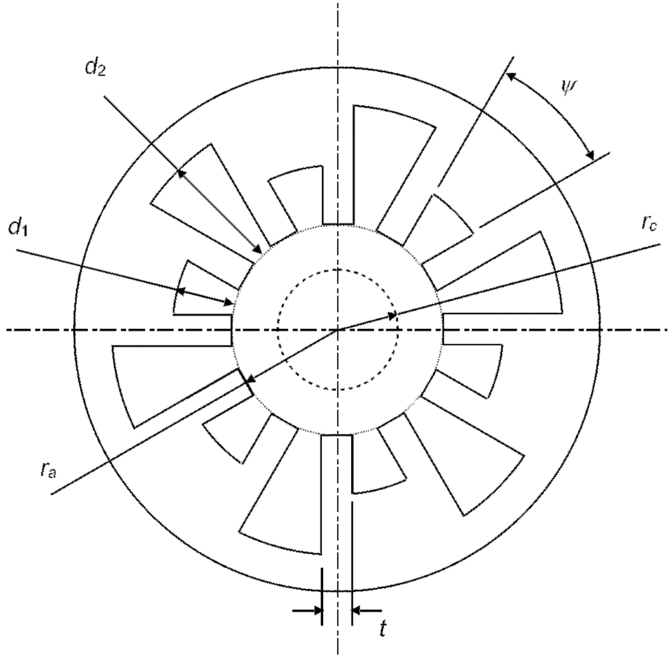
WS 15.2 Rising sun anode model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Data given in Collins, G. B. (1948).
Microwave Magnetrons. New York, McGraw-Hill, p.106



$N_v := 18$

$r_a := 4.602 \cdot mm$

$r_c := 2.715 \cdot mm$

$d_2 := 6.755 \cdot mm$

$d_1 := 3.806 \cdot mm$

$\theta := 0.068$ half angle subtended by each gap

$\lambda_\pi := \dfrac{c}{10 \cdot GHz}$

Calculate subsidiary dimensions

$$p := \frac{2\cdot\pi\cdot r_a}{N_v} \qquad w := 2\cdot\theta\cdot r_a \qquad t := p - w \qquad \psi := \frac{2\cdot\pi}{N_v} \qquad r_1 := r_a - \frac{t}{\psi} \qquad h := 0.1\cdot\lambda_\pi \qquad d_{21} := \frac{d_2}{d_1} \qquad R_a := \frac{r_a}{r_c}$$

Admittance of a resonator

$$Y(k, r_1, r_2) := j\cdot\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\left(\frac{h}{\psi\cdot r_1}\right)\cdot\left(\frac{J0(k\cdot r_1)\cdot Y1(k\cdot r_2) - J1(k\cdot r_2)\cdot Y0(k\cdot r_1)}{J1(k\cdot r_1)\cdot Y1(k\cdot r_2) - J1(k\cdot r_2)\cdot Y1(k\cdot r_1)}\right)$$

Equation 15.57

$$Y'(k, r_1, r_2) := \frac{d}{dk}Y(k, r_1, r_2)$$

Large resonators

$$k_L := \left| \begin{array}{l} r_2 \leftarrow r_1 + d_2 \\ kL \leftarrow \dfrac{\pi}{4\cdot d_2} \\ k \leftarrow \mathrm{root}(Y(kL, r_1, r_2), kL) \end{array} \right.$$

$$C_{0L} := \frac{Y'(k_L, r_1, r_1 + d_2)}{2\cdot j\cdot c} \qquad C_{0L} = 0.082\cdot pF$$

$$L_{1L} := \frac{1}{C_{0L}\cdot(c\cdot k_L)^2} \qquad L_{1L} = 4.744\cdot nH$$

Small resonators

$$nH := H\cdot 10^{-9}$$

$$k_S(d_{21}) := \left| \begin{array}{l} r_2 \leftarrow r_1 + d_2\cdot d_{21}^{-1} \\ ks \leftarrow \dfrac{\pi}{4\cdot d_1} \\ k \leftarrow \mathrm{root}(Y(ks, r_1, r_2), ks) \end{array} \right.$$

