WS 3.5 Gap field

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Section 3.5.3 Interaction Gap Fields

This worksheet can be used to investigate the effects of the field profile in the gap for varying gap lengths and fixed gap voltage

Gap voltage $V_0 := 1$ Tunnel radius $a := 1$

a) Uniform field

Axial electric field at r = a

$$E_0(z) := \begin{cases} \frac{V_0}{g} & \text{if } |z| < \frac{g}{2} \\ 0 & \text{otherwise} \end{cases}$$

Limits of integration as approximation to infinity

$$\text{lim} := 100$$

Fourier transform

$$g_0(\beta) := V_0 \cdot \text{sinc}\left(\beta \cdot \frac{g}{2}\right)$$

Equation 3.78

Axial component of electric field within the gap

$$E0(r, z) := \frac{1}{2 \cdot \pi} \cdot \int_{-\text{lim}}^{\text{lim}} \frac{I0(\beta \cdot r)}{I0(\beta \cdot a)} \cdot g_0(\beta) \cdot \exp(-j \cdot \beta \cdot z)\, d\beta$$

Equation 3.79

b) Hyperbolic cosine field

Axial electric field at r = a

$$E_1(k, z) := \begin{cases} \frac{k \cdot V_0 \cdot \cosh(k \cdot z)}{2 \cdot \sinh\left(k \cdot \frac{g}{2}\right)} & \text{if } |z| < \frac{g}{2} \\ 0 & \text{otherwise} \end{cases}$$

Equation 3.89

Fourier transform

$$g_1(k, \beta) := \left[\frac{\sinh\left[(k + j \cdot \beta) \cdot \frac{g}{2}\right]}{(k + j \cdot \beta)} + \frac{\sinh\left[(k - j \cdot \beta) \cdot \frac{g}{2}\right]}{(k - j \cdot \beta)} \right] \cdot \frac{k \cdot V_0}{2 \cdot \sinh\left(k \cdot \frac{g}{2}\right)}$$

Axial component of electric field within the gap

$$E1(k, r, z) := \frac{1}{2 \cdot \pi} \cdot \int_{-\text{lim}}^{\text{lim}} \frac{I0(\beta \cdot r)}{I0(\beta \cdot a)} \cdot g_1(k, \beta) \cdot \exp(-j \cdot \beta \cdot z)\, d\beta$$

c) Knife edge field

Axial electric field at r = a

$$E_2(z) := \begin{cases} \dfrac{V_0}{\pi} \cdot \dfrac{1}{\sqrt{\left(\frac{g}{2}\right)^2 - z^2}} & \text{if } |z| < \dfrac{g}{2} \\ 0 & \text{otherwise} \end{cases}$$

Equation 3.90

Fourier transform

$$g_2(\beta) := V_0 \cdot J0\left(\beta \cdot \frac{g}{2}\right)$$

Axial component of electric field within the gap

$$E2(r, z) := \frac{1}{2 \cdot \pi} \cdot \int_{-\lim}^{\lim} \frac{I0(\beta \cdot r)}{I0(\beta \cdot a)} \cdot g_2(\beta) \cdot \exp(-j \cdot \beta \cdot z)\, d\beta$$

Define the gap length $g \equiv 1$

and the field shape parameter for a hyperbolic cosine field $k \equiv \frac{2}{g}$

Plotting range $z1 := -2 \cdot a, -1.95 \cdot a .. 2 \cdot a$

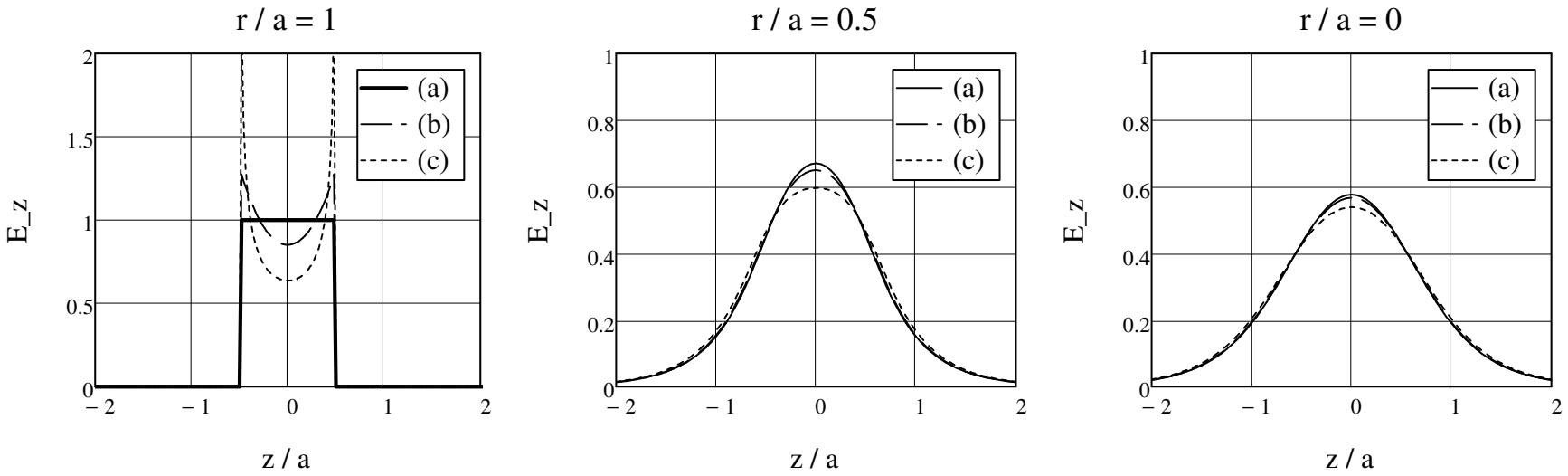

Figure 3.17