WS 2.1 Ridged Waveguide

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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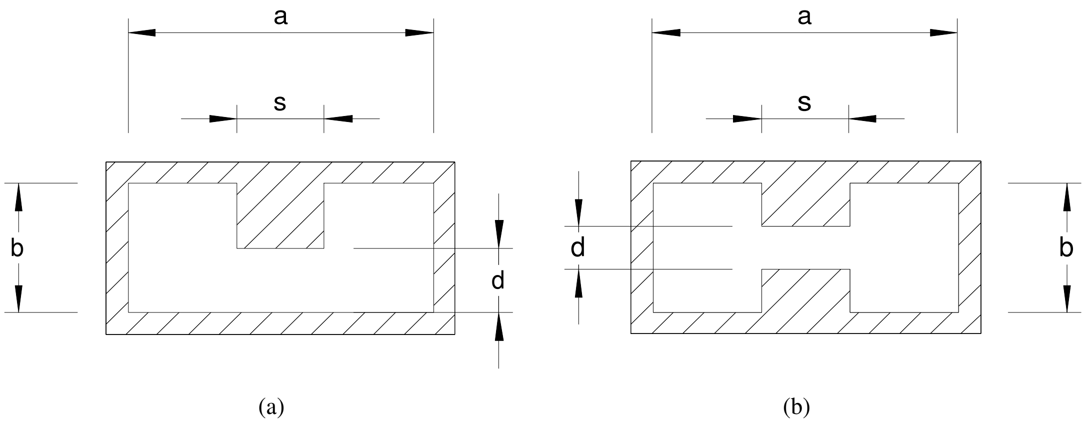


Figure 2.11

Section 2.3.3 Ridged waveguides

Calculation of cut-off wavelength

Define the aspect ratio of the guide (b/a) $b\_a := 0.5$

Note: $b\_a = \frac{b}{a};\ s\_a = \frac{s}{a};\ d\_b = \frac{d}{b}$

Define the admittances at the step for the outer region, the inner region, and the fringing fields, for a double-ridged guide

$$Y_1 = -j\frac{1}{b}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cot\left(\beta_0 a\left(1-s/a\right)/2\right)$$

$$Y_1(s\_a, \beta 0a) := -\cot[0.5\cdot\beta 0a\cdot(1 - s\_a)]$$

Equation 2.78

$$Y_2 = j\frac{1}{d}\sqrt{\frac{\varepsilon_0}{\mu_0}}\tan\left(\beta_0 a\left(s/a\right)/2\right)$$

$$Y_2(s\_a, d\_b, \beta 0a) := \frac{1}{d\_b}\cdot\tan(0.5\cdot\beta 0a\cdot s\_a)$$

Equation 2.79

$$B = \frac{\beta_0}{2\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\left[\left(\frac{d}{b}+\frac{b}{d}\right)\ln\left(\frac{1+d/b}{1-d/b}\right) - 2\ln\left(\frac{4d/b}{1-d^2/b^2}\right)\right]$$

$$B1(d\_b, \beta 0a) := \frac{\beta 0a\cdot b\_a}{2\cdot\pi}\cdot\left[\left(d\_b + \frac{1}{d\_b}\right)\cdot\ln\left(\frac{1 + d\_b}{1 - d\_b}\right) - 2\cdot\ln\left(\frac{4\cdot d\_b}{1 - d\_b^2}\right)\right]$$

Equation 2.80

Define the function whose root is the value of $\beta_0\cdot a$ at cut-off and solve for that value

