WS 13.6 Klystron design worksheet

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet can be used to investigate the design of a klystron (see Section 13.4). The data provided is based on:

T. G. Lee, *et al.*, "A fifty megawatt klystron for the Stanford Linear Collider," in *International Electron Devices Meeting,* 1983, pp. 144-147.

1. Statement of requirements

Centre frequency	Peak RF power	Saturated gain	Pulse length	Pulse repetition frequency
$f_0 := 2.856 \cdot \mathrm{GHz}$	$P_{pk} := 50 \cdot \mathrm{MW}$	50 dB	$\tau := 5 \cdot 10^{-6} \cdot \mathrm{sec}$	$prf := 180 \cdot \mathrm{Hz}$

Physical constants

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \frac{\mathrm{C}}{\mathrm{kg}}$ Perveance $\mu P := \mu A \cdot V^{-1.5}$ $dB := 1$

2. Compute the beam current and voltage.

Estimate the conversion efficiency $\eta_e := 50\%$ See figures 13.11 and 13.13

DC beam power $P_{dc} := \frac{P_{pk}}{\eta_e}$

Choose the perveance of the gun $perv := 2 \cdot \mu P$

The perveance was selected to reduce the problems with gun breakdown which might arise if a lower perveance were chosen.

Compute the anode voltage $V_a := \left(\frac{P_{dc}}{perv}\right)^{0.4}$ $V_a = 302 \cdot kV$

Compute the beam current $I_0 := \frac{P_{dc}}{V_a}$ $I_0 = 331\ A$

3. Calculate the beam velocity and the electronic propagation constant

Choose the beam filling factor (b_a) $b\_a := \frac{2}{3}$

$$V_0 := \begin{array}{|l} V_0 \leftarrow V_a \\ \text{for } n \in 0..3 \\ \quad \begin{array}{|l} u_n \leftarrow c \cdot \left[1 - \frac{1}{\left(1 + \frac{\eta \cdot V_n}{c^2}\right)^2}\right]^{0.5} \\ V_{n+1} \leftarrow V_0 - \frac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left(\frac{1}{2} - \ln(b\_a)\right) \end{array} \\ \text{return } V_{n+1} \end{array}$$

$$u_0 := c \cdot \left[1 - \frac{1}{\left(1 + \frac{\eta \cdot V_0}{c^2}\right)^2}\right]^{0.5} \qquad Rel := \frac{1}{\sqrt{1 - \frac{u_0^{\,2}}{c^2}}}$$

$V_0 = 278.1 \cdot kV$ $u_0 = 2.284 \times 10^8 \frac{m}{s}$ $Rel = 1.544$

4. **Find the beam and tunnel radii**

$\omega_0 := 2\cdot\pi\cdot f_0$ $\beta_e := \dfrac{\omega_0}{u_0}$ $\beta_e = 78.6\,\dfrac{1}{m}$ $\gamma_e := \sqrt{\beta_e^{\,2} - \dfrac{\omega_0^{\,2}}{c^2}}$ $\lambda_e := \dfrac{2\cdot\pi}{\beta_e}$

Choose the normalised beam radius $\gamma eb := 0.5$

Beam radius $b := \dfrac{\gamma eb}{\gamma_e}$ Tunnel radius $a := \dfrac{b}{b\_a}$ $b = 9.83\cdot mm$

$a = 14.74\cdot mm$

5. **Calculate the plasma frequency and the reduced plasma frequency**

Plasma frequency $\omega_p := \sqrt{\dfrac{\eta}{\varepsilon_0}\cdot\dfrac{I_0}{\pi\cdot b^2\cdot u_0}\cdot\dfrac{1}{Rel^3}}$

Brillouin field $B_B := \dfrac{\sqrt{2}\cdot\omega_p\cdot Rel}{\eta}$ $B_B = 0.063\,T$

Choose the ratio of the focusing field to the Brillouin field to give adequate beam stiffness $mB := 2$

