WS 4.5 Tape helix model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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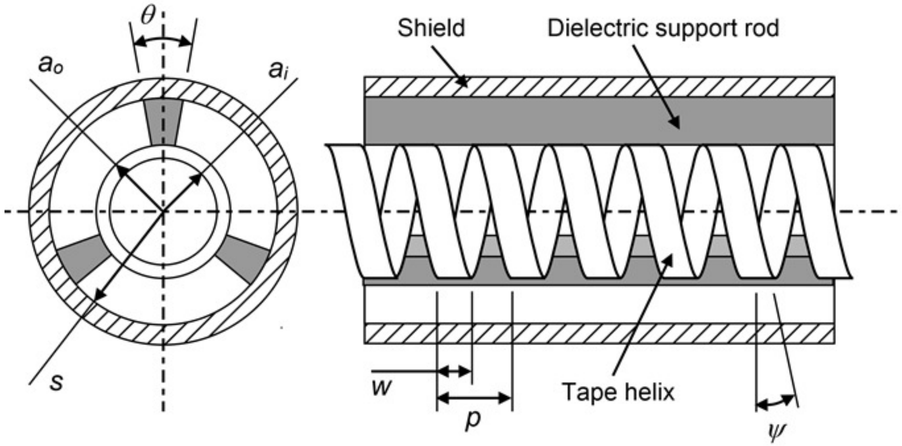


The support rods whose relative permittivity is ε_r are represented by an equivalent uniform dielectric cylinder.

$a_i := 1.465 \cdot mm$ $\quad a_o := 1.59 \cdot mm$ $\quad s := 2.92 \cdot mm$ $\quad w := 0.89 \cdot mm$ $\quad \psi := 9.5 \cdot deg$ $\quad N_r := 3$ $\quad \theta := \frac{0.68 \cdot mm}{s}$ $\quad \varepsilon_r := 5.2$

Section 4.3.4 Equivalent circuit of helix slow-wave structures

Mean helix radius $a := \dfrac{a_i + a_o}{2}$ $a = 1.528 \cdot mm$

Tape thickness $t := a_o - a_i$ $t = 0.125\,mm$

Helix pitch $p := 2 \cdot \pi \cdot a \cdot \tan(\psi)$ $p = 1.606\,mm$

Ratios of parameters $ap := \dfrac{a}{p}$ $sp := \dfrac{s}{p}$ $gp := \dfrac{p - w}{p}$ $aip := \dfrac{a_i}{p}$

Equivalent permittivity $\varepsilon_2 := 1 + \dfrac{N_r \cdot \theta}{2 \cdot \pi} \cdot (\varepsilon_r - 1)$ Equation 4.82 $\varepsilon_2 = 1.467$

Sheath helix model

$$L_0(\beta p) := \frac{\mu_0}{2 \cdot \pi} \cdot I1(\beta p \cdot ap) \cdot K1(\beta p \cdot ap)\cot(\psi)^2 \qquad L_1(\beta p) := L_0(\beta p) \cdot \left(1 - \frac{I1(\beta p \cdot ap) \cdot K1(\beta p \cdot sp)}{I1(\beta p \cdot sp) \cdot K1(\beta p \cdot ap)}\right)$$

Equations 4.76 and 4.78

$$C_0(\beta p) := \frac{2 \cdot \pi \cdot \varepsilon_0}{I0(\beta p \cdot ap) \cdot K0(\beta p \cdot ap)} \qquad C_1(\beta p) := C_0(\beta p) \cdot \left(1 - \frac{I0(\beta p \cdot ap) \cdot K0(\beta p \cdot sp)}{I0(\beta p \cdot sp) \cdot K0(\beta p \cdot ap)}\right)^{-1}$$

Equations 4.77 and 4.79

$$\varepsilon_{eff}(\beta p) := 1 + (\varepsilon_2 - 1) \cdot (\beta p \cdot ap) \cdot I0(\beta p \cdot ap) \cdot K1(\beta p \cdot ap) \cdot \left(1 + \frac{I1(\beta p \cdot ap) \cdot K0(\beta p \cdot sp)}{K1(\beta p \cdot ap) \cdot I0(\beta p \cdot sp)}\right)$$

Equation 4.80

Tape helix equivalent circuit

The capacitances for phase shifts 0, π/2 and π per turn without dielectric are calculated using Worksheet 4.4

