# Worksheet 7.5 Periodic Electrostatic Focusing



This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.



This worksheet calculates the beam edge profile for a beam with periodic electrostatic focusing. See section 7.6.3

The parameters $\beta$, $U_1$ are set and the differential equations governing the beam edge are solved given the initial normalised radius (R) and slope (dR/dθ) of the beam edge. The axial variation of the electric field is as cos Z .

The maximum value of the normalised time θ and the number of integrations steps are set.

$\beta := 0.01$ $U_1 := 0.351$ $R_0 := 1$ $R1_0 := 0$ $\theta_{max} := 12 \cdot \pi$ $nsteps := 1000$

The section below can be collapsed to allow the input parameters and the results to be displayed on the same screen.

$$\theta = \beta_0 \cdot u_0 \cdot t$$

$$R = \frac{r}{r_0} \qquad \beta = \frac{1}{2} \cdot \frac{\beta_p^{\;2}}{\beta_0^{\;2}} \qquad U_1 = \frac{V_1}{V_0}$$

$$Z = \beta_0 \cdot z$$

$$\frac{d^2R}{d\theta^2} = \beta \frac{1}{R} + \frac{1}{4} U_1 R \cos(Z) \qquad \boxed{\text{Equation 7.111}}$$

$$\frac{d^2Z}{d\theta^2} = -\frac{1}{2} U_1 \sin(Z) \qquad \boxed{\text{Equation 7.112}}$$

The differential equation is solved for the initial conditions R = $R_0$ and dR/dθ = $R1_0$

$$D(\theta, R1) := \begin{pmatrix} R1_1 \\ \dfrac{\beta}{R1_0} - \dfrac{U_1}{4} \cdot R1_0 \cdot \cos\left(R1_2\right) \\ R1_3 \\ \dfrac{1}{2} \cdot U_1 \cdot \sin\left(R1_2\right) \end{pmatrix} \qquad \begin{pmatrix} R1_0 \\ R1_1 \\ R1_2 \\ R1_3 \end{pmatrix} = \begin{pmatrix} R \\ \dfrac{dR}{d\theta} \\ Z \\ \dfrac{dZ}{d\theta} \end{pmatrix}$$

The paraxial ray equations referred to the injection radius are expressed as simultaneous first order equations

$$R1 := \begin{pmatrix} R_0 \\ R1_0 \\ 0 \\ \sqrt{1 - U_1} \end{pmatrix}$$

$$\Theta := \text{AdamsBDF}\left(R1, 0, \theta_{max}, \text{nsteps}, D\right)$$

The solution to the differential equations is contained in the matrix Θ. The first column contains the normalised time θ, and the remaining columns contain R, dr/dθ, Z and dZ/dθ.

$$R_{min} := \min\left(\Theta^{\langle 1\rangle}\right) \qquad R_{max} := \max\left(\Theta^{\langle 1\rangle}\right) \qquad ripple := \frac{\max\left(\Theta^{\langle 1\rangle}\right) - \min\left(\Theta^{\langle 1\rangle}\right)}{\max\left(\Theta^{\langle 1\rangle}\right) + \min\left(\Theta^{\langle 1\rangle}\right)} \qquad R_{mean} := 0.5\cdot\left(\max\left(\Theta^{\langle 1\rangle}\right) + \min\left(\Theta^{\langle 1\rangle}\right)\right)$$

Find the ripple as a function of β and U1 and, hence find the value of U1 that minimises the ripple.

$$\mathrm{Ripple}\left(\beta, U_1\right) := \left| \begin{array}{l} R1 \leftarrow \begin{pmatrix} R_0 \\ R1_0 \\ 0 \\ \sqrt{1 - U_1} \end{pmatrix} \\ D(\theta, R1) \leftarrow \begin{pmatrix} R1_1 \\ \dfrac{\beta}{R1_0} - \dfrac{U_1}{4}\cdot R1_0\cdot\cos\left(R1_2\right) \\ R1_3 \\ \dfrac{1}{2}\cdot U_1\cdot\sin\left(R1_2\right) \end{pmatrix} \\ \Theta \leftarrow \mathrm{AdamsBDF}\left(R1, 0, \theta_{max}, nsteps, D\right) \\ \mathrm{Ripple} \leftarrow \dfrac{\max\left(\Theta^{\langle 1\rangle}\right) - \min\left(\Theta^{\langle 1\rangle}\right)}{\max\left(\Theta^{\langle 1\rangle}\right) + \min\left(\Theta^{\langle 1\rangle}\right)} \\ \mathrm{return\ \ Ripple} \end{array} \right.$$

$$\mathrm{Rip}\left(U_1\right) := \left|\mathrm{Ripple}\left(\beta, U_1\right)\right| \qquad U_{1min} := \mathrm{Minimize}\left(\mathrm{Rip}, U_1\right)$$

