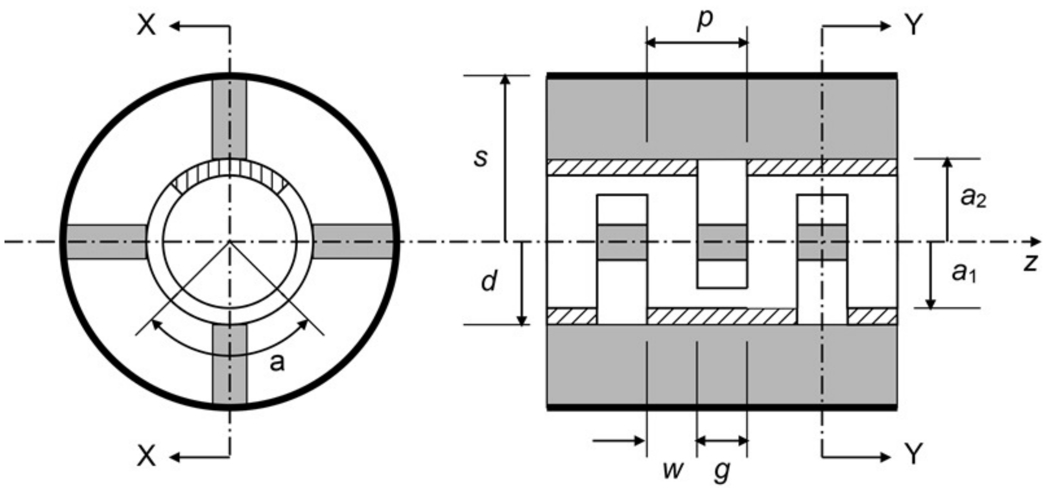
WS 4.5 Ring-Bar Structure

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.



Section on Y - Y

Section on X - X

Section 4.4 Ring-bar structure

The period of the structure is P and the pitch p where P = 2p

$a := 2.375 \cdot mm$ $s := 4.35 \cdot mm$ $p := 1.76 \cdot mm$ $w := 0.88 \cdot mm$ $t := 0.34 \cdot mm$

Relative permittivity of equivalent uniform dielectric $\varepsilon_2 := 2.6$

Angle subtended on the axis by the bars $\chi := 90 \cdot deg$

$a_1 := a - 0.5 \cdot t$ $a_2 := a + 0.5 \cdot t$ $gap := p - w = 0.88 \cdot mm$

Equivalent circuit model

Scale converts from axial F/m to azimuthal F/m $scale := \dfrac{p}{2 \cdot \pi \cdot a}$

The capacitances for phase shifts 0, π/2 and π per pitch p without dielectric loading are found using Worksheet 4.4

$C_e := 10.83 \cdot \varepsilon_0$ $C_{\pi 2} := 32.04 \cdot \varepsilon_0$ $C_o := 49.05 \cdot \varepsilon_0$

$C_0 := C_e \cdot scale$ $C_1 := 0.25 \cdot \left(C_o - C_e\right) \cdot scale$ $C_2 := 0.25 \cdot \left(C_{\pi 2} - C_e - 2 \cdot C_1\right) \cdot scale$

$C1(\phi) := C_0 + 2 \cdot C_1 \cdot (1 - \cos(0.5 \cdot \phi)) + 2 \cdot C_2 \cdot (1 - \cos(\phi))$ Equation 4.44

$L1(\phi) := \dfrac{1}{c^2 \cdot C1(\phi)}$ Equation 4.48

The transverse phase velocity without dielectric loading is c. The inductance is unaffected by dielectric loading.

The capacitances for phase shifts 0, π/2 and π per pitch p with dielectric loading are found using Worksheet 4.4

$CK_e := 27.51 \cdot \varepsilon_0$ $\quad CK_{\pi 2} := 58.25 \cdot \varepsilon_0$ $\quad CK_o := 83.16 \cdot \varepsilon_0$

$CK_0 := CK_e \cdot scale$ $\quad CK_1 := 0.25 \cdot (CK_o - CK_e) \cdot scale$ $\quad CK_2 := 0.25 \cdot (CK_{\pi 2} - CK_e - 2 \cdot CK_1) \cdot scale$

$CK(\phi) := CK_0 + 2 \cdot CK_1 \cdot (1 - \cos(0.5 \cdot \phi)) + 2 \cdot CK_2 \cdot (1 - \cos(\phi))$

Effective relative permittivity and transverse phase velocities of the even and odd modes with dielectric loading.

