Worksheet 12.5 # IOT Model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet combines a simple model of an IOT a described in Section 12.6 with a large-signal disc model of the output gap for purposes of comparison.
See also:
Carter, R. G. (2010). "Simple model of an inductive output tube."
IEEE Transactions on Electron Devices **ED-57**(3): 720-725.

Tube Data

Frequency	$f := 1.3 \cdot GHz$	Tunnel radius	$a := 11 \cdot mm$
Anode voltage	$V_a := 25 \cdot kV$	Beam radius	$b := 6.5 \cdot mm$
Grid bias voltage	$V_{g0} := -105 \cdot V$	Output gap length	$gap := 11 \cdot mm$

This section can be collapsed to allow the input data and the results to be seen on the same screen

Characteristics of the electron gun and the input circuit

Starting from the data in Table I of the paper cited [1] we aim to find the characteristics of the electron gun. Note that it is probably satisfactory to assume that the beam voltage is 25kV for all currents. The small changes are most likely to be due to the source impedance of the EHT supply.

[1] J.F. Orrett et al., IOT Testing at ERLP, Proc. EPAC 2006, pp.1382-84

Beam current (mA) Input power (W) Output power (kW) Efficiency (%) Gain (dB)

$$\text{Data\_I}_b := \begin{pmatrix} 440 \\ 550 \\ 642 \\ 741 \\ 822 \\ 891 \\ 966 \\ 1042 \end{pmatrix} \cdot \text{mA} \quad \text{Data\_P}_{in} := \begin{pmatrix} 45 \\ 84 \\ 120 \\ 152 \\ 182 \\ 216 \\ 250 \\ 275 \end{pmatrix} \cdot \text{W} \quad \text{Data\_P}_{out} := \begin{pmatrix} 2.8 \\ 5.2 \\ 7.4 \\ 9.0 \\ 10.6 \\ 12.0 \\ 14.0 \\ 15.0 \end{pmatrix} \cdot \text{kW} \quad \text{Data\_}\eta_e := \begin{pmatrix} 25.3 \\ 37.6 \\ 46.0 \\ 48.4 \\ 51.4 \\ 53.7 \\ 58.0 \\ 57.4 \end{pmatrix} \quad \text{Data\_GdB} := \begin{pmatrix} 17.9 \\ 17.9 \\ 17.9 \\ 17.7 \\ 17.6 \\ 17.4 \\ 17.4 \\ 17.3 \end{pmatrix}$$

Define the grid voltage as a function of the amplitude of the DC and RF grid voltages and the phase angle.

$$V_g(V_{g0}, V_{g1}, \theta) := V_{g0} + V_{g1} \cdot \cos(\theta)$$

Define the instantaneous beam current as a function of d.c. and r.f. grid voltages and the phase angle. The values for μ, I_0 and for the power are adjusted later to fit the model to the data. Note: the equation has been written in this form because Mathcad does not accept variable exponents on dimensional quantities

$$I(V_{g0}, V_{g1}, \theta) := \left| \begin{array}{l} I \leftarrow I_0 \cdot \left(1 + \dfrac{\mu \cdot V_g(V_{g0}, V_{g1}, \theta)}{V_a}\right)^{pwr} \quad \text{if } V_a + \mu \cdot V_g(V_{g0}, V_{g1}, \theta) > 0 \\ I \leftarrow 0 \quad \text{otherwise} \\ \text{return } I \end{array} \right.$$

Equation 6.31

Compute the DC current and the first harmonic of the RF current as a function of drive level using Fourier analysis.

$$I_b(V_{g0},V_{g1}) := \frac{1}{2\cdot\pi}\cdot\int_0^{2\cdot\pi} I(V_{g0},V_{g1},\theta)\,d\theta \qquad I_1(V_{g0},V_{g1}) := \frac{1}{\pi}\cdot\int_0^{2\cdot\pi} I(V_{g0},V_{g1},\theta)\cdot\cos(\theta)\,d\theta$$

The input impedance is

$$R_{in}(V_{g0},V_{g1}) := \begin{vmatrix} \dfrac{V_{g1}}{I_1(V_{g0},V_{g1})} & \text{if } V_{g1} \neq 0 \\ R_s & \text{otherwise} \end{vmatrix}$$

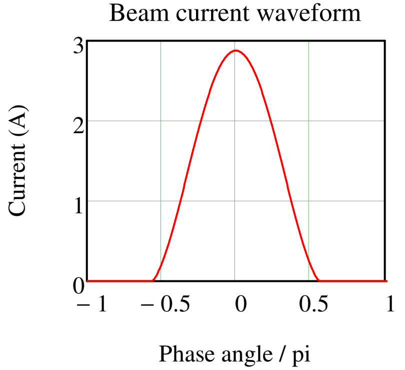
Equation 12.42

The input power measured using a directional coupler in the input line with source impedance R_s is

$$P_{in}(V_{g0},V_{g1}) := \frac{1}{8}\cdot\left(\frac{R_{in}(V_{g0},V_{g1}) + R_s}{R_{in}(V_{g0},V_{g1})}\right)^2\cdot\frac{{V_{g1}}^2}{R_s}$$

