WS 16.1 CFA model

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Data from H. L. McDowell, "Crossed-field amplifier simulations using a moving wavelength computer code," *Plasma Science, IEEE Transactions on,* vol. 30, pp. 962-979, 2002. The tube is not identified but it appears to be either SFD-261 or SFD-262

Data input Tube_type := "SFD-262"

Cell pitch	Anode height	Anode-cathode dist.	SWS length	Drift length	Cell gap
$p := 0.19 \cdot cm$	$L_a := 1.194 \cdot cm$	$d_{ac} := 0.216 \cdot cm$	$L_c := 11.75 \cdot cm$	$L_d := 1.9 \cdot cm$	$w := 1 \cdot mm$
Frequency	Phase shift/cell	Interaction impedance	Attenuation	Input power	Magnetic field
$f_0 := 3.3 \cdot GHz$	$\phi_c := 95 \cdot deg$	$Z_c := 21.5 \cdot \Omega$	$Attn := 0.177 \cdot dB \cdot cm^{-1}$	$P_{in} := 7 \cdot kW$	$B_z := 0.3 \cdot T$

Anode voltage used to check the formulae and as a starting point for the solution

$V_a := 13.25 kV$

Transit time offset (cells)

$n_0 := 0$

SPOKE := 0

0 = Rigid spoke model

1 = Guiding centre model

Define the charge to mass ratio of the electron and the decibel

$\eta := 1.759 \cdot 10^{11} \cdot C \cdot kg^{-1}$ $\qquad dB \equiv 1$

Calculate auxiliary data

Number of cells in the complete anode, number of active cells

$N_a := \mathrm{round}\left(\frac{L_c + L_d}{p}\right)$ $\qquad N_c := \mathrm{round}\left(\frac{L_c}{p}\right)$ $\qquad N_a = 72$ $\qquad N_c = 62$

Vane thickness

$t := p - w$ $\qquad t = 0.9 \cdot mm$

Calculate the anode and cathode radii and the normalised anode radius

$r_a := \frac{L_c + L_d}{2 \cdot \pi}$ $\qquad r_c := r_a - d_{ac}$ $\qquad R_a := \frac{r_a}{r_c}$ $\qquad r_a = 21.7 \cdot mm$ $\qquad r_c = 19.6 \cdot mm$ $\qquad R_a = 1.11$

Calculate the voltage attenuation per cell and the cold loss

$\alpha := \frac{Attn}{20 \cdot \log(e)}$ $\qquad \alpha = 2.038 \frac{1}{m}$ $\qquad A_c := \exp(-\alpha \cdot p)$ $\qquad A_c = 0.996$ $\qquad Cold\_Loss := Attn \cdot L_c$ $\qquad Cold\_Loss = 2.08 \cdot dB$

Calculate the number of spokes and the synchronous frequency. Note that for operation away from synchronism M_s is not an integer.

$\omega_0 := 2 \cdot \pi \cdot f_0$ $\qquad M_s := \frac{N_a \cdot \phi_c}{2 \cdot \pi}$ $\qquad \omega_s := \frac{\omega_0}{M_s}$ $\qquad n := M_s$ $\qquad M_s = 19$

Cyclotron frequency

$\omega_c(B_z) := \eta \cdot B_z$

Calculate the circuit voltage at the input power

$V_{in} := \sqrt{2 \cdot P_{in} \cdot Z_c}$ $\qquad V_{in} = 0.549 \cdot kV$ $\qquad \frac{V_{in}}{V_a} = 0.041$ $\qquad \frac{\omega_s}{\omega_c(B_z)} = 0.021$

EQUIVALENT MAGNETRON

The CFA is modelled by assuming that each cell can be regarded as one cell of an equivalent magnetron having the same dimensions, anode voltage and magnetic field. The anode current and the r.f. power delivered to the cell are computed from the equivalent magnetron with the same local r.f. voltage on the slow-wave structure. The spoke properties are modelled using either the rigid spoke model or the guiding centre model.

