## Worksheet 10.3 Multi-element depressed collector model (IOT)



This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.



**This sheet calculates the properties of an IOT with a multi-element depressed collector**

Frequency $f := 1.3 \cdot GHz$

Anode voltage $V_a := 25 \cdot kV$

Grid bias voltage $V_{g0} := -105 \cdot V$

Tunnel radius $a := 11 \cdot mm$

Beam radius $b := 6.5 \cdot mm$

Output gap length $gap := 11 \cdot mm$

Gridded gun parameters

$$I(V_{.g}, \theta) = I_0 \cdot \left(1 + \frac{\mu \cdot V_g(V_g, \theta)}{V_a}\right)^{pwr}$$

$I_0 \equiv 0.9 \cdot A$ $\mu \equiv 150$ $pwr \equiv 1.5$

Source resistance $R_s \equiv 220 \cdot \Omega$

Load resistance $R_L \equiv 28 \cdot k\Omega$

Maximum RF grid voltage $V_{g1\_max} := 350 \cdot V$

Number of collector stages N_stages := 5

Normalised electrode voltages

Vc :=

| | 0 |
|---|---|
| 0 | 0 |
| 1 | 0.2 |
| 2 | 0.4 |
| 3 | 0.6 |
| 4 | 0.8 |
| 5 | 1 |
| 6 | |

Collection probability function

probability :=

- Constant
- Linear
- Sine
- Sine squared

The section below can be collapsed to hide the detailed calcualtions if required

The fraction of the current collected by each stage is estimated by assuming a probability distribution. Current begins to be collected on an electrode when the electron voltage exceeds the stage voltage and rises to 100% when the electron voltage is just less than the voltage of the next stage. Current not collected at the stage voltage is collected at the voltage of the preceding stage. For the final stage the presence of a fictitious electrode is assumed such that the voltage difference between it and the nth voltage is equal to that between the nth and (n-1)th stages. See Section 10.3.2

Define the probability distributions

$$p(x) := \begin{array}{|l} 1 \;\; \text{if probability} = 1 \\ x \;\; \text{if probability} = 2 \\ \sin\left(\frac{\pi \cdot x}{2}\right) \;\; \text{if probability} = 3 \\ \sin\left(\frac{\pi \cdot x}{2}\right)^2 \;\; \text{if probability} = 4 \end{array}$$

Variable probability

$$pc(Ve) := \begin{array}{|l} Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ \text{for } n \in N\_stages .. 0 \\ \quad pc_n \leftarrow \begin{array}{|l} p\left(\frac{Ve - Vc_n}{Vc_{n+1} - Vc_n}\right) \;\; \text{if } Ve \geq Vc_n \wedge Ve < Vc_{n+1} \\ 1 \;\; \text{if } n = N\_stages \wedge Ve \geq Vc_{n+1} \\ 1 \;\; \text{if } n = 0 \wedge Ve < Vc_1 \\ (1 - pc_{n+1}) \;\; \text{if } n < N\_stages \wedge (Ve \geq Vc_{n+1} \wedge Ve < Vc_{n+2}) \\ 0 \;\; \text{otherwise} \end{array} \\ \text{return } pc \end{array}$$

Constant probability

$$pc0(Ve) := \begin{array}{|l} Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ \text{for } n \in N\_stages .. 0 \\ \quad pc_n \leftarrow \begin{array}{|l} 1 \;\; \text{if } Ve \geq Vc_n \wedge Ve < Vc_{n+1} \\ 1 \;\; \text{if } n = N\_stages \wedge Ve \geq Vc_{n+1} \\ 1 \;\; \text{if } n = 0 \wedge Ve < Vc_1 \\ 0 \;\; \text{otherwise} \end{array} \\ \text{return } pc \end{array}$$

Plot the probability of collection by each electrode as a function of the electron energy with rectangles showing constant probability for comparison.

