WS 14.2 Helix TWT large-signal model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet provides a simple large-signal model of a helix TWT. Small-signal analysis (see Worksheet 14.1) is used to provide an initial estimate of the RF electric field acting on the electrons. The electrons in one wavelength are represented by a set of rigid disks of equal dimensions whose charges are equal. The motion of the disks is tracked through the helix with time as the independent variable using numerical integration. Dimensionless variables $\theta = \beta_e \cdot z$ and $\phi = \omega \cdot t$ are used.The RF power transferred to the helix is computed using conservation of energy.

The model can be run with, and without, space-charge and the backward wave. Three wavelengths of electrons are tracked where the outer wavelengths are guard wavelengths to ensure correct calculation of the space-charge forces. The results are computed from the electrons in the central wavelength. Good results are normally obtained with 24 electrons per wavelength.

The results in the time domain are converted into the space domain by finding the times at which the electrons cross NP regularly spaced planes and their velocities at those times. The RF beam current harmonics are computed by superimposing the harmonics of the current pulses of individual discs repeated at the signal frequency. This is a complex model which takes over a minute to produce the results.

The results obtained with this sheet differ slightly from those given in the book because of small changes to the model since those results were generated.

Tube data

Anode voltage

$V_a := 6.0 \cdot kV$

Beam current

$I_0 := 135 \cdot mA$

Frequency

$f := 11.7 \cdot GHz$

RF input power

$P_{in} := 10 \cdot mW$

Beam radius

$b := 0.34 \cdot mm$

Helix radius

$a := 0.68 \cdot mm$

Helix length

$L_h := 100 \cdot mm$

Propagation constant

$\beta 0 := 1700 \cdot m^{-1}$

Pierce impedance

$Z_P := 37 \cdot \Omega$

Cold loss per wavelength

$loss := 0.0 \cdot dB$

Parameters of the large-signal model

Number of discs per wavelength $Nd := 24$

Space-charge forces (1 = YES, 0 = NO) $SCF := 1$

Backward wave (1 = YES, 0 = NO) $BW := 0$

Saturation effects (1 = YES, 0 = NO) $SAT := 1$

When saturation effects are to be included the model is run first without saturation effects. The positions of the trapping and saturation planes in the green result fields are copied into the fields below and the model run with saturation effects included. The new plane positions are inserted and the process is repeated until convergence is obtained.

Trapping plane $z_t := 69 \cdot mm$

Saturation plane $z_s := 89 \cdot mm$

The section below can be collapsed to allow the data and the results to be seen on the screen together.

Small-signal model

Define constants

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \frac{C}{kg}$

Perveance $\mu P := \mu A \cdot V^{-1.5}$

$dB \equiv 1$

$dBm := 1$

Calculate the beam velocity, allowing for space-charge potential depression, and the electronic propagation constant.

$$V_0 := \begin{array}{|l} V_1 \leftarrow V_a \\ \text{for } n \in 1..5 \\ \quad \begin{array}{|l} u_n \leftarrow c \cdot \left[1 - \frac{1}{\left(1 + \frac{\eta \cdot V_n}{c^2}\right)^2}\right]^{0.5} \\ V_{n+1} \leftarrow V_1 - \frac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left(\frac{1}{2} - \ln\left(\frac{b}{a}\right)\right) \end{array} \\ \text{return } V_{n+1} \end{array}$$

$$u_0 := c \cdot \left[1 - \frac{1}{\left(1 + \frac{\eta \cdot V_0}{c^2}\right)^2}\right]^{0.5}$$

$$Rel := \frac{1}{\sqrt{1 - \frac{u_0^{\,2}}{c^2}}}$$

$V_0 = 5.94 \cdot kV$

$u_0 = 4.53 \times 10^7 \cdot m \cdot s^{-1}$

$Rel = 1.012$

$$Perv := \frac{I_0}{V_a^{\,1.5}}$$

$Perv = 0.29 \cdot \mu P$

$\omega := 2 \cdot \pi \cdot f$

$\beta_e := \frac{\omega}{u_0}$

$\lambda_e := \frac{2 \cdot \pi}{\beta_e}$

$\gamma_e := \frac{\beta_e}{Rel}$

$\gamma_e \cdot b = 0.545$

Calculate the plasma frequency and the reduced plasma frequency

Plasma frequency $\omega_p := \sqrt{\frac{\eta}{\varepsilon_0} \cdot \frac{I_0}{\pi \cdot b^2 \cdot u_0} \cdot \frac{1}{Rel^3}}$

Ratio of magnetic field to the Brillouin field chosen to approximate confined flow $mB := 10$

$$\omega_q := \left| \begin{array}{l} \gamma a \leftarrow \gamma_e \cdot a \\ \gamma b \leftarrow \gamma_e \cdot b \\ \tau b(p) \leftarrow \gamma b \cdot \left[\dfrac{\dfrac{1}{p^2} - 1}{\dfrac{1}{p^2 - 2\cdot\left(mB^2 - 1\right)} - 1} \right]^{\frac{1}{2}} \\ fn1 \leftarrow \dfrac{1}{\gamma b} \cdot \dfrac{I1(\gamma b)\cdot K0(\gamma a) + I0(\gamma a)\cdot K1(\gamma b)}{I0(\gamma b)\cdot K0(\gamma a) - I0(\gamma a)\cdot K0(\gamma b)} \\ fn2(p) \leftarrow \dfrac{1 - \dfrac{1}{p^2}}{\tau b(p)} \cdot \dfrac{I1(\tau b(p))}{I0(\tau b(p))} \\ fn(p) \leftarrow \dfrac{1}{fn1} - \dfrac{1}{fn2(p)} \\ p0 \leftarrow 0.9 \\ \omega_q \leftarrow root(fn(p0), p0)\cdot \omega_p \end{array} \right.$$

$$\beta_q := \frac{\omega_q}{u_0}$$

$$\frac{\omega_q}{\omega} = 0.059$$

Propagation constants of the fast and slow space-charge waves

$$\beta_f := \beta_e - \beta_q \qquad \beta_s := \beta_e + \beta_q \qquad \gamma_f := \frac{\beta_f}{Rel} \qquad \gamma_s := \frac{\beta_s}{Rel}$$