Equation 12.43

Display the beam current waveform with RF grid voltage V_{RF} $\quad V_{RF} := 300\cdot V \quad P_{in}(V_{g0},V_{RF}) = 209\,W$



Plotting range for RF grid voltage $\quad V_{g1} := 0\cdot V, 10\cdot V .. 350\cdot V$

The values of μ, I_0 and pwr (the exponent for space-charge limited flow) are adjusted to fit the model to the data

$\mu \equiv 150$ $\quad I_0 \equiv 0.9 \cdot A$ $\quad pwr \equiv 1.5$

The source impedance is adjusted to match it to the input impedance

$R_s \equiv 220 \cdot \Omega$

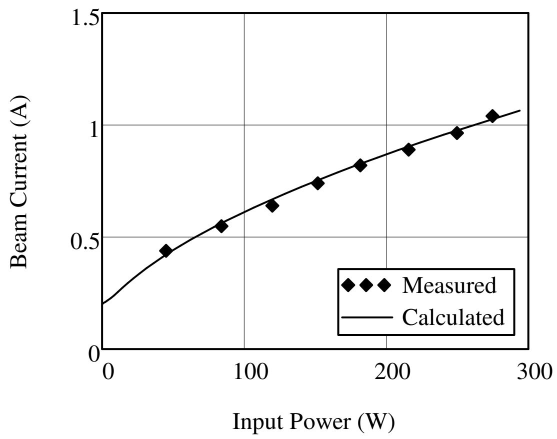
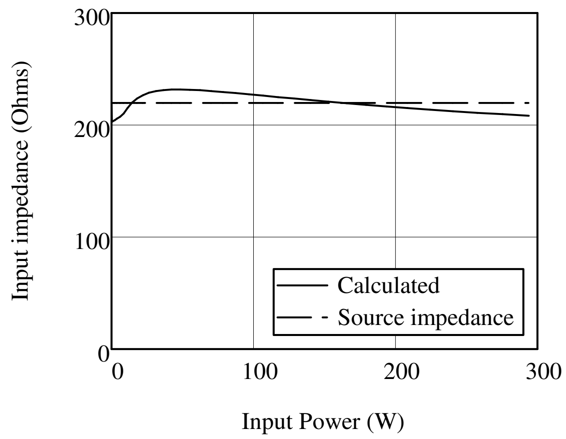


Figure 12.21



Cut-off voltage

$$\frac{-V_a}{\mu} = -167\,V$$

Quiesecent current

$$I_b(V_{g0}, 0) = 203 \cdot mA$$

Simple model of the output gap

This section can be collapsed to allow the input data and the results to be seen on the same screen

Calculate the DC beam velocity (u_0).

Define the charge/mass ratio and the rest energy (eV) of the electron

$\eta := 1.759 \cdot 10^{11} \cdot \frac{C}{kg}$ $\qquad V_R := 511 \cdot kV$

$$V_0 := \left| \begin{array}{l} V_0 \leftarrow V_a \\ \text{for } n \in 0..3 \\ \quad \left| \begin{array}{l} u_n \leftarrow c \cdot \left[1 - \dfrac{1}{\left(1 + \dfrac{V_n}{V_R}\right)^2} \right]^{0.5} \\ V_{n+1} \leftarrow V_0 - \dfrac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left(\dfrac{1}{2} - \ln\left(\dfrac{b}{a}\right) \right) \end{array} \right. \\ \text{return } V_{n+1} \end{array} \right.$$

Equation 1.4

Equation 7.8

$V_0 = 24.8 \cdot kV$

$$u_0 := c \cdot \left[1 - \frac{1}{\left(1 + \dfrac{V_0}{V_R}\right)^2} \right]^{0.5}$$

Equation 1.4

$u_0 = 9.018 \times 10^7 \dfrac{m}{s}$

Define the gap coupling factor as a function of electron velocity

$\omega := 2 \cdot \pi \cdot f$ $\qquad \beta_e := \dfrac{\omega}{u_0}$ $\qquad \beta e(u) := \dfrac{\omega}{u}$

$$M(u) := \left(\frac{\sin(0.5 \cdot \beta e(u) \cdot gap)}{0.5 \cdot \beta e(u) \cdot gap} \right) \cdot \left(\frac{2 \cdot I1(\beta e(u) \cdot b)}{\beta e(u) \cdot b \cdot I0(\beta e(u) \cdot a)} \right)$$

Equations 11.36 and 11.38

Use an iterative calculation to find mutually consistent values of the effective gap coupling factor and the exit velocity (u_s)

A piecewise constant approximation to the velocity is used $M_{eff} = \dfrac{M(u_0) + M(u_s)}{2}$

A damping term has been added to ensure fast convergence.

