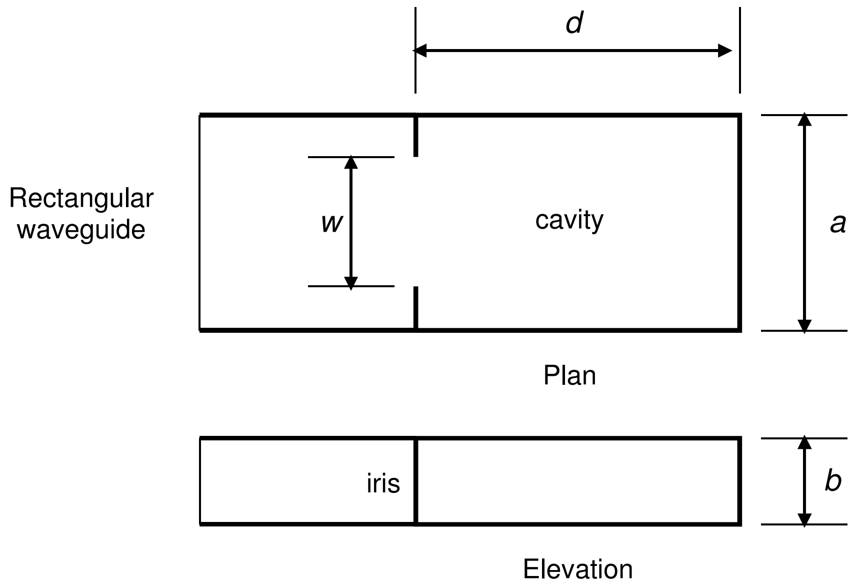
WS 3.6 Iris-coupled cavity

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Section 3.6.2 Iris coupling

This sheet illustrates the properties of a rectangular cavity coupled to a rectangular waveguide having the same width and height by an inductive iris of width *w* in a thin wall.



Set the waveguide and cavity dimensions (WG10)

$a := 72 \cdot mm$ $\quad b := 34 \cdot mm$ $\quad d := 1.0a$ $\quad w := 0.15 \cdot a$

The region below can be collapsed to allow the results to appear on the same screen as the data.

Calculate the surface resistance

Conductivity of copper $\sigma := 5.959\cdot 10^{7}\cdot S\cdot m^{-1}$ r.m.s. surface roughness $\Delta := 0.1\cdot \mu m$

$$\delta s(\omega) := \sqrt{\frac{2}{\omega\cdot\sigma\cdot\mu_0}} \qquad R_s(\omega) := \frac{1}{\sigma\cdot\delta s(\omega)}$$

Equation 3.50

Effect of r.m.s. surface roughness Δ $R_{rs}(x) := 1 + \frac{2}{\pi}\cdot \text{atan}\left(1.4\cdot x^2\right)$ Equation 3.56

Surface resistance $R_r(\omega) := R_s(\omega)\cdot R_{rs}\left(\frac{\Delta}{\delta s(\omega)}\right)$

Calculate cavity parameters

Resonant frequency

$$\omega_0 := c\cdot\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} \qquad f_0 := \frac{\omega_0}{2\cdot\pi}$$

Skin depth $\delta s(\omega_0) = 1.202\cdot \mu m$ $f_0 = 2.944\cdot GHz$

R/Q

$$R\_Q := \frac{4}{\pi}\cdot\frac{b}{\sqrt{a^2 + d^2}}\cdot\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$R\_Q = 160.2\cdot\Omega$

Unloaded Q

$$Q := \frac{\pi}{4\cdot R_r(\omega_0)}\cdot\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\left[\frac{2\cdot b\cdot\left(a^2 + d^2\right)^{1.5}}{a\cdot d\cdot\left(a^2 + d^2\right) + 2\cdot b\cdot\left(a^3 + d^3\right)}\right]$$