Electronic admittance of the beam

$$Y_e := \frac{I_0}{Rel\cdot(Rel + 1)\cdot V_0}\cdot\frac{\omega}{\omega_q} \qquad \boxed{\text{Equation 11.80}} \qquad Z_e := \frac{1}{Y_e}$$

Phase velocity of the slow space-charge wave

$$vp_s := \frac{\omega}{\beta_s}$$

Calculate the characteristic impedance and the coupling impedance of the helix

$$\gamma_0 := \frac{\beta 0}{Rel} \qquad Z_c := Z_P\cdot I0\left(\gamma_0\cdot a\right)^2 \qquad Y_c := \frac{1}{Z_c} \qquad \gamma_1 := \frac{1}{2}\cdot\left(\gamma_0 + \gamma_s\right) \qquad \mu_c := \frac{2}{\gamma_1\cdot b}\cdot\frac{I1\left(\gamma_1\cdot b\right)}{I0\left(\gamma_1\cdot a\right)}$$

$$Z_c = 67.8\,\Omega$$

$$\beta_0 := \beta 0\cdot\left(1 - j\cdot\frac{loss}{40\cdot\pi\cdot\log(e)}\right)$$

$$\mu_c = 0.767$$

Pierce parameters

$$CP := \left(\frac{\mu_c^{\,2}\cdot Z_c\cdot I_0}{4\cdot V_a}\right)^{\frac{1}{3}} \qquad QC := \left(\frac{\omega_q}{\omega+\omega_q}\cdot\frac{1}{2\cdot CP}\right)^2 \qquad bP := \frac{1}{CP}\cdot\left(\frac{\beta_0}{\beta_e}-1\right) \qquad \alpha := \frac{\text{loss}\cdot\beta_0}{40\cdot\pi\cdot\log(e)} \qquad dP := \frac{1}{CP}\cdot\frac{\alpha}{\beta_e}$$

Coupled-mode matrix

$$CM := \begin{pmatrix} \beta_0 & 0 & \frac{1}{2}\cdot\mu_c\cdot Z_c\cdot Y_e\cdot\beta_f & -\frac{1}{2}\cdot\mu_c\cdot Z_c\cdot Y_e\cdot\beta_s \\ 0 & -\beta_0 & -\frac{1}{2}\cdot\mu_c\cdot Z_c\cdot Y_e\cdot\beta_f & \frac{1}{2}\cdot\mu_c\cdot Z_c\cdot Y_e\cdot\beta_s \\ \frac{1}{2}\cdot\mu_c\cdot\beta_0 & -\frac{1}{2}\cdot\mu_c\cdot\beta_0 & \beta_f & 0 \\ \frac{1}{2}\cdot\mu_c\cdot\beta_0 & -\frac{1}{2}\cdot\mu_c\cdot\beta_0 & 0 & \beta_s \end{pmatrix}$$

Equation 11.132

Note: this equation is wrong in the book and has been corrected

The eigenvalues of the matrix are

$$\beta := \left| \begin{array}{l} \beta \leftarrow \text{sort}(\text{eigenvals}(CM)) \\ \beta \leftarrow \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_3 \\ \beta_2 \end{pmatrix} \ \text{if } \operatorname{Im}(\beta_3) < 0 \\ \beta \end{array} \right.$$

Note: The roots have been sorted in the order:

(1) Backward wave
(2) Fast wave
(3) Decaying wave
(4) Growing wave

$$\beta = \begin{pmatrix} -1700 \\ 1505 \\ 1720-71i \\ 1720+71i \end{pmatrix} m^{-1}$$

Growing wave propagation constant $\beta_g := \beta_3$

Dimensionless connection matrix which relates the amplitudes of the coupled modes (W) to those of the uncoupled modes (U)

$$CC := \begin{array}{|l} Yc \leftarrow Y_c \cdot \Omega \\ Ye \leftarrow Y_e \cdot \Omega \\ C1 \leftarrow \begin{pmatrix} 1 & 1 & 0 & 0 \\ Yc & -Yc & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & Ye & -Ye \end{pmatrix} \\ C2 \leftarrow \begin{array}{|l} \text{for } j \in 0..3 \\ \quad \begin{array}{|l} C2_{0,j} \leftarrow 1 \\ C2_{1,j} \leftarrow \dfrac{\beta_j \cdot Yc}{\beta_0} \\ C2_{3,j} \leftarrow \left[\dfrac{(\beta_j)^2 - \beta_0{}^2}{\beta_j \cdot \beta_0}\right] \cdot \dfrac{Yc}{\mu_c} \\ C2_{2,j} \leftarrow C2_{3,j} \cdot \dfrac{\beta_e - \beta_j}{\beta_q \cdot Ye} \end{array} \\ C2 \end{array} \\ CC \leftarrow C2^{-1} \cdot C1 \end{array}$$

Equation 14.14

Equations 14.26 to 14.29

Equation 14.31

Calculate the ratio of electron velocity to the phase velocity of the growing wave and to the phase velocity of the helix

$$u0\_vg := \frac{Re(\beta_g)}{\beta_e} \qquad u0\_vg = 1.060$$

$$u0\_vp := \frac{Re(\beta_0)}{\beta_e} \qquad u0\_vp = 1.048$$

Calculate the phase of the bunch relative to the growing wave. The phase shift of π is a result of the sign convention used for the current in the large-signal model.

$$\phi IV := 1 + \frac{1}{\pi} \arg\left[\frac{\beta_0{}^2 - (\beta_g)^2}{\beta_0 \cdot \beta_g}\right]$$