$\varepsilon_{eff}(\phi) := \dfrac{CK(\phi)}{C1(\phi)}$ $\quad v_{pe}(\phi) := \dfrac{c}{\sqrt{\varepsilon_{eff}(\phi)}}$ $\quad v_{po}(\phi) := \dfrac{c}{\sqrt{\varepsilon_{eff}(\phi - 2 \cdot \pi)}}$

Characteristic impedances of even and odd modes with and without dielectric loading.

$Z_e(\phi) := \sqrt{\dfrac{L1(\phi)}{CK(\phi)}}$ $\quad Z_o(\phi) := Z_e(\phi - 2 \cdot \pi)$ $\quad Z_{e0}(\phi) := \sqrt{\dfrac{L1(\phi)}{C1(\phi)}}$ $\quad Z_{o0}(\phi) := Z_{e0}(\phi - 2 \cdot \pi)$

Define the transverse path length for one half of the structure. $Lt := \pi \cdot a$

Calculate the dispersion curves

(a) Ignoring coupling between lines and assume that the effective permittivity is that of the substrate at the centre frequency

$\omega 0(\phi) := \dfrac{c \cdot \phi}{\sqrt{(1 + \varepsilon_2) \cdot 0.5} \cdot Lt}$ $\quad f0(\phi) := \dfrac{\omega 0(\phi)}{2 \cdot \pi \cdot GHz}$ $\quad f0(\pi) = 14.974$ $\quad v_{p0}(\phi) := \dfrac{\omega 0(\phi) \cdot p}{\phi}$

(b) Including coupling between lines and assuming that the effective length is Lt

Define the dispersion functions <u>with dielectric loading</u> (Equation 4.57)

$$F1(\omega,\phi) := \left(\tan\left(\frac{\phi}{2}\right)\right)^2 - \frac{Z_o(\phi)}{Z_e(\phi)}\cdot\tan\left(\frac{\omega\cdot Lt}{2\cdot v_{pe}(\phi)}\right)\cdot\tan\left(\frac{\omega\cdot Lt}{2\cdot v_{po}(\phi)}\right)$$

$$F2(\omega,\phi) := \tan\left(\frac{\phi}{2}\right)^2 - \frac{Z_o(\phi)}{Z_e(\phi)}\cdot\cot\left(\frac{\omega\cdot Lt}{2\cdot v_{pe}(\phi)}\right)\cdot\cot\left(\frac{\omega\cdot Lt}{2\cdot v_{po}(\phi)}\right)$$

Solve for the lower and upper branches

$$\omega1(\phi) := \left|\begin{array}{l}\omega_1 \leftarrow \omega0(\phi)\\ \omega1 \leftarrow \mathrm{root}\left(F1\left(\omega_1,\phi\right),\omega_1\right)\end{array}\right.$$

$$f1(\phi) := \frac{\omega1(\phi)}{2\cdot\pi\cdot GHz}$$

Phase velocity $v_p(\phi) := \dfrac{\omega1(\phi)\cdot p}{\phi}$

$$\omega2(\phi) := \left|\begin{array}{l}\omega_1 \leftarrow \omega0(2\pi-\phi)\\ \omega1 \leftarrow \mathrm{root}\left(F2\left(\omega_1,\pi-\phi\right),\omega_1\right)\end{array}\right.$$

$$f2(\phi) := \frac{\omega2(\phi)}{2\cdot\pi\cdot GHz}$$

Group velocity $v_g(\phi) := p\cdot\left(\dfrac{d}{d\phi}\omega1(\phi)\right)$

Define the dispersion function <u>without dielectric loading</u> (Equation 4.57)

$$F3(\omega,\phi) := \tan\left(\frac{\phi}{2}\right)^2 - \frac{Z_{o0}(\phi)}{Z_{e0}(\phi)}\cdot\tan\left(\frac{\omega\cdot Lt}{2\cdot c}\right)\cdot\tan\left(\frac{\omega\cdot Lt}{2\cdot c}\right)$$

$$\omega3(\phi) := \left|\begin{array}{l}\omega_1 \leftarrow \omega0(\phi)\\ \omega3 \leftarrow \mathrm{root}\left(F3\left(\omega_1,\phi\right),\omega_1\right)\end{array}\right.$$

