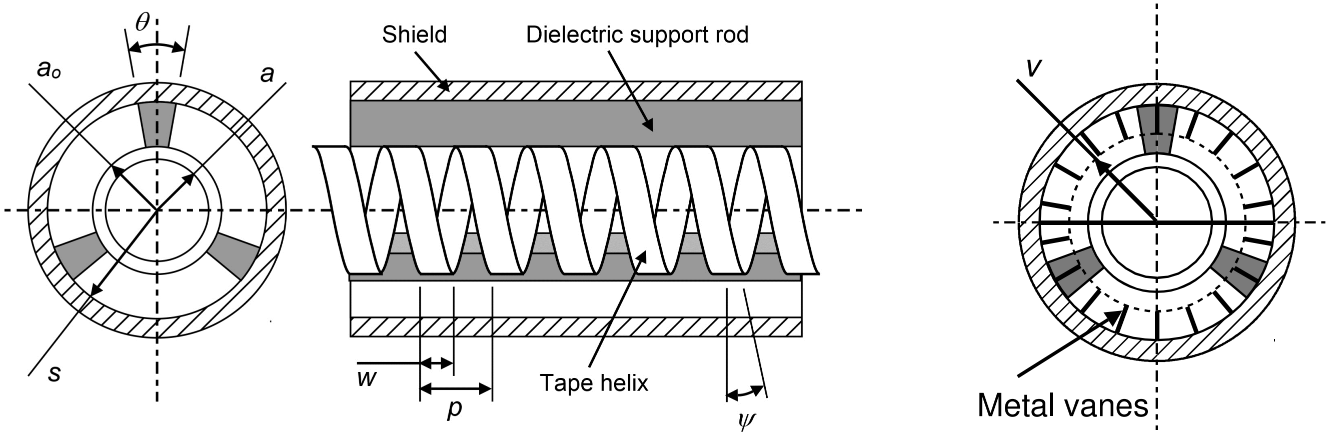
WS 4.3 Sheath helix model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

The contents of this sheet are provided for educational purposes only and no warranty is expressed or implied that they are suitable for use as professional design tools.



The helix is represented by in this model by a thin helically conducting cylinder, and the support rods whose relative permittivity is ε_r are represented by an equivalent uniform dielectric cylinder. The dispersion may be adjusted by the addition of metal vanes

$a := 3\cdot mm$ $\quad s := 2\cdot a$ $\quad v := 1.5\cdot a$ $\quad \psi := 10\cdot deg$ $\quad N_r := 3$ $\quad \theta := 20\cdot deg$ $\quad \varepsilon_r := 10$

Section 4.3.1 Equivalent circuit model of the sheath helix

This sheet implements the method described in S. F. Paik, "Design formulas for helix dispersion shaping," *IEEE Transactions on Electron Devices,* vol. 16, pp. 1010-1014, (1969) with corrections given in Basu, B. "Equivalent circuit analysis of a dielectric-supported helix in a metal shell." *International Journal of Electronics* **47**(3): 311-314 (1979).

Sheath helix in free space

The dispersion equation is

$$\frac{(\gamma_0 a)^2}{(ka)^2}\tan^2\psi = \frac{I_1(\gamma_0 a)K_1(\gamma_0 a)}{I_0(\gamma_0 a)K_0(\gamma_0 a)}$$

Equation 4.75

This can be expressed in terms of the equivalent circuit parameters where it is assumed that $\beta = \gamma_0$. Note $\beta a = \beta$ /a etc.

Series inductance

$$L_0(\beta a) := \frac{\mu_0}{2\cdot\pi}\cdot I1(\beta a)\cdot K1(\beta a)\cot(\psi)^2$$

Equation 4.76

Shunt capacitance

$$C_0(\beta a) := \frac{2\cdot\pi\cdot\varepsilon_0}{I0(\beta a)\cdot K0(\beta a)}$$

Equation 4.77

Phase velocity

$$v_{p0}(\beta a) := \frac{1}{\sqrt{L_0(\beta a)\cdot C_0(\beta a)}}$$

$$Z_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Group velocity

$$v_{g0}(\beta a) := \frac{d}{d(\beta a)}\left(\beta a\cdot v_{p0}(\beta a)\right)$$

