WS 14.3 Helix TWT design

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet explores the design of a helix TWT (see section 14.4.1)

Define constants

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1}$ $dB := 1$

Basic performance requirements

Minimum RF output power $P_{RF} := 1000 \cdot W$ Minimum saturated gain $G_{sat} := 33 \cdot dB$

Frequency band $f_1 := 2.0 \cdot GHz$ to $f_2 := 4.0 \cdot GHz$

i) Calculate the DC beam power

If the band edges are the 1 dB points, then the band centre power is $P_0 := 1.259 \cdot P_{RF}$

Choose the synchronous frequency

$f_0 := 3.0 \cdot GHz$ or, in radians $\omega_0 := 2 \cdot \pi \cdot f_0$ $\omega_1 := 2 \cdot \pi \cdot f_1$ $\omega_2 := 2 \cdot \pi \cdot f_2$

Estimate the effiiciency of the tube at the band centre on the basis of experience and comparison with other similar tubes. This estimate can be revised later from large-signal calculations

$\eta_e := 0.26$

ii) Choose a trial value of the gun perveance. This is likely to lie in the range 0.1 to 2.0 μA $V^{-1.5}$

$Perv := 1.9 \cdot \mu A \cdot V^{-1.5}$

The DC beam power required is then given by

$$P_{DC} := \frac{P_0}{\eta_e}$$

The anode voltage and beam current can now be calculated

$$V_a := \left(\frac{P_{DC}}{Perv}\right)^{0.4} \qquad V_a = 5.8 \cdot kV$$

$$I_0 := \frac{P_{DC}}{V_a} \qquad I_0 = 0.84 \cdot A$$

iii) Choose the beam filling factor b/a which is typically in the range 0.5 to 0.7

$$b\_a := 0.5$$

Calculate the beam velocity and the effective beam voltage within the helix allowing for space-charge potential depression.

$$u_0 := \left| \begin{array}{l} V_D \leftarrow 0 \\ \text{for } n \in 0..10 \\ \quad \left| \begin{array}{l} u_n \leftarrow c \cdot \left[1 - \dfrac{1}{\left[1 + \dfrac{\eta \cdot (V_a - V_D)}{c^2} \right]^2} \right]^{0.5} \\ V_D \leftarrow \dfrac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot (1 - 2 \cdot \ln(b\_a)) \end{array} \right. \\ \text{return } u_n \end{array} \right.$$

$$V_0 := \frac{c^2}{\eta} \cdot \left(\frac{1}{\sqrt{1 - \dfrac{u_0^2}{c^2}}} - 1 \right)$$

$$Rel := \frac{1}{\sqrt{1 - \dfrac{u_0^2}{c^2}}}$$

$$V_0 = 4.92 \cdot kV$$

$$u_0 = 4.130 \times 10^7 \frac{m}{s}$$

The electronic and free space propagation constants at the band centre are given by

$$\beta_e(\omega) := \frac{\omega}{u_0}$$

$$\beta_e(\omega_0) = 456.387 \frac{1}{m}$$

and the radial propagation constant is

$$\gamma_e(\omega) := \sqrt{\beta_e(\omega)^2 - \frac{\omega^2}{c^2}}$$

$$\gamma_e(\omega_0) = 452.035 \frac{1}{m}$$

iv) Choose a value for the normalised beam radius at the band centre which is typically in the range 0.5 to 1.0. Recall that choosing a large value results in lower coupling impedance and that the value at the top of the band will be greater than that chosen at the centre.

$\gamma eb := 0.7$

Calculate the beam radius and the helix radius

$b := \frac{\gamma eb}{\gamma_e(\omega_0)}$ and $a := \frac{b}{b\_a}$ $b = 1.55 \cdot mm$ $a = 3.10 \cdot mm$

The charge density in the beam is calculated using

$\rho := \frac{I_0}{\pi \cdot b^2 \cdot u_0}$ $\rho = 2.689 \times 10^{-3} \cdot C \cdot m^{-3}$

v) The plasma frequency is calculated using

$\omega_p := \sqrt{\eta \cdot \frac{\rho}{\varepsilon_0}}$ $\omega_p = 7.309 \times 10^9 \frac{1}{s}$