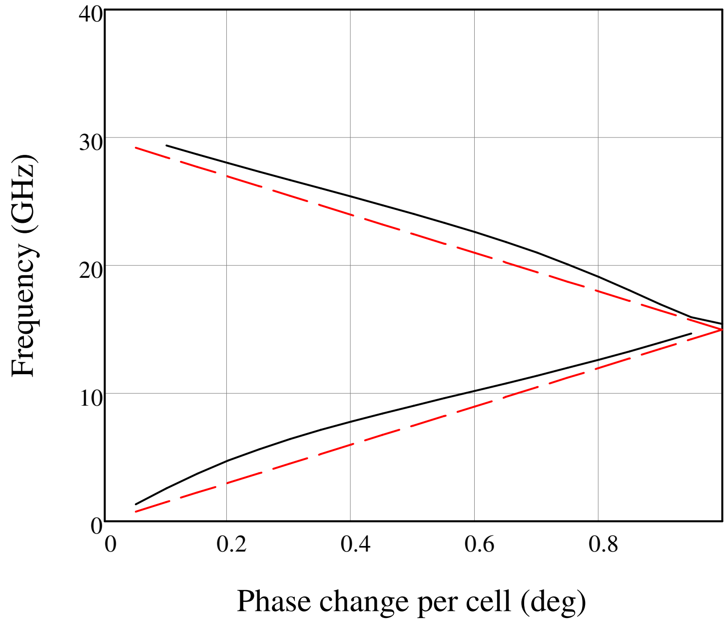
$$f3(\phi) := \frac{\omega3(\phi)}{2\cdot\pi\cdot GHz}$$

Phase velocity $v_{p3}(\phi) := \dfrac{\omega3(\phi)\cdot p}{\phi}$

$$k3(\phi) := \frac{\omega3(\phi)}{c}$$

Group velocity $v_{g3}(\phi) := p\cdot\left(\dfrac{d}{d\phi}\omega3(\phi)\right)$

Plot the dispersion curves $\phi 1 := 0.05\cdot\pi, 0.1\cdot\pi .. \pi$



Propagation constants and relative amplitudes of the even and odd modes with dielectric loading

$$k_e(\phi) := \frac{\omega 1(\phi)}{v_{pe}(\phi)} \qquad k_o(\phi) := \frac{\omega 1(\phi)}{v_{po}(\phi)}$$

Axial propagation constant $\beta_0(\phi) := \dfrac{\phi}{p}$

Calculate the power flow for the whole structure and the Pierce impedance assuming that the field is constant in the gap.

Define the line to ground voltage of the even mode $V_a := 1\cdot V$

Without dielectric loading

Ratio of mode amplitudes

$$b\_a3(\phi) := -\sqrt{\frac{Z_{o0}(\phi)}{Z_{e0}(\phi)}}$$

$\boxed{\text{Equation 4.56}}$

Axial power flow

$$P0_z(\phi) := \frac{4Lt}{p}\cdot V_a{}^2\cdot v_{g3}(\phi)\cdot\left[\frac{1}{c\cdot Z_{e0}(\phi)}\cdot\left(1 + \mathrm{sinc}\left(k_e(\phi)\cdot Lt\right)\right) + \frac{b\_a3(\phi)^2}{c\cdot Z_{o0}(\phi)}\cdot\left(1 - \mathrm{sinc}\left(k_o(\phi)\cdot Lt\right)\right)\right]$$