Equation 3.64 $Q = 14463$

Shunt resistance $R_C := R\_Q\cdot Q$ $R_C = 2.317\cdot M\Omega$

Calculate waveguide parameters

Cut-off frequency $\beta_c := \dfrac{\pi}{a}$ $\omega_c := c \cdot \beta_c$ $f_c := \dfrac{\omega_c}{2 \cdot \pi}$ $f_c = 2.082 \cdot \text{GHz}$

Guide wavelength $\beta_g(\omega) := \sqrt{\dfrac{\omega^2}{c^2} - \beta_c{}^2}$

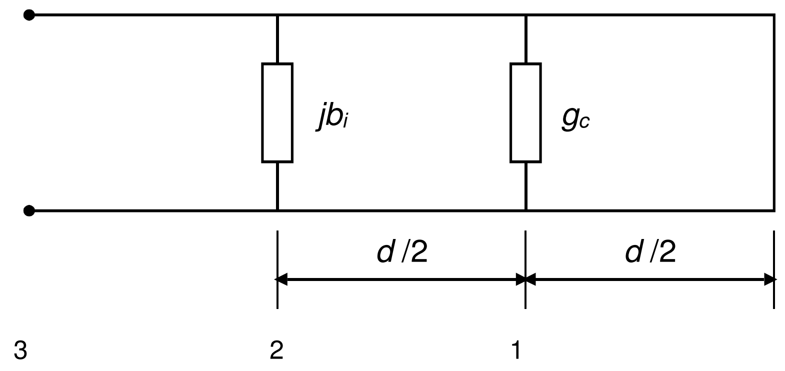
Characteristic impedance $Z_{PV}(\omega) := 2 \cdot \dfrac{b}{a} \cdot \left(\dfrac{\omega}{c \cdot \beta_g(\omega)} \cdot \sqrt{\dfrac{\mu_0}{\varepsilon_0}} \right)$ $Z_{PV}(\omega_0) = 503.2\,\Omega$

Calculate normalised iris susceptance

$$bi(\omega, w) := -\frac{2 \cdot \pi}{\beta_g(\omega) \cdot a} \cdot \cot\left(\frac{\pi \cdot w}{2 \cdot a}\right)^2$$

1. Distributed element equivalent circuit model

The cavity loss is represented by a shunt resistor R_C at the centre of the cavity. The normalised input admittance is calculated using transmission line analysis.



Normalised cavity conductance $g_C(\omega) := \dfrac{Z_{PV}(\omega)}{R_C}$

Admittance at 1 $y1_1(\omega) := \dfrac{1}{j\cdot\tan\left(\beta_g(\omega)\cdot\dfrac{d}{2}\right)}$

Admittance at 2 $y1_2(\omega) := y1_1(\omega) + g_C(\omega)$

Admittance at 3 $y1_3(\omega) := \dfrac{y1_2(\omega) + j\cdot\tan\left(\beta_g(\omega)\cdot\dfrac{d}{2}\right)}{1 + j\cdot y1_2(\omega)\cdot\tan\left(\beta_g(\omega)\cdot\dfrac{d}{2}\right)}$

Input admittance and reflection coefficient

$$y_{in}(\omega, w) := y1_3(\omega) + j\cdot bi(\omega, w) \qquad z_{in}(\omega, w) := \frac{1}{y_{in}(\omega, w)}$$

$$\rho(\omega, w) := \frac{z_{in}(\omega, w) - 1}{z_{in}(\omega, w) + 1} \qquad S_{11}(\omega, w) := 20\cdot\log(|\rho(\omega, w)|) \qquad \phi 1(\omega, w) := \operatorname{atan2}(\operatorname{Re}(\rho(\omega, w)), \operatorname{Im}(\rho(\omega, w)))$$

Find the detuned resonant frequency where the input admittance is real

$$\omega_1(w) := \operatorname{root}\left(\operatorname{Im}\left(y_{in}\left(\omega_0, w\right)\right), \omega_0\right)$$

$$\frac{\omega_1(w)}{\omega_0} = 0.99547$$

Shift the phase so that the diameter of the resonant circle lies on the real axis of the Smith chart

$$\phi(\omega,w) := \begin{cases} \left(\phi1(\omega,w) - \phi1\left(\omega_1(w),w\right)\right) & \text{if } \left|\phi1\left(\omega_1(w),w\right)\right| \le \frac{\pi}{2} \\ \phi1(\omega,w) - \phi1\left(\omega_1(w),w\right) - \pi & \text{otherwise} \end{cases}$$

