WS 11.6 Large Signal Model of the interaction between a modulated beam and a gap

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet is designed for investigation of the interaction between an electron beam, represented by ND discs of charge in one electron wavelength, and the RF field of a gap centred at z = 0 (see Section 11.8). The beam can be pre-modulated with an idealised bunching waveform defined by the parameter NB which does not have to be an integer (see Equation 11.175). If NB = 0 the beam is unmodulated. Space-charge forces can be included if required.

Define the parameters of the model

Normalised position	$\theta = \beta_e \cdot z$	Normalised time	$\phi = \omega \cdot t$		
Beam voltage	$V_0 := 25 \cdot kV$	Normalised tunnel radius	$\gamma a := 1.0$	Bunch centre starting position	$\theta_0 := -2\pi$
Beam current	$I_0 := 3.7 \cdot A$	Normalised beam radius	$\beta b := 0.629$	Initial time	$\phi_0 := \theta_0$
Frequency	$f := 1 \cdot GHz$	Normalised gap length	$\beta gap := 1.0$	Final position	$\theta_f := 2\pi$
Beam bunching	$NB := 6$	Number of discs	$ND := 24$	Final time	$\phi_f := 5\theta_f$

Normalised gap voltage
$\left(X = \dfrac{M \cdot V_g}{V_0} \right)$

$X \equiv 0.8$

Phase of gap voltage
($\Phi_1 = 0$ for accelerating field at $\theta = 0$ and $\phi = 0$)

$\Phi_1 \equiv 180 \cdot \mathrm{deg}$

Sections below can be collapsed if required to allow data and results to be viewed on the same screen

Define constants

Charge/mass ratio of the electron $\quad \eta := 1.759 \cdot 10^{11} \cdot \dfrac{C}{kg} \quad \mu S := 10^{-6} \cdot S \quad \mu P := \mu A \cdot V^{-1.5} \quad Perv := I_0 \cdot V_0^{-1.5}$

Electron velocity

$$u_0 := c \cdot \left[1 - \frac{1}{\left(1 + \frac{\eta \cdot V_0}{c^2} \right)^2} \right]^{0.5} \qquad Rel := \frac{1}{\sqrt{1 - \frac{u_0^2}{c^2}}} \qquad u_0 = 9.049 \times 10^7 \frac{m}{s}$$

Propagation constants

$$\omega := 2 \cdot \pi \cdot f \qquad \beta_e := \frac{\omega}{u_0} \qquad \lambda_e := \frac{2 \cdot \pi}{\beta_e} \qquad \gamma_e(\beta_e) := \sqrt{\beta_e^2 - \frac{\omega^2}{c^2}}$$

Physical dimensions

$$a := \frac{\gamma a}{\gamma_e(\beta_e)} \qquad b := \frac{\beta b}{\beta_e} \qquad gap := \frac{\beta gap}{\beta_e}$$

$a = 15.11 \cdot mm$

$b = 9.06 \cdot mm$

$gap = 14.40 \cdot mm$

Small-signal gap coupling factor

$$M(\beta_e) := \frac{2 \cdot I1(\gamma_e(\beta_e) \cdot b)}{(\gamma_e(\beta_e) \cdot b) \cdot I0(\gamma_e(\beta_e) \cdot a)} \cdot \left(\frac{\sin(0.5 \cdot \beta_e \cdot gap)}{0.5 \cdot \beta_e \cdot gap} \right) \qquad M(\beta_e) = 0.792$$

Gap voltage

$$V_g := \frac{X \cdot V_0}{M(\beta_e)} \qquad V_g = 25.3\,kV$$

Define the theoretical waveforms for an optimally-bunched beam using n harmonics

Equation 11.175

Current waveform $\quad IIn(\phi,n) := (1+\cos(\phi))^n$

DC current $\quad I0n(n) := \frac{1}{2\cdot\pi}\cdot\int_{-\pi}^{\pi} IIn(\phi,n)\,d\phi$

Normalised current waveform $\quad I_n(\phi,n) := \frac{IIn(\phi,n)}{I0n(n)}$

$$I(\phi) := I_0\cdot I_n(\phi,NB)$$

Amplitudes of harmonics

$$Ih(n) := \left| \begin{array}{l} \text{for } nn \in 0..n \\ \quad Ih_{nn} \leftarrow \frac{1}{\pi}\cdot\int_{-\pi}^{\pi} I_n(\phi,n)\cdot\cos(nn\cdot\phi)\,d\phi \\ Ih_0 \leftarrow 0.5\cdot Ih_0 \\ Ih \end{array}\right.$$

