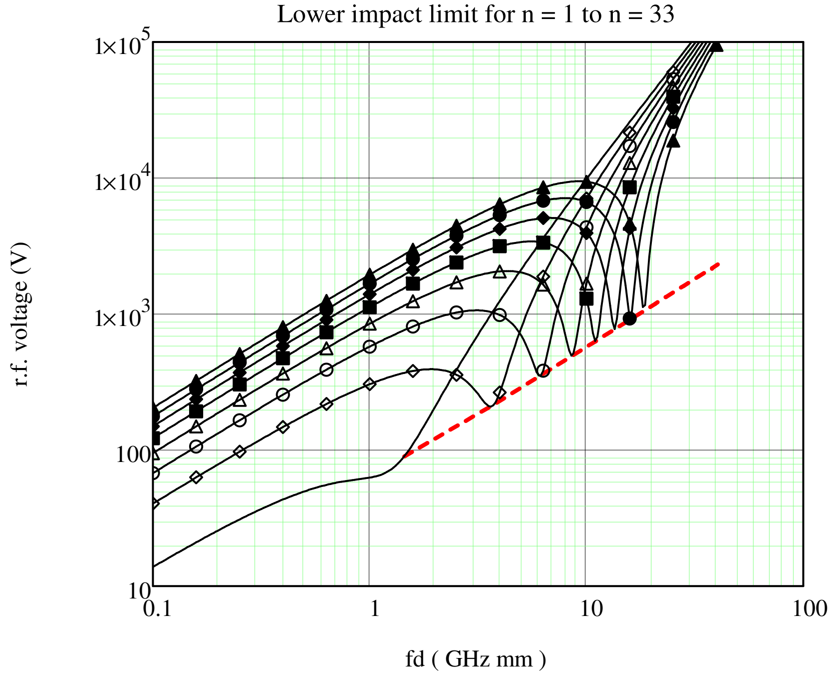
$$Vd(n) := Vi_1(n, V_0, V_{min}, 2\cdot\pi\cdot fd(n))$$

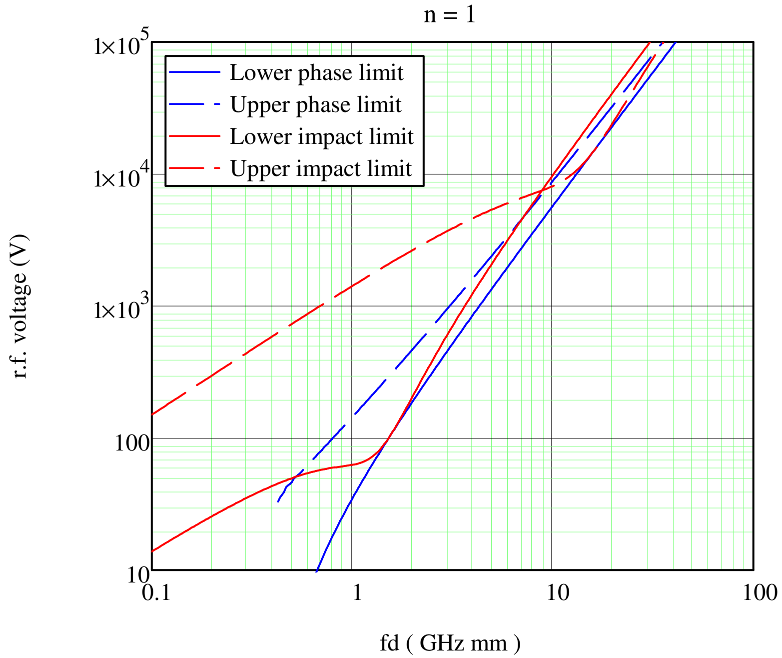
Plot curves for which the impact voltage is equal to the lower voltage for which $\delta = 1$

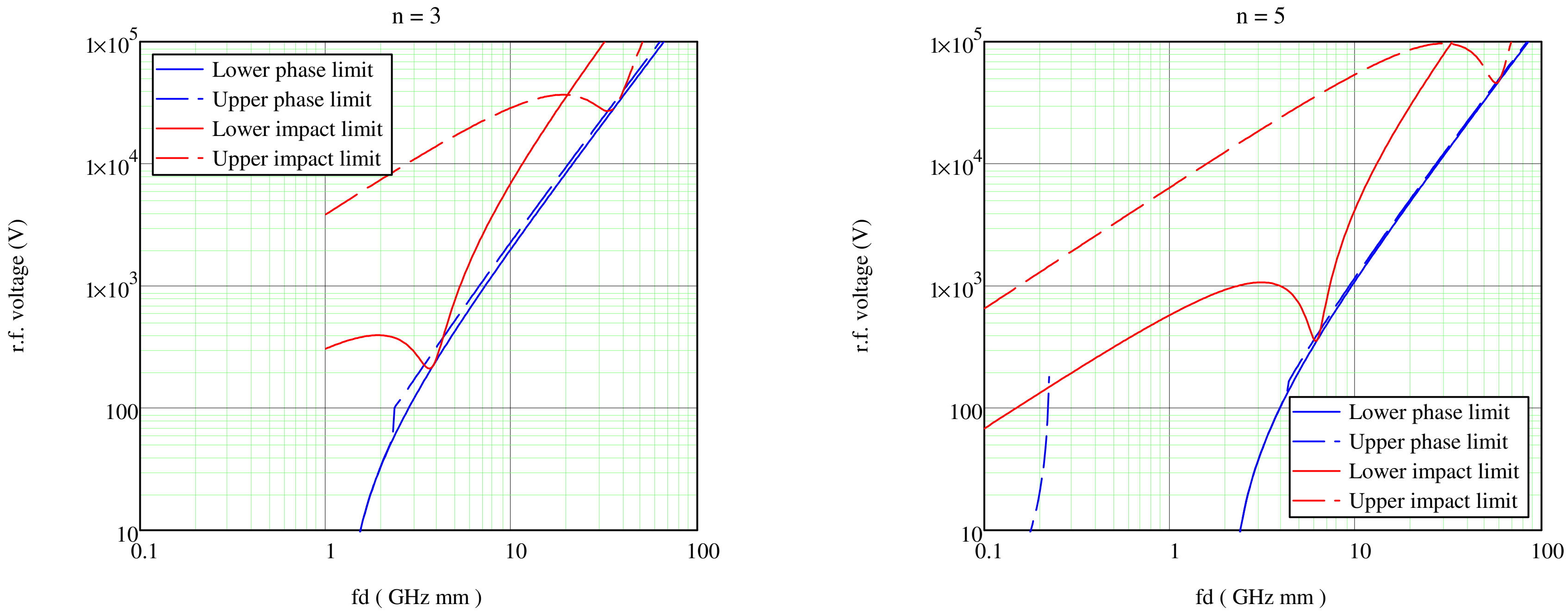
$V_i := V_{min}$

$ny := -1, -0.99 .. 2 \qquad nn := 1, 3 .. 33 \qquad nny := 0, 0.01 .. 2$

Plot the Hatch diagrams for the first three modes
(Note: this is very slow)






n = 3
1×10^5
1×10^4
1×10^3
100
10
0.1
1
10
100
r.f. voltage (V)
fd (GHz mm)
Lower phase limit
Upper phase limit
Lower impact limit
Upper impact limit
n = 5
1×10^5
1×10^4
1×10^3
100
10
0.1
1
10
100
r.f. voltage (V)
fd (GHz mm)
Lower phase limit
Upper phase limit
Lower impact limit
Upper impact limit