Equation 14.28

Dimensionless transfer matrix which relates the uncoupled modes at the start of a section to those at the end of the section

$$
TT := \left|
\begin{array}{l}
\beta z \leftarrow \beta \cdot L_h \\
SS \leftarrow \begin{pmatrix} \exp(-j \cdot \beta z_0) & 0 & 0 & 0 \\ 0 & \exp(-j \cdot \beta z_1) & 0 & 0 \\ 0 & 0 & \exp(-j \cdot \beta z_2) & 0 \\ 0 & 0 & 0 & \exp(-j \cdot \beta z_3) \end{pmatrix} \\
T \leftarrow CC^{-1} \cdot SS \cdot CC
\end{array}
\right.
$$

Equation 14.32

Equation 14.33

Transfer matrix which relates the uncoupled modes at the start of a section to the wave amplitudes at position z is

$$
RR(z) := \left|
\begin{array}{l}
\beta z \leftarrow \beta \cdot z \\
SS \leftarrow \begin{pmatrix} \exp(-j \cdot \beta z_0) & 0 & 0 & 0 \\ 0 & \exp(-j \cdot \beta z_1) & 0 & 0 \\ 0 & 0 & \exp(-j \cdot \beta z_2) & 0 \\ 0 & 0 & 0 & \exp(-j \cdot \beta z_3) \end{pmatrix} \\
RR \leftarrow SS \cdot CC
\end{array}
\right.
$$

Set up initial conditions

Voltage of the input signal $V_{in} := \sqrt{2 \cdot P_{in} \cdot Z_c}$ $V_{in} = 1.16\,V$

Uncoupled amplitudes at the input

$$U1 := \begin{pmatrix} 1 \\ \dfrac{-RR(L_h)_{0,0}}{RR(L_h)_{0,1}} \\ 0 \\ 0 \end{pmatrix} \cdot V_{in}$$

This definition of U1 ensures that the output of the tube is matched so that the amplitude of the backward wave at the output is zero. It is necessary because a single section with high gain has been used so that reflection of power at the output can cause errors which would not occur in a severed tube.

Check wave amplitudes at the output $W2 := RR(L_h) \cdot U1$

$$W2 = \begin{pmatrix} 0.0 \\ 0.1 + 0.0i \\ -0.0 - 0.0i \\ -434.6 - 427.5i \end{pmatrix} V$$

Uncoupled amplitudes in the output waveguide

$U2 := TT \cdot U1$ Equation 14.33

Computation of the signal growth along the length of the tube

The forward power on the helix as a function of position along the structure is computed from the forward wave amplitudes at the start of the section using equations (14.26) and (14.27).

$$W1(z) := RR(z)\cdot U1 \qquad P_f(z) := \frac{\mathrm{Re}\left(W1(z)_1\cdot\overline{W1(z)_1}\cdot\beta_1 + W1(z)_2\cdot\overline{W1(z)_2}\cdot\beta_2 + W1(z)_3\cdot\overline{W1(z)_3}\cdot\beta_3\right)}{2\cdot\mathrm{Re}\left(\beta_0\right)\cdot Z_c} \qquad P_f(0) = 4.19\cdot\mathrm{mW}$$

Small-signal gain for the full length of the helix

$$\mathrm{Gain}(z) := 10\cdot\log\left(\left|\frac{P_f(z)}{P_{in}}\right|\right) \qquad \mathrm{Gain}\left(L_h\right) = 54.4\cdot\mathrm{dB}$$

Plotting range $\qquad z1 := 0, 0.001\cdot L_h .. L_h$

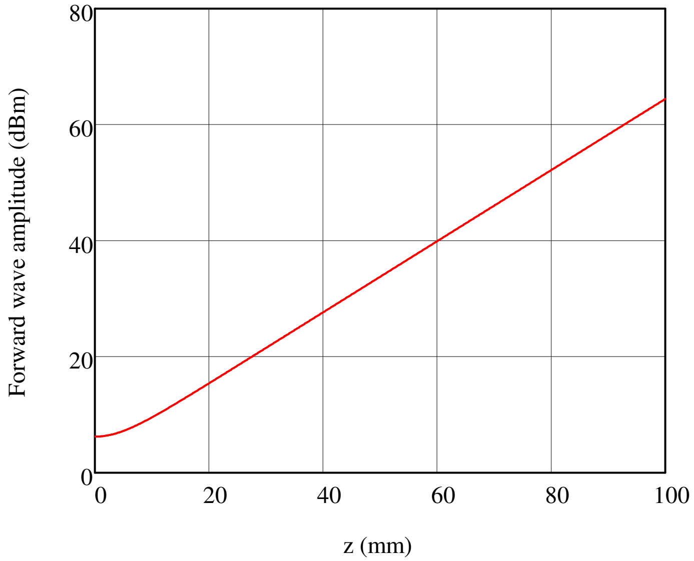


Figure 14.10

Large-signal disk model

It will not normally be necessary to change these settings

Number of wavelengths tracked (odd) $N\lambda := 3$

Number of harmonics for current calculations $NH := 6$

Bunch centre starting position $\theta_0 := -(N\lambda + 1)\pi$

Number of reference planes $NP := 200$

Number of integration steps $nmax := 100$

Final values of the normalised position and time $\theta_f := \beta_e \cdot L_h$ $\quad \phi_f := \theta_f + 10\cdot\pi$

Number of discs $ND := Nd \cdot N\lambda$

Disk charge $Q := \dfrac{2\cdot\pi\, I_0}{\omega \cdot Nd}$

Normalised disk thickness $\theta_d := \dfrac{2\cdot\pi\cdot N\lambda}{ND}$

Disk length $\Delta L := \dfrac{\theta_d}{\beta_e}$

Normalised initial positions and velocities of the disks

$$\theta := \left| \begin{array}{l} \text{for } j \in 0, 2 .. 2\cdot(ND-1) \\ \quad \left| \begin{array}{l} \theta_j \leftarrow \theta_0 + N\lambda\pi\cdot\left(1 - \dfrac{j+1}{ND}\right) \\ \theta_{j+1} \leftarrow 1 \end{array} \right. \\ \theta \end{array} \right.$$

Compute the space-charge function

The Space-Charge Field is found from the equations given in
J.R. Hechtel, "The effect of potential beam energy on the performance of linear beam devices",
IEEE Transactions on Electron Devices ED-17, pp.999-1009, Nov. 1970.