Magnetic field $B_0 := mB\cdot B_B$ $B_0 = 0.126\,T$

Calculate the plasma frequency reduction factor (see Worksheet 11.2)

$$\tau b(\beta b, m, p) := \beta b\cdot\left[\frac{\dfrac{1}{p^2} - 1}{\dfrac{1}{p^2 - 2\cdot\left(m^2 - 1\right)} - 1}\right]^{\frac{1}{2}}$$

$$fn1(\beta b, A) := \frac{1}{\beta b}\cdot\frac{I1(\beta b)\cdot K0(A\cdot\beta b) + I0(A\cdot\beta b)\cdot K1(\beta b)}{I0(\beta b)\cdot K0(A\cdot\beta b) - I0(A\cdot\beta b)\cdot K0(\beta b)}$$

$$fn2(\beta b, m, p) := \frac{1 - \dfrac{1}{p^2}}{\tau b(\beta b, m, p)}\cdot\frac{I1(\tau b(\beta b, m, p))}{I0(\tau b(\beta b, m, p))}$$

$$fn(\beta b, A, m, p) := \frac{1}{fn1(\beta b, A)} - \frac{1}{fn2(\beta b, m, p)}$$

$p := 0.9$ $P(\beta b, A, m) := root(fn(\beta b, A, m, p), p)$

Reduced plasma frequency $\omega_q := P\left(\gamma eb, \frac{1}{b\_a}, mB\right)\cdot\omega_p$ $\beta_q := \frac{\omega_q}{u_0}$ $\lambda_q := \frac{2\cdot\pi}{\beta_q}$ $\frac{\omega_q}{\omega_0} = 0.077$

Propagation constants of the fast and slow space-charge waves at the centre frequency

$\beta_f := \beta_e - \beta_q$ $\beta_s := \beta_e + \beta_q$ $\gamma_f := \sqrt{\beta_f^{\,2} - \frac{\omega_0^{\,2}}{c^2}}$ $\gamma_s := \sqrt{\beta_s^{\,2} - \frac{\omega_0^{\,2}}{c^2}}$ $\lambda_q = 1.038\,m$

Electronic admittance $Y_e := \frac{I_0}{Rel\cdot(Rel+1)\cdot V_0}\cdot\frac{\omega_0}{\omega_q}$ Equation 11.80 $Y_e = 3.935\times 10^{-3}\,\frac{1}{\Omega}$

6. Compute the cathode disk radius.

Choose the cathode current density $J_c := 5.6\cdot A\cdot cm^{-2}$

Beam current density $J_b := \frac{I_0}{\pi\cdot b^2}$

Calculate the cathode disc radius $r_c := b\cdot\sqrt{\frac{J_b}{J_c}}$ Area convergence $\left(\frac{r_c}{b}\right)^2 = 19.5$ $r_c = 43.4\cdot mm$

7. Choose the gap length and calculate the gap coupling factor

Choose the gap length $gap := 1.0\cdot a$ $gap = 14.74\cdot mm$

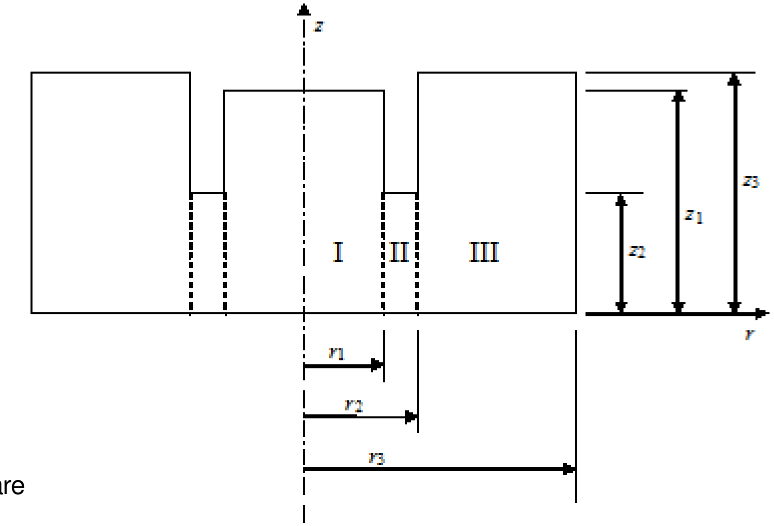
For initial calculations it is assumed that all the gaps are the same and the couling factor does not vary with frequency.

