WS 1.3 Linearity and intermodulation distortion

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Section 1.6.4 Single carrier transfer characteristics

Simple functions which can be made a good fit to experimental data for TWTs are given in:

A. A. M. Saleh, "Frequency-Independent and Frequency-Dependent Nonlinear Models of TWT Amplifiers," *IEEE Transactions on Communications,* vol. 29, pp. 1715-1720, (1981).

Input power in dB normalised to saturation at r = 1 $D1(r) := 20 \cdot \log(r)$

Empirical constants defining the AM/AM and AM/PM transfer curves are adjusted to fit simulated or experimental data

$\alpha_a := 2$ $\beta_a := 1$ $\alpha_\phi := -2.53$ $\beta_\phi := 2.8$

AM/AM curve $$Am(r) := \frac{\alpha_a \cdot r}{1 + \beta_a \cdot r^2}$$ Equation 1.56

AM/PM curve $$\Phi m(r) := \frac{\alpha_\phi \cdot r^2}{1 + \beta_\phi \cdot r^2}$$ Equation 1.57

AM/AM conversion at the second harmonic might be modelled by a similar function with constants $\alpha_h := 3$ $\beta_h := 1.5$

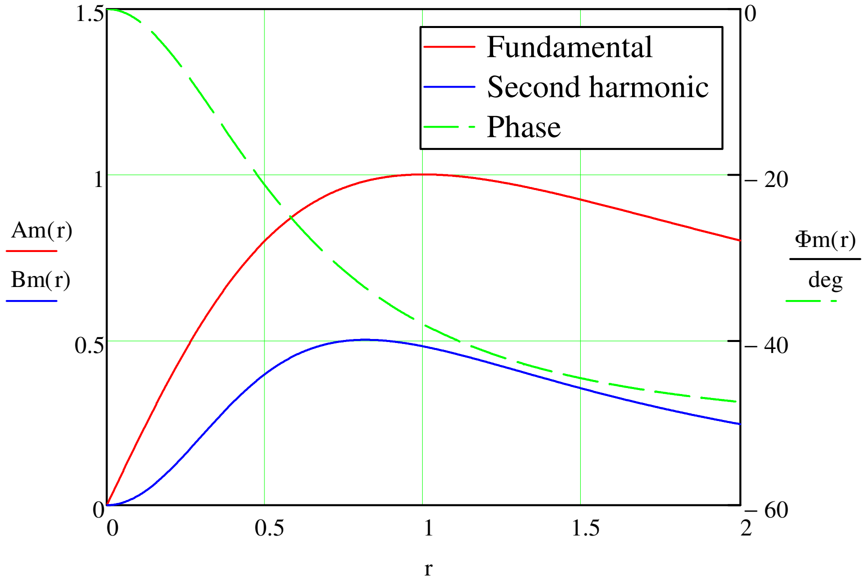
$$Bm(r) := \frac{\alpha_h \cdot r^2}{\left(1 + \beta_h \cdot r^2\right)^2}$$ Equation 1.58

Output power in dB normalised to saturation $D2(r) := 20 \cdot \log(Am(r))$

Output power in dB at the second harmonic normalised to saturation $Dh(r) := 20 \cdot \log(Bm(r))$

Linear extrapolations of the small-signal curves $Dhh(r) := 2 \cdot D1(r) + 20 \cdot \log(\alpha_h)$ $D22(r) := D1(r) + 20 \cdot \log(\alpha_a)$