Define the first ten zeros of the Bessel function $J_0(z)$.

$$\mu B := \frac{1}{a}\cdot\begin{pmatrix} 2.405 & 5.520 & 8.654 & 11.791 & 14.931 & 18.071 & 21.212 & 24.352 & 27.494 & 30.635 \end{pmatrix}^T$$

Charge density $\rho_0 := -\dfrac{1}{\pi\cdot b^2\cdot\Delta L}$ — ρ_0 is calculated for a disk charge of -1C.
Thus the electric field must be multiplied by the charge of the source disk

The space-charge field is calculated at intervals over a normalised distance of 2π from the centre of the disc.

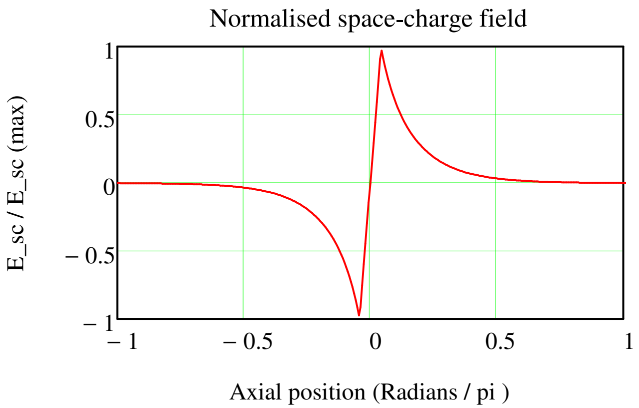
$$\begin{aligned}
ESn := \; & \text{for } n \in 0..240 \\
& \quad \theta_n \leftarrow \frac{n}{120}\cdot\pi \\
& \quad z_n \leftarrow \frac{\theta_n}{\beta_e} \\
& \quad ES_n \leftarrow \left(\frac{4\cdot Q\cdot\rho_0}{\varepsilon_0}\right)\cdot\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\exp(-\mu B_m\cdot z_n)\cdot\sinh\left(\frac{\mu B_m\cdot\Delta L}{2}\right)\right] \text{ if } \theta_n \ge 0.5\cdot\theta_d \\
& \quad ES_n \leftarrow \left(\frac{4\cdot Q\cdot\rho_0}{\varepsilon_0}\right)\cdot\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\left(\exp\left(-\mu B_m\cdot\frac{\Delta L}{2}\right)\cdot\sinh(\mu B_m\cdot z_n)\right)\right] \text{ otherwise} \\
& ES
\end{aligned}$$

The space-charge table is converted into a continuous function using linear interpolation

$$\theta n := \left| \begin{array}{l} \text{for } n \in 0..240 \\ \quad \theta_n \leftarrow \frac{n}{120} \cdot \pi \\ \theta \end{array} \right.$$

$$Es(\theta) := \text{sign}(\theta) \cdot \text{linterp}(\theta n, ESn, |\theta|)$$

$$ES(\theta) := \left| \begin{array}{ll} Es(\theta + 2 \cdot N\lambda \cdot \pi) & \text{if } \theta < -N\lambda \cdot \pi \\ Es(\theta - 2 \cdot N\lambda \cdot \pi) & \text{if } \theta > N\lambda \cdot \pi \\ Es(\theta) & \text{otherwise} \end{array} \right.$$



Interaction field

The interaction field on the axis is found using small-signal theory because Mathcad is too slow to make iteration possible.

All quantities are defined in terms of absolute position and time

$$W_{in} = \begin{pmatrix} 0.000 \\ 0.115 \\ 0.525 + 0.012i \\ 0.525 - 0.012i \end{pmatrix} V$$

Initial wave amplitudes $\quad W_{in} := W1(0)$

Forward wave field

$$E_f(\theta,\phi) := \left| \begin{array}{l} Ef \leftarrow \left| \begin{array}{ll} \mathrm{Re}\left[\sum_{n=1}^{3}\left[j\cdot\beta_n\cdot W_{in_n}\cdot\exp\left[j\cdot\left(\phi - \frac{\beta_n}{\beta_e}\cdot\theta\right)\right]\right]\right] & \text{if } \theta \ge 0 \wedge \theta \le \theta_f \\ 0 & \text{otherwise} \end{array}\right. \\ \text{return } Ef\cdot\mu_c \end{array}\right.$$

Backward wave field

$$E_b(\theta,\phi) := \left| \begin{array}{l} Eb \leftarrow \left| \begin{array}{ll} \mathrm{Re}\left[j\cdot\beta_0\cdot W_{in_0}\cdot\exp\left[j\cdot\left(\phi - \frac{\beta_0}{\beta_e}\cdot\theta\right)\right]\right] & \text{if } \theta \ge 0 \wedge \theta \le \theta_f \\ 0 & \text{otherwise} \end{array}\right. \\ \text{return } Eb\cdot\mu_c \end{array}\right.$$

$$Sat(z) := \left[\left| \begin{array}{l} Sat \leftarrow \left| \begin{array}{ll} \exp\left[-\mathrm{Im}(\beta_3)\cdot\frac{(z - z_t)^2}{2\cdot(z_s - z_t)}\right] & \text{if } z \ge z_t \wedge SAT = 1 \\ 1 & \text{otherwise} \end{array}\right. \\ \text{return } Sat \end{array}\right. \right]$$

Define an approximate function to model saturation using quadratic variation of growth rate starting from the position of first trapping. Note that this differs from the cubic function in equation (14.65). The quadratic function gives better consistency with the large-signal results in this case.

Interaction field (V/m) $\quad E_z(\theta,\phi) := \left(E_f(\theta,\phi) + BW\cdot E_b(\theta,\phi)\right)\cdot Sat\left(\frac{\theta}{\beta_e}\right)$

The Coefficients of the Differential Equations for the motions of the electrons are defined.

