WS 17.1 Gyrotron model

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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Data from:
Thumm, M., S. Alberti, et al. (2007). "EU Megawatt-Class 140-GHz CW Gyrotron."
IEEE Transactions on Plasma Science, **35**(2): 143-153.

Define the electron beam and the magnetic field

$V_a := 81 \cdot kV$ $\quad I_0 := 40 \cdot A$ $\quad r_0 := 10.1 \cdot mm$ $\quad \alpha := 1.3$ $\quad$ Space charge $SC := 0$ $\quad$ Cyclotron harmonic $sn := 1$

Define the circular waveguide assuming propagation in the $TE_{m,p}$ mode

$a := 20.48 \cdot mm$ $\quad L_c := 14.5 \cdot mm$ $\quad Q_0 := 48650$ $\quad Q_L := 1150$ $\quad$ Azimuthal index $mn := 28$ $\quad$ Radial index $p := 8$

The section below can be collapsed to hide the details of the calculations

Define constants

Charge/mass ratio of the electron $\eta := 1.758820088 \cdot 10^{11} \cdot C \cdot kg^{-1}$ Electron rest energy $V_R := \frac{c^2}{\eta}$ $Z_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}}$

Find the electron velocity allowing for space-charge potential depression

$$\Delta V := \left| \begin{array}{l} \text{for } n \in 0, 5 \\ \quad \left| \begin{array}{l} Va_n \leftarrow V_a - \Delta V \\ u_n \leftarrow c \cdot \sqrt{1 - \frac{1}{\left(1 + \frac{Va_n}{V_R}\right)^2}} \\ \Delta V \leftarrow SC \cdot 60 \cdot \frac{I_0 \cdot c \cdot \sqrt{1 + \alpha^2}}{u_n} \cdot \ln\left(\frac{a}{r_0}\right) \cdot \frac{V}{A} \end{array} \right. \\ \Delta V \end{array} \right.$$

$$u_0 := c \cdot \sqrt{1 - \frac{1}{\left(1 + \frac{V_a - \Delta V}{V_R}\right)^2}} \qquad \frac{u_0}{c} = 0.505 \qquad \gamma_0 := \frac{1}{\sqrt{1 - \left(\frac{u_0}{c}\right)^2}} \qquad \gamma_0 = 1.159$$

$$\Delta V = 0 \cdot kV$$

$$u_z := \frac{u_0}{\sqrt{1 + \alpha^2}} \qquad u_\theta := \alpha \cdot u_z \qquad \frac{u_z}{c} = 0.308 \qquad \frac{u_\theta}{c} = 0.4$$

Define the roots of the Bessel function for the cavity modes

$Jn1(n, x) := \frac{d}{dx} Jn(n, x)$ Define the first derivative of the Bessel function $J_n(x)$

Zeroes of the Bessel function $J'_{mn}(x)$ for the $TE_{mn,p}$ mode as a data table which can be populated as necessary using the equations below to search for the roots. Trace can be used on the graph to find approximate values of the roots.

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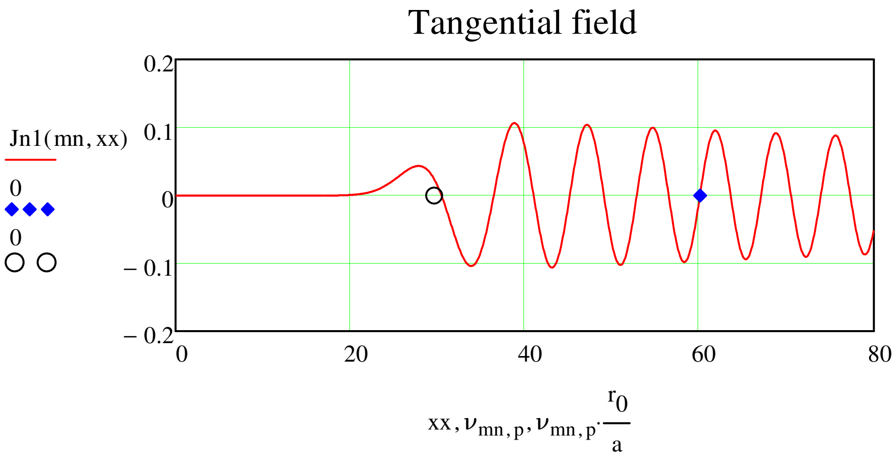
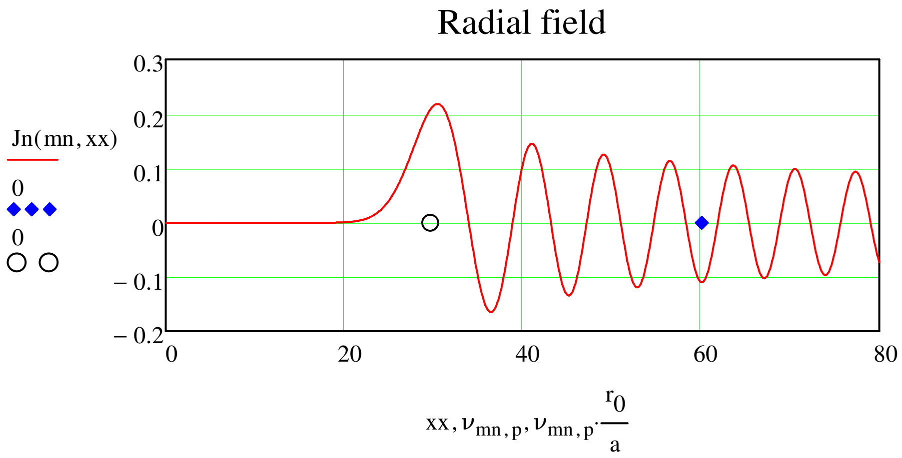
Approximate root $x := 60$ $\mathrm{root}(Jn1(mn,x),x) = 60.10147963373113000$ Equation 17.19

