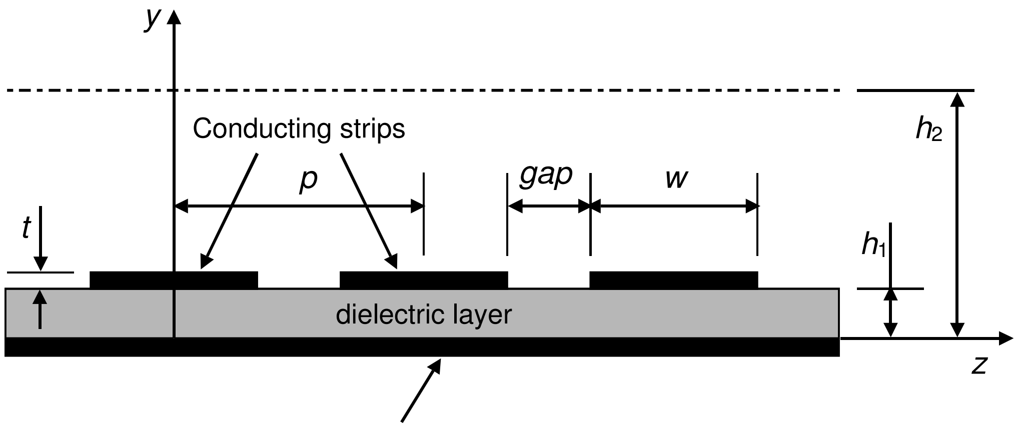
WS 4.1 Capacitance of a planar grid

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

The sheet calculates and displays the potential map using a finite difference calculation on a square mesh and calculates the capacitance per unit length of one strip divided by ε_0

Figure 4.7



Relative permittivity of the dielectric layer is ε_2.

In this model t = 0.

Structure dimensions

$h_1 := 0.062 \cdot in$ $h_2 := 2.6 \cdot h_1$ $L := 0.537 \cdot in$ $w := 0.045 \cdot in$ $gap := 0.057 \cdot in$ $p := w + gap$ $\varepsilon_2 := 6.5$

$p = 2.591 \cdot mm$ $h_1 = 1.57 \cdot mm$ $L = 13.6 \cdot mm$ $w = 1.14 \cdot mm$ $gap = 1.45 \cdot mm$ $p = 2.59 \cdot mm$ $h_2 = 4.09 \cdot mm$

Choose mesh size for FD calculations $mesh := 0.142 \cdot mm$

The mesh is a square mesh and the boundaries are taken to be on the closest mesh lines.

$NH1 := \frac{h_1}{mesh}$ $NH2 := \frac{h_2}{mesh}$ $NP := \frac{p}{mesh}$ $NW := \frac{w}{mesh}$

$NH1 = 11.09$ $NH2 = 28.834$ $NP = 18.245$ $NW = 8.049$

These numbers should be as close to integers as possible

$Nh1 := round(NH1)$ $Nh2 := round(NH2)$ $Np := round(NP)$ $Nw := round(NW)$ $Nw2 := \frac{Nw}{2}$ $Np2 := \frac{Np}{2}$

$Nh1 = 11$ $Nh2 = 29$ $Np = 18$ $Nw = 8$

NB: Np and Nw must be even numbers

Calculation of capacitances using a finite difference solution of Laplace's equation

This sheet models a section of the grid of width p between the centres of adjacent strips.