Plot the characteristics on linear scales



Plotting range for r $\quad r := 0.01, 0.04 .. 4$

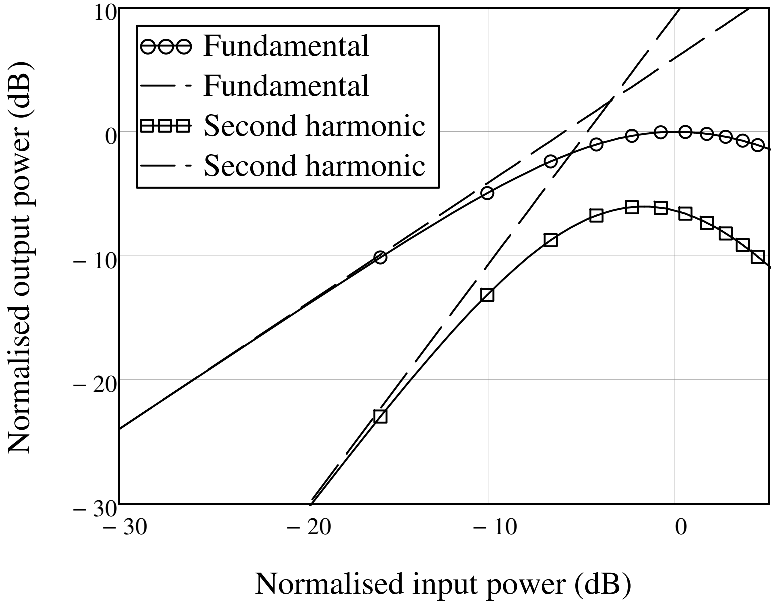


Figure 1.4

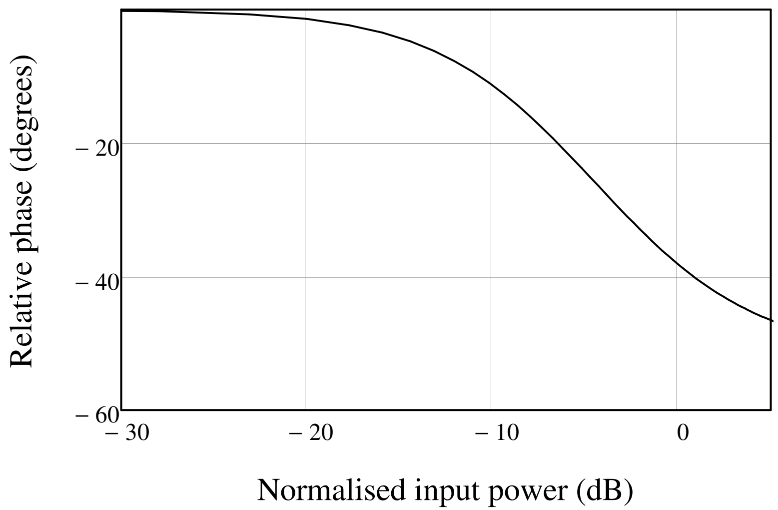


Figure 1.5

Compare Figure 14.23(a)

Compare Figure 14.24

Two-tone intermodulation products

Define the input signal in the time domain with two equal carriers with amplitude $\frac{V_1}{\sqrt{2}}$ at frequencies (n+1) and (n-1)

$n := 10$

so that the total carrier power is equal to that of a single carrier with amplitude V_1

Envelope $r(V_1, \phi) := \sqrt{2} \cdot V_1 \cdot \sin(\phi)$

Note $\phi = \Delta\omega \cdot t$

Input waveform $V_{in}(V_1, \phi) := r(V_1, \phi) \cdot \cos(n \cdot \phi)$

Equation 1.50

Plot the input waveform with $V_1 := \sqrt{2}$

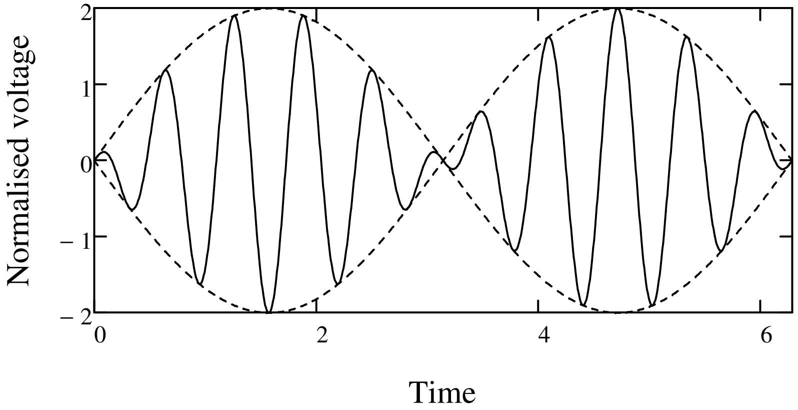


Figure 1.16(a)

The output voltage in the time domain is found using the AM/AM and AM/PM curves defined above.

$$V2\left(V_1, \phi, \phi s\right) := Am\left(r\left(V_1, \phi\right)\right) \cdot \cos\left(n \cdot \phi + \phi s \cdot \Phi m\left(r\left(V_1, \phi\right)\right)\right)$$

Equation 1.51

The parameter ϕs is zero if the AM/PM characteristics are ignored and 1 if they are included

Plot the output voltage waveform

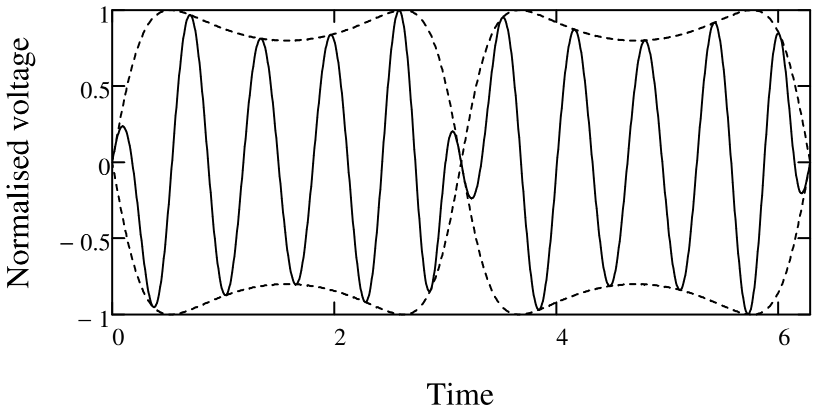


Figure 1.16(b)

