WS 3.3 Re-entrant cavity with beam hole

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

The definitions of the matrices used can be found in

Carter, R. G., J. J. Feng, et al. (2007). "Calculation of the properties of reentrant cylindrical cavity resonators." IEEE Transactions on Microwave Theory and Techniques **55**(12): 2531-2538.

Section 3.5.1 Method of moments model of a cylindrical re-entrant cavity resonator

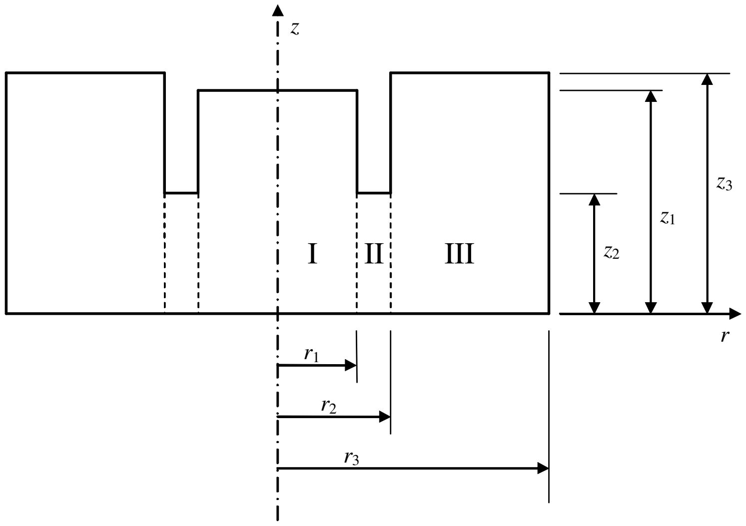
Define physical constants

$\varepsilon_0 = 8.854\times 10^{-12}\cdot\frac{F}{m}$ $\mu_0 = 1.257\times 10^{-6}\cdot\frac{H}{m}$ $\eta := 1.759\cdot 10^{11}\cdot\frac{C}{kg}$

$c = 2.998\times 10^{8}\,\frac{m}{s}$ $Z_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}}$ $Y_0 := \frac{1}{Z_0}$

Define the cavity dimensions

The cavity is divided into three regions numbered I, II and III starting from the axis. The height of region i is z_i and its outer radius is r_i



$r_1 := 5 \cdot mm$ $\quad r_2 := 7.0 \cdot mm$ $\quad r_3 := 26.11 \cdot mm$

$z_1 := 20 \cdot mm$ $\quad z_2 := 2.5 \cdot mm$ $\quad z_3 := 10 \cdot mm$

The length z1 may be multiplied by a constant if required. If z1 = z2 = z3 then the results for a pill-box cavity are reproduced exactly.

Set the gap voltage and the conductivity of the cavity walls

$V_0 := 1000 \cdot V$ $\quad \sigma := 5.959 \cdot 10^7 \cdot \frac{S}{m}$

Set the parameters of the computation. NN2 sets the number of basis functions used in region 2. The numbers of basis functions in the other regions are chosen so that the shortest wavelengths are approximately the same. This gives the greatest accuracy.

$NN2 := 16$ $\quad NN3 := \mathrm{round}\left[\frac{z_3}{z_2} \cdot (NN2 + 1) - 1\right]$ $\quad NN1 := \mathrm{round}\left[\frac{z_1}{z_2} \cdot (NN2 + 1) - 1\right]$ $\quad N1 := NN1 \cdot 1$ $\quad N2 := NN2 \cdot 1$ $\quad N3 := NN3 \cdot 1$

Define the frequency which is used as an initial guess $\quad f_0 \equiv 3000 \cdot MHz$

The region below may be collapsed to allow the input data and the results to appear on the screen simultaneously

Define the radial variation functions for the inner (1) middle (2) and outer (3) regions

Region I (inner, beam hole)

$$\beta 1 := \frac{\pi}{z_1}$$

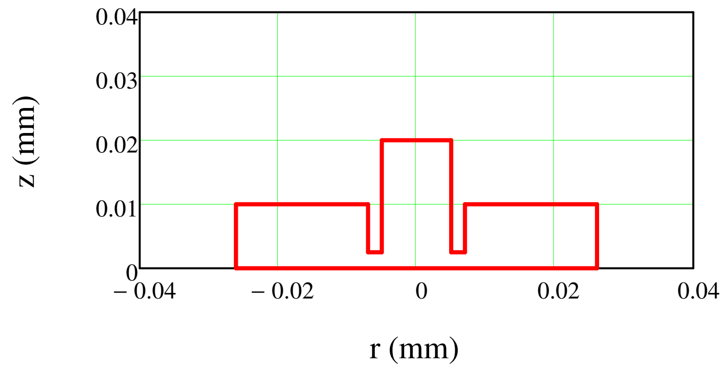
$$\gamma 1(k) := \left| \begin{array}{l} \text{for } n \in 0..N1 \\ \quad \gamma 1_n \leftarrow \sqrt{\left|k^2 - n^2 \cdot \beta 1^2\right|} \\ \text{return } \gamma 1 \end{array} \right.$$

Note: approximations are used for the Bessel functions when the arguments are large

$$GI(k) := \left| \begin{array}{l} \gamma \leftarrow \gamma 1(k) \\ \gamma r \leftarrow \gamma \cdot r_1 \\ jkY_0 \leftarrow j \cdot k \cdot Y_0 \\ \text{for } n \in 0..N1 \\ \quad \left| \begin{array}{l} G1_{n,n} \leftarrow \dfrac{J1(\gamma r_n)}{\gamma_n \cdot J0(\gamma r_n)} \text{ if } (|k|)^2 \geq n^2 \cdot \beta 1^2 \\ G1_{n,n} \leftarrow \dfrac{I1(\gamma r_n)}{\gamma_n \cdot I0(\gamma r_n)} \text{ if } (|k|)^2 < n^2 \cdot \beta 1^2 \wedge \gamma r_n < 700 \\ G1_{n,n} \leftarrow \dfrac{8 \cdot (\gamma r_n) - 3}{\gamma_n \cdot \left[8 \cdot (\gamma r_n) + 1\right]} \text{ if } (|k|)^2 < n^2 \cdot \beta 1^2 \wedge \gamma r_n \geq 700 \end{array} \right. \\ jkY_0 \cdot G1 \end{array} \right.$$

Plot cavity shape

$$rr := \begin{pmatrix} -r_3 \\ -r_3 \\ -r_2 \\ -r_2 \\ -r_1 \\ -r_1 \\ r_1 \\ r_1 \\ r_2 \\ r_2 \\ r_3 \\ r_3 \\ -r_3 \end{pmatrix} \qquad zz := \begin{pmatrix} 0 \\ z_3 \\ z_3 \\ z_2 \\ z_2 \\ z_1 \\ z_1 \\ z_2 \\ z_2 \\ z_3 \\ z_3 \\ 0 \\ 0 \end{pmatrix}$$