$\nu :=$

	0	1	2	3	4	5	6	7	8
0	0.000	3.832	7.016	10.173	13.324	16.471	19.616	22.760	25.904
1	0.000	1.841	5.331	8.536	11.706	14.864	18.016	21.164	24.311
2	0.000	3.054	6.706	9.969	13.170	16.348	19.513	22.672	25.826
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	5.318	9.282	12.682	15.964	19.196	22.401	25.590	28.768
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	7.501	11.735	15.268	18.637	21.932	25.184	28.410	...

$\nu_{mn,p} = 60.10147963$





Showing the positions of the cavity wall and the annular electron beam

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Cut-off frequency

$$\beta_{mp} := \frac{\nu_{mn,p}}{a} \qquad \omega_{mp} := c\cdot\beta_{mp} \qquad f_{mp} := \frac{\omega_{mp}}{2\cdot\pi}$$

The guiding centre radius can be chosen to coincide with the innermost peak of the field by enabling the shaded fields

$$xx := 0.5\cdot\left(\nu_{mn,1} + \nu_{mn,0}\right)$$

$$\text{Given} \qquad xx < \nu_{mn,1} \qquad xx > \nu_{mn,0} \qquad fJ(xx) := \left|\frac{d}{dxx}Jn(mn, xx)\right| \qquad y1 := \left|\text{Maximize}(fJ, xx)\right| \qquad y1 = \blacksquare$$

$$r1_0 := \frac{y1}{\beta_{mp}} \qquad \frac{r1_0}{a} = \blacksquare \qquad \beta_{mp}\cdot r_0 = \blacksquare \qquad r1_0 = \blacksquare\cdot mm$$

Calculate the cyclotron frequency

$$\omega_c := \frac{\eta\cdot B_z}{\gamma_0} \qquad f_c := \frac{\omega_c}{2\cdot\pi}$$

Calculate the Larmor radius and compare it with the guiding centre radius

$$r_L := \frac{u_\theta}{\omega_c} \qquad r_L = 0.14\cdot mm$$

Note: Theory assumes that the beam thickness is small compared with the guiding centre radius.

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Plot the cold dispersion diagram with parameters normalised to the cut-off frequency and propagation constant

$\beta g := -4\cdot\beta_{mp}, -3.9\cdot\beta_{mp} .. 4\beta_{mp}$

$\Omega_g(\beta_g) := \sqrt{1 + \frac{\beta_g{}^2}{\beta_{mp}{}^2}}$

$\Omega_b(\beta_g) := \frac{\beta_g}{\beta_{mp}} \cdot \frac{u_z}{c}$

$\Omega_c(\beta_g) := \frac{\beta_g}{\beta_{mp}} \cdot \frac{u_z}{c} + \frac{sn\cdot\omega_c}{\omega_{mp}}$

$\beta_{max} := 2$

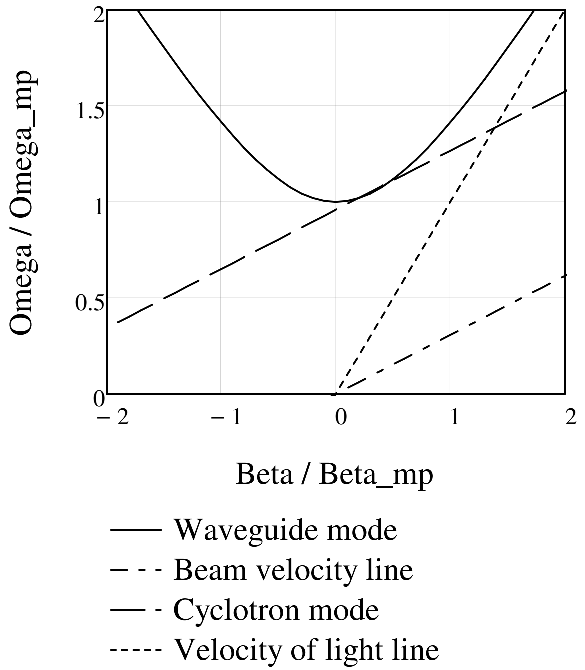

Figure 17.2 for the gyrotron interaction

$V_a - \Delta V = 81\cdot kV$

$f_{mp} = 140.0\cdot GHz$

$f_c = 134.3\cdot GHz$

$\frac{f_c}{f_{mp}} = 0.959$

$\frac{2\cdot r_L}{r_0} = 0.028$

$\sqrt{(\nu_{mn,p})^2 - mn^2} = 53.181$

$\frac{2\cdot r_L}{r_0} = 0.028$

The section below can be collapsed to hide the details of the calculations

Coupling constant

$$KK := \left| \begin{array}{l} z \leftarrow \beta_{mp} \cdot r_L \\ K \leftarrow \dfrac{\alpha \cdot u_z \cdot (\gamma_0 - 1)}{\pi \cdot \gamma_0 \cdot c} \cdot \dfrac{I_0 \cdot Z_0}{V_a} \cdot \left[\dfrac{Jn(sn, \beta_{mp} \cdot r_0) \cdot \dfrac{d}{d(z)} Jn(sn, z)}{\beta_{mp} \cdot a \cdot J0(\beta_{mp} \cdot a)} \right]^2 \end{array} \right.$$