Find the diameter of the resonant circle and the coupling coefficient from (24) in Kajfez, D. (1984). "Q-factor measurement with network analyser." IEEE Transactions on Microwave Theory and Techniques **MTT-32**(7): 666-670.

$$\mathrm{dia}(w) := \begin{cases} \left(1 + \left|\rho\left(\omega_1(w),w\right)\right|\right) & \text{if } \left|\phi1\left(\omega_1(w),w\right)\right| \le \frac{\pi}{2} \\ \left(1 - \left|\rho\left(\omega_1(w),w\right)\right|\right) & \text{otherwise} \end{cases}$$

$$\kappa(w) := \frac{\mathrm{dia}(w)}{2 - \mathrm{dia}(w)}$$

$$\kappa(w) = 3.735$$

Find the co-ordinates of the points which are at angles +/- α

$$\alpha := 45 \cdot \mathrm{deg}$$

$$\Gamma1(w) := \sqrt{1 + \mathrm{dia}(w)^2 \cos(\alpha)^2 - 2 \cdot \mathrm{dia}(w) \cdot \cos(\alpha)^2}$$

$$\Gamma1(w) = 0.817$$

$$\theta1(w) := \pi - \mathrm{asin}\left(\frac{\mathrm{dia}(w)}{\Gamma1(w)} \cdot \sin(\alpha) \cdot \cos(\alpha)\right)$$

$$\pi - \theta1(w) = 1.309$$

$$\theta2(w) := \pi - \mathrm{asin}\left(\frac{\mathrm{dia}(w)}{\Gamma1(w)} \cdot \sin(-\alpha) \cdot \cos(\alpha)\right)$$

$$\pi - \theta2(w) = -1.309$$

Find the loaded Q from the frequencies at these two points

$$
Q_L(w) := \left|
\begin{array}{l}
\omega_{1U} \leftarrow 1.1\cdot\omega_1(w) \\
\omega_{1L} \leftarrow 0.9\cdot\omega_1(w) \\
\omega_{2U} \leftarrow 1.1\cdot\omega_1(w) \\
\omega_{2L} \leftarrow 0.9\cdot\omega_1(w) \\
\Gamma1 \leftarrow \Gamma1(w) \\
f(\omega) \leftarrow (|\rho(\omega, w)| - \Gamma1) \\
\text{for } n \in 1..1000 \\
\quad \left|
\begin{array}{l}
\omega1_n \leftarrow 0.5\cdot(\omega_{1U} + \omega_{1L}) \\
\omega_{1U} \leftarrow \omega1_n \ \text{ if } f(\omega1_n) > 0 \\
\omega_{1L} \leftarrow \omega1_n \ \text{ if } f(\omega1_n) < 0 \\
\omega2_n \leftarrow 0.5\cdot(\omega_{2U} + \omega_{2L}) \\
\omega_{2U} \leftarrow \omega2_n \ \text{ if } f(\omega2_n) < 0 \\
\omega_{2L} \leftarrow \omega2_n \ \text{ if } f(\omega2_n) > 0 \\
Q_n \leftarrow \left|
\begin{array}{l}
\dfrac{\omega_0\cdot\tan(\alpha)}{\omega1_n - \omega2_n} \ \text{ if } \omega1_n - \omega2_n \neq 0 \\
0 \ \text{ otherwise}
\end{array}
\right. \\
(\text{break}) \ \text{ if } Q_n \neq 0 \wedge |Q_n - Q_{n-1}| < 0.0001
\end{array}
\right. \\
\text{return } Q_n
\end{array}
\right.
$$