Region 2 (middle, drift tube)

$$\beta 2 := \frac{\pi}{z_2}$$

$$\gamma 2(k) := \left| \begin{array}{l} \text{for } n \in 0..N2 \\ \quad \gamma 2_n \leftarrow \sqrt{\left|k^2 - n^2 \cdot \beta 2^2\right|} \\ \text{return } \gamma 2 \end{array} \right.$$

$$U11(k) := \left| \begin{array}{l} \gamma \leftarrow \gamma 2(k) \\ \gamma r1 \leftarrow \gamma \cdot r_1 \\ \gamma r2 \leftarrow \gamma \cdot r_2 \\ arg \leftarrow \gamma r2 - \gamma r1 \\ jkY_0 \leftarrow j \cdot k \cdot Y_0 \\ \text{for } n \in 0..N2 \\ \quad \left| \begin{array}{l} U11_{n,n} \leftarrow \dfrac{J0(\gamma r2_n) \cdot Y1(\gamma r1_n) - Y0(\gamma r2_n) \cdot J1(\gamma r1_n)}{J0(\gamma r2_n) \cdot Y0(\gamma r1_n) - Y0(\gamma r2_n) \cdot J0(\gamma r1_n)} \text{ if } (|k|)^2 \ge n^2 \cdot \beta 2^2 \\ U11_{n,n} \leftarrow -\dfrac{I0(\gamma r2_n) \cdot K1(\gamma r1_n) + K0(\gamma r2_n) \cdot I1(\gamma r1_n)}{I0(\gamma r2_n) \cdot K0(\gamma r1_n) - K0(\gamma r2_n) \cdot I0(\gamma r1_n)} \text{ if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n < 700 \\ U11_{n,n} \leftarrow -\dfrac{(64 \cdot \gamma r1_n \cdot \gamma r2_n + 3) \cdot \cosh(arg_n) + 8 \cdot (\gamma r1_n + 3 \cdot \gamma r2_n) \cdot \sinh(arg_n)}{(64 \cdot \gamma r1_n \cdot \gamma r2_n - 1) \cdot \sinh(arg_n) - 8 \cdot arg_n \cdot \cosh(arg_n)} \text{ if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n \ge 700 \\ UU11_{n,n} \leftarrow \dfrac{jkY_0}{\gamma_n} \cdot U11_{n,n} \end{array} \right. \\ UU11 \end{array} \right.$$

$$
\begin{aligned}
U12(k) := \;& \gamma \leftarrow \gamma 2(k) \\
& \gamma r1 \leftarrow \gamma \cdot r_1 \\
& \gamma r2 \leftarrow \gamma \cdot r_2 \\
& arg \leftarrow \gamma r2 - \gamma r1 \\
& jkY_0 \leftarrow j \cdot k \cdot Y_0 \\
& \text{for } n \in 0..N2 \\
& \quad U12_{n,n} \leftarrow \frac{2}{\pi \cdot \gamma r1_n} \cdot \frac{1}{J0(\gamma r2_n) \cdot Y0(\gamma r1_n) - Y0(\gamma r2_n) \cdot J0(\gamma r1_n)} \text{ if } (|k|)^2 \geq n^2 \cdot \beta 2^2 \\
& \quad U12_{n,n} \leftarrow \frac{1}{\gamma r1_n} \cdot \frac{1}{I0(\gamma r2_n) \cdot K0(\gamma r1_n) - K0(\gamma r2_n) \cdot I0(\gamma r1_n)} \text{ if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n < 700 \\
& \quad U12_{n,n} \leftarrow \sqrt{\frac{r_2}{r_1}} \cdot \left[\frac{64 \cdot (\gamma r1_n \cdot \gamma r2_n)}{(64 \cdot \gamma r1_n \cdot \gamma r2_n - 1) \cdot \sinh(arg_n) - 8 \cdot arg_n \cdot \cosh(arg_n)} \right] \text{ if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n \geq 700 \\
& \quad UU12_{n,n} \leftarrow \frac{jkY_0}{\gamma_n} \cdot U12_{n,n} \\
& UU12
\end{aligned}
$$

$$
\begin{aligned}
U21(k) := \; & \gamma \leftarrow \gamma 2(k) \\
& \gamma r1 \leftarrow \gamma \cdot r_1 \\
& \gamma r2 \leftarrow \gamma \cdot r_2 \\
& arg \leftarrow \gamma r2 - \gamma r1 \\
& jkY_0 \leftarrow j \cdot k \cdot Y_0 \\
& \text{for } n \in 0..N2 \\
& \quad U21_{n,n} \leftarrow \frac{-2}{\pi \cdot \gamma r2_n} \cdot \frac{1}{J0(\gamma r2_n) \cdot Y0(\gamma r1_n) - Y0(\gamma r2_n) \cdot J0(\gamma r1_n)} \quad \text{if } (|k|)^2 \geq n^2 \cdot \beta 2^2 \\
& \quad U21_{n,n} \leftarrow \frac{-1}{\gamma r2_n} \cdot \frac{1}{I0(\gamma r2_n) \cdot K0(\gamma r1_n) - K0(\gamma r2_n) \cdot I0(\gamma r1_n)} \quad \text{if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n < 700 \\
& \quad U21_{n,n} \leftarrow -\sqrt{\frac{r_1}{r_2}} \cdot \frac{64 \cdot \gamma r1_n \cdot \gamma r2_n}{(64 \cdot \gamma r1_n \cdot \gamma r2_n - 1) \cdot \sinh(arg_n) - 8 \cdot arg_n \cdot \cosh(arg_n)} \quad \text{if } (|k|)^2 < n^2 \cdot \beta 2^2 \wedge \gamma r2_n \geq 700 \\
& \quad UU21_{n,n} \leftarrow \frac{jkY_0}{\gamma_n} \cdot U21_{n,n} \\
& UU21
\end{aligned}
$$

$$
\begin{aligned}
U22(k) := \;&\gamma \leftarrow \gamma 2(k)\\
&\gamma r1 \leftarrow \gamma \cdot r_1\\
&\gamma r2 \leftarrow \gamma \cdot r_2\\
&arg \leftarrow \gamma r2 - \gamma r1\\
&jkY_0 \leftarrow j \cdot k \cdot Y_0\\
&\text{for } n \in 0..N2\\
&\quad U22_{n,n} \leftarrow \frac{J1(\gamma r2_n)\cdot Y0(\gamma r1_n) - Y1(\gamma r2_n)\cdot J0(\gamma r1_n)}{J0(\gamma r2_n)\cdot Y0(\gamma r1_n) - Y0(\gamma r2_n)\cdot J0(\gamma r1_n)} \text{ if } (|k|)^2 \geq n^2\cdot \beta 2^2\\
&\quad U22_{n,n} \leftarrow \frac{I1(\gamma r2_n)\cdot K0(\gamma r1_n) + K1(\gamma r2_n)\cdot I0(\gamma r1_n)}{I0(\gamma r2_n)\cdot K0(\gamma r1_n) - K0(\gamma r2_n)\cdot I0(\gamma r1_n)} \text{ if } (|k|)^2 < n^2\cdot \beta 2^2 \wedge \gamma r2_n < 700\\
&\quad U22_{n,n} \leftarrow \frac{(64\cdot \gamma r1_n \cdot \gamma r2_n + 3)\cdot \cosh(arg_n) - 8\cdot(3\cdot \gamma r1_n + \gamma r2_n)\cdot \sinh(arg_n)}{(64\cdot \gamma r1_n \cdot \gamma r2_n - 1)\cdot \sinh(arg_n) - 8\cdot \overline{arg_n}\cdot \cosh(arg_n)} \text{ if } (|k|)^2 < n^2\cdot \beta 2^2 \wedge \gamma r2_n \geq 700\\
&\quad UU22_{n,n} \leftarrow \frac{jkY_0}{\gamma_n}\cdot U22_{n,n}\\
&UU22
\end{aligned}
$$

$$
U(k) := \left|
\begin{array}{l}
U11 \leftarrow U11(k) \\
U12 \leftarrow U12(k) \\
U21 \leftarrow U21(k) \\
U22 \leftarrow U22(k) \\
nn \leftarrow N2 + 1 \\
\text{for } i \in 0..N2 \\
\quad \text{for } j \in 0..N2 \\
\qquad \left|
\begin{array}{l}
U_{i,j} \leftarrow U11_{i,j} \\
U_{i,(j+nn)} \leftarrow U12_{i,j} \\
U_{(i+nn),j} \leftarrow U21_{i,j} \\
U_{(i+nn),(j+nn)} \leftarrow U22_{i,j}
\end{array}
\right. \\
U \cdot \Omega
\end{array}
\right.
$$

