

## WS 4.8 Space-Harmonic Slow-Wave Structure

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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R. G. Carter and S. K. Liu, "Method for Calculating the Properties of Coupled-Cavity Slow-Wave Structures from Their Dimensions," *IEE Proceedings-H Microwaves Antennas and Propagation*, vol. 133, pp. 330-334, Oct 1986.

Physical constants

$$\epsilon_0 = 8.854 \times 10^{-12} \cdot \frac{\text{F}}{\text{m}}$$

$$\mu_0 = 1.257 \times 10^{-6} \cdot \frac{\text{H}}{\text{m}}$$

$$c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$Z_0 := \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$Y_0 := \frac{1}{Z_0}$$

### Define the structure dimensions

Cavity dimensions

$$r_a := 3.25 \cdot \text{mm}$$

$$r_c := 4.5 \cdot \text{mm}$$

$$R_c := 20 \cdot \text{mm}$$

$$H_c := 9 \cdot \text{mm}$$

$$G_c := 3 \cdot \text{mm}$$

$$L_c := 12 \cdot \text{mm}$$

Slot dimensions

$$R_s := 12 \cdot \text{mm}$$

$$r_s := 5 \cdot \text{mm}$$

$$\alpha_1 := 77 \cdot \text{deg}$$

$$\alpha_2 := 100 \cdot \text{deg}$$

### Experimental data for structures with the dimensions given above

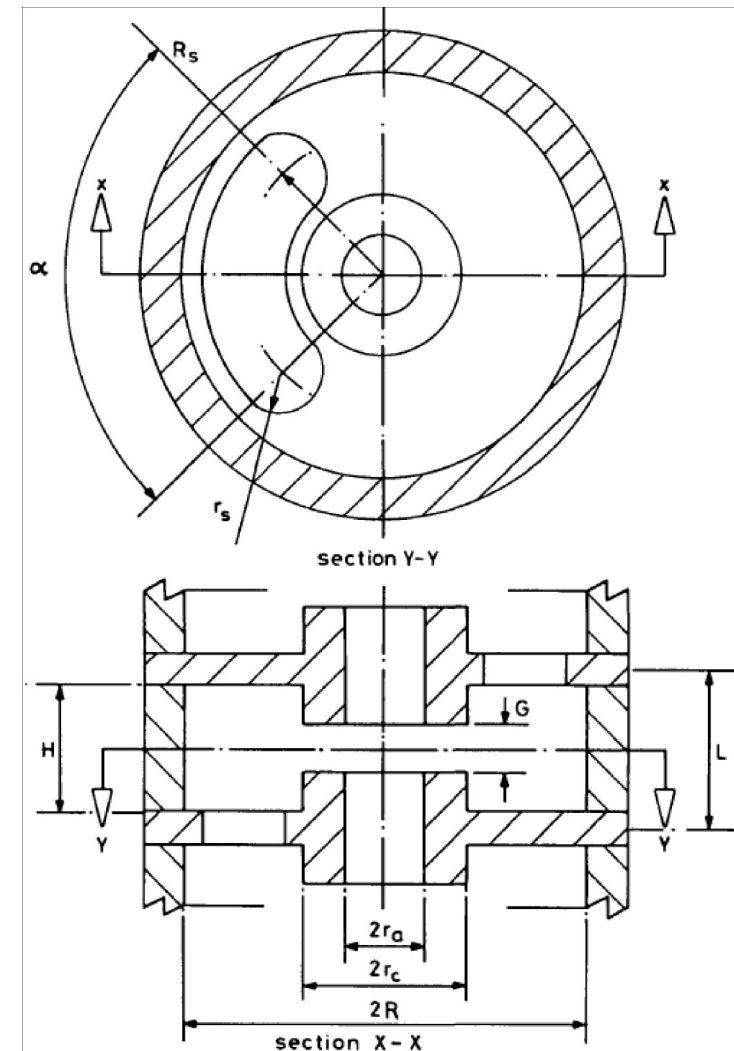
Phase shift per cavity /  $\pi$ ; Frequency (GHz); Total admittance (mS) for slot angles 77 deg and 100 deg