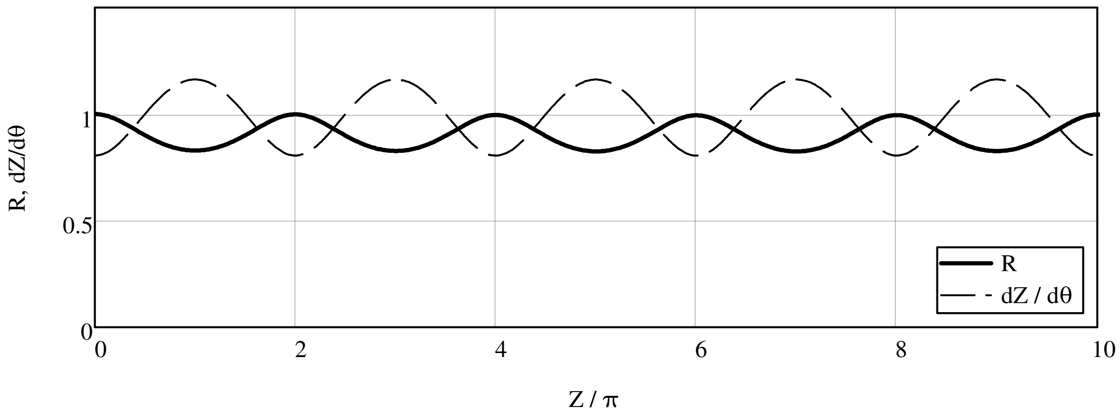


Figure 7.15

$\beta = 0.01$    $U_1 = 0.351$

$R_{min} = 0.825$    $R_{max} = 1$

$R_{mean} = 0.912$    $ripple = 9.6 \cdot \%$

**Minimum ripple solutions**

The minimum ripple solution is obtained by finding the minimum value of $U_1$ for which $R_{max} = 1$.
Copy the results into the data table. The first column is β, the second is the value of $U_1$ for minimum ripple and the third is the ripple.

$U_{1min} = 0.350$    $Ripple(\beta, U_{1min}) = \blacksquare$

X0 :=

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | $1 \cdot 10^{-3}$ | 0.106 | 0.027 |
| 2 | $2.5 \cdot 10^{-3}$ | 0.17 | 0.043 |
| 3 | $5 \cdot 10^{-3}$ | 0.243 | 0.062 |
| 4 | 0.01 | 0.35 | 0.093 |
| 5 | 0.015 | 0.434 | 0.119 |
| 6 | 0.02 | 0.506 | 0.145 |
| 7 | 0.025 | 0.57 | 0.171 |
| 8 | 0.03 | 0.63 | 0.199 |
| 9 | 0.035 | 0.686 | 0.231 |
| 10 | 0.04 | 0.731 | 0.269 |

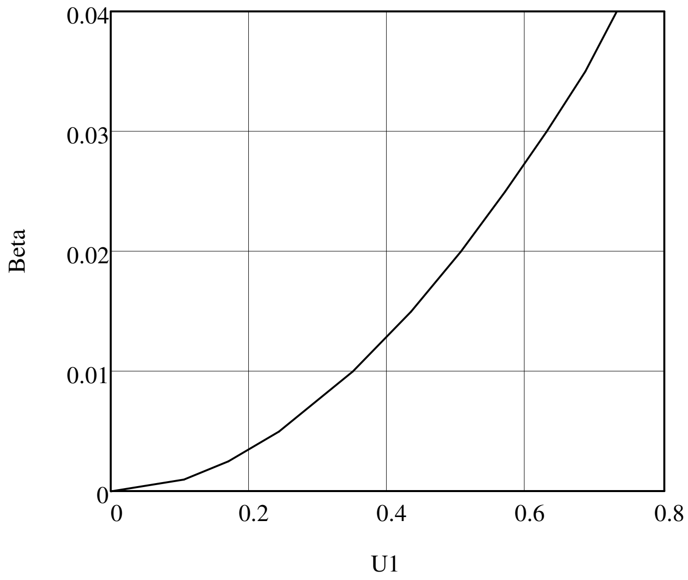


Figure 7.16

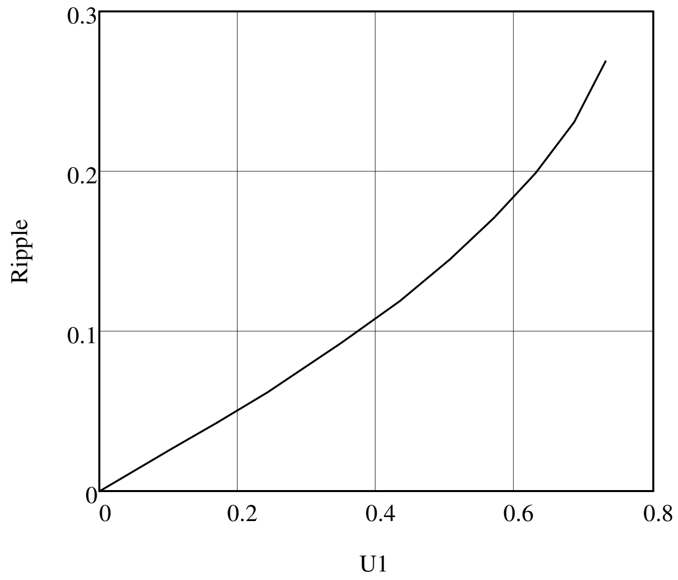


Figure 7.17