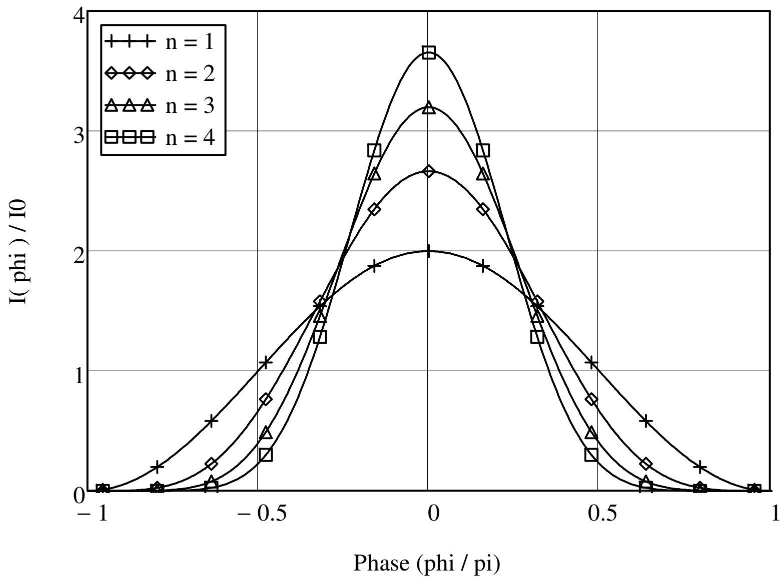


Figure 11.30

Table 11.2

$NB = 6.0$

$$Ih(NB) = \begin{pmatrix} 1.000 \\ 1.714 \\ 1.071 \\ 0.476 \\ 0.143 \\ 0.026 \\ 0.002 \end{pmatrix}$$

Disc model of the electron beam

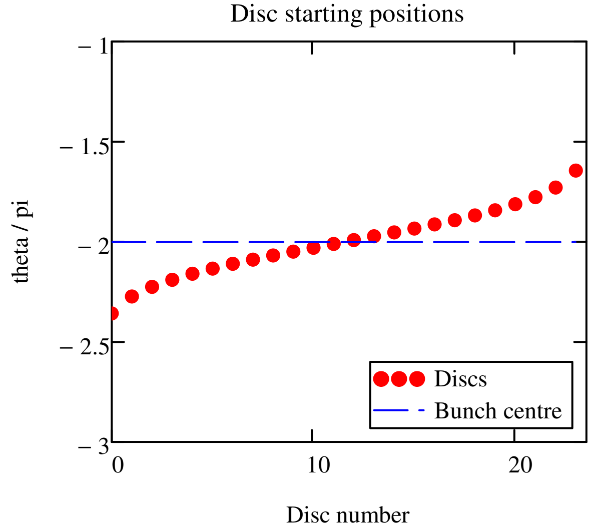
The beam is modelled as ND identical rigid discs. The motions of the electrons at the central planes of the discs are tracked. We define their initial positions and velocities using the disc thickness ΔL. As the Mathcad ODE solver rkfixed does not accept variables with dimensions the dimensionless variables: $\theta = \beta_e \cdot z$ and $\phi = \omega t$ are used.

Disc thickness $\Delta L := \dfrac{\lambda_e}{ND}$ Normalised disk thickness $\theta_d := \beta_e \cdot \Delta L$ Disk charge $Q := \dfrac{2\cdot\pi\, I_0}{\omega \cdot ND}$

Normalised disk starting positions and velocities relative to the gap centre for the prebunched current waveform defined above.

$$\theta := \left| \begin{array}{l} \alpha \leftarrow 0 \\ f1(\alpha) \leftarrow \dfrac{1}{2\cdot\pi} \cdot \displaystyle\int_{-\pi}^{\alpha} I_n(\phi, NB)\, d\phi \\ \text{for } j \in 0, 2 .. (2\cdot ND - 1) \\ \quad \left| \begin{array}{l} \theta_j \leftarrow \text{root}\left(f1(\alpha) - \dfrac{j+1}{2ND}, \alpha \right) + \theta_0 \\ \theta_{j+1} \leftarrow 1 \end{array} \right. \\ \theta \end{array} \right.$$

$j := 0, 2 .. 2\cdot ND - 1$



Check the D.C. beam current calculated from the disks is the same as that previously computed.

$f \cdot ND \cdot Q = 3.700\,A$ $I_0 = 3.700\,A$

Space-Charge Field (Not used to generate the results presented below)

The space-charge field is found from the equations given in:
J.R. Hechtel, "The effect of potential beam energy on the performance of linear beam devices",
IEEE Transactions on Electron Devices ED-17, pp.999-1009, Nov. 1970

The first ten zeros of the Bessel function $J_0(z)$.

$$\mu B := \frac{1}{a}\cdot(2.405\ \ 5.520\ \ 8.654\ \ 11.791\ \ 14.931\ \ 18.071\ \ 21.212\ \ 24.352\ \ 27.494\ \ 30.635)^T$$

Calculate the axial electric field of a disc having a charge of 1 C at 100 points up to one electronic wavelength from the centre of the disc

$$ESn := \left|\begin{array}{l} \text{for } n \in 0..100 \\ \quad \left|\begin{array}{l} \theta_n \leftarrow 0.02\cdot n\cdot\pi \\ z_n \leftarrow \dfrac{\theta_n}{\beta_e} \\ ES_n \leftarrow \left[\dfrac{-4}{\varepsilon_0\cdot(\pi\cdot b^2\cdot\Delta L)}\right]\cdot\displaystyle\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\exp(-\mu B_m\cdot z_n)\cdot\sinh\left(\frac{\mu B_m\cdot\Delta L}{2}\right)\right] \text{ if } \theta_n \ge 0.5\cdot\theta_d \\ ES_n \leftarrow \left[\dfrac{-4}{\varepsilon_0\cdot(\pi\cdot b^2\cdot\Delta L)}\right]\cdot\displaystyle\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\left(\exp\left(-\mu B_m\cdot\frac{\Delta L}{2}\right)\cdot\sinh(\mu B_m\cdot z_n)\right)\right] \text{ otherwise} \end{array}\right. \\ ES \end{array}\right.$$

