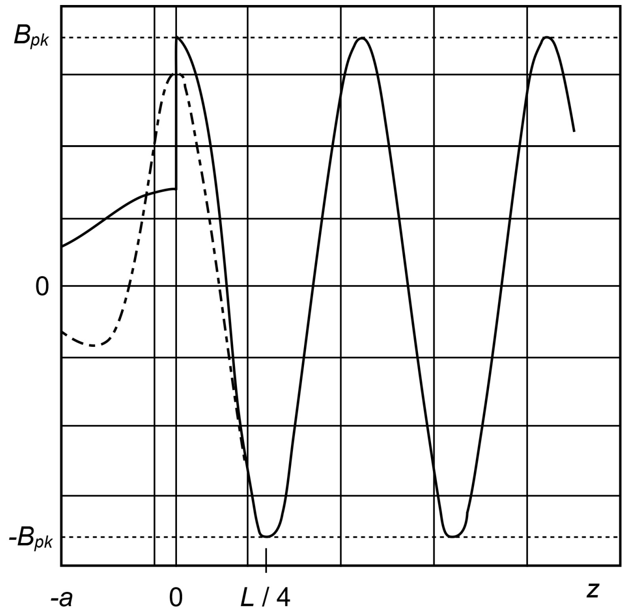
Worksheet 9.2 Beam entry into a PPM stack

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet calculates the beam edge profile for a p.p.m. focused beam using the method described by Harker (see Section 7.6.1)
Harker, K. J. (1955). "Periodic focusing of beams from partially shielded cathodes." IRE Transactions on Electron Devices **ED-2**(4): 13-19.

The beam entry conditions are the idealised as decribed in:
Rawls, J. L., J. R. Ashley, et al. (1967). "PPM focusing of convergent beams emerging from partially shielded cathodes." IEEE Transactions on Electron Devices **14**(6): 301-305.

The initial beam is assumed to have constant radius and the flux linked to it is equal to the cathode flux. It enters the PPM stack at a peak of the magnetic field as shown in the figure. The model allows the beam edge profile to be computed both with and without the entry conditions.



The PPM stack is defined using Harker's parameters

Magnetic field parameter	$\alpha := 0.1$	Equation 7.89	Control := Uniform stack / Entry field	
Space charge parameter	$\beta 1 := 0.0537$	Equation 7.90		
Cathode flux parameter	$K_1 := 0.2$	Equation 7.51	Initial radius	$R_0 := 1.929$
			Initial radial velocity	$R1_0 := -0.2656$

In the entry region the flux linked to the beam is constant and the beam radius follows the universal beam spreading curve. This can be investigated in three stages:

1. Select Uniform stack and set the initial radius $R_0 = 1$ and the radial velocity $R1_0 = 0$. Adjust the parameters to achieve minimum ripple flow with $R_{max} = 1$ (or any other desired condition.

2. Select Entry field and set the initial radius $R_0 = 1$ and the radial velocity $R1_0 = 0$. Note the radius R0 and the radial velocity R10 at the entrance to the PPM field.

3. Select Entry field and set R_0 =R0 and $R1_0$ = -R10. Now it should be found that R0 = 1 and R10 = 0. The properties of the beam within the stack are displayed. The effect of changes in the initial conditions and the stack parameters can be investigated.

Define the magnetic field and the cathode flux parameter as functions of the normalised axial position θ

$$B(\theta) := \left| \begin{array}{l} B \leftarrow \cos(\theta) \\ B \leftarrow 0 \ \text{ if } \ \theta < 2.0\cdot\pi \wedge \text{Control} = 2 \\ \text{return } B \end{array} \right.$$

$$KK(\theta) := \left| \begin{array}{l} K \leftarrow K_1 \\ K \leftarrow 0 \ \text{ if } \ \theta < 2.0\cdot\pi \wedge \text{Control} = 2 \\ \text{return } K \end{array} \right.$$

The differential equation is solved for the given initial conditions

$$R1 := \begin{pmatrix} R_0 \\ R1_0 \end{pmatrix}$$

$$D(\theta, R1) := \begin{bmatrix} R1_1 \\ \dfrac{\beta 1}{R1_0} + \dfrac{2 \cdot \alpha \cdot KK(\theta)}{\left(R1_0\right)^3} - 2 \cdot \alpha B(\theta)^2 \cdot R1_0 \end{bmatrix}$$

Equation 7.53

$$\Theta := \text{AdamsBDF}(R1, 0, 10 \cdot \pi, 1000, D)$$

The results are stored in the matrix Θ. In order the columns contain: the normalised time θ, the normalised radius R, and the normalised radial velocity dR/dθ. Calculate the maximum, minimum and mean beam radius and the ripple within the PPM stack, i.e. where n > 200

$$R0 := \left(\Theta^{\langle 1 \rangle}\right)_{200}$$

$$R10 := \left(\Theta^{\langle 2 \rangle}\right)_{200}$$

$$rr := \begin{array}{|l} \text{for } n \in 0..800 \\ \quad rr_n \leftarrow \left(\Theta^{\langle 1 \rangle}\right)_{n+200} \\ \text{return } rr \end{array}$$

$$R_{min} := \min(rr)$$

$$R_{mean} := 0.5 \cdot (\max(rr) + \min(rr))$$

$$R_{max} := \max(rr)$$

$$ripple := \frac{R_{max} - R_{min}}{R_{max} + R_{min}}$$

$R_{max} = 1$ $R_{min} = 0.947$ $R_{mean} = 0.974$ $ripple = 2.74 \cdot \%$ $R0 = 1$ $R10 = 0$