

WS 4.1 Capacitance of a planar grid

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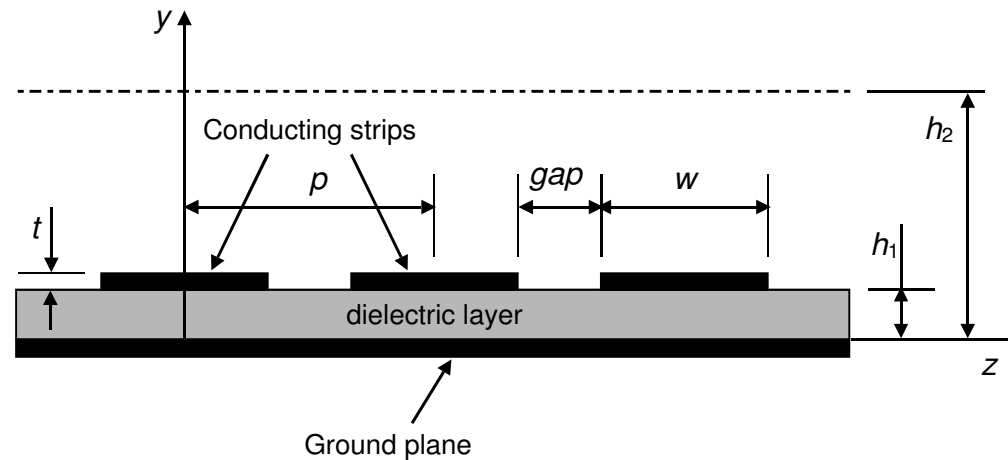
This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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The sheet calculates and displays the potential map using a finite difference calculation on a square mesh and calculates the capacitance per unit length of one strip divided by ϵ_0

Figure 4.7



Relative permittivity of the dielectric layer is ϵ_2 .

In this model $t = 0$.

Structure dimensions

$$h_1 := 0.062 \cdot \text{in}$$

$$h_2 := 2.6 \cdot h_1$$

$$L_{\text{w}} := 0.537 \cdot \text{in}$$

$$w := 0.045 \cdot \text{in}$$

$$\text{gap} := 0.057 \cdot \text{in}$$

$$p := w + \text{gap}$$

$$\epsilon_2 := 6.5$$

$$p = 2.591 \cdot \text{mm}$$

$$h_1 = 1.57 \cdot \text{mm}$$

$$L = 13.6 \cdot \text{mm}$$

$$w = 1.14 \cdot \text{mm}$$

$$\text{gap} = 1.45 \cdot \text{mm}$$

$$p = 2.59 \cdot \text{mm}$$

$$h_2 = 4.09 \cdot \text{mm}$$

Choose mesh size for FD calculations

$$\text{mesh} := 0.142 \cdot \text{mm}$$

The mesh is a square mesh and the boundaries are taken to be on the closest mesh lines.

$$NH1 := \frac{h_1}{\text{mesh}}$$

$$NH2 := \frac{h_2}{\text{mesh}}$$

$$NP := \frac{p}{\text{mesh}}$$

$$NW := \frac{w}{\text{mesh}}$$

$$NH1 = 11.09$$

$$NH2 = 28.834$$

$$NP = 18.245$$

$$NW = 8.049$$

These numbers should be as close to integers as possible

$$Nh1 := \text{round}(NH1)$$

$$Nh2 := \text{round}(NH2)$$

$$Np := \text{round}(NP)$$

$$Nw := \text{round}(NW)$$

$$Nw2 := \frac{Nw}{2}$$

$$Np2 := \frac{Np}{2}$$

$$Nh1 = 11$$

$$Nh2 = 29$$

$$Np = 18$$

$$Nw = 8$$

NB: Np and Nw must be even numbers

Calculation of capacitances using a finite difference solution of Laplace's equation

This sheet models a section of the grid of width p between the centres of adjacent strips.

