WS 20.1 Power converters

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet can be used to explore the properties of different types of simple power converter. For further information see
Mohan, N., T. M. Undeland, et al. (2003). *Power Electronics: Converters, Applications and Design*. Hoboken, NJ, Wiley & Sons Inc.

Peak AC source voltage $V_{pk} := 1$ Load resistance $R := 1$

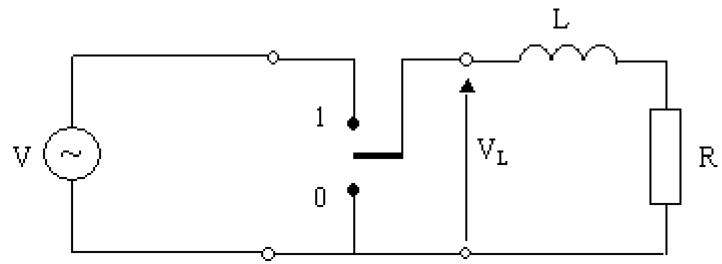
DC source voltage $V_0 := 1$ Series inductance $L := 1$

Frequency $f := 1$ Time constant $\tau := \frac{L}{R}$ $\tau = 1$

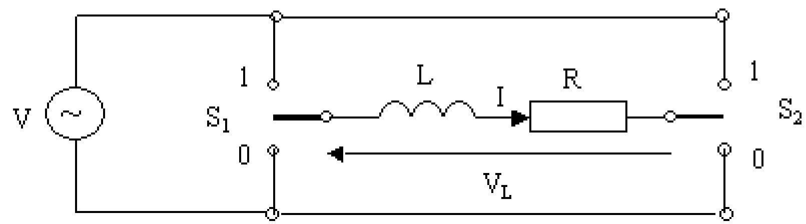
Period $T := \frac{1}{f}$ $T = 1$

Circuits

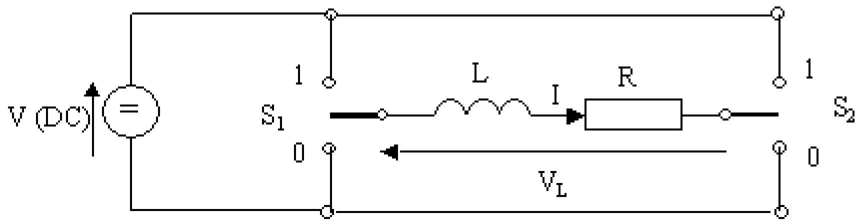
Circuit A



Circuit B



Circuit C



AC to DC Converters (Rectifiers)

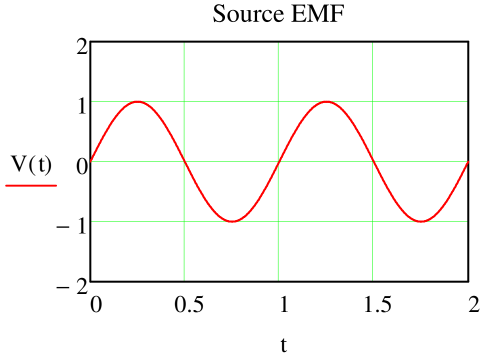
1. Half-wave uncontrolled rectifier (Circuit A)

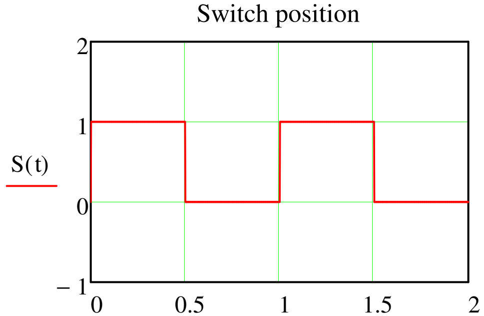
Source EMF $V(t) := V_{pk} \cdot \sin(2 \cdot \pi \cdot f \cdot t)$

Switch position $S(t) := \begin{cases} 1 & \text{if } (\sin(2 \cdot \pi \cdot f \cdot t) > 0) \\ 0 & \text{otherwise} \end{cases}$

Output voltage $V_L(t) := S(t) \cdot V(t)$

DC load voltage $V_{av} = \dfrac{V_{pk}}{2 \cdot \pi} \cdot \displaystyle\int_0^{\pi} \sin(\theta)\, d\theta$ $\qquad V_{av} := \dfrac{V_{pk}}{\pi}$ $\qquad V_{av} = 0.318$