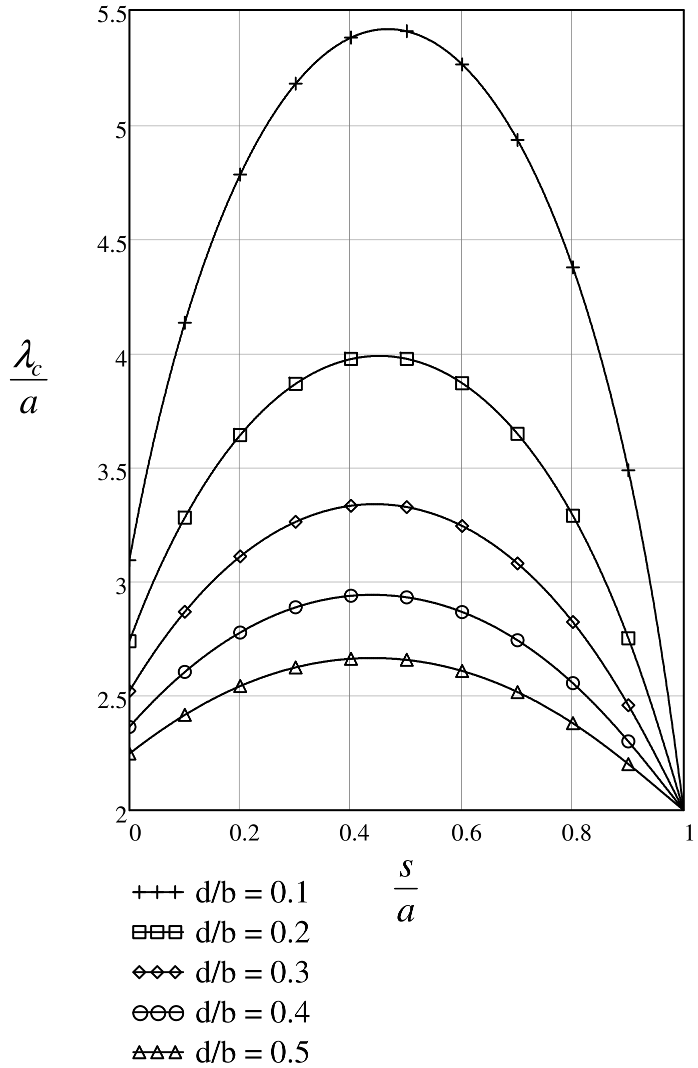
$$Fn(s\_a, d\_b, \beta 0a) := Y_1(s\_a, \beta 0a) + Y_2(s\_a, d\_b, \beta 0a) + B1(d\_b, \beta 0a)$$

Equation 2.81

Initial value of $\beta_0\cdot a$ $\beta 0a := \pi$

Find $\beta_c\cdot a$ $\beta ca(s\_a, d\_b) := root(Fn(s\_a, d\_b, \beta 0a), \beta 0a)$

Calculate $\frac{\lambda_c}{a}$ $\lambda c\_a(s\_a, d\_b) := \frac{2\cdot\pi}{\beta ca(s\_a, d\_b)}$



Set b/a = 0.5 for comparison with Figure 2.13 which is Figure 2 in Hopfer, S. (1955). "The Design of Ridged Waveguides." IRE Transactions on Microwave Theory and Techniques **3**(5): 20-29.

The agreement with Hopfer is excellent. The results for single-ridged guide can be created by doubling the value of b/a.

For comparison with Hopfer's Figure 5 for a single ridged guide b/a = 0.9

Calculation of characteristic admittance

The equivalent circuit of the TE01 mode is given in Figure 2.5. In the high frequency limit the impedance of the shunt reactance tends to infinity and the properties of the waveguide are represented by a transmission line with series inductance and shunt capacitance in which the phase velocity is equal to the velocity of light. Thus, in the high frequency limit, the charateristic admitttance of the waveguide is given by

$$Y_g = c \cdot C_g$$

where C_g is the capacitance per unit length. If we assume that this is effectively the capacitance between the ridges when d/b is << 1 then making use of Equation 2.96 the capacitance per unit length for a doubled-ridged guide is approximately