The Brillouin field is given by

$B_B := \sqrt{2} \cdot \frac{\omega_p}{\eta}$ so that $B_B = 0.059 \cdot T$

The magnetic field multiplication factor is chosen to give the best compromise between adequate beam stiffness and the size and weight of the magnet. Assume that

$m_B := 1.1$

so that the RMS PPM field required is $B_0 := m_B \cdot B_B$ and $B_0 = 0.065\,T$

The peak value of the PPM field is then $B_{pk} := B_0 \cdot \sqrt{2}$ giving $B_{pk} = 0.091 \cdot T$

$\omega_L := \frac{\eta}{2} \cdot B_0$ $\qquad$ $\beta_L := \frac{\omega_L}{u_0}$

$\beta_p := \frac{\omega_p}{u_0}$ $\qquad$ $\sqrt{\frac{2 \cdot \beta_L^{\,2}}{\beta_p^{\,2}}} = 1.100$

For acceptable ripple without excessive demagnetising field α should be in the range 0.1 to 0.2

$\alpha := 0.15$ $\qquad$ $\beta_{ppm} := \frac{\beta_L}{\sqrt{\alpha}}$ $\qquad$ $\text{Period} := \frac{2 \cdot \pi}{\beta_{ppm}}$ $\qquad$ $\text{Period} = 17.7 \cdot \text{mm}$

Check that the constant in eq. (29) in Bliss, E. E. (1962). Traveling-wave tube design. Electron Tube design, Radio Corporation of America: 898-928 is in the range 8 to 10.

$B_{pk} = 914.1 \cdot G$ $\qquad$ $\frac{2 \cdot \pi}{\beta_{ppm}} = 0.696 \cdot \text{in}$ $\qquad$ $V_0 = 4919\,V$ $\qquad$ $C_B := \frac{\text{Period} \cdot B_{pk}}{\sqrt{V_0}}$ $\qquad$ $C_B = 9.1 \cdot \frac{\text{in} \cdot G}{\sqrt{V}}$

vi) Computation of the reduced plasma frequency

$$\omega_q(\omega) := \begin{array}{|l} \gamma a \leftarrow \gamma_e(\omega)\cdot a \\ \gamma b \leftarrow \gamma_e(\omega)\cdot b \\ \tau b(p) \leftarrow \gamma b\cdot\left[\dfrac{\dfrac{1}{p^2}-1}{\dfrac{1}{p^2-2\cdot\left(m_B{}^2-1\right)}-1}\right]^{\frac{1}{2}} \\ fn1 \leftarrow \dfrac{1}{\gamma b}\cdot\dfrac{I1(\gamma b)\cdot K0(\gamma a)+I0(\gamma a)\cdot K1(\gamma b)}{I0(\gamma b)\cdot K0(\gamma a)-I0(\gamma a)\cdot K0(\gamma b)} \\ fn2(p) \leftarrow \dfrac{1-\dfrac{1}{p^2}}{\tau b(p)}\cdot\dfrac{I1(\tau b(p))}{I0(\tau b(p))} \\ fn(p) \leftarrow \dfrac{1}{fn1}-\dfrac{1}{fn2(p)} \\ p0 \leftarrow 0.9 \\ \omega_q \leftarrow root(fn(p0), p0)\cdot\omega_p \end{array}$$

$$\beta_q(\omega) := \frac{\omega_q(\omega)}{u_0} \qquad \beta_q(\omega_0) = 66.8\,\frac{1}{m}$$

The ratio of the reduced plasma frequency to the centre frequency is

$$\frac{\omega_q(\omega_0)}{\omega_0} = 0.146$$

Calculate the electronic impedance of the beam

$$Z_e(\omega) := 2\cdot\frac{\omega_q(\omega)\cdot V_0}{\omega\cdot I_0} \qquad Z_e(\omega_0) = 1.72\cdot k\Omega$$

$$Y_e(\omega) := Z_e(\omega)^{-1}$$

Estimate the conversion efficiency at the synchronous point and at the band edges