ω / c

$$k(\beta a) := \frac{\beta a}{c\cdot\sqrt{L_0(\beta a)\cdot C_0(\beta a)}}$$

βp

$$\beta p(\beta a) := 2\cdot\pi\cdot\beta a\cdot\tan(\psi)$$

Transverse impedance and Pierce impedance according to Paik equations (2) and (3)

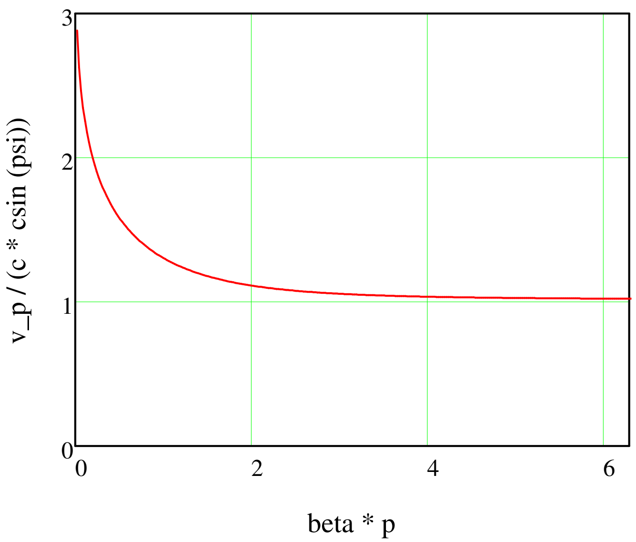
$$Z_t(\beta a) := \sqrt{\frac{L_0(\beta a)}{C_0(\beta a)}} \qquad Z_{p0}(\beta a) := \frac{v_{p0}(\beta a)}{v_{g0}(\beta a)}\cdot\frac{1}{I0(\beta a)^2}\cdot Z_t(\beta a)$$

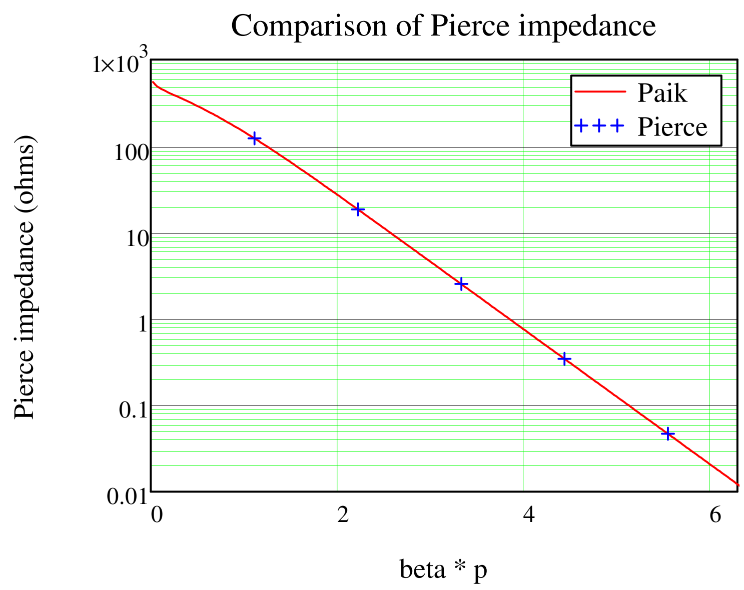
see Equation 4.9

Pierce impedance according to PiercePierce, J. R. (1950). Traveling-Wave Tubes. Princeton, N.J., D. van Nostrand.

$$ZP(z) := \frac{c}{v_{p0}(z)}\cdot\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{1}{z\cdot\pi}\cdot\frac{K0(z)}{I0(z)}\cdot\left(\frac{I1(z)}{I0(z)} - \frac{I0(z)}{I1(z)} + \frac{K0(z)}{K1(z)} - \frac{K1(z)}{K0(z)} + \frac{4}{z}\right)^{-1}$$

The two formulae give identical results





Sheath helix with a shield and dielectric loading $sa := \frac{s}{a}$ $va := \frac{v}{a}$

Series inductance $$L_1(\beta a, sa) := L_0(\beta a)\cdot\left[1 - \left(\frac{I1(\beta a)\cdot K1(\beta a\cdot sa)}{I1(\beta a\cdot sa)\cdot K1(\beta a)}\right)\right]$$ Equation 4.78

