WS 18.2 Crossed-field multipactor model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

The equations used are taken from
S. Riyopoulos, D. Chernin and D. Dialetis, Theory of electron multipactor in crossed fields,
Phys Plasmas, Vol.2, pp.3194-3213, August 1995.

The electrons move under the influence of the RF electric field in the *y* direction between parallel plates separated by distance D. There is a uniform static magnetic field in the *z* direction leading to motion of the electrons in the *x-y* plane.

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1}$

Frequency

$f := 400 \cdot MHz$ $\omega := 2 \cdot \pi \cdot f$

Plate separation

$D := 10 \cdot mm$

Initial electron energy

$V_{x0} := 0 \cdot V$ $V_{y0} := 1 \cdot V$

Specify the magnetic field and find the normalised cyclotron frequency

Single surface multipactor

$B0_z := 0.005 \cdot T$ $\Omega 0 := \frac{\eta \cdot B0_z}{\omega}$ $\Omega 0 = 0.35$

Two-surface multipactor

$B1_z := 0.002 \cdot T$ $\Omega 1 := \frac{\eta \cdot B1_z}{\omega}$ $\Omega 1 = 0.14$

Specify RF voltage and find the normalised electric field ($\mathcal{E}$ in Riyopoulos but E here)

$V0_{RF} := 0.1 \cdot kV$ $E0 := \frac{\eta \cdot V0_{RF}}{\omega^2 \cdot D^2}$ $E0 = 0.028$

$V1_{RF} := 1.5 \cdot kV$ $E1 := \frac{\eta \cdot V1_{RF}}{\omega^2 \cdot D^2}$ $E1 = 0.418$

Number of RF cycles (1, 2, 3, etc.)

$N0 := 1$

Number of RF cycles (0.5, 1.5, 2.5 etc.)

$N1 := 0.5$

The section below can collapsed to conceal the detailed calculations

Specify the normalised initial velocity components of the electrons

$$u_{x0} := \frac{\sqrt{2\cdot\eta\cdot V_{x0}}}{\omega\cdot D} \qquad u_{y0} := \frac{\sqrt{2\cdot\eta\cdot V_{y0}}}{\omega\cdot D} \qquad u_{x0} = 0 \qquad u_{y0} = 0.024$$

The working equations, numbered as in the paper, are as follows with $\tau = \omega t$ where ϕ is the RF phase at which the electron is launched.
The dimensions are normalised to D

$$y(\tau,\phi,E,\Omega) := \frac{E}{\Omega^2-1}\cdot\left(\sin(\phi+\tau) - \sin(\phi)\cdot\cos(\Omega\cdot\tau) - \frac{1}{\Omega}\cdot\cos(\phi)\cdot\sin(\Omega\cdot\tau)\right) \ldots + \frac{u_{y0}}{\Omega}\cdot\sin(\Omega\cdot\tau) - \frac{u_{x0}}{\Omega}\cdot(1-\cos(\Omega\cdot\tau)) \tag{7a}$$

$$x(\tau,\phi,E,\Omega) := \frac{E}{\Omega^2-1}\cdot\left[-\Omega\cdot\cos(\phi+\tau) - \sin(\phi)\cdot\sin(\Omega\cdot\tau) - \frac{1}{\Omega}\cdot\cos(\phi)\cdot\left(1-\Omega^2-\cos(\Omega\cdot\tau)\right)\right] \ldots + \frac{u_{y0}}{\Omega}\cdot(1-\cos(\Omega\cdot\tau)) + \frac{u_{x0}}{\Omega}\cdot\sin(\Omega\cdot\tau) \tag{7b}$$

$$u_y(\tau,\phi,E,\Omega) := \frac{E}{\Omega^2-1}\cdot(\cos(\phi+\tau) + \Omega\cdot\sin(\phi)\cdot\sin(\Omega\cdot\tau) - \cos(\phi)\cdot\cos(\Omega\cdot\tau)) \ldots + u_{y0}\cdot\cos(\Omega\cdot\tau) - u_{x0}\cdot\sin(\Omega\cdot\tau) \tag{9a}$$