Capture ratio

$CR := 0.7$

$$\eta e(\omega) := CR\cdot\left[1-\left(\frac{1-\dfrac{\omega_q(\omega)}{\omega}}{1+\dfrac{\omega_q(\omega)}{\omega}}\right)^2\right]\cdot\frac{V_0}{V_a}$$

Equation 14.56

$\eta e(\omega_1) = 0.28$ $\qquad \eta e(\omega_0) = 0.265$ $\qquad \eta e(\omega_2) = 0.25$

vii) Synchronous point

The slow-wave propagation constant is

$$\beta_s(\omega) := \frac{\left(\omega + \omega_q(\omega)\right)}{u_0} \qquad \gamma_s(\omega) := \sqrt{\beta_s(\omega)^2 - \frac{\omega^2}{c^2}} \qquad \beta_s(\omega_0) = 523.229\,\frac{1}{m}$$

For synchronism at frequency f_0 the phase velocity of the wave on the helix is

$$v_s := \frac{\omega_0}{\beta_s(\omega_0)}$$ so that $$v_s = 3.603\times 10^7\,\frac{m}{s}$$ and $$\frac{u_0}{v_s} = 1.146 \qquad \frac{v_s}{c} = 0.120$$

viii) Calculate the dimensions of the helix

Specify the shield radius and the number, wedge angle and relative permittivity of the support rods. Also the tape helix factors (see section 4.3.4)

$$r_s := 2\cdot a \qquad N := 3 \qquad \theta := 20\cdot deg \qquad \varepsilon_r := 10 \qquad \alpha_C := 1 \qquad \alpha_L := 1$$

The effective permittivity of the support rods is

$$\varepsilon_2 := 1 + \left(\frac{N\cdot\theta}{2\cdot\pi}\right)\cdot\left(\varepsilon_r - 1\right)$$ Equation 4.82

Check the approximate cut-off frequency of the TE_{11} coaxial mode

$$f_{11} := \frac{c}{\pi\cdot a\cdot\left(1 + \frac{r_s}{a}\right)} \qquad f_{11} = 10.3\cdot GHz$$

Equivalent circuit parameters for the helix

$$L_1(\gamma,\psi) := \left| \begin{array}{l} \gamma a \leftarrow \gamma \cdot a \\ \gamma s \leftarrow \gamma \cdot r_s \\ L_0 \leftarrow \dfrac{\mu_0}{2\cdot\pi} \cdot I1(\gamma a)\cdot K1(\gamma a)\cot(\psi)^2 \\ L_1 \leftarrow L_0 \cdot \left(1 - \dfrac{I1(\gamma a)\cdot K1(\gamma s)}{I1(\gamma s)\cdot K1(\gamma a)}\right) \end{array} \right.$$

$$C_1(\gamma) := \left| \begin{array}{l} \gamma a \leftarrow \gamma \cdot a \\ \gamma s \leftarrow \gamma \cdot r_s \\ C_0 \leftarrow \dfrac{2\cdot\pi\cdot\varepsilon_0}{I0(\gamma a)\cdot K0(\gamma a)} \\ D \leftarrow \gamma a \cdot I0(\gamma a)\cdot K1(\gamma a)\cdot\left(1 + \dfrac{I1(\gamma a)\cdot K0(\gamma s)}{K1(\gamma a)\cdot I0(\gamma s)}\right) \\ \varepsilon_{eff} \leftarrow 1 + (\varepsilon_2 - 1)\cdot D \\ C_1 \leftarrow C_0 \cdot \varepsilon_{eff} \cdot \left(1 - \dfrac{I0(\gamma a)\cdot K0(\gamma s)}{I0(\gamma s)\cdot K0(\gamma a)}\right)^{-1} \end{array} \right.$$

Phase velocity

$$vp(\gamma,\psi) := \frac{1}{\sqrt{\alpha_L \cdot \alpha_C \cdot L_1(\gamma,\psi)\cdot C_1(\gamma)}}$$

Find the pitch angle which gives the required phase velocity at synchronism

$$\psi_a := 10\cdot deg \qquad \psi := root\left[\left(vp\left(\gamma_s(\omega_0),\psi_a\right) - \frac{\omega_0}{\beta_s(\omega_0)}\right),\psi_a\right] \qquad \psi = 8.7\,deg$$