Top boundary condition. This variable is 0 for a conducting top plane and 2 for a symmetry boundary

$$H2_{\text{sym}} := 2$$

Maximum number of iterations

$$\text{Niter} := 1000$$

Right hand boundary condition. This variable is 0 for an anti-symmetry boundary and 2 for a symmetry boundary

$$R_{\text{sym}} := 2$$

Voltages on left-hand and right-hand strips

$$V1 := 100$$

$$V2 := 100$$

The zero, $\pi/2$ and π modes can be modelled as follows

Zero mode: $V1 = 100$ $V2 = 100$ $R_{\text{sym}} = 2$

$\pi/2$ mode $V1 = 100$ $V2 = 0$ $R_{\text{sym}} = 0$

π mode $V1 = 100$ $V2 = -100$ $R_{\text{sym}} = 2$

The section below can be collapsed to allow the data and the results to be seen together



```

VM :=
  for i ∈ 0..Np
    for j ∈ 0..Nh2
       $V_{i,j} \leftarrow 0$ 
    for n ∈ 1..Niter
      for i ∈ 0..Np
         $V_{i,0} \leftarrow 0$ 
        for j ∈ 1..(Nh1 - 1)
           $V_{0,j} \leftarrow (2 \cdot V_{1,j} + V_{0,j+1} + V_{0,j-1}) \cdot 0.25$ 
           $V_{Np,j} \leftarrow (R_{sym} \cdot V_{Np-1,j} + V_{Np,j+1} + V_{Np,j-1}) \cdot 0.25$ 
          for i ∈ 1..Np - 1
             $V_{i,j} \leftarrow (V_{i+1,j} + V_{i,j+1} + V_{i-1,j} + V_{i,j-1}) \cdot 0.25$ 
        for i ∈ 0..Nw2
           $V_{i,Nh1} \leftarrow V1$ 
        for i ∈ (Nw2 + 1)..(Np - Nw2 - 1)
          
$$V_{i,Nh1} \leftarrow \frac{V_{i,Nh1-1} \cdot \epsilon_2 + 0.5 \cdot (V_{i-1,Nh1} + V_{i+1,Nh1}) \cdot (\epsilon_2 + 1) + V_{i,Nh1+1}}{2 \cdot \epsilon_2 + 2}$$

        for i ∈ (Np - Nw2)..Np
           $V_{i,Nh1} \leftarrow V2$ 

```

```

for j ∈ (Nh1 + 1) .. (Nh2 - 1)
     $V_{0,j} \leftarrow (2 \cdot V_{1,j} + V_{0,j+1} + V_{0,j-1}) \cdot 0.25$ 
     $V_{Np,j} \leftarrow (R_{sym} \cdot V_{Np-1,j} + V_{Np,j+1} + V_{Np,j-1}) \cdot 0.25$ 
    for i ∈ 1 .. Np - 1
         $V_{i,j} \leftarrow (V_{i+1,j} + V_{i,j+1} + V_{i-1,j} + V_{i,j-1}) \cdot 0.25$ 
 $V_{0,Nh2} \leftarrow (2 \cdot V_{1,Nh2} + H2_{sym} \cdot V_{0,Nh2-1}) \cdot 0.25$ 
 $V_{Np,Nh2} \leftarrow (R_{sym} \cdot V_{Np-1,Nh2} + H2_{sym} \cdot V_{Np,Nh2-1}) \cdot 0.25$ 
for i ∈ 1 .. Np - 1
     $V_{i,Nh2} \leftarrow (V_{i+1,Nh2} + V_{i-1,Nh2} + H2_{sym} \cdot V_{i,Nh2-1}) \cdot 0.25$ 
VM ← V
return VM

```

Calculate the charge per unit length on the edge of the grid, and the capacitance per unit length divided by ϵ_0 from the total charge.