Equation 17.26

$$KK = 1.438 \times 10^{-7}$$

Dispersion equation

$$\left(\beta_0^{\,2} - \beta^2 - \beta_{mp}^{\,2}\right) \cdot \left(\beta_0 - \beta \cdot \frac{u_z}{c} - sn \cdot \frac{\omega_c}{c}\right)^2 + KK \cdot \left(\beta_0^{\,2} - \beta^2\right) \cdot \beta_{mp}^{\,2} = 0$$

Equation 17.25

by collecting terms, yields

$$\left[\begin{array}{l} \left(-\dfrac{u_z^{\,2}}{c^2}\right) \cdot \beta^4 + \dfrac{2 \cdot u_z \cdot \left(\beta_0 - \dfrac{sn \cdot \omega_c}{c}\right)}{c} \cdot \beta^3 + \left[\dfrac{u_z^{\,2} \cdot \left(\beta_0^{\,2} - \beta_{mp}^{\,2}\right)}{c^2} - KK \cdot \beta_{mp}^{\,2} - \left(\beta_0 - \dfrac{1}{c} \cdot sn \cdot \omega_c\right)^2 \right] \cdot \beta^2 \ldots \\ + \left[-\dfrac{2 \cdot u_z \cdot \left(\beta_0^{\,2} - \beta_{mp}^{\,2}\right) \cdot \left(\beta_0 - \dfrac{sn \cdot \omega_c}{c}\right)}{c} \right] \cdot \beta + \left(\beta_0^{\,2} - \beta_{mp}^{\,2}\right) \cdot \left(\beta_0 - \dfrac{sn \cdot \omega_c}{c}\right)^2 + KK \cdot \beta_0^{\,2} \cdot \beta_{mp}^{\,2} \end{array} \right] = 0$$

Solve for the roots of the quartic equation in propagation constant (beta) for real omega

$$v(\beta_0) := \begin{bmatrix} \left[\left(\beta_0{}^2 - \beta_{mp}{}^2\right)\cdot\left(\beta_0 - \dfrac{sn\cdot\omega_c}{c}\right)^2 + KK\cdot\beta_0{}^2\cdot\beta_{mp}{}^2\right]\cdot m^4 \\ -\dfrac{2\cdot u_z\cdot\left(\beta_0{}^2 - \beta_{mp}{}^2\right)\cdot\left(\beta_0 - \dfrac{sn\cdot\omega_c}{c}\right)}{c}\cdot m^3 \\ \left[\dfrac{u_z{}^2\cdot\left(\beta_0{}^2 - \beta_{mp}{}^2\right)}{c^2} - KK\cdot\beta_{mp}{}^2 - \left(\beta_0 - \dfrac{1}{c}\cdot sn\cdot\omega_c\right)^2\right]\cdot m^2 \\ \dfrac{2\cdot u_z\cdot\left(\beta_0 - \dfrac{sn\cdot\omega_c}{c}\right)}{c}\cdot m \\ -\dfrac{u_z{}^2}{c^2} \end{bmatrix}$$

$\beta\beta(\beta_0) := \mathrm{sort}(\mathrm{Re}(\mathrm{polyroots}(v(\beta_0))))$ $\qquad\gamma\gamma(\beta_0) := \mathrm{sort}(\mathrm{Im}(\mathrm{polyroots}(v(\beta_0))))$

$x1 := 0.8, 0.801 .. 1.2$

$$\mathrm{polyroots}(v(1.001\cdot\beta_{mp})) = \begin{pmatrix} -132.797 \\ 137.532 \\ 365.239 \\ 422.331 \end{pmatrix}$$

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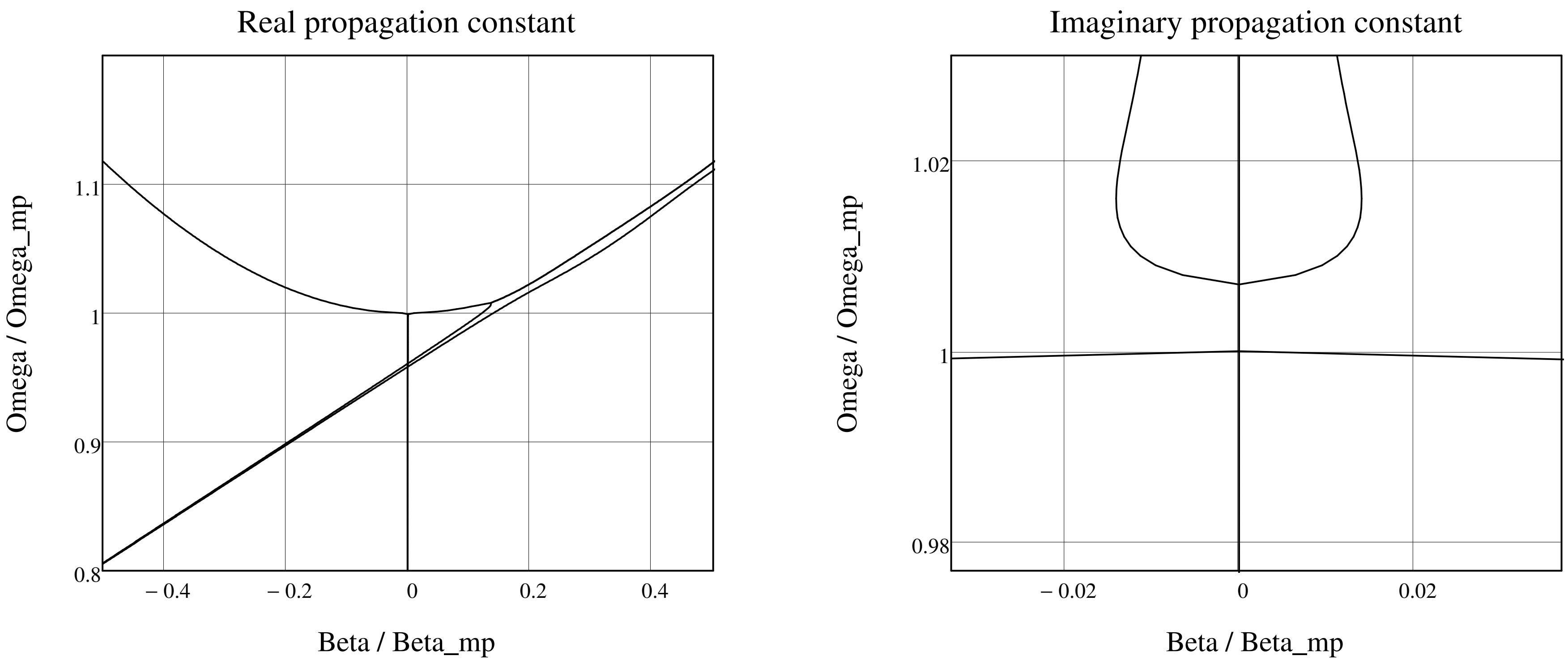

