

Worksheet 11.1 Calculation of coupling factors and beam loading for gridded and ungridded gaps

© 2018 Richard G Carter

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This resource is provided free of charge by Cambridge University Press with permission of the author, but is subject to copyright. You are permitted to view, print and download this resource for your own personal use only, provided any copyright lines are not removed or altered in any way. Any other use, including but not limited to, distribution of the resource in modified form, or via electronic or other media, is strictly prohibited unless you have permission from the author and provided you give appropriate acknowledgement of the source.

The contents of this sheet are provided for educational purposes only and no warranty is expressed or implied that they are suitable for use as professional design tools.

1. Gridded Gaps

The coupling factor and beam loading conductance are calculated in terms of the gap transit angle $\theta = \beta_e g$

$$M(\theta) := \text{sinc}(0.5 \cdot \theta)$$

Equation 11.7

$$Gg(\theta) := -\frac{1}{4} \cdot \theta \cdot \frac{d}{d\theta} M(\theta)^2$$

Equation 11.22

$$\theta := 0, 0.1 \dots 4 \cdot \pi$$

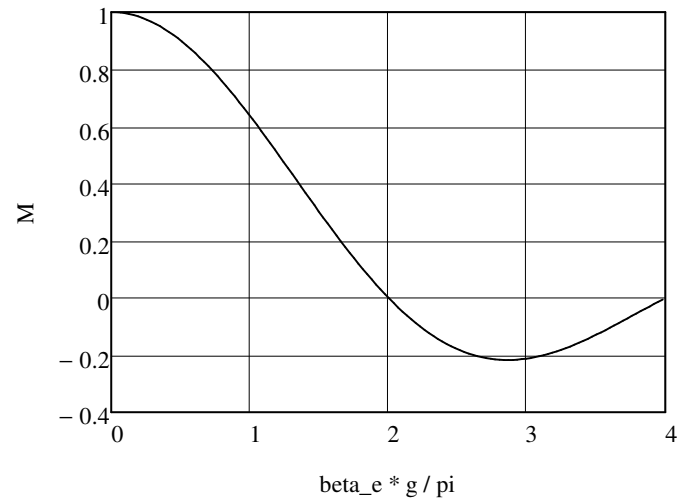


Figure 11.3

$$Gb(\theta) := 0.5 \cdot \text{sinc}(0.5 \cdot \theta) \cdot (\text{sinc}(0.5 \cdot \theta) - \cos(0.5 \cdot \theta))$$

Equation 11.20

$$Bb(\theta) := 0.5 \cdot \frac{\cos(0.5 \cdot \theta)}{0.5 \cdot \theta} \cdot (\text{sinc}(0.5 \cdot \theta) - \cos(0.5 \cdot \theta))$$

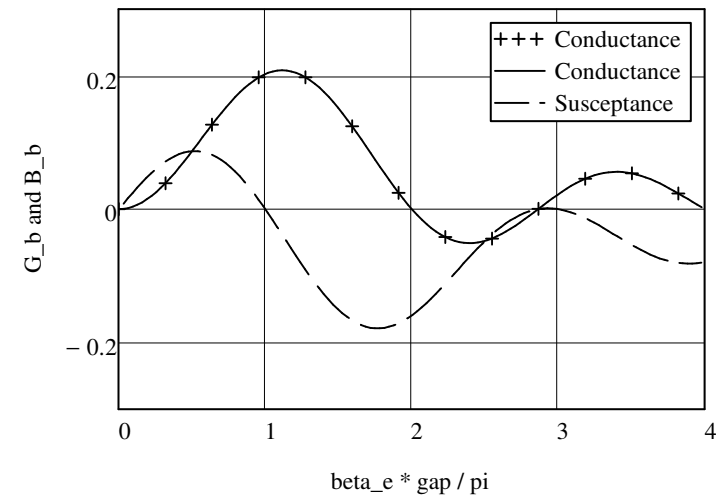


Figure 11.5

2. Coupling in an ungridded gap

Radial Coupling Factor

See Garland M. Branch: Electron Beam Coupling in Interaction Gaps of Cylindrical Symmetry
IRE Trans ED Vol.ED-8, pp.193-207 (1961)

a. Brillouin Beam: interaction concentrated on beam surface.

