Worksheet 11.4 Continuous interaction between an electron beam and a slow-wave structure

Revision 1 - 12 April 2018

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet illustrates the properties of continuous interaction between an electron beam and a slow-wave structure in a helix TWT (see Section 11.5). Typical values of the parameters are used but the variation of the reduced plasma frequency with frequency and the radial coupling factor are ignored for simplicity. The frequency dependences of the coupling impedance and the electronic admittance of the beam are represented by approximations.

Set the lowest index for matrices $\mathrm{ORIGIN} := 1$

1. Computation of the coupled-mode diagram for typical parameters

Define the tube parameters in terms of Ω, the frequency normalised to the synchronous point

$\beta_e(\Omega) := 717 \cdot \Omega$ $\quad$ $\beta_q := 67$

$\mu_c := 0.5$ $\quad$ $Z_c(\Omega) := \dfrac{46}{\Omega^2}$ $\quad$ $Y_e(\Omega) := \dfrac{\Omega}{8302}$

For the forward circuit wave $\quad$ $\beta_0(\Omega) := (\beta_e(1) + \beta_q) \cdot \Omega$

$A(\Omega) := \dfrac{1}{2} \cdot \mu_c \cdot Z_c(\Omega) \cdot Y_e(\Omega)$ $\quad$ Equation 11.133 $\quad$ $B := \dfrac{1}{2} \cdot \mu_c$ $\quad$ Equation 11.134

Define the coupled-mode matrix as a function of normalised frequency

$$C(\Omega) := \begin{bmatrix} \beta_0(\Omega) & 0 & A(\Omega)\cdot(\beta_e(\Omega) - \beta_q) & -A(\Omega)\cdot(\beta_e(\Omega) + \beta_q) \\ 0 & -\beta_0(\Omega) & -A(\Omega)\cdot(\beta_e(\Omega) - \beta_q) & A(\Omega)\cdot(\beta_e(\Omega) + \beta_q) \\ B\cdot\beta_0(\Omega) & -B\cdot\beta_0(\Omega) & \beta_e(\Omega) - \beta_q & 0 \\ B\cdot\beta_0(\Omega) & -B\cdot\beta_0(\Omega) & 0 & \beta_e(\Omega) + \beta_q \end{bmatrix}$$

Equation 11.132

This equation is wrong in the book.

Corrected 12 April 2018

The matrix for the uncoupled modes is found by setting $\mu_c = 0$

$$C0(\Omega) := \begin{pmatrix} \beta_0(\Omega) & 0 & 0 & 0 \\ 0 & -\beta_0(\Omega) & 0 & 0 \\ 0 & 0 & \beta_e(\Omega) - \beta_q & 0 \\ 0 & 0 & 0 & \beta_e(\Omega) + \beta_q \end{pmatrix}$$

The solutions of the determinantal equation are the eigenvalues of C. These are sorted into acending order of their real parts and plotted as a dispersion diagram. Note that this means that the indices refer to different waves if the lines cross.

For the uncoupled modes

$\beta0(\Omega) := \mathrm{sort}(\mathrm{eigenvals}(C0(\Omega)))$

$$\beta0(1) = \begin{pmatrix} -784.0 \\ 650.0 \\ 784.0 \\ 784.0 \end{pmatrix}$$

$\Omega := 0.1, 0.11 .. 3$

For the coupled modes

$\beta(\Omega) := \mathrm{sort}(\mathrm{eigenvals}(C(\Omega)))$

$$\beta(1) = \begin{pmatrix} -784.0 \\ 648.8 \\ 784.6 - 14.5i \\ 784.6 + 14.5i \end{pmatrix}$$

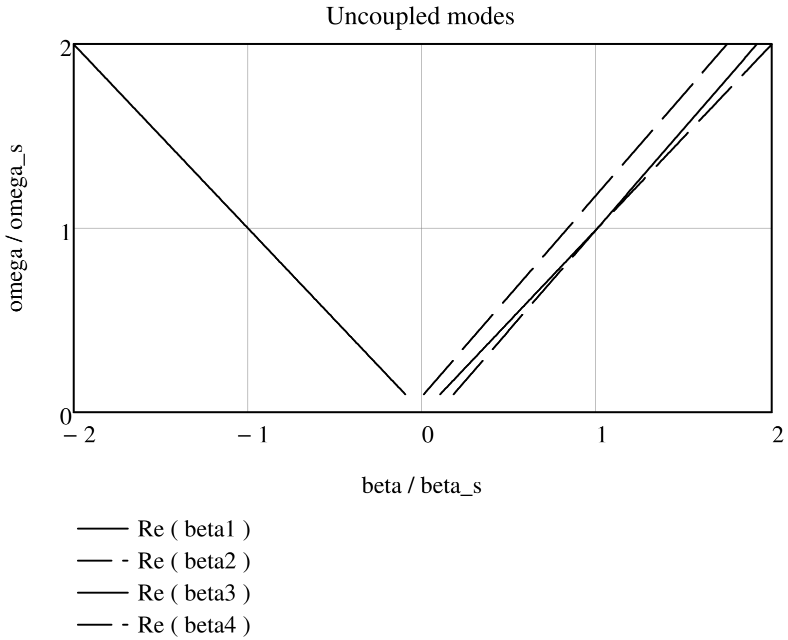


Figure 11.19(a)