Figure 17.5

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Solve for the roots of the quartic equation in frequency (Beta_0 = omega / c) for real values of propagation constant (beta)

$$\left(\beta_0{}^2-\beta^2-\beta_{mp}{}^2\right)\cdot\left(\beta_0-\beta\cdot\frac{u_z}{c}-sn\cdot\frac{\omega_c}{c}\right)^2+KK\cdot\left(\beta_0{}^2-\beta^2\right)\cdot\beta_{mp}{}^2=0$$

by collecting terms, yields

$$\begin{bmatrix}\beta_0{}^4+\left(-\frac{2\cdot\beta\cdot u_z}{c}-\frac{2\cdot sn\cdot\omega_c}{c}\right)\cdot\beta_0{}^3+\left[KK\cdot\beta_{mp}{}^2-\beta^2-\beta_{mp}{}^2+\left(\frac{\beta\cdot u_z}{c}+\frac{sn\cdot\omega_c}{c}\right)^2\right]\cdot\beta_0{}^2\ \ldots \\ +\left(\frac{2\cdot\beta\cdot u_z}{c}+\frac{2\cdot sn\cdot\omega_c}{c}\right)\cdot\left(\beta^2+\beta_{mp}{}^2\right)\cdot\beta_0-\left[\left(\frac{\beta\cdot u_z}{c}+\frac{sn\cdot\omega_c}{c}\right)^2\cdot\left(\beta^2+\beta_{mp}{}^2\right)+KK\cdot\beta^2\cdot\beta_{mp}{}^2\right]\end{bmatrix}=0$$

Solve for the roots of the quartic equation

$$v2(\beta):=\begin{bmatrix}-\left[\left(\frac{\beta\cdot u_z}{c}+\frac{sn\cdot\omega_c}{c}\right)^2\cdot\left(\beta^2+\beta_{mp}{}^2\right)+KK\cdot\beta^2\cdot\beta_{mp}{}^2\right]\cdot m^4 \\ \left(\frac{2\cdot\beta\cdot u_z}{c}+\frac{2\cdot sn\cdot\omega_c}{c}\right)\cdot\left(\beta^2+\beta_{mp}{}^2\right)\cdot m^3 \\ \left[KK\cdot\beta_{mp}{}^2-\beta^2-\beta_{mp}{}^2+\left(\frac{\beta\cdot u_z}{c}+\frac{sn\cdot\omega_c}{c}\right)^2\right]\cdot m^2 \\ \left(-\frac{2\cdot\beta\cdot u_z}{c}-\frac{2\cdot sn\cdot\omega_c}{c}\right)\cdot m \\ 1\end{bmatrix}$$

$$\beta\beta0(\beta):=\mathrm{sort}(\mathrm{Re}(\mathrm{polyroots}(v2(\beta))))$$

$$\gamma\gamma0(\beta):=\mathrm{sort}(\mathrm{Im}(\mathrm{polyroots}(v2(\beta))))$$

