WS 5.1 Plane parallel space-charge limited diodes

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Physical constants

Charge on the electron $q_e := 1.602 \cdot 10^{-19} \cdot C$

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1}$

Rest energy of the electron $V_R := \frac{c^2}{\eta}$ $V_R = 510.947 \cdot kV$

Boltzmann's constant $k := 1.38 \cdot 10^{-23} \cdot J \cdot K^{-1}$

1. Plane parallel space-charge limited diode with no initial velocity (Section 5.4)

Normalised voltage $U = \frac{V}{V_R}$ Equation 5.57

Normalised position $X = \sqrt{\frac{-J_a}{2\varepsilon_0 \cdot c \cdot V_R}} \cdot x$ Equation 5.58

Non-relativistic

$$X1(U) := \frac{1}{3} \cdot (2U)^{\frac{3}{4}}$$ Equation 5.63

Relativistic

$$X2(U) := \frac{1}{2} \cdot \int_0^U \left(2 \cdot U + U^2\right)^{\frac{-1}{4}} dU$$ Equation 5.60

Relativistic series expansion

$$X3(U) := \left(U^2 + 2 \cdot U\right)^{-\frac{1}{4}} \cdot \left(\frac{2}{3} \cdot U + \frac{1}{21} \cdot U^2 + \frac{3}{21 \cdot 11} U^3\right)$$ Equation 5.62 (Three terms)

Note that there is an error in the last term in Equation 5.62 in the book. The correct equation is

$$X = \left(U^2 + 2U\right)^{-\frac{1}{4}} \left(\frac{2}{3} U + \frac{1}{21} U^2 + \frac{1 \cdot 3}{21 \cdot 11} U^3 + \cdots (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-5)(2n-3)}{21 \cdot 11 \cdot 15 \ldots (4n-5)(4n-1)} U^n\right)$$