8. Design the cavities using Fujisawa's formulae (see Section 3.5.2)

Choose the cavity height (see Figure 3.15)
This should be as large as possible
consistent with a solution for r_3 below

$$z_3 := 0.6 \cdot gap$$

$$z_2 := \frac{gap}{2}$$



Define the outer radius of the drift tube with a wall thickness of 5 mm

$$r_1 := a \qquad r_2 := a + 5 \cdot mm$$

$$z_1 := 2 \cdot z_3$$

The equivalent circuit parameters for the singly re-entrant cavity in the diagram are

$$C_I := 2\varepsilon_0 \cdot \int_{-20\cdot\beta_e}^{20\cdot\beta_e} \frac{r_1 \cdot I1(\beta \cdot r_1)}{\beta \cdot I0(\beta \cdot r_1)} \cdot \mathrm{sinc}(\beta \cdot z_2)\, d\beta \qquad C_{II} := \frac{2 \cdot \varepsilon_0 \cdot \pi \cdot \left(r_2{}^2 - r_1{}^2\right)}{gap}$$

$$C_{III}(r_3) := 4 \cdot \varepsilon_0 \cdot r_2 \cdot \ln\left[\frac{e \cdot \sqrt{(r_3 - r_2)^2 + z_3{}^2}}{gap}\right] \qquad C_0(r_1) := C_I + C_{II} + C_{III}(r_1)$$

$$L_0(r_3) := \frac{\mu_0 \cdot z_3}{2 \cdot \pi} \cdot \ln\left(\frac{r_3}{r_2}\right) \qquad \omega_c(r_3) := \frac{1}{\sqrt{L_0(r_3) \cdot C_0(r_3)}}$$

Find the outer radius of the cavity

$$r3 := 5 \cdot r_2 \qquad r_3 := \mathrm{root}(\omega_c(r3) - \omega_0, r3)$$

$$z_3 = 8.8\,mm$$

$$z_2 = 7.4\,mm$$

$$r_1 = 14.7\,mm$$

$$r_2 = 19.7\,mm$$

$$r_3 = 42.0\,mm$$

R/Q for a doubly re-entrant cavity $R\_Q := 2\cdot\sqrt{\frac{L_0(r_3)}{C_0(r_3)}}$ $R\_Q = 47.9\cdot\Omega$

It may be necessary to adjust the length of the gap and/or the tunnel radius to find an acceptable solution. The results can be checked using Worksheet 3.3.

$$Qe := \begin{array}{|l} \text{for } n \in 0..NCAV \\ \quad Qe_n \leftarrow 95000 \\ Qe_0 \leftarrow 0 \\ \text{return } Qe \end{array}$$

Assume a value for Q_0 allowing for increased surface loss (see Worksheet 3.3) $Q_0 := 4000$

9. Calculate the impedances of the cavities

Assume that the properties of all the cavities are the same apart from their resonant frequencies

$$Zc(\omega_c,\omega) := \frac{R\_Q}{\frac{1}{Q_0} + j\cdot\left(\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega}\right)} \qquad Yc(\omega_c,\omega) := \frac{1}{Zc(\omega_c,\omega)}$$

10. Calculate the gap coupling factor and the beam loading conductance

Calculate the gap coupling factor $M(\beta_e,\gamma_e) := \frac{2\cdot I1(\gamma_e\cdot b)}{\gamma_e\cdot b\cdot I0(\gamma_e\cdot a)}\cdot\frac{\sin(0.5\cdot\beta_e\cdot gap)}{0.5\cdot\beta_e\cdot gap}$ $M(\beta_e,\gamma_e) = 0.851$

Calculate the beam loadings conductance $G_b := \frac{Y_e}{4}\cdot\left(M(\beta_f,\gamma_f)^2 - M(\beta_s,\gamma_s)^2\right)$ $G_b = 1.303\times 10^{-4}\,S$

For a matched input the source conductance must be $Ge_1 := G_b + Yc(\omega_0,\omega_0)$

The external Q of the input cavity is $Qe_1 := \frac{1}{Ge_1\cdot R\_Q}$ $Qe_1 = 154$

11. Find the output parameters

RF input power $P_{in} := 100 \cdot W$

Assume that at the penultimate cavity $I_1 = 1.2 \cdot I_0$

Assume that at the output cavity $I_1 := 1.6 \cdot I_0$ See figure 13.10(a)

