WS 2.2 Capacitive Iris in Rectangular Waveguide

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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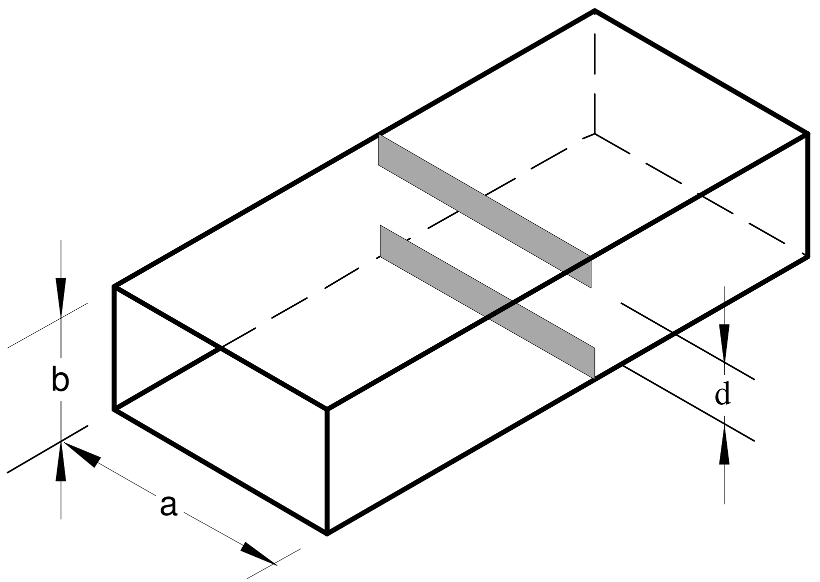

Figure 2.18

Section 2.4.2 Capacitive iris in a rectangular waveguide

see also Section 2.4.1 Height step in a rectangular waveguide

The iris is assumed to be thin and symmetrical. The height of the gap is *d.* The iris can be represented by a capacitive susceptance whose magnitude is given in Marcuvitz, N. (1951) McGraw-Hill, pp.218-220. The first approximation is

$$\frac{B_i}{Y_g}\cdot\frac{\lambda_g}{b} = -4\ln\left(\sin\left(\frac{\pi d}{2b}\right)\right)$$

Equation 2.96

$$\mathrm{Term1}(d\_b) := -4\cdot\ln\left(\sin\left(\frac{\pi\cdot d\_b}{2}\right)\right)$$

More accurate results can be obtained by adding extra terms given

$$Q_2(b\_\lambda_g) := \frac{1}{\sqrt{1-\left(b\_\lambda_g\right)^2}} - 1$$

$$\mathrm{Term2}(d\_b, b\_\lambda_g) := \frac{4\cdot Q_2(b\_\lambda_g)\cdot\cos(0.5\cdot\pi\cdot d\_b)^4}{1 + Q_2(b\_\lambda_g)\cdot\sin(0.5\cdot\pi\cdot d\_b)^4} + \mathrm{Term1}(d\_b)$$

$$\mathrm{Term3}(d\_b, b\_\lambda_g) := \frac{4}{16}\cdot\left(b\_\lambda_g\right)^2\cdot\left(1 - 3\cdot\sin(0.5\cdot\pi\cdot d\_b)^2\right)^2\cdot\cos(0.5\cdot\pi\cdot d\_b)^4 + \mathrm{Term2}(d\_b, b\_\lambda_g)$$