Relative permittivity of uniform cylinder $$\varepsilon_2 := 1 + \left(\frac{N_r\cdot\theta}{2\cdot\pi}\right)\cdot\left(\varepsilon_r - 1\right)$$ $\varepsilon_2 = 2.50$ Equation 4.82

Effective relative permittivity (Paik) $$P\varepsilon_{eff}\left(\beta a, sa, \varepsilon_2\right) := 1 + \left(\varepsilon_2 - 1\right)\cdot\beta a\cdot I0(\beta a)\cdot K1(\beta a)$$

Effective relative permittivity (Basu) $$\varepsilon_{eff}\left(\beta a, sa, \varepsilon_2\right) := 1 + \left(\varepsilon_2 - 1\right)\cdot\beta a\cdot I0(\beta a)\cdot K1(\beta a)\cdot\left(1 + \frac{I1(\beta a)\cdot K0(\beta a\cdot sa)}{K1(\beta a)\cdot I0(\beta a\cdot sa)}\right)$$ Equation 4.80

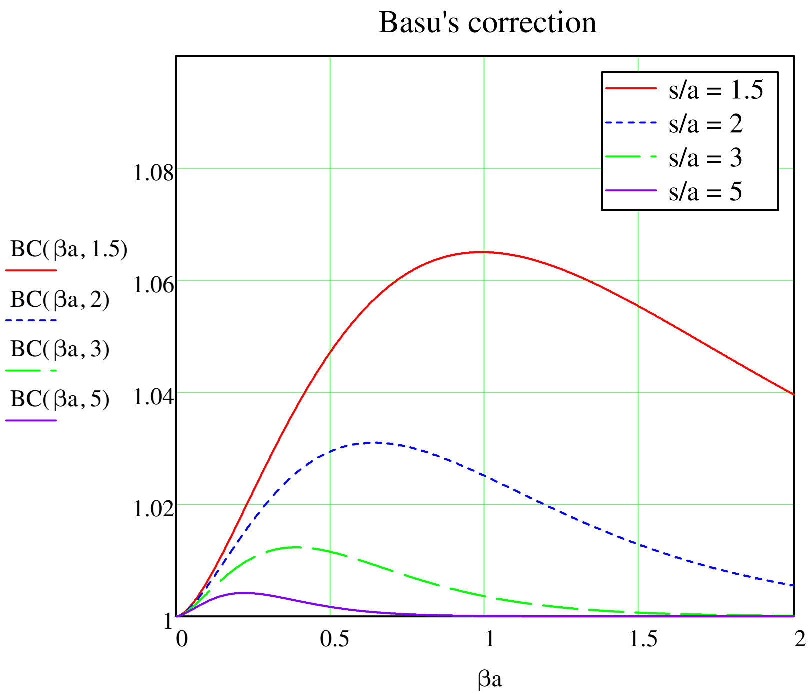
Basu's correction to the effective relative permittivity $$BC(\beta a, sa) := \frac{\varepsilon_{eff}\left(\beta a, sa, \varepsilon_2\right)}{P\varepsilon_{eff}\left(\beta a, sa, \varepsilon_2\right)}$$

Shunt capacitance $$C_1\left(\beta a, sa, \varepsilon_2\right) := C_0(\beta a)\cdot\varepsilon_{eff}\left(\beta a, sa, \varepsilon_2\right)\cdot\left[1 - \left(\frac{I0(\beta a)\cdot K0(\beta a\cdot sa)}{I0(\beta a\cdot sa)\cdot K0(\beta a)}\right)\right]^{-1}$$ Equation 4.81

Phase velocity $$v_p(\beta a, sa, va) := \frac{1}{\sqrt{L_1(\beta a, sa)\cdot C_1\left(\beta a, va, \varepsilon_2\right)}}$$

Group velocity $$v_g(\beta a, sa, va) := \frac{d}{d(\beta a)}\left(\beta a\cdot v_p(\beta a, sa, va)\right)$$

Pierce impedance $$Z_P(\beta a, sa, va) := \frac{1}{I0(\beta a)^2}\cdot\frac{v_p(\beta a, sa, va)}{v_g(\beta a, sa, va)}\cdot\sqrt{\frac{L_1(\beta a, sa)}{C_1\left(\beta a, va, \varepsilon_2\right)}}$$