$\boxed{\text{Equation 4.60}}$

Electric field in the gap

$$E0_g(\phi, x) := \frac{4\cdot V_a}{gap}\cdot\left(\cos(k3(\phi)\cdot x)\cdot\sin\left(\frac{\phi}{2}\right) + b\_a3(\phi)\cdot\sin(k3(\phi)\cdot x)\cdot\cos\left(\frac{\phi}{2}\right)\right)$$

$\boxed{\text{Equation 4.61}}$

Electric field of the fundamental space harmonic

$$E0_{g0}(\phi) := \frac{1}{\pi\cdot a}\cdot\int_{\frac{-Lt}{2}}^{\frac{Lt}{2}} E0_g(\phi, x)\, dx$$

Radial propagation constant

$$\gamma_3(\phi) := \sqrt{\beta_0(\phi)^2 - \left(\frac{\omega3(\phi)}{c}\right)^2}$$

Electric field of the fundamental space harmonic on the axis

$$E0_0(\phi) := E0_{g0}(\phi)\cdot\mathrm{sinc}\left(\beta_0(\phi)\cdot\frac{gap}{2}\right)\cdot\frac{gap}{p}\cdot\frac{1}{I0\left(\gamma_3(\phi)\cdot a_1\right)}$$

Pierce impedance

$$Z0_P(\phi) := \frac{E0_0(\phi)^2}{2\cdot\beta_0(\phi)^2\cdot P0_z(\phi)}$$

With dielectric loading

Ratio of mode amplitudes

$$b\_a1(\phi) := -\sqrt{\frac{Z_o(\phi)\cdot\sin(k_e(\phi)\cdot Lt)}{Z_e(\phi)\cdot\sin(k_o(\phi)\cdot Lt)}}$$

Axial power flow

$$P_z(\phi) := \frac{4Lt}{p}\cdot V_a^{\,2}\cdot v_g(\phi)\cdot\left[\frac{1}{v_{pe}(\phi)\cdot Z_e(\phi)}\cdot(1+\mathrm{sinc}(k_e(\phi)\cdot Lt)) + \frac{b\_a1(\phi)^2}{v_{po}(\phi)\cdot Z_o(\phi)}\cdot(1-\mathrm{sinc}(k_o(\phi)\cdot Lt))\right]$$

Electric field in the gap

$$E_g(\phi, x) := \frac{4\cdot V_a}{gap}\cdot\left(\cos(k_e(\phi)\cdot x)\cdot\sin\left(\frac{\phi}{2}\right) + b\_a1(\phi)\cdot\sin(k_o(\phi)\cdot x)\cdot\cos\left(\frac{\phi}{2}\right)\right)$$

Electric field of the fundamental space harmonic

$$E_{g0}(\phi) := \frac{1}{\pi\cdot a}\cdot\int_{\frac{-Lt}{2}}^{\frac{Lt}{2}} E_g(\phi, x)\,dx$$

Radial propagation constant

$$\gamma_1(\phi) := \sqrt{\beta_0(\phi)^2 - \left(\frac{\omega1(\phi)}{c}\right)^2}$$

Electric field of the fundamental space harmonic on the axis

$$E_0(\phi) := E_{g0}(\phi)\cdot\mathrm{sinc}\left(\beta_0(\phi)\cdot\frac{gap}{2}\right)\cdot\frac{gap}{p}\cdot\frac{1}{I0(\gamma_1(\phi)\cdot a_1)}$$

Pierce impedance

$$Z_P(\phi) := \frac{E_0(\phi)^2}{2\cdot\beta_0(\phi)^2\cdot P_z(\phi)}$$

Experimental data derived from

D. T. Lopes and C. C. Motta, "Characterization of Ring-Bar and Contrawound Helix Circuits for High-Power Traveling-Wave Tubes," *Electron Devices, IEEE Transactions on,* vol. 55, pp. 2498-2504, (2008) fig. 11. The 'a' series points are without dielectric loading, 'b' series are with dielectric loading

