WS 15.4 Magnetron design

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet implements the design procedure for a magnetron decribed in Section 15.7.3. Note that the results differ slightly from those given in the book

The specification is based on that described in
Shibata, C., H. Tamai, et al. (1991). "High-power (500 kW) c.w. magnetron for industrial heating."
Electrical Engineering in Japan 111(2): 94-100.

Outline Statement of Requirements

Frequency	$f := 912.6 \cdot MHz$
RF output power	$P_{RF} := 500 \cdot kW$
Duty cycle	$Duty := 1.0$

Define constants

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1}$

Thermal conductivity of copper $\kappa_{th} := 401 \cdot W \cdot m^{-1} \cdot K^{-1}$

Angular frequency $\omega := 2 \cdot \pi \cdot f$

Wavelength $\lambda := \frac{c}{f}$ $\lambda = 328.5 \cdot mm$

i) Estimate the efficiency and compute the anode voltage and current

Estimated RF efficiency $\eta_0 := 0.75$

The figures in the book correspond to 75% not 80% as stated on p.620.

Choose the DC impedance $R_{DC} := 2.9 \cdot k\Omega$

See Table 15.4

DC input power $P_{DC} := \frac{P_{RF}}{\eta_0}$ $P_{DC} = 667 \cdot kW$

Anode voltage $V_a := \sqrt{P_{DC} \cdot R_{DC}}$ $V_a = 44.0 \cdot kV$

Anode current $I_a := \frac{P_{DC}}{V_a}$ $I_a = 15.2\ A$

ii) Find the normalised magnetic field and the normalised theshold voltage

Estimated circuit efficiency $\eta_c := 0.9$

Electronic efficiency $\eta_e := \frac{\eta_0}{\eta_c}$ $\eta_e = 0.83$

Normalised magnetic field (B / B_0) $BB0 := \frac{3 - \eta_e}{2 \cdot (1 - \eta_e)}$ From equation 15.49 $BB0 = 6.5$

Normalised theshold voltage (V_T / V_0) $VTV0 := 2 \cdot BB0 - 1$ Equation 15.40 $VTV0 = 12.0$

iii) Find the threshold, characteristic and cut-off voltages

Estimate V_a / V_T $\quad VaVT := 1.0$

Threshold voltage $\quad V_T := \dfrac{V_a}{VaVT}$ $\quad V_T = 44.0 \cdot kV$

Characteristic voltage $\quad V_0 := \dfrac{V_T}{VTV0}$ $\quad V_0 = 3.7 \cdot kV$

Cut-off voltage $\quad V_H := V_0 \cdot BB0^2$ $\quad$ Equation 15.25 $\quad V_H = 155 \cdot kV$

iv) Investigate possible anode parameters

Anode radius $\quad r_a(N_v) := \dfrac{\sqrt{2 \cdot \eta \cdot V_0}}{2 \cdot \omega} \cdot N_v$ $\quad$ Equation 15.146

Choose the Modified Slater Factor $\quad R' := 2.0$

$$ff(N_v) := \left(\frac{N_v}{R'}\right) \cdot \sqrt{1 - \frac{V_a}{V_H}}$$

Normalised anode radius ($R_a = r_a / r_c$) $\quad R_a(N_v) := \dfrac{ff(N_v) + 1}{ff(N_v) - 1}$ $\quad$ From equation 15.154

Cathode radius $\quad r_c(N_v) := \dfrac{r_a(N_v)}{R_a(N_v)}$

Choose the anode length $\quad L_a := 0.16 \cdot \lambda$ $\quad L_a = 52.6 \cdot mm$

Estimate returned current fraction	$\alpha := 0.75$		
Cathode current density	$J_c(N_v) := \dfrac{I_a}{2\cdot\pi\cdot r_c(N_v)\cdot L_a\cdot(1-\alpha)}$		
Maximum anode dissipation	$P_a := P_{DC} - P_{RF}$		$P_a = 167\,kW$
Anode pitch	$p := \dfrac{\pi\sqrt{2\cdot\eta\cdot V_0}}{\omega}$	From equation 15.22	$p = 20\,mm$
Choose anode vane thickness	$t := 12\cdot mm$		
Total cross-sectional area of the vanes	$A_v(N_v) := N_v\cdot L_a\cdot t$		
Choose thermal path length	$L_{th} := 10\cdot mm$		
Choose cooling channel temperature	$T_r := 293\cdot K$		
Vane tip temperature	$T_{tip}(N_v) := \dfrac{P_a\cdot L_{th}}{\kappa_{th}\cdot A_v(N_v)} + T_r$	Equation 15.156	
Choose a range of numbers of vanes	$Nr_v := 8, 10 .. 24$		

Table 15.5

$Nr_v =$	$r_a(Nr_v) =$ (·mm)	$r_c(Nr_v) =$ (·mm)	$J_c(Nr_v) =$ ($\cdot A\cdot cm^{-2}$)	$T_{tip}(Nr_v) =$ (K)
8	25.0	13.6	1.35	1117
10	31.3	19.3	0.95	952
12	37.6	25.2	0.73	842
14	43.8	31.2	0.59	764
16	50.1	37.2	0.49	705
18	56.4	43.3	0.42	659
20	62.6	49.4	0.37	622
22	68.9	55.5	0.33	593
24	75.1	61.7	0.30	568

Choose the number of vanes to give acceptable cathode loading and vane tip temperature $N_v := 14$

Anode radius $r_a(N_v) = 43.8\ \text{mm}$ Cathode radius $r_c(N_v) = 31.2\ \text{mm}$

v) Find the magnetic field $R_a(N_v) = 1.41$

Characteristic field $$B_0 := \frac{4\cdot\omega}{\eta\cdot N_v\cdot\left(1-\frac{1}{R_a(N_v)^2}\right)}$$ Equation 15.23 $B_0 = 0.019\,T$

Magnetic field $B_z := BB0\cdot B_0$ $B_z = 0.122\,T$

Cyclotron frequency $\omega_c := \eta\cdot B_z$ $\frac{\omega_c}{\omega} = 3.76$

vi) Check the threshold voltage for the $\pi - 1$ mode for which $n = \frac{N_v}{2} - 1$

Characteristic voltage $$V0 := \left(\frac{N_v}{N_v - 2}\right)^2 \cdot V_0$$

Characteristic field $$B0 := \left(\frac{N_v}{N_v - 2}\right) \cdot B_0$$

Theshold voltage $$VT := V0 \cdot \left(\frac{2 \cdot B_z}{B0} - 1\right)$$ $$VT = 50.6\,\text{kV}$$