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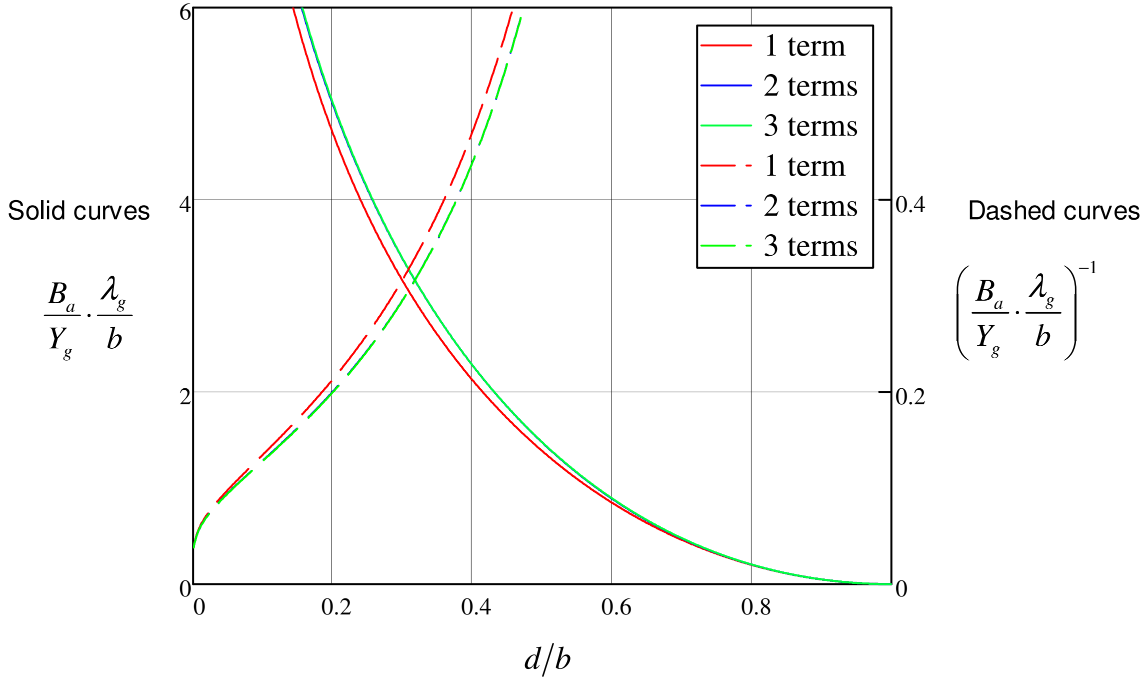
For a standard waveguide (e.g. WG16) $b\_\lambda_g = 0$ at cut-off and 0.45 at the top of the normal working band (12.4 GHz). Comparison of the effect of using one, two, or three, terms (see the figure below) shows that the second term is a small correction and the third term is negligible. Thus the normalised susceptance depends only on the ratio h/b to a good approximation.



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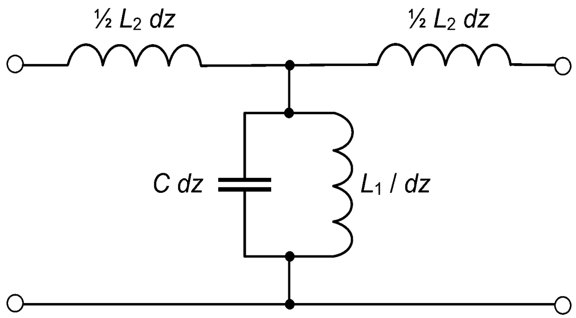
Equivalent circuit

Now suppose that the iris can be represented by a short length of waveguide having the same cross-sectional dimensions as the iris and a thickness δ which represents the fringing fields. The equivalent circuit of the iris is that for a TE mode shown in the figure below



From the theory of the TE mode we know that the inductances and the characteristic impedance can be expressed in terms of the capacitance as

$$L_1 = \frac{1}{\omega_c^2 C} \qquad L_2 = \frac{1}{c^2 C} \qquad Z_g = \frac{\beta_0}{\beta_g}\frac{1}{cC} \qquad \boxed{\text{Equation 2.36}}$$

where $\beta_g^2 = \beta_0^2 - \beta_c^2 \qquad \beta_c = \dfrac{\pi}{a}$

For the moment let us assume that the impedance of L_2 is small compared with Z_g. Then the admittance of the iris is

$$Y = j\omega C_i \delta - \frac{j\delta}{\omega L_1} = j\omega C_i \delta \left(1 - \frac{1}{\omega^2 L_1 C_i}\right) = j\frac{C_i \delta}{\omega}\left(\omega^2 - \omega_c^2\right) = jcC_i\delta\frac{\beta_g^2}{\beta_0}$$

where C_i is the capacitance per unit length of the iris.

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The admittance added by the iris is found by subtracting the admittance of the waveguide whose height is *b*

$$Y_i = jcC_i\delta\frac{\beta_g^2}{\beta_0} - jcC\delta\frac{\beta_g^2}{\beta_0} = jcC\delta\frac{\beta_g^2}{\beta_0}\left(\frac{C_i}{C}-1\right)$$

Normalising to the waveguide impedance

$$\frac{B}{Y_g} = \beta_g\delta\left(\frac{C_i}{C}-1\right)$$

Now analysis of the equivalent circuit for a rectangular waveguide shows that the capacitance per unit length is inversely proportional to *h* regardless of the choice of definition of impedance. Thus

$$\frac{B}{Y_g} = \beta_g\left(\frac{b}{d}-1\right) = \frac{2\pi\delta}{\lambda_g}\left(\frac{b}{d}-1\right)$$

