Worksheet 13.5 Klystron spent-beam characteristics

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet computes the spent-beam chararacteristics of a klystron based on some simplifying assumptions

The normalised beam current waveforms at the output gap assume that the waveform at saturation is the ideal bunching waveform with two harmonics. The drive level is denoted by X which is unity at saturation. It is assumed that the first harmonic amplitude is proportional to X and the second harmonic to X squared. This is equivalent to assuming that the modulation is sinusoidal at the penultimate gap so that all the harmonic content is generated by that gap. No attempt has been made to model the change in the bunching length with drive level.

This model appears to be a good approximation for braod-band klystrons such as those used for UHF TV.

Normalised parameters calculated at the centre frequency and the DC beam velocity

Beam radius $\beta eb := 0.6$ Efficiency at saturation $\eta_{sat} := 0.53$

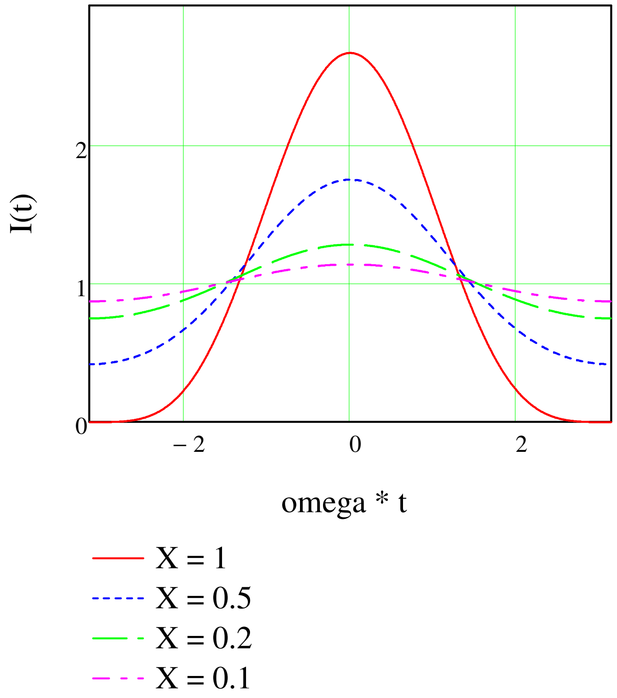
Tunnel radius $\beta ea := 1.0$

Outout gap length $\beta eg := 1.0$

Beam current waveform at the output gap $I(\theta, X) := 1 + 1.33 \cdot X \cdot \cos(\theta) + 0.33 \cdot X^2 \cdot \cos(2\theta)$ Equation 13.52

Normalised fundamental RF beam current $I_1(X) := 1.33 \cdot X$ Equation 13.53

Plot the normalised current waveforms for a range of drive levels



Reciprocal of the mean of the DC and spent electron velocities normalised to the DC velocity as a function of the normalised spent electron energy (Vs / V0).

$$U(V_s) := \frac{2}{1 + \sqrt{V_s}} \qquad \left(U = \frac{2 \cdot u_0}{u_0 + u_s} \right)$$

Effective gap coupling factor

$$M(U) := \frac{2 \cdot I1(\beta eb \cdot U)}{\beta eb \cdot U \cdot I0(\beta ea)} \cdot \mathrm{sinc}(0.5 \cdot U \cdot \beta eg)$$

At saturation the effective normalised gap voltage is

$$V_{eff\_sat} := \frac{2 \cdot \eta_{sat}}{I_1(1)} \qquad \boxed{\text{Equation 13.54}}$$

The normalised energy of the slowest electrons is

$$V_{min\_sat} := 1 - V_{eff\_sat}$$

The normalised load resistance is

$$R_L := \frac{V_{eff\_sat}}{M(U(V_{min\_sat}))^2 \cdot I_1(1)} \qquad \boxed{\text{Equation 13.55}}$$