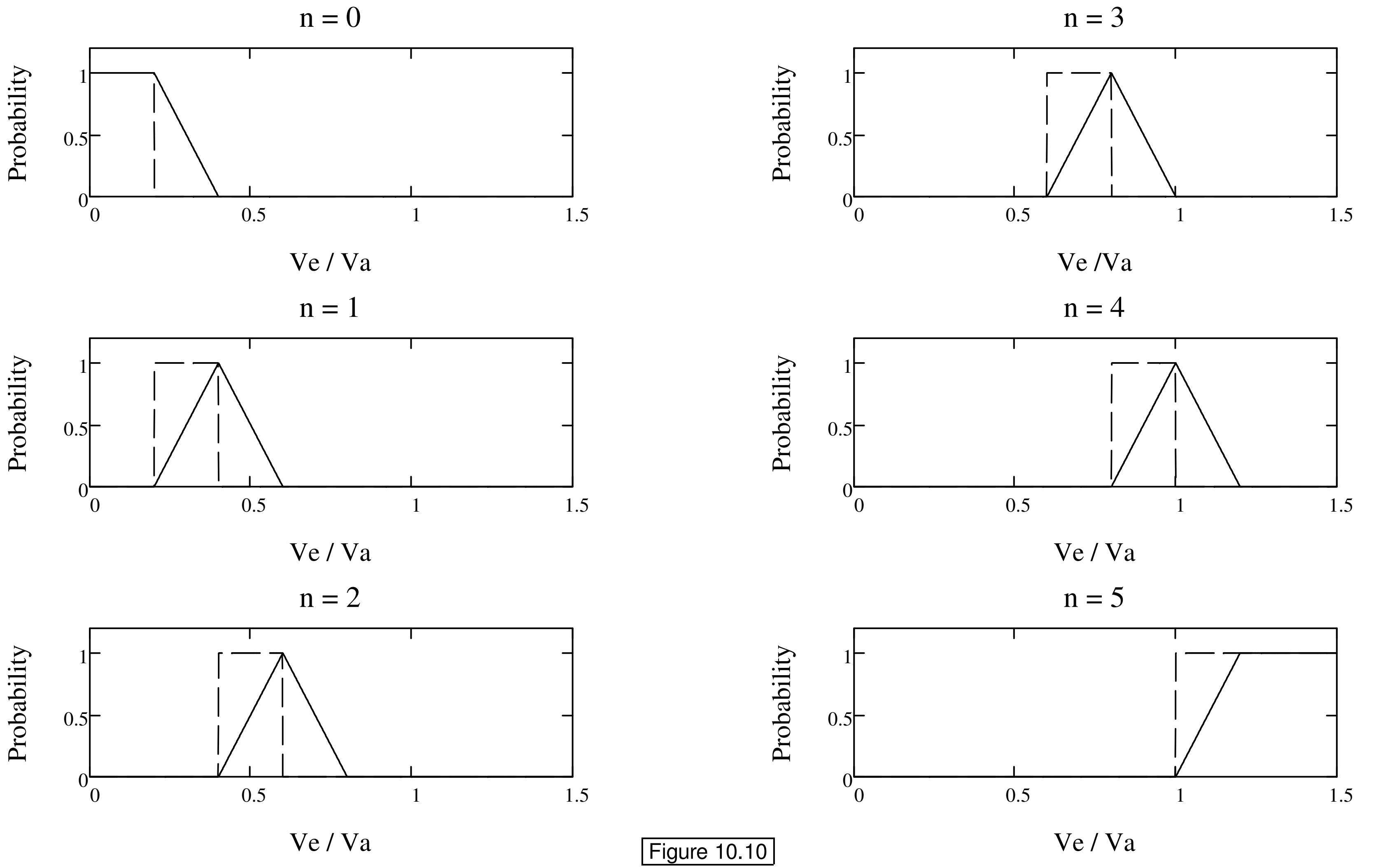



Figure 10.10

**Calculate the DC collector efficiency as a function of electron energy**

$$\eta_{DC}(Ve) := \frac{1}{Ve} \cdot \sum_{n=0}^{N\_stages} \left(pc(Ve)_n \cdot Vc_n\right)$$

$\boxed{\text{Equation 10.26}}$

Ve is the electron energy in eV divided by the maximum collection potential.