Find the characteristic voltage and field and define the threshold voltage

Equations 15.22, 15.23 and 15.40

$$V_0 := \frac{1}{2\cdot\eta}\cdot r_a^2\cdot\omega_s^2 \qquad B_0 := \frac{2\cdot\omega_s}{\eta}\cdot\left(1-\frac{r_c^2}{r_a^2}\right)^{-1} \qquad V_T(B_z) := V_0\cdot\left(\frac{2\cdot B_z}{B_0}-1\right)$$

$V_0 = 1.598\cdot\mathrm{kV}$ $\quad B_0 = 0.066\,\mathrm{T}$ $\quad V_T(B_z) = 13\cdot\mathrm{kV}$ $\quad \frac{B_z}{B_0} = 4.57$

Find the hub radius and the normalised hub radius as functions of the anode voltage and the magnetic field.

$$Vab(r_b, B_z) := \frac{\eta}{4}\cdot B_z^2\cdot r_b^2\cdot\left(1-\frac{r_c^4}{r_b^4}\right)\cdot\ln\left(\frac{r_a}{r_b}\right)+\frac{\eta}{8}\cdot B_z^2\cdot r_b^2\cdot\left(1-\frac{r_c^2}{r_b^2}\right)^2$$

Equation 15.2

$$r_b(V_a, B_z) := \left| \begin{array}{l} b \leftarrow r_c \\ r_b \leftarrow \mathrm{root}\left(Vab(b, B_z) - V_a, b\right) \\ \mathrm{return}\ r_b \end{array} \right. \qquad R_b(V_a, B_z) := \frac{r_b(V_a, B_z)}{r_c}$$

$r_b(V_a, B_z) = 20.0\cdot\mathrm{mm}$ $\quad R_b(V_a, B_z) = 1.024$

Find the voltage and angular velocity on the surface of the hub

Equations 15.1 and 15.4

$$V_b(V_a, B_z) := \frac{\eta}{8}\cdot B_z^2\cdot r_b(V_a, B_z)^2\cdot\left[1-\left(\frac{r_c}{r_b(V_a, B_z)}\right)^2\right]^2 \qquad \omega_b(V_a, B_z) := \frac{\sqrt{2\cdot\eta\cdot V_b(V_a, B_z)}}{r_b(V_a, B_z)}$$

$V_b(V_a, B_z) = 1.68\cdot\mathrm{kV}$ $\quad \frac{\omega_b(V_a, B_z)}{\omega_s} = 1.11$

Find the d.c. potential and the radial d.c. electric field

Equations 15.2 and 15.3

$$V0(r,V_a,B_z) := \begin{array}{|l} r_b \leftarrow r_b(V_a,B_z) \\ V \leftarrow \frac{\eta}{8}\cdot B_z^2\cdot r^2\cdot\left[1-\left(\frac{r_c}{r}\right)^2\right]^2 \text{ if } r \le r_b \\ V \leftarrow \frac{\eta}{4}\cdot B_z^2\cdot r_b^2\cdot\left[1-\left(\frac{r_c}{r_b}\right)^4\right]\cdot\ln\left(\frac{r}{r_b}\right) + \frac{\eta}{8}\cdot B_z^2\cdot r_b^2\cdot\left[1-\left(\frac{r_c}{r_b}\right)^2\right]^2 \text{ otherwise} \\ \text{return } V \end{array}$$

$$E0(r,V_a,B_z) := \begin{array}{|l} r_b \leftarrow r_b(V_a,B_z) \\ E \leftarrow -\frac{\eta}{4}\cdot B_z^2\cdot r\cdot\left[1-\left(\frac{r_c}{r}\right)^4\right] \text{ if } r \le r_b \\ E \leftarrow -\frac{\eta}{4}\cdot B_z^2\cdot\frac{r_b^2}{r}\cdot\left[1-\left(\frac{r_c}{r_b}\right)^4\right] \text{ otherwise} \\ \text{return } E \end{array}$$