Basu's correction
s/a = 1.5
s/a = 2
s/a = 3
s/a = 5
BC(βa, 1.5)
BC(βa, 2)
BC(βa, 3)
BC(βa, 5)
1.08
1.06
1.04
1.02
1
0
0.5
1
1.5
2
βa

Plotting range $\beta a := 0.01, 0.02 .. 12$

$a = 3\,\text{mm}$ $\varepsilon_2 = 2.5$ $\psi = 0.175$

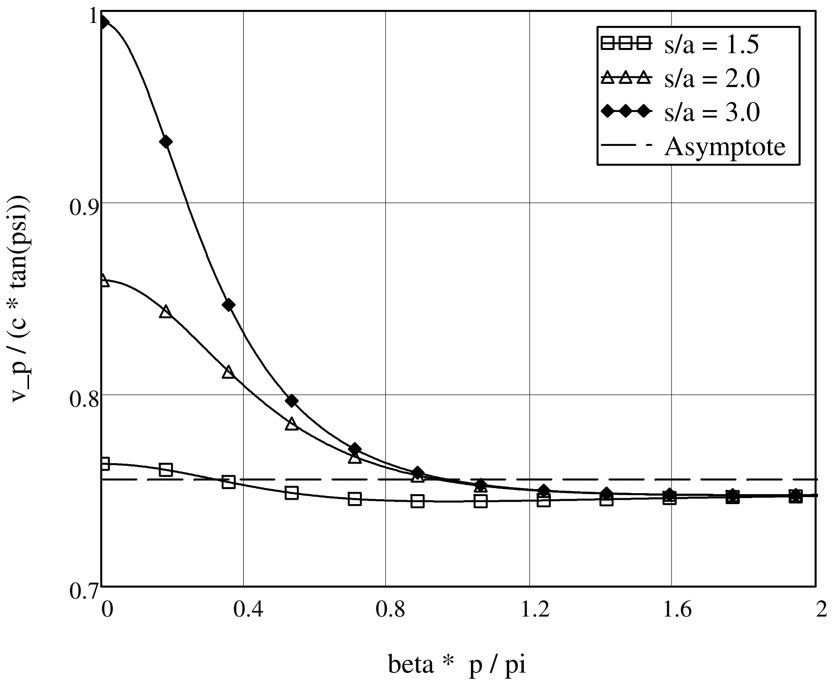


Figure 4.16(a)

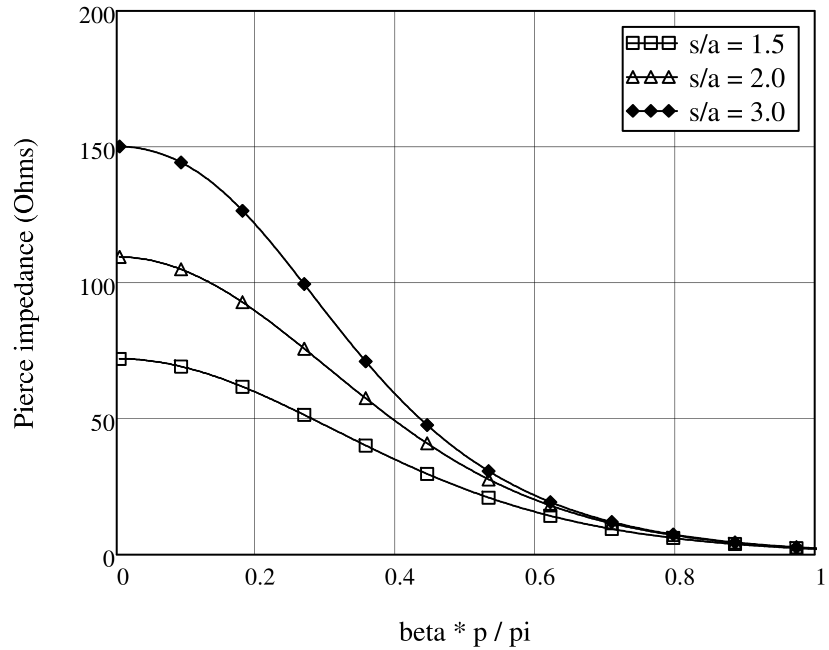


Figure 4.16(b)