$$C_{0S}(d_{21}) := \frac{Y'\left(k_S(d_{21}), r_1, r_1 + d_2\cdot d_{21}^{-1}\right)}{2\cdot j\cdot c} \qquad C_{0S}(d_{21}) = 0.056\cdot pF$$

$$L_{1S}(d_{21}) := \frac{1}{C_{0S}(d_{21})\cdot\left(c\cdot k_S(d_{21})\right)^2} \qquad L_{1S}(d_{21}) = 1.859\cdot nH$$

Calculate the fringing capacitance

$$\mu := \frac{N_v \cdot w}{2 \cdot \pi \cdot r_a}$$ Equation 15.64

$$C_f := \left(\frac{h \cdot \varepsilon_0}{\pi}\right) \cdot \left(1 + \ln\left(\frac{4}{\pi \cdot \mu}\right)\right)$$ Equation 15.63

$$C_f = 0.018 \cdot pF$$

$$C_{1L} := C_{0L} + C_f$$ $$C_{1L} = 0.101\,pF$$

$$C_{1S}(d_{21}) := C_{0S}(d_{21}) + C_f$$ $$C_{1S}(d_{21}) = 0.075\,pF$$

Resonator frequencies

$$\omega_{1L} := \frac{1}{\sqrt{L_{1L} \cdot C_{1L}}}$$

$$\omega_{1S}(d_{21}) := \frac{1}{\sqrt{L_{1S}(d_{21}) \cdot (C_{1S}(d_{21}))}}$$

$$f_{1L} := \frac{\omega_{1L}}{2 \cdot \pi \cdot GHz}$$ $$f_{1L} = 7.278$$

$$f_{1S}(d_{21}) := \frac{\omega_{1S}(d_{21})}{2 \cdot \pi \cdot GHz}$$ $$f_{1S}(d_{21}) = 13.501$$

Estimate anode/cathode capacitance per vane

Upper bound $$C_U := \frac{p}{r_a} \cdot \varepsilon_0 \cdot h \cdot \frac{1}{\ln(R_a)}$$ $$C_U = 0.018 \cdot pF$$

Lower bound $$C_L := \frac{t}{r_a} \cdot \varepsilon_0 \cdot h \cdot \frac{1}{\ln(R_a)}$$ $$C_L = 0.011 \cdot pF$$

Best estimate $$C_0 := \sqrt{C_U \cdot C_L}$$ $$C_0 = 0.014 \cdot pF$$

Dispersion diagrams for the uncoupled cavities

Equation 15.56

$$\omega_S(\phi, d_{21}) := \left| \begin{array}{l} \omega \leftarrow 0 \quad \text{if } \phi = 0 \vee \phi = 2\cdot\pi \\ \omega \leftarrow \omega_{1S}(d_{21})\cdot\left[1 + \dfrac{C_0}{2\cdot C_{1S}(d_{21})\cdot(1-\cos(\phi))}\right]^{-0.5} \quad \text{otherwise} \\ \text{return } \omega \end{array} \right.$$

$$\omega_L(\phi) := \left| \begin{array}{l} \omega \leftarrow 0 \quad \text{if } \phi = 0 \vee \phi = 2\cdot\pi \\ \omega \leftarrow \omega_{1L}\cdot\left[1 + \dfrac{C_0}{2\cdot C_{1L}\cdot(1-\cos(\phi))}\right]^{-0.5} \quad \text{otherwise} \\ \text{return } \omega \end{array} \right.$$

$$f_S(\phi, d_{21}) := \frac{\omega_S(\phi, d_{21})}{\omega_{1L}} \qquad \frac{C_0}{2\cdot C_{1S}(d_{21})} = 0.092$$

$$f_L(\phi) := \frac{\omega_L(\phi)}{\omega_{1L}} \qquad \frac{C_0}{2\cdot C_{1L}} = 0.068$$