Choose the final drift length $d_5 := 0.5 \cdot \lambda_e$ See figure 13.10(b) $\frac{d_5}{\lambda_q} = 0.039$

The effective gap coupling factor is $M_{eff} := 0.6$ See figure 11.32 given $M(\beta_e, \gamma_e) = 0.851$

Hence the output gap voltage required is $V_{g6} := \frac{2 \cdot P_{pk}}{I_1 \cdot M_{eff}}$ $V_{g6} = 314.3\,kV$

The output load resistance is $R_L := \frac{V_{g6}^2}{2 \cdot P_{pk}}$ $R_L = 988\,\Omega$

And the external Q of the output cavity is $Qe_{NCAV} := \frac{R_L}{R\_Q}$ $Ge_{NCAV} := \frac{1}{R_L}$ $Qe_{NCAV} = 20.6$

12. Design the interaction structure

The positions of the cavities are defined in terms of the reduced plasma wavelength. The distances between the first few cavities are small compared with the reduced plasma wavelength. The drift before the penultimate cavity is chosen to be a little more than a quarter of a plasma wavelength to increase the efficiency. The drift length between the penultimate cavity and the output cavity can be determined from Figure 13.10(b).

The positions and frequencies of the cavities are adjusted using the small-signal gain curve to achieve small-signal gain exceeding 56 dB at the band centre and a normalised RF current of at least 1.2 at the entrance to the penultimate cavity.

Cavity frequency

$$\omega c := \begin{pmatrix} 0 \\ 0.998 \\ 0.996 \\ 1.002 \\ 1.003 \\ 1.01 \\ 1.0 \end{pmatrix} \cdot \omega_0$$

Gap position

$$zg := \begin{pmatrix} 0 \\ 0 \\ 0.05 \\ 0.1 \\ 0.15 \\ 0.45 \\ 0.49 \end{pmatrix} \cdot \lambda_q$$

Number of cavities

$NCAV \equiv 6$

The section below may be collapsed to allow the data and the results to be seen on the same screen

13. Compute the small signal gain using the method in Section 13.2.4

$$X(\omega) := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad \left| \begin{array}{l} MM_n \leftarrow M(\beta_e, \gamma_e) \\ YY_n \leftarrow G_b + Yc(\omega c_n, \omega) + Ge_n \end{array} \right. \\ Vg_{1,0} \leftarrow \dfrac{\sqrt{8 \cdot Ge_1 \cdot P_{in}}}{|YY_1|} \\ \text{for } n \in 2..NCAV \\ \quad \left| \begin{array}{l} I1_n \leftarrow 0 \\ \text{for } m \in 1..(n-1) \\ \quad I1_n \leftarrow I1_n + j \cdot Y_e \cdot MM_m \cdot Vg_{m,0} \cdot \sin\left[\beta_q \cdot (zg_n - zg_m)\right] \cdot \exp\left[-j \cdot \beta_e \cdot (zg_n - zg_m)\right] \\ Vg_{n,0} \leftarrow -\dfrac{MM_n}{YY_n} \cdot I1_n \\ Vg_{n,1} \leftarrow I1_n \cdot \Omega \end{array} \right. \\ Vg \end{array} \right.$$

$$Vg(\omega) := X(\omega)^{\langle 0 \rangle} \qquad I_{rf}(\omega) := \frac{X(\omega)^{\langle 1 \rangle}}{\Omega}$$