Now use an iterative calculation to find self-consistent values of the spent-beam voltage and the effective gap voltage as a function of X

$$Vs(X) := \left| \begin{array}{l} Vs_0 \leftarrow 1.0 \\ \text{for } n \in 1..10 \\ \quad \left| \begin{array}{l} U \leftarrow \dfrac{2}{1 + \sqrt{Vs_{(n-1)}}} \\ Vs_n \leftarrow 1 - M(U)^2 \cdot I_1(X) \cdot R_L \end{array} \right. \\ Vs_{10} \end{array} \right.$$

$$V_{geff}(X) := 1 - Vs(X)$$

$$V_g(X) := \frac{V_{geff}(X)}{M(U(Vs(X)))}$$

The RF output power is proportional to $\frac{1}{2} \cdot M_{eff}{}^2 I_1(X)^2 \cdot R_L$. Thus the output power relative to saturation, the output back-off in dB and the efficiency can be calculated as functions of X

$$P(X) := \left(\frac{M(U(Vs(X))) \cdot I_1(X)}{M(U(Vs(1))) \cdot I_1(1)} \right)^2$$ $\boxed{\text{Equation 13.58}}$ $dB := 1$ $PdB(X) := 10 \cdot \log(P(X))$ $PdB(0.7) = -3.102 \cdot dB$

Find the value of X corresponding to an output backoff of D dB. $X1 := 1$ $X(D) := \mathrm{root}(PdB(X1) - D, X1)$ $X(-3) = 0.708$

$$\eta_e(X) := \frac{1}{2} \cdot I_1(X) \cdot V_{geff}(X)$$ $\eta_e(0.7) = 25.9 \cdot \%$

Define the spent-beam voltage and the integrated current from as functions of phase angle and plot the spent-beam curve for comparison with the one plotted in McCune (1986) for a klystron with very similar parameters. The curves bear a marked similarity to one another.

$$V_s(\theta, X) := 1 - V_{geff}(X) \cdot \cos(\theta)$$ $\boxed{\text{Equation 13.56}}$

$$Ic(\theta, X) := \frac{1}{\pi} \cdot \int_{\theta}^{\pi} I(\theta, X)\, d\theta$$ $\boxed{\text{Equation 13.57}}$

$\theta := 0, 0.05 .. \pi$

Computations based on ideal bunching and saturated efficiency

$\eta_{sat} = 53 \cdot \%$

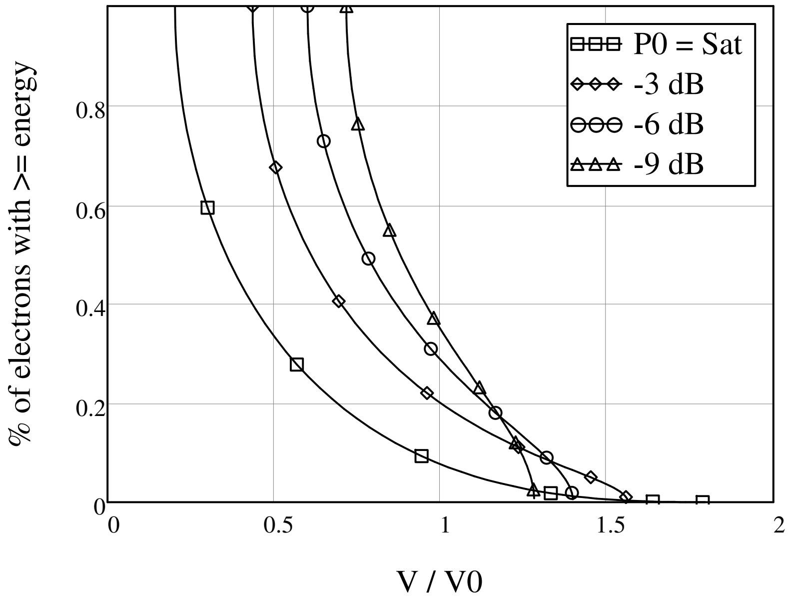


Figure 13.15

Normalised load resistance $R_L = 0.953$

Normalised effective gap voltage $V_{geff}(1) = 0.797$

Normalised gap voltage $V_g(1) = 1.005$

Note:
This figure differs slightly from the one in the book because of changes to the model.