WS 19.3 PPM stack design

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

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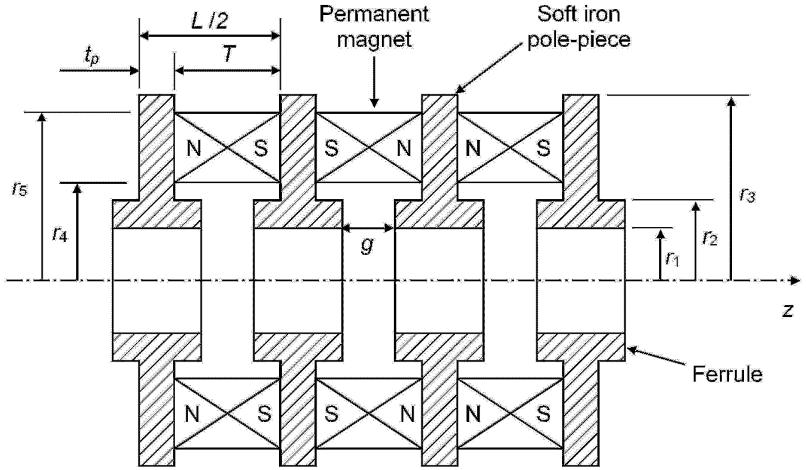
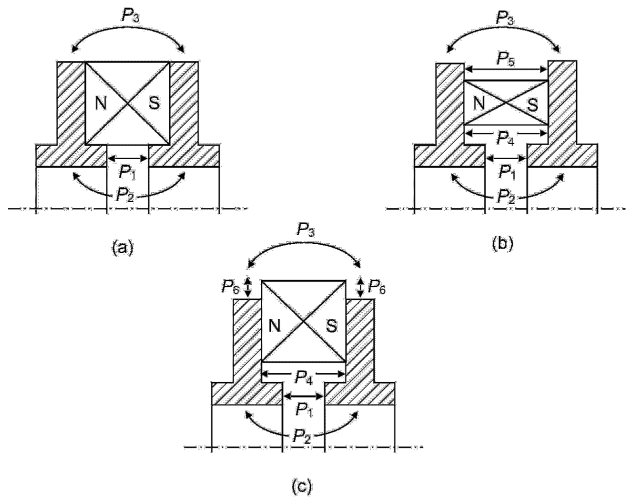
This sheet provides a model of periodic permanent magnet stack (see Section 19.7.3). It is based on
M. Santra, *et al.*, "An Improved Analysis of PPM Focusing Structures Including the Effect of Magnetic Saturation in the Iron Pole Pieces,"
IEEE Transactions on Electron Devices, vol. 56, pp. 974-980, May 2009.





Define the dimensions of the stack

Ferrule inner radius	$r_1 := 1.6 \cdot mm$	Magnet period	$L_m := 8.5 \cdot mm$
Ferrule outer radius	$r_2 := 3.05 \cdot mm$	Magnet thickness	$T_m := 2.95 \cdot mm$
Pole-piece outer radius	$r_3 := 7.5 \cdot mm$	Gap length	$g_f := 2.25 \cdot mm$
Magnet inner radius	$r_4 := 3.5 \cdot mm$		
Magnet outer radius	$r_5 := 7.5 \cdot mm$	Pole-piece thickness	$t_p := \frac{L_m}{2} - T_m$ $t_p = 1.3\,mm$

Define the demagnetisation line of the permanent magnet material

$B_r := 8500 \cdot G$ $H_c := 8000 \cdot Oe$ In SI units $B_r = 0.85\,T$ $H_c = 637\,kA \cdot m^{-1}$

Define the permeances

$$P_1(r_1, r_2, g_f) := \frac{\mu_0 \cdot \pi \cdot \left(r_2^{\,2} - r_1^{\,2}\right)}{g_f}$$

Equation 19.46 $P_1(r_1, r_2, g_f) = 1.183 \times 10^{-8} \cdot H$

Number of terms for the summations $nmax := 21$ $n := 1, 3 .. nmax$

$$P_2(r_1, L_m, g_f) := \sum_n \left[\left(\frac{4 \cdot \mu_0 \cdot L_m \cdot r_1}{\pi \cdot n^2 \cdot g_f} \right) \cdot \frac{I1\left(\frac{2 \cdot n \cdot \pi \cdot r_1}{L_m}\right)}{I0\left(\frac{2 \cdot n \cdot \pi \cdot r_1}{L_m}\right)} \cdot \sin\left(\frac{n\,\pi \cdot g_f}{L_m}\right) \right]$$