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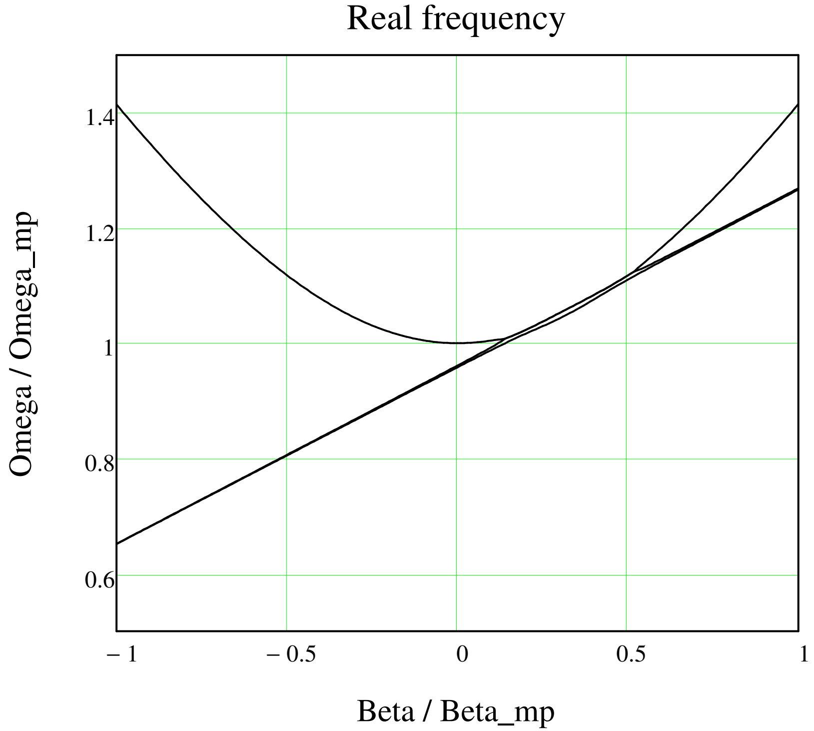
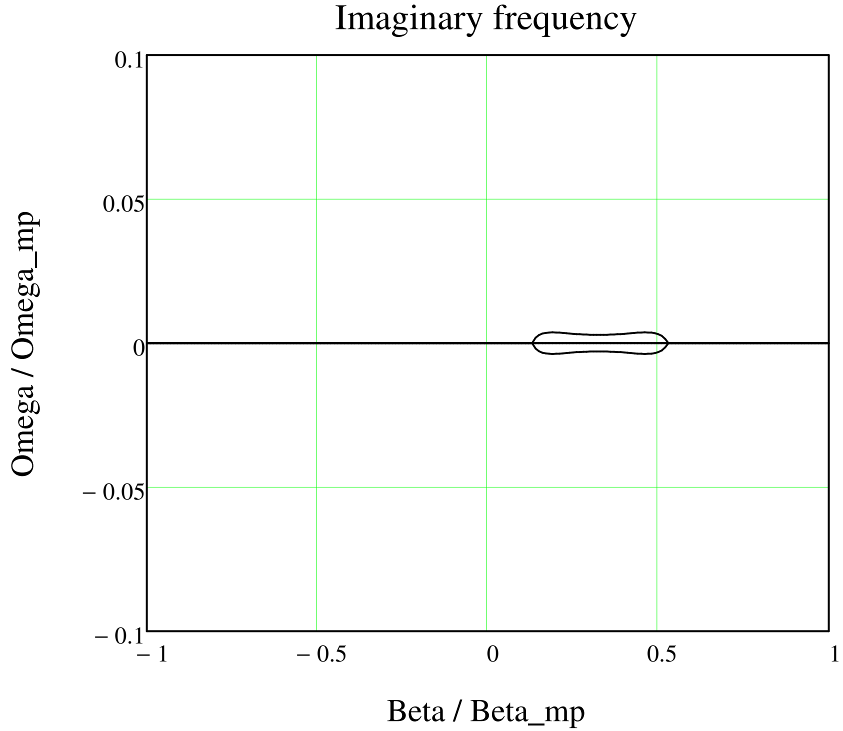
$x2 := -1, -0.99 .. 1$





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Non-linear model

Based on the equations in B. Danly and R. J. Temkin, "Generalized nonlinear harmonic gyrotron theory," *Physics of Fluids,* vol. 29, p. 561, 1986.

Frequency $f_0 := 139.8 \cdot GHz$

Rotation $Rot := -1$

The field parameter is adjusted to give the correct d.c. current

Number of electrons $n_e := 60$

Normalised field strength $F_a := 0.1032$

$B_z \equiv 5.56 \cdot T$

The section below can be collapsed to hide the details of the calculations

$\beta_{T0} := \frac{u_\theta}{c}$ $\quad \beta_{ll0} := \frac{u_z}{c}$ $\quad \frac{\omega_c}{\omega_{mp}} = 0.959$ $\quad \omega := 2 \cdot \pi \cdot f_0$

$\beta_{T0} = 0.4$ $\quad \beta_{ll0} = 0.308$ $\quad \lambda := \frac{2 \cdot \pi \cdot c}{\omega}$ $\quad \frac{L_c}{\lambda} = 6.762$

$w = \frac{\beta_{T0}^{\ 2}}{2} \cdot u$ $\quad p1_T(w) = \beta_{T0} \cdot \left[1 - \frac{1}{\beta_{T0}^{\ 2}} \cdot \left(2 \cdot w - w^2\right)\right]^{\frac{1}{2}}$

Equation 17.35

$p1_T(u) := \beta_{T0} \cdot \left[1 - \left[u - \left(\frac{\beta_{T0}}{2} \cdot u\right)^2\right]\right]^{\frac{1}{2}}$

$\mu := \pi \cdot \frac{\beta_{T0}^{\ 2}}{\beta_{ll0}} \cdot \frac{L_c}{\lambda}$

Equation 17.39

$f(\zeta) := \exp\left[-\left(\frac{2 \cdot \zeta}{\mu}\right)^2\right]$

Equation 17.45

$\delta_0 := 1 - \frac{sn \cdot \omega_c}{\omega}$

Equation 17.36

$\Delta := \frac{2 \cdot \delta_0}{\beta_{T0}^{\ 2}}$

Equation 17.40

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$$\Delta B(B_z) := \frac{2}{\beta_{T0}^{\,2}} \cdot \left(1 - \frac{sn \cdot \eta \cdot B_z}{\omega \cdot \gamma_0}\right)$$ $\boxed{\text{Equation 17.40}}$