Normalised positions at which ES was calculated

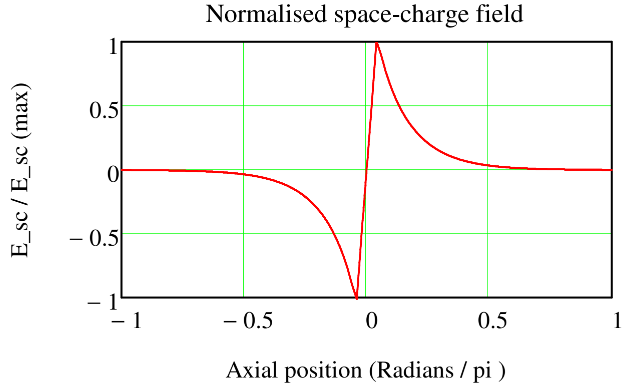
$$\theta n := \left|\begin{array}{l} \text{for } n \in 0..100 \\ \quad \theta_n \leftarrow 0.02\cdot n\cdot\pi \\ \theta \end{array}\right.$$

Define a continuous function for the field by linear interpolation

$$Es(\theta) := \text{sign}(\theta)\cdot\text{linterp}(\theta n, ESn, |\theta|)$$

$$ES(\theta) := \left|\begin{array}{ll} Es(\theta + 2\cdot\pi) & \text{if } \theta < -\pi \\ Es(\theta - 2\cdot\pi) & \text{if } \theta > \pi \\ Es(\theta) & \text{otherwise} \end{array}\right.$$

$$\theta 1 := (-\pi), (-0.999\cdot\pi)..\pi$$



The space-charge field of adjacent bunches is included by assuming that the field is periodic in z. This is not correct but tests with an initially unmodulated beam and three wavelengths of electrons give almost identical results for the trajectories and the current harmonics except well beyond the first bunch and at microperveance greater than 2.

The field of the gap

The field of the gap is found for unit gap voltage is found from the Fourier Transform of the field in the gap (assumed to be constant). The average of the field over the beam is used. Linear interpolation on the values calculated at regular intervals is used to provide a fast look-up function.

$$En := \left| \begin{array}{l} \text{for } n \in 0..100 \\ \quad \left| \begin{array}{l} \theta_n \leftarrow 0.02\cdot n\cdot\pi \\ \gamma(\beta) \leftarrow \sqrt{\beta^2 - \omega^2 c^{-2}} \\ E_n \leftarrow -\dfrac{V}{\pi}\cdot\displaystyle\int_0^{\frac{20\cdot\pi}{gap}} \frac{2\cdot I1(\gamma(\beta)\cdot b)}{(\gamma(\beta)\cdot b)\cdot I0(\gamma(\beta)\cdot a)}\cdot\left(\frac{\sin(0.5\cdot\beta\cdot gap)}{0.5\cdot\beta\cdot gap}\right)\cdot\cos\left(\frac{\beta}{\beta_e}\cdot\theta_n\right) d\beta \end{array}\right. \\ E \end{array}\right.$$

$$Egap(\theta) := linterp(\theta n, En, |\theta|)$$

Equation 3.79

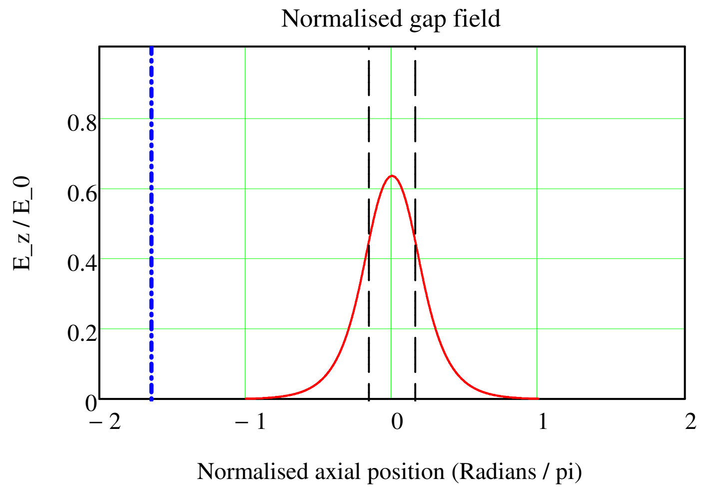
CHECK the beam coupling factor

$$\frac{-2}{\beta_e\cdot V}\cdot\int_0^{2\pi} Egap(\theta)\cdot\cos(\theta)\, d\theta = 0.792$$

$$M(\beta_e) = 0.792$$

Electric field of the gap as function of normalised position and time

$$E_z(\theta,\phi) := \mathrm{Re}\left[\frac{V_g}{V}\cdot \mathrm{Egap}(\theta)\exp\left[j\cdot\left(\phi-\Phi_1\right)\right]\right]$$



The blue chain-dotted line is the initial position of the first electron in the bunch. It should lie outside the range of the gap field.