Equation 19.47 $P_2(r_1, L_m, g_f) = 3.868 \times 10^{-9}\,H$

$$P_3(r_3,L_m,T_m) := \sum_n \left[\left(\frac{4\cdot\mu_0\cdot L_m\cdot r_3}{\pi\cdot n^2\cdot T_m}\right)\cdot\frac{K1\left(\frac{2\cdot n\cdot\pi\cdot r_3}{L_m}\right)}{K0\left(\frac{2\cdot n\cdot\pi\cdot r_3}{L_m}\right)}\cdot\sin\left(\frac{n\,\pi\cdot T_m}{L_m}\right)\right]$$

Equation 19.48

$$P_3(r_3,L_m,T_m) = 3.225\times 10^{-8}\,\mathrm{H}$$

$$P_4(r_2,r_4,T_m) := \frac{\mu_0\cdot\pi\cdot\left(r_4^{\,2}-r_2^{\,2}\right)}{T_m}$$

Equation 19.49

$$P_4(r_2,r_4,T_m) = 3.944\times 10^{-9}\,\mathrm{H}$$

$$P_5(r_3,r_5,T_m) := \begin{cases}\dfrac{\mu_0\cdot\pi\cdot\left(r_3^{\,2}-r_5^{\,2}\right)}{T_m} & \text{if } r_3 > r_5\\ 0 & \text{otherwise}\end{cases}$$

Equation 19.50

$$P_5(r_3,r_5,T_m) = 0\,\mathrm{H}$$

$$R_6(r_3,r_5,t_p) := \left|\begin{array}{l} R_6 \leftarrow \dfrac{1}{2\cdot\mu_0\cdot\pi\cdot t_p}\cdot\ln\left(\dfrac{r_5}{r_3}\right) \quad \text{if } r_5 > r_3\\ R_6 \leftarrow 0 \quad \text{otherwise}\\ R_6\end{array}\right.$$

Equation 19.51
(reciprocal)
Note: this equation is
incorrect in the book

$$R_6(r_3,r_5,t_p) = 0\,\frac{1}{\mathrm{H}}$$

$$P_{3m}(r_3,r_5,L_m,T_m,t_p) := \left(\frac{1}{P_3(r_3,L_m,T_m)} + 2\cdot R_6(r_3,r_5,t_p)\right)^{-1}$$

Equation 19.52

$$P_{3m}(1\cdot r_3,r_5,L_m,T_m,t_p) = 3.225\times 10^{-8}\,\mathrm{H}$$

Calculate the total permeance

$$P_t(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := P_1(r_1,r_2,g_f) + P_2(r_1,L_m,g_f) + P_{3m}(r_3,r_5,L_m,T_m,t_p) + P_4(r_2,r_4,T_m) + P_5(r_3,r_5,T_m)$$

$$P_t(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 5.19\times 10^{-8}\,H$$

Magnet cross-sectional area

$$A_m(r_4,r_5) := \pi\cdot\left(r_5^{\,2} - r_4^{\,2}\right)$$

Find the working point of the magnet (H_d, B_d) and the flux density in the gap (B_g)

$$K_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := \frac{T_m}{A_m(r_4,r_5)}\cdot P_t(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)$$

Equation 19.53

$$K_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 1.108\times 10^{-6}\,T\cdot A^{-1}\cdot m$$

$$H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := \frac{B_r}{K_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) + \dfrac{B_r}{H_c}}$$

Equation 19.54

$$H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 348\cdot kA\cdot m^{-1}$$

$$B_g(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := \frac{\mu_0\cdot T_m\cdot H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)}{g_f}$$

$$B_g(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 0.573\,T$$

$$B_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := K_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)\cdot H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)$$

$$B_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 0.385\,\mathrm{T}$$

Energy product of the magnet

$$B_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)\cdot H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 1.341\times 10^5\cdot \mathrm{J}\cdot \mathrm{m}^{-3}$$

Find the flux density distribution in the beam tunnel

$$B_z(r,z,r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := \sum_n \left(\frac{4\cdot \sin\left(\frac{n\cdot\pi\cdot g_f}{L_m}\right)}{n\cdot\pi} \cdot \frac{I0\left(\frac{2\cdot n\cdot\pi\cdot r}{L_m}\right)}{I0\left(\frac{2\cdot n\cdot\pi\cdot r_1}{L_m}\right)} \cdot \cos\left(\frac{2\cdot n\cdot\pi\cdot z}{L_m}\right)\right)\cdot B_g(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)$$