Region 3 (outer)

$$\beta 3 := \frac{\pi}{z_3}$$

$$\gamma 3(k) := \left| \begin{array}{l} \text{for } n \in 0..N3 \\ \quad \gamma 3_n \leftarrow \sqrt{\left| k^2 - n^2 \cdot \beta 3^2 \right|} \\ \text{return } \gamma 3 \end{array} \right.$$

$$\mathrm{GIII}(k) := \left[\begin{array}{l} \gamma \leftarrow \gamma 3(k) \\ \gamma r2 \leftarrow \gamma \cdot r_2 \\ \gamma r3 \leftarrow \gamma \cdot r_3 \\ jkY_0 \leftarrow j \cdot k \cdot Y_0 \\ \text{for } n \in 0..N3 \\ \quad G3_{n,n} \leftarrow \dfrac{J1(\gamma r2_n) \cdot Y0(\gamma r3_n) - Y1(\gamma r2_n) \cdot J0(\gamma r3_n)}{\gamma_n \cdot \left(J0(\gamma r2_n) \cdot Y0(\gamma r3_n) - Y0(\gamma r2_n) \cdot J0(\gamma r3_n)\right)} \text{ if } (|k|)^2 \geq n^2 \cdot \beta 3^2 \\ \quad G3_{n,n} \leftarrow \dfrac{I1(\gamma r2_n) \cdot K0(\gamma r3_n) + K1(\gamma r2_n) \cdot I0(\gamma r3_n)}{\gamma_n \cdot \left(I0(\gamma r2_n) \cdot K0(\gamma r3_n) - K0(\gamma r2_n) \cdot I0(\gamma r3_n)\right)} \text{ if } (|k|)^2 < n^2 \cdot \beta 3^2 \wedge \gamma r3_n < 700 \\ \quad G3_{n,n} \leftarrow \dfrac{-1}{\gamma_n} \cdot \left(\dfrac{8 \cdot \gamma r2_n + 3}{8 \cdot \gamma r2_n - 1} \right) \text{ if } (|k|)^2 < n^2 \cdot \beta 3^2 \wedge \gamma r3_n \geq 700 \\ jkY_0 \cdot G3 \end{array} \right]$$

Calculate the matrices P and Q for matching the fields at r_1 and r_2

$$
\begin{aligned}
P1 := \;& \text{for } n1 \in 0..N1 \\
& \quad \text{for } n2 \in 0..N2 \\
& \qquad \text{if } n1 = 0 \\
& \qquad\quad P_{n1,n2} \leftarrow \frac{z_2}{z_1} \;\text{ if } n2 = 0 \\
& \qquad\quad P_{n1,n2} \leftarrow 0 \;\text{ otherwise} \\
& \qquad \text{otherwise} \\
& \qquad\quad P_{n1,n2} \leftarrow \frac{z_2}{z_1}\cdot\left(1 + \frac{\sin(2\cdot n2\cdot\pi)}{2\cdot n2\cdot\pi}\right) \;\text{ if } n2\cdot\beta 2 = n1\cdot\beta 1 \\
& \qquad\quad P_{n1,n2} \leftarrow 0 \;\text{ if } \left|\sin\left(n1\cdot\beta 1\cdot z_2\right)\right| < 10^{-14} \wedge n2\cdot\beta 2 \neq n1\cdot\beta 1 \\
& \qquad\quad P_{n1,n2} \leftarrow \frac{2}{z_1}\cdot\frac{n1\cdot\beta 1}{(n1\cdot\beta 1)^2 - (n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n1\cdot\beta 1\cdot z_2\right) \;\text{ otherwi} \\
& P
\end{aligned}
$$

$$
\begin{aligned}
P2 := \;& \text{for } n3 \in 0..N3 \\
& \quad \text{for } n2 \in 0..N2 \\
& \qquad \text{if } n3 = 0 \\
& \qquad\quad P_{n3,n2} \leftarrow \frac{z_2}{z_3} \;\text{ if } n2 = 0 \\
& \qquad\quad P_{n3,n2} \leftarrow 0 \;\text{ otherwise} \\
& \qquad \text{otherwise} \\
& \qquad\quad P_{n3,n2} \leftarrow \frac{z_2}{z_3}\cdot\left(1 + \frac{\sin(2\cdot n2\cdot\pi)}{2\cdot n2\cdot\pi}\right) \;\text{ if } n2\cdot\beta 2 = n3\cdot\beta 3 \\
& \qquad\quad P_{n3,n2} \leftarrow 0 \;\text{ if } \left|\sin\left(n3\cdot\beta 3\cdot z_2\right)\right| < 10^{-14} \wedge n2\cdot\beta 2 \neq n3\cdot\beta 3 \\
& \qquad\quad P_{n3,n2} \leftarrow \frac{2}{z_3}\cdot\frac{n3\cdot\beta 3}{(n3\cdot\beta 3)^2 - (n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n3\cdot\beta 3\cdot z_2\right) \;\text{ otherwise} \\
& P
\end{aligned}
$$

$$
Q1 := \left|\begin{array}{l}
\text{for } n2 \in 0..N2 \\
\quad \text{for } n1 \in 0..N1 \\
\quad\quad \text{if } n2 = 0 \\
\quad\quad\quad Q_{n2,n1} \leftarrow 1 \quad \text{if } n1 = 0 \\
\quad\quad\quad Q_{n2,n1} \leftarrow \dfrac{1}{z_2}\cdot\dfrac{n1\cdot\beta 1}{(n1\cdot\beta 1)^2-(n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n1\cdot\beta 1\cdot z_2\right) \quad \text{other} \\
\quad\quad \text{otherwise} \\
\quad\quad\quad Q_{n2,n1} \leftarrow 1+\dfrac{\sin(2\cdot n1\cdot\pi)}{2\cdot n1\cdot\pi} \quad \text{if } n2\cdot\beta 2 = n1\cdot\beta 1 \\
\quad\quad\quad Q_{n2,n1} \leftarrow 0 \quad \text{if } \left|\sin\left(n1\cdot\beta 1\cdot z_2\right)\right| < 10^{-14} \wedge n2\cdot\beta 2 \neq n1\cdot\beta 1 \\
\quad\quad\quad Q_{n2,n1} \leftarrow \dfrac{2}{z_2}\cdot\dfrac{n1\cdot\beta 1}{(n1\cdot\beta 1)^2-(n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n1\cdot\beta 1\cdot z_2\right) \quad \text{other} \\
Q
\end{array}\right.
$$

$$
Q2 := \left|\begin{array}{l}
\text{for } n2 \in 0..N2 \\
\quad \text{for } n3 \in 0..N3 \\
\quad\quad \text{if } n2 = 0 \\
\quad\quad\quad Q_{n2,n3} \leftarrow 1 \quad \text{if } n3 = 0 \\
\quad\quad\quad Q_{n2,n3} \leftarrow \dfrac{1}{z_2}\cdot\dfrac{n3\cdot\beta 3}{(n3\cdot\beta 3)^2-(n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n3\cdot\beta 3\cdot z_2\right) \quad \text{otherwise} \\
\quad\quad \text{otherwise} \\
\quad\quad\quad Q_{n2,n3} \leftarrow 1+\dfrac{\sin(2\cdot n3\cdot\pi)}{2\cdot n3\cdot\pi} \quad \text{if } n2\cdot\beta 2 = n3\cdot\beta 3 \\
\quad\quad\quad Q_{n2,n3} \leftarrow 0 \quad \text{if } \left|\sin\left(n3\cdot\beta 3\cdot z_2\right)\right| < 10^{-14} \wedge n2\cdot\beta 2 \neq n3\cdot\beta 3 \\
\quad\quad\quad Q_{n2,n3} \leftarrow \dfrac{2}{z_2}\cdot\dfrac{n3\cdot\beta 3}{(n3\cdot\beta 3)^2-(n2\cdot\beta 2)^2}\cdot(-1)^{n2}\cdot\sin\left(n3\cdot\beta 3\cdot z_2\right) \quad \text{otherwise} \\
Q
\end{array}\right.
$$

Define the matrix whose determinant is set to zero to find the resonant value of *k*.