Define the radial and azimuthal RF electric fields as a function of the normalised radius R = r/rc in a frame of reference synchronous with a rotating wave of amplitude V_1 .

$$E_r(R,\theta,V_a,V_1,B_z) := \begin{array}{|l} \left[-\frac{n\cdot V_1}{R\cdot r_c}\cdot\frac{\left(R^n + R^{-n}\right)}{\left(R_a^{\ n} - R_a^{\ -n}\right)}\cdot\cos(n\cdot\theta) + E0(R\cdot r_c,V_a,B_z)\right] \text{ if } R \ge 1 \wedge R \le R_a \\ 0 \text{ otherwise} \end{array}$$

Equation 15.18

$$E_\theta(R,\theta,V_1) := \begin{array}{|l} \left[\frac{n\cdot V_1}{R\cdot r_c}\cdot\frac{\left(R^n - R^{-n}\right)}{\left(R_a^{\ n} - R_a^{\ -n}\right)}\cdot\sin(n\cdot\theta)\right] \text{ if } R \ge 1 \wedge R \le R_a \\ 0 \text{ otherwise} \end{array}$$

Equation 15.19

Rigid Spoke model

Find the conduction angle and the spoke phase in terms of the threshold voltage, the anode voltage and the r.f. wave voltage

$$\theta_{c0}(V_a, V_1, B_z) := \left| \begin{array}{l} \theta \leftarrow \frac{1}{n} \cdot \operatorname{acos}\left(\frac{V_T(B_z) - V_a}{V_1}\right) \\ \theta \leftarrow \frac{\pi}{n} \ \text{ if } \operatorname{Im}(\theta) \neq 0 \end{array} \right.$$

Equation 15.106

$$n \cdot \theta_{c0}(V_a, V_{in}, B_z) = 117 \cdot \mathrm{deg}$$

$$\theta_{s0}(V_a, V_1, B_z) := 0.5 \cdot \theta_{c0}(V_a, V_1, B_z)$$

$$n \cdot \theta_{s0}(V_a, V_{in}, B_z) = 58 \cdot \mathrm{deg}$$

Note: This angle is the physical position of the spoke in the rotating co-ordinate system in which it is stationary with respect to the angle at which the wave voltage is maximum. It is assumed that electrons can only reach the anode if the local anode potential exceeds the threshold potential. The angle defined by Vaughan is the phase angle with respect to the zero of the r.f. voltage. Thus his phase is $\theta = \left(n \cdot \theta_s - \frac{\pi}{2}\right)$

Guiding Centre Spoke Model

$$VV1(V_1, r) := V_1 \cdot \left[\left(\frac{r}{r_c}\right)^n - \left(\frac{r}{r_c}\right)^{-n}\right] \cdot \left[\left(\frac{r_a}{r_c}\right)^n - \left(\frac{r_a}{r_c}\right)^{-n}\right]^{-1}$$

Equation 15.17

Find the radial and tangential velocities at radius r and angle θ in the laboratory frame

$$v_r(R, \theta, V_1, B_z) := \frac{E_\theta(R, \theta, V_1)}{B_z}$$

$$v_\theta(R, \theta, V_a, V_1, B_z) := -\frac{E_r(R, \theta, V_a, V_1, B_z)}{B_z}$$