The rows represent, in order, the position in radians and the normalised velocity of the electrons.

$$\text{Const} := -\frac{\eta}{\omega \cdot u_0}$$

$$D(\phi,\theta) := \left| \begin{array}{l} \text{for } j \in 0,2..2\cdot(ND-1) \\ \quad \left| \begin{array}{l} D_j \leftarrow \theta_{j+1} \\ D_{j+1} \leftarrow \left\| \begin{array}{ll} \left[\text{Const}\cdot\left[1-\left(\frac{u_0}{c}\cdot\theta_{j+1}\right)^2\right]^{1.5} \cdot E_z(\theta_j,\phi) \right] & \text{if } SCF \neq 1 \\ \left[\text{Const}\cdot\left[1-\left(\frac{u_0}{c}\cdot\theta_{j+1}\right)^2\right]^{1.5} \cdot \left[E_z(\theta_j,\phi) + \sum_{i=0}^{ND-1} \left(ES\left(\theta_j - \theta_{2\cdot i}\right)\right) \right] \right] & \text{otherwise} \end{array} \right. \end{array} \right. \\ D \end{array} \right.$$

Definitions of normalised variables

$\phi = \omega \cdot t \qquad \theta = \beta_e \cdot z \qquad \theta' = \frac{v}{u_0}$

$\frac{d}{dt} z = v \qquad \frac{d}{d\phi}\theta = \frac{v}{u_0}$

$\frac{d}{dt} v = -\eta \cdot E \qquad \frac{d}{d\phi}\frac{v}{u_0} = \frac{\eta \cdot E}{\omega \cdot u_0}$

The Equations are Solved using with *nmax* time steps starting from 0. The final time is t_f

The variable tol specifies the tolerance on the solution of the differential equations.
10E-6 works well normally but much smaller values may be needed at low drive levels $\quad tol := 10^{-6}$

$$Z := \text{AdamsBDF}\left(\theta, 0, \phi_f, \text{nmax}, D, \text{tol}\right)$$

The results are in a single table (Z) in which the first column (0) is the time and the other columns (1-12) are the positions and velocities of the electrons in the same order as before at each value of n.

Extract the vector of phase, the matrices containing the normalised positions and velocities of the disks and the vector of the final velocities of the electrons

$$\phi n := \left| \begin{array}{l} \text{for } n \in 0..\text{nmax} \\ \quad \phi_n \leftarrow Z_{n,0} \\ \phi \end{array} \right. \qquad \theta n := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..\text{nmax} \\ \quad\quad \theta_{n,j} \leftarrow Z_{n,2\cdot j+1} \\ \theta \end{array} \right. \qquad un := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..\text{nmax} \\ \quad\quad u_{n,j} \leftarrow Z_{n,2\cdot j+2} \\ u \end{array} \right. \qquad umax := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad u_j \leftarrow Z_{\text{nmax},2\cdot j+2} \\ u \end{array} \right.$$

Define a set of equally-spaced reference planes in θ.

$$\theta p := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \theta_p \leftarrow \dfrac{p}{NP}\cdot\theta_f \\ \text{return } \theta \end{array} \right. \qquad zp := \frac{\theta p}{\beta_e}$$

Find the normalised velocities of the electrons as they cross each reference plane

$$up := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } p \in 0..NP \\ \quad\quad \text{for } n \in 1..\text{nmax} \\ \quad\quad\quad \left| \begin{array}{l} \text{flag} \leftarrow 0 \\ \text{flag} \leftarrow 1 \text{ if } \theta n_{n,j} > \theta p_p \\ up_{p,j} \leftarrow un_{n-1,j} + \dfrac{\theta p_p - \theta n_{n-1,j}}{\theta n_{n,j} - \theta n_{n-1,j}}\cdot\left(un_{n,j} - un_{n-1,j}\right) \text{ if flag} = 1 \\ (\text{break}) \text{ if flag} = 1 \end{array} \right. \\ \text{return } up \end{array} \right.$$

Find the phase of the wave on the helix as a function of position from the small signal model

$$V_h(\theta,\phi) := \sum_{n=1}^{3}\left[W_{in_n}\cdot\exp\left[j\cdot\left(\phi - \frac{\beta_n-\beta_3}{\beta_e}\cdot\theta\right)\right]\right] \qquad \phi V(\theta) := \arg\left(V_h(\theta,0)\right) - \frac{\mathrm{Re}\left(\beta_3\right)}{\beta_e}\cdot\theta$$

Find the phase when each electron crosses each reference plane referred to the phase of the growing wave determined from the small-signal model.

```
φp := | for j ∈ 0..(ND − 1)
      |   for p ∈ 0..NP
      |     for n ∈ 1..nmax
      |       | flag ← 0
      |       | flag ← 1  if θn[n,j] > θp[p]
      |       | φp[p,j] ← [φn[n−1] + (θp[p] − θn[n−1,j])/(θn[n,j] − θn[n−1,j])·(φn[n] − φn[n−1])] + φV(θp[p])  if flag = 1
      |       | (break)  if flag = 1
      | for p ∈ 1..NP
      |   | φp[p,j] ← φp[p,j] + 2·π  if φp[p−1,j] − φp[p,j] > π
      |   | φp[p,j] ← φp[p,j] − 2·π  if φp[p,j] − φp[p−1,j] > π
      | return φp
```

$$\phi p_{p,j} \leftarrow \left[\phi n_{n-1} + \frac{\theta p_p - \theta n_{n-1,j}}{\theta n_{n,j} - \theta n_{n-1,j}}\cdot\left(\phi n_n - \phi n_{n-1}\right)\right] + \phi V\left(\theta p_p\right) \quad \text{if flag} = 1$$

Define the serial numbers of the electrons in the central group used to calculate the performance of the tube.