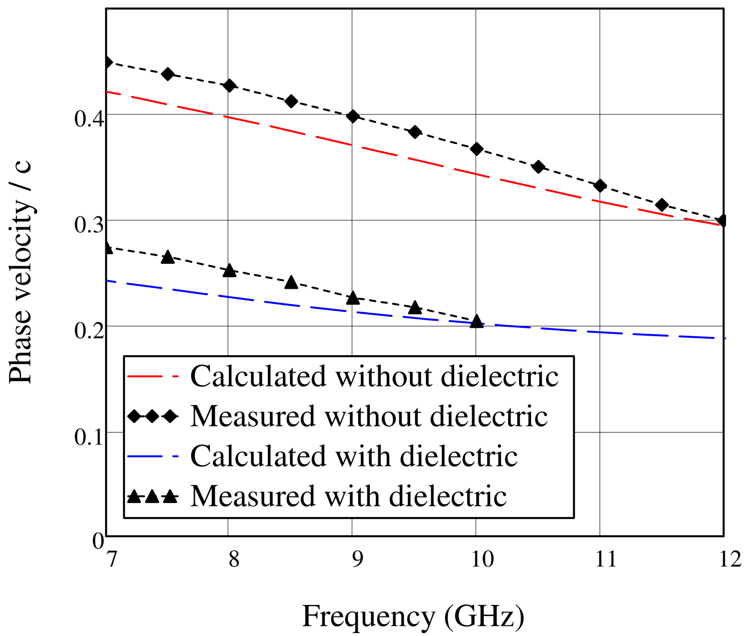
Lopes' parameters $a = 2.375\,\mathrm{mm}$ $r := \frac{s}{a} = 1.832$ $\cot\psi := \frac{\pi \cdot a}{p} = 4.239$ $\eta := \frac{\pi \cdot w}{p} = 1.571$

Note: Lopes' p is the structure period which is double the pitch defined above.

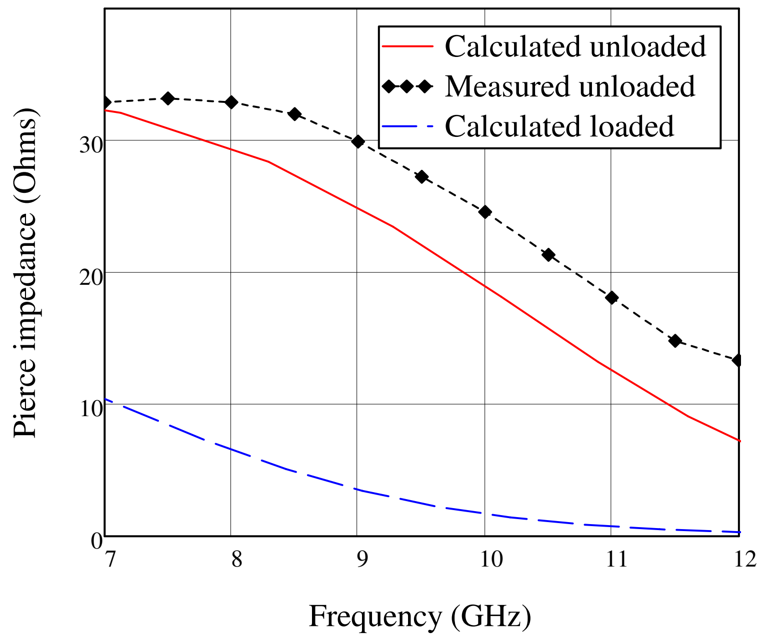
$$fa := \begin{pmatrix} 7 \\ 7.5 \\ 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \\ 11 \\ 11.5 \\ 12 \end{pmatrix} \quad vpa := \begin{pmatrix} 0.449 \\ 0.438 \\ 0.427 \\ 0.412 \\ 0.398 \\ 0.383 \\ 0.367 \\ 0.350 \\ 0.332 \\ 0.314 \\ 0.299 \end{pmatrix} \quad K_0 := \frac{40 \cdot \Omega}{135} \cdot \begin{pmatrix} 111 \\ 112 \\ 111 \\ 108 \\ 101 \\ 92 \\ 83 \\ 72 \\ 61 \\ 50 \\ 45 \end{pmatrix} \quad fb := \begin{pmatrix} 7 \\ 7.5 \\ 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \end{pmatrix} \quad vpb := \begin{pmatrix} 0.274 \\ 0.265 \\ 0.252 \\ 0.241 \\ 0.226 \\ 0.217 \\ 0.204 \end{pmatrix}$$

$$\phi a := \text{for } n \in 0..10 \quad \phi_n \leftarrow \frac{2 \cdot \pi \cdot fa_n \cdot \mathrm{GHz} \cdot p}{vpa_n \cdot c}$$

$$\phi b := \text{for } n \in 0..6 \quad \phi_n \leftarrow \frac{2 \cdot \pi \cdot fb_n \cdot \mathrm{GHz} \cdot p}{vpb_n \cdot c}$$