Equations 15.112 and 15.111

Find the conduction angle and the spoke tip angle

$$\theta_{c1}(V_a, V_1, B_z) := \left| \begin{array}{l} R_b \leftarrow \dfrac{r_b(V_a, B_z)}{r_c} \\ ff(R) \leftarrow \dfrac{v_\theta\left(R, \dfrac{\pi}{n}, V_a, V_1, B_z\right)}{(R \cdot r_c \cdot \omega_s)} - 1 \\ r_e \leftarrow r_b(V_a, B_z) \ \text{ on error } \operatorname{root}(ff(R1), R1, R_b, R_a) \cdot r_c \\ f \leftarrow \dfrac{1}{VV1(V_1, r_b(V_a, B_z))} \cdot \left[\begin{array}{l} (V0(r_e, V_a, B_z) - V0(r_b(V_a, B_z), V_a, B_z) + VV1(V_1, r_e) \cdot \cos(\pi)) \ \ldots \\ + \dfrac{\omega_s \cdot \eta \cdot B_z}{2 \cdot \eta} \cdot \left(r_b(V_a, B_z)^2 - r_e^{\,2}\right) \end{array} \right] \\ \theta \leftarrow \dfrac{\operatorname{acos}(f)}{n} \\ \text{return } \theta \end{array} \right.$$

$$n \cdot \theta_{c1}(V_a, V_{in}, B_z) = 61 \cdot deg$$

Equation 15.123

$$\theta_{s1}(V_a, V_1, B_z) := \left| \begin{array}{l} f \leftarrow \dfrac{1}{VV1(V_1, r_a)} \cdot \left[\begin{array}{l} V0(r_b(V_a, B_z), V_a, B_z) - V0(r_a, V_a, B_z) \ \ldots \\ + VV1(V_1, r_b(V_a, B_z)) \cdot \cos(0.5 \cdot n \cdot \theta_{c1}(V_a, V_1, B_z)) + \dfrac{\omega_s \cdot \eta \cdot B_z}{2 \cdot \eta} \cdot \left[r_a^{\,2} - (r_b(V_a, B_z))^2 \right] \end{array} \right] \\ \theta \leftarrow \dfrac{\operatorname{acos}(f)}{n} \end{array} \right.$$

$$n \cdot \theta_{s1}(V_a, V_{in}, B_z) = 83 \cdot deg$$

Equation 15.123

Select the spoke model

$$\theta_c(V_a, V_{in}, B_z) := \left| \begin{array}{l} \theta \leftarrow \theta_{c0}(V_a, V_{in}, B_z) \ \text{ if } \ SPOKE = 0 \\ \theta \leftarrow \theta_{c1}(V_a, V_{in}, B_z) \ \text{ if } \ SPOKE = 1 \end{array} \right.$$

$$\theta_s(V_a, V_{in}, B_z) := \left| \begin{array}{l} \theta \leftarrow \theta_{s0}(V_a, V_{in}, B_z) \ \text{ if } \ SPOKE = 0 \\ \theta \leftarrow \theta_{s1}(V_a, V_{in}, B_z) \ \text{ if } \ SPOKE = 1 \end{array} \right.$$

Calculate the energy of the electron impact on the anode from the guiding centre equations
When this function is plotted against the spoke phase it is found to vary approximately sinusoidally from a maximum when the phase is zero to a small minimum value when the phase is 180 deg.

$$V_{ia}(V_a, V_1, B_z) := \frac{1}{2\cdot\eta}\cdot\left(v_r\left(R_a, \theta_s(V_a, V_1, B_z), V_1, B_z\right)^2 + v_\theta\left(R_a, \theta_s(V_a, V_1, B_z), V_a, V_1, B_z\right)^2\right)$$

Equation 15.126

$$\frac{V_{ia}(V_a, V_{in}, B_z)}{V_a} = 11.1\cdot\%$$

Vaughan's estimate of the kinetic energy dissipated on the cathode divided by the anode current

$$V_{ic}(V_a, V_1, B_z) := \frac{0.04\cdot V_a}{\sin\left(n\cdot\theta_s(V_a, V_1, B_z)\right)}$$

