WS 1.1 Coupled modes

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Coupled-mode theory: Section 1.3.8

The contents of this sheet are based on Briggs, R. J., "Electron-stream interaction with plasmas", MIT Press, (1964) pp.39-42.

The working equations with their implementations in Mathcad are:

$$\left(\omega-\beta v_{g1}\right)\left(\omega-\beta v_{g2}\right)=k_0^2$$

Equation (1.28)

$$\omega=\frac{1}{2}\left\{\left(v_{g1}+v_{g2}\right)\beta\pm\sqrt{\left(v_{g1}-v_{g2}\right)^2\beta^2+4k_0^2}\right\}\quad(\text{real }\beta)$$

Equation (1.30)

$$\omega 1\left(\beta,v_{g1},v_{g2},k_0\right):=\frac{1}{2}\cdot\left[\left(v_{g1}+v_{g2}\right)\cdot\beta+\sqrt{\left(v_{g1}-v_{g2}\right)^2\cdot\beta^2+4k_0}\right]$$

$$\omega 2\left(\beta,v_{g1},v_{g2},k_0\right):=\frac{1}{2}\cdot\left[\left(v_{g1}+v_{g2}\right)\cdot\beta-\sqrt{\left(v_{g1}-v_{g2}\right)^2\cdot\beta^2+4k_0}\right]$$

$$\beta=\frac{1}{2v_{g1}v_{g2}}\left\{\left(v_{g1}+v_{g2}\right)\omega\pm\sqrt{\left(v_{g1}-v_{g2}\right)^2\omega^2+4v_{g1}v_{g2}k_0^2}\right\}\quad(\text{real }\omega)$$

Equation (1.31)

$$\beta 1\left(\omega,v_{g1},v_{g2},k_0\right):=\frac{1}{2\cdot v_{g1}\cdot v_{g2}}\cdot\left[\left(v_{g1}+v_{g2}\right)\cdot\omega+\sqrt{\left(v_{g1}-v_{g2}\right)^2\cdot\omega^2+4v_{g1}\cdot v_{g2}\cdot k_0}\right]$$

$$\beta 2\left(\omega,v_{g1},v_{g2},k_0\right):=\frac{1}{2\cdot v_{g1}\cdot v_{g2}}\cdot\left[\left(v_{g1}+v_{g2}\right)\cdot\omega-\sqrt{\left(v_{g1}-v_{g2}\right)^2\cdot\omega^2+4v_{g1}\cdot v_{g2}\cdot k_0}\right]$$

Case A: $v_{g1} > 0$; $v_{g2} > 0$; $k_0 > 0$

$v_{g1} := 1.5$ $v_{g2} := 0.5$ $k_0 := 0.3$

$R\omega1(\beta, k_0) := \mathrm{Re}(\omega1(\beta, v_{g1}, v_{g2}, k_0))$

$I\omega1(\beta, k_0) := \mathrm{Im}(\omega1(\beta, v_{g1}, v_{g2}, k_0))$

$R\omega2(\beta, k_0) := \mathrm{Re}(\omega2(\beta, v_{g1}, v_{g2}, k_0))$

$I\omega2(\beta, k_0) := \mathrm{Im}(\omega2(\beta, v_{g1}, v_{g2}, k_0))$

$R\beta1(\omega, k_0) := \mathrm{Re}(\beta1(\omega, v_{g1}, v_{g2}, k_0))$

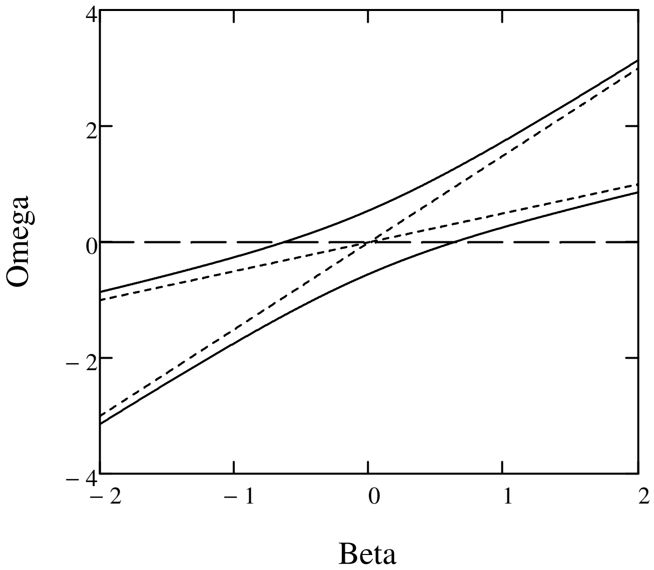
$I\beta1(\omega, k_0) := \mathrm{Im}(\beta1(\omega, v_{g1}, v_{g2}, k_0))$