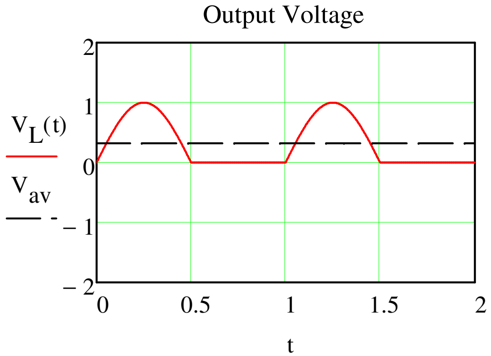


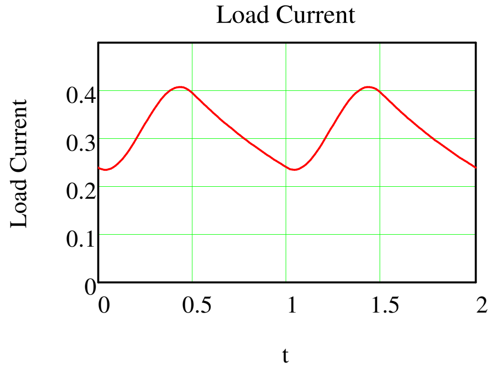


Solve equation for I(t)

$$D(t, I) := \frac{(V_L(t) - R \cdot I)}{L} \qquad \tau = 1$$

$$I_0 := \frac{V_{av}}{R} \qquad Z := \text{rkfixed}(I, -10, 2, 1000, D)$$





2. Half-wave controlled rectifier (Circuit A)

Source EMF $V(t) := V_{pk} \cdot \sin(2 \cdot \pi \cdot t)$

Define the **firing angle** α
(Usually expressed in degrees)

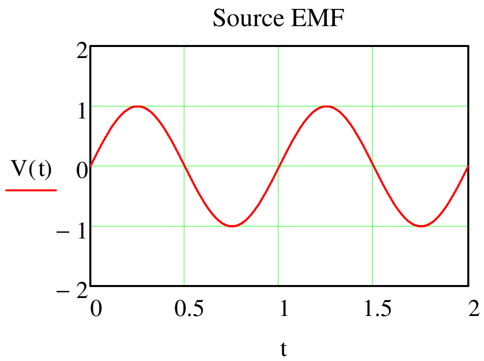
$\alpha := 60 \cdot deg$

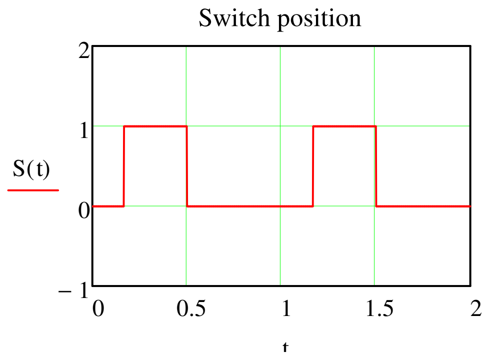
Switch position
$$S(t) := \begin{array}{|l} 1 \quad \text{if } (\sin(2 \cdot \pi \cdot t) > 0) \wedge (\sin(2 \cdot \pi \cdot t - \alpha) > 0) \\ 0 \quad \text{otherwise} \end{array}$$

Output voltage $V_L(t) := S(t) \cdot V(t)$

DC load voltage
$$V_{av} = \frac{V_{pk}}{2\pi} \cdot \int_{\alpha}^{\pi} \sin(\theta)\, d\theta \qquad V_{av} := \frac{1}{2} \cdot \frac{V_{pk}}{\pi} \cdot (1 + \cos(\alpha))$$

$V_{av} = 0.239$



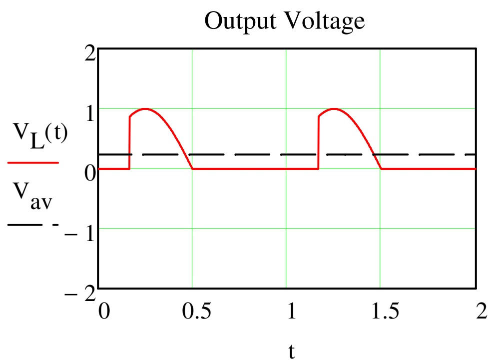


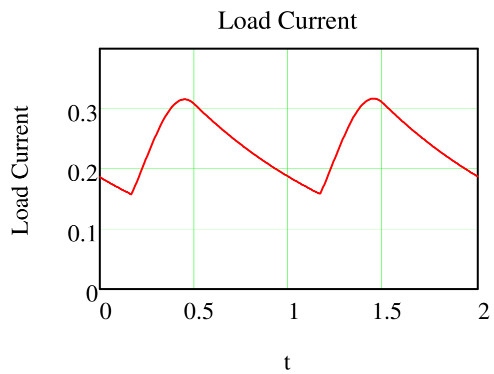
Solve equation for I(t)

