Worksheet 8.1 Electron trajectories in crossed fields

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Planar geometry (Section 8.2)

Define the electron velocity, the cyclotron frequency and the value of r.

$u_0 := 1$ $\omega_c := 1$ $r_1 := -1.5$

$x(r,t,u_0) := -r\cdot\sin(\omega_c\cdot t) + u_0\cdot t$ $y(r,t) := r\cdot\cos(\omega_c\cdot t)$ Equation 8.6

$u_{x0}(r) := u_0 - \omega_c\cdot r$ $u_{x0}(r_1) = 2.5$ Equation 8.6

Plot the trajectory for $t := 0, 0.1 .. 4\cdot\pi$

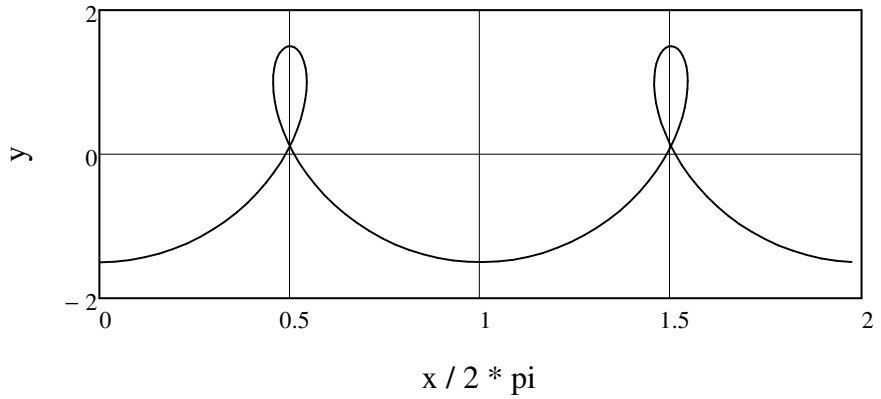

Figure 8.3

Note: Positive values of r produce identical graphs with a horizontal shift of 0.5

Cylindrical geometry (Section 8.4)

Express the differential equations as a set of simultaneous first order equations

$$\frac{dR}{d\alpha} = R'$$

$$\frac{dR'}{d\alpha} = \frac{1}{8}\frac{A^2 U}{\ln(A)}\left(1-\frac{1}{A^2}\right)^2\frac{1}{R} + \frac{1}{4}R\left\{\frac{1}{R^4}-1\right\}$$ Equation 8.62

$$\frac{d\theta}{d\alpha} = \frac{1}{2}\left(1-\frac{1}{R^2}\right)$$ Equation 8.63

Define the parameter values

$A := 2$ $U := 0.5$

$$D(\alpha, z) := \begin{bmatrix} z_1 \\ \frac{1}{8}\cdot\frac{A^2\cdot U}{\ln(A)}\cdot\left(1-\frac{1}{A^2}\right)^2\cdot\frac{1}{z_0}+\frac{1}{4}\cdot z_0\cdot\left[\frac{1}{(z_0)^4}-1\right] \\ \frac{1}{2}\cdot\left[1-\frac{1}{(z_0)^2}\right] \end{bmatrix}$$

Initial conditions

$$z := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$R = 1$

$\frac{dR}{d\alpha} = 0$

$\theta = 0$

$Z := \text{AdamsBDF}(z, 0, 40, 100, D)$

The solution of the equations is in the matrix Z. The first column is the normalised time (α) and the remaining columns are R, dR/dα and θ

Plot the trajectory for $\theta := 0, 1 \cdot deg .. 360 \cdot deg$

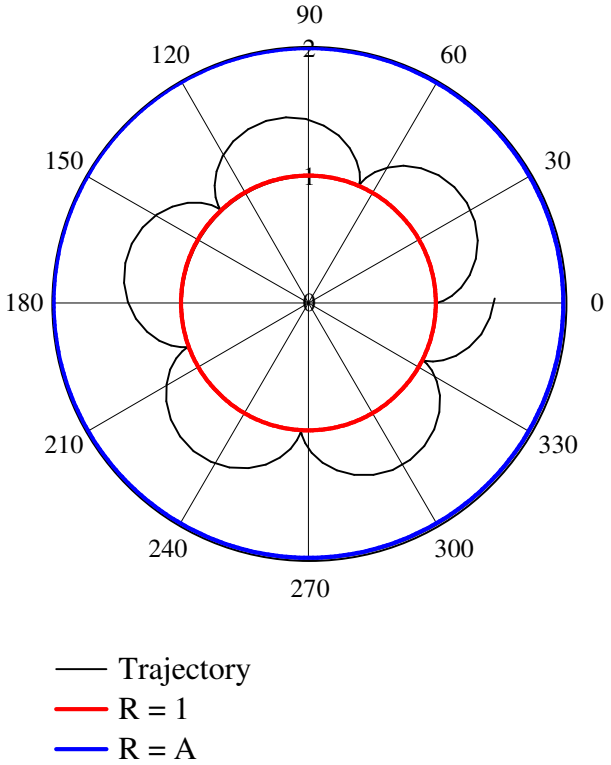

Figure 8.11