Hence the normalised susceptance plotted by Marcuvitz is

$$\frac{B}{Y_g}\frac{\lambda_g}{b} = 2\pi\left(\frac{b}{d}-1\right)\frac{\delta}{b} = 2\pi\left(1-\frac{d}{b}\right)\frac{\delta}{d}$$

Equation 2.95

Therefore the normalised susceptance depends only on the dimensions of the aperture including the, as yet undetermined, parameter δ. We note that the susceptance is positive and, therefore, capacitive

The condition that the series inductance is negligible is

$$\frac{\omega L_2}{Z_g} = \frac{\omega}{c^2 C}\frac{\beta_g}{\beta_0}cC = \beta_g \to 0$$

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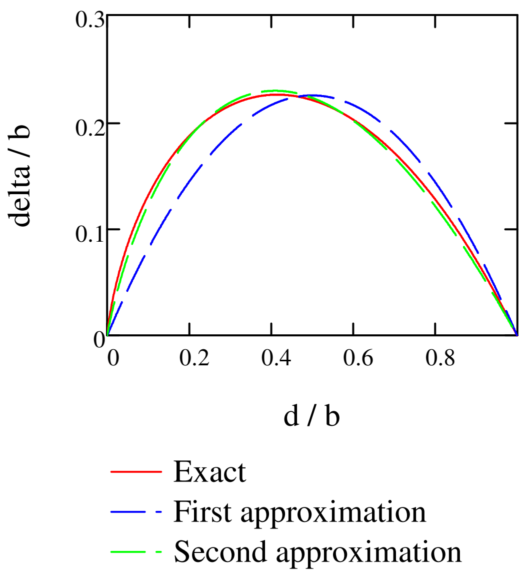
Thus we expect the expression derived above to be comparable with that computed as Term1 above. Hence the parameter $\frac{\delta}{b}$ is given by

$$\frac{\delta}{b} = -\frac{2}{\pi}\ln\left(\sin\left(\frac{\pi d}{2b}\right)\right)\cdot\left(\frac{d}{b}-1\right)^{-1} \qquad \delta\_b(d\_b) := \frac{\mathrm{Term1}(d\_b)}{2\cdot\pi\cdot\left(\frac{1}{d\_b}-1\right)}$$

The curve can be approximated by $\frac{\delta}{b} \approx 0.9\frac{d}{b}\left(1-\frac{d}{b}\right)$ and, more accurately by $\frac{\delta}{b} = 2.8\frac{d}{b}\left(1-\left(\frac{d}{b}\right)^{\frac{1}{4}}\right)$

$$\delta1(x) := 0.9\cdot[x\cdot(1-x)]^{1} \qquad \delta2(x) := 2.8\cdot\left[x\cdot\left(1-x^{0.25}\right)\right]$$



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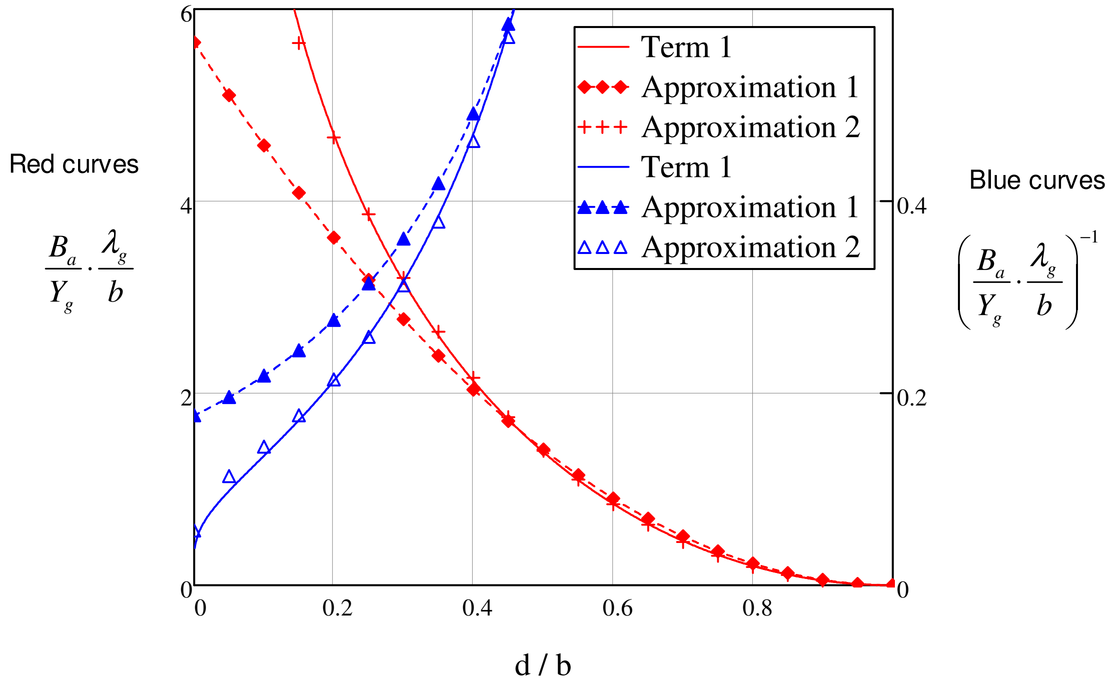
The first and second approximations to the normalised susceptance are