Space-charge (0 or 1) $SCF := 0$

Turn space-charge force off until the electrons reach the origin to avoid non-physical dispersion of the bunched beam.

$$SC(\theta) := \begin{cases} SCF & \text{if } \theta \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Equations of motion

The Coefficients of the Differential Equations for the motions of the electrons are defined. The rows represent, in order, the position in radians and the normalised velocity of the electrons.

$$D(\phi,\theta) := \left| \begin{array}{l} \text{for } j \in 0,2..2\cdot(ND-1) \\ \quad \left| \begin{array}{l} D_j \leftarrow \theta_{j+1} \\ D_{j+1} \leftarrow -\dfrac{\eta}{\omega\cdot u_0}\cdot\left[1-\left(\dfrac{u_0}{c}\cdot\theta_{j+1}\right)^2\right]^{1.5}\cdot\left(E_z\left(\theta_j,\phi\right)+Q\cdot SC\left(\theta_j\right)\cdot\displaystyle\sum_{i=0}^{ND-1} ES\left(\theta_j-\theta_{2\cdot i}\right)\right) \end{array}\right. \\ D \end{array}\right.$$

Definitions of normalised variables

$\phi = \omega\cdot t$ $\quad \theta = \beta_e\cdot z$ $\quad \theta' = \dfrac{v}{u_0}$

$\dfrac{d}{dt}z = v$ $\quad \dfrac{d}{d\phi}\theta = \dfrac{v}{u_0}$

$\dfrac{d}{dt}v = -\eta\cdot E$ $\quad \dfrac{d}{d\phi}\dfrac{v}{u_0} = \dfrac{\eta\cdot E}{\omega\cdot u_0}$

The Equations are Solved using with *nmax* time steps starting from ϕ_0 which is defined in such a way that the centre electron would cross the gap centre at t = 0 if it travelled with a constant velocity u_0. The final time is ϕ_f

Number of integration steps $nmax := 100$

The variable tol specifies the tolerance on the solution of the differential equations. 10E-6 works well normally but much smaller values may be needed at low drive levels $tol := 10^{-6}$

$$Z := \mathrm{AdamsBDF}(\theta, \phi_0, \phi_f, nmax, D, tol)$$

The results are in a single table (Z) in which the first column (0) is the time and the other columns are the positions and velocities of the electrons in the same order as before at each value of *n.*

Extract the vector of phase, the matrices containing the normalised positions and velocities of the disks and the vector of the final velocities of the electrons

$$\phi n := \left| \begin{array}{l} \text{for } n \in 0..nmax \\ \quad \phi_n \leftarrow Z_{n,0} \\ \phi \end{array} \right.$$

$$\theta n := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad \theta_{n,j} \leftarrow Z_{n,2\cdot j+1} \\ \theta \end{array} \right.$$

$$un := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad u_{n,j} \leftarrow Z_{n,2\cdot j+2} \\ u \end{array} \right.$$

$$umax := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad u_j \leftarrow Z_{nmax,2\cdot j+2} \\ u \end{array} \right.$$

Kinetic Energy of the bunch at each time step.

$$KE := \left| \begin{array}{l} \text{for } n \in 0..nmax \\ \quad KE_n \leftarrow \dfrac{Q\cdot c^2}{\eta}\cdot\displaystyle\sum_{j=0}^{ND-1}\left[\dfrac{1}{\sqrt{1-\dfrac{(un_{n,j}\cdot u_0)^2}{c^2}}}-1\right] \\ KE \end{array} \right.$$

Check that the frequency times the intial KE is equal to the DC beam power

$P_{DC} := V_0\cdot I_0 = 92.5\cdot kW$ $\qquad KE_0\cdot f = 92.5\cdot kW$

Calculate the RF power transferred to the gap from the change in KE

$$P_{RF} := (KE_0 - KE_{nmax})\cdot f \qquad P_{RF} = 53.2\cdot kW$$

Calculate the efficiency $\eta_e := \dfrac{P_{RF}}{P_{DC}}$ $\qquad \eta_e = 57.5\cdot\%$

Define a set of equally-spaced planes in θ and compute the times and velocities at which the electrons cross them using linear interpolation.

Reference plane interval $\Delta\theta := 0.01\cdot\pi$

Number of reference planes $NP := \dfrac{\theta_f - \theta_0}{\Delta\theta}$ $NP = 400.00$

$$\theta p := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad \theta_p \leftarrow \dfrac{p}{NP}\cdot(\theta_f - \theta_0) + \theta_0 \\ \text{return } \theta \end{array} \right.$$

