WS 2.3 Inductive Iris in Rectangular Waveguide

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

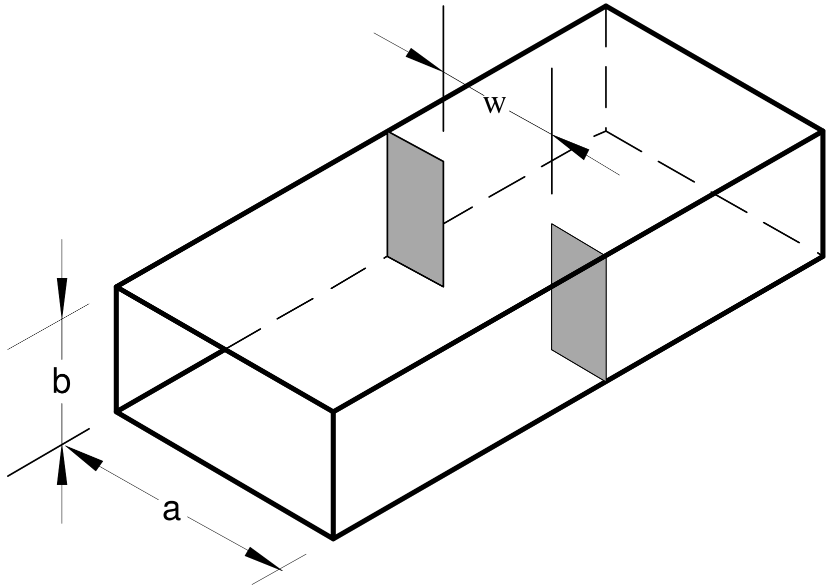


Figure 2.16

Section 2.4.3

The iris is assumed to be thin and symmetrical. The width of the gap is *w.* The iris can be represented by an inductive susceptance whose magnitude is given by Marcuvitz, N. *Waveguide Handbook*, McGraw-Hill (1951) pp.221-223. The reactance can be calculated from eq. (1a)

$$\mathrm{Term1}(w\_a) := \tan(0.5\cdot\pi\cdot w\_a)^2$$

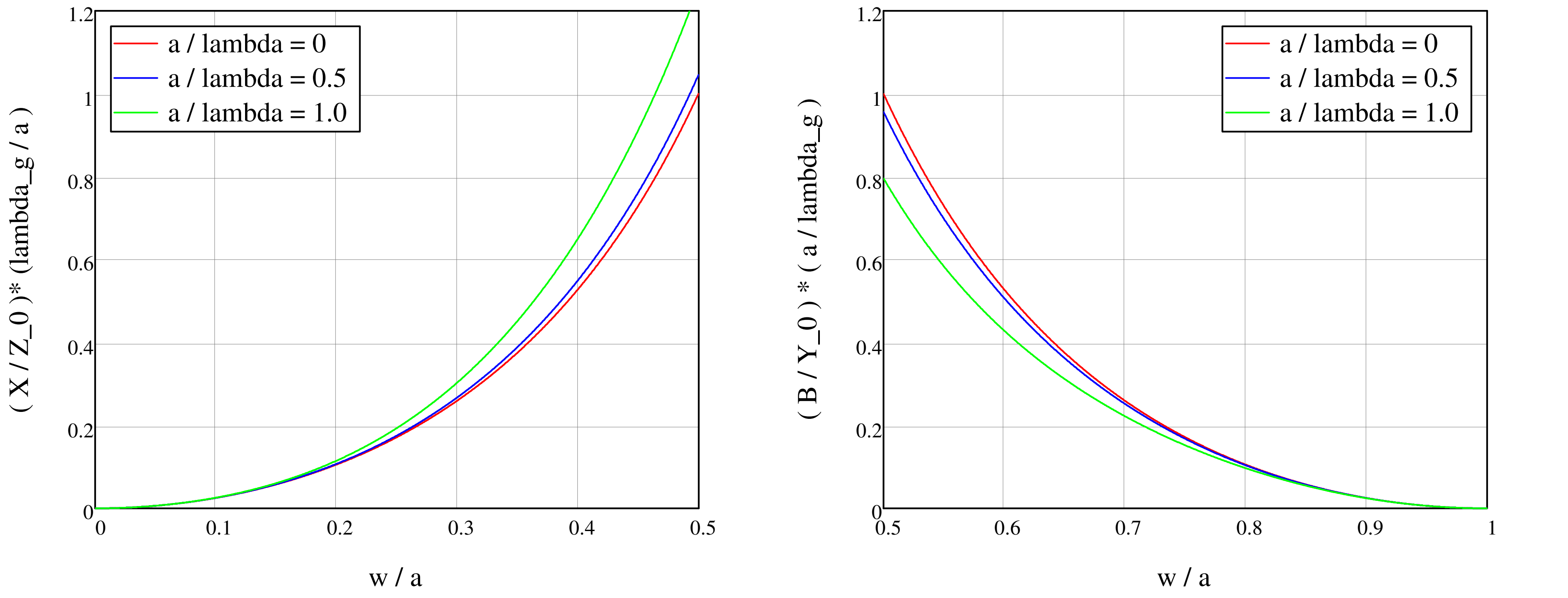
$$\mathrm{Term2}(w\_a, a\_\lambda) := \mathrm{Term1}(w\_a)\cdot\frac{3}{4}\cdot\left[\frac{1}{\sqrt{1-\left(\frac{2}{3}\cdot a\_\lambda\right)^2}} - 1\right]\cdot\sin(\pi\cdot w\_a)^2$$