The amplitudes of the harmonics of the output waveform are found by Fourier analysis

$$V_{10}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n+1) \cdot \phi] \, d\phi \qquad V_{10}(1,1) = 0.461$$

$$V_{01}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n-1) \cdot \phi] \, d\phi \qquad V_{01}(1,1) = -0.461$$

$$V_{21}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n+3)\phi] \, d\phi \qquad V_{21}(1,1) = 0.164$$

$$V_{12}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n-3)\phi] \, d\phi \qquad V_{12}(1,1) = -0.164$$

$$V_{32}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n+5)\phi] \, d\phi \qquad V_{32}(1,1) = 0.057$$

$$V_{23}(V_1, \phi s) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} V2(V_1, \phi, \phi s) \cdot \sin[(n-5)\phi] \, d\phi \qquad V_{23}(1,1) = -0.057$$

Total carrier, third order and fifth order intermodulation powers in dB

$$D_{10}(V_1, \phi s) := 10 \cdot \log\left(2\, V_{10}(V_1, \phi s)^2\right) \qquad D_{21}(V_1, \phi s) := 10 \cdot \log\left(2\, V_{21}(V_1, \phi s)^2\right) \qquad D_{32}(V_1, \phi s) := 10 \cdot \log\left(2\, V_{32}(V_1, \phi s)^2\right)$$

Plotting range for the input voltage $V_1 := 0, 0.01 .. 10.0$

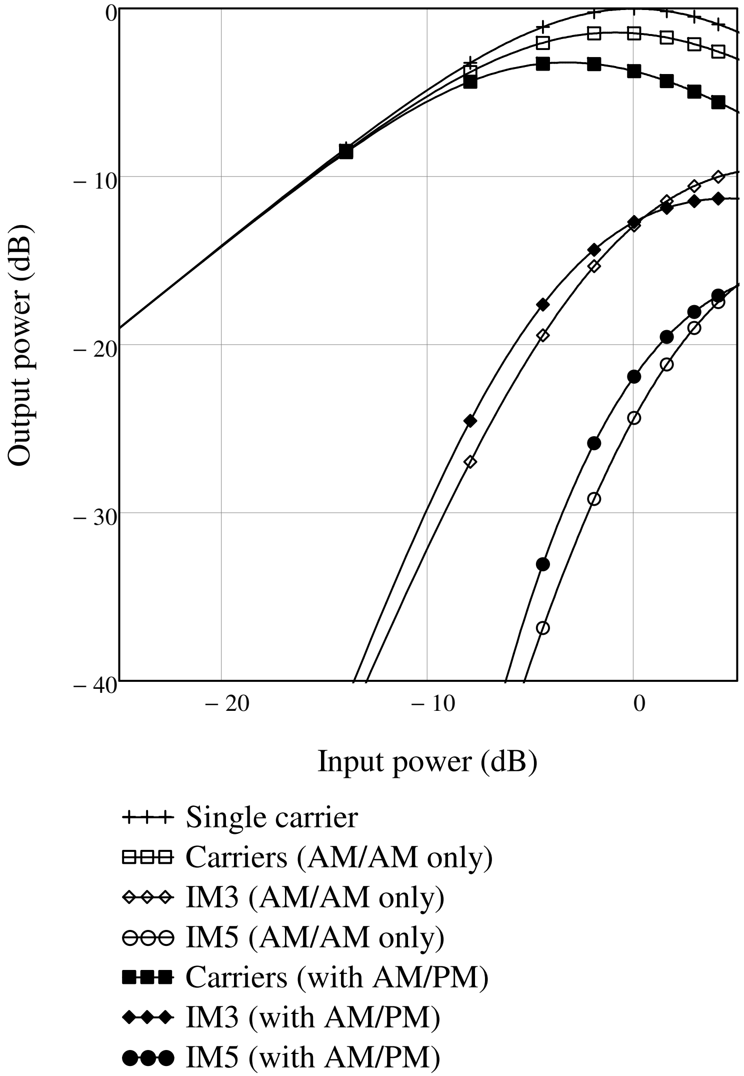


Figure 1.15 (corrected)

Compare Figure 2 in: Stette, G. R.
"Calculation of Intermodulation from a Single Carrier Amplitude Characteristic."
IEEE Transactions on Communications **22**(3): 319-323.(1974).