$$\phi p := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } p \in 0..NP \\ \quad\quad \text{for } n \in 1..nmax \\ \quad\quad\quad \left| \begin{array}{l} flag \leftarrow 0 \\ flag \leftarrow 1 \ \text{ if } \ \theta n_{n,j} > \theta p_p \\ \phi p_{p,j} \leftarrow \phi n_{n-1} + \dfrac{\theta p_p - \theta n_{n-1,j}}{\theta n_{n,j} - \theta n_{n-1,j}}\cdot(\phi n_n - \phi n_{n-1}) - \theta p_p \ \text{ if } \ flag = 1 \\ (\text{break}) \ \text{ if } \ flag = 1 \end{array} \right. \\ \text{return } \phi p \end{array} \right.$$

$$up := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } p \in 0..NP \\ \quad\quad \text{for } n \in 1..nmax \\ \quad\quad\quad \left| \begin{array}{l} flag \leftarrow 0 \\ flag \leftarrow 1 \ \text{ if } \ \theta n_{n,j} > \theta p_p \\ up_{p,j} \leftarrow un_{n-1,j} + \dfrac{\theta p_p - \theta n_{n-1,j}}{\theta n_{n,j} - \theta n_{n-1,j}} \cdot \left(un_{n,j} - un_{n-1,j}\right) \ \text{ if } \ flag = 1 \\ (\text{break}) \ \text{ if } \ flag = 1 \end{array} \right. \\ \text{return } up \end{array} \right.$$

Calculate the **total kinetic energy** of the electrons as they cross each plane.

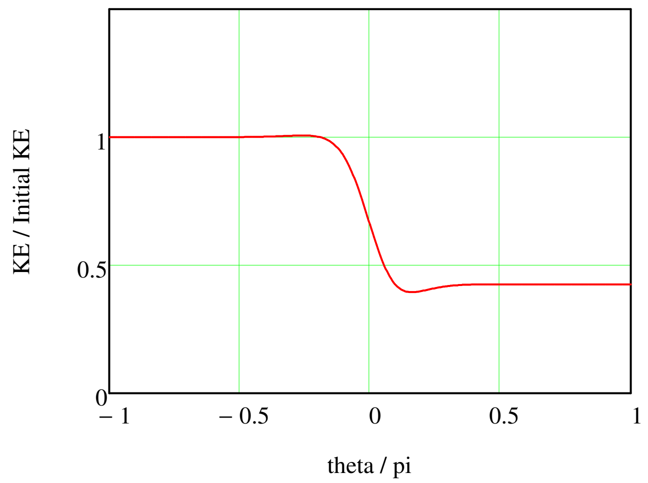
$$KEp := \left| \begin{array}{l} \text{for } p \in 0..NP \\ \quad KE_p \leftarrow \dfrac{Q \cdot c^2}{\eta} \cdot \displaystyle\sum_{j=0}^{ND-1} \left[\dfrac{1}{\sqrt{1 - \dfrac{\left(up_{p,j} \cdot u_0\right)^2}{c^2}}} - 1 \right] \\ KE \end{array} \right.$$

$$np := 0..NP$$

Electronic efficiency

$$\frac{\left(KEp_0 - KEp_{NP}\right) \cdot f}{P_{DC}} = 57.5 \cdot \%$$

$$\eta_e = 57.5 \cdot \%$$



Calculate the complex current harmonics at each plane by superimposing the Fourier components of the discs.
For simplicity each disc is treated as having constant length. Number of current harmonics $NH := 6$

$$
\begin{aligned}
Ip := \;& \text{for } p \in 0..NP \\
& \quad Ip_{p,0} \leftarrow \frac{Q}{2\cdot\pi\cdot\Delta L}\cdot\sum_{j=0}^{ND-1}\theta_d \\
& \quad \text{for } h \in 1..NH \\
& \quad\quad Ip_{p,h} \leftarrow \frac{2\cdot Q}{\pi\cdot h\cdot\Delta L}\cdot\sum_{j=0}^{ND-1}\left(\sin\left(\frac{h\cdot\theta_d}{2}\right)\cdot\exp\left(j\cdot h\cdot\phi p_{p,j}\right)\right) \\
& \text{return } Ip\cdot u_0
\end{aligned}
$$

Instantaneous current = $\dfrac{Q\cdot up_{p,j}}{\Delta L}$

Pulse phase duration = $\dfrac{\theta_d}{up_{p,j}}$

Effective gap coupling factor

$$M_{eff} := \frac{2\cdot P_{RF}}{Ih(NB)_1\cdot I_0\cdot V_g}$$

$$V_{ss}(u) := \frac{c^2}{\eta}\cdot\left(\frac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}-1\right)$$

Approximate model of the beam/gap interaction

The exit energy is calculated as a function of ϕ using piecewise constant velocity and iteration to find the exit velocity.
The results are compared with those from the disk model.

$$u_e(V_e) := c\cdot\left[1-\frac{1}{\left(1+\frac{\eta\cdot V_e}{c^2}\right)^2}\right]^{0.5}$$

$$\beta e(V_e) := \frac{\omega}{u_e(V_e)}$$

$$Vss2(\phi, X, \Phi, V_s) := 1+\frac{1}{2}\cdot\left(1+\frac{M(\beta e(V_0\cdot V_s))}{M(\beta_e)}\right)\cdot X\cdot\cos(\phi-\Phi)$$