Equation 15.127

$$\frac{V_{ic}(V_a, V_{in}, B_z)}{V_a} = 4.7\cdot\%$$

Calculate the electronic efficiency

$$\eta_e(V_a, V_1, B_z) := 1 - \frac{V_{ia}(V_a, V_1, B_z) + V_{ic}(V_a, V_1, B_z)}{V_a}$$

$$\eta_e(V_a, V_{in}, B_z) = 84.2\cdot\%$$

Calculate the d.c. anode current
The current is calculated at the base of the spoke which is taken to be on the surface of the hub. The charge density is equal to the hub charge density.

$$\rho(V_a, B_z) := \frac{\varepsilon_0\cdot\eta}{2}\cdot B_z^2\cdot\left[1 + \left(\frac{r_c}{r_b(V_a, B_z)}\right)^4\right]$$

Equation 15.96

Note: This equation does not make the correction to the charge density proposed by Riyopoulos.

Σ

$$I_{dc}(V_a, V_1, B_z) := \left| \begin{array}{l} R_b \leftarrow R_b(V_a, B_z) \\ I \leftarrow \dfrac{n\cdot\rho(V_a, B_z)\cdot L_a\cdot V_1}{B_z}\cdot\left(\dfrac{R_b^{\,n} - R_b^{\,-n}}{R_a^{\,n} - R_a^{\,-n}}\right)\cdot\left(1 - \cos\left(n\cdot\theta_c(V_a, V_1, B_z)\right)\right) \\ I \leftarrow 0 \ \text{ if } \ V_1 < V_T(B_z) - V_a \\ \text{return } I \end{array} \right.$$

Equation 15.108

$$I_{dc}(V_a, V_{in}, B_z) = 10.4\,A$$

RF power delivered to the anode

$$P_1(V_a, V_1, B_z) := \left(V_a - V_{ia}(V_a, V_1, B_z) - V_{ic}(V_a, V_1, B_z)\right) \cdot I_{dc}(V_a, V_1, B_z)$$

$$P_1(V_a, V_{in}, B_z) = 115.8 \cdot kW$$

This is the power which would be delivered to the anode in an equivalent magnetron. This is used to calculate the power delivered to one cell based on the local value of the r.f. voltage.

$$P_c(V_a, V_1, B_z) := \frac{P_1(V_a, V_1, B_z)}{N_a}$$

$$P_c(V_a, V_{in}, B_z) = 1.61 \cdot kW$$

CALCULATE THE PROPERTIES OF THE CFA

The build-up of power along the slow-wave circuit is calculated by assuming that all the power delivered to a cell is added to the growing wave. This ignores any power which is accumulated as a backward wave and emerges from the input of the tube. The arrays P1 and V1 hold the forward wave power and voltage in the cells. The cells are numbered from 0 to (Nc -1).

An approximate correction for transit time effects is made by suppressing the interaction in cells 0 to n_0. The result gives a roughly piecewise linear approximation to the curve of power growth along the structure

$$P1(V_a, P_{in}, B_z) := \left| \begin{array}{l} P1_0 \leftarrow P_{in} \\ \text{for } n \in 0..N_c - 2 \\ \quad \left| \begin{array}{l} V1_n \leftarrow \sqrt{2 \cdot Z_c \cdot P1_n} \\ \Delta P_c \leftarrow \left| \begin{array}{ll} \dfrac{\left(V_a - V_{ia}(V_a, V1_n, B_z) - V_{ic}(V_a, V1_n, B_z)\right) \cdot I_{dc}(V_a, V1_n, B_z)}{N_a} & \text{if } n > n_0 \\ 0 & \text{otherwise} \end{array} \right. \\ P1_{n+1} \leftarrow \left(P1_n + \Delta P_c\right) \cdot \exp(-2 \cdot \alpha \cdot p) \end{array} \right. \\ \text{return } P1 \end{array} \right.$$