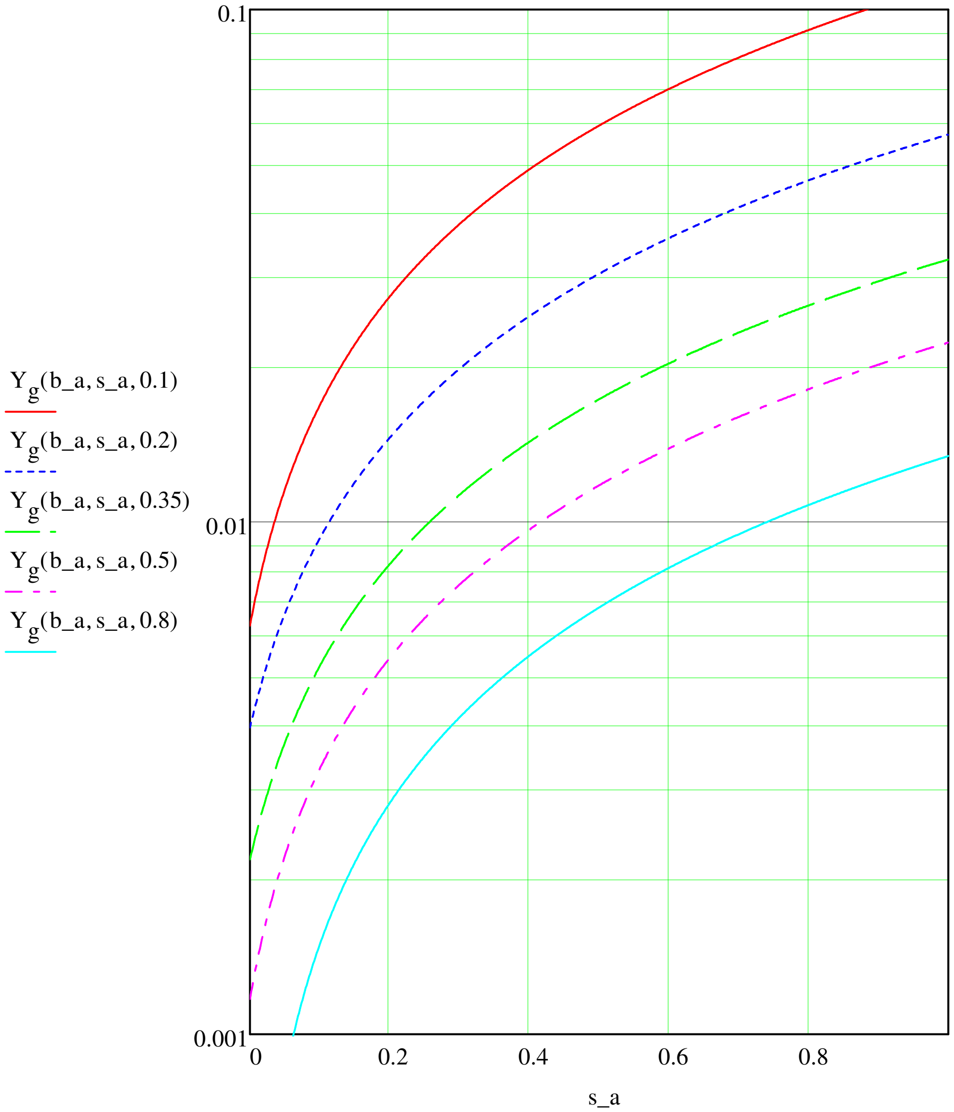
$$C_g = \frac{\varepsilon_0 s}{d} - \frac{2\varepsilon_0}{\pi} \ln\left[\sin\left(\frac{\pi d}{2b}\right)\right]$$

$$C_g(b\_a, s\_a, d\_b) := \varepsilon_0 \cdot \frac{s\_a}{d\_b \cdot b\_a} - \frac{2 \cdot \varepsilon_0}{\pi} \cdot \ln\left(\sin\left(\frac{\pi \cdot d\_b}{2}\right)\right)$$

For single-ridged guide the values of b/a and Yg must be doubled. If the number of ridges is $N_r := 1$

$$Y_g(b\_a, s\_a, d\_b) := \frac{2}{N_r} c \cdot C_g(b\_a, s\_a, d\_b)$$

The admittance values are multiplied by $\frac{\lambda}{\lambda_g}$ for lower frequencies



For comparison with Hopfer's Figure 16 for a single-ridged guide set b/a = 0.9 and N_r = 1.

There is good agreement for d/b < 0.5 and for 0.3 < s/a < 0.7 where the capacitance is dominated by that of the ridge.