Combining equations (3.68), (3.72) and (3.74)

$$\left[h^{II}\right]=\left[Q_1\right]\left[G^{I}\right]\left[P_1\right]\left[e^{II}\right]=\left[V_{11}\right]\left[e^{II}\right] \qquad V11(k) := Q1\cdot GI(k)\cdot P1$$

Combining equations (3.69), (3.73) and (3.75)

$$\left[hh^{II}\right]=\left[Q_2\right]\left[G^{III}\right]\left[P_2\right]\left[ee^{II}\right]=\left[V_{22}\right]\left[ee^{II}\right] \qquad V22(k) := Q2\cdot GIII(k)\cdot P2$$

Combining these equations with (3.71) gives

$$\begin{bmatrix} e^{II} \\ \cdots \\ ee^{II} \end{bmatrix}=\left[U\right]^{-1}\begin{bmatrix} h^{II} \\ \cdots \\ hh^{II} \end{bmatrix}=\left[U\right]^{-1}\begin{bmatrix} V_{11} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & V_{22} \end{bmatrix}\begin{bmatrix} e^{II} \\ \cdots \\ ee^{II} \end{bmatrix}=\left[U\right]^{-1}\left[VV\right]\begin{bmatrix} e^{II} \\ \cdots \\ ee^{II} \end{bmatrix}$$

Construct the matrix [VV]

$$
\mathrm{VV}(k) := \left|
\begin{array}{l}
\mathrm{V11} \leftarrow \mathrm{V11}(k) \\
\mathrm{V22} \leftarrow \mathrm{V22}(k) \\
\mathrm{mm} \leftarrow \mathrm{N2} + 1 \\
\text{for } i \in 0..\mathrm{N2} \\
\quad \text{for } j \in 0..\mathrm{N2} \\
\quad\quad \left| \begin{array}{l}
V_{i,j} \leftarrow 0 \\
V_{i,j} \leftarrow \mathrm{V11}_{i,j} \\
V_{(i+mm),(j+mm)} \leftarrow \mathrm{V22}_{i,j}
\end{array} \right. \\
V \cdot \Omega
\end{array}
\right.
$$

Define the Unit Matrix

$$
I := \left|
\begin{array}{l}
\text{for } i \in 0..2 \cdot \mathrm{N2} + 1 \\
\quad I_{i,i} \leftarrow 1 \\
I
\end{array}
\right.
$$

$$
[W]\begin{bmatrix} e^{II} \\ \cdots \\ ee^{II} \end{bmatrix} = [U]^{-1}[VV] - [I]\begin{bmatrix} e^{II} \\ \cdots \\ ee^{II} \end{bmatrix} = 0 \qquad \text{Equation 3.76}
$$

$$
\mathrm{W1}(k) := \mathrm{U}(k)^{-1} \cdot \mathrm{VV}(k) - I
$$

Find the root kk of the determinant of [WW] using an initial guessed value k0 which is set by the guessed frequency f0 below.

$$
k_0 := \frac{2 \cdot \pi \cdot f_0}{c} \qquad kk := \mathrm{root}\left(\left|\mathrm{W1}\left(k_0\right)\right|, k_0\right) \qquad kk = 62.883 \cdot m^{-1}
$$

Check the accuracy of the solution $\left|\mathrm{W1}(kk)\right| = 9.241 \times 10^{-6}$

The resonant frequency is $f := \dfrac{kk \cdot c}{2 \cdot \pi}$ $\qquad f = 3000.350 \cdot \mathrm{MHz}$

Alternative solution using the secant method setting the error in kk for convergence $err := 10^{-12}$

$$
k := \left|
\begin{array}{l}
kk_0 \leftarrow k_0 \\
kk_1 \leftarrow k_0 \cdot 1.5 \\
y_0 \leftarrow \left|W1\left(kk_0\right)\right| \\
y_1 \leftarrow \left|W1\left(kk_1\right)\right| \\
i \leftarrow 0 \\
\text{while } \left|\dfrac{kk_i - kk_{i+1}}{kk_{i+1}}\right| > err \\
\quad \left|
\begin{array}{l}
kk_{i+2} \leftarrow \dfrac{kk_{i+1} \cdot y_i - kk_i \cdot y_{i+1}}{y_i - y_{i+1}} \\
\text{pause}\left(\text{"i=\{0\}, kk(i+2) =\{1\}, DetF = \{2\}"}, i, kk_{i+2}, \left|W1\left(kk_{i+2}\right)\right|\right) \\
y_{i+2} \leftarrow \left|W1\left(kk_{i+2}\right)\right| \\
i \leftarrow i + 1
\end{array}
\right. \\
kk_{i+1}
\end{array}
\right.
$$

$k = 62.883 \cdot m^{-1}$

Find the eigenvectors for the solution

$WW := U(k)^{-1} \cdot VV(k)$ $\quad evec := \mathrm{eigenvec}(WW, 1) \cdot 10^3$ $\quad hvec := U(k) \cdot evec$

$$eII := \left| \begin{array}{l} \text{for } n \in 0..N2 \\ \quad e_n \leftarrow evec_n \\ e \cdot V \cdot m^{-1} \end{array} \right. \quad eeII := \left| \begin{array}{l} \text{for } n \in 0..N2 \\ \quad ee_n \leftarrow evec_{n+N2+1} \\ ee \cdot V \cdot m^{-1} \end{array} \right. \quad hII := \left| \begin{array}{l} \text{for } n \in 0..N2 \\ \quad h_n \leftarrow hvec_n \\ h \cdot A \cdot m^{-1} \end{array} \right. \quad hhII := \left| \begin{array}{l} \text{for } n \in 0..N2 \\ \quad hh_n \leftarrow hvec_{n+N2+1} \\ hh \cdot A \cdot m^{-1} \end{array} \right.$$

$eI := P1 \cdot eII$ $\quad eIII := P2 \cdot eeII$ $\quad hI := GI(k) \cdot eI$ $\quad hIII := GIII(k) \cdot eIII$

Calculate the fields on the interfaces between the regions

$$EI(z) := \sum_{n=0}^{N1} \left(eI_n \cdot \cos(n \cdot \beta 1 \cdot z) \right) \qquad EIII(z) := \sum_{n=0}^{N3} \left(eIII_n \cdot \cos(n \cdot \beta 3 \cdot z) \right)$$

$$HI(z) := \sum_{n=0}^{N1} \left(hI_n \cdot \cos(n \cdot \beta 1 \cdot z) \right) \qquad HIII(z) := \sum_{n=0}^{N3} \left(hIII_n \cdot \cos(n \cdot \beta 3 \cdot z) \right)$$

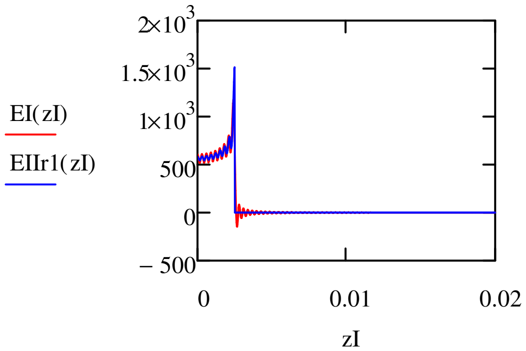
$$EIIr1(z) := \left| \begin{array}{ll} \sum_{n=0}^{N2} \left(eII_n \cdot \cos(n \cdot \beta 2 \cdot z) \right) & \text{if } |z| \le z_2 \\ 0 & \text{otherwise} \end{array} \right. \qquad EIIr2(z) := \left| \begin{array}{ll} \sum_{n=0}^{N2} \left(eeII_n \cdot \cos(n \cdot \beta 2 \cdot z) \right) & \text{if } |z| \le z_2 \\ 0 & \text{otherwise} \end{array} \right.$$