$$X\lambda g\_Z0a(w\_a, a\_\lambda) := \mathrm{Term1}(w\_a) + \mathrm{Term2}(w\_a, a\_\lambda)$$

Marcuvitz includes two further terms but the use of just these two gives good agreement with the published curves.

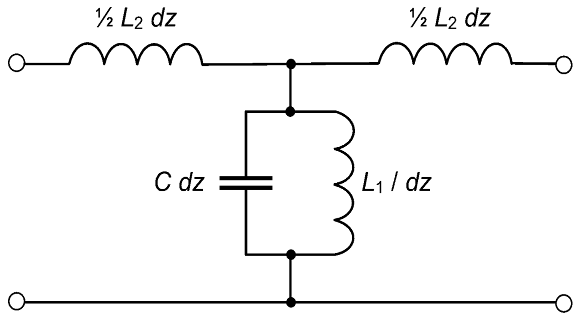
For a standard waveguide (e.g. WG16) $\frac{a}{\lambda} = 0.5$ at cut-off, 0.63 at the bottom and 0.95 at the top of the normal working band (8.2 and 12.4 GHz) where λ is the free-space wavelength. When a / λ = 0 Term2 = 0.

Comparison of the three curves (see the figures below) shows that the second term is a small correction. Thus to a good approximation the normalised susceptance depends only on the ratio w/a.


(X / Z_0)* (lambda_g / a)
w / a
a / lambda = 0
a / lambda = 0.5
a / lambda = 1.0
0
0.2
0.4
0.6
0.8
1
1.2
0
0.1
0.2
0.3
0.4
0.5
(B / Y_0) * (a / lambda_g)
w / a
a / lambda = 0
a / lambda = 0.5
a / lambda = 1.0
0
0.2
0.4
0.6
0.8
1
1.2
0.5
0.6
0.7
0.8
0.9
1

Equivalent circuit

Now suppose that the iris can be represented by a short length of waveguide having the same cross-sectional dimensions as the iris and thickness δ which represents the fringing fields. The equivalent circuit of the iris is that for a TE mode shown in the figure below



From the theory of the TE mode we know that the inductances and the characteristic impedance can be expressed in terms of the capacitance as

$$L_1 = \frac{1}{\omega_c^2 C} \qquad L_2 = \frac{1}{c^2 C} \qquad Z_g = \frac{\beta_0}{\beta_g}\frac{1}{cC}$$

where

$$\beta_g^2 = \beta_w^2 - \beta_c^2 \qquad \beta_c = \frac{\pi}{a} \qquad \text{for the waveguide} \qquad \beta_w = \frac{\omega}{c}$$

$$\beta_g^2 = \beta_w^2 - \beta_i^2 \qquad \beta_i = \frac{\pi}{w} \qquad \text{for the iris}$$

For the moment let us assume that the impedance of L_2 is small compared with Z_g. Then the admittance of the iris is

$$Y = j\omega C_i\delta - \frac{j\delta}{\omega L_1} = j\omega C_i\delta\left(1 - \frac{1}{\omega^2 L_1 C_i}\right) = j\frac{C_i\delta}{\omega}\left(\omega^2 - \omega_i^2\right) = jcC_i\delta\frac{\beta_i^2}{\beta_0}$$

where C_i is the capacitance per unit length of the iris.

The admittance added by the iris is found by subtracting the admittance when w = a

$$Y_i = jcC_i\delta\frac{\beta_i^2}{\beta_0} - jcC\delta\frac{\beta_g^2}{\beta_0} = jcC\delta\frac{\beta_g^2}{\beta_0}\left(\frac{\beta_i^2}{\beta_g^2}\frac{C_i}{C} - 1\right)$$

Normalising to the waveguide impedance

$$\frac{B}{Y_g} = \beta_g\delta\left(\frac{\beta_i^2}{\beta_g^2}\frac{C_i}{C} - 1\right)$$

