

WS 18.2 Crossed-field multipactor model

© 2018 Richard G Carter

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This resource is provided free of charge by Cambridge University Press with permission of the author, but is subject to copyright. You are permitted to view, print and download this resource for your own personal use only, provided any copyright lines are not removed or altered in any way. Any other use, including but not limited to, distribution of the resource in modified form, or via electronic or other media, is strictly prohibited unless you have permission from the author and provided you give appropriate acknowledgement of the source.

The contents of this sheet are provided for educational purposes only and no warranty is expressed or implied that they are suitable for use as professional design tools.

The equations used are taken from
S. Riyopoulos, D. Chernin and D. Dialetis, Theory of electron multipactor in crossed fields,
Phys Plasmas, Vol.2, pp.3194-3213, August 1995.

The electrons move under the influence of the RF electric field in the y direction between parallel plates separated by distance D . There is a uniform static magnetic field in the z direction leading to motion of the electrons in the x - y plane.

Charge/mass ratio of the electron $\eta := 1.759 \cdot 10^{11} \cdot \text{C} \cdot \text{kg}^{-1}$

Frequency

$$f := 400 \cdot \text{MHz}$$

$$\omega := 2 \cdot \pi \cdot f$$

Plate separation

$$D := 10 \cdot \text{mm}$$

Initial electron energy

$$V_{x0} := 0 \cdot \text{V}$$

$$V_{y0} := 1 \cdot \text{V}$$

Specify the magnetic field and find the normalised cyclotron frequency

Single surface multipactor

$$B_{0z} := 0.005 \cdot \text{T}$$

$$\Omega_0 := \frac{\eta \cdot B_{0z}}{\omega}$$

$$\Omega_0 = 0.35$$

Two-surface multipactor

$$B_{1z} := 0.002 \cdot \text{T}$$

$$\Omega_1 := \frac{\eta \cdot B_{1z}}{\omega}$$

$$\Omega_1 = 0.14$$

Specify RF voltage and find the normalised electric field (\mathcal{E} in Riyopoulos but E here)

$$V_{0\text{RF}} := 0.1 \cdot \text{kV}$$

$$E_0 := \frac{\eta \cdot V_{0\text{RF}}}{\omega^2 \cdot D^2}$$

$$E_0 = 0.028$$

$$V_{1\text{RF}} := 1.5 \cdot \text{kV}$$

$$E_1 := \frac{\eta \cdot V_{1\text{RF}}}{\omega^2 \cdot D^2}$$

$$E_1 = 0.418$$

Number of RF cycles (1, 2, 3, etc.)

$$N_0 := 1$$

Number of RF cycles (0.5, 1.5, 2.5 etc.)

$$N_1 := 0.5$$

The section below can collapsed to conceal the detailed calculations



Specify the normalised initial velocity components of the electrons

$$u_{x0} := \frac{\sqrt{2 \cdot \eta \cdot V_{x0}}}{\omega \cdot D} \quad u_{y0} := \frac{\sqrt{2 \cdot \eta \cdot V_{y0}}}{\omega \cdot D} \quad u_{x0} = 0 \quad u_{y0} = 0.024$$

The working equations, numbered as in the paper, are as follows with $\tau = \omega t$ where ϕ is the RF phase at which the electron is launched. The dimensions are normalised to D

$$y(\tau, \phi, E, \Omega) := \frac{E}{\Omega^2 - 1} \cdot \left(\sin(\phi + \tau) - \sin(\phi) \cdot \cos(\Omega \cdot \tau) - \frac{1}{\Omega} \cdot \cos(\phi) \cdot \sin(\Omega \cdot \tau) \right) \dots \quad (7a)$$

$$+ \frac{u_{y0}}{\Omega} \cdot \sin(\Omega \cdot \tau) - \frac{u_{x0}}{\Omega} \cdot (1 - \cos(\Omega \cdot \tau))$$

$$x(\tau, \phi, E, \Omega) := \frac{E}{\Omega^2 - 1} \cdot \left[-\Omega \cdot \cos(\phi + \tau) - \sin(\phi) \cdot \sin(\Omega \cdot \tau) - \frac{1}{\Omega} \cdot \cos(\phi) \cdot (1 - \Omega^2 - \cos(\Omega \cdot \tau)) \right] \dots \quad (7b)$$

$$+ \frac{u_{y0}}{\Omega} \cdot (1 - \cos(\Omega \cdot \tau)) + \frac{u_{x0}}{\Omega} \cdot \sin(\Omega \cdot \tau)$$