Find the small-signal gain

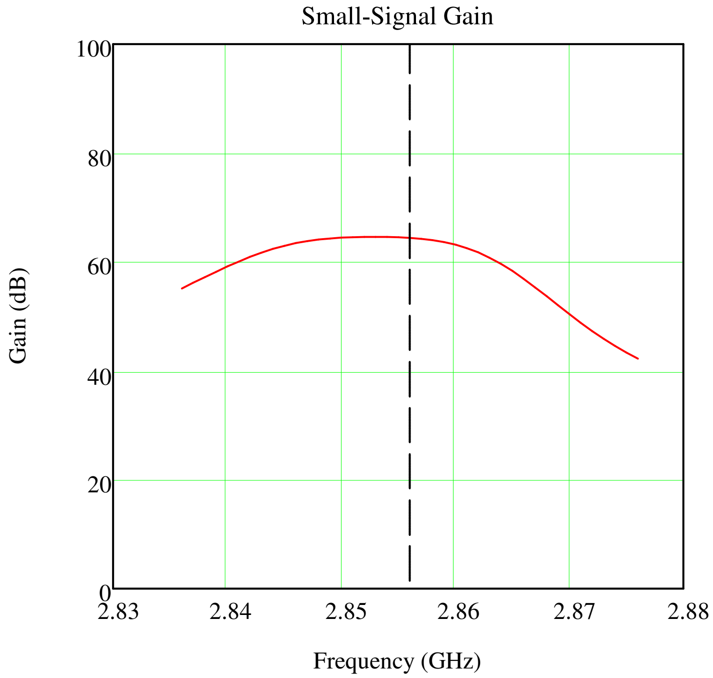
$$P_{out}(\omega) := \frac{1}{2} \cdot \left(\left| Vg(\omega)_6 \right| \right)^2 \cdot Ge_6$$

$$Gain(\omega) := 10 \cdot \log\left(\frac{P_{out}(\omega)}{P_{in}} \right)$$

Find the normalised amplitudes of the gap voltages and the RF beam current at each gap

$$ModV := \left| \begin{array}{l} \text{for } n \in 0..NCAV \\ \quad ModV_n \leftarrow \left| Vg(\omega_0)_n \right| \\ \dfrac{ModV}{V_0} \end{array} \right. \qquad ModI := \left| \begin{array}{l} \text{for } n \in 0..NCAV \\ \quad ModI_n \leftarrow \left| I_{rf}(\omega_0)_n \right| \\ \dfrac{ModI}{I_0} \end{array} \right.$$

$f := 2.836\cdot GHz, 2.837\cdot GHz .. 2.876\cdot GHz$



Small-signal gain at the band centre

$\text{Gain}(\omega_0) = 64.4\ \text{dB}$

Normalised gap voltages (Vg / V0) and RF currents (I1 / I0) at the gaps.
Note that the first element of each matrix is not used.

$$\text{ModV} = \begin{pmatrix} 0.000 \\ 0.004 \\ 0.017 \\ 0.066 \\ 0.403 \\ 2.816 \\ 2.650 \end{pmatrix} \qquad \text{ModI} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.004 \\ 0.010 \\ 0.073 \\ 1.214 \\ 3.000 \end{pmatrix}$$

The complete data for the tube for input to Worksheets 13.1 and 13.3

Anode voltage	Beam current	Centre frequency	Input power	Magnetic field
$V_a = 302\,\text{kV}$	$I_0 = 331\,\text{A}$	$f_0 = 2856\,\text{MHz}$	$P_{in} = 100\,\text{W}$	$B_0 = 0.126\,\text{T}$

Tunnel radius	Beam radius	Number of cavities
$a = 14.7\,\text{mm}$	$b = 9.83\,\text{mm}$	$NCAV = 6$

Cavity frequency

$$\frac{\omega c}{2\cdot\pi} = \begin{pmatrix} 0 \\ 2850 \\ 2845 \\ 2862 \\ 2865 \\ 2885 \\ 2856 \end{pmatrix} \text{MHz}$$

Cavity harmonic

$$nh := \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

External Q

$$Qe = \begin{pmatrix} 0 \\ 154 \\ 95000 \\ 95000 \\ 95000 \\ 95000 \\ 21 \end{pmatrix}$$

Unloaded Q

$$Q_0 = 4000$$

R/Q

$$R\_Q = 47.9\,\Omega$$

Gap length

$$gap = 14.7\,\text{mm}$$

Gap position

$$zg = \begin{pmatrix} 0 \\ 0 \\ 0.052 \\ 0.104 \\ 0.156 \\ 0.467 \\ 0.508 \end{pmatrix} \text{m}$$

Cavity dimensions for input into Worksheet 3.3

$r_1 = 14.7\,\text{mm}$ $\quad z_1 = 17.7\,\text{mm}$

$r_2 = 19.7\,\text{mm}$ $\quad z_2 = 7.4\,\text{mm}$

$r_3 = 42.0\,\text{mm}$ $\quad z_3 = 8.8\,\text{mm}$