$$
\begin{aligned}
u_s(V_{g0}, V_{g1}) := \; & us_0 \leftarrow u_0 \cdot 0.5 \\
& Mu0 \leftarrow M(u_0) \\
& \text{for } i \in 1..100 \\
& \quad Mu \leftarrow 0.5 \cdot \left(Mu0 + M\left(us_{i-1}\right)\right) \\
& \quad us_i \leftarrow c \cdot \left[1 - \frac{1}{\left[1 + \dfrac{\eta \cdot \left(V_a - Mu^2 \cdot I_1(V_{g0}, V_{g1}) \cdot R_L\right)}{c^2}\right]^2}\right]^{0.5} \\
& \quad us_i \leftarrow 0.5 \cdot \left(us_i + us_{i-1}\right) \\
& \quad \text{break if } \left|\frac{us_i - us_{i-1}}{u_0}\right| < 10^{-6} \\
& us_i \leftarrow 0 \text{ if } i = 100 \\
& \text{return } us_i
\end{aligned}
$$

Equation 11.168

Calculate the ouput r.f. power

$$P_{out}(V_{g0},V_{g1}) := \frac{1}{2}\cdot\left[I_1(V_{g0},V_{g1})\cdot\left(\frac{M(u_0)+M(u_s(V_{g0},V_{g1}))}{2}\right)\right]^2\cdot R_L$$

$$Gain(V_{g0},V_{g1}) := 10\cdot\log\left(\frac{P_{out}(V_{g0},V_{g1})}{P_{in}(V_{g0},V_{g1})}\right)$$

$$\eta_e(V_{g0},V_{g1}) := \frac{P_{out}(V_{g0},V_{g1})}{I_b(V_{g0},V_{g1})\cdot V_a}$$

Calculate the normalised spent beam distribution

$$V_{sn}(V_{g0},V_{g1},\theta) := \left| \begin{array}{l} us0 \leftarrow u_s(V_{g0},V_{g1}) \\ V_{eff} \leftarrow \left(M\left(\frac{u_0+us0}{2}\right)\right)^2\cdot I_1(V_{g0},V_{g1})\cdot R_L \\ V_s(\theta) \leftarrow 1-\frac{V_{eff}}{V_a}\cdot\cos(\theta) \\ \text{return } V_s(\theta) \end{array}\right.$$

$$I_{sn}(V_{g0},V_{g1},\theta) := \left| \begin{array}{l} I1(\theta) \leftarrow I(V_{g0},V_{g1},\theta) \\ Ib \leftarrow I_b(V_{g0},V_{g1}) \\ I_s(\theta) \leftarrow \frac{-1}{\pi\cdot Ib}\cdot\int_{\pi}^{\theta} I1(\theta)\,d\theta \\ \text{return } I_s(\theta) \end{array}\right.$$

Calculate the ouput r.f. power and adjust the load resistance (R_L) to fit the data.

$R_L \equiv 28\cdot k\Omega$

Additional data sets calculated using the disk model below have also been plotted for comparison.

Note: Small changes to the model have made small differences between the figures below and those in the book. Note that the load resistances calculated by the two models are not identical.

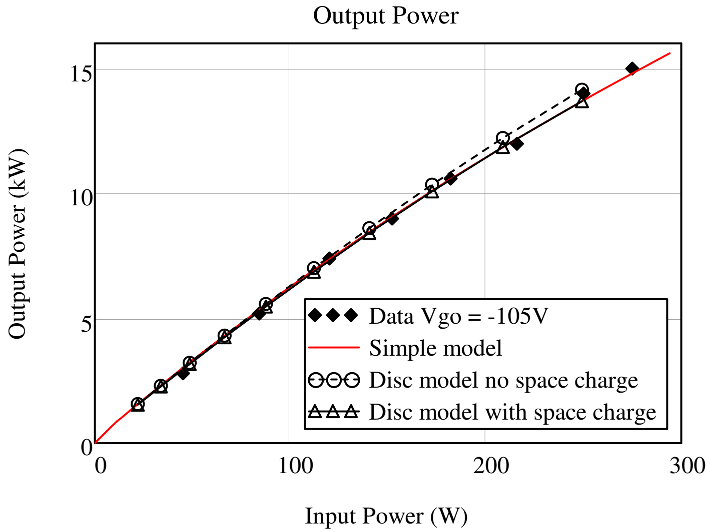
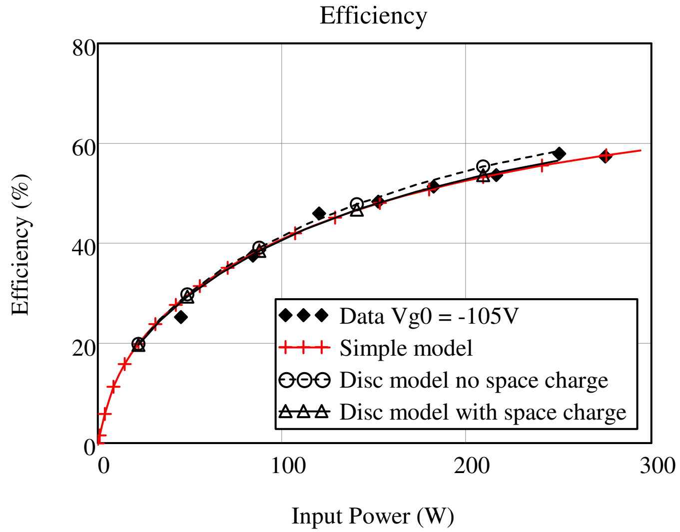