$$D(t, I) := \frac{(V_L(t) - R \cdot I)}{L}$$

$\tau = 1$

$I_0 := 0 \qquad Z := \text{rkfixed}(I, -5 \cdot \tau, 2, 1000, D)$





3. Full-wave uncontrolled rectifier (Circuit B)

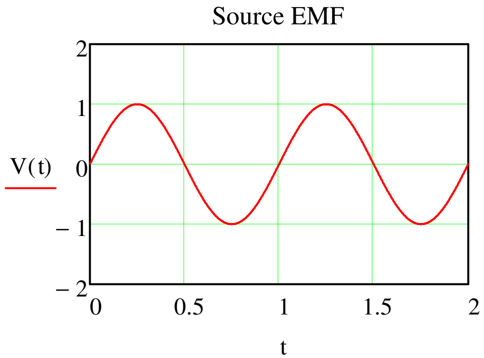
Source EMF $V(t) := V_{pk} \cdot \sin(2 \cdot \pi \cdot t)$

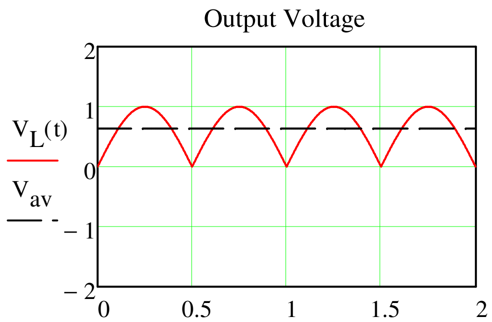
Switch 1 position $S_1(t) := \begin{array}{|l} 1 \;\; \text{if} \;\; (\sin(2 \cdot \pi \cdot t) > 0) \\ 0 \;\; \text{otherwise} \end{array}$

Switch 2 position $S_2(t) := \begin{array}{|l} 1 \;\; \text{if} \;\; (\sin(2 \cdot \pi \cdot t) < 0) \\ 0 \;\; \text{otherwise} \end{array}$

Output voltage $V_L(t) := \left(S_1(t) - S_2(t)\right) \cdot V(t)$

DC load voltage $V_{av} = \dfrac{V_{pk}}{\pi} \cdot \displaystyle\int_0^{\pi} \sin(\theta)\, d\theta$ $\qquad V_{av} := \dfrac{2 \cdot V_{pk}}{\pi}$ $\qquad V_{av} = 0.637$

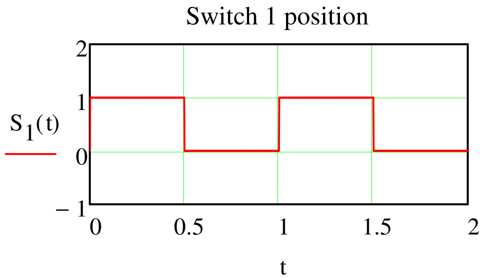


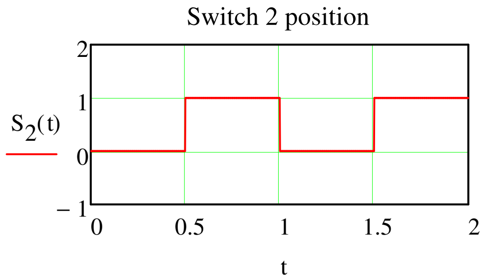


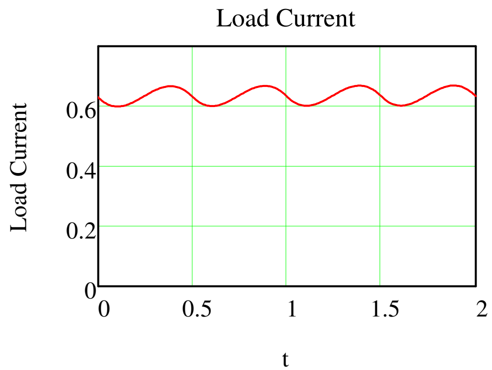
Solve equation for I(t)

$D(t, I) := \dfrac{\left(V_L(t) - R \cdot I\right)}{L}$ $\qquad \tau = 1$

$I_0 := 0$ $\qquad Z := \text{rkfixed}(I, -5 \cdot \tau, 2, 1000, D)$







4. Full-wave half-controlled rectifier (Circuit B)

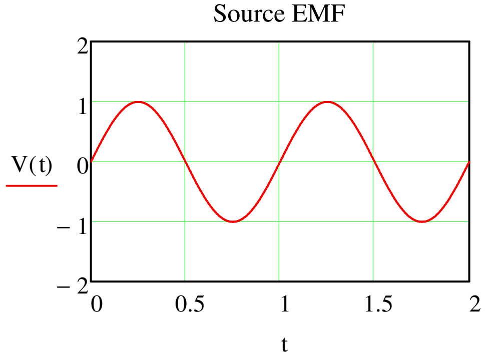
Source EMF $V(t) := V_{pk} \cdot \sin(2 \cdot \pi \cdot t)$ $\alpha := 60 \cdot deg$