$\phi_{ex} :=$	$\begin{pmatrix} 1.000 \\ 1.167 \\ 1.333 \\ 1.500 \\ 1.667 \\ 1.833 \\ 2.000 \end{pmatrix}$	$f_{77} :=$	$\begin{pmatrix} 3.783 \\ 3.886 \\ 4.038 \\ 4.217 \\ 4.432 \\ 4.583 \\ 4.636 \end{pmatrix}$	$Y_{77} :=$	$\begin{pmatrix} 0 \\ 0.561 \\ 0.948 \\ 1.098 \\ 0.945 \\ 0.546 \\ 0 \end{pmatrix}$	$f_{100} :=$	$\begin{pmatrix} 3.468 \\ 3.568 \\ 3.731 \\ 3.998 \\ 4.283 \\ 4.519 \\ 4.607 \end{pmatrix}$	$Y_{100} :=$	$\begin{pmatrix} 0 \\ 0.781 \\ 1.429 \\ 1.653 \\ 1.449 \\ 0.847 \\ 0 \end{pmatrix}$
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### Import cavity parameters from the method of moments model of the cavity WS 3.3

$$f_C := 4.638 \cdot \text{GHz}$$

$$R_{QC} := 76.9 \cdot \Omega$$



Calculate the slot width, the effective slot length and resonant frequency and the effective slot thickness

$$w_s := 2 \cdot r_s \quad l_s(\alpha) := \alpha \cdot R_s + \frac{\pi \cdot r_s}{2} \quad f_s(\alpha) := \frac{c}{2 \cdot l_s(\alpha)} \quad \text{Equations 4.121 and 4.122} \quad f_s(\alpha 1) = 6.251 \text{ GHz}$$

$$t_s := L_c - H_c + w_s \quad \text{Equation 4.123} \quad f_s(\alpha 2) = 5.205 \text{ GHz}$$

Slot R/Q

$$R_{QS} := \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{2 \cdot r_s}{\pi \cdot t_s} \quad \text{Equation 4.124} \quad R_{QS} = 92.2 \Omega$$

Slot coupling factor

$$k(\alpha) := \frac{l_s(\alpha)}{2 \cdot \pi \cdot R_s} \quad \text{Equation 4.125}$$

Dispersion equation

$$a_k(\alpha) := 2 \cdot k(\alpha) \cdot \frac{f_C \cdot R_{QS}}{f_s(\alpha) \cdot R_{QC}} \quad \text{Equation 4.118}$$

$$F1(f, \phi, \alpha) := 1 - \cos(\phi) - \frac{1}{k(\alpha) \cdot a_k(\alpha)} \cdot \left[ 1 - \left( \frac{f}{f_C} \right)^2 \right] \cdot \left[ 1 + a_k(\alpha) - \left( \frac{f}{f_s(\alpha)} \right)^2 \right] \quad \text{Equation 4.115}$$

Solve for the frequency as a function of phase shift per cavity for the lower and upper pass-bands

$$f1(\phi, \alpha) := \text{root}(F1(f_C, \phi, \alpha), f_C) \quad f_2 := 3 \cdot f_C \quad f2(\phi, \alpha) := \text{root}(F1(f_2, \phi, \alpha), f_2)$$

Total impedance and admittance

$$Z(\phi, \alpha) := \frac{2 \cdot R_{QC}}{k(\alpha) \cdot a_k(\alpha) \cdot \sin(-\phi)} \cdot \frac{f1(\phi, \alpha)}{f_C} \cdot \left[ 1 + a_k(\alpha) - \left( \frac{f1(\phi, \alpha)}{f_s(\alpha)} \right)^2 \right] \quad Y(\phi, \alpha) := \frac{k\Omega}{Z(\phi, \alpha)} \quad \text{Equation 4.120}$$