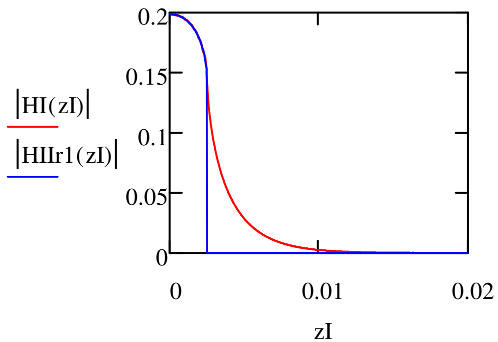
$$HIIr1(z) := \left| \begin{array}{ll} \sum_{n=0}^{N2} \left(hII_n \cdot \cos(n \cdot \beta 2 \cdot z) \right) & \text{if } |z| \le z_2 \\ 0 & \text{otherwise} \end{array} \right. \qquad HIIr2(z) := \left| \begin{array}{ll} \sum_{n=0}^{N2} \left(hhII_n \cdot \cos(n \cdot \beta 2 \cdot z) \right) & \text{if } |z| \le z_2 \\ 0 & \text{otherwise} \end{array} \right.$$

Plot the fields on the interfaces to check the continuity

$zI := 0, 0.001 \cdot z_1 .. z_1$

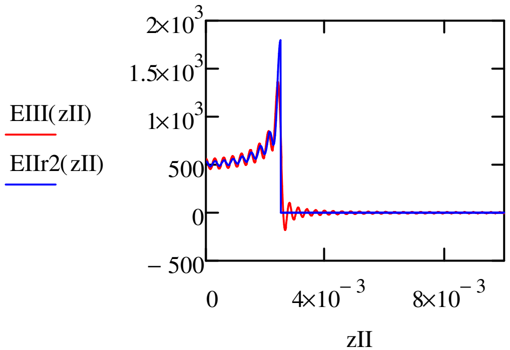
Boundary between regions I and II

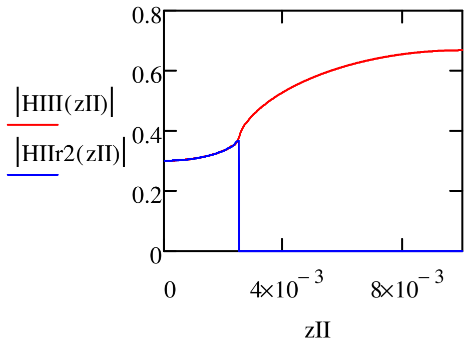




$zII := 0, 0.001 \cdot z_3 .. z_3$

Boundary between regions II and III





Calculate the gap voltage by integrating the z component of E

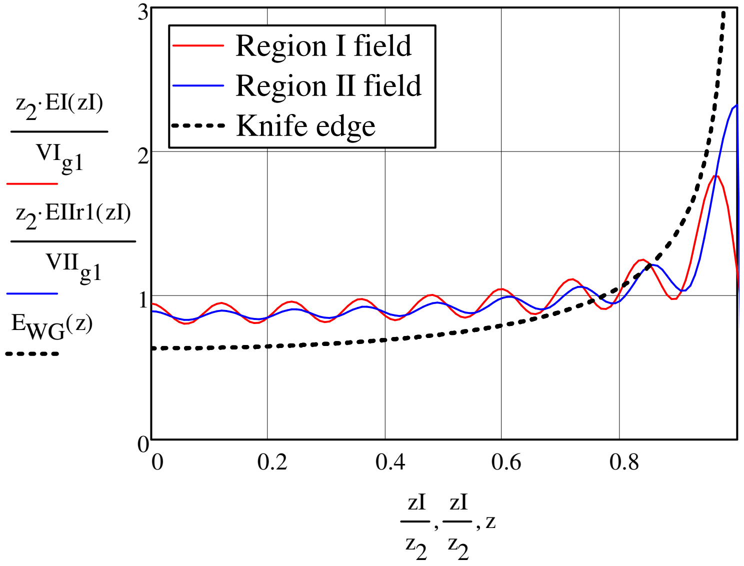
r = 0 $\quad V_{g0} := \int_0^{z_1} EI(z)\,dz \qquad V_{g0} = 1.626\,V$

r = r_1 $\quad VI_{g1} := \int_0^{z_2} EI(z)\,dz \qquad VI_{g1} = 1.602\,V \qquad VII_{g1} := \int_0^{z_2} EIIr1(z)\,dz \qquad VII_{g1} = 1.626\,V$

r = r_2 $\quad VII_{g2} := \int_0^{z_2} EIIr2(z)\,dz \qquad VII_{g2} = 1.596\,V \qquad VIII_{g2} := \int_0^{z_2} EIII(z)\,dz \qquad VIII_{g2} = 1.567\,V$

Knife edge field $\quad E_{WG}(z) := \frac{2}{\pi}\cdot\frac{1}{\sqrt{1-z^2}}$ $\quad$ Equation 3.89

Compare the normalised voltage variation at r = r_1 computed from the fields in regions 1 and 2 with the theoretical field for a knife-edge gap.



Calculate the field components in the three regions

Region I

Radial field variations

$$RI(r) := \left| \begin{array}{l} r \leftarrow r \\ \gamma \leftarrow \gamma 1(k) \\ \text{for } n \in 0..N1 \\ \quad \left| \begin{array}{l} RI_n \leftarrow \dfrac{J0(\gamma_n \cdot r)}{J0(\gamma_n \cdot r_1)} \quad \text{if } k^2 \geq n^2 \cdot \beta 1^2 \\ RI_n \leftarrow \dfrac{I0(|\gamma_n| \cdot r)}{I0(\gamma_n \cdot r_1)} \quad \text{otherwise} \end{array} \right. \\ \text{return } RI \end{array} \right.$$

$$RId(r) := \left| \begin{array}{l} r \leftarrow r \\ \gamma \leftarrow \gamma 1(k) \\ \text{for } n \in 0..N1 \\ \quad \left| \begin{array}{l} RId_n \leftarrow \dfrac{J1(\gamma_n \cdot r)}{J1(\gamma_n \cdot r_1)} \quad \text{if } k^2 \geq n^2 \cdot \beta 1^2 \\ RId_n \leftarrow \dfrac{I1(|\gamma_n| \cdot r)}{I1(|\gamma_n| \cdot r_1)} \quad \text{otherwise} \end{array} \right. \\ \text{return } RId \end{array} \right.$$

Field components

$$EI_z(r,z) := \left| \begin{array}{l} RI \leftarrow RI(r) \\ \gamma \leftarrow \gamma 1(k) \\ E \leftarrow \displaystyle\sum_{n=0}^{N1} \left(eI_n \cdot RI_n \cdot \cos(n \cdot \beta 1 \cdot z) \right) \\ \text{return } E \end{array} \right.$$

$$HI_\theta(r,z) := \left| \begin{array}{l} RId \leftarrow RId(r) \\ \gamma \leftarrow \gamma 1(k) \\ H \leftarrow \displaystyle\sum_{n=0}^{N1} \left(hI_n \cdot RId_n \cdot \cos(n \cdot \beta 1 \cdot z) \right) \\ \text{return } H \end{array} \right.$$

$$EI_r(r,z) := \left| \begin{array}{l} RId \leftarrow RId(r) \\ \gamma \leftarrow \gamma 1(k) \\ E \leftarrow \displaystyle\sum_{n=0}^{N1} -\left(hI_n \cdot RId_n \cdot n \cdot \beta 1 \cdot \sin(n \cdot \beta 1 \cdot z) \right) \\ \text{return } \dfrac{E}{j \cdot k \cdot Y_0} \end{array} \right.$$