**Calculate the IOT spent beam data using the simple model in Worksheet 13.5**

**Define the grid voltage** as a function of the amplitude of the DC and RF grid voltages and the phase angle.

$$V_g\left(V_{g1}, \theta\right) := V_{g0} + V_{g1} \cdot \cos(\theta)$$

**Define the instantaneous beam current** as a function of d.c. and r.f. grid voltages and the phase angle. The values for $\mu$, $I_0$ and for the power are adjusted later to fit the model to the data. Note: the equation has been written in this form because Mathcad does not accept variable exponents on dimensional quantities

$$I\left(V_{g1}, \theta\right) := \left| \begin{array}{l} I \leftarrow I_0 \cdot \left(1 + \dfrac{\mu \cdot V_g\left(V_{g1}, \theta\right)}{V_a}\right)^{pwr} \quad \text{if } V_a + \mu \cdot V_g\left(V_{g1}, \theta\right) > 0 \\ I \leftarrow 0 \quad \text{otherwise} \\ \text{return } I \end{array} \right.$$

$\boxed{\text{Equation 6.31}}$

**Compute the DC current and the first harmonic of the RF current** as a function of drive level using Fourier analysis.

$$I_b\left(V_{g1}\right) := \frac{1}{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} I\left(V_{g1}, \theta\right) d\theta \qquad I_1\left(V_{g1}\right) := \frac{1}{\pi} \cdot \int_0^{2 \cdot \pi} I\left(V_{g1}, \theta\right) \cdot \cos(\theta)\, d\theta$$

**The input impedance is**

$$R_{in}(V_{g1}) := \begin{cases} \dfrac{V_{g1}}{I_1(V_{g1})} & \text{if } V_{g1} \neq 0 \\ R_s & \text{otherwise} \end{cases}$$

Equation 12.42

**The input power measured using a directional coupler in the input line with source impedance $R_s$ is**

$$P_{in}(V_{g1}) := \frac{1}{8} \cdot \left( \frac{R_{in}(V_{g1}) + R_s}{R_{in}(V_{g1})} \right)^2 \cdot \frac{V_{g1}^{\ 2}}{R_s}$$

Equation 12.43

**Calculate the DC beam velocity ($u_0$).**

Define the charge/mass ratio and the rest energy (eV) of the electron

$$\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1} \qquad V_R := 511 \cdot kV$$

$$V_0 := \begin{array}{|l} V_0 \leftarrow V_a \\ \text{for } n \in 0..3 \\ \quad \begin{array}{|l} u_n \leftarrow c \cdot \left[ 1 - \dfrac{1}{\left(1 + \dfrac{V_n}{V_R}\right)^2} \right]^{0.5} \\ V_{n+1} \leftarrow V_0 - \dfrac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left( \dfrac{1}{2} - \ln\left( \dfrac{b}{a} \right) \right) \end{array} \\ \text{return } V_{n+1} \end{array}$$

Equation 1.4

Equation 7.8

$$V_0 = 24.8 \cdot kV$$

$$u_0 := c \cdot \left[ 1 - \frac{1}{\left(1 + \frac{V_0}{V_R}\right)^2} \right]^{0.5}$$

Equation 1.4

$$u_0 = 9.018 \times 10^7 \frac{m}{s}$$

**Define the gap coupling factor as a function of electron velocity**

$$\omega := 2\cdot\pi\cdot f \qquad \beta_e := \frac{\omega}{u_0} \qquad \beta e(u) := \frac{\omega}{u}$$

$$M(u) := \left(\frac{\sin(0.5\cdot\beta e(u)\cdot gap)}{0.5\cdot\beta e(u)\cdot gap}\right)\cdot\left(\frac{2\cdot I1(\beta e(u)\cdot b)}{\beta e(u)\cdot b\cdot I0(\beta e(u)\cdot a)}\right)$$

Equations 11.36 and 11.38

**Use an iterative calculation to find mutually consistent values of the effective gap coupling factor and the exit velocity ($u_s$).**