Equation 11.168

$$V_s := 0.2 \qquad fn2(V_s, \phi, X, \Phi) := V_s - Vss2(\phi, X, \Phi, V_s) \qquad Vs2(\phi, X, \Phi) := root(fn2(V_s, \phi, X, \Phi), V_s)$$

$$P_{RF2}(X, \Phi) := \frac{V_0}{2\cdot\pi}\cdot\int_{-\pi}^{\pi} I(\phi)\cdot(1-Vs2(\phi, X, \Phi))\, d\phi$$

Equation 11.179

$$M_{eff2}(X, \Phi) := \frac{1}{2}\cdot\left(M(\beta_e)+M(\beta e(V_0\cdot Vs2(0, X, \Phi)))\right)$$

Equation 11.182

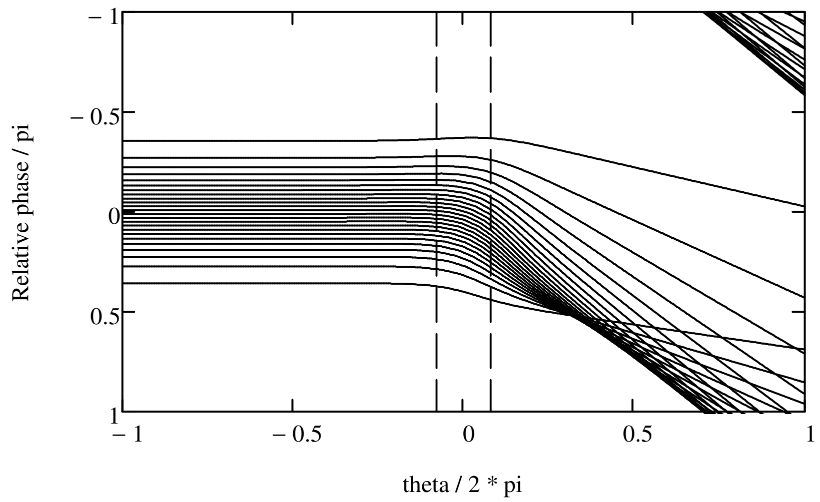
$$\eta_{e1}(X, \Phi) := -\frac{1}{2}\cdot Ih(NB)_1\cdot\frac{M_{eff2}(X, \Phi)}{M(\beta_e)}\cdot X\cdot\cos(0-\Phi)$$

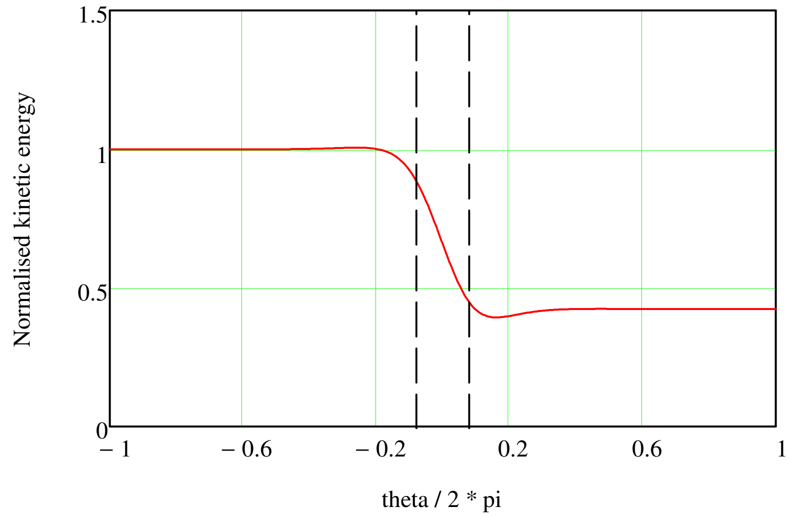
$$\eta_{e2}(X, \Phi) := \frac{P_{RF2}(X, \Phi)}{I_0\cdot V_0}$$

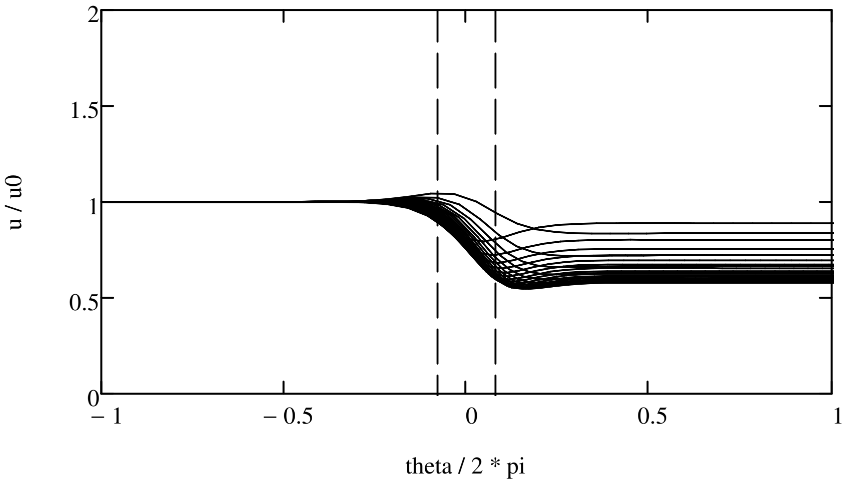
Plotting ranges for graphs $\quad j := 0..ND-1 \quad x := 0, 0.1..1.5 \quad \phi := \phi_0..1.2\phi_f \quad \phi_g := -1..1 \quad pp := 0..NP$