$R\beta2(\omega, k_0) := \mathrm{Re}(\beta2(\omega, v_{g1}, v_{g2}, k_0))$

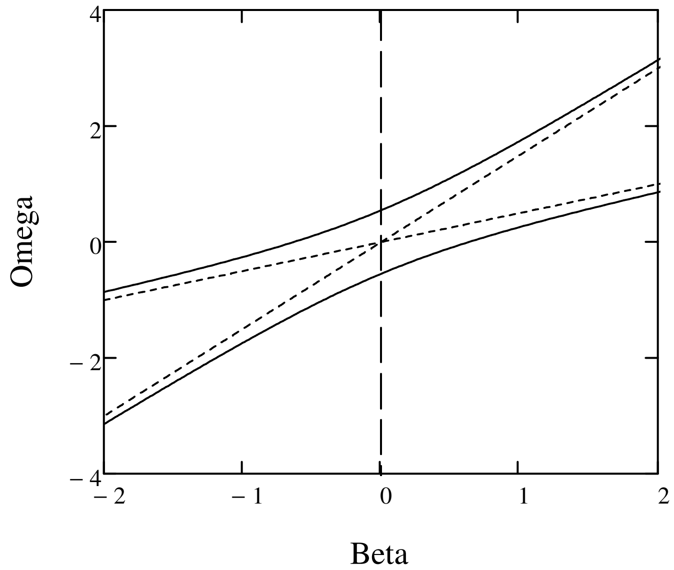
$I\beta2(\omega, k_0) := \mathrm{Im}(\beta2(\omega, v_{g1}, v_{g2}, k_0))$

Figure 1.8

Real beta



Real omega



Case B: $v_{g1} > 0$; $v_{g2} < 0$; $k_0 > 0$

$v_{g1} := \frac{4}{3}$ $v_{g2} := \frac{-4}{3}$ $k_0 := 1$ Note: These values overwrite those defined above

$R\omega 1(\beta, k_0) := \mathrm{Re}(\omega 1(\beta, v_{g1}, v_{g2}, k_0))$

$I\omega 1(\beta, k_0) := \mathrm{Im}(\omega 1(\beta, v_{g1}, v_{g2}, k_0))$

$R\omega 2(\beta, k_0) := \mathrm{Re}(\omega 2(\beta, v_{g1}, v_{g2}, k_0))$

$I\omega 2(\beta, k_0) := \mathrm{Im}(\omega 2(\beta, v_{g1}, v_{g2}, k_0))$

$R\beta 1(\omega, k_0) := \mathrm{Re}(\beta 1(\omega, v_{g1}, v_{g2}, k_0))$

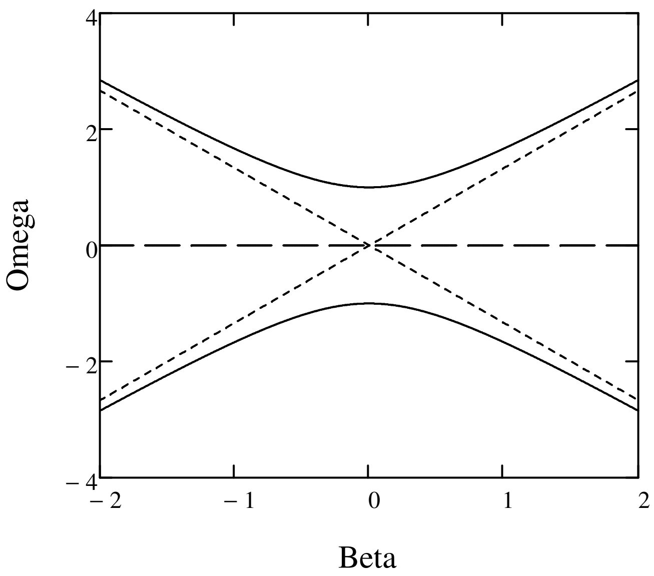
$I\beta 1(\omega, k_0) := \mathrm{Im}(\beta 1(\omega, v_{g1}, v_{g2}, k_0))$