$$NG := 0.5\cdot(N\lambda - 1) \qquad N1 := NG\cdot Nd \qquad N2 := N1 + Nd - 1$$

Find the mean velocity of the electrons normalised to the phase velocity of the growing wave

$$umean := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad um_p \leftarrow \frac{1}{Nd}\cdot\sum_{j=N1}^{N2} up_{p,j} \\ um\cdot\frac{Re(\beta_3)}{\beta_e} \end{array} \right.$$

Calculate the complex current harmonics at each plane, normalised to the DC beam current, by superimposing the Fourier components of the discs. For simplicity each disc is treated as having constant charge and length. The currents are referred to the phase of the growing wave.

$$Ip := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \left| \begin{array}{l} Ip_{p,0} \leftarrow \sum_{j=N1}^{N2} \left(\frac{1}{2}\cdot\theta_d\right) \\ \text{for } h \in 1..NH \\ \quad Ip_{p,h} \leftarrow \sum_{j=N1}^{N2} \left(\theta_d\cdot \text{sinc}\left(\frac{h\cdot\theta_d}{2\cdot\pi\cdot up_{p,j} + 0\cdot 10^{-6}}\right)\cdot\exp\left(j\cdot h\cdot\phi p_{p,j}\right)\right) \end{array} \right. \\ \text{return } \frac{Ip}{\pi} \end{array} \right.$$

Instantaneous current = $\frac{Q_j\cdot up_{p,j}}{\Delta L}$

Pulse phase duration = $\frac{\theta_d}{up_{p,j}}$

Find the normalised amplitude and the phase of the first harmonic of the RF beam current

$$Ip1 := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad Ip1_p \leftarrow \left| \left(Ip^{\langle 1 \rangle}\right)_p \right| \\ Ip1 \end{array} \right.$$

$$\phi I1 := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \phi I_p \leftarrow \arg\left[\left(Ip^{\langle 1 \rangle}\right)_p\right] \\ \phi I \end{array} \right.$$

Find the first maximum of I_1/I_0

$$Ip1_{max} := \left| \begin{array}{l} \text{for } p \in 0..NP-1 \\ \quad \left| \begin{array}{l} I_{max} \leftarrow Ip1_p \ \text{ if } Ip1_p > Ip1_{p+1} \\ (\text{break}) \ \text{ if } I_{max} \neq 0 \end{array} \right. \\ (\text{break}) \ \text{ if } I_{max} \neq 0 \\ \text{return } I_{max} \end{array} \right.$$

Calculate the total kinetic power of the electrons as they cross each reference plane.

$$P_{DC} := I_0 \cdot V_0$$

$$Prp := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad P_p \leftarrow \dfrac{P_{DC}}{Nd \cdot \left(\dfrac{1}{\sqrt{1 - \dfrac{u_0^{\,2}}{c^2}}} - 1 \right)} \cdot \displaystyle\sum_{j=N1}^{N2} \left[\dfrac{1}{\sqrt{1 - \dfrac{u_0^{\,2}}{c^2} \left(up_{p,j}\right)^2}} - 1 \right] \\ P \end{array} \right.$$

CHECK the initial beam power is equal to the DC power $\quad Prp_0 = 801.4\,W \quad P_{DC} = 801.4\,W$

The RF power on the helix at each plane is equal to the DC beam power plus the initial RF power minus the residual beam power at that plane.
This ignores the effects of harmonics

$$P_{RF} := P_f(0) + P_{DC} - Prp$$

Find the maximum RF power, the saturated gain and the efficiency

$$P_{max} := \max(P_{RF}) \qquad Max\_power := 10 \cdot \log\left(\frac{P_{max}}{1 \cdot mW}\right) \qquad Sat\_gain := 10 \cdot \log\left(\frac{P_{max}}{P_{in}}\right) \qquad \eta_e := \frac{P_{max}}{I_0 \cdot V_a}$$

Find the initiation plane at which the large-signal phase first equals the small-signal phase

$$p_i := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \left| \begin{array}{l} p_i \leftarrow p \ \text{ if } \ \dfrac{-\phi I1_p}{\pi} \ge \phi IV \\ (\text{break}) \ \text{ if } \ p_i \ne 0 \end{array} \right. \\ (\text{break}) \ \text{ if } \ p_i \ne 0 \\ \text{return } p_i \end{array} \right.$$

$$zi := p_i \cdot \frac{L_h}{NP}$$

Find the plane at which trapping commences where the velocity of the slowest electron equals the phase velocity of the growing wave obtained from the small-signal model.

$$p_t := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \left| \begin{array}{l} \text{for } j \in N1..N2 \\ \quad \left| \begin{array}{l} p_t \leftarrow p \ \text{ if } \ up_{p,j} \cdot \dfrac{\mathrm{Re}(\beta_g)}{\beta_e} \le 1 \\ (\text{break}) \ \text{ if } \ p_t \ne 0 \end{array} \right. \\ (\text{break}) \ \text{ if } \ p_t \ne 0 \end{array} \right. \\ \text{return } p_t \end{array} \right.$$

$$zt := p_t \cdot \left(\frac{L_h}{NP} \right)$$

Find the plane of saturation

$$p_s := \left| \begin{array}{l} \text{for } p \in 0.5 \cdot NP..NP-1 \\ \quad \left| \begin{array}{l} p_s \leftarrow p \ \text{ if } \ P_{RF_p} > P_{RF_{p+1}} \\ (\text{break}) \ \text{ if } \ p_s \ne 0 \end{array} \right. \\ (\text{break}) \ \text{ if } \ p_s \ne 0 \\ \text{return } p_s \end{array} \right.$$

$$zs := p_s \cdot \frac{L_h}{NP}$$

Calculate the spent beam distribution curve at plane P

$$Vs(P) := \left| \begin{array}{l} \text{for } j \in 0..Nd-1 \\ \quad us_j \leftarrow up_{P,j+Nd} \\ uss \leftarrow \mathrm{reverse}(\mathrm{sort}(us)) \\ \text{for } j \in 0..Nd-1 \\ \quad Vs_j \leftarrow (uss_j)^2 \\ \text{return } Vs \end{array} \right.$$

$$Gain(zs) = 47.990 \cdot dB$$

Plotting ranges

$p1 := 0..NP$ $nj := 0..ND$ $j1 := 0..Nd-1$

Gain compression

$$Compression := Gain(zs) - Sat\_gain$$

Results

$Perv = 0.290 \cdot \mu P$

Small-signal results

Pierce parameters (C, QC, b and d)	$CP = 0.061$	$QC = 0.212$	$bP = 0.784$	$dP = 0.000$
Relative phase of the bunch / π	$\phi IV = 0.406$			
Beam velocity / phase velocity of helix	$u0\_vp = 1.048$	Beam velocity / phase velocity of growing wave		$u0\_vg = 1.060$