$$u_y(\tau, \phi, E, \Omega) := \frac{E}{\Omega^2 - 1} \cdot (\cos(\phi + \tau) + \Omega \cdot \sin(\phi) \cdot \sin(\Omega \cdot \tau) - \cos(\phi) \cdot \cos(\Omega \cdot \tau)) \dots \quad (9a)$$

$$+ u_{y0} \cdot \cos(\Omega \cdot \tau) - u_{x0} \cdot \sin(\Omega \cdot \tau)$$

$$u_x(\tau, \phi, E, \Omega) := \frac{E}{\Omega^2 - 1} \cdot (\Omega \cdot \sin(\phi + \tau) - \Omega \cdot \sin(\phi) \cdot \cos(\Omega \cdot \tau) - \cos(\phi) \cdot \sin(\Omega \cdot \tau)) \dots \quad (9b)$$

$$+ u_{y0} \cdot \sin(\Omega \cdot \tau) + u_{x0} \cdot \cos(\Omega \cdot \tau)$$



To find the launch phase such that the electron trajectory is resonant we choose an initial guessed value of ϕ and then seek the value of ϕ which makes $y = 0$ or 1. A multiplying factor of 1000 is used to increase the accuracy of the solution. It will be seen that only certain combinations of the parameters lead to valid solutions

Single surface

$$\phi_0 := 0.5\pi \quad \phi_0(\Omega) := \text{root}\left[1000 \cdot \left(y(2 \cdot N_0 \cdot \pi, \phi_0, E_0, \Omega)\right), \phi_0\right]$$

$$\phi_0(\Omega_0) = 93.3 \cdot \text{deg}$$

Calculate the impact energy in eV

$$V_{0i} := \frac{(\omega \cdot D)^2}{2 \cdot \eta} \cdot \left(u_x(2 \cdot N_0 \cdot \pi, \phi_0(\Omega_0), E_0, \Omega_0)^2 + u_y(2 \cdot N_0 \cdot \pi, \phi_0(\Omega_0), E_0, \Omega_0)^2 \right) \quad V_{1i} := \frac{(\omega \cdot D)^2}{2 \cdot \eta} \cdot \left(u_x(2 \cdot N_1 \cdot \pi, \phi_1(\Omega_1), E_1, \Omega_1)^2 + u_y(2 \cdot N_1 \cdot \pi, \phi_1(\Omega_1), E_1, \Omega_1)^2 \right)$$

$$V_{0i} = 0.7 \cdot \text{V}$$

Plotting ranges $\tau_{00} := 0, 0.001 \dots 2N_0 \pi$

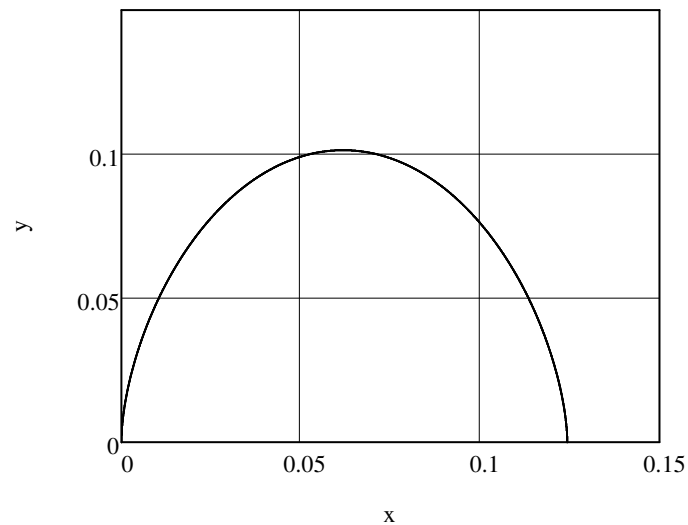


Figure 18.18

Two-surface

$$\phi_1 := 0.5 \cdot \pi \quad \phi_1(\Omega) := \text{root}\left[1000 \cdot \left(y(2 \cdot N_1 \cdot \pi, \phi_1, E_1, \Omega) - 1\right), \phi_1\right]$$

$$\phi_1(\Omega_1) = 84.7 \cdot \text{deg}$$

$$V_{1i} = 44.3 \cdot \text{V}$$

$\tau_{11} := 0, 0.001 \dots 2 \cdot N_1 \pi$ $x_\pi := x(\pi, \phi_1(\Omega_1), E_1, \Omega_1)$