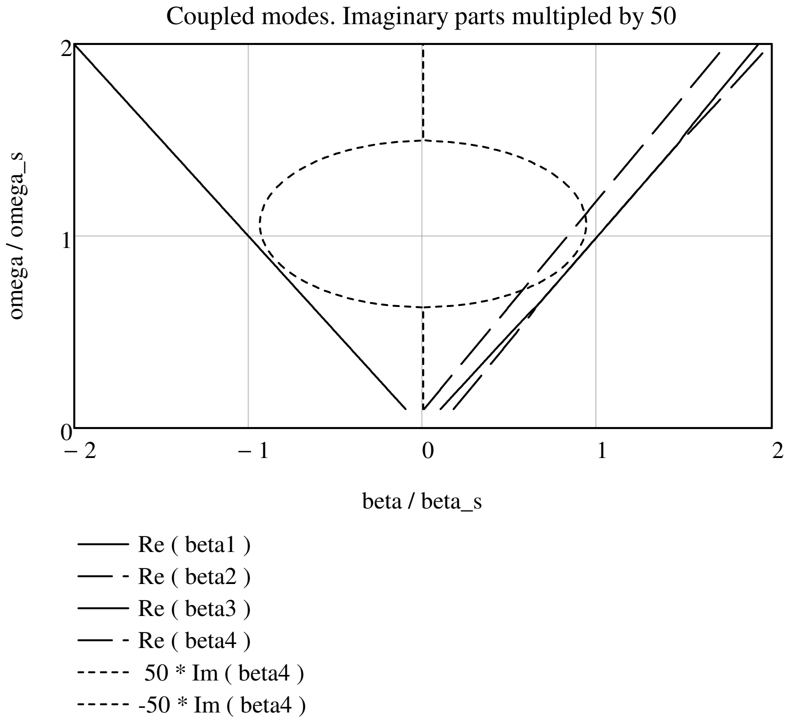


Figure 11.18(b)

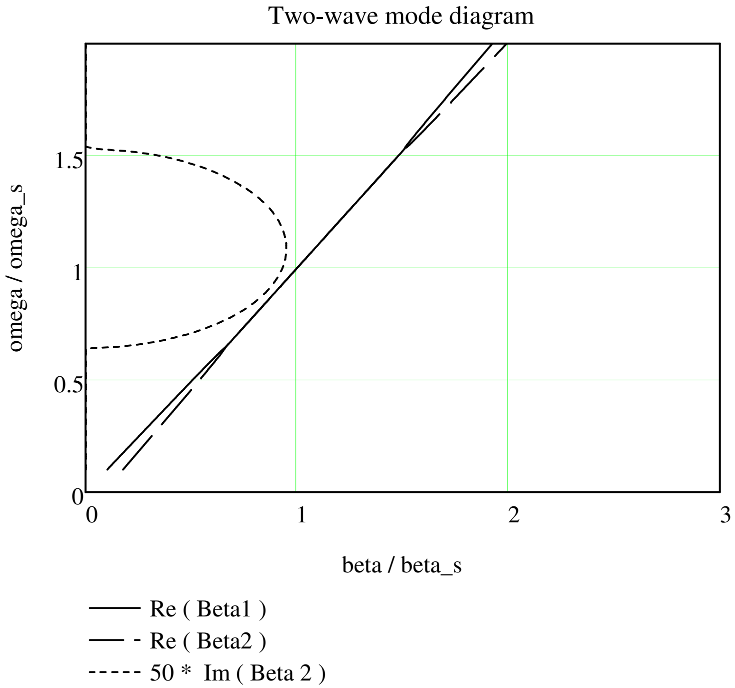
2. Computation from the two-wave model for comparison

Define the coupled-mode matrix by omitting the backward wave and the fast space-charge wave, compute its eigenvalues and plot the coupled-mode diagram

$$C2(\Omega) := \begin{pmatrix} C(\Omega)_{1,1} & C(\Omega)_{1,4} \\ C(\Omega)_{4,1} & C(\Omega)_{4,4} \end{pmatrix}$$

Equation 11.143

$$\beta2(\Omega) := \mathrm{sort}(\mathrm{eigenvals}(C2(\Omega)))$$



Compare the two-wave and the four-wave solutions

$$\frac{\beta 2(0.5)}{\beta_0(0.5)} = \begin{pmatrix} 1.010 \\ 1.076 \end{pmatrix}$$

$$\frac{\beta(0.5)}{\beta_0(0.5)} = \begin{pmatrix} -1.000 \\ 0.742 \\ 1.012 \\ 1.075 \end{pmatrix}$$

$$\frac{\beta 2(1.0)}{\beta_0(1.0)} = \begin{pmatrix} 1.000 - 0.019i \\ 1.000 + 0.019i \end{pmatrix}$$

