Worksheet 11.5 Discrete interaction between an electron beam and a slow-wave structure

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet illustrates the properties of discreter interaction between an electron beam and a slow-wave structure in a coupled-cavity TWT (see Section 11.6). Typical values of the parameters are used but the variation of the reduced plasma frequency with frequency and the radial coupling factor are ignored for simplicity. The frequency dependences of the coupling impedance and the electronic admittance of the beam are represented by approximations.

Define the origin for matrices $\quad \mathrm{ORIGIN} := 1$

Define typical parameter values where Ω is the frequency normalised to the band centre. The variation of the plasma frequency reduction factor and the structure impedance with frequency have been ignored.

Uncoupled propagation constants

$\beta_e(\Omega) := 717\cdot\Omega$ $\beta_q := 67$ $\beta_0(\Omega) := 783\cdot\Omega$

Cell pitch and gap and the radial coupling factor

$p := \dfrac{\pi}{2\cdot(\beta_e(1) + \beta_q)}$ $g := \dfrac{p}{2}$ $\mu_c := 0.5$

Gap coupling factors for the fast and slow space-charge waves

$M(\beta) := \mu_c\cdot\dfrac{\left(\sin\left(\beta\cdot\dfrac{g}{2}\right)\right)}{\left(\beta\cdot\dfrac{g}{2}\right)}$ $MF(\Omega) := M\left(\beta_e(\Omega) - \beta_q\right)$ $MS(\Omega) := M\left(\beta_e(\Omega) + \beta_q\right)$

Electronic admittance of the beam

$Y_e(\Omega) := \dfrac{\Omega}{8302}$ $Y_b := 10^{-5}$

Iterative and total impedances of the structure

$Z_s(\Omega) := \dfrac{46}{\Omega^2}$ $Z_T(\Omega) := 4\cdot Z_s(\Omega)\cdot\left(\sin\left(\beta_0(\Omega)\cdot 0.5\cdot p\right)\right)^2$ $N(\Omega) := \sqrt{\dfrac{Z_T(\Omega)}{Z_s(\Omega)}}$

Unit matrix

$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Set up the submatrices

$$B_2(\Omega) := \begin{pmatrix} 0.5 \cdot MF(\Omega) \\ 0.5 \cdot MS(\Omega) \end{pmatrix} \qquad B_3(\Omega) := -\left(-MF(\Omega) \cdot Y_e(\Omega) \quad MS(\Omega) \cdot Y_e(\Omega)\right)$$

$$S_2(\Omega) := -\begin{pmatrix} \frac{1}{2} \cdot N(\Omega)\, Z_s(\Omega) \\ -\frac{1}{2} \cdot N(\Omega)\, Z_s(\Omega) \end{pmatrix} \qquad S_3(\Omega) := (N(\Omega) \quad N(\Omega))$$

Find the partitions of the gap matrix G

$$G1(\Omega) := I - Y_b \cdot S_2(\Omega) \cdot S_3(\Omega) \qquad G2(\Omega) := S_2(\Omega) \cdot B_3(\Omega)$$

$$G3(\Omega) := B_2(\Omega) \cdot S_3(\Omega) \qquad G4(\Omega) := I + B_2(\Omega) \cdot Z_T(\Omega) \cdot B_3(\Omega)$$

Set up the gap matrix G and the drift matrix D for half a cell

$$G(\Omega) := \begin{pmatrix} G1(\Omega)_{1,1} & G1(\Omega)_{1,2} & G2(\Omega)_{1,1} & G2(\Omega)_{1,2} \\ G1(\Omega)_{2,1} & G1(\Omega)_{2,2} & G2(\Omega)_{2,1} & G2(\Omega)_{2,2} \\ G3(\Omega)_{1,1} & G3(\Omega)_{1,2} & G4(\Omega)_{1,1} & G4(\Omega)_{1,2} \\ G3(\Omega)_{2,1} & G3(\Omega)_{2,2} & G4(\Omega)_{2,1} & G4(\Omega)_{2,2} \end{pmatrix}$$

Equation 11.154

Setting $Y_b = 0$ makes no visible difference.

$$D(\Omega) := \begin{bmatrix} \exp\left(-j\cdot\beta_0(\Omega)\cdot\frac{p}{2}\right) & 0 & 0 & 0 \\ 0 & \exp\left(j\cdot\beta_0(\Omega)\cdot\frac{p}{2}\right) & 0 & 0 \\ 0 & 0 & \exp\left[-j\cdot\left(\beta_e(\Omega)-\beta_q\right)\cdot\frac{p}{2}\right] & 0 \\ 0 & 0 & 0 & \exp\left[-j\cdot\left(\beta_e(\Omega)+\beta_q\right)\cdot\frac{p}{2}\right] \end{bmatrix}$$

