WS 3.2 Cavity resonators

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Section 3.3 Pill-box cavity resonators

Calculate the surface resistance Conductivity of copper $\sigma := 5.959\cdot 10^{7}\cdot S\cdot m^{-1}$

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu_0}{2\sigma}}$$ $$\delta s(\omega) := \sqrt{\frac{2}{\omega\cdot\sigma\cdot\mu_0}}$$ $$R_s(\omega) := \frac{1}{\sigma\cdot\delta s(\omega)}$$ Equation 3.50

Section 3.3.1 Surface roughness

Effect of r.m.s. surface roughness Δ

$$\frac{R_r}{R_s} = 1 + \frac{2}{\pi}\arctan\left[1.4\left(\frac{\Delta}{\delta}\right)^2\right]$$ $$R_{rs}(x) := 1 + \frac{2}{\pi}\cdot\mathrm{atan}\left(1.4\cdot x^2\right)$$ Equation 3.56

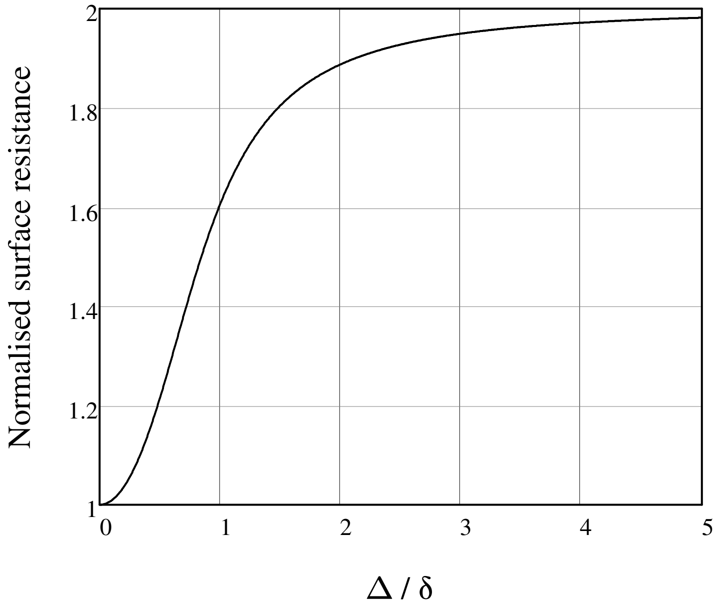

Figure 3.12

Pill-box cavities (TM_{010} mode)

Specify the cavity radius (a) and height (h)

$a := 40 \cdot mm$ $\quad h := 10 \cdot mm$

Find the first zero of the Bessel Function $J_0(z)$ from guess $z1 := 2$ $\quad ka := root(J0(z1), z1)$ $\quad ka = 2.4048$

Calculate the resonant frequency $\quad \omega_0 := ka \cdot \frac{c}{a}$ $\quad$ Equation 3.44 $\quad f_0 := \frac{\omega_0}{2 \cdot \pi}$ $\quad f_0 = 2.869 \cdot GHz$

Calculate the stored electric energy for a given field on the axis and the R/Q $\quad E_0 := 1000 \cdot \frac{V}{m}$

$$W_E := \varepsilon_0 \cdot \pi \cdot h \cdot a^2 \cdot E_0^{\,2} \cdot \frac{1}{ka^2} \cdot \int_0^{ka} J0(kr)^2 \cdot kr \, d(kr)$$

Equation 3.46 $\quad W_E = 5.997 \times 10^{-11} \, J$

$$R\_Q := \frac{h^2 \cdot E_0^{\,2}}{2 \cdot \omega_0 \cdot W_E}$$

Equation 3.14 $\quad R\_Q = 46.3 \, \Omega$

Calculate the unloaded Q

Specify r.m.s. surface roughness $\Delta := 0.1 \cdot \mu m$ $\quad$ Skin depth $\quad \delta s(\omega_0) = 1.217 \cdot \mu m$

Surface resistance $\quad R_r(\omega) := R_s(\omega) \cdot R_{rs}\left(\frac{\Delta}{\delta s(\omega)}\right)$ $\quad R_r(\omega_0) = 0.014 \, \Omega$

Unloaded Q $\quad Q := \frac{ka}{2 \cdot R_r(\omega_0)} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{h}{a + h}$ $\quad$ Equation 3.55 $\quad Q = 6533$

Shunt impedance $\quad R_c := R\_Q \cdot Q$ $\quad R_c = 302.2 \cdot k\Omega$

Section 3.4 Rectangular cavity resonators (TM_{110} mode)

Specify the transverse dimensions (a and b) and the height h

$a := 50 \cdot mm$ $b := 60 \cdot mm$ $h := 12 \cdot mm$

Calculate the resonant frequency

$$\beta_C := \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$ Equation 3.58 $\omega_0 := c \cdot \beta_C$ $f := \frac{\omega_0}{2 \cdot \pi}$ $f = 3.902 \cdot GHz$

Calculate the R/Q

$$W_E := \frac{\varepsilon_0}{8} \cdot a \cdot b \cdot h \cdot E_0^{\,2}$$ Equation 3.62 $W_E = 3.984 \times 10^{-11} J$

$$R\_Q := \frac{h^2 \cdot E_0^{\,2}}{2 \cdot \omega_0 \cdot W_E}$$ Equation 3.14 $R\_Q = 73.7\,\Omega$

Calculate the unloaded Q

Specify r.m.s. surface roughness $\Delta := 0.1 \cdot \mu m$ Skin depth $\delta s(\omega_0) = 1.044 \cdot \mu m$

Surface resistance $R_r(\omega) := R_s(\omega) \cdot R_{rs}\left(\frac{\Delta}{\delta s(\omega)}\right)$ $R_r(\omega_0) = 0.016\,\Omega$

Unloaded Q $$Q := \frac{\pi}{4 \cdot R_r(\omega_0)} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \left[\frac{2 \cdot h \cdot \left(a^2 + b^2\right)^{1.5}}{a \cdot b \cdot \left(a^2 + b^2\right) + 2 \cdot h \cdot \left(a^3 + b^3\right)}\right]$$ Equation 3.64 $Q = 7880$

Shunt impedance $R_c := R\_Q \cdot Q$ $R_c = 580.8 \cdot k\Omega$