$R\beta 2(\omega, k_0) := \mathrm{Re}(\beta 2(\omega, v_{g1}, v_{g2}, k_0))$

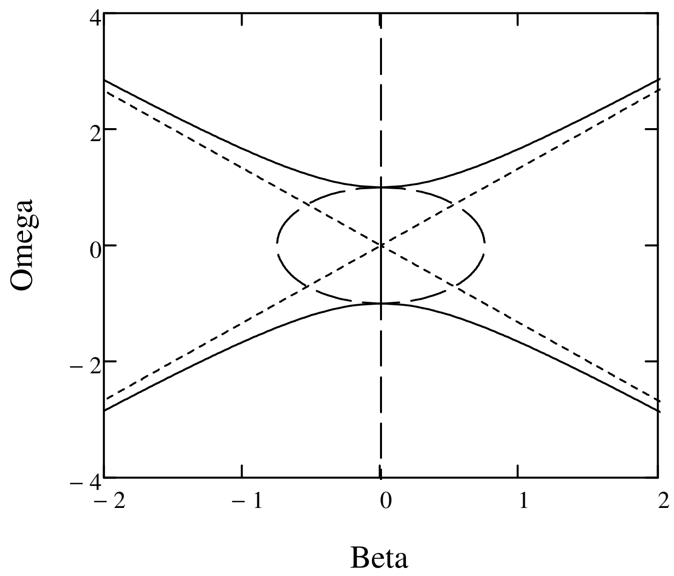
$I\beta 2(\omega, k_0) := \mathrm{Im}(\beta 2(\omega, v_{g1}, v_{g2}, k_0))$

Figure 1.9

Real beta



Real omega



Case C: $v_{g1} > 0$; $v_{g2} > 0$; $k_0 < 0$

$v_{g1} := 1.5$ $\quad$ $v_{g2} := 0.5$ $\quad$ $k_0 := -0.3$

$R\omega1(\beta, k_0) := \mathrm{Re}(\omega1(\beta, v_{g1}, v_{g2}, k_0))$ $\quad$ $R\beta1(\omega, k_0) := \mathrm{Re}(\beta1(\omega, v_{g1}, v_{g2}, k_0))$

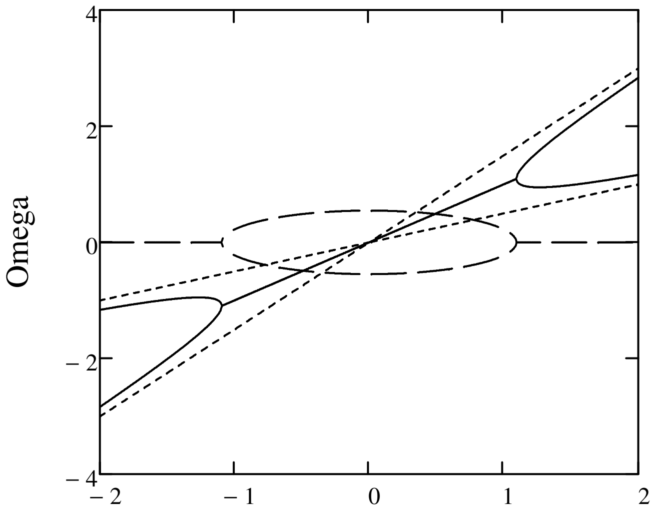
$I\omega1(\beta, k_0) := \mathrm{Im}(\omega1(\beta, v_{g1}, v_{g2}, k_0))$ $\quad$ $I\beta1(\omega, k_0) := \mathrm{Im}(\beta1(\omega, v_{g1}, v_{g2}, k_0))$

$R\omega2(\beta, k_0) := \mathrm{Re}(\omega2(\beta, v_{g1}, v_{g2}, k_0))$ $\quad$ $R\beta2(\omega, k_0) := \mathrm{Re}(\beta2(\omega, v_{g1}, v_{g2}, k_0))$