Helix pitch $\quad p_h := 2\cdot\pi\cdot a\cdot\tan(\psi) \qquad p_h = 3.0\,mm$

Propagation constant

$$\beta_0(\omega) := \left| \begin{array}{l} \gamma(\beta) \leftarrow \sqrt{\beta^2 - \dfrac{\omega^2}{c^2}} \\ \beta \leftarrow \beta_s(\omega_0) \\ \beta_0 \leftarrow \mathrm{root}\left(\dfrac{\beta}{\sqrt{L_1(\gamma(\beta),\psi)\cdot C_1(\gamma(\beta))}} - \omega, \beta\right) \end{array} \right.$$

Phase velocity and group velocity

$$v_p(\omega) := \frac{\omega}{\beta_0(\omega)}$$

$$v_g(\omega) := \left(\frac{d}{d\omega}\beta_0(\omega)\right)^{-1}$$

Check synchronism

$$\beta_0(\omega_0) - \beta_s(\omega_0) = 0\,\mathrm{m}^{-1}$$

Frequency at the pi mode

$$f_\pi := \frac{1}{2\cdot\pi}\cdot\mathrm{root}\left(\frac{\beta_s(\omega_0)\cdot p_h}{\pi} - 1, \omega_0\right)$$

$$f_\pi = 6.21\,\mathrm{GHz}$$

Transverse impedance, characteristic impedance and Pierce impedance

$$Z_t(\gamma) := \sqrt{\frac{\alpha_L\cdot L_1(\gamma,\psi)}{\alpha_C\cdot C_1(\gamma)}}$$

$$Z_c(\omega) := \left| \begin{array}{l} \gamma \leftarrow \sqrt{\beta_0(\omega)^2 - \dfrac{\omega^2}{c^2}} \\ \dfrac{v_p(\omega)}{v_g(\omega)}\cdot Z_t(\gamma) \end{array} \right.$$

$$Y_c(\omega) := \frac{1}{Z_c(\omega)}$$

$$Z_c(\omega_0) = 83.8\,\Omega$$

$$\gamma_0(\omega) := \sqrt{\beta_0(\omega)^2 - \frac{\omega^2}{c^2}}$$

$$Z_p(\omega) := \frac{Z_c(\omega)}{I0(\gamma_0(\omega)\cdot a)^2}$$

Compute the coupling factor corresponding to the approximate propagation constant of the growing wave

$$\gamma_1(\omega) := \frac{1}{2}\cdot(\gamma_0(\omega) + \gamma_s(\omega))$$

$$\mu_c(\omega) := \frac{2}{\gamma_1(\omega)\cdot b}\cdot\frac{I1(\gamma_1(\omega)\cdot b)}{I0(\gamma_1(\omega)\cdot a)}$$

$$\mu_c(\omega_0) = 0.616$$

ix) Calculate the approximate gain per unit length using the two-wave model. Eq (11.145)

$$\beta_w(\omega) := \frac{1}{2}\cdot\left(\beta_s(\omega) + \beta_0(\quad \alpha_w(\omega) := \frac{1}{2}\cdot\sqrt{\mu_c(\omega)^2\cdot Z_c(\omega)\cdot Y_e(\omega)\cdot\beta_0(\omega)\cdot\beta_s(\omega) - \left(\beta_0(\omega) - \beta_s(\omega)\right)^2}\right.$$

$$\mathrm{Gain}(\omega) := 20\cdot\log(e)\cdot\alpha_w(\omega) \qquad \mathrm{Gain}(\omega_0) = 308\cdot\mathrm{dB}\cdot\mathrm{m}^{-1}$$

Assuming that the tube has $N_s := 2$ sections, and that the launching loss is 6 dB, the sever loss is 6 dB and the gain compression is 3 dB then the active tube length (calculated at the lowest frequency where the wavelength is least) is

$$\mathrm{Lh}(\omega) := \frac{\left[G_{sat} + 6 + \left(N_s - 1\right)\cdot 6 + 3\right]}{\mathrm{Gain}(\omega)} \qquad \mathrm{Lh}(\omega_1) = 174\cdot\mathrm{mm}$$