Dispersion diagram for the rising sun anode

Define $\alpha_1(d_{21}) := \dfrac{2\cdot C_{1S}(d_{21})}{C_0}$ $\qquad \alpha_2 := \dfrac{2\cdot C_{1L}}{C_0}$

Expand equation 15.83 as a quadratic equation in ω^2. The coefficients are in ascending powers of ω^2

$$V_{rs}(\phi, d_{21}) := \begin{bmatrix} \alpha_1(d_{21})\cdot\alpha_2\cdot\omega_{1S}(d_{21})^2\cdot\omega_{1L}^2\cdot\left(1-\cos(\phi)^2\right)\cdot s^4 \\ -\alpha_1(d_{21})\cdot\alpha_2\cdot\left(\omega_{1S}(d_{21})^2 + \omega_{1L}^2\right)\cdot\left(1-\cos(\phi)^2\right)\cdot s^2 - \alpha_1(d_{21})\cdot\omega_{1S}(d_{21})^2\cdot s^2 - \alpha_2\cdot\omega_{1L}^2\cdot s^2 \\ \alpha_1(d_{21})\cdot\alpha_2\cdot\left(1-\cos(\phi)^2\right) + \alpha_1(d_{21}) + \alpha_2 + 1 \end{bmatrix}$$

The roots of the equation are

$$\omega(\phi, d_{21}) := \frac{1}{s}\cdot\sqrt{\text{polyroots}(V_{rs}(\phi, d_{21}))} \qquad f_{rs1}(\phi, d_{21}) := \frac{\omega(\phi, d_{21})_0}{\omega_{1L}} \qquad f_{rs2}(\phi, d_{21}) := \frac{\omega(\phi, d_{21})_1}{\omega_{1L}}$$

Experimental data

$$\lambda_e := \begin{pmatrix} 1.944 \\ 1.456 \\ 1.381 \\ 1.360 \\ 0.754 \\ 0.766 \\ 0.800 \\ 0.920 \\ 1.000 \end{pmatrix} \cdot \lambda_\pi \qquad f_e := \frac{c}{GHz \cdot \lambda_e} \qquad f_e = \begin{pmatrix} 5.144 \\ 6.868 \\ 7.241 \\ 7.353 \\ 13.263 \\ 13.055 \\ 12.5 \\ 10.87 \\ 10 \end{pmatrix}$$

Lower mode $f_{1e} := \begin{pmatrix} 0 \\ 5.144 \\ 6.868 \\ 7.241 \\ 7.353 \\ 7.353 \\ 7.241 \\ 6.868 \\ 5.144 \\ 0 \end{pmatrix}$ Upper mode $f_{2e} := \begin{pmatrix} 10 \\ 10.87 \\ 12.5 \\ 13.055 \\ 13.263 \\ 13.263 \\ 13.055 \\ 12.5 \\ 10.87 \\ 10 \end{pmatrix}$