$U_a := 0, 0.01 .. 2$

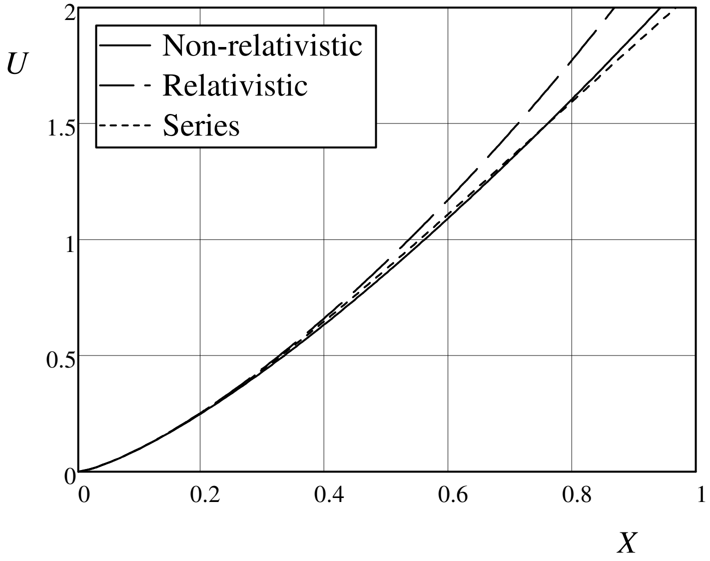


Figure 5.7

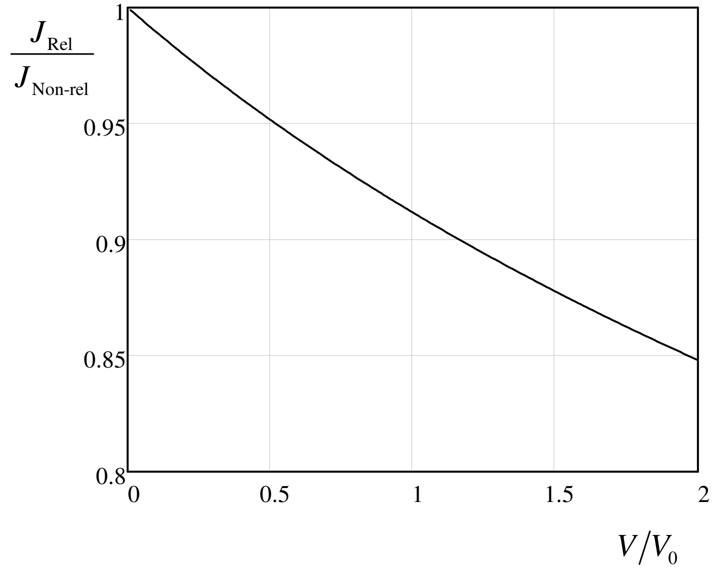


Figure 5.8

2. Planar diode with thermal velocities (Section 5.3)

Normalised position $X = \sqrt{\dfrac{-J_a}{\varepsilon_0 \cdot \sqrt{2 \cdot \eta \cdot V_T^{\,3}}}} \cdot x$

where V_T is the volt equivalent of the cathode temperature

Normalised voltage $U = \dfrac{V}{V_T}$

Note that the symbol υ used in the book is represented here by U.

Without thermal velocities $X_0(U) := \dfrac{2}{3} \cdot U^{\frac{3}{4}}$

Equation 5.51

Between the potential minimum and the anode (Section 5.3.1)

$$\left(\frac{dU}{dX}\right)^2 = 2\sqrt{\pi} \cdot \left[\exp(U) - 1 - \exp(U)\,P\left(\sqrt{U}\right) + \frac{2}{\sqrt{\pi}}\sqrt{U}\right]$$

Equation 5.37

Between the cathode and the potential minimum (Section 5.3.2)

$$\left(\frac{dU}{dX}\right)^2 = 2\sqrt{\pi} \cdot \left[\exp(U) - 1 + \exp(U)\,P\left(\sqrt{U}\right) - \frac{2}{\sqrt{\pi}}\sqrt{U}\right]$$

Equation 5.48

The function P(x) can be computed using a Mathcad built-in function.

$$P(x) := 2 \cdot \mathrm{pnorm}\left(x, 0, \sqrt{0.5}\right) - 1$$

Define the functions appearing onthe right-hand sides of these equations. Alternative definitions are needed for large values of U to avoid floating point overflows caused by the exponential functions

$$E_1(U) := \left|\begin{array}{l} E \leftarrow \sqrt{\exp(U) \cdot \left(1 - P\left(\sqrt{U}\right)\right) + \dfrac{2}{\sqrt{\pi}} \cdot \sqrt{U} - 1} \quad \text{if } U \le 50 \\ E \leftarrow \sqrt{\dfrac{2}{\sqrt{\pi}} \cdot \sqrt{U} - 1} \quad \text{otherwise} \\ \text{return } E \end{array}\right.$$

$$E_2(U) := \left|\begin{array}{l} E \leftarrow \sqrt{\exp(U) - 1 + \exp(U) \cdot P\left(\sqrt{U}\right) - \dfrac{2}{\sqrt{\pi}} \cdot \sqrt{U}} \quad \text{if } U \le 50 \\ E \leftarrow \infty \quad \text{otherwise} \\ \text{return } E \end{array}\right.$$

Integrating the equations

$$X_1(U) := \frac{1}{\sqrt{2\sqrt{\pi}}} \cdot \int_0^U E_1(U)^{-1}\, dU \qquad X_2(U) := \frac{1}{\sqrt{2\sqrt{\pi}}} \cdot \int_0^U E_2(U)^{-1}\, dU$$