Figure 12.22



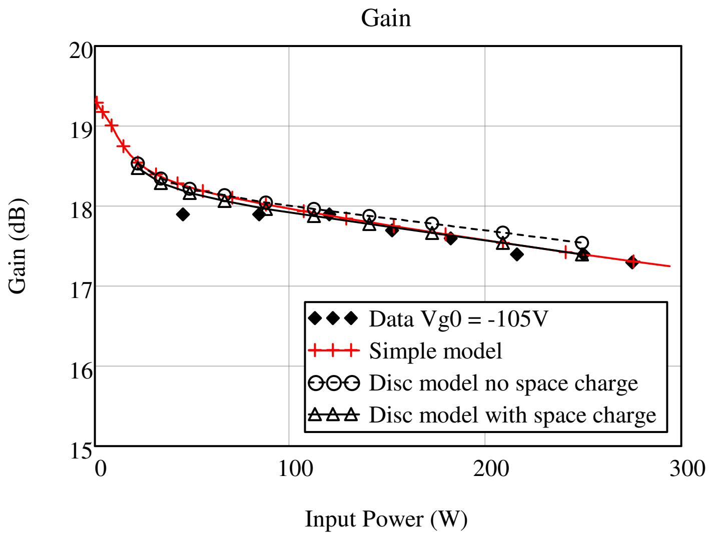
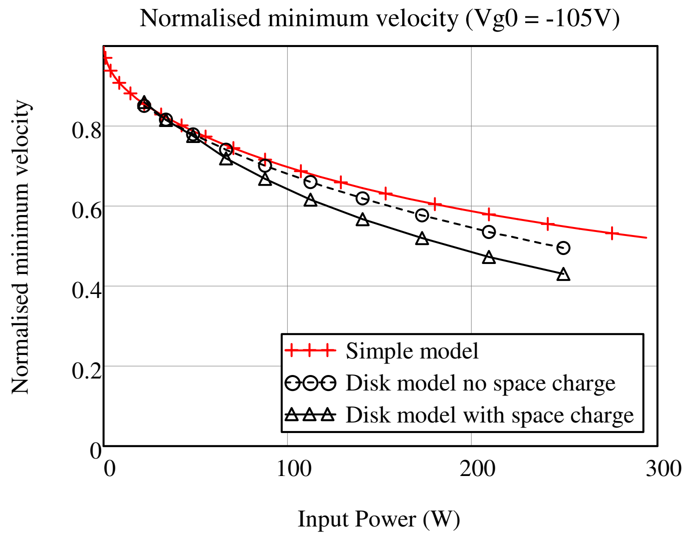
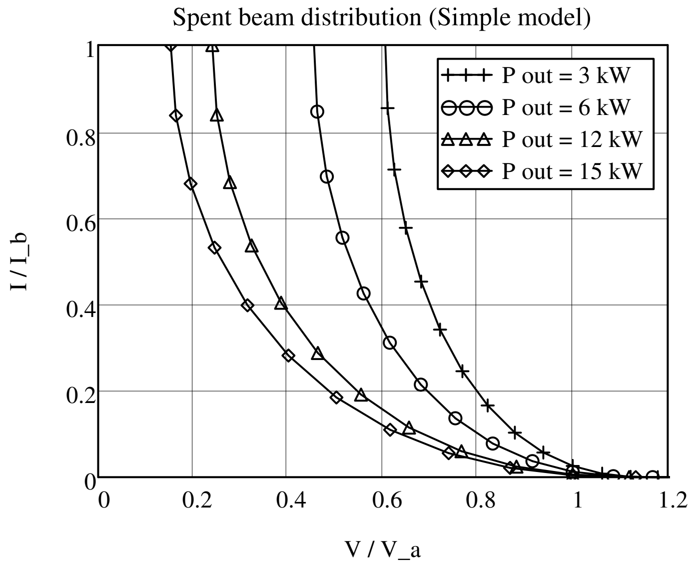


Figure 12.22



Plotting range $\theta1 := 0, 0.05\cdot\pi .. \pi$



$P_{out}(V_{g0}, 342\cdot V) = 15.0\cdot kW$

$P_{out}(V_{g0}, 302\cdot V) = 12.0\cdot kW$

$P_{out}(V_{g0}, 208\cdot V) = 6.0\cdot kW$

$P_{out}(V_{g0}, 145\cdot V) = 3.0\cdot kW$

Disk model of the output gap

The bunch is represented by a set of rigid disks of equal dimensions and charges whose initial positions are chosen to match the current profile defined above. The motion of the disks is tracked through the output gap with time as the independent variable using a Rung-Kutta integration. Dimensionless variables $\theta = \beta_e \cdot z$ and $\phi = \omega \cdot t$ are used.The model requires a single value of the RF grid voltage and can be run with and without space-charge. The output gap voltage is assumed to be the same as in the simple model for the same RF input power. The initial position of the bunch should be far enough from the gap to ensure that all electrons start outside the field of the gap. The final time should be large enough to ensure that all electrons have left the gap field. The phase of the gap voltage is chosen so that an electron at the bunch centre would cross the gap centre at the maximum retarding field if it travelled with constant velocity u_0. The phase of the gap field can be altered but should normally be set to zero.