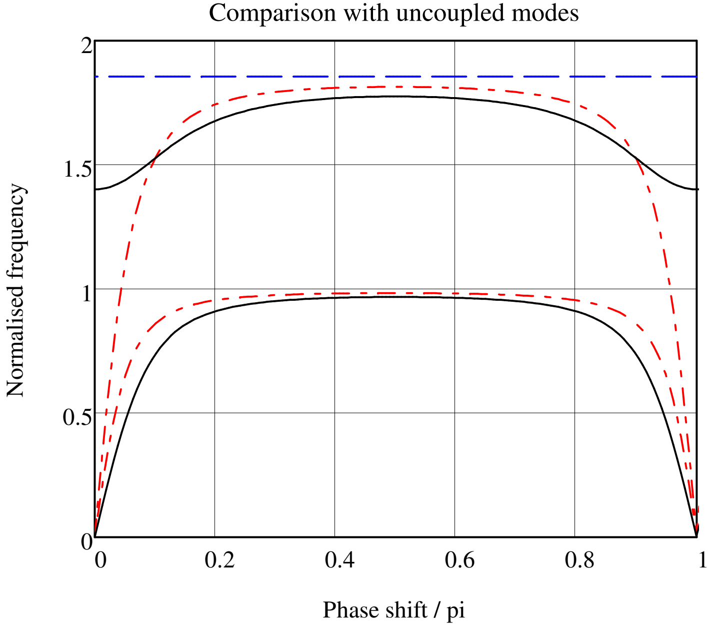
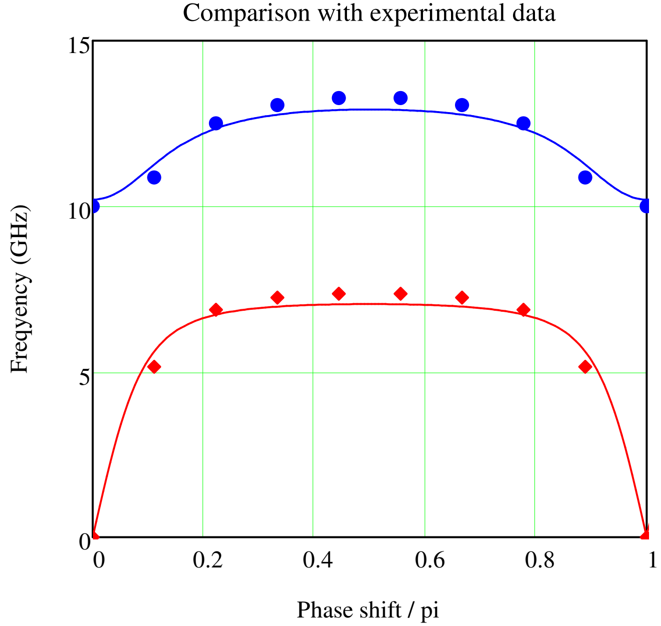


Figure 15.23



$d21 := 1, 1.5 .. 3$

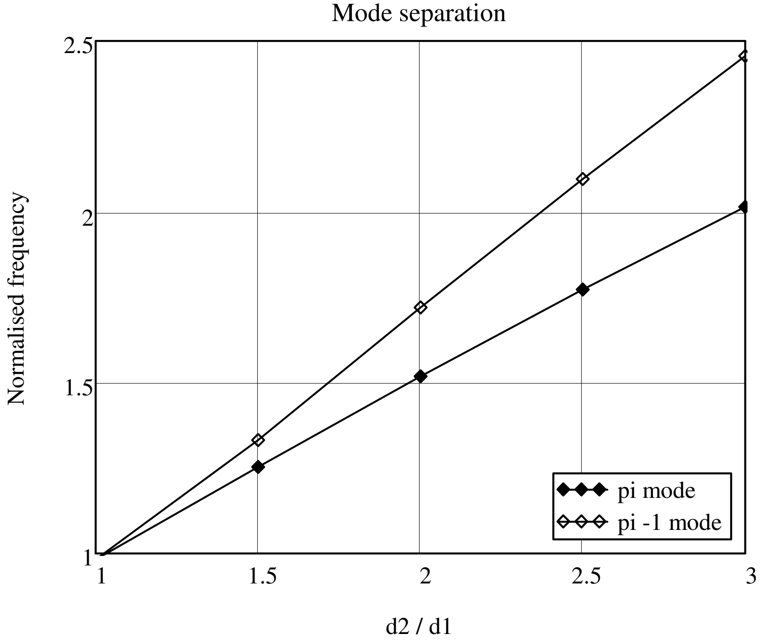


Figure 15.24