$C_e := 10.03 \cdot \varepsilon_0$ $\quad C_{\pi 2} := 22.02 \cdot \varepsilon_0$ $\quad C_o := 32.45 \cdot \varepsilon_0$

$C1_0 := C_e$ $\quad C1_1 := 0.25 \cdot \left(C_o - C_e\right)$ $\quad C1_2 := 0.25 \cdot \left(C_{\pi 2} - C_e - 2 \cdot C1_1\right)$

$$C1(\phi) := C1_0 + 2 \cdot C1_1 \cdot (1 - \cos(\phi)) + 2 \cdot C1_2 \cdot (1 - \cos(2\phi))$$

Equation 4.39

The capacitances for phase shifts 0, π/2 and π per turn with dielectric are calculated using Worksheet 4.4

$CK_e := 14.56 \cdot \varepsilon_0$ $\quad CK_{\pi 2} := 28.19 \cdot \varepsilon_0$ $\quad CK_o := 40.13 \cdot \varepsilon_0$

$CK_0 := CK_e$ $\quad CK_1 := 0.25 \cdot \left(CK_o - CK_e\right)$ $\quad CK_2 := 0.25 \cdot \left(CK_{\pi 2} - CK_e - 2 \cdot CK_1\right)$

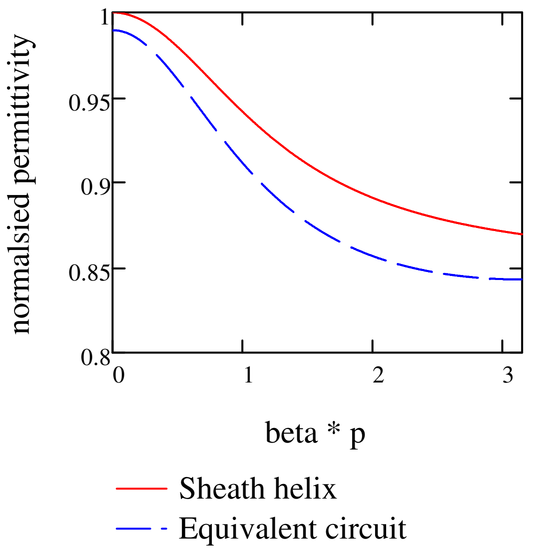
$$CK(\phi) := CK_0 + 2 \cdot CK_1 \cdot (1 - \cos(\phi)) + 2 \cdot CK_2 \cdot (1 - \cos(2\phi))$$

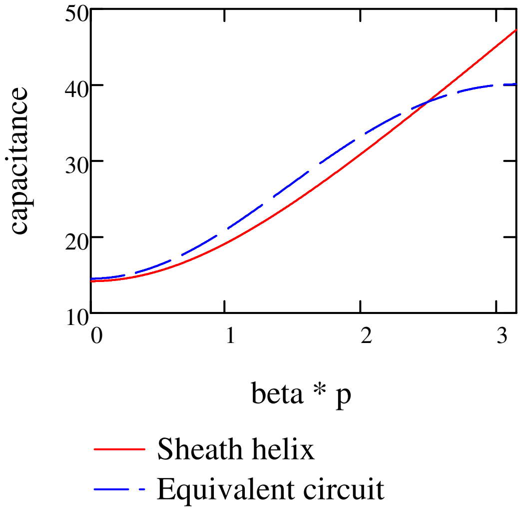
Equation 4.39

The effective relative permittivity as a function of phase shift per turn is

$$\varepsilon 1_{eff}(\phi) := \frac{CK(\phi)}{C1(\phi)}$$

Comparison between the parameters of the sheath helix and the equivalent circuit





Phase velocity without dielectric loading calculated from the sheath helix equations with corrections for the difference between a tape helix and a sheath helix.