Good results are normally obtained with the following parameters: ND = 24, nmax = 30, θ_0 = -2π, ϕ_f = 4π. When space charge is included it is found that the results depend on the choice of the drift length because the bunch starts to spread so that there is a spread of velocities as it enters the gap. The bunch starting position must be before the start of the drift region.

Number of discs	$ND := 50$	Number of integration steps	$nmax := 50$	Phase of gap field	$\Phi_0 \equiv 0 \cdot deg$
RF grid voltage	$V_{g1} := 300 \cdot V$	Space charge force = 1 for ON, = 0 for OFF	$SCF := 0$		
Bunch centre starting position and initial and final times	$\theta_0 := -4 \cdot \pi$	$\phi_0 := \theta_0$	$\phi_f := 4 \cdot \pi$		

The detailed calculations can be hidden to allow the data and results to be viewed on the screen simultaneously

This section can be collapsed to allow the input data and the results to be seen on the same screen

The Bunch is modelled as ND rigid discs. The motions of the electrons at the disc centres are followed. We define their initial positions and velocities using the disc thickness ΔL. As the Mathcad ODE solver rkfixed does not accept variables with dimensions the dimensionless variables: q = b_ez and f = wt are used.

Calculate the bunch half length

$$\theta_b := \left| \begin{array}{l} \theta \leftarrow \left| \begin{array}{ll} \operatorname{acos}\left(\dfrac{-V_{g0} - \dfrac{V_a}{\mu}}{V_{g1}}\right) & \text{if } \left|-V_{g0} - \dfrac{V_a}{\mu}\right| \le V_{g1} \\ \pi & \text{otherwise} \end{array} \right. \\ \theta \end{array} \right.$$

$$\frac{\theta_b}{\pi} = 0.566$$

Calculate the normalised disk thickness (q_d)

$$\theta_d := \frac{2 \cdot \theta_b}{ND}$$

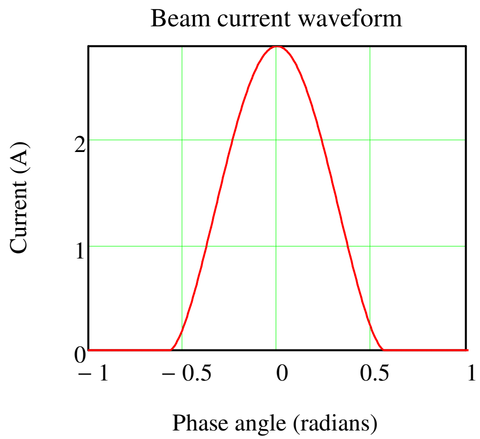
Calculate disk charge (equal for all the disks)

$$Q := \left| \begin{array}{l} \text{for } j \in 0..ND-1 \\ \quad Q_j \leftarrow \dfrac{2 \cdot \pi \, I_b(V_{g0}, V_{g1})}{\omega \cdot ND} \\ \text{return } Q \end{array} \right.$$

Define normalised disk starting positions and velocities relative to the gap centre for the given waveform.

$$f1(\alpha) := \frac{\displaystyle\int_{-\pi}^{\alpha} I(V_{g0}, V_{g1}, \theta)\, d\theta}{\displaystyle\int_{-\pi}^{\pi} I(V_{g0}, V_{g1}, \theta)\, d\theta}$$

$$\theta := \left| \begin{array}{l} \alpha \leftarrow 0 \\ \text{for } j \in 0, 2..(2 \cdot ND - 2) \\ \quad \left| \begin{array}{l} \theta_j \leftarrow \operatorname{root}\left(f1(\alpha) - \dfrac{j+1}{2ND}, \alpha\right) + \theta_0 \\ \theta_{j+1} \leftarrow 1 \end{array} \right. \\ \text{for } j \in 0, 2..(2 \cdot ND - 2) \\ \quad \theta_j \leftarrow \theta_j \\ \theta \end{array} \right.$$



Check the D.C. beam current calculated from the disks is the same as that previously computed.

$$f \cdot Q_0 \cdot ND = 0.889\,A \qquad I_b(V_{g0}, V_{g1}) = 0.889\,A$$

Compute the space-charge lookup function ES(q)

Define the first ten zeros of $J_0(z)$. Note that the fields of adjacent bunches have been omitted and this, therefore, may overestimate the space-charge force. However, the field at the next bunch is effectively zero if the bunch length is less than half a wavelength. The space-charge field is calculated at intervals from which an interpolated function is derived using equations from:
Hechtel, J. R. (1970). "The effect of potential beam energy on the performance of linear beam devices."
IEEE Transactions on Electron Devices **17**(11): 999-1009.

$$\mu B := \frac{1}{a}\cdot(2.405\ \ 5.520\ \ 8.654\ \ 11.791\ \ 14.931\ \ 18.071\ \ 21.212\ \ 24.352\ \ 27.494\ \ 30.635)^T \qquad \Delta L := \frac{\theta_d}{\beta_e} \qquad \rho_0 := \frac{1}{\pi\cdot b^2\cdot\Delta L}$$