A damping term has been added to ensure fast convergence. **A** piecewise constant approximation to the velocity is used

$$M_{eff} = \frac{M(u_0) + M(u_s)}{2}$$

Equation 11.168

$$u_s(V_{g1}) := \left|\begin{array}{l} us_0 \leftarrow u_0\cdot 0.5 \\ Mu0 \leftarrow M(u_0) \\ \text{for } i \in 1..100 \\ \quad \left|\begin{array}{l} Mu \leftarrow 0.5\cdot\left(Mu0 + M\left(us_{i-1}\right)\right) \\ us_i \leftarrow c\cdot\left[1 - \dfrac{1}{\left[1 + \dfrac{\eta\cdot\left(V_a - Mu^2\cdot I_1(V_{g1})\cdot R_L\right)}{c^2}\right]^2}\right]^{0.5} \\ us_i \leftarrow 0.5\cdot\left(us_i + us_{i-1}\right) \\ \text{break if } \left|\dfrac{us_i - us_{i-1}}{u_0}\right| < 10^{-6} \end{array}\right. \\ us_i \leftarrow 0 \text{ if } i = 100 \\ \text{return } us_i \end{array}\right.$$

**Calculate the ouput r.f. power and efficiency**

$$P_{out}(V_{g1}) := \frac{1}{2}\cdot\left[I_1(V_{g1})\cdot\left(\frac{M(u_0)+M(u_s(V_{g1}))}{2}\right)\right]^2\cdot R_L \qquad P_{DC}(V_{g1}) := I_b(V_{g1})\cdot V_a \qquad \eta_e(V_{g1}) := \frac{P_{out}(V_{g1})}{P_{DC}(V_{g1})}$$

**Calculate the normalised spent beam distribution**

$$V_{sn}(V_{g1},\theta) := \left|\begin{array}{l} us0 \leftarrow u_s(V_{g1}) \\ V_{eff} \leftarrow \left(\frac{M(u_0)+M(u_s(V_{g1}))}{2}\right)^2\cdot I_1(V_{g1})\cdot R_L \\ V_s(\theta) \leftarrow 1-\frac{V_{eff}}{V_a}\cdot\cos(\theta) \\ \text{return } V_s(\theta) \end{array}\right.$$

$$I_{sn}(V_{g1},\theta) := \left|\begin{array}{l} I1(\theta) \leftarrow I(V_{g1},\theta) \\ Ib \leftarrow I_b(V_{g1}) \\ I_s(\theta) \leftarrow \frac{-1}{\pi\cdot Ib}\cdot\int_{\pi}^{\theta} I1(\theta)\,d\theta \\ \text{return } I_s(\theta) \end{array}\right.$$

Check the total current and the total power entering the collector

$$I_{b\_max} := I_b(V_{g1\_max})$$

$$P_{ent}(V_{g1}) := \frac{1}{\pi\cdot I_{b\_max}}\cdot\int_0^{\pi} I(V_{g1},\theta)\cdot V_{sn}(V_{g1},\theta)\,d\theta$$

Check $\quad 1-P_{ent}(V_{g1\_max}) = 0.586 \qquad\qquad \eta_e(V_{g1\_max}) = 0.586$

Calculate the rf efficiency of the collector as a function of the RF grid voltage $V_{g1}$

$$P_{rec}(V_{g1}) := \frac{1}{\pi \cdot I_{b\_max}} \cdot \int_0^{\pi} I(V_{g1}, \theta) \cdot V_{sn}(V_{g1}, \theta) \cdot \eta_{DC}(V_{sn}(V_{g1}, \theta))\, d\theta$$

$$\eta_c(V_{g1}) := \frac{P_{rec}(V_{g1})}{P_{ent}(V_{g1})}$$

$$\eta_c(V_{g1\_max}) = 0.517$$

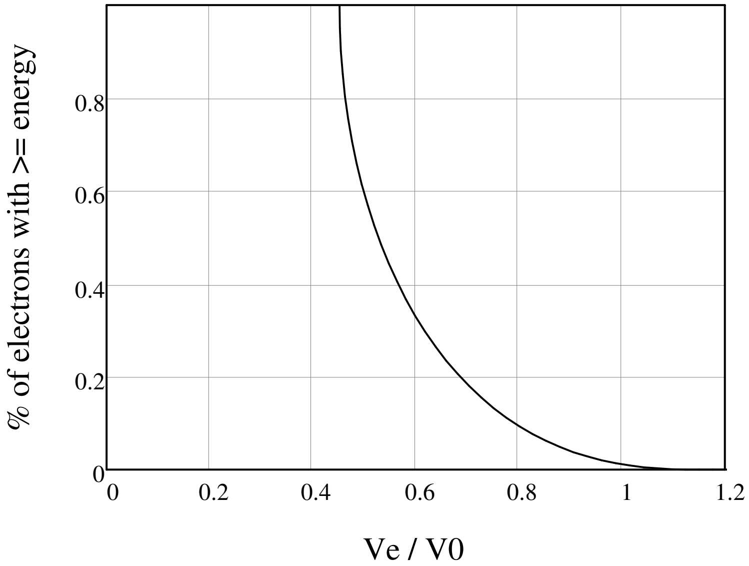
Compute the RF efficiency as a function of drive level

