Worksheet 7.2 Solenoid Focusing

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

The radial motion of an electron on the edge of the beam can be found by solving the differential equation below subject to the initial conditions in R and R' where R = r/b and b is the equilibrium beam radius. The effect of changing the magnetic field, the beam current and the initial conditions can be investigated. The independent variable is

$$\theta = \frac{\omega_L}{u_0}\cdot z$$

Set the values of the beam stiffness parameter m, the initial conditions for the normalised radius and normalised slope and the final value of θ.

$m_B := 1.0$ $R_0 := 1.1$ $R1_0 := 0$ $\theta_{end} := 10\cdot\pi$

Definitions for the magnetic field and beam current relative to their uniform values. The effects of different functions can be investigated by selecting them.

B_control :=
Uniform
Step
Ramp

k_control :=
Uniform
Step
Ramp

$B\_step := 2.0$

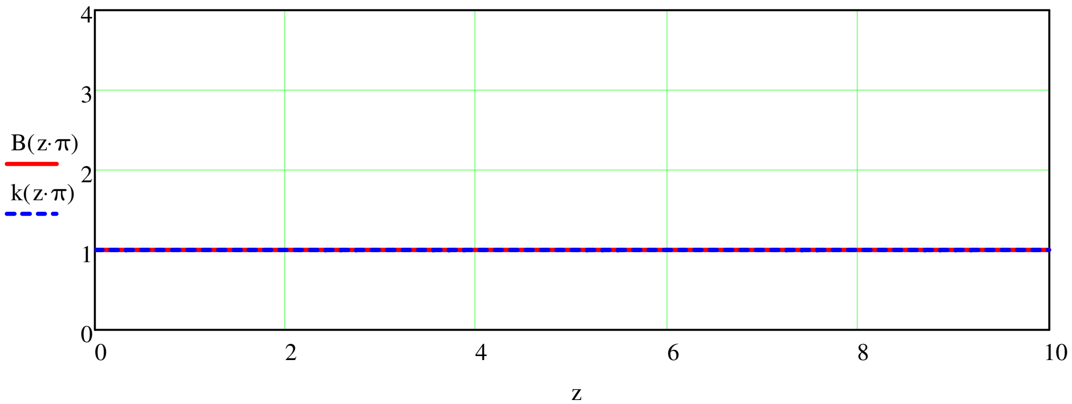
$k\_step := 2.0$

$B\_slope := 1.0$

$k\_slope := 1.0$

$$B(\theta) := \left| \begin{array}{l} B \leftarrow \left| \begin{array}{l} 1 \quad \text{if} \quad \theta < 2\cdot\pi \\ 1 \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge B\_control = 1 \\ B\_step \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge B\_control = 2 \\ \left[1 + \dfrac{B\_slope\cdot(\theta - 2\cdot\pi)}{2.\pi}\right] \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge B\_control = 3 \end{array} \right. \\ \text{return } B \end{array} \right.$$

$$k(\theta) := \left| \begin{array}{l} k \leftarrow \left| \begin{array}{l} 1 \quad \text{if} \quad \theta < 2\cdot\pi \\ 1 \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge k\_control = 1 \\ k\_step \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge k\_control = 2 \\ \left[1 + \dfrac{k\_slope\cdot(\theta - 2\cdot\pi)}{2.\pi}\right] \quad \text{if} \quad \theta \geq 2\cdot\pi \wedge k\_control = 3 \end{array} \right. \\ \text{return } k \end{array} \right.$$



The differential equation is solved for the initial conditions $R = R_0$ and $dR/d\theta = R1_0$

$$D(\theta, R) := \begin{bmatrix} R_1 \\ \dfrac{k(\theta)}{{m_B}^2 \cdot R_0} + \left[\left(1 - \dfrac{1}{{m_B}^2}\right) \cdot \dfrac{1}{(R_0)^3} - R_0\right] \cdot B(\theta) \end{bmatrix}$$

$$\frac{dR}{d\theta} = R_1$$

$$\frac{dR_1}{d\theta} = f(R, m_B)$$

The equation derived from equation 7.53 is expressed as a pair of simultaneous first-order differential equations,

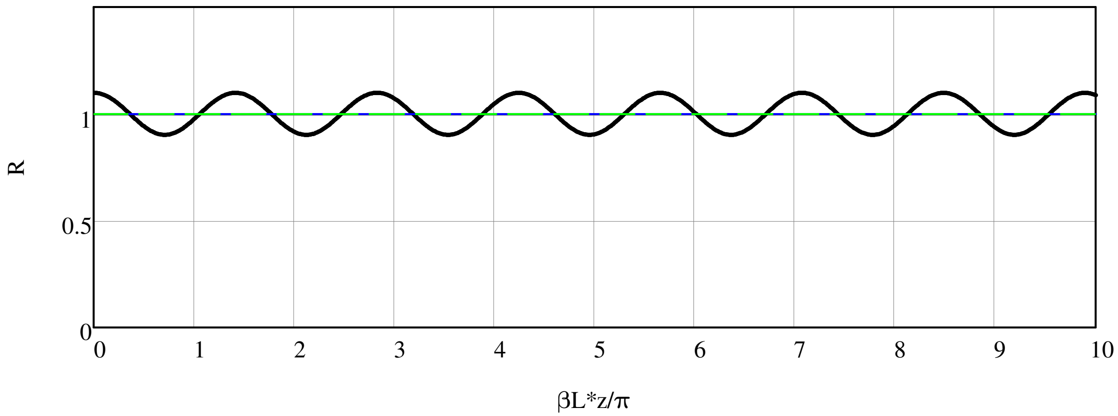
$$R0 := \begin{pmatrix} R_0 \\ R1_0 \end{pmatrix}$$

Initial conditions

$$\Theta := \text{AdamsBDF}(R0, 0, 10 \cdot \pi, 1000, D)$$

Numerical solution of the differential equation. The first column of the matrix Θ contains values of θ, the second column contains the normalised radius and the third column contains the normalised slope of the beam edge.