$$\mu_B(A, \gamma b) := \frac{I_0(\gamma b)}{I_0(A \cdot \gamma b)} \quad \text{Equation 11.40}$$

$$\text{where } A = \frac{a}{b}$$

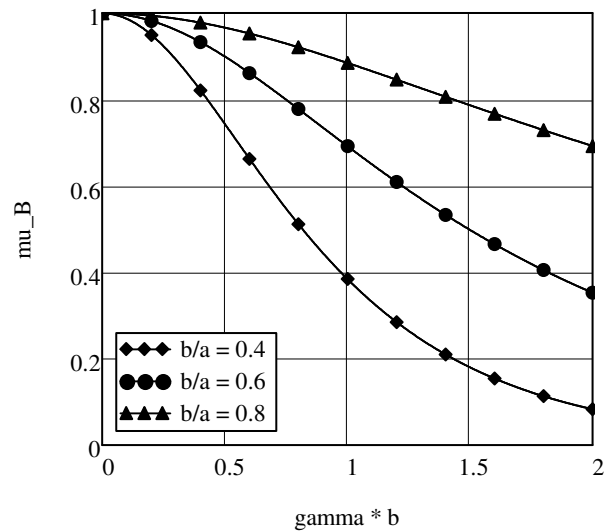


Figure 11.7(a)

b. Confined flow: Coupling averaged over area of beam

$$\mu_c(A, \gamma b) := \frac{2 \cdot I_1(\gamma b)}{\gamma b \cdot I_0(A \cdot \gamma b)} \quad \text{Equation 11.38}$$

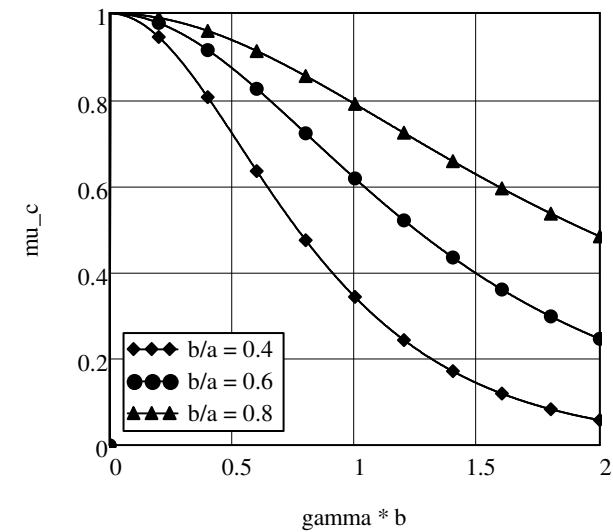


Figure 11.7(b)

Computation of Beam Loading Admittance of an ungridded gap by a beam in confined flow

Define the propagation constant, the coupling coefficient and the normalised gap loading conductance in terms of the transit angle θ (in radians) and the dimensions of the beam and the gap

$$\theta = \beta_c \cdot g \quad \mu_d(\theta) := \left(\frac{\sin(0.5 \cdot \theta)}{0.5 \cdot \theta} \right)$$

$$\beta_c(g, \theta) := \frac{\theta}{g}$$

$$\mu_r(a, b, g, \theta) := \frac{2 \cdot \Pi(\beta_c(g, \theta) \cdot b)}{\beta_c(g, \theta) \cdot b \cdot \text{IO}(\beta_c(g, \theta) \cdot a)}$$

$$M(a, b, g, \theta) := \mu_r(a, b, g, \theta) \cdot \mu_d(\theta)$$

$$G_b(a, b, g, \theta) := \frac{-1}{4} \cdot \theta \cdot \frac{d}{d\theta} M(a, b, g, \theta)^2$$

$$a := 1$$

$$b := 0.7$$

$$G_b(g, \theta) := G_b(a, b, g, \theta)$$

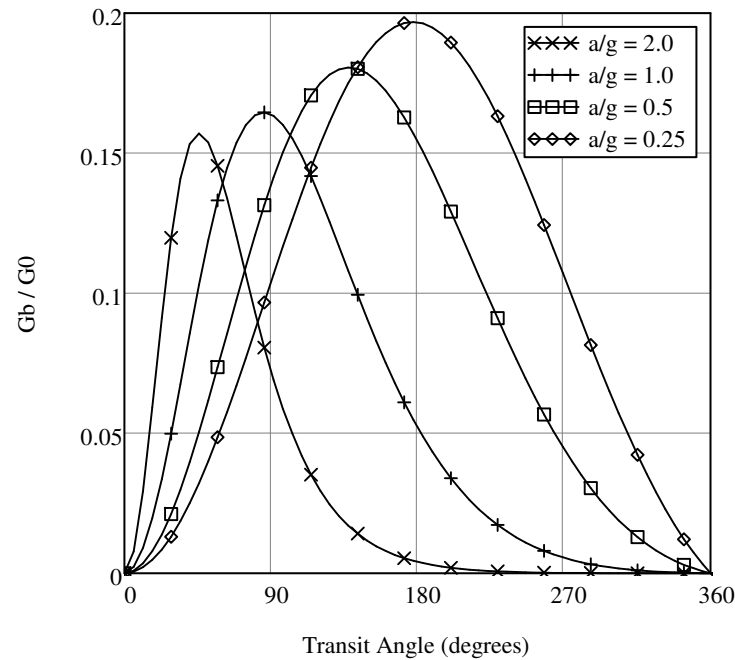


Figure 11.16
(G_b/G_0 only)