Results of the disk model calculations

$P_{RF} = 53.2 \cdot kW$ $\quad \eta_e = 57.5 \cdot \%$ $\quad M(\beta_e) = 0.792$ $\quad M_{eff} = 0.664$ $\quad Perv = 0.936 \cdot \mu P$






2
1.5
1
0.5
0
u / u0
− 1
− 0.5
0
0.5
1
theta / 2 * pi

Investigation of the effect of gap voltage on the effective gap coupling factor

The results of the calculations for different values of X are entered in the data matrices below for plotting

Disc model

$$\frac{M_{eff}}{M(\beta_e)} = 0.839$$

M_{eff} / M for NB = 1, 2 and 6

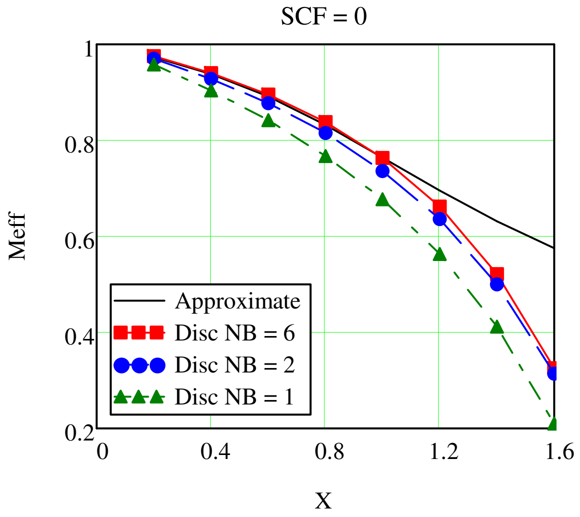
$$X1 := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1.0 \\ 1.2 \\ 1.4 \\ 1.6 \end{pmatrix} \quad Meff1 := \begin{pmatrix} 0.959 \\ 0.905 \\ 0.843 \\ 0.768 \\ 0.678 \\ 0.564 \\ 0.413 \\ 0.211 \end{pmatrix} \quad Meff2 := \begin{pmatrix} 0.971 \\ 0.929 \\ 0.878 \\ 0.816 \\ 0.737 \\ 0.637 \\ 0.501 \\ 0.315 \end{pmatrix} \quad Meff6 := \begin{pmatrix} 0.976 \\ 0.941 \\ 0.896 \\ 0.839 \\ 0.764 \\ 0.663 \\ 0.522 \\ 0.326 \end{pmatrix}$$

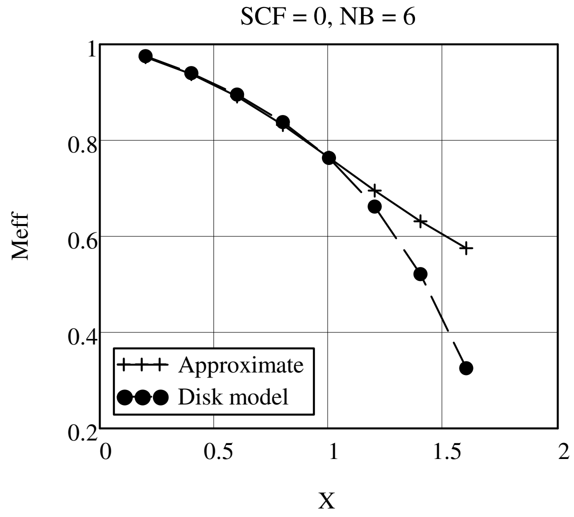
Approximate model

$$\frac{M_{eff2}(X, \Phi_1)}{M(\beta_e)} = 0.833$$

M_{eff2} / M

$$Mapp := \begin{pmatrix} 0.974 \\ 0.939 \\ 0.892 \\ 0.833 \\ 0.765 \\ 0.696 \\ 0.632 \\ 0.576 \end{pmatrix}$$


SCF = 0
Meff
X
Approximate
Disc NB = 6
Disc NB = 2
Disc NB = 1
1
0.8
0.6
0.4
0.2
0
0.4
0.8
1.2
1.6


SCF = 0, NB = 6
Meff
X
Approximate
Disk model
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2

Investigation of the values of efficiency calculated using the disc model and two theoretical approximations

The results from the disc model without space-charge and from approximate theoretical models are computed and the results stored in the data matrices for plotting.