Capacitance per unit length of a coaxial line

$$C_{00}(\varepsilon_2) := \frac{2\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_2}{\ln\left(\frac{s}{a_o}\right)}$$ Equation 4.84

$$\alpha_{C0} := \frac{C1(0)}{C_{00}(1)}$$ Equation 4.96 $\alpha_{C0} = 0.970$

$$v_{p0}(\beta p, \alpha_L) := \frac{1}{\sqrt{\alpha_L\cdot\alpha_{C0}\cdot L_1(\beta p)\cdot C_1(\beta p)}}$$

Find the value of α_L from the asymptotic phase velocity at $\beta p = \pi$ in the absence of dielectric (Equation 4.88)

$$\alpha_L := \left(\frac{v_{p0}(\pi, 1)}{c \cdot \tan(\psi)}\right)^2$$

$\alpha_L = 1.10$

Calculate the phase and group velocity with dielectric loading using the effective permittivity calculated using the equivalent circuit

$$\alpha_C := \frac{CK(0)}{C_{00}(\varepsilon_2)}$$

Equation 4.104

$\alpha_C = 0.960$

$$v_p(\beta p) := \frac{1}{\sqrt{\alpha_L \cdot \alpha_C \cdot L_1(\beta p) \cdot C_1(\beta p) \cdot \varepsilon 1_{eff}(\beta p)}}$$

$$v_g(\beta p) := \frac{d}{d(\beta p)}\left(\beta p \cdot v_p(\beta p)\right)$$

Frequency

$$f(\beta p) := \frac{\beta p}{2 \cdot \pi \cdot p} \cdot \frac{v_p(\beta p)}{GHz}$$

Calculate the characteristic impedance and the Pierce impedance

$$Z_c(\beta p) := \frac{v_p(\beta p)}{v_g(\beta p)} \cdot \sqrt{\frac{\alpha_L \cdot L_1(\beta p)}{\alpha_C \cdot C_1(\beta p) \cdot \varepsilon 1_{eff}(\beta p)}}$$

Equation 4.103

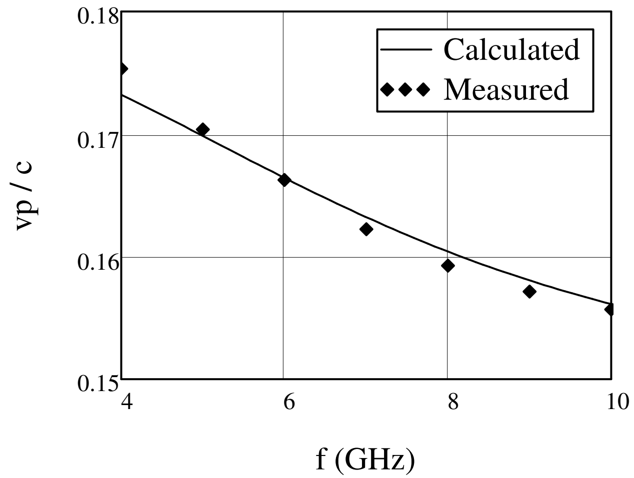
$$Z_P(\beta p) := \left(\frac{\sin(0.5 \cdot \beta p \cdot \cos(\psi))}{0.5 \cdot \beta p \cdot \cos(\psi)} \cdot \frac{\sin(0.5 \cdot \beta p \cdot gp \cdot \cos(\psi))}{0.5 \cdot \beta p \cdot gp \cdot \cos(\psi)} \cdot \frac{\cos(\psi)^2}{I0(\beta p \cdot aip)}\right)^2 \cdot Z_c(\beta p)$$

Equation 4.102

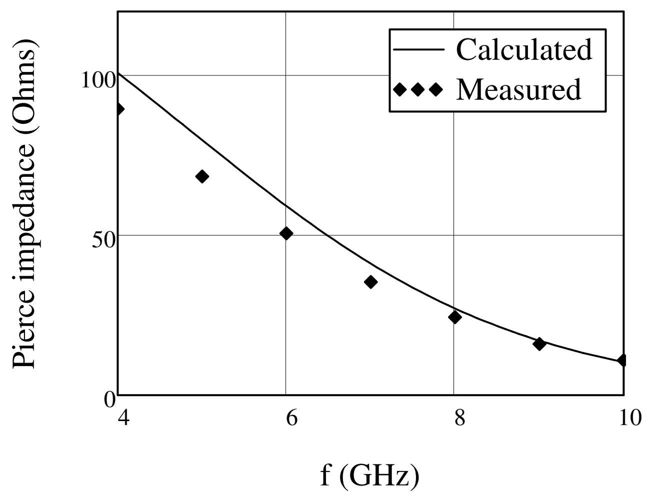
Experimental data

From D'Agostino, S., F. Emma, et al. "Accurate analysis of helix slow-wave structures." IEEE Transactions on Electron Devices 45(7): 1605-1613 (1998) Figures 7 and 8 (SWS-2). Note that the rectangular rods have been replaced by equivalent wedge-shaped rods.