$$U := 0, 0.1 .. 36$$

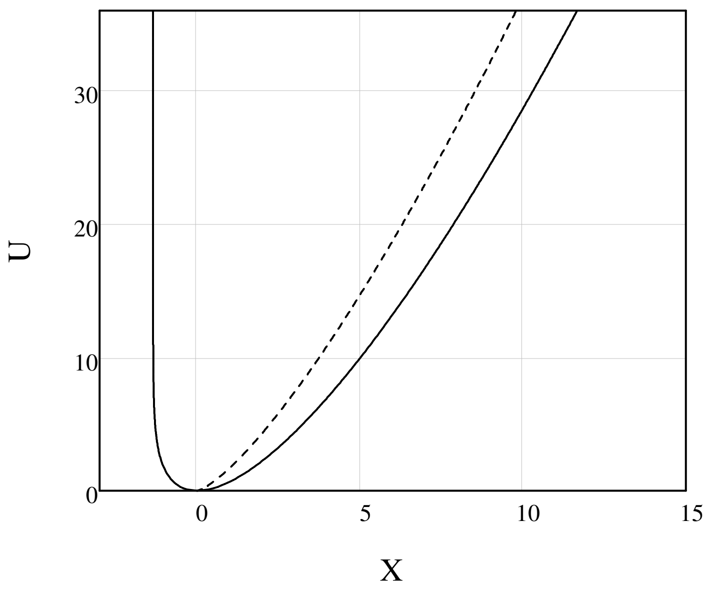


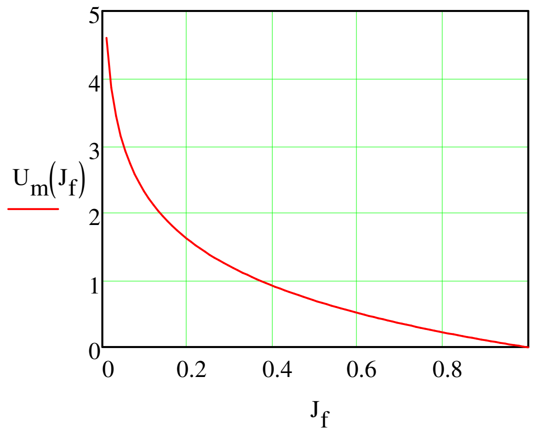
Figure 5.4

Calculation of the current-voltage characteristics of a diode with thermal velocities

Specify the cathode temperature $T_k := 1300 \cdot K$ $V_T := \dfrac{k \cdot T_k}{q_e}$ $V_T = 0.112\,V$

Find the potential at the minimum relative to the cathode when $J_f = \dfrac{J_a}{J_s}$ and $U = \dfrac{V}{V_T}$

$$V_m = V_T \ln\left(\frac{J_s}{J_a}\right) \quad \boxed{\text{Equation 5.53}} \quad U_m(J_f) := \ln\left(\frac{1}{J_f}\right)$$



Find the normalising constant for distance

$$X = \sqrt{\frac{-J_a}{\varepsilon_0 \cdot \sqrt{2 \cdot \eta \cdot V_T{}^3}}} \cdot x \quad \boxed{\text{Equation 5.31}} \quad C_2(J_f, J_s) := \sqrt{\frac{J_f \cdot J_s}{\varepsilon_0 \cdot \sqrt{2 \cdot \eta \cdot V_T{}^3}}}$$

Find the distance of the minimum from the cathode as function of J_f and J_s

$$x_2(J_f, J_s) := \frac{X_2(U_m(J_f))}{C_2(J_f, J_s)}$$

Find the normalised distance between the minimum and the anode

$$X_{ma}(J_f, J_s, d) := C_2(J_f, J_s) \cdot d - X_2(U_m(J_f))$$

$J_f := 0.01, 0.02 .. 1$

$J_{s1} := 0.1 \cdot \frac{A}{cm^2}$ $J_{s2} := 1.0 \cdot \frac{A}{cm^2}$ $J_{s3} := 10 \cdot \frac{A}{cm^2}$