The output power is the power in the last cell

$$P_{out}(V_a, P_{in}, B_z) := P1(V_a, P_{in}, B_z)_{N_c-1} \qquad P_{out}(V_a, P_{in}, B_z) = 203 \cdot kW$$

$$Gain(V_a, P_{in}, B_z) := 10 \cdot \log\left(\frac{P_{out}(V_a, P_{in}, B_z)}{P_{in}}\right) \qquad Gain(V_a, P_{in}, B_z) = 14.6 \cdot dB$$

Calculate the wave voltage in each cell from the power flow

$$V1(V_a, P_{in}, B_z) := \sqrt{2 \cdot Z_c \cdot P1(V_a, P_{in}, B_z)}$$

The anode current is calculated by summing the current drawn from the hub. Note that current is drawn in all cells so that the total current is correct even when some of the current does not reach the anode until later cells.

$$Ia(V_a, P_{in}, B_z) := \left| \begin{array}{l} \text{for } n \in 0..(N_c - 1) \\ \quad I_n \leftarrow \dfrac{I_{dc}(V_a, V1(V_a, P_{in}, B_z)_n, B_z)}{N_a} \\ I_{N_c} \leftarrow \displaystyle\sum_{n=0}^{N_c-1} I_n \\ \text{return } I \end{array} \right.$$

$$I_a(V_a, P_{in}, B_z) := Ia(V_a, P_{in}, B_z)_{N_c} \qquad I_a(V_a, P_{in}, B_z) = 23.4\ A$$

Calculate the power added electronic efficiency

$$P_{gen}(V_a, P_{in}, B_z) := P_{out}(V_a, P_{in}, B_z) - P_{in} \qquad P_{gen}(V_a, P_{in}, B_z) = 196 \cdot kW$$

$$\eta e(V_a, P_{in}, B_z) := \frac{P_{gen}(V_a, P_{in}, B_z)}{V_a \cdot I_a(V_a, P_{in}, B_z)} \qquad \eta e(V_a, P_{in}, B_z) = 63.2 \cdot \%$$

Calculate the anode, cathode and r.f. dissipation. The first two of these ignore the effects of transit time and use the local current

$$PA := \left| \begin{array}{l} \text{for } n \in 0..\left(N_c - 1\right) \\ \quad P_n \leftarrow Ia\left(V_a, P_{in}, B_z\right)_n \cdot V_{ia}\left(V_a, V1\left(V_a, P_{in}, B_z\right)_n, B_z\right) \\ P_{N_c} \leftarrow \sum_{n=0}^{N_c-1} P_n \\ \text{return } P \end{array} \right.$$

Anode dissipation on each vane. Element Nc is the total dissipation.

$$\frac{PA_{N_c}}{V_a \cdot I_a\left(V_a, P_{in}, B_z\right)} = 15.1 \cdot \%$$

$$\frac{V_{ia}\left(V_a, \sqrt{2 \cdot Z_c \cdot 208 \cdot kW}, B_z\right)}{V_a} = 17.698 \cdot \%$$

$$PC := \left| \begin{array}{l} \text{for } n \in 1..\left(N_c - 1\right) \\ \quad P_n \leftarrow Ia\left(V_a, P_{in}, B_z\right)_n \cdot V_{ic}\left(V_a, V1\left(V_a, P_{in}, B_z\right)_n, B_z\right) \\ P_{N_c} \leftarrow \sum_{n=0}^{N_c-1} P_n \\ \text{return } P \end{array} \right.$$

Cathode dissipation for each vane. Element Nc is the total dissipation.

$$\frac{PC_{N_c}}{V_a \cdot I_a\left(V_a, P_{in}, B_z\right)} = 5.3 \cdot \%$$

$$\frac{V_{ic}\left(V_a, \sqrt{2 \cdot Z_c \cdot 208 \cdot kW}, B_z\right)}{V_a} = 5.435 \cdot \%$$

$$PL := \left| \begin{array}{l} \text{for } n \in 1..\left(N_c - 1\right) \\ \quad P_n \leftarrow P1\left(V_a, P_{in}, B_z\right)_n \cdot \left(1 - \exp(-2 \cdot \alpha \cdot p)\right) \\ P_{N_c} \leftarrow \sum_{n=0}^{N_c-1} P_n \\ \text{return } P \end{array} \right.$$