Compare fig.6 in
Craig, E.J., The beam loading admittance of gridless klystron gaps,
IEEE Trans. Vol.ED-14, No.5, pp.273-278 (1967)

Beam Loading Admittance for a relativistic beam

Based on E. J. Craig, "Relativistic beam-loading admittance," *IEEE Transactions on Electron Devices*, vol. 16, pp. 139-139, 1969 and using his notation. The results are for an ungridded gap assuming uniform field in the gap.

Assume that the normalised reduced plasma frequency is $\omega_{q_w} := 0.05$

Ratio of beam velocity to velocity of light

$u0_c := 0.01$

$$R := \frac{1}{\sqrt{1 - u0_c^2}} \quad \gamma = \frac{\beta}{R}$$

a = tunnel radius

b = beam radius

gap = gap length

beta = beta_e

gap := 2

$$\text{ReI}(\beta, \gamma, a, b, \text{gap}) := \frac{2 \cdot \sin(0.5 \cdot \beta \cdot \text{gap})^2 (I_0(\gamma \cdot b)^2 - I_1(\gamma \cdot b)^2)}{(\beta \cdot \text{gap})^2 \cdot I_0(\gamma \cdot a)^2}$$

Equation 11.101

$$G2(\beta, a, b, \text{gap}) := \frac{1}{2 \cdot R \cdot (R + 1) \cdot \omega_{q_w}} \left[\text{ReI} \left[(1 - \omega_{q_w}) \cdot \beta, \frac{(1 - \omega_{q_w}) \cdot \beta}{R}, a, b, \text{gap} \right] - \text{ReI} \left[(1 + \omega_{q_w}) \cdot \beta, \frac{(1 + \omega_{q_w}) \cdot \beta}{R}, a, b, \text{gap} \right] \right]$$

Equation 11.100
(Real part)

Zeroes of the Bessel function J_0

Number of terms in the series

nmax := 3

```

λ := for n ∈ 1..nmax
    x ← n · π
    λ_n ← root(J0(x), x)
return λ
    
```

```

r(β, a, gap) := for n ∈ 1..nmax
    r_n ← (λ_n · gap) / a · √(1 - ((a · β) / λ_n)^2 · u0_c^2)
return r
    
```

Equation 11.103

$$\begin{aligned} \underline{N}(\beta, a, \text{gap}) := & \text{for } n \in 1..nmax \\ & N_n \leftarrow \left[1 - \left(\frac{a \cdot \beta}{\lambda_n} \right)^2 \cdot u0_c^2 \right]^{-1} \\ & \text{return } N \end{aligned}$$

$$\begin{aligned} x(a, b, \text{gap}) := & \text{for } n \in 1..nmax & \text{Equation 11.106} \\ & x_n \leftarrow \left(\frac{\text{gap} \cdot J0\left(\frac{b \cdot \lambda_n}{a}\right)}{a \cdot J1(\lambda_n)} \right)^2 \\ & \text{return } x \end{aligned}$$

$$\begin{aligned} y(a, b, \text{gap}) := & \text{for } n \in 1..nmax & \text{Equation 11.107} \\ & x_n \leftarrow \left(\frac{\text{gap} \cdot J1\left(\frac{b \cdot \lambda_n}{a}\right)}{a \cdot J1(\lambda_n)} \right)^2 \\ & \text{return } x \end{aligned}$$

$$\begin{aligned} \underline{A}(\beta, a, b, \text{gap}) := & \text{for } n \in 1..nmax & \text{Equation 11.104} \\ & A_n \leftarrow 2 \cdot N(\beta, a, \text{gap})_n \cdot (x(a, b, \text{gap})_n + y(a, b, \text{gap})_n) \cdot \frac{(r(\beta, a, \text{gap})_n - 1 + \exp(-r(\beta, a, \text{gap})_n))}{r(\beta, a, \text{gap})_n} \\ & \text{return } A \end{aligned}$$