The calculation assumes that the electrode voltages are +/- 100 or zero.

$$Q1 := \left[\sum_{i=0}^{0.5 \cdot Np} VM_{i,1} - \left(\frac{VM_{0,1} + VM_{Np2,1}}{2} \right) \right] \cdot \epsilon_2 \quad Q1 = 455.1$$

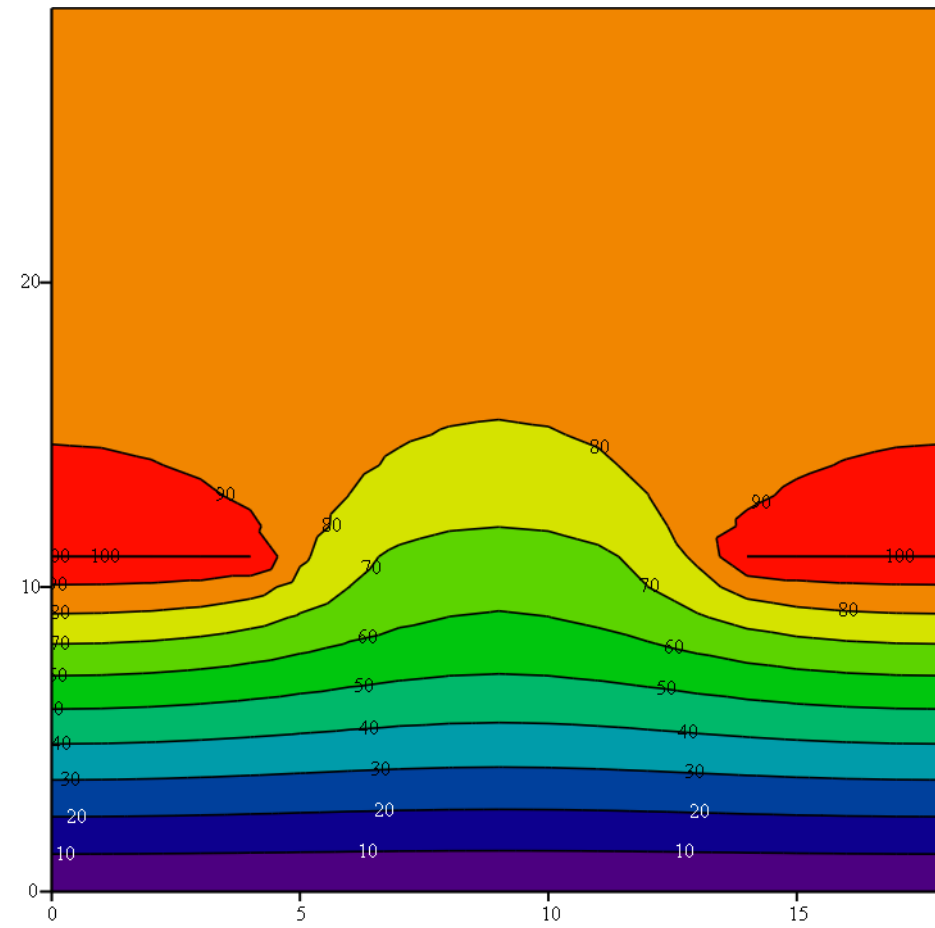
$$Q2 := \left[\sum_{j=0}^{Nh1} (VM_{Np2-1,j} - VM_{Np2+1,j}) - \frac{(VM_{Np2-1,0} - VM_{Np2+1,0})}{2} - \left(\frac{VM_{Np2-1,Nh1} - VM_{Np2+1,Nh1}}{2} \right) \right] \cdot \frac{\epsilon_2}{2} \quad Q2 = -0$$

$$Q3 := \left[\sum_{j=Nh1}^{Nh2} (VM_{Np2-1,j} - VM_{Np2+1,j}) - \frac{(VM_{Np2-1,Nh1} - VM_{Np2+1,Nh1})}{2} - \left(\frac{VM_{Np2-1,Nh2} - VM_{Np2+1,Nh2}}{2} \right) \right] \cdot \frac{1}{2} \quad Q3 = -0.1$$

$$Q4 := \left[\sum_{i=0}^{0.5 \cdot Np} VM_{i,Nh2-1} - \left(\frac{VM_{0,Nh2-1} + VM_{Np2,Nh2-1}}{2} \right) \right] - \left[\sum_{i=0}^{0.5 \cdot Np} VM_{i,Nh2} - \left(\frac{VM_{0,Nh2} + VM_{Np2,Nh2}}{2} \right) \right] \quad Q4 = 0.0$$

$$\text{Capacitance} := \frac{Q1 + Q2 + Q3 + Q4}{50}$$





Capacitance = 9.101