$m_B = 1$ $R_0 = 1.1$ $R1_0 = 0$



When the beam is not in equilibrium it scallops around the equilibrium radius.

The maximum and minimum values of R can be found by using the Trace facility.

Estimation of beam stiffness

The beam stiffness can be estimated by calculating the change in the equilibrium radius when the beam current is multiplied by a factor *k.*

$$R1(k, m_B) := \left[\frac{k + \sqrt{k^2 + 4\cdot m_B{}^2\cdot\left(m_B{}^2 - 1\right)}}{2\cdot m_B{}^2}\right]^{0.5}$$

Equation 7.65

Plot the dependence of the normalised equilibrium radius on the normalised current for various values of *m.* The results are close to those obtained from integration of the differential equation with a ramp function for the current.

$k1 := 0, 0.01 .. 4$

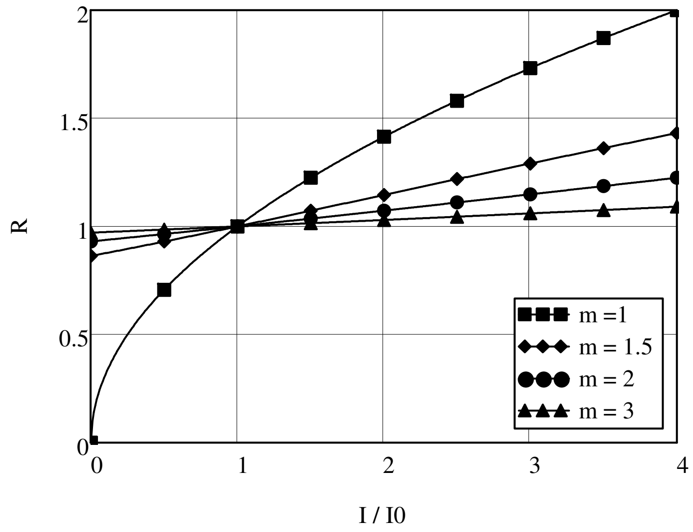


Figure 7.5

The normalised RF bulk and surface currents can be estimated by finding the proportions of (I - I_0) which lie inside and outside R = 1.

$$I_B(k, m_B) := \frac{\left[\left(\frac{k}{R1(k, m_B)^2}\right) - 1\right]}{k - 1}$$ Equation 7.69 $$I_S(k, m_B) := 1 - I_B(k, m_B)$$

Define the range of values of m and plot the normalised body and surface currents for different depths of modulation. Note: I_1/I_0 = (k - 1)

$mB := 1, 1.01 .. 5$

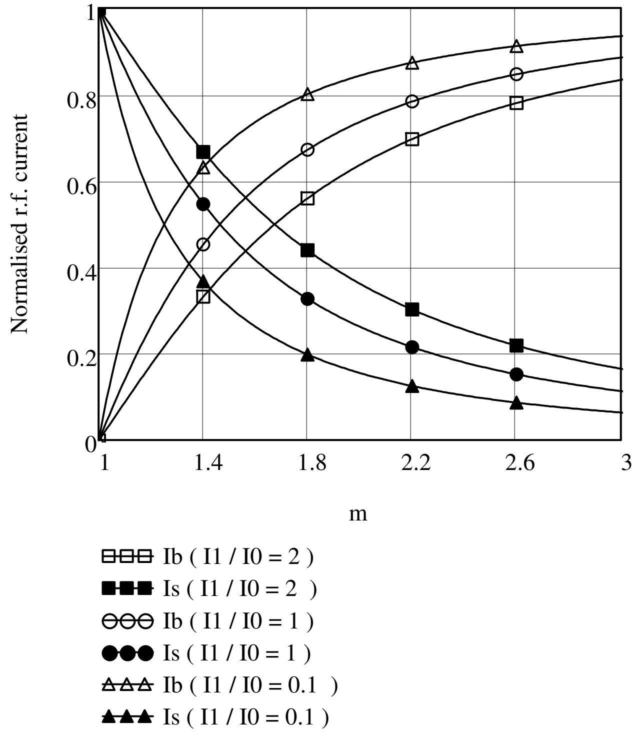


Figure 7.6

Define the parameter $\Omega 1$ to enable I_B and I_S to be plotted for comparison with Figure 2 in Brewer, G. R. (1956). "Some effects of magnetic field strength on space-charge-wave propagation." Proceedings of the IRE **44**(7): 896-903.

$$\Omega 1(m_B) := \sqrt{\frac{\left(m_B{}^2 - 1\right)}{2}}$$