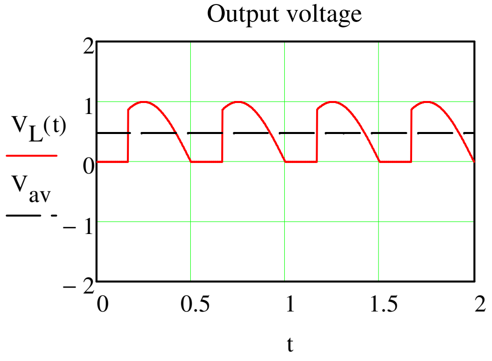
Switch 1 position $S_1(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) > 0) \wedge \sin(2\cdot\pi\cdot t - \alpha) > 0 \\ 0 & \text{otherwise} \end{cases}$

Switch 2 position $S_2(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) < 0) \wedge \sin(2\cdot\pi\cdot t - \alpha) < 0 \\ 0 & \text{otherwise} \end{cases}$

Output voltage $V_L(t) := (S_1(t) - S_2(t)) \cdot V(t)$

DC load voltage $V_{av} = \frac{V_{pk}}{\pi} \cdot \int_{\alpha}^{\pi} \sin(\theta)\, d\theta$ $V_{av} := \frac{V_{pk}}{\pi} \cdot (1 + \cos(\alpha))$ $V_{av} = 0.477$

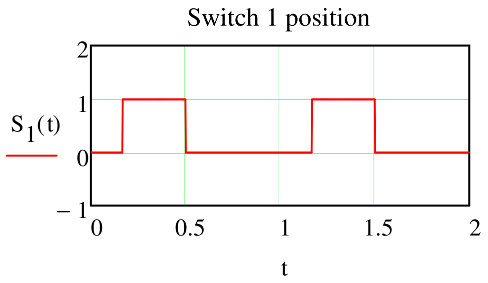


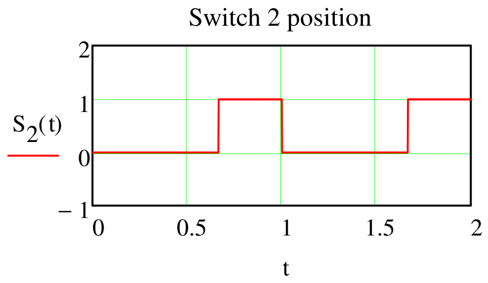


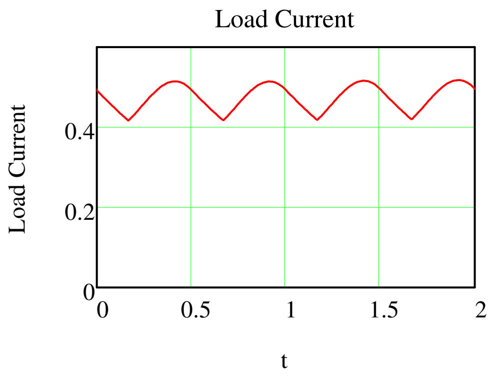
Solve equation for I(t)

$D(t, I) := \frac{(V_L(t) - R \cdot I)}{L}$ $\tau = 1$

$I_0 := 0$ $Z := \mathrm{rkfixed}(I, -5\cdot\tau, 2, 1000, D)$







5. Full-wave fully-controlled rectifier (Circuit B)

$\alpha := 60 \cdot deg$

Note: If $\alpha > 90$ degrees V_{av} is negative.

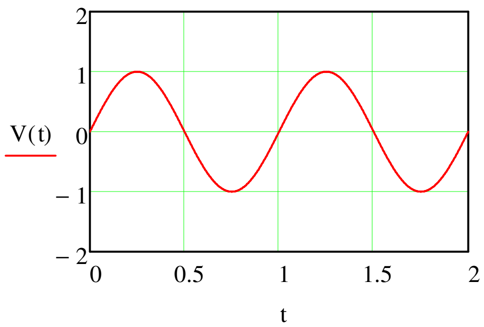
Source EMF $\quad V(t) := V_{pk} \cdot \sin(2 \cdot \pi \cdot t)$

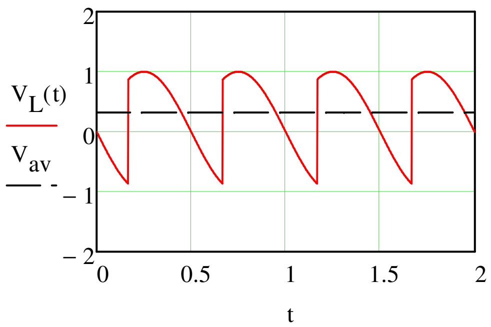
Switch 1 position $\quad S_1(t) := \begin{cases} 1 & \text{if } \sin(2 \cdot \pi \cdot t - \alpha) > 0 \\ 0 & \text{otherwise} \end{cases}$