$$\eta_{rf}(V_{g1}) := \frac{\eta_e(V_{g1})}{1 - \eta_c(V_{g1}) \cdot (1 - \eta_e(V_{g1}))}$$

Equation 10.13

Plotting range $\theta := 0, 0.05 .. \pi$

RF grid voltage $V_{g1} := 220 \cdot V$ $P_{out}(V_{g1}) = 6.65 \cdot kW$




Compare Figure 12.19(b)

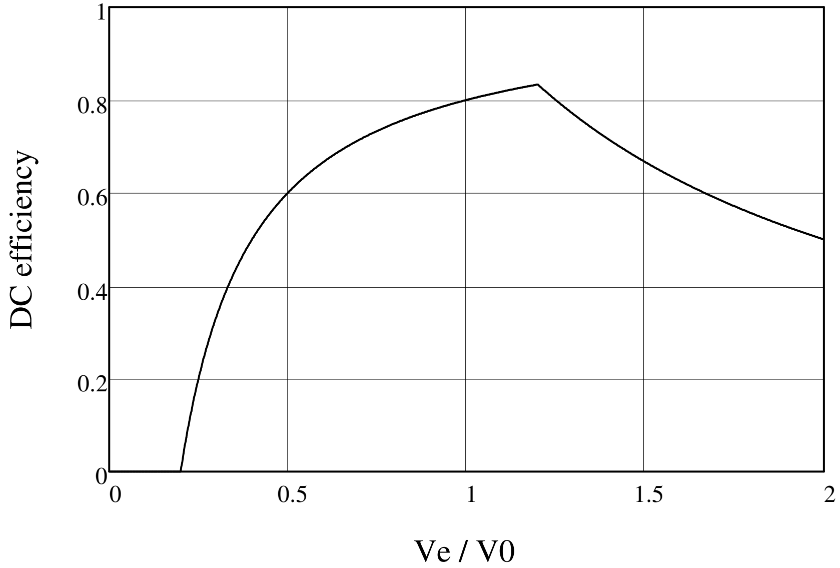



Figure 10.11

Plotting range $Vg1 := 0 \cdot V, 10 \cdot V .. V_{g1\_max}$

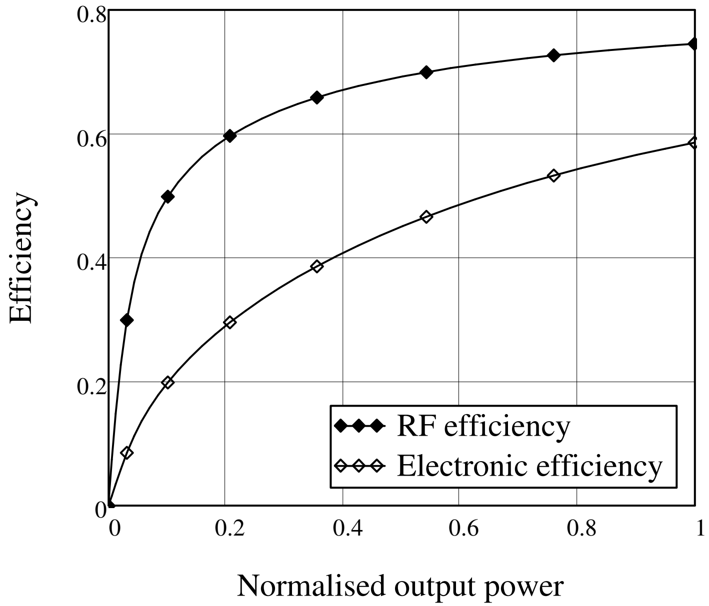



Figure 12.20

Note this figure differs slightly from the one in the book because of small changes in the model.