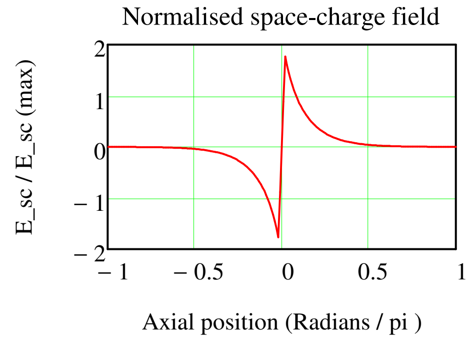
ρ_0 is calculated for a disc charge of 1C. Thus the electric field must be multiplied by the charge of the source disc.

$$\begin{aligned}
ESn := \;&\text{for } n \in 0..100\\
&\quad \theta_n \leftarrow 0.02\cdot n\cdot\pi\\
&\quad z_n \leftarrow \frac{\theta_n}{\beta_e}\\
&\quad ES_n \leftarrow \left(\frac{4\cdot\rho_0}{\varepsilon_0}\right)\cdot\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\exp(-\mu B_m\cdot z_n)\cdot\sinh\left(\frac{\mu B_m\cdot\Delta L}{2}\right)\right] \text{ if } \theta_n \ge 0.5\cdot\theta_d\\
&\quad ES_n \leftarrow \left(\frac{4\cdot\rho_0}{\varepsilon_0}\right)\cdot\sum_{m=0}^{9}\left[\frac{1}{\mu B_m}\cdot\left(\frac{J1(\mu B_m\cdot b)}{\mu B_m\cdot a\cdot J1(\mu B_m\cdot a)}\right)^2\cdot\left(\exp\left(-\mu B_m\cdot\frac{\Delta L}{2}\right)\cdot\sinh(\mu B_m\cdot z_n)\right)\right] \text{ otherwise}\\
&ES
\end{aligned}$$

$$\begin{aligned}
\theta n := \;&\text{for } n \in 0..100\\
&\quad \theta_n \leftarrow 0.02\cdot n\cdot\pi\\
&\theta
\end{aligned}$$

$$ES(\theta) := \text{sign}(\theta)\cdot\text{linterp}(\theta n, ESn, |\theta|)$$

$$\theta 1 := (-\pi), (-0.99\cdot\pi)..\pi$$



The Interaction Field on the axis is found from the Fourier Transform of the field in the gap (assumed to be constant). The average of the field over the beam is used. Linear interpolation on the values calculated at regular intervals is used to provide a fast look-up function.

$$V_{gap} := I_1(V_{g0}, V_{g1}) \cdot M\left(\frac{u_0 + u_s(V_{g0}, V_{g1})}{2}\right) \cdot R_L \qquad E_0 := \frac{V_{gap}}{gap}$$

$$En := \left| \begin{array}{l} \text{for } n \in 0..100 \\ \quad \left| \begin{array}{l} \theta_n \leftarrow 0.02 \cdot n \cdot \pi \\ E_n \leftarrow \frac{-V_{gap}}{\pi} \cdot \displaystyle\int_0^{\frac{20 \cdot \pi}{gap}} \frac{2 \cdot I1(\beta \cdot b)}{(\beta \cdot b) \cdot I0(\beta \cdot a)} \cdot \left(\frac{\sin(0.5 \cdot \beta \cdot gap)}{0.5 \cdot \beta \cdot gap}\right) \cdot \cos\left(\frac{\beta}{\beta_e} \cdot \theta_n\right) d\beta \end{array} \right. \\ E \end{array} \right.$$

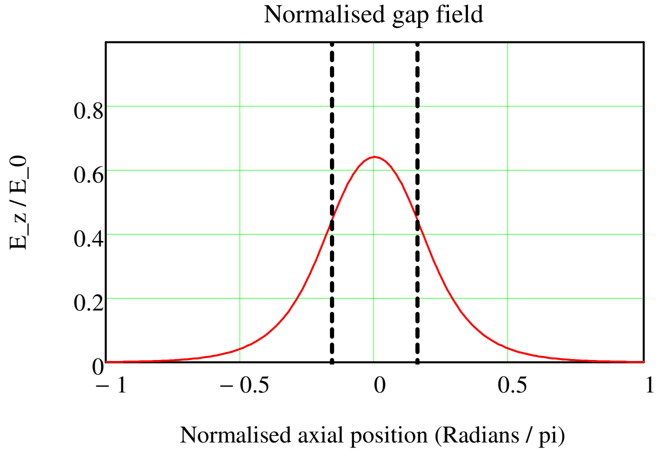
$$E_z(\theta) := \text{linterp}(\theta n, En, |\theta|)$$

CHECK

$$\frac{-1}{\beta_e \cdot V_{gap}} \cdot \int_{-2\pi}^{2\pi} E_z(\theta) \cdot \cos(\theta)\, d\theta = 0.792$$

$$M(u_0) = 0.792$$

$$\theta_{gap} := 0.5 \cdot \beta_e \cdot gap = 0.498$$



Set space-charge field to zero until the disc enters the field of the gap

$$SC(\theta) := \left| \begin{array}{ll} SCF & \text{if } \theta > -\pi \\ 0 & \text{otherwise} \end{array} \right.$$

The Coefficients of the Differential Equations for the motions of the electrons are defined.