Now analysis of the equivalent circuit for a rectangular waveguide shows that the capacitance per unit length is proportional to *w* regardless of the choice of definition of impedance. Thus

$$\frac{B}{Y_g} = \beta_g\delta\left(\frac{\beta_i^2}{\beta_g^2}\frac{w}{a} - 1\right) = \frac{\delta}{\beta_g}\left(\frac{w}{a}\beta_i^2 - \beta_g^2\right)$$

Expressing this in terms of wavelengths

$$\frac{B}{Y_g} = \frac{\lambda_g}{a}\frac{a\delta}{2\pi}\left[\frac{w}{a}\left[\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{w}\right)^2\right] - \left[\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2\right]\right]$$

Equation 2.100

Hence the normalised susceptance plotted by Marcuvitz is

$$\frac{B}{Y_g}\frac{a}{\lambda_g} = \frac{\pi\delta}{2a}\left[\frac{w}{a}\left[\left(\frac{2a}{\lambda}\right)^2 - \left(\frac{a}{w}\right)^2\right] - \left[\left(\frac{2a}{\lambda}\right)^2 - 1\right]\right]$$

$$\frac{B}{Y_g}\frac{a}{\lambda_g} = \frac{\pi\delta}{2a}\left(1 - \frac{a}{w}\right) \quad + \quad \frac{\pi\delta}{2a}\left(\frac{2a}{\lambda}\right)^2\left(\frac{w}{a} - 1\right)$$

We note that both terms are negative and therefore the variation with a/λ given by the second term is in the wrong sense to reproduce the variation given by the exact expression. Since we have ignored the series inductance we have assumed that the waveguide is close to cut-off and therefore λ=2a. Hence

$$\frac{B}{Y_g}\frac{a}{\lambda_g} = \frac{\pi\delta}{2a}\left(\frac{w}{a} - \frac{a}{w}\right)$$ Equation 2.101

Therefore the normalised susceptance depends only on the dimensions of the aperture including the, as yet undetermined, parameter *d*. We note that the susceptance is negative and, therefore, inductive.

The condition that the series inductance is negligible is

$$\frac{\omega L_2}{Z_g} = \frac{\omega}{c^2 C}\frac{\beta_g}{\beta_0} cC = \beta_g \rightarrow 0$$

The expression derived is not a good fit to the exact curves. However, we note that only part of the longitudinal current in the waveguide is intercepted by the iris. That fraction is given by

$$\mathrm{k(w\_a)} := \sin\left(\frac{\pi}{2}\cdot \mathrm{w\_a}\right)$$ Equation 2.102

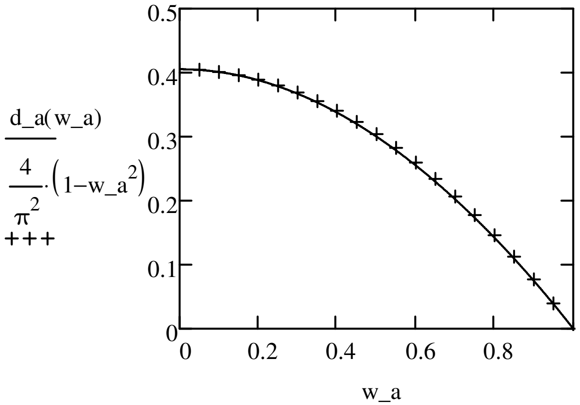
Since the equivalent circuit assumes that the whole of the current passes through the iris the impedance must be reduced (and the admittance increased) to take account of this. Thus the admittance should be divided by k

The value of the parameter δ/a can be determined by taking the ratio of the approximate and exact expressions when a/λ = 0.

$$\mathrm{d\_a(w\_a)} := -\frac{\tan\left(\frac{\pi}{2}\cdot \mathrm{w\_a}\right)^{-2}}{0.5\cdot\pi\cdot\left(\mathrm{w\_a}-\frac{1}{\mathrm{w\_a}}\right)\cdot\sin\left(\frac{\pi}{2}\cdot\mathrm{w\_a}\right)^{-1}}$$

From the curve it is apparent that a good fit is obtained using (2.104)

$$\frac{\delta}{a}\approx\frac{4}{\pi^2}\left(1-\left(\frac{w}{a}\right)^2\right)$$



so the normalised inductive suceptance is given approximately by combining (2.103) with (2.104)

$$\frac{B}{Y_g}\frac{a}{\lambda_g}\approx\frac{2}{\pi}\frac{a}{w}\left(1-\left(\frac{w}{a}\right)^2\right)^2\frac{1}{\sin\left(\pi w/2a\right)}$$

Approximate Reactance

$$\mathrm{Approx(w\_a)} := \left[\frac{\pi}{2}\cdot\frac{\mathrm{w\_a}}{\left(1-\mathrm{w\_a}^2\right)^2}\right]\cdot\sin\left(\frac{\pi\cdot\mathrm{w\_a}}{2}\right)$$

Then, comparing the exact and approximate normalised reactance and susceptance