Region II

$$bb := \left| \begin{array}{l} \gamma \leftarrow \gamma2(k) \\ \text{for } n \in 0..N2 \\ \quad \left| \begin{array}{l} b_n \leftarrow \dfrac{Y0(\gamma_n \cdot r_2) \cdot eII_n - Y0(\gamma_n \cdot r_1) \cdot eeII_n}{(J0(\gamma_n \cdot r_1) \cdot Y0(\gamma_n \cdot r_2) - Y0(\gamma_n \cdot r_1) \cdot J0(\gamma_n \cdot r_2)) \cdot (\gamma_n)^2} \quad \text{if } k^2 \geq n^2 \cdot \beta2^2 \\ b_n \leftarrow \dfrac{K0(\gamma_n \cdot r_2) \cdot eII_n - K0(\gamma_n \cdot r_1) \cdot eeII_n}{(I0(\gamma_n \cdot r_1) \cdot K0(\gamma_n \cdot r_2) - K0(\gamma_n \cdot r_1) \cdot I0(\gamma_n \cdot r_2)) \cdot (\gamma_n)^2} \quad \text{otherwise} \end{array} \right. \\ b \end{array} \right.$$

$$cc := \left| \begin{array}{l} \gamma \leftarrow \gamma2(k) \\ \text{for } n \in 0..N2 \\ \quad \left| \begin{array}{l} c_n \leftarrow \dfrac{-J0(\gamma_n \cdot r_2) \cdot eII_n + J0(\gamma_n \cdot r_1) \cdot eeII_n}{(J0(\gamma_n \cdot r_1) \cdot Y0(\gamma_n \cdot r_2) - Y0(\gamma_n \cdot r_1) \cdot J0(\gamma_n \cdot r_2)) \cdot (\gamma_n)^2} \quad \text{if } k^2 \geq n^2 \cdot \beta2^2 \\ c_n \leftarrow \dfrac{-In(0, \gamma_n \cdot r_2) \cdot eII_n + In(0, \gamma_n \cdot r_1) \cdot eeII_n}{(I0(\gamma_n \cdot r_1) \cdot K0(\gamma_n \cdot r_2) - K0(\gamma_n \cdot r_1) \cdot I0(\gamma_n \cdot r_2)) \cdot (\gamma_n)^2} \quad \text{otherwise} \end{array} \right. \\ c \end{array} \right.$$

$$
\begin{aligned}
EII_z(r,z) := \;& \gamma \leftarrow \gamma 2(k) \\
& EII \leftarrow 0 \\
& \text{for } n \in 0..N2 \\
& \quad EII \leftarrow EII + (\gamma_n)^2 \cdot \left(J0(\gamma_n \cdot r)\cdot bb_n + Y0(\gamma_n \cdot r)\cdot cc_n\right)\cdot \cos(n\cdot\beta 2\cdot z) \quad \text{if } k^2 \ge n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& \quad EII \leftarrow EII + (\gamma_n)^2 \cdot \left(I0(\gamma_n \cdot r)\cdot bb_n + K0(\gamma_n \cdot r)\cdot cc_n\right)\cdot \cos(n\cdot\beta 2\cdot z) \quad \text{if } k^2 < n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& EII
\end{aligned}
$$

$$
\begin{aligned}
HII_\theta(r,z) := \;& \gamma \leftarrow \gamma 2(k) \\
& jkY_0 \leftarrow j\cdot k\cdot Y_0 \\
& HII \leftarrow 0 \\
& \text{for } n \in 0..N2 \\
& \quad HII \leftarrow HII + \gamma_n \cdot \left(J1(\gamma_n \cdot r)\cdot bb_n + Y1(\gamma_n \cdot r)\cdot cc_n\right)\cdot \cos(n\cdot\beta 2\cdot z) \quad \text{if } k^2 \ge n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& \quad HII \leftarrow HII + \gamma_n \cdot \left(I1(\gamma_n \cdot r)\cdot bb_n - K1(\gamma_n \cdot r)\cdot cc_n\right)\cdot \cos(n\cdot\beta 2\cdot z) \quad \text{if } k^2 < n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& jkY_0\cdot HII
\end{aligned}
$$

$$
\begin{aligned}
EII_r(r,z) := \;& \gamma \leftarrow \gamma 2(k) \\
& E \leftarrow 0 \\
& \text{for } n \in 0..N2 \\
& \quad E \leftarrow E - \gamma_n \cdot \left(J1(\gamma_n \cdot r)\cdot bb_n + Y1(\gamma_n \cdot r)\cdot cc_n\right)\cdot n\cdot\beta 2\cdot \sin(n\cdot\beta 2\cdot z) \quad \text{if } k^2 \ge n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& \quad E \leftarrow E - \gamma_n \cdot \left(I1(\gamma_n \cdot r)\cdot bb_n - K1(\gamma_n \cdot r)\cdot cc_n\right)\cdot n\cdot\beta 2\cdot \sin(n\cdot\beta 2\cdot z) \quad \text{if } k^2 < n^2\cdot\beta 2^2 \wedge |z| \le z_2 \\
& \text{return } E
\end{aligned}
$$

Region III

$$\mathrm{RIII}(r) := \left|\begin{array}{l} r \leftarrow r \\ \gamma \leftarrow \gamma 3(k) \\ \text{for } n \in 0..N3 \\ \quad \left|\begin{array}{l} R3_n \leftarrow \dfrac{J0(\gamma_n \cdot r)\cdot Y0(\gamma_n \cdot r_3) - Y0(\gamma_n \cdot r)\cdot J0(\gamma_n \cdot r_3)}{J0(\gamma_n \cdot r_2)\cdot Y0(\gamma_n \cdot r_3) - Y0(\gamma_n \cdot r_2)\cdot J0(\gamma_n \cdot r_3)} \text{ if } k^2 \ge n^2 \cdot \beta 3^2 \\ R3_n \leftarrow \dfrac{I0(|\gamma_n| \cdot r)\cdot K0(|\gamma_n| \cdot r_3) - K0(|\gamma_n| \cdot r)\cdot I0(|\gamma_n| \cdot r_3)}{I0(|\gamma_n| \cdot r_2)\cdot K0(|\gamma_n| \cdot r_3) - K0(|\gamma_n| \cdot r_2)\cdot I0(|\gamma_n| \cdot r_3)} \text{ otherwise} \end{array}\right. \\ \text{return } R3 \end{array}\right.$$

$$\mathrm{RIIId}(r) := \left|\begin{array}{l} r \leftarrow r \\ \gamma \leftarrow \gamma 3(k) \\ \text{for } n \in 0..N3 \\ \quad \left|\begin{array}{l} R3d_n \leftarrow \dfrac{J1(\gamma_n \cdot r)\cdot Y0(\gamma_n \cdot r_3) - Y1(\gamma_n \cdot r)\cdot J0(\gamma_n \cdot r_3)}{J1(\gamma_n \cdot r_2)\cdot Y0(\gamma_n \cdot r_3) - Y1(\gamma_n \cdot r_2)\cdot J0(\gamma_n \cdot r_3)} \text{ if } k^2 \ge n^2 \cdot \beta 3^2 \\ R3d_n \leftarrow \dfrac{I1(|\gamma_n| \cdot r)\cdot K0(|\gamma_n| \cdot r_3) + K1(|\gamma_n| \cdot r)\cdot I0(|\gamma_n| \cdot r_3)}{I1(|\gamma_n| \cdot r_2)\cdot K0(|\gamma_n| \cdot r_3) + K1(|\gamma_n| \cdot r_2)\cdot I0(|\gamma_n| \cdot r_3)} \text{ otherwise} \end{array}\right. \\ \text{return } R3d \end{array}\right.$$