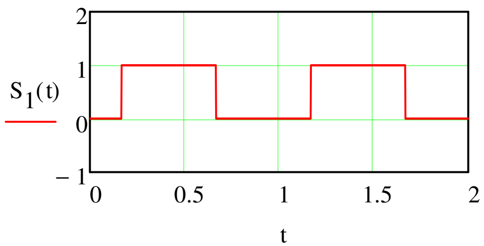
Switch 2 position $\quad S_2(t) := \begin{cases} 1 & \text{if } S_1(t) = 0 \\ 0 & \text{otherwise} \end{cases}$

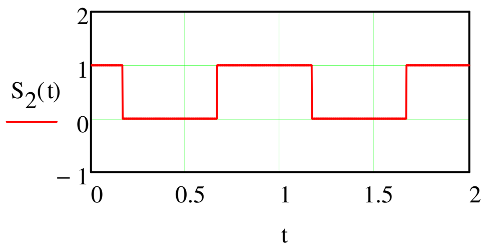
Output voltage $\quad V_L(t) := \left(S_1(t) - S_2(t)\right) \cdot V(t)$

DC load voltage $\quad V_{av} = \frac{V_{pk}}{\pi} \cdot \int_{\alpha}^{\pi+\alpha} \sin(\theta)\, d\theta \qquad V_{av} := \frac{2 \cdot V_{pk}}{\pi} \cdot \cos(\alpha) \qquad V_{av} = 0.318$





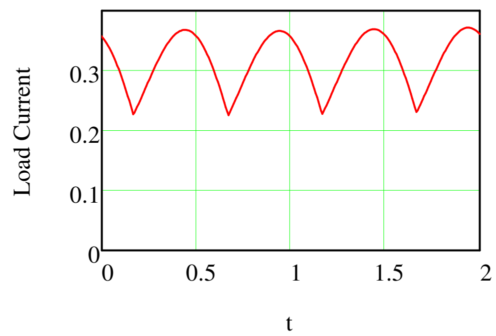




Solve equation for I(t)

$D(t, I) := \frac{\left(V_L(t) - R \cdot I\right)}{L} \qquad \tau = 1$

$I_0 := 0 \qquad Z := \text{rkfixed}(I, -5 \cdot \tau, 2, 1000, D)$



DC to AC converters (Inverters)

6. Square Wave Inverter (Circuit C)

Switch 1 position $S_1(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) > 0) \\ 0 & \text{otherwise} \end{cases}$

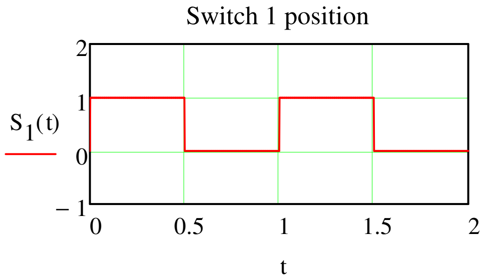
Switch 2 position $S_2(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) < 0) \\ 0 & \text{otherwise} \end{cases}$

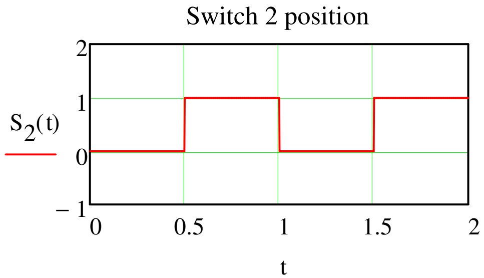
Output voltage $V_L(t) := \left(S_1(t) - S_2(t)\right)\cdot V_0$

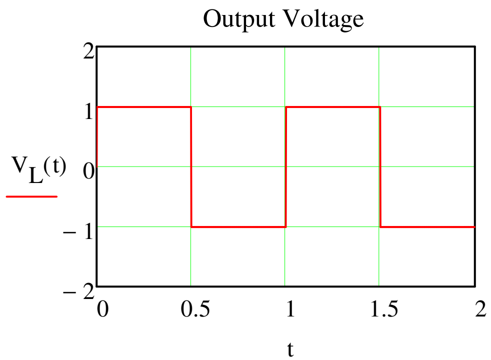
Solve equation for I(t)