Define initial energies and angular positions

$$\upsilon := \left| \begin{array}{l} \text{for } i \in 0, 2 .. 2(n_e - 1) \\ \quad \left| \begin{array}{l} \upsilon_i \leftarrow 0 \\ \upsilon_{i+1} \leftarrow \frac{\pi}{n_e} \cdot i \end{array} \right. \\ \text{return } \upsilon \end{array} \right.$$

Note: The working variable u: for subscripts n = 0, 2, 4, etc u represents the energy (u) and for subscripts n = 1, 3, 5 etc. u represents the angular position (theta).

$$\zeta_0 := \frac{\sqrt{3} \cdot \mu}{2} \qquad \zeta_0 = 9.571$$

Define the coefficients of the differential equations

$$D(\zeta, \upsilon) := \left| \begin{array}{l} \text{for } i \in 0, 2 .. (2 \cdot n_e - 2) \\ \quad \left| \begin{array}{l} D_i \leftarrow 2 \cdot \left[\frac{2^{sn} \cdot sn!}{sn^{sn} \cdot \beta_{T0}^{(sn-1)}} \right] \cdot F_a \cdot f(\zeta) \cdot \frac{p1_T(\upsilon_i)}{\beta_{T0}} \cdot Jn1(sn, sn \cdot p1_T(\upsilon_i)) \cdot \sin(\upsilon_{i+1}) \\ D_{i+1} \leftarrow \Delta - \upsilon_i - sn \cdot \left[\frac{2^{sn} \cdot sn!}{sn^{sn} \cdot \beta_{T0}^{(sn-1)}} \right] \cdot F_a \cdot f(\zeta) \cdot \frac{\beta_{T0} \cdot \left(1 - 0.5 \cdot \beta_{T0}^{\,2} \cdot \upsilon_i\right)}{p1_T(\upsilon_i)^2} \cdot Jn(sn, sn \cdot p1_T(\upsilon_i)) \cdot \cos(\upsilon_{i+1}) \end{array} \right. \\ D \end{array} \right.$$

$\boxed{\text{Equation 17.41}}$

$\boxed{\text{Equation 17.42}}$

Solve the simultaneous differential equations

Number of integration planes $nmax := 10$

$$Z := \mathrm{AdamsBDF}\left(\upsilon, -\zeta_0, \zeta_0, nmax, D\right)$$

The first column of this array contains the axial position, the odd numbered columns are the values of u and the even numbered columns the values of theta.

Unpack the array and calculate r1 = r / rL

$$\zeta := \left| \begin{array}{l} \text{for } j \in 0..nmax \\ \quad \zeta_j \leftarrow Z_{j,0} \\ \zeta \end{array} \right.$$

$$\upsilon 1 := \left| \begin{array}{l} \text{for } i \in 0,1..\left(n_e - 1\right) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad \upsilon_{n,i} \leftarrow Z_{n,2 \cdot i+1} \\ \upsilon \end{array} \right.$$

$$\theta := \left| \begin{array}{l} \text{for } i \in 0,1..\left(n_e - 1\right) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad \theta_{n,i} \leftarrow Z_{n,2 \cdot (i+1)} \\ \theta \end{array} \right.$$

$$r1 := \left| \begin{array}{l} \text{for } i \in 0,1..\left(n_e - 1\right) \\ \quad \text{for } n \in 0..nmax \\ \quad\quad \left| \begin{array}{l} \gamma \leftarrow \gamma_0 \cdot \left(1 - \dfrac{{\beta_{T0}}^2}{2} \cdot \upsilon 1_{n,i}\right) \\ r_{n,i} \leftarrow \sqrt{\dfrac{\left(1 - {\beta_{ll0}}^2\right) \cdot \gamma^2 - 1}{\left(1 - {\beta_{ll0}}^2\right) \cdot {\gamma_0}^2 - 1}} \end{array} \right. \\ r \end{array} \right.$$

Equation 17.49

Equation 17.48

Tangential and electronic efficiencies at the reference planes

$$\eta_T := \left| \begin{array}{l} \text{for } n \in 0..nmax \\ \quad \eta_n \leftarrow \dfrac{1}{n_e} \cdot \sum_{i=0}^{n_e - 1} \upsilon 1_{n,i} \\ \eta \end{array} \right.$$

$$z1 := \left| \begin{array}{l} \text{for } j \in 0..10 \\ \quad z1_j \leftarrow 0.1 \cdot j \cdot L_c \\ \dfrac{z1}{L_c} \end{array} \right.$$

$$\eta_e := \frac{{\beta_{T0}}^2}{2 \cdot \left(1 - \dfrac{1}{\gamma_0}\right)} \cdot \eta_T$$

Equation 17.50

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Define the reference current for calculation of the normalised current

$$I_R := \left[\frac{0.2385\cdot 10^{-3}}{A}\cdot\left(\frac{Q_L}{\gamma_0}\right)\cdot\beta_{T0}^{\,2\cdot(sn-3)}\cdot\frac{\lambda}{L_c}\cdot\left(\frac{sn^{sn}}{2^{sn}\cdot sn!}\right)^2\cdot\frac{Jn\left(mn+Rot\cdot sn,\nu_{mn,p}\cdot\frac{r_0}{a}\right)^2}{\left[\left(\nu_{mn,p}\right)^2-mn^2\right]\cdot Jn\left(mn,\nu_{mn,p}\right)^2}\right]^{-1}$$

Equation 17.53

$$I_n := \frac{F_a^{\,2}}{\eta_{T_{10}}}$$

Equation 17.51

$$I_a := I_n\cdot I_R$$

Equation 17.52

$$Q_E := \left(\frac{1}{Q_L}-\frac{1}{Q_0}\right)^{-1} \qquad \eta_{overall} := \eta_{e_{10}}\cdot\frac{Q_L}{Q_E}$$