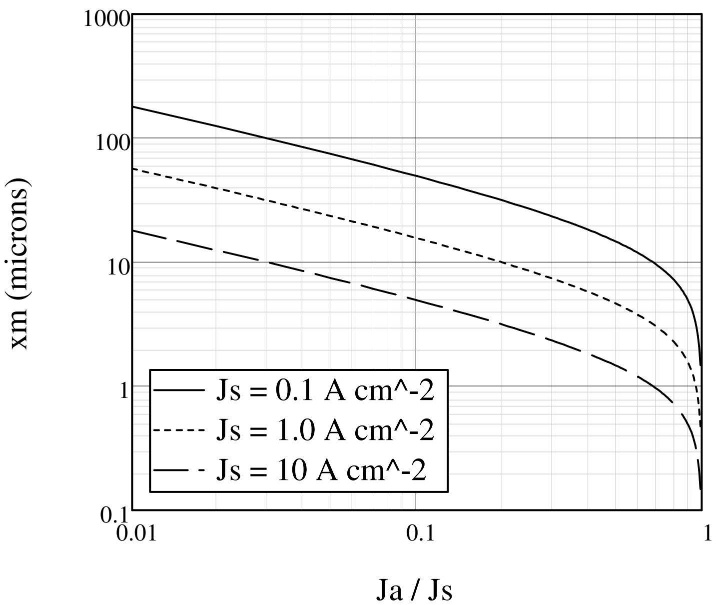


Figure 5.5

Find the normalised potential difference between the cathode and the anode

$$U_{ak}(J_f, J_s, d) := \left| \begin{array}{l} U \leftarrow 0 \\ U_{ak} \leftarrow \left| \begin{array}{ll} \mathrm{root}\left(\frac{X_1(U)}{X_{ma}(J_f, J_s, d)} - 1, U \right) - U_m(J_f) & \text{if } X_{ma}(J_f, J_s, d) > 0 \\ \ln(J_f) & \text{otherwise} \end{array} \right. \\ U_{ak} \end{array} \right.$$

Find the potential difference between the anode and the cathode
without thermal velocities for comparison

$$J_s = -\frac{4}{9} \cdot \frac{\varepsilon_0 \sqrt{2\eta}}{x^2} \cdot V^{\frac{3}{2}}$$

Equation 5.16

$$U_0(J_f, J_s, d) := \frac{1}{V_T} \cdot \left(\frac{9 \cdot J_f \cdot J_s \cdot d^2}{4 \cdot \varepsilon_0 \cdot \sqrt{2 \cdot \eta}} \right)^{\frac{2}{3}}$$