$$\mathrm{EIII}_z(r,z) := \left|\begin{array}{l} \mathrm{RIII} \leftarrow \mathrm{RIII}(r) \\ \gamma \leftarrow \gamma 3(k) \\ E \leftarrow \displaystyle\sum_{n=0}^{N3} \left(\mathrm{eIII}_n \cdot \mathrm{RIII}_n \cdot \cos(n \cdot \beta 3 \cdot z)\right) \\ \text{return } E \end{array}\right.$$

$$\mathrm{HIII}_\theta(r,z) := \left|\begin{array}{l} \mathrm{RIIId} \leftarrow \mathrm{RIIId}(r) \\ \gamma \leftarrow \gamma 3(k) \\ H \leftarrow \displaystyle\sum_{n=0}^{N3} \left(j \cdot \mathrm{hIII}_n \cdot \mathrm{RIIId}_n \cdot \cos(n \cdot \beta 3 \cdot z)\right) \\ \text{return } H \end{array}\right.$$

$$\mathrm{EIII}_r(r,z) := \left|\begin{array}{l} \mathrm{RIIId} \leftarrow \mathrm{RIIId}(r) \\ \gamma \leftarrow \gamma 3(k) \\ E \leftarrow \displaystyle\sum_{n=0}^{N3} -\left(\mathrm{hIII}_n \cdot \mathrm{RIIId}_n \cdot n \cdot \beta 3 \cdot \sin(n \cdot \beta 3 \cdot z)\right) \\ \text{return } \dfrac{E}{j \cdot k \cdot Y_0} \end{array}\right.$$

Find the magnetic stored energy in a singly re-entrant cavity. First integrate the squared magnetic field over z as a function of r since all the cross products integrate to zero. Then compute the R/Q for singly and doubly re-entrant cavities.

$$\mathrm{HIsq}(r) := \left| \begin{array}{l} \mathrm{RId} \leftarrow \mathrm{RId}(r) \\ \mathrm{H1sq} \leftarrow \sum_{n=1}^{N1} \left(\left|\mathrm{hI}_n \cdot \mathrm{RId}_n\right|\right)^2 \\ \mathrm{H1sq} \leftarrow \frac{1}{2}\cdot \mathrm{H1sq} + \left(\left|\mathrm{hI}_0 \cdot \mathrm{RId}_0\right|\right)^2 \\ \mathrm{H1sq} \leftarrow \mathrm{H1sq}\cdot z_1 \\ \text{return H1sq} \end{array} \right.$$

$$\mathrm{HIIsq}(r) := \left| \begin{array}{l} \gamma \leftarrow \gamma 2(k) \\ \mathrm{HIIsq} \leftarrow 0 \\ \text{for } n \in 1..N2 \\ \quad \left| \begin{array}{l} \mathrm{HIIsq} \leftarrow \mathrm{HIIsq} + \left[\gamma_n\cdot\left(\mathrm{J1}(\gamma_n\cdot r)\cdot \mathrm{bb}_n + \mathrm{Y1}(\gamma_n\cdot r)\cdot \mathrm{cc}_n\right)\right]^2 \text{ if } k^2 \ge n^2\cdot \beta 2^2 \\ \mathrm{HIIsq} \leftarrow \mathrm{HIIsq} + \left[\gamma_n\cdot\left(\mathrm{I1}(\gamma_n\cdot r)\cdot \mathrm{bb}_n - \mathrm{K1}(\gamma_n\cdot r)\cdot \mathrm{cc}_n\right)\right]^2 \text{ otherwise} \end{array} \right. \\ \mathrm{HIIsq} \leftarrow \frac{1}{2}\cdot \mathrm{HIIsq} + \left[\gamma_0\cdot\left(\mathrm{J1}(\gamma_0\cdot r)\cdot \mathrm{bb}_0 + \mathrm{Y1}(\gamma_0\cdot r)\cdot \mathrm{cc}_0\right)\right]^2 \\ \mathrm{HIIsq} \leftarrow \left(k\cdot Y_0\right)^2\cdot \mathrm{HIIsq}\cdot z_2 \\ \text{return HIIsq} \end{array} \right.$$

$$\mathrm{HIIIsq}(r) := \left| \begin{array}{l} \mathrm{RIIId} \leftarrow \mathrm{RIIId}(r) \\ \mathrm{H3sq} \leftarrow \sum_{n=1}^{N3} \left(\left|\mathrm{hIII}_n \cdot \mathrm{RIIId}_n\right|\right)^2 \\ \mathrm{H3sq} \leftarrow \frac{1}{2}\cdot \mathrm{H3sq} + \left(\left|\mathrm{hIII}_0 \cdot \mathrm{RIIId}_0\right|\right)^2 \\ \text{return } \mathrm{H3sq}\cdot z_3 \end{array} \right.$$

Stored energy in the three regions

$$\mathrm{WMI} := \frac{\mu_0}{2}\cdot\int_0^{r_1} 2\cdot\pi\cdot r\cdot \mathrm{HIsq}(r)\, dr \qquad \mathrm{WMII} := \frac{\mu_0}{2}\cdot\int_{r_1}^{r_2} 2\cdot\pi\cdot r\, \mathrm{HIIsq}(r)\, dr \qquad \mathrm{WMIII} := \frac{\mu_0}{2}\cdot\int_{r_2}^{r_3} 2\cdot\pi\cdot r\, \mathrm{HIIIsq}(r)\, dr$$

$$\mathrm{WM} := \mathrm{WMI} + \mathrm{WMII} + \mathrm{WMIII} \qquad \mathrm{WMI} = 2.222\times 10^{-15}\,\mathrm{J} \qquad \mathrm{WMII} = 8.076\times 10^{-15}\,\mathrm{J} \qquad \mathrm{WMIII} = 1.416\times 10^{-12}\,\mathrm{J}$$

$$\mathrm{WM} = 1.427\times 10^{-12}\,\mathrm{J} \qquad \mathrm{RQS} := \frac{{V_{g0}}^2}{4\cdot\pi\cdot f\cdot \mathrm{WM}} \qquad \mathrm{RQD} := 2\cdot \mathrm{RQS} \qquad \boxed{\mathrm{RQS} = 49.143\,\Omega}$$

Note: this value of R/Q differs slightly from that in Table 3.6

Define the functions for computing the losses in the cavity and the Q factor

Surface resistance $$R_s(k) := \sqrt{\frac{k\cdot c\cdot \mu_0}{2\cdot\sigma}}$$

For region I

$$PI_{top} := \frac{R_s(k)}{2}\cdot\int_0^{r_1} 2\cdot\pi\cdot r\cdot\left(\left|HI_\theta(r,z_1)\right|\right)^2 dr \qquad PI_{top} = 1.003\times 10^{-15}\,W$$

$$PI_{tube} := \frac{R_s(k)}{2}\cdot\int_{z_2}^{z_1} 2\cdot\pi\cdot r_1\cdot\left(\left|HI_\theta(r_1,z)\right|\right)^2 dz \qquad PI_{tube} = 2.908\times 10^{-9}\,W$$

$$PI_{bott} := \frac{R_s(k)}{2}\cdot\int_0^{r_1} 2\cdot\pi\cdot r\cdot\left(\left|HI_\theta(r,0\cdot m)\right|\right)^2 dr \qquad PI_{bott} = 9.495\times 10^{-9}\,W$$