$$\mathrm{Approx1(h\_b)} := 1.8\cdot\pi\cdot(1 - \mathrm{h\_b})^2 \qquad \mathrm{Approx2(h\_b)} := 5.6\cdot\pi\cdot(1 - \mathrm{h\_b})\cdot\left(1 - \mathrm{h\_b}^{0.25}\right)$$

Then, comparing the exact and approximate normalised susceptance



Thus the first approximate model derived from the equivalent circuit gives good accuracy provided that d/b > 0.5 and that it is permissible to neglect the effect of the series inductance.

Comparison between the results for a height step and a thin iris. For a height step

$$\frac{B}{Y_{PV}} \cdot \frac{\lambda_g}{b} = \left(\frac{d}{b} + \frac{b}{d}\right) \ln\left(\frac{1+d/b}{1-d/b}\right) \quad - \quad 2\ln\left(\frac{4d/b}{1-d^2/b^2}\right)$$

Equation 2.89

$$\mathrm{B(d\_b)} := \left(\mathrm{d\_b} + \frac{1}{\mathrm{d\_b}}\right) \cdot \ln\left(\frac{1 + \mathrm{d\_b}}{1 - \mathrm{d\_b}}\right) - 2 \cdot \ln\left(\frac{4 \cdot \mathrm{d\_b}}{1 - \mathrm{d\_b}^2}\right)$$

and

$$\delta\mathrm{s\_b(d\_b)} := \frac{\mathrm{B(d\_b)}}{\left[2 \cdot \pi \cdot \left(\frac{1}{\mathrm{d\_b}} - 1\right)\right]}$$

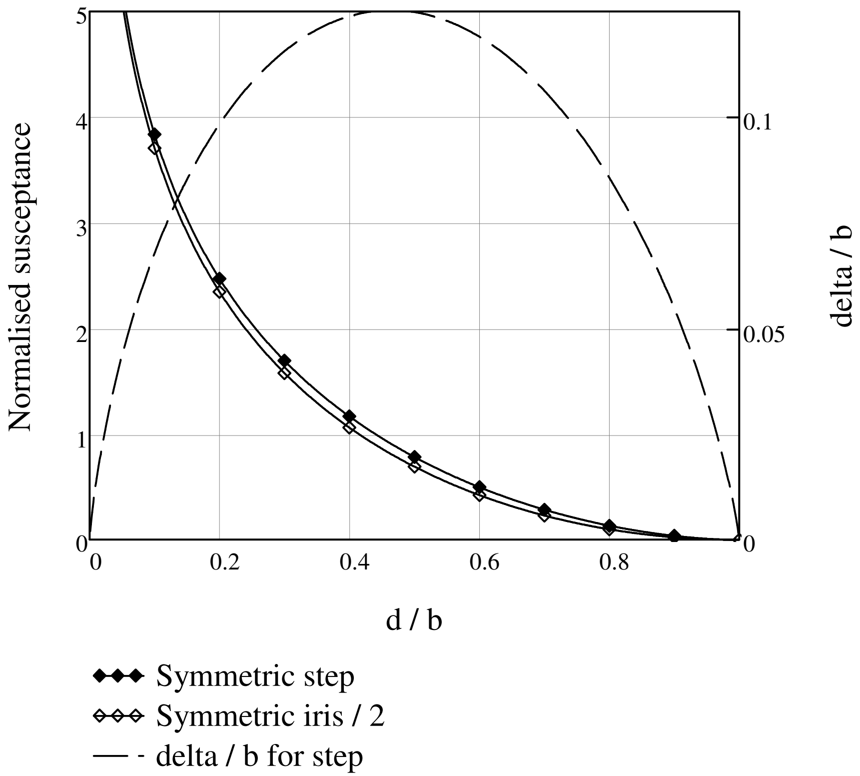


Figure 2.20

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