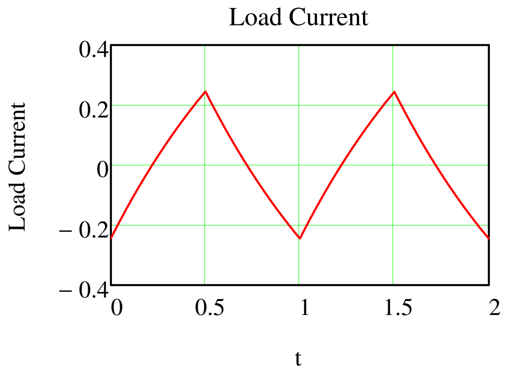
$$D(t, I) := \frac{\left(V_L(t) - R\cdot I\right)}{L} \qquad \tau = 1$$

$$I_0 := 0 \qquad Z := \text{rkfixed}(I, -5\cdot\tau, 2, 5000, D)$$









7. Quasi-Square Wave Inverter (Circuit C)

$\beta := 30 \cdot deg$

Switch 1 position $S_1(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) > \sin(\beta)) \\ 0 & \text{otherwise} \end{cases}$

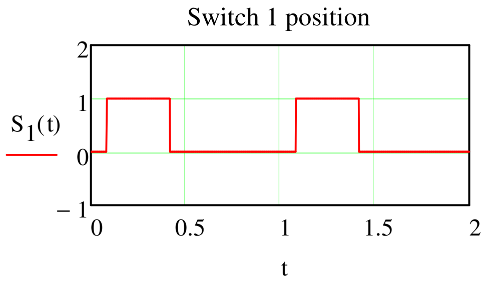
Switch 2 position $S_2(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) < -\sin(\beta)) \\ 0 & \text{otherwise} \end{cases}$

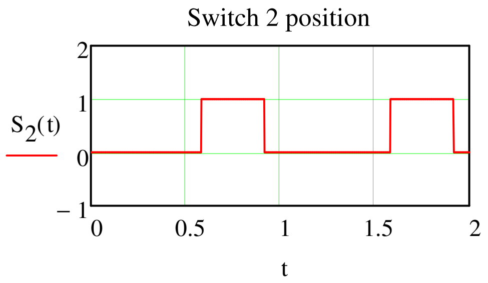
Output voltage $V_L(t) := (S_1(t) - S_2(t)) \cdot V_0$

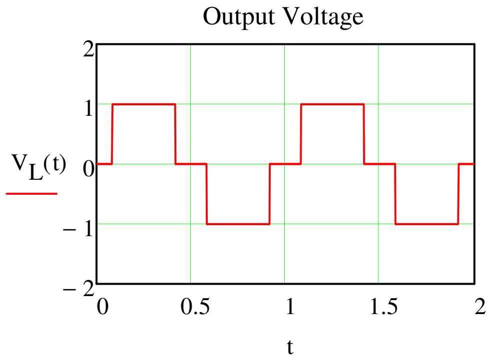
Solve equation for I(t)

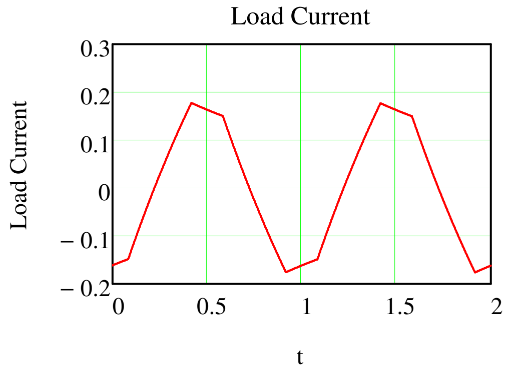
$I_0 := 0 \qquad D(t, I) := \dfrac{(V_L(t) - R \cdot I)}{L}$

$Z := \text{rkfixed}(I, -5\cdot\tau, 2, 5000, D)$





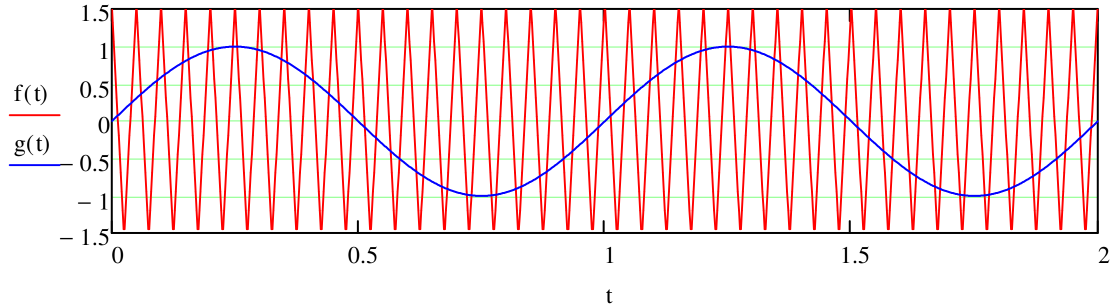




8. Pulse Width Modulated (PWM) Inverter (Alternate Switching, Circuit C)

Define a triangular wave f(t) with a frequency much higher than the waveform to be generated g(t)