$Q_L(w) = 3048$

Calculate the unloaded Q and compare it with the value calculated above

$Q_0(w) := Q_L(w)\cdot(1 + \kappa(w))$

$\boxed{Q = 14463}$

$Q_0(w) = 14431$

Plot the results

$$\omega_{start} := \left(1 - \frac{5}{Q_L(w)}\right)\cdot\omega_1(w) = 1.839\times 10^{10}\ \frac{1}{s}$$

$$\omega_{stop} := \left(1 + \frac{5}{Q_L(w)}\right)\cdot\omega_1(w) = 1.845\times 10^{10}\ \frac{1}{s}$$

$$\omega_{step} := \frac{\omega_{stop} - \omega_{start}}{200}$$

$$dB := 1$$

Marker co-ordinates

$$\Gamma m(w) := \begin{pmatrix} 1 \\ \Gamma 1(w) \\ 1 \\ \Gamma 1(w) \\ 1 \end{pmatrix} \qquad \theta m(w) := \begin{pmatrix} 0.5\cdot\pi \\ \theta 1(w) \\ \pi \\ \theta 2(w) \\ 1.5\cdot\pi \end{pmatrix}$$

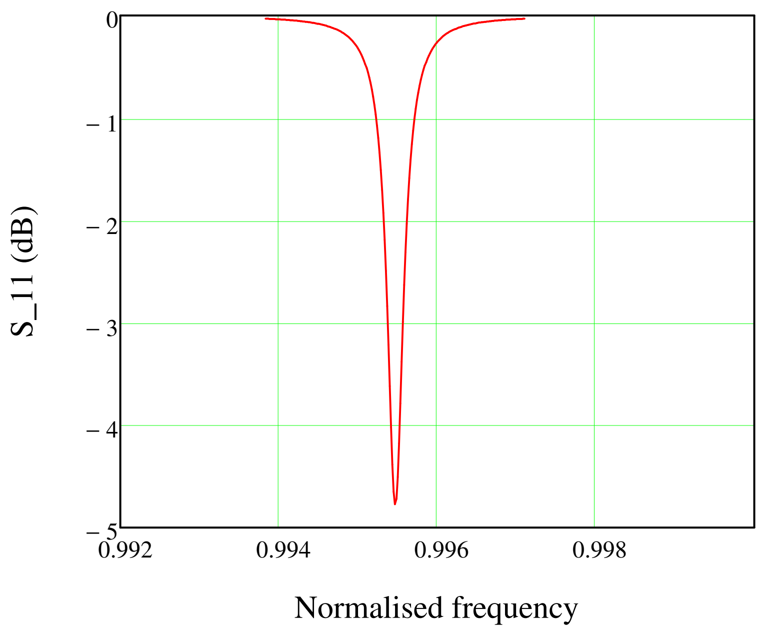
$$\omega p := \omega_{start}, \left(\omega_{start} + \omega_{step}\right) .. \omega_{stop}$$

Results: Coupling factor, Unloaded and loaded Q, resonant frequency, and S11 at resonance

$\kappa(w) = 3.735$ $\quad Q_0(w) = 14431$ $\quad Q_L(w) = 3048$

$\frac{\omega_1(w)}{2 \cdot \pi} = 2.931 \cdot \mathrm{GHz}$ $\quad \frac{\omega_1(w)}{\omega_0} = 0.995$ $\quad S_{11}(\omega_1(w), w) = -4.8\,\mathrm{dB}$

Note: this calculation only converges reliably for a limited range of input parameters