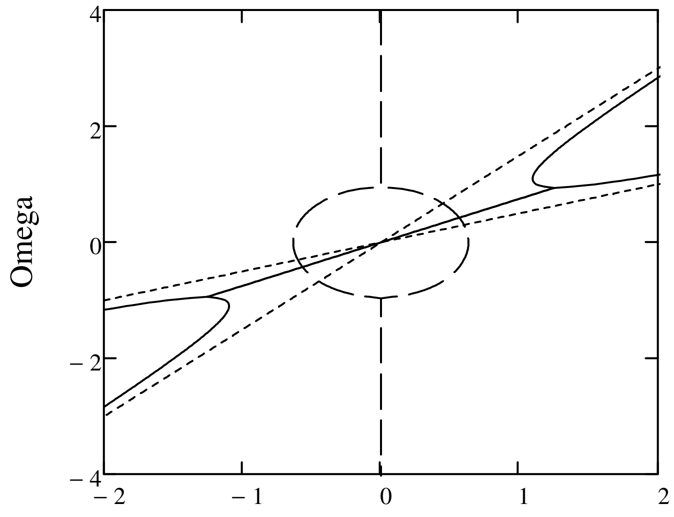
$I\omega2(\beta, k_0) := \mathrm{Im}(\omega2(\beta, v_{g1}, v_{g2}, k_0))$ $\quad$ $I\beta2(\omega, k_0) := \mathrm{Im}(\beta2(\omega, v_{g1}, v_{g2}, k_0))$

Figure 1.10

Real beta



Real omega



Case D: $v_{g1} > 0$; $v_{g2} < 0$; $k_0 < 0$

$$v_{g1} := \frac{4}{3} \qquad v_{g2} := \frac{-4}{3} \qquad k_0 := -1$$

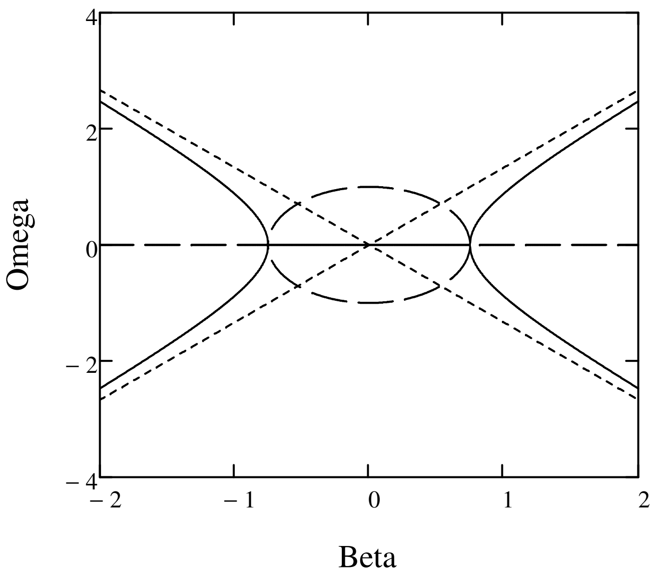
$$R\omega1(\beta, k_0) := \mathrm{Re}\left(\omega1(\beta, v_{g1}, v_{g2}, k_0)\right) \qquad R\beta1(\omega, k_0) := \mathrm{Re}\left(\beta1(\omega, v_{g1}, v_{g2}, k_0)\right)$$

$$I\omega1(\beta, k_0) := \mathrm{Im}\left(\omega1(\beta, v_{g1}, v_{g2}, k_0)\right) \qquad I\beta1(\omega, k_0) := \mathrm{Im}\left(\beta1(\omega, v_{g1}, v_{g2}, k_0)\right)$$

$$R\omega2(\beta, k_0) := \mathrm{Re}\left(\omega2(\beta, v_{g1}, v_{g2}, k_0)\right) \qquad R\beta2(\omega, k_0) := \mathrm{Re}\left(\beta2(\omega, v_{g1}, v_{g2}, k_0)\right)$$

$$I\omega2(\beta, k_0) := \mathrm{Im}\left(\omega2(\beta, v_{g1}, v_{g2}, k_0)\right) \qquad I\beta2(\omega, k_0) := \mathrm{Im}\left(\beta2(\omega, v_{g1}, v_{g2}, k_0)\right)$$

Real beta



Real omega

Figure 1.11