Circuit loss in each cell. Element Nc is the total loss.

$$\frac{PL_{N_c}}{\left(V_a \cdot I_a\left(V_a, P_{in}, B_z\right)\right)} = 13.9 \cdot \%$$

$$PL_{N_c} = 43.2 \cdot kW$$

$$PA_{N_c} + PC_{N_c} = 63.2 \cdot kW$$

Energy balance calculation to check the accuracy of the results

$$\frac{P_{out}(V_a, P_{in}, B_z) + PA_{N_c} + PC_{N_c} + PL_{N_c}}{V_a \cdot I_a(V_a, P_{in}, B_z) + P_{in}} = 97.5 \cdot \%$$

Revised calculation of dissipation from electron impact based on energy balance. Note that this calculation does not give the share of the dissipation on the anode and the cathode, nor the distribution along the slow-wave structure.

$$P_i := V_a \cdot I_a(V_a, P_{in}, B_z) + P_{in} - P_{out}(V_a, P_{in}, B_z) - PL_{N_c}$$

$$P_i = 71.1 \cdot kW$$

$$\frac{P_i}{\left(V_a \cdot I_a(V_a, P_{in}, B_z)\right)} = 22.9 \cdot \%$$

Stored results for 3.1 GHz (Phase = 82 deg; Zc = 32.3 Ohms) and 3.5 GHz (Phase = 109 deg; Zc = 14.32 Ohms. This data was obtained by using the scaling factors for the SFD-261 in McDowell (1978) without any allowance for transit time effects.

$$Va31 := \begin{pmatrix} 13.2 \\ 13.4 \\ 13.6 \\ 13.8 \\ 14 \\ 14.2 \\ 14.4 \end{pmatrix} \text{kV} \quad Ia31 := \begin{pmatrix} 0.0 \\ 5.5 \\ 15.6 \\ 22.7 \\ 29.2 \\ 35.4 \\ 41.5 \end{pmatrix} \text{A} \quad Pout31 := \begin{pmatrix} 4.4 \\ 52.6 \\ 140.5 \\ 202.8 \\ 259.8 \\ 315.2 \\ 370.3 \end{pmatrix} \text{kW}$$

$$Va35 := \begin{pmatrix} 11.6 \\ 11.8 \\ 12.0 \\ 12.2 \\ 12.4 \\ 12.6 \\ 12.8 \\ 13.0 \\ 13.2 \\ 13.4 \\ 13.6 \\ 13.8 \\ 14.0 \\ 14.2 \end{pmatrix} \text{kV} \quad Ia35 := \begin{pmatrix} 0.0 \\ 2.9 \\ 7.6 \\ 11.8 \\ 15.8 \\ 19.8 \\ 23.5 \\ 27.0 \\ 30.4 \\ 33.7 \\ 36.9 \\ 40.0 \\ 43.0 \\ 46.1 \end{pmatrix} \text{A} \quad Pout35 := \begin{pmatrix} 4.4 \\ 27.0 \\ 65.3 \\ 99.5 \\ 133.3 \\ 167.7 \\ 201.3 \\ 234.1 \\ 266.6 \\ 299.0 \\ 331.6 \\ 364.5 \\ 397.8 \\ 431.6 \end{pmatrix} \text{kW}$$