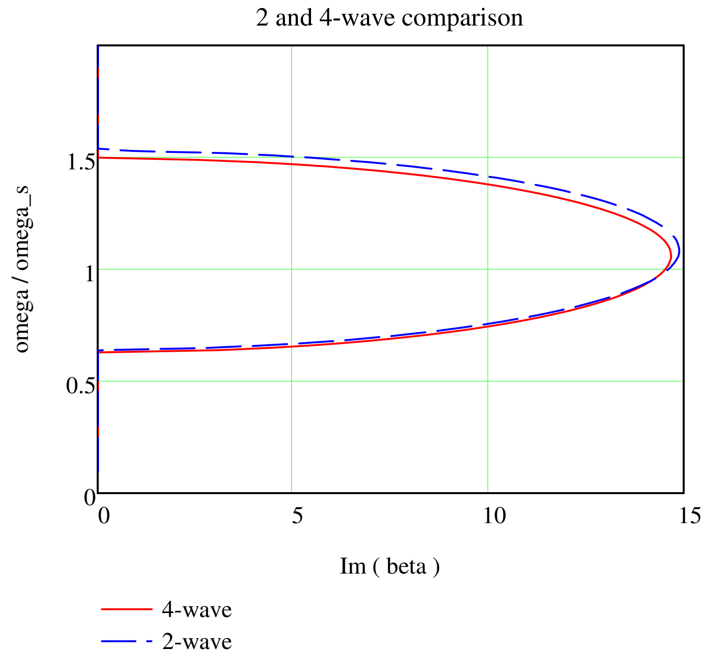
$$\frac{\beta(1.0)}{\beta_0(1.0)} = \begin{pmatrix} -1.000 \\ 0.828 \\ 1.001 - 0.019i \\ 1.001 + 0.019i \end{pmatrix}$$

$$\frac{\beta 2(1.5)}{\beta_0(1.5)} = \begin{pmatrix} 0.986 - 0.005i \\ 0.986 + 0.005i \end{pmatrix}$$

$$\frac{\beta(1.5)}{\beta_0(1.5)} = \begin{pmatrix} -1.000 \\ 0.856 \\ 0.985 \\ 0.988 \end{pmatrix}$$

$$\frac{\beta 2(2.0)}{\beta_0(2.0)} = \begin{pmatrix} 0.962 \\ 0.996 \end{pmatrix}$$

$$\frac{\beta(2.0)}{\beta_0(2.0)} = \begin{pmatrix} -1.000 \\ 0.871 \\ 0.961 \\ 0.997 \end{pmatrix}$$



TWT small signal gain

Two-wave approximation with $x = \beta_- / \beta_0$ $y = 2\alpha / \beta_0$ $C' = \mu_c{}^2 Z_c / Z_e$

$x := 0.8, 0.801 .. 1.2$

$y(x, C') := \sqrt{C' \cdot x - (x-1)^2}$ $\boxed{\text{Equation 11.149}}$

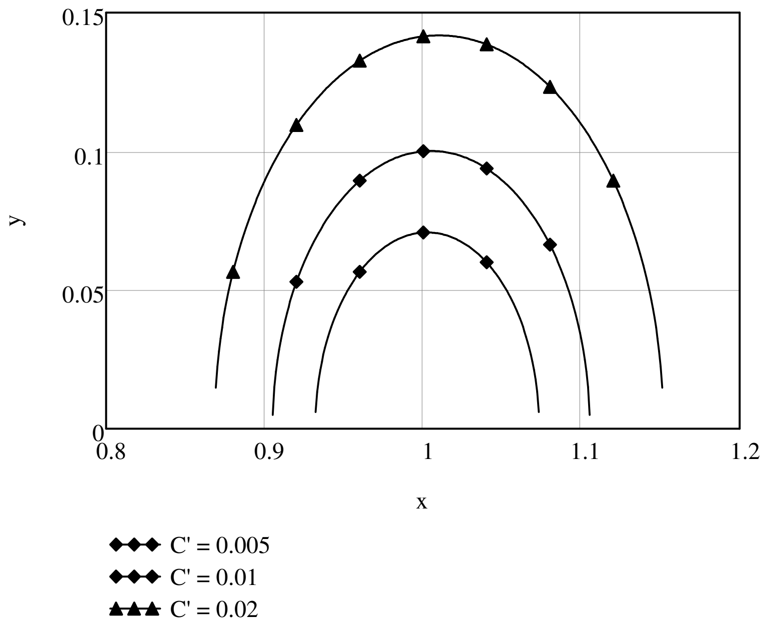


Figure 11.20