Compare Lopez (2008) fig. 11(a)



Compare Lopez (2008) fig. 11(b)

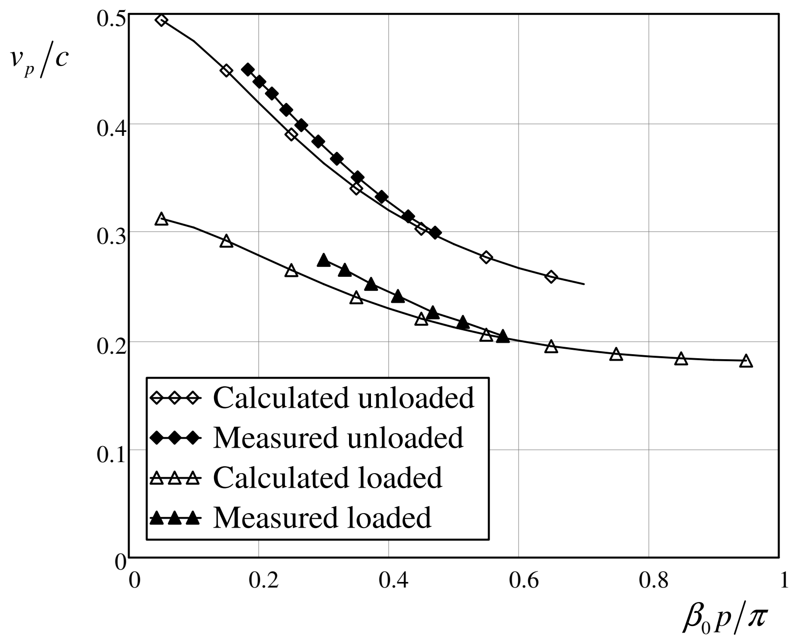


Figure 4.23(a)

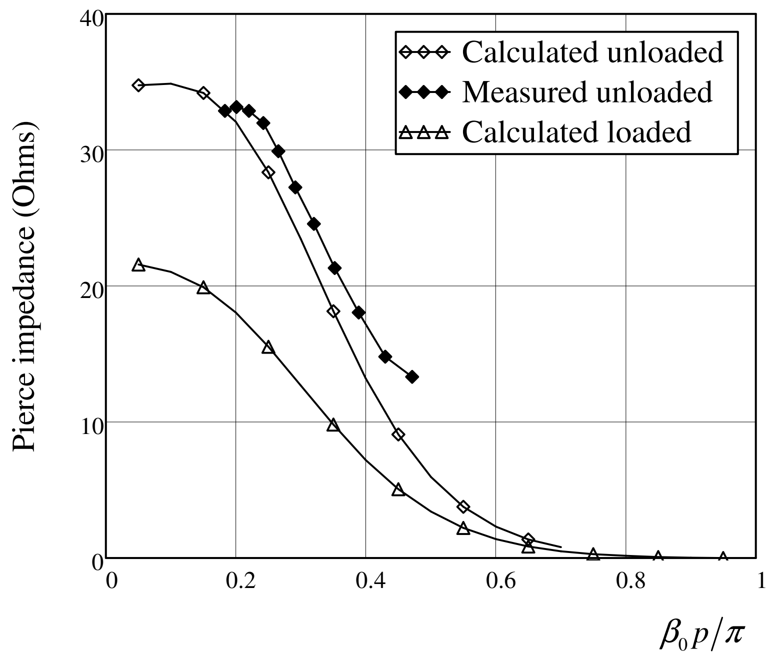


Figure 4.23(b)