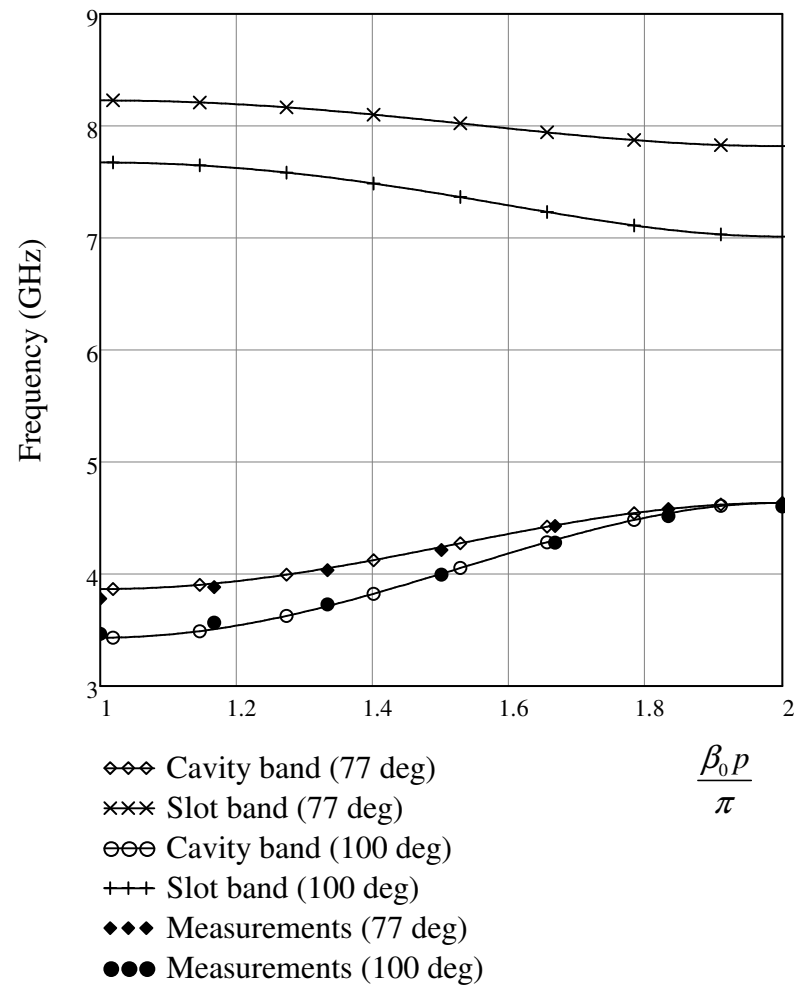


Figure 4.31a

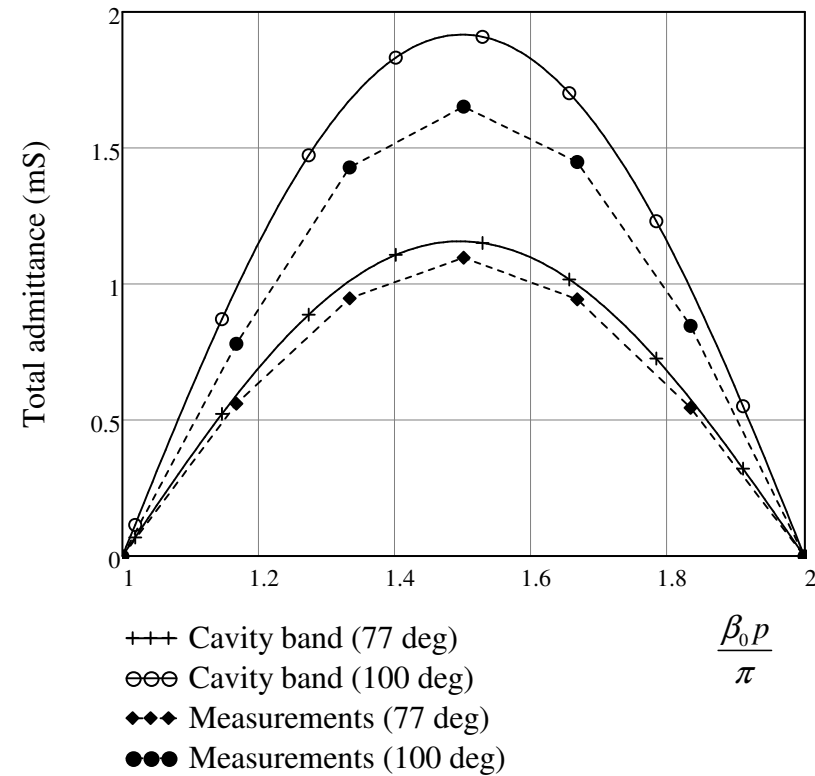


Figure 4.31b