Large-signal results

Saturated RF power	$P_{max} = 133 \cdot W$	Efficiency	$\eta_e = 16.4 \cdot \%$
Saturated gain	$Sat\_gain = 41.2 \cdot dB$	Gain compression	$Compression = 6.8 \cdot dB$
Maximum normalised RF beam current	$Ip1_{max} = 1.102$		
Initiation, trapping and saturation planes	$zi = 22.5 \cdot mm$	$zt = 69.0 \cdot mm$	$zs = 89.5 \cdot mm$
Plotting plane and phase offset	$P := p_s$	$Offset := 2$	

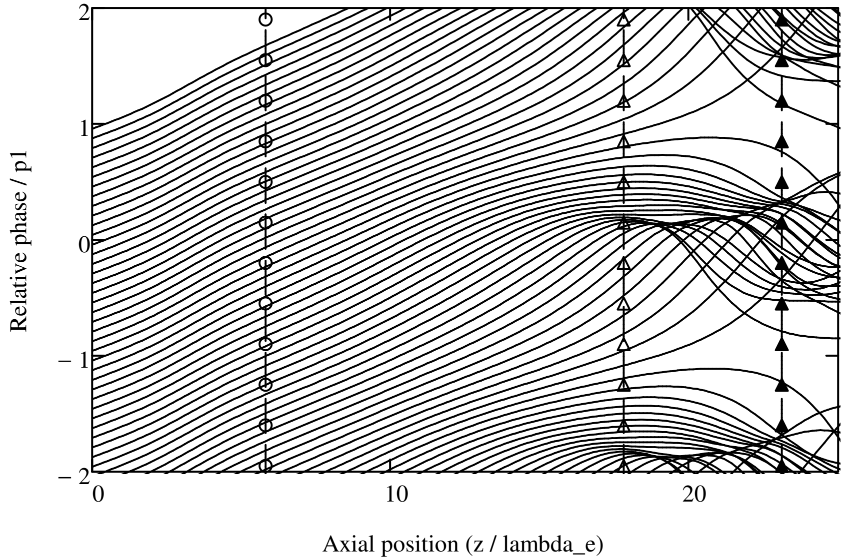


Figure 14.13

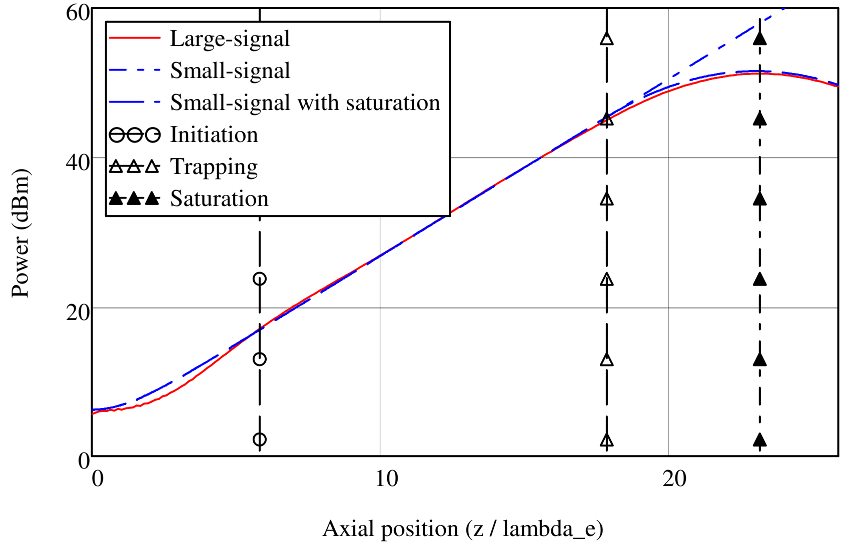


Figure 14.10

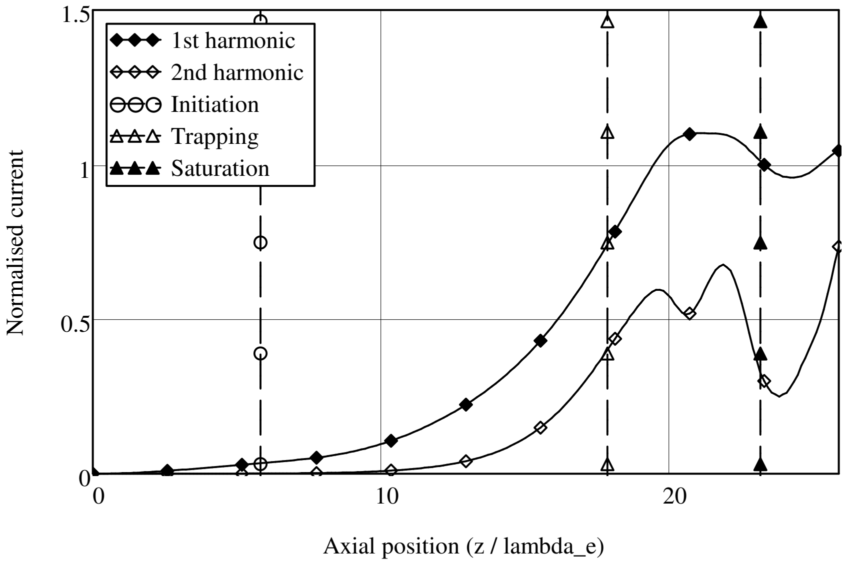


Figure 14.11

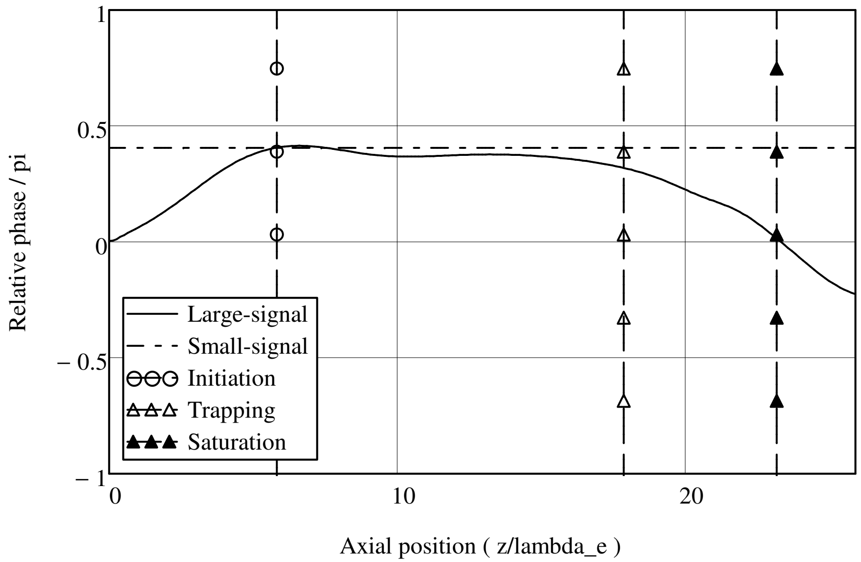


Figure 14.12

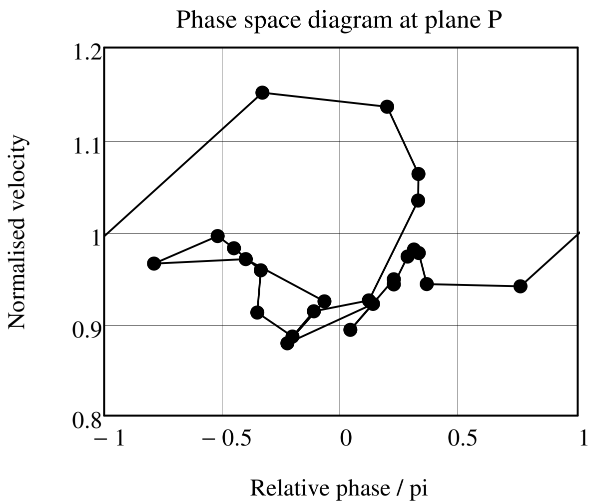


Figure 14.14

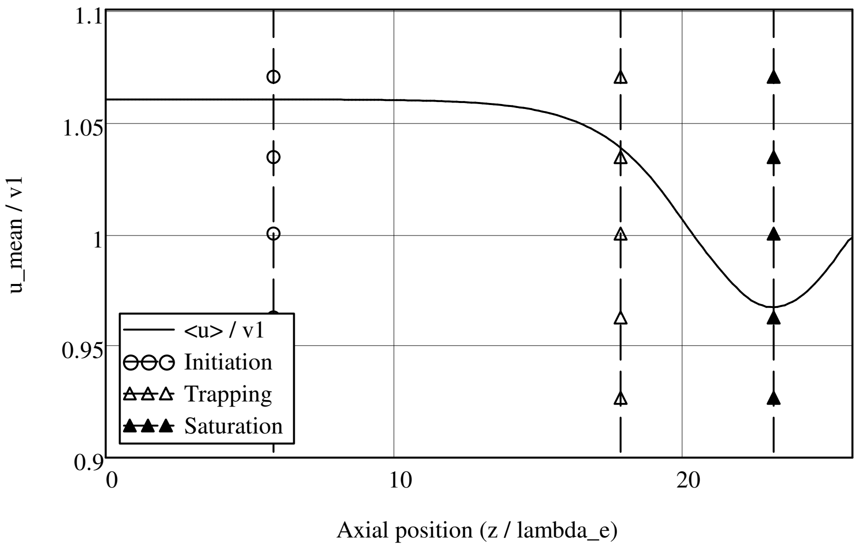


Figure 14.15

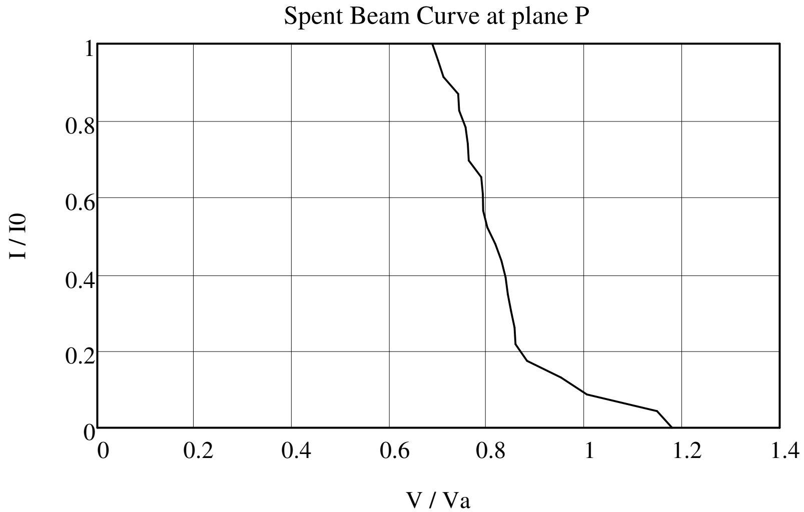


Figure 14.16