$$u_x(\tau,\phi,E,\Omega) := \frac{E}{\Omega^2-1}\cdot(\Omega\cdot\sin(\phi+\tau) - \Omega\cdot\sin(\phi)\cdot\cos(\Omega\cdot\tau) - \cos(\phi)\cdot\sin(\Omega\cdot\tau)) \ldots + u_{y0}\cdot\sin(\Omega\cdot\tau) + u_{x0}\cdot\cos(\Omega\cdot\tau) \tag{9b}$$

To find the launch phase such that the electron trajectory is resonant we choose an initial guessed value of ϕ and then seek the value of ϕ which makes y = 0 or 1. A multiplying factor of 1000 is used to increase the accuracy of the solution. It will be seen that only certain combinations of the parameters lead to valid solutions

Single surface

$\phi_0 := 0.5\pi$

$\phi0(\Omega) := \mathrm{root}\left[1000\cdot\left(y\left(2\cdot N0\cdot\pi, \phi_0, E0, \Omega\right)\right), \phi_0\right]$

$\phi0(\Omega0) = 93.3\cdot\mathrm{deg}$

Two-surface

$\phi_1 := 0.5\cdot\pi$

$\phi1(\Omega) := \mathrm{root}\left[1000\cdot\left(y\left(2\cdot N1\cdot\pi, \phi_1, E1, \Omega\right) - 1\right), \phi_1\right]$

$\phi1(\Omega1) = 84.7\cdot\mathrm{deg}$

Calculate the impact energy in eV

$$V0_i := \frac{(\omega\cdot D)^2}{2\cdot\eta}\cdot\left(u_x(2\cdot N0\cdot\pi, \phi0(\Omega0), E0, \Omega0)^2 + u_y(2\cdot N0\cdot\pi, \phi0(\Omega0), E0, \Omega0)^2\right)$$

$V0_i = 0.7\cdot V$

$$V1_i := \frac{(\omega\cdot D)^2}{2\cdot\eta}\cdot\left(u_x(2\cdot N1\cdot\pi, \phi1(\Omega1), E1, \Omega1)^2 + u_y(2\cdot N1\cdot\pi, \phi1(\Omega1), E1, \Omega1)^2\right)$$

$V1_i = 44.3\cdot V$

Plotting ranges $\tau00 := 0, 0.001 .. 2N0\,\pi$

$\tau11 := 0, 0.001 .. 2\cdot N1\,\pi$ $\quad x_\pi := x(\pi, \phi1(\Omega1), E1, \Omega1)$

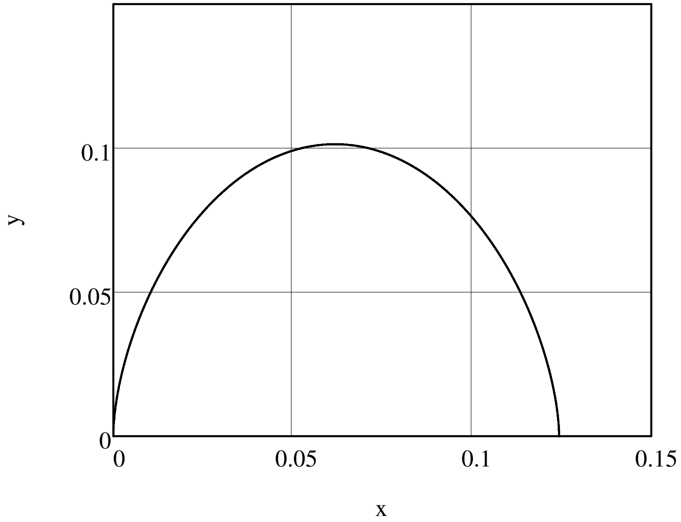

Figure 18.18