Equation 11.155

Set up the cell wave matrix P

Equation 11.156

$$P(\Omega) := D(\Omega)\cdot G(\Omega)\cdot D(\Omega)$$

Find the eigenvalues of P and the corresponding propagation constants

$$\lambda(\Omega) := \mathrm{eigenvals}(P(\Omega)) \qquad \beta(\Omega) := \frac{j}{p}\cdot\ln(\lambda(\Omega))$$

Shift the negative branch into the second positive Brillouin zone

$$\beta1(\Omega, n) := \left| \begin{array}{l} \beta(\Omega)_n \ \text{ if } \ \mathrm{Re}\left(\beta(\Omega)_n\right) > 0 \\ \left[\left(\beta(\Omega)_n\right) + \frac{2\cdot\pi}{p}\right] \ \text{ otherwise} \end{array} \right.$$

Reconstruct the vector and find the real and imaginary parts sorted in order of magnitude. Note that this process, which is necessary to obtain clean dispersion diagrams, produces real and imaginary parts which do not necessarily correspond to each other.

$$\beta2(\Omega) := \begin{pmatrix} \beta1(\Omega,1) \\ \beta1(\Omega,2) \\ \beta1(\Omega,3) \\ \beta1(\Omega,4) \end{pmatrix} \qquad R\beta(\Omega) := \mathrm{Re}(\mathrm{sort}(\beta2(\Omega))) \qquad I\beta(\Omega) := \mathrm{Re}(\mathrm{sort}(-j\ \beta2(\Omega)))$$

$$\Omega := 0.2, 0.21 .. 3.0 \qquad \Omega1 := 0.2, 0.21 .. 2.0 \qquad \Omega2 := 1.8, 1.805 .. 2.4$$

Coupled modes. Imaginary part multipled by 50

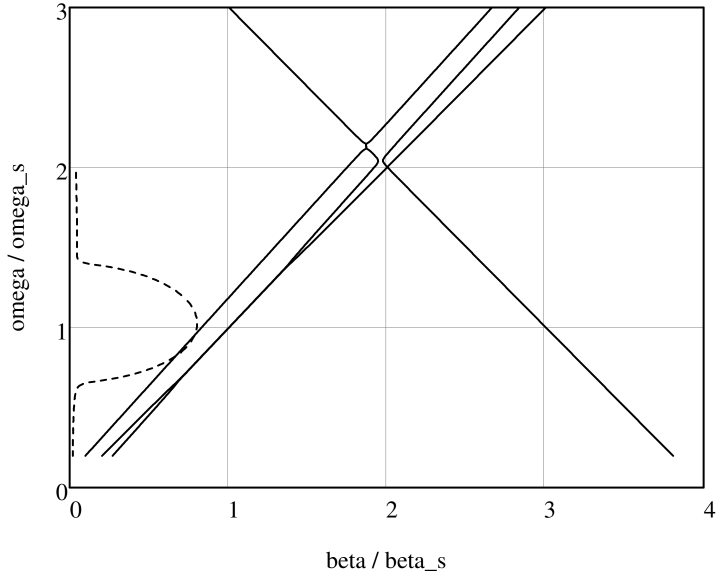

Figure 11.22

Detail of the coupled-mode diagram

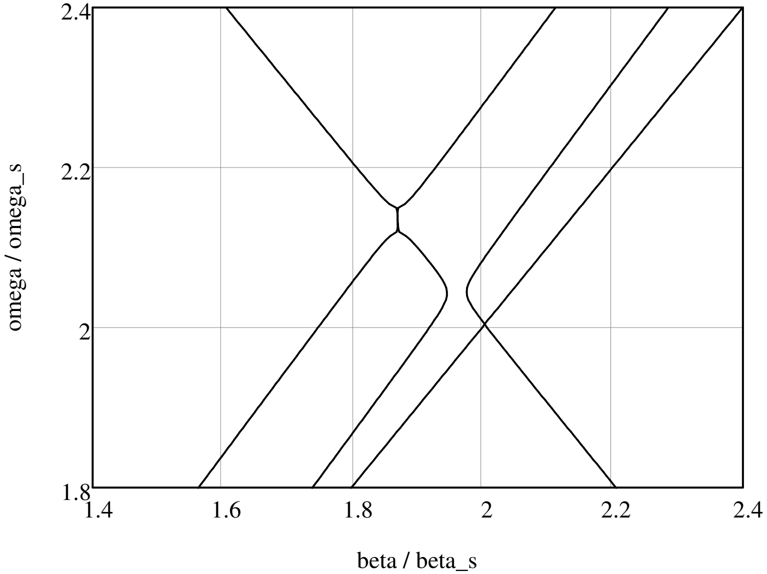

Figure 11.23