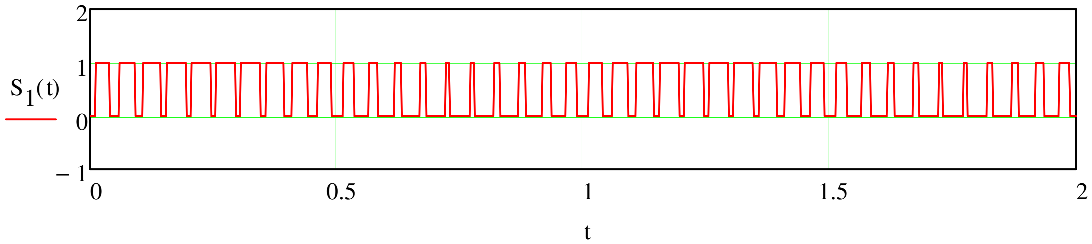
$f(t) := \operatorname{asin}(\cos(40\cdot\pi\cdot t))$ $\qquad$ $g(t) := \sin(2\cdot\pi\cdot t)$



Switch 1 position

$$S_1(t) := \begin{cases} 1 & \text{if } (\sin(2\cdot\pi\cdot t) > f(t)) \\ 0 & \text{otherwise} \end{cases}$$

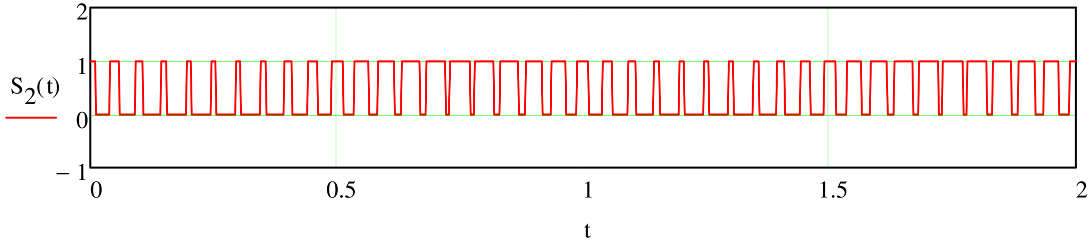
Switch 1 position



Switch 2 position

$$S_2(t) := \begin{cases} 0 & \text{if } S_1(t) = 1 \\ 1 & \text{otherwise} \end{cases}$$

Switch 2 position



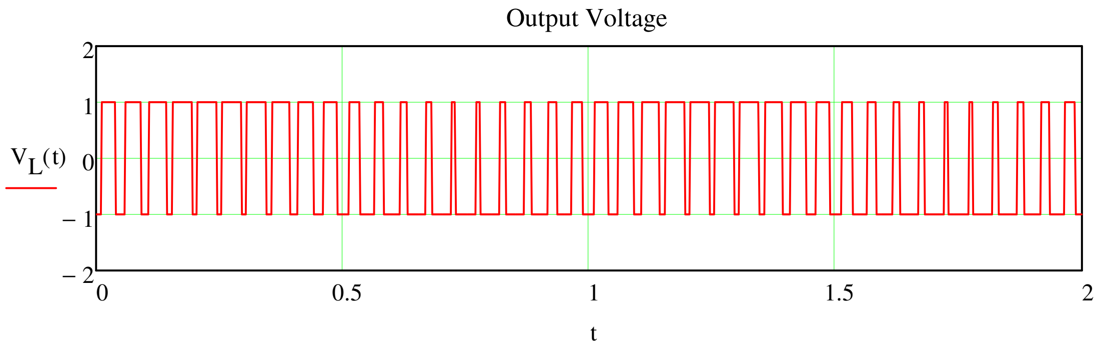
Output voltage

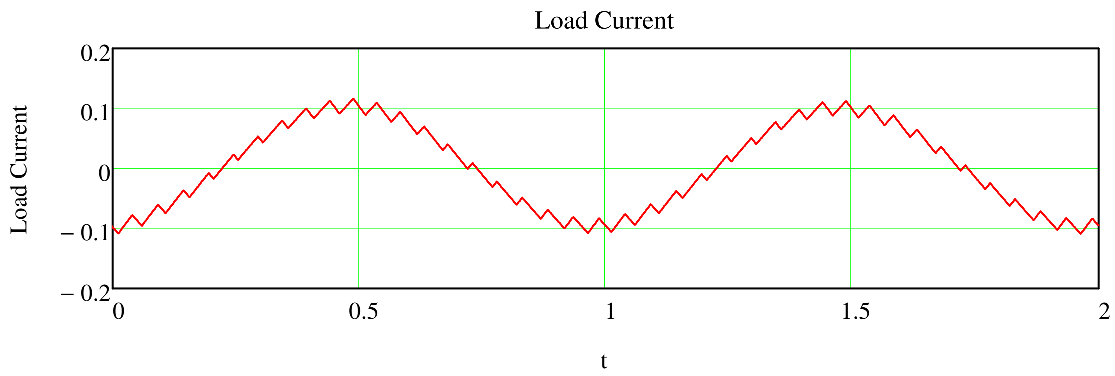
$$V_L(t) := \left(S_1(t) - S_2(t)\right)\cdot V_0$$