The rows represent, in order, the position in radians and the normalised velocity of the electrons.

Definitions of normalised variables

$$\phi = \omega \cdot t \qquad \theta = \beta_e \cdot z \qquad \theta' = \frac{v}{u_0}$$

$$\frac{d}{dt} z = v \qquad \frac{d}{d\phi}\theta = \frac{v}{u_0}$$

$$\frac{d}{dt} v = \eta \cdot E \qquad \frac{d}{d\phi}\frac{v}{u_0} = \frac{\eta \cdot E}{\omega \cdot u_0}$$

$$D(\phi,\theta) := \left| \begin{array}{l} \text{for } j \in 0,2..2\cdot(ND-1) \\ \quad \left| \begin{array}{l} D_j \leftarrow \theta_{j+1} \\ D_{j+1} \leftarrow \dfrac{\eta}{\omega \cdot u_0} \cdot \left[1 - \left(\dfrac{u_0}{c}\cdot \theta_{j+1}\right)^2\right]^{1.5} \cdot \left[E_z(\theta_j)\cdot \cos(\phi + \Phi_0) + SC(\theta_j)\cdot \displaystyle\sum_{i=0}^{ND-1} \left(Q_i \cdot ES(\theta_j - \theta_{2\cdot i})\right)\right] \end{array} \right. \\ D \end{array} \right.$$

The Equations are Solved using a Runge-Kutta method with *nmax* time steps starting from ϕ_0 which is defined in such a way that the centre electron would cross the gap centre at f = 0 if it travelled with a constant velocity u_0. The final time is f_f

$$n := 0..nmax \qquad Z := \text{rkfixed}(\theta, \phi_0, \phi_f, nmax, D) \qquad \Delta\phi := \frac{\phi_f - \phi_0}{nmax}$$

The results are in a single table (Z) in which the first column (0) is the time and the other columns (1-12) are the positions and velocities of the electrons in the same order as before at each value of n.

Extract the vector of phase, the matrices containing the normalised positions and velocities of the disks and the vector of the final velocities of the electrons

$$\phi n := \left| \begin{array}{l} \text{for } n \in 0..nmax \\ \quad \phi_n \leftarrow Z_{n,0} \\ \phi \end{array} \right. \qquad \theta n := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad \theta_{n,j} \leftarrow Z_{n,2\cdot j+1} \\ \theta \end{array} \right. \qquad un := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad u_{n,j} \leftarrow Z_{n,2\cdot j+2} \\ u \end{array} \right. \qquad umax := \left| \begin{array}{l} \text{for } j \in 0..(ND-1) \\ \quad u_j \leftarrow Z_{nmax,2\cdot j+2} \\ u \end{array} \right.$$

The Kinetic Energy of the bunch is calculated using the relativistically correct formulae by summing the energies of the disks.
The effect of potential energy stored in the bunch is ignored.

$$KE := \left| \begin{array}{l} \text{for } n \in 0..nmax \\ \quad KE_n \leftarrow \sum_{j=0}^{ND-1} \left[\frac{c^2 Q_j}{\eta} \cdot \left[\frac{1}{\sqrt{1 - \frac{(un_{n,j} \cdot u_0)^2}{c^2}}} - 1 \right] \right] \\ KE \end{array} \right.$$

Check that the frequency times the intial KE is equal to the DC beam power

$$P_{DC} := V_0 \cdot I_b(V_{g0}, V_{g1}) = 22.07 \cdot kW \qquad KE_0 \cdot f = 22.07 \cdot kW$$

Calculate the RF output power from the change in KE

$$P_{RF} := (KE_0 - KE_{nmax}) \cdot f \qquad P_{RF} = 12.2\,kW$$

$$\eta e := \frac{P_{RF}}{P_{DC}} \qquad \eta e = 55.4\,\%$$

Calculate the spent beam distribution

$$Sbeam := \left| \begin{array}{l} S \leftarrow \text{reverse}\left[\text{csort}\left[\text{augment}\left[\frac{c^2}{\eta \cdot V_a} \cdot \left[\frac{1}{\sqrt{1 - \left(\frac{umax \cdot u_0}{c}\right)^2}} - 1\right], \frac{f \cdot Q}{I_b(V_{g0}, V_{g1})}\right], 0\right]\right] \\ \text{for } n \in 1..(ND - 1) \\ \quad S_{n,1} \leftarrow S_{(n-1),1} + S_{n,1} \\ \text{return } S \end{array} \right.$$

RESULTS

$SCF = 0$

$V_{g1} = 300\,V$ $P_{RF} = 12.22 \cdot kW$ $\min(umax) = 0.536$ $\eta e = 55.4\,\%$ $\dfrac{V_{gap}^{\;2}}{2 \cdot P_{RF}} = 30.1\,k\Omega$