$$f1 := \begin{pmatrix} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{pmatrix} \cdot GHz \qquad vp1 := c \cdot \left[0.15 + \frac{0.03}{142} \begin{pmatrix} 120 \\ 96.5 \\ 77 \\ 58 \\ 44 \\ 34 \\ 27 \end{pmatrix} \right] \qquad K_p := \frac{120}{142} \cdot \begin{pmatrix} 106 \\ 81 \\ 60 \\ 42 \\ 29 \\ 19 \\ 13 \end{pmatrix} \cdot \Omega \qquad \beta p1 := \frac{2 \cdot \pi \cdot f1 \cdot p}{vp1} \qquad \beta p1 = \begin{pmatrix} 0.768 \\ 0.988 \\ 1.215 \\ 1.452 \\ 1.69 \\ 1.927 \\ 2.162 \end{pmatrix}$$



Compare d'Agostino fig. 8



Compare d'Agostino fig. 9

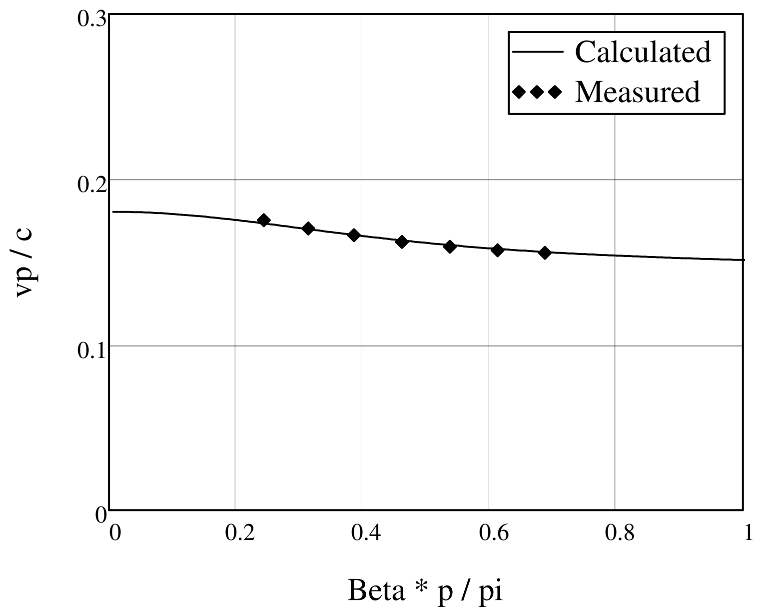


Figure 4.21(a)

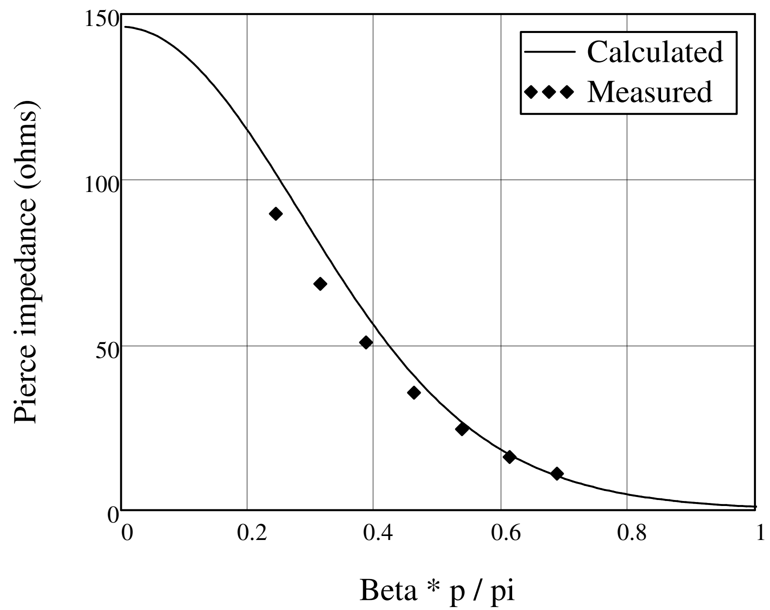


Figure 4.21(b)