d1 := 0.01·mm   d2 := 0.1·mm   d3 := 1.0·mm   d4 := 10·mm

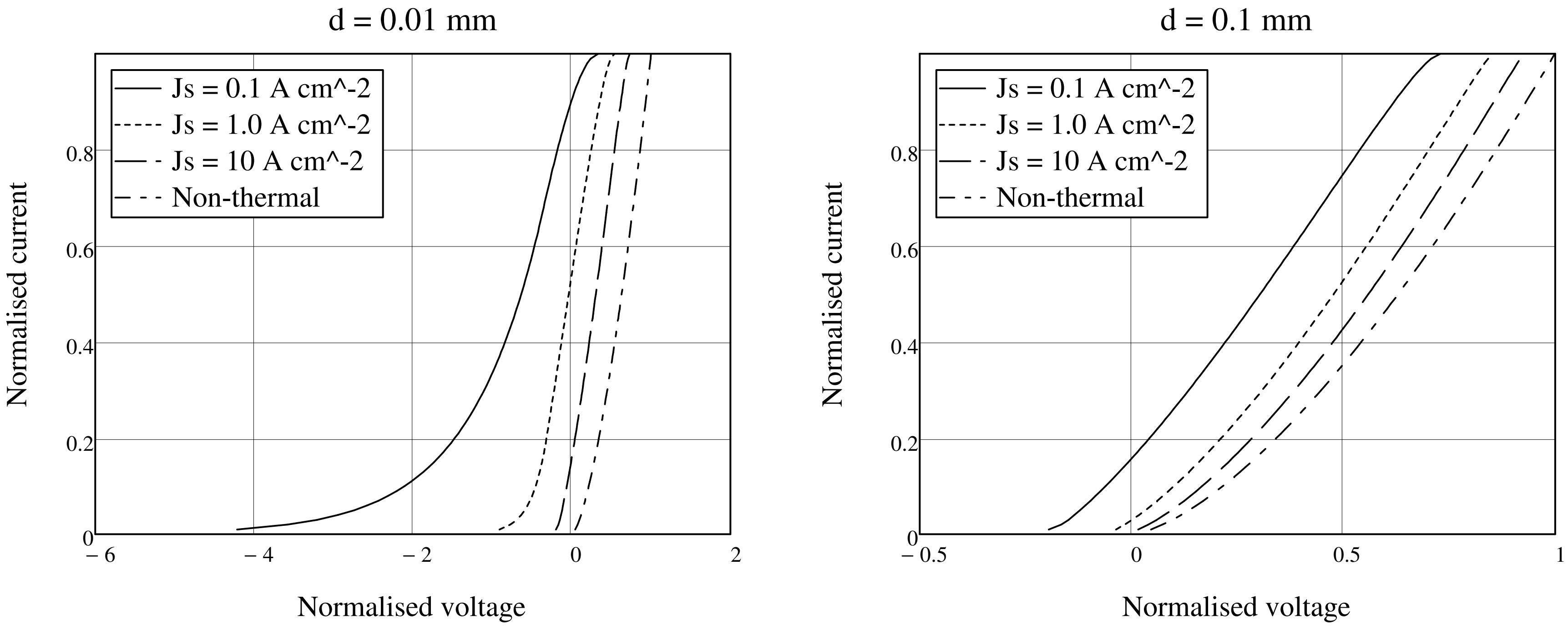


Figure 5.6(a) and (b)

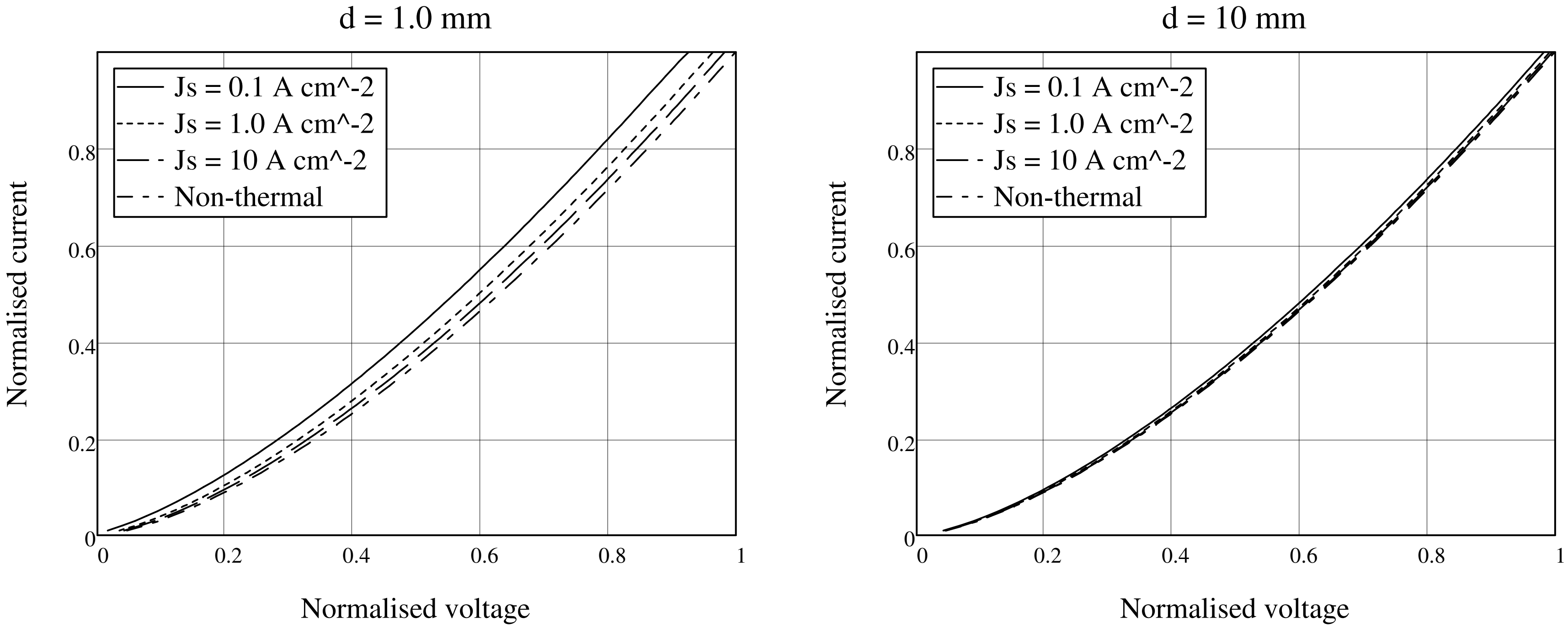


Figure 5.6(c) and (d)