1. Φ=180 deg and various values of X.
2. Beta * g = 1.0, NB = 6, X = 1.0 and various phases

$$X1 = \begin{pmatrix} 0.200 \\ 0.400 \\ 0.600 \\ 0.800 \\ 1.000 \\ 1.200 \\ 1.400 \\ 1.600 \end{pmatrix} \qquad \eta_{ed} := \begin{pmatrix} 0.167 \\ 0.323 \\ 0.461 \\ 0.575 \\ 0.655 \\ 0.682 \\ 0.627 \\ 0.447 \end{pmatrix}$$

$$\Phi1 := \begin{pmatrix} 0 \\ 30 \\ 60 \\ 90 \\ 120 \\ 150 \\ 180 \\ 210 \\ 240 \\ 270 \\ 300 \\ 330 \\ 360 \end{pmatrix} \cdot deg \qquad \eta e1 := \begin{pmatrix} -0.942 \\ -0.803 \\ -0.502 \\ -0.128 \\ 0.233 \\ 0.514 \\ 0.660 \\ 0.612 \\ 0.337 \\ -0.103 \\ -0.554 \\ -0.860 \\ -0.942 \end{pmatrix}$$

$X3 := 0.2, 0.4 .. 1.6$

$\Phi 2 := 0, 30 \cdot deg .. 360 \cdot deg$

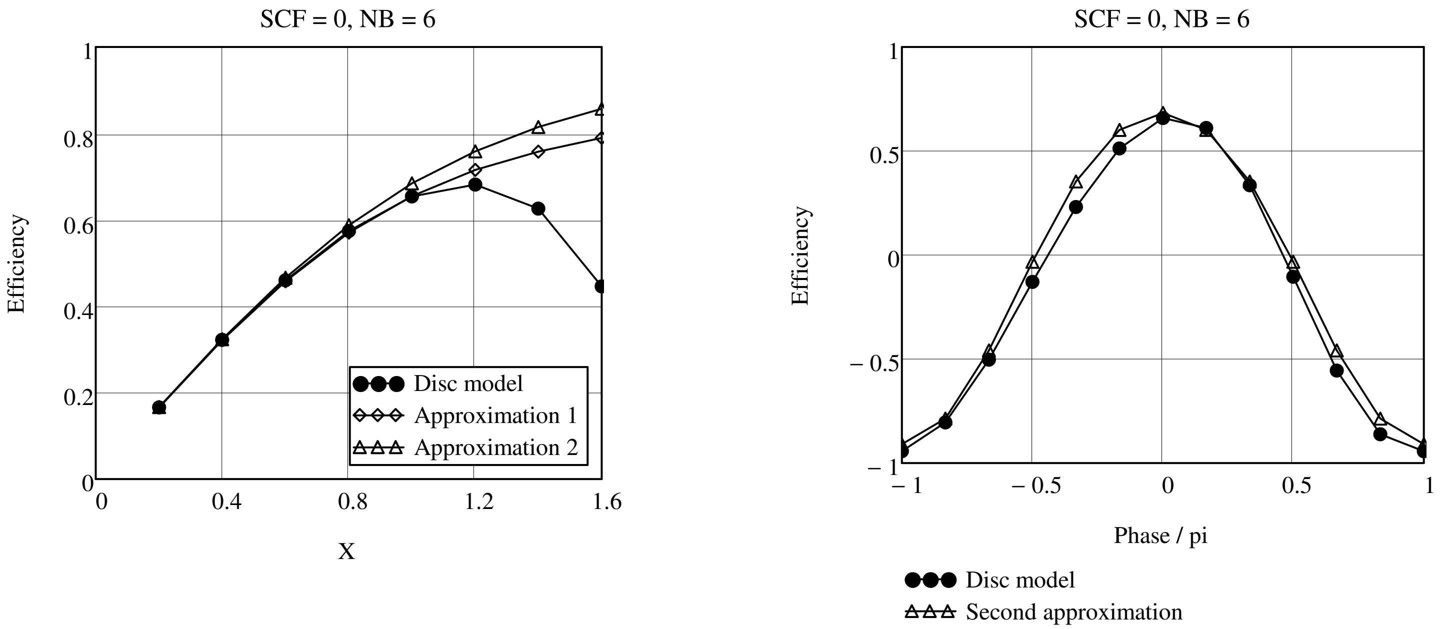


Figure 11.31

Investigation of maximum energy transfer from a bunched beam to an output gap

The calculations were carried out using 24 discs, no space-charge. For each choice of I1/I0, gamma*a and beta*g X and PHI were adjusted for maximum efficiency. The results are entered in the yellow cells of the embedded spreadsheet for plotting. The figures in the table were computed with an earlier version of the model and may differ slightly from those calculated now.

$\mathrm{Ih(NB)}_1 = 1.714$ $\eta_e = 57.5\cdot\%$ $\frac{V_g}{V_0} = 1.010$

Approximate curve (in red) $\mathrm{Me5(M)} := 0.5\cdot\left(M + M^6\right)$

$\begin{pmatrix} M1 \\ Me1 \\ M2 \\ Me2 \\ M3 \\ Me3 \\ M4 \\ Me4 \end{pmatrix} :=$

Calculation of effective gap factors for maximum energy transfer to the gap from a bunched beam										b / a = 0.6
I1/I0	gamma*a	Beta*g	X1	PHI	Vg / Va	M	eta		Meff	
1.33	1	0.1	1.2	185	1.455	0.825	54.8		0.57	
1.33	1	0.2	1.2	185	1.456	0.824	54.69		0.57	
1.33	1	0.4	1.2	185	1.463	0.82	54.25		0.56	

M1 is the small-signal coupling factor and Me1 the effective coupling factor for the first pair of values of I1 / I0 and gamma * a, and so on.

Figure 11.32