Equation 19.55

$$Bz(r,z) := B_z(r,z,r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)$$

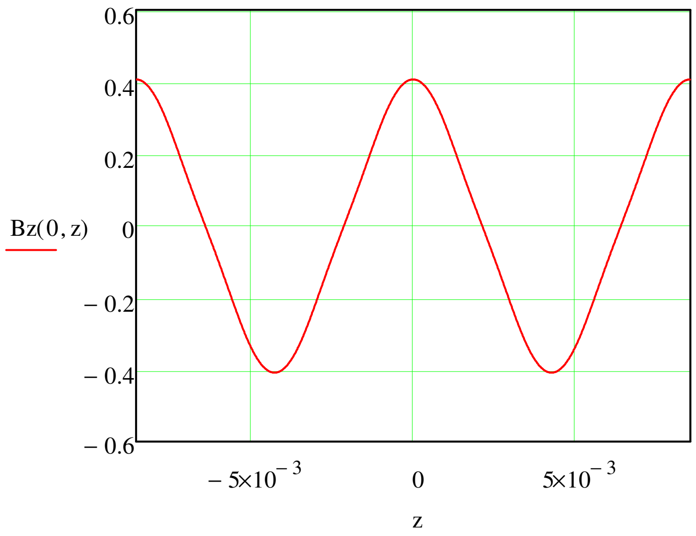
CHECK the magneto-motive force

Find the peak field on the axis

$$\frac{g_f}{\mu_0}\cdot B_g(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 1.027\times 10^3\cdot \mathrm{A}$$

$$B_{pk}(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) := B_z(0,0,r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p)$$

$$T_m\cdot H_d(r_1,r_2,r_3,r_4,r_5,L_m,g_f,T_m,t_p) = 1.027\times 10^3\,\mathrm{A}$$


0.6
0.4
0.2
0
− 0.2
− 0.4
− 0.6
Bz(0, z)
− 5×10−3
0
5×10−3
z

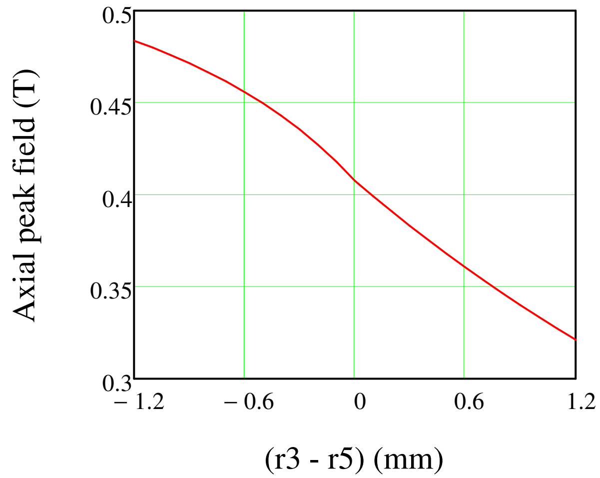
Peak field as a function of the pole-piece outer radius with all other dimensions fixed

$$B_{pk1}(r_3) := B_{pk}(r_1, r_2, r_3, r_4, r_5, L_m, g_f, T_m, t_p)$$

$$B_{pk1}(r_3) = 0.408\,T$$

Plotting range

$$r3 := 6.3 \cdot mm, 6.4 \cdot mm .. 8.7 \cdot mm$$



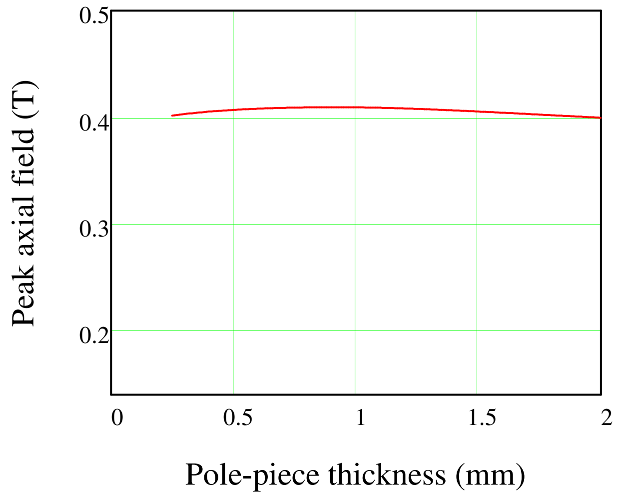
Compare Figure 4 in Santra et al.

Peak field as a function of pole-piece thickness with all other dimensions fixed

$$B_{pk2}(t_p) := B_{pk}\left[r_1, r_2, r_3, r_4, r_5, 2 \cdot (T_m + t_p), g_f, T_m, t_p\right]$$

$$B_{pk2}(t_p) = 0.408\,T$$

$$tp := 0.25 \cdot mm, 0.26 \cdot mm .. 2.0 \cdot mm$$



Compare Figure 6 in Santra et al.