Solve equation for I(t)

$I_0 := 0$ $\quad\quad$ $D(t, I) := \dfrac{\left(V_L(t) - R \cdot I\right)}{L}$

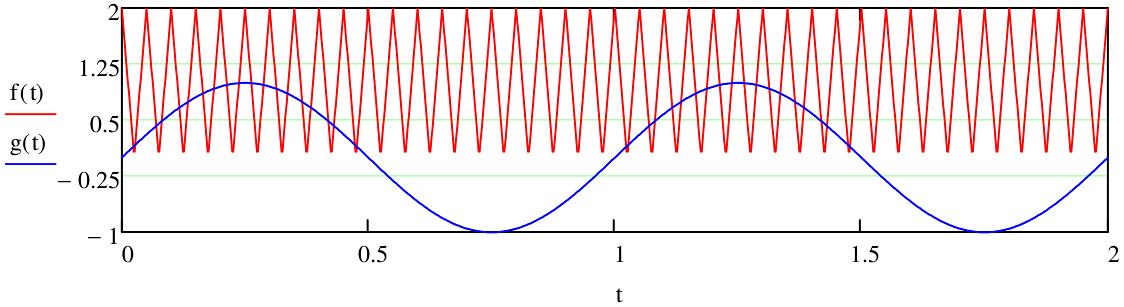
$Z := \mathrm{rkfixed}(I, -5 \cdot \tau, 2, 5000, D)$





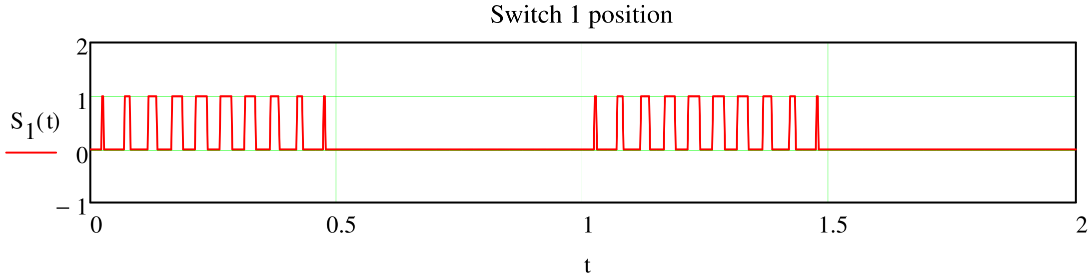
9. Pulse Width Modulated (PWM) Inverter (Sequential Switching, Circuit C)

$$f(t) := \frac{2}{\pi}\,\mathrm{asin}(\cos(40\cdot\pi\cdot t)) + 1 \qquad g(t) := \sin(2\cdot\pi\cdot t)$$



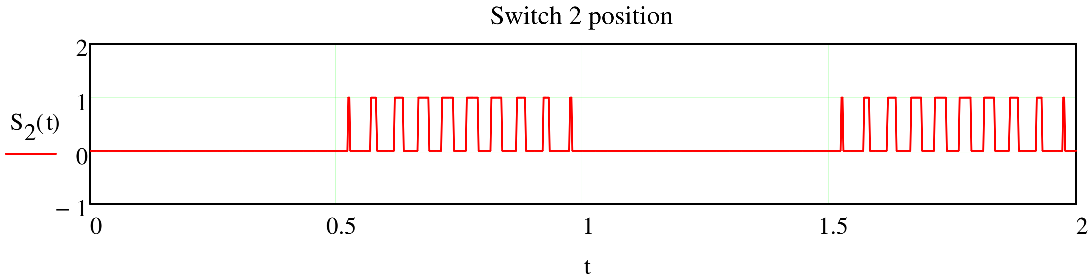
Switch 1 position

$$S_1(t) := \begin{cases} 1 & \text{if } (g(t) > f(t)) \wedge g(t) > 0 \\ 0 & \text{otherwise} \end{cases}$$



Switch 2 position

$$S_2(t) := \begin{cases} 1 & \text{if } -g(t) > f(t) \wedge g(t) < 0 \\ 0 & \text{otherwise} \end{cases}$$



Output voltage

$$V_L(t) := \left(S_1(t) - S_2(t)\right)\cdot V_0$$

Solve equation for I(t)

$I_0 := -0.01 \quad D(t, I) := \frac{\left(V_L(t) - R \cdot I\right)}{L}$

$Z := \text{rkfixed}(I, -5 \cdot \tau, 2, 5000, D)$