Vane loaded helix $\frac{s}{a} = 2$

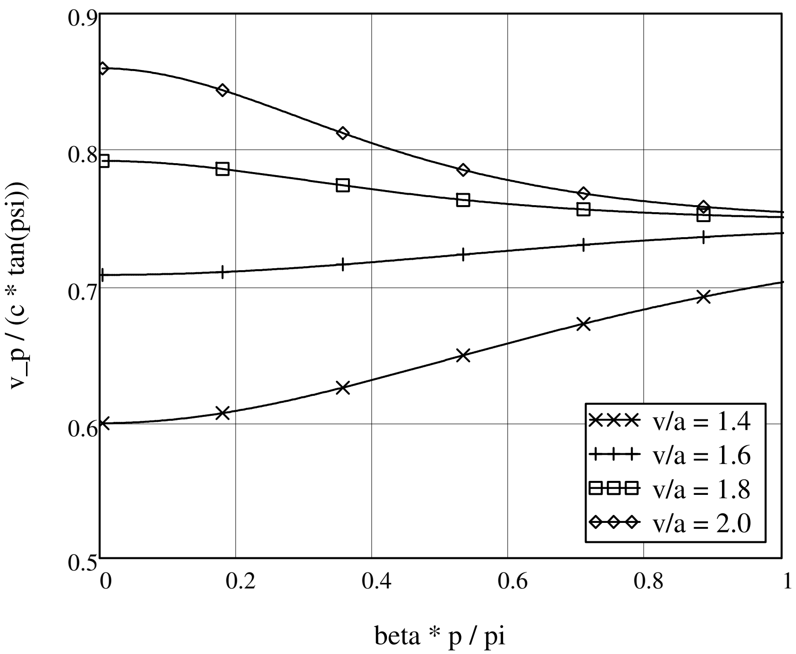


Figure 4.17(a)

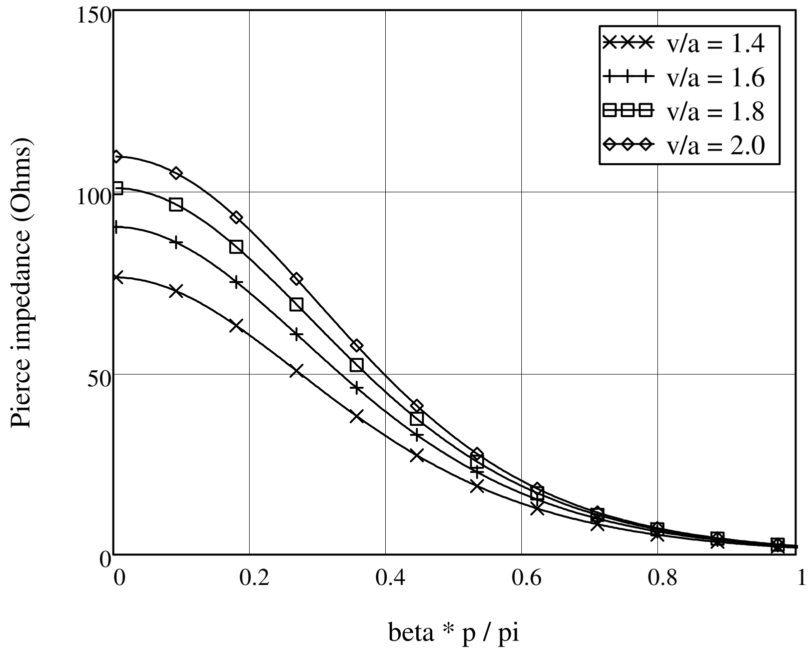


Figure 4.1(b)

Properties in the low frequency limit

In the low frequency limit the capacitance is equal to that of a coaxial line and the inductance is that of a coaxial line with a helical inner conductor.

Coaxial line shunt capacitance divided by ε_0

$$C0(sa, \varepsilon_2) := \frac{2\cdot\pi\cdot\varepsilon_2}{\ln(sa)}$$

Coaxial line with helical inner conductor series inductance divided by μ_0

$$L0(sa) := \frac{1}{4\cdot\pi}\cdot\left(1 - \frac{1}{sa^2}\right)\cdot\cot(\psi)^2$$

Plotting range $sa := 1.1, 1.2 .. 4$