The actual length of the helix must include the length of the attenuator(s) at the sever. In this region the gain is small. For a good match each attenuator should be about 3 wavelengths long. Thus the total length of the attenuators is

$$L_a := \left(N_s - 1\right)\cdot 6\cdot\frac{2\pi}{\beta_w(\omega_0)} \qquad L_a = 72\cdot\mathrm{mm}$$

Thus the total length of the helix is $L_a + \mathrm{Lh}(2\cdot\pi\cdot f_1) = 246\cdot\mathrm{mm}$

Define plotting ranges

$$n := 0..6 \qquad f3 := (1.500\ \ 2.000\ \ 2.500\ \ 3.000\ \ 3.500\ \ 4.000\ \ 4.500)^T \qquad \omega 3 := 2\cdot\pi\cdot f3\cdot\mathrm{GHz}$$

Helix phase velocity $vh_n := \dfrac{\omega 3_n}{\beta_0(\omega 3_n)}$ Slow space-charge wave velocity $vs_n := \dfrac{\omega 3_n}{\beta_s(\omega 3_n)}$

Approximate small-signal gain allowing for launching and sever loss $Gh_n := \mathrm{Gain}(\omega 3_n)\cdot\mathrm{Lh}(2\cdot\pi\cdot f_1) - 12$

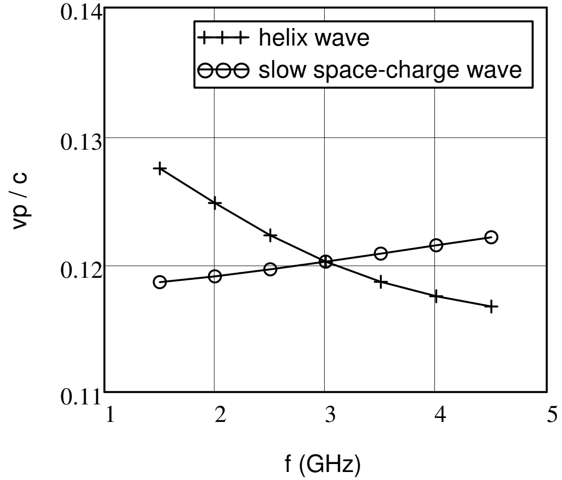

Figure 14.26(a)

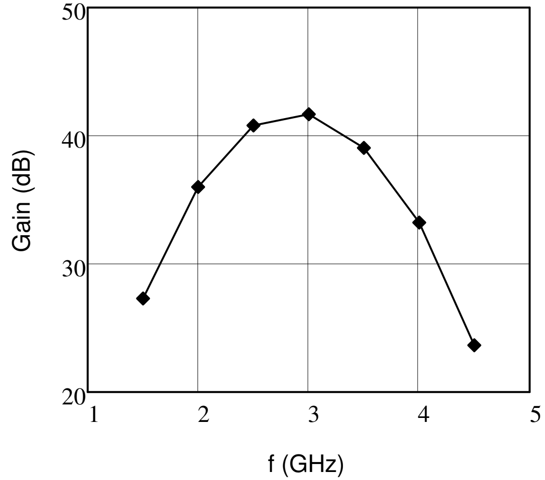

Figure 14.26(b)

Note this is the estimated small-signal gain for the whole tube, not a single section as stated in the book

Tube parameters (Table 14.3)

$\eta_e = 26\,\%$

$b = 1.55\cdot mm$

$Perv = 1.9\,\mu A\cdot V^{-1.5}$

$a = 3.10\cdot mm$

$V_a = 5.79\cdot kV$

$B_B = 0.059\,T$

$I_0 = 0.84\,A$

$B_{pk} = 0.091\,T$

$\frac{b}{a} = 0.50$

$Period = 17.7\,mm$

$V_0 = 4.92\cdot kV$

$Lh(\omega_1) = 174\,mm$

$\gamma_e(\omega_0)\cdot b = 0.70$

$\frac{v_s}{c} = 0.120$