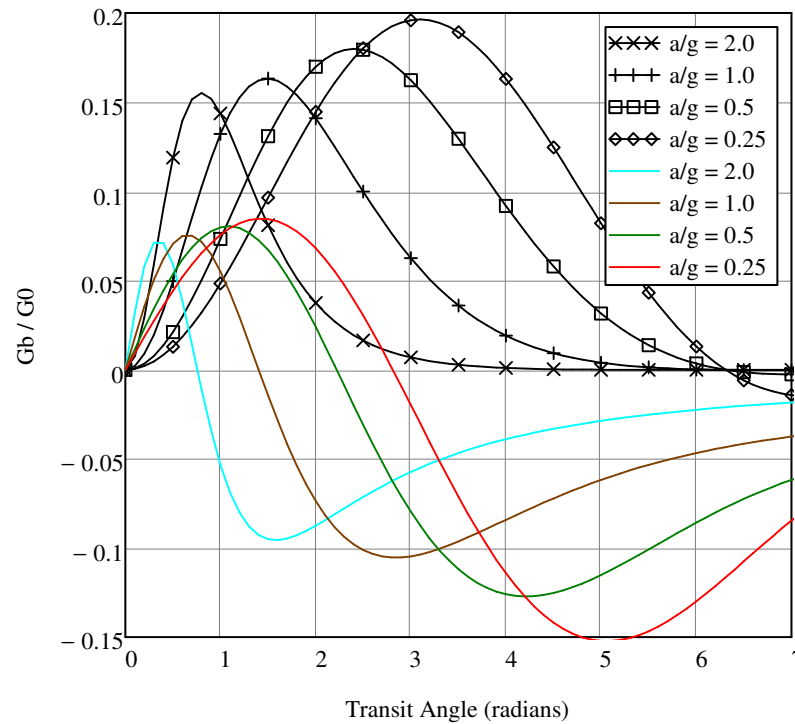
$$\begin{aligned} B(\beta, a, b, \text{gap}) := & \text{for } n \in 1..nmax & \text{Equation 11.105} \\ & B_n \leftarrow \left[N(\beta, a, \text{gap})_n \cdot (x(a, b, \text{gap})_n + y(a, b, \text{gap})_n) \cdot \frac{3 - 2 \cdot r(\beta, a, \text{gap})_n - (3 + r(\beta, a, \text{gap})_n) \cdot \exp(-r(\beta, a, \text{gap})_n)}{(r(\beta, a, \text{gap})_n)^3} \right. \\ & \quad \left. + \frac{2 \cdot x(a, b, \text{gap})_n \cdot (r(\beta, a, \text{gap})_n - 1 + \exp(-r(\beta, a, \text{gap})_n))}{(r(\beta, a, \text{gap})_n)^3} \right] \dots \\ & \text{return } B \end{aligned}$$

$$\text{ImI}(\beta, \gamma, a, b, \text{gap}) := \left[\frac{(\beta \cdot \text{gap} - \sin(\beta \cdot \text{gap})) (I_0(\gamma \cdot b)^2 - I_1(\gamma \cdot b)^2)}{\beta^2 \cdot \text{gap}^2 \cdot I_0(\gamma \cdot a)^2} + 2 \cdot \beta \cdot \text{gap} \cdot \sum_{n=1}^{n_{\max}} \left[\frac{A(\beta, a, b, \text{gap})_n}{\left[\beta^2 \cdot \text{gap}^2 + (r(\beta, a, \text{gap})_n)^2 \right]^2} - \frac{B(\beta, a, b, \text{gap})_n}{\left[\beta^2 \cdot \text{gap}^2 + (r(\beta, a, \text{gap})_n)^2 \right]} \right] \right]$$

Equation 11.102

$$B2(\beta, a, b, \text{gap}) := \frac{1}{2 \cdot R \cdot (R + 1) \cdot \omega_q \omega} \left[\text{ImI} \left[(1 - \omega_q \omega) \cdot \beta, \frac{(1 - \omega_q \omega) \cdot \beta}{R}, a, b, \text{gap} \right] - \text{ImI} \left[(1 + \omega_q \omega) \cdot \beta, \frac{(1 + \omega_q \omega) \cdot \beta}{R}, a, b, \text{gap} \right] \right]$$

 Equation 11.100
(Imaginary part)

 $\beta := 0, 0.05 \dots 7$


The results are independent of the reduced plasma frequency if $\frac{\omega_q}{\omega} < 0.1$.

$u0_c = 0.01$

$\omega_q \omega = 0.05$

Figure 11.16

Computed in the limit of small beam velocity