Equation 17.54

$$P_{out} := \eta_{overall}\cdot V_a\cdot I_a$$

$$P_{out} = 1007\cdot kW$$

Find the starting current and the minimum starting current

$$x_s(\Delta,\mu) := \frac{\mu\cdot\Delta}{4}$$

Equation 17.57

$$I_{st}(\Delta,\mu) := \left(\frac{4}{\pi\cdot\mu^2}\right)\cdot\frac{\left(\exp\left(2\,x_s(\Delta,\mu)^2\right)\right)}{\mu\cdot x_s(\Delta,\mu)-sn}$$

Equation 17.56

$$x_m := \frac{1}{2}\cdot\left(\frac{sn}{\mu}+\sqrt{\frac{sn^2}{\mu^2}+1}\right)$$

$$I_{min} := \left(\frac{4}{\pi\cdot\mu^2}\right)\cdot\frac{\left(\exp\left(2x_m^{\,2}\right)\right)}{\mu\cdot x_m-sn}$$

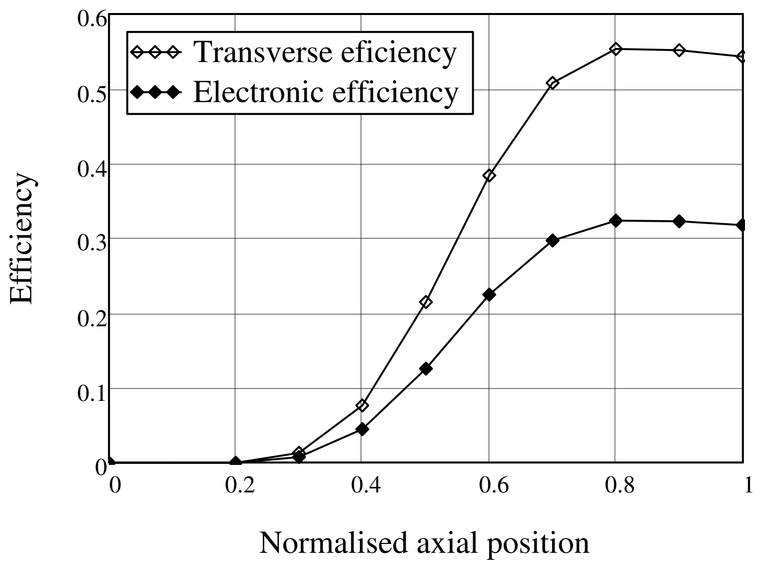

Figure 17.16

Normalised parameters

$F_a = 0.103$

$\Delta = 0.49$ $\mu = 11.1$ $I_n = 0.020$

$SC = 0$ $\alpha = 1.3$

Results

$V_a = 81.0 \cdot kV$ $I_0 = 40.0\,A$ $I_a = 40.0\,A$

$\eta_{T_{10}} = 54.4 \cdot \%$ $\eta_{e_{10}} = 31.8 \cdot \%$ $P_{out} = 1007 \cdot kW$

$I_{st}(\Delta, \mu) = 0.028$ $I_{st}(\Delta, \mu) \cdot I_R = 57.7\,A$ $I_{min} \cdot I_R = 7.7\,A$

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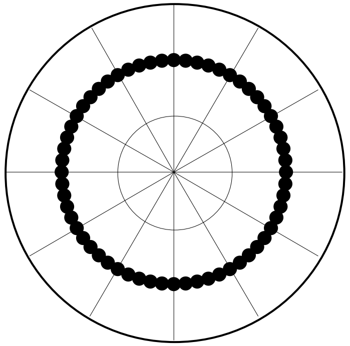
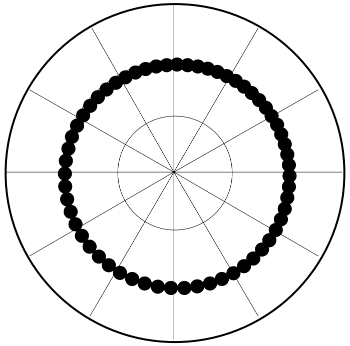
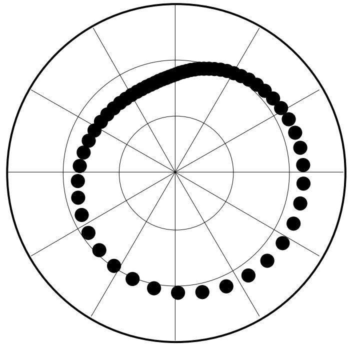
$n1 := 0 .. n_e - 1$



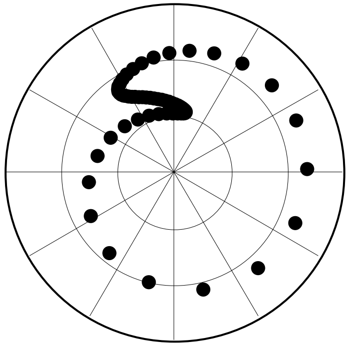
$z1_0 = 0$



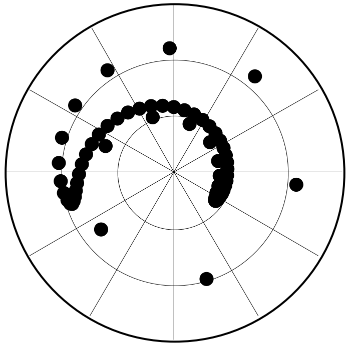
$z1_2 = 0.2$ $\quad \eta_{e_2} = 0.007 \cdot \%$



