

## **WS 3.2 Cavity resonators**

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This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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### Section 3.3 Pill-box cavity resonators

Calculate the surface resistance

Conductivity of copper

$$\sigma := 5.959 \cdot 10^7 \cdot \text{S} \cdot \text{m}^{-1}$$

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\omega \mu_0}{2\sigma}}$$

$$\delta_s(\omega) := \sqrt{\frac{2}{\omega \cdot \sigma \cdot \mu_0}}$$

$$R_s(\omega) := \frac{1}{\sigma \cdot \delta_s(\omega)}$$

Equation 3.50

#### Section 3.3.1 Surface roughness

Effect of r.m.s. surface roughness  $\Delta$

$$\frac{R_r}{R_s} = 1 + \frac{2}{\pi} \arctan \left[ 1.4 \left( \frac{\Delta}{\delta} \right)^2 \right]$$

$$R_{rs}(x) := 1 + \frac{2}{\pi} \cdot \text{atan}(1.4 \cdot x^2)$$

Equation 3.56

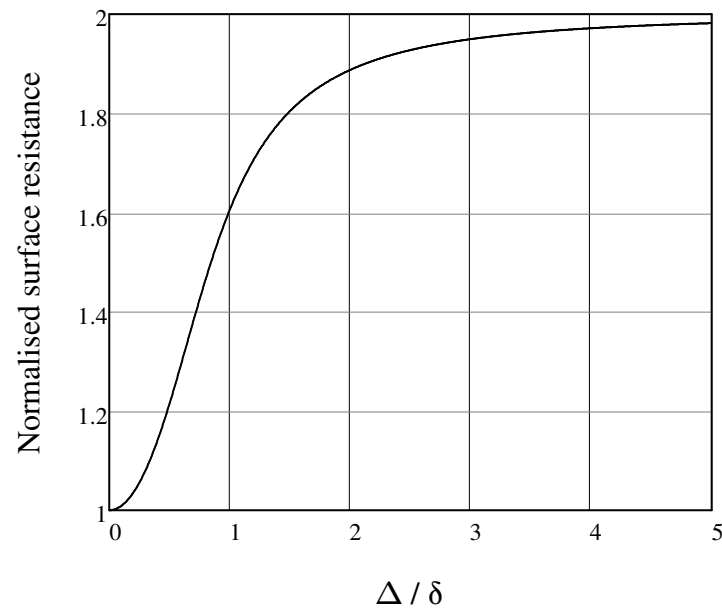


Figure 3.12

**Pill-box cavities (TM<sub>010</sub> mode)**

Specify the cavity radius (a) and height (h)

$$a := 40 \cdot \text{mm}$$

$$h := 10 \cdot \text{mm}$$

 Find the first zero of the Bessel Function J<sub>0</sub>(z) from guess

$$z1 := 2$$

$$ka := \text{root}(J_0(z1), z1)$$

$$ka = 2.4048$$

Calculate the resonant frequency

$$\omega_0 := ka \cdot \frac{c}{a}$$

Equation 3.44

$$f_0 := \frac{\omega_0}{2 \cdot \pi}$$

$$f_0 = 2.869 \cdot \text{GHz}$$

Calculate the stored electric energy for a given field on the axis and the R/Q

$$E_0 := 1000 \cdot \frac{\text{V}}{\text{m}}$$

$$W_E := \epsilon_0 \cdot \pi \cdot h \cdot a^2 \cdot E_0^2 \cdot \frac{1}{ka^2} \cdot \int_0^{ka} J_0(kr)^2 \cdot kr \, d(kr)$$

Equation 3.46

$$W_E = 5.997 \times 10^{-11} \text{ J}$$

$$R_Q := \frac{h^2 \cdot E_0^2}{2 \cdot \omega_0 \cdot W_E}$$

Equation 3.14

$$R_Q = 46.3 \, \Omega$$

Calculate the unloaded Q

Specify r.m.s. surface roughness

$$\Delta := 0.1 \cdot \mu\text{m}$$

Skin depth

$$\delta_s(\omega_0) = 1.217 \cdot \mu\text{m}$$

Surface resistance

$$R_r(\omega) := R_s(\omega) \cdot R_{rs} \left( \frac{\Delta}{\delta_s(\omega)} \right)$$

$$R_r(\omega_0) = 0.014 \, \Omega$$

Unloaded Q

$$Q := \frac{ka}{2 \cdot R_r(\omega_0)} \cdot \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{h}{a + h}$$

Equation 3.55

$$Q = 6533$$

Shunt impedance

$$R_c := R_Q \cdot Q$$

$$R_c = 302.2 \cdot \text{k}\Omega$$

### Section 3.4 Rectangular cavity resonators (TM<sub>110</sub> mode)

Specify the transverse dimensions (a and b) and the height h

$$a := 50 \cdot \text{mm}$$

$$b := 60 \cdot \text{mm}$$

$$h := 12 \cdot \text{mm}$$

Calculate the resonant frequency

$$\beta_C := \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

Equation 3.58

$$\omega_0 := c \cdot \beta_C$$

$$f := \frac{\omega_0}{2 \cdot \pi}$$

$$f = 3.902 \cdot \text{GHz}$$

Calculate the R/Q

$$W_E := \frac{\epsilon_0}{8} \cdot a \cdot b \cdot h \cdot E_0^2$$

Equation 3.62

$$W_E = 3.984 \times 10^{-11} \text{ J}$$

$$R_Q := \frac{h^2 \cdot E_0^2}{2 \cdot \omega_0 \cdot W_E}$$

Equation 3.14

$$R_Q = 73.7 \Omega$$

Calculate the unloaded Q

Specify r.m.s. surface roughness  $\Delta := 0.1 \cdot \mu\text{m}$

Skin depth

$$\delta_s(\omega_0) = 1.044 \cdot \mu\text{m}$$

Surface resistance

$$R_r(\omega) := R_s(\omega) \cdot R_{rs} \left( \frac{\Delta}{\delta_s(\omega)} \right)$$

$$R_r(\omega_0) = 0.016 \Omega$$

Unloaded Q

$$Q := \frac{\pi}{4 \cdot R_r(\omega_0)} \cdot \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \left[ \frac{2 \cdot h \cdot (a^2 + b^2)^{1.5}}{a \cdot b \cdot (a^2 + b^2) + 2 \cdot h \cdot (a^3 + b^3)} \right]$$

Equation 3.64

$$Q = 7880$$

Shunt impedance

$$R_c := R_Q \cdot Q$$

$$R_c = 580.8 \cdot \text{k}\Omega$$