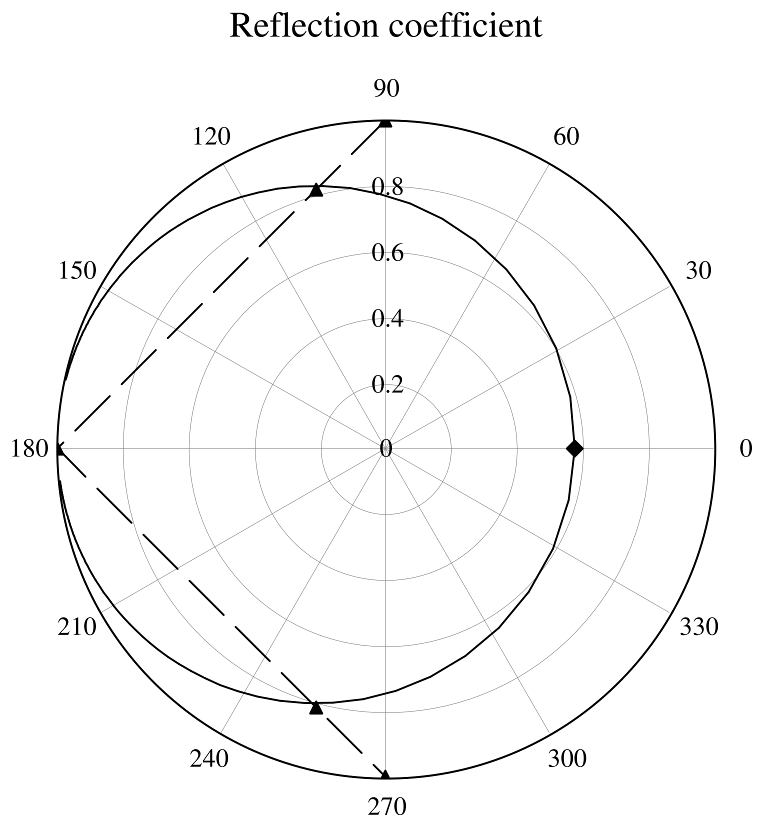
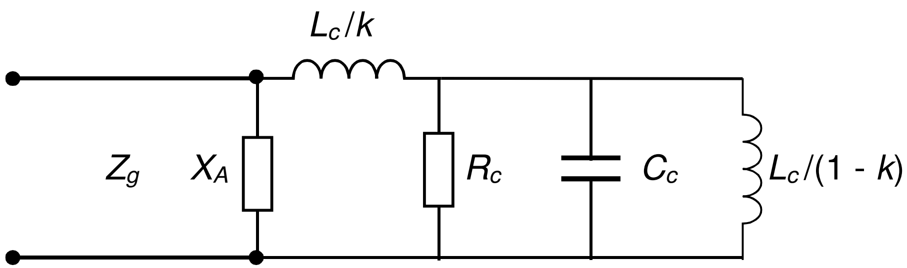


Figure 3.24

Markers show the loaded resonant frequency and 3dB points

2. Lumped element equivalent circuit model

The cavity inductance is divided into two parts determined by the parameter k since only part of the current circulating in the cavity is intercepted by the iris.



Lumped element parameters

$$C_C := \frac{1}{\omega_0 \cdot R\_Q} \qquad L_C := \frac{R\_Q}{\omega_0}$$

$C_C = 0.338 \cdot pF$ $\qquad L_C = 0.0087 \cdot \mu H$ $\qquad R_C = 2.32 \cdot M\Omega$

Admittance and impedance of the cavity

$$Y2_1(k,\omega) := j \cdot \omega \cdot C_C + \frac{1}{R_C} + \frac{1-k}{j \cdot \omega \cdot L_C} \qquad Z2_1(k,\omega) := \frac{1}{Y2_1(k,\omega)}$$

Impedance presented by the cavity at the plane of the iris

$$Z2_2(k,\omega) := \frac{j \cdot \omega \cdot L_C}{k} + Z2_1(k,\omega)$$

Normalised admittance at the plane of the iris

$$y2_2(k,\omega) := \frac{Z_{PV}(\omega)}{Z2_2(k,\omega)}$$

Resonant frequency

$$\omega_2(k,w) := \mathrm{root}\left(\mathrm{Im}\left(y2_2(k,\omega_0) + j \cdot bi(\omega_0, w)\right), \omega_0\right)$$

Current fraction to give the correct resonant frequency

$kk := 0.3$ $\qquad k := \mathrm{root}\left(\omega_1(w) - \omega_2(kk,w), kk\right)$ $\qquad k = 0.321$

Check $\quad \dfrac{\omega_2(k,w)}{\omega_1(w)} = 1$