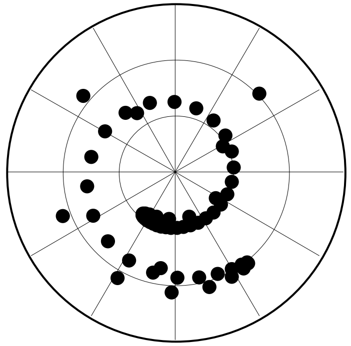
$z1_4 = 0.4$ $\quad \eta_{e_4} = 4.49 \cdot \%$



$z1_6 = 0.6$ $\quad \eta_{e_6} = 22.5 \cdot \%$



$z1_8 = 0.8$ $\quad \eta_{e_8} = 32.4 \cdot \%$



$z1_{10} = 1$ $\quad \eta_{e_{10}} = 31.8 \cdot \%$

Figure 17.15

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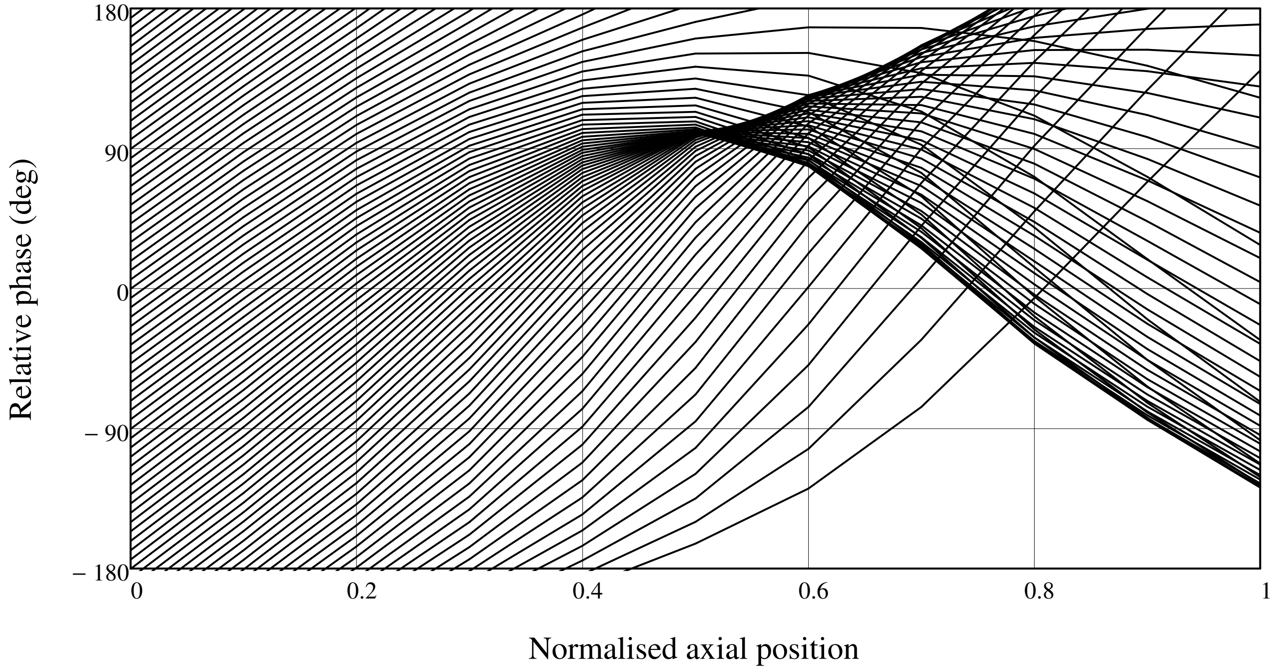

Figure 17.14

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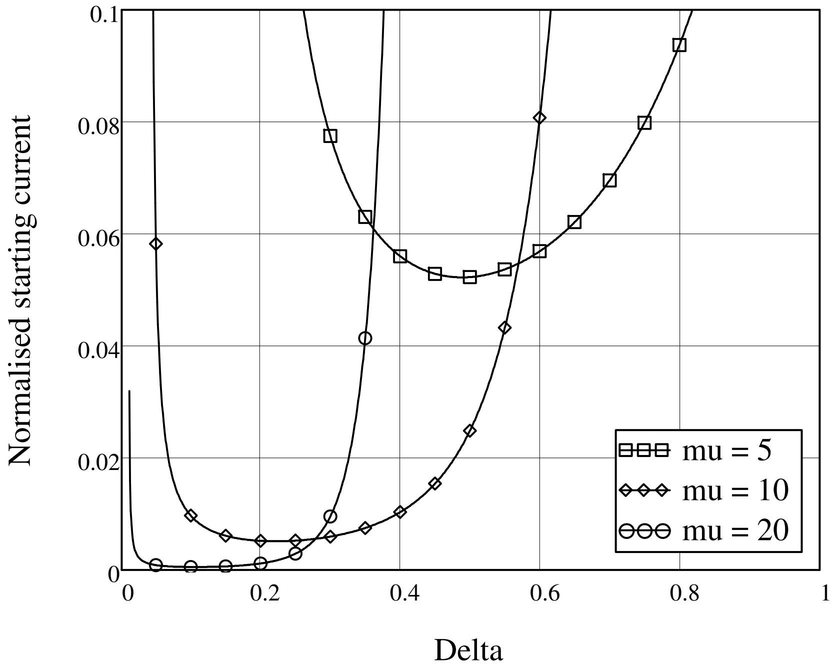

Figure 17.13

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