Top boundary condition. This variable is 0 for a conducting top plane and 2 for a symmetry boundary $H2sym := 2$

Maximum number of iterations $Niter := 1000$

Right hand boundary condition. This variable is 0 for an anti-symmetry boundary and 2 for a symmetry boundary $Rsym := 2$

Voltages on left-hand and right-hand strips

$V1 := 100$ $V2 := 100$

The zero, π/2 and π modes can be modelled as follows

Zero mode:	V1 = 100	V2 = 100	Rsym = 2
π/2 mode	V1 = 100	V2 = 0	Rsym = 0
π mode	V1 = 100	V2 = -100	Rsym = 2

The section below can be collapsed to allow the data and the results to be seen together

```
VM := | for  i ∈ 0..Np
      |   for  j ∈ 0..Nh2
      |     V[i,j] ← 0
      | for  n ∈ 1..Niter
      |   | for  i ∈ 0..Np
      |   |   V[i,0] ← 0
      |   | for  j ∈ 1..(Nh1 − 1)
      |   |   | V[0,j] ← (2·V[1,j] + V[0,j+1] + V[0,j−1])·0.25
      |   |   | V[Np,j] ← (Rsym·V[Np−1,j] + V[Np,j+1] + V[Np,j−1])·0.25
      |   |   | for  i ∈ 1..Np − 1
      |   |   |   V[i,j] ← (V[i+1,j] + V[i,j+1] + V[i−1,j] + V[i,j−1])·0.25
      |   | for  i ∈ 0..Nw2
      |   |   V[i,Nh1] ← V1
      |   | for  i ∈ (Nw2 + 1)..(Np − Nw2 − 1)
      |   |   V[i,Nh1] ← (V[i,Nh1−1]·ε2 + 0.5·(V[i−1,Nh1] + V[i+1,Nh1])·(ε2 + 1) + V[i,Nh1+1]) / (2·ε2 + 2)
      |   | for  i ∈ (Np − Nw2)..Np
      |   |   V[i,Nh1] ← V2
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$$V_{i,Nh1} \leftarrow \frac{V_{i,Nh1-1}\cdot\varepsilon_2 + 0.5\cdot\left(V_{i-1,Nh1} + V_{i+1,Nh1}\right)\cdot\left(\varepsilon_2 + 1\right) + V_{i,Nh1+1}}{2\cdot\varepsilon_2 + 2}$$

for $j \in (Nh1 + 1)..(Nh2 - 1)$

$V_{0,j} \leftarrow \left(2 \cdot V_{1,j} + V_{0,j+1} + V_{0,j-1}\right) \cdot 0.25$