For comparison the results of the simple calculation are

$P_{in}(V_{g0}, V_{g1}) = 208.8\,W$ $P_{out}(V_{g0}, V_{g1}) = 11.86 \cdot kW$ $\dfrac{u_s(V_{g0}, V_{g1})}{u_0} = 0.580$ $\eta_e(V_{g0}, V_{g1}) = 53.3 \cdot \%$ $R_L = 28.0 \cdot k\Omega$

The results of calculations using the disk model are copied into the vectors below for plotting above for comparison with the simple model

Without space-charge | **With space-charge**

Vg1 ≡	Pin ≡	PoutNSC ≡	uminNSC ≡	ηeNSC ≡	PoutSC ≡	uminSC ≡	ηeSC ≡
100	22.0	1.57	0.851	19.9	1.55	0.861	19.7
125	33.8	2.31	0.817	24.9	2.28	0.816	24.6
150	48.5	3.22	0.780	29.8	3.18	0.776	29.4
175	66.3	4.32	0.742	34.6	4.25	0.720	34.0
200	87.5	5.58	0.702	39.2	5.48	0.669	38.5
225	112.1	7.02	0.661	43.7	6.88	0.617	42.8
250	140.4	8.62	0.620	47.9	8.42	0.568	46.8
275	172.6	10.36	0.578	51.8	10.09	0.521	50.4
300	208.8	12.22	0.536	55.4	11.86	0.474	53.7
325	249.2	14.16	0.496	58.5	13.70	0.431	56.6

$$GainNSC \equiv 10 \cdot \log\left(\frac{PoutNSC \cdot 1000}{Pin}\right) \qquad GainSC \equiv 10 \cdot \log\left(\frac{PoutSC \cdot 1000}{Pin}\right)$$

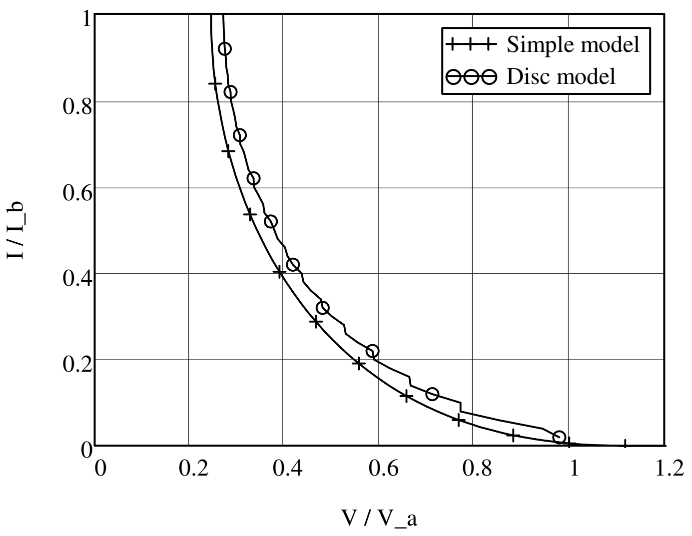
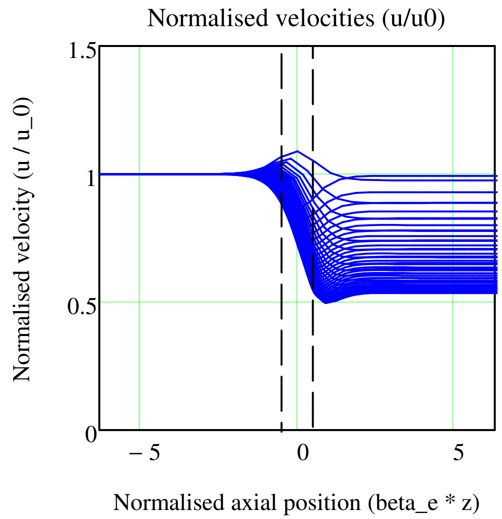
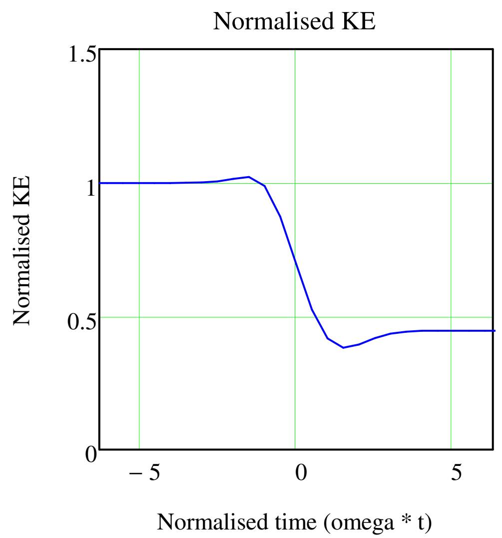


Figure 12.19(a)





$ND = 50$ $SCF = 0$ $V_{g1} = 300\,V$

$P_{in}(V_{g0}, V_{g1}) = 208.8\,W$ $P_{out}(V_{g0}, V_{g1}) = 11.9\cdot kW$