For region II

$$PII_{top} := \frac{R_s(k)}{2}\cdot\int_{r_1}^{r_2} 2\cdot\pi\cdot r\cdot\left(\left|HII_\theta(r,z_2)\right|\right)^2 dr \qquad PII_{top} = 3.865\times 10^{-8}\,W$$

$$PII_{bott} := \frac{R_s(k)}{2}\cdot\int_{r_1}^{r_2} 2\cdot\pi\cdot r\cdot\left(\left|HII_\theta(r,0\cdot m)\right|\right)^2 dr \qquad PII_{bott} = 3.49\times 10^{-8}\,W$$

For region III

$$PIII_{top} := \frac{R_s(k)}{2} \cdot \int_{r_2}^{r_3} 2\cdot\pi\cdot r\cdot\left(\left|HIII_\theta(r,z_3)\right|\right)^2 dr \qquad PIII_{top} = 1.754\times 10^{-6}\,W$$

$$PIII_{side} := \frac{R_s(k)}{2} \cdot \int_{0}^{z_3} 2\cdot\pi\cdot r_3\cdot\left(\left|HIII_\theta(r_3,z)\right|\right)^2 dz \qquad PIII_{side} = 7.487\times 10^{-7}\,W$$

$$PIII_{tube} := \frac{R_s(k)}{2} \cdot \int_{z_2}^{z_3} 2\cdot\pi\cdot r_2\cdot\left(\left|HIII_\theta(r_2,z)\right|\right)^2 dz \qquad PIII_{tube} = 8.349\times 10^{-7}\,W$$

$$PIII_{bott} := \frac{R_s(k)}{2} \cdot \int_{r_2}^{r_3} 2\cdot\pi\cdot r\cdot\left(\left|HIII_\theta(r,0\cdot m)\right|\right)^2 dr \qquad PIII_{bott} = 1.393\times 10^{-6}\,W$$

Compute the total power dissipation Q and shunt impedance for singly and doubly re-entrant cavities

$$PS := PI_{top} + PI_{tube} + PI_{bott} + PII_{top} + PII_{bott} + PIII_{top} + PIII_{side} + PIII_{bott} + PIII_{tube} \qquad PS = 4.817\times 10^{-6}\,W$$

$$PD := 2\cdot\left(PI_{top} + PI_{tube} + PII_{top} + PIII_{tube} + PIII_{top} + PIII_{side}\right) \qquad PD = 6.759\times 10^{-6}\,W$$

$$QS := \frac{k\cdot c\cdot WM}{PS} \qquad QD := \frac{k\cdot c\cdot 2WM}{PD} \qquad RcS := QS\cdot RQS \qquad RcD := QD\cdot RQD \qquad QS = 5583 \qquad QD = 7959$$

Results $N1 = 135$ $N2 = 16$ $N3 = 67$

$f = 3000.3501 \cdot MHz$ $QS = 5583$ $RQS = 49.1\,\Omega$ $RcS = 274.4 \cdot k\Omega$ Singly reentrant cavity (height = z_3)

$QD = 7959$ $RQD = 98.3\,\Omega$ $RcD = 782.2 \cdot k\Omega$ Doubly reentrant cavity cavity (height = $2z_3$)

Arrow plot of the electric field with scale factor Δ

$\Delta := 1 \cdot mm$ $Nr := \mathrm{floor}\left(\frac{r_3}{\Delta}\right)$ $Nz := \mathrm{floor}\left(\frac{z_3}{\Delta}\right)$ $Nr = 26$ $Nz = 10$

$nr := 0..Nr$ $nz := 0..Nz$ $r_{nr} := nr \cdot \Delta$ $z_{nz} := nz \cdot \Delta$

$$E_r(r,z) := \begin{cases} EI_r(r,z) & \text{if } r \le r_1 \wedge z < z_1 \\ EII_r(r,z) & \text{if } r > r_1 \wedge r \le r_2 \wedge z < z_2 \\ EIII_r(r,z) & \text{if } r > r_2 \wedge r \le r_3 \wedge z < z_3 \\ 0 & \text{otherwise} \end{cases}$$

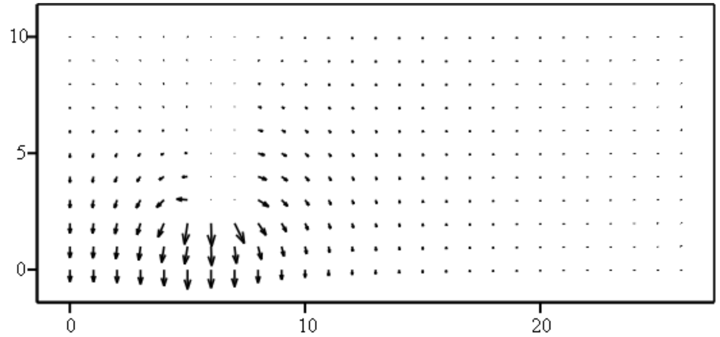
$$E_z(r,z) := \begin{cases} -EI_z(r,z) & \text{if } r \le r_1 \wedge z \le z_1 \\ -EII_z(r,z) & \text{if } r > r_1 \wedge r \le r_2 \wedge z \le z_2 \\ -EIII_z(r,z) & \text{if } r > r_2 \wedge r \le r_3 \wedge z \le z_3 \\ 0 & \text{otherwise} \end{cases}$$

$$H_\theta(r,z) := \begin{cases} HI_\theta(r,z) & \text{if } r \le r_1 \wedge z \le z_1 \\ HII_\theta(r,z) & \text{if } r > r_1 \wedge r \le r_2 \wedge z \le z_2 \\ HIII_\theta(r,z) & \text{if } r > r_2 \wedge r \le r_3 \wedge z \le z_3 \\ 0 & \text{otherwise} \end{cases}$$

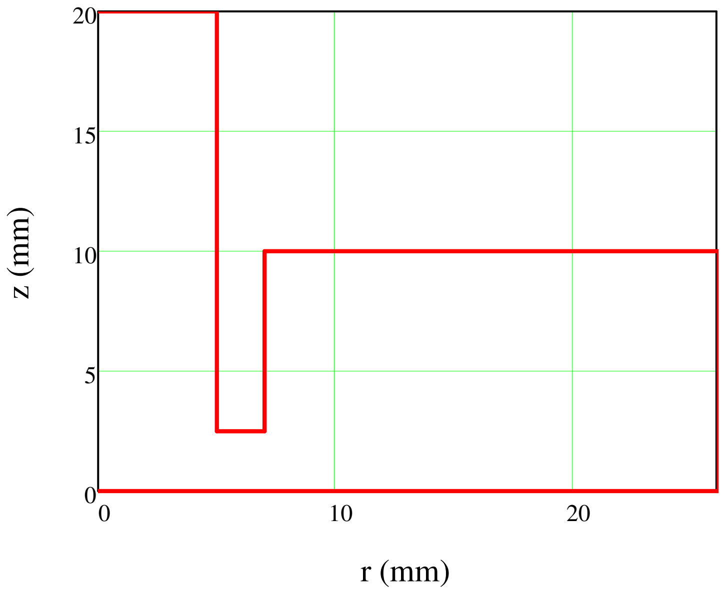
$$EE_{nr,nz} := E_r(r_{nr}, z_{nz}) + j \cdot E_z(r_{nr}, z_{nz})$$

$$E := \overrightarrow{EE}$$

Figure 3.16



E



3 GHz Re-entrant cavities: Gap = 5 mm, Beam hole radius = 5 mm, Drift tube radius = 6.25 mm
The results obtained from individual calculations are pasted in here for plotting

h_g :=	r_3 :=	Qre :=	R_Q :=	Rsh :=
1	38.61	3712	23.5	87.2
2	35.75	6537	52.6	343.5
3	31.35	8051	82.7	665.8
4	26.11	7959	103.3	822.1
5	21.28	6992	111	776.2
6	17.53	5854	109.8	642.5
7	14.70	4738	102	483.2
8	12.37	3615	86.8	314
9	10.36	2455	64	157.2
10	8.485	1174	32.3	38

Figure 3.15