$V_{Np,j} \leftarrow \left(Rsym \cdot V_{Np-1,j} + V_{Np,j+1} + V_{Np,j-1}\right) \cdot 0.25$

for $i \in 1..Np - 1$

$V_{i,j} \leftarrow \left(V_{i+1,j} + V_{i,j+1} + V_{i-1,j} + V_{i,j-1}\right) \cdot 0.25$

$V_{0,Nh2} \leftarrow \left(2 \cdot V_{1,Nh2} + H2sym \cdot V_{0,Nh2-1}\right) \cdot 0.25$

$V_{Np,Nh2} \leftarrow \left(Rsym \cdot V_{Np-1,Nh2} + H2sym \cdot V_{Np,Nh2-1}\right) \cdot 0.25$

for $i \in 1..Np - 1$

$V_{i,Nh2} \leftarrow \left(V_{i+1,Nh2} + V_{i-1,Nh2} + H2sym \cdot V_{i,Nh2-1}\right) \cdot 0.25$

$VM \leftarrow V$

return VM

Calculate the charge per unit length on the edge of the grid, and the capacitance per unit length divided by ε_0 from the total charge.
The calculation assumes that the electrode voltages are +/- 100 or zero.

$$Q1 := \left[\sum_{i=0}^{0.5\cdot Np} VM_{i,1} - \left(\frac{VM_{0,1} + VM_{Np2,1}}{2}\right)\right]\cdot \varepsilon_2 \qquad Q1 = 455.1$$

$$Q2 := \left[\sum_{j=0}^{Nh1} \left(VM_{Np2-1,j} - VM_{Np2+1,j}\right) - \frac{\left(VM_{Np2-1,0} - VM_{Np2+1,0}\right)}{2} - \left(\frac{VM_{Np2-1,Nh1} - VM_{Np2+1,Nh1}}{2}\right)\right]\cdot \frac{\varepsilon_2}{2} \qquad Q2 = -0$$

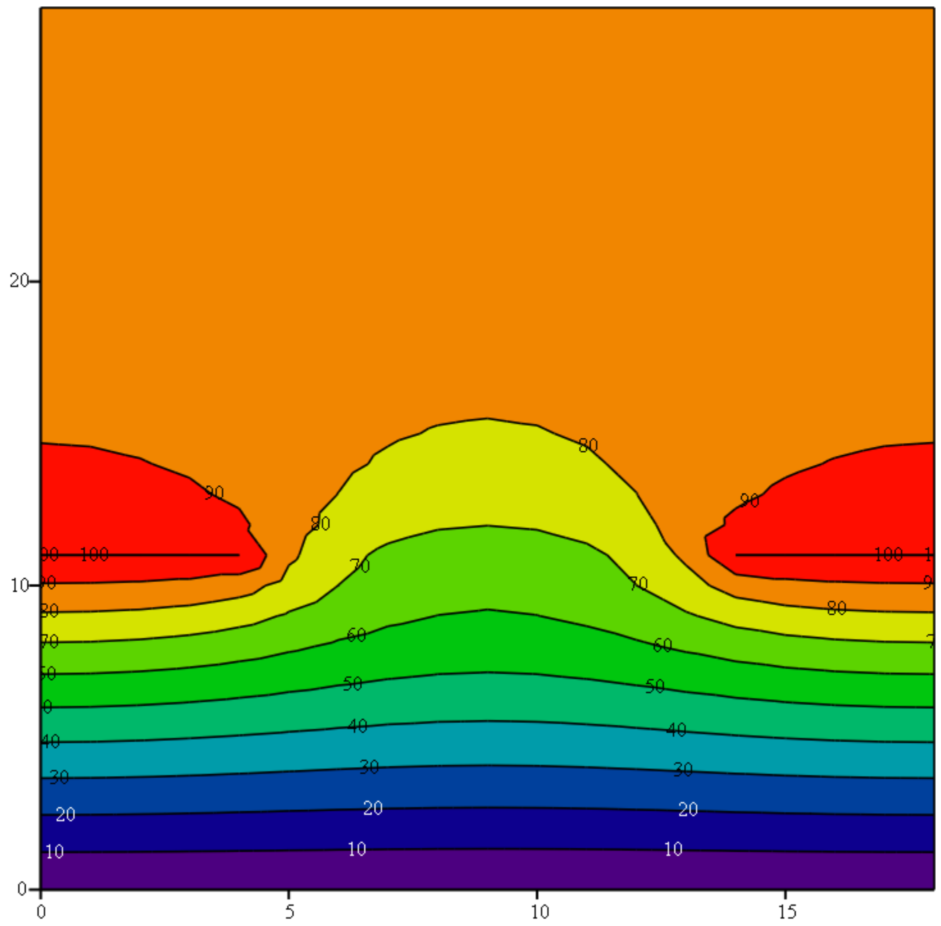
$$Q3 := \left[\sum_{j=Nh1}^{Nh2} \left(VM_{Np2-1,j} - VM_{Np2+1,j}\right) - \frac{\left(VM_{Np2-1,Nh1} - VM_{Np2+1,Nh1}\right)}{2} - \left(\frac{VM_{Np2-1,Nh2} - VM_{Np2+1,Nh2}}{2}\right)\right]\cdot \frac{1}{2}$$

$$Q3 = -0.1$$

$$Q4 := \left[\sum_{i=0}^{0.5\cdot Np} VM_{i,Nh2-1} - \left(\frac{VM_{0,Nh2-1} + VM_{Np2,Nh2-1}}{2}\right) - \left[\sum_{i=0}^{0.5\cdot Np} VM_{i,Nh2} - \left(\frac{VM_{0,Nh2} + VM_{Np2,Nh2}}{2}\right)\right]\right]$$

$$Q4 = 0.0$$

$$Capacitance := \frac{Q1 + Q2 + Q3 + Q4}{50}$$



VM

Capacitance = 9.101