Experimental data for plotting

Experimental data from McDowell (2002) Fig.4					
3.3 GHz					
x	I	y1	kV	y2	kW
27	13.2	39	12.6	28	95.2
36	17.6	44	13.0	37	125.9
44	21.5	48	13.2	46	156.5
52	25.4	50	13.4	55	187.1
3.1 GHz					
27	13.2	46	13.1	26.5	90.1
36	17.6	51	13.4	34.5	117.3
44	21.5	53	13.6	43.5	148.0
52	25.4	55	13.7	51.5	175.2
3.5 GHz					
27	13.2	34	12.3	23.5	79.9
36	17.6	39	12.6	30	102.0
44	21.5	43	12.9	37.5	127.6
44	21.5	43	12.9	37.5	127.6

$$I_{ex31} := \begin{pmatrix} 13.2 \\ 17.6 \\ 21.5 \\ 25.4 \end{pmatrix} \qquad V_{ex31} := \begin{pmatrix} 13.1 \\ 13.4 \\ 13.6 \\ 13.7 \end{pmatrix} \qquad P_{ex31} := \begin{pmatrix} 90.1 \\ 117.3 \\ 148.0 \\ 175.2 \end{pmatrix}$$

$$I_{ex33} := \begin{pmatrix} 13.2 \\ 17.6 \\ 21.5 \\ 25.4 \end{pmatrix} \qquad V_{ex33} := \begin{pmatrix} 12.6 \\ 13.0 \\ 13.2 \\ 13.4 \end{pmatrix} \qquad P_{ex33} := \begin{pmatrix} 95.2 \\ 125.9 \\ 156.5 \\ 187.1 \end{pmatrix}$$

$$I_{ex35} := \begin{pmatrix} 13.2 \\ 17.6 \\ 21.5 \\ 21.5 \end{pmatrix} \qquad V_{ex35} := \begin{pmatrix} 12.3 \\ 12.6 \\ 12.9 \\ 12.9 \end{pmatrix} \qquad P_{ex35} := \begin{pmatrix} 79.9 \\ 102.0 \\ 127.6 \\ 127.6 \end{pmatrix}$$

Minimum input powers for spoke formation at various anode voltages computed using the trajectory model below

$$\mathrm{Pinlim} := \begin{pmatrix} 3.5 \\ 6.0 \\ 10.0 \\ 14.0 \\ 20.0 \end{pmatrix} \qquad \mathrm{Poutlim} := \begin{pmatrix} P_{out}(12.75\cdot kV, 3.5\cdot kW, B_z) \\ P_{out}(13\cdot kV, 6kW, B_z) \\ P_{out}(13.25\cdot kV, 10\cdot kW, B_z) \\ P_{out}(13.5\cdot kV, 14\cdot kW, B_z) \\ P_{out}(13.75\cdot kV, 20\cdot kW, B_z) \end{pmatrix} \cdot \frac{1}{kW}$$

Plotting parameters

$$Va := 12.4\cdot kV, 12.6\cdot kV .. 14.4\cdot kV$$

$$Va1 := 11.4\cdot kV, 11.6\cdot kV .. 13.4\cdot kV \qquad n1 := 0 .. (N_c - 1)$$

$$Va2 := 13.4\cdot kV, 13.6\cdot kV .. 15.0\cdot kV \qquad Pin := 5\cdot kW, 10\cdot kW .. 40\cdot kW$$

Data for power growth with distance from fig.7 in McDowell (2002)

$$z := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 11.8 \end{pmatrix} \cdot cm \quad Ppk := \begin{pmatrix} 4.5 \\ 6 \\ 7.5 \\ 12 \\ 17.5 \\ 26 \\ 37 \\ 49 \\ 60 \\ 70 \\ 79 \\ 90 \\ 99 \end{pmatrix} \cdot \frac{160}{102}$$

Tube_type = "SFD-262" $P_{in} = 7 \cdot kW$ $B_z = 0.3\,T$ $f_0 = 3.3 \cdot GHz$ SPOKE = 0

Comparison with the experimental results in McDowell (2002). Note that the calculated curves at 3.1 and 3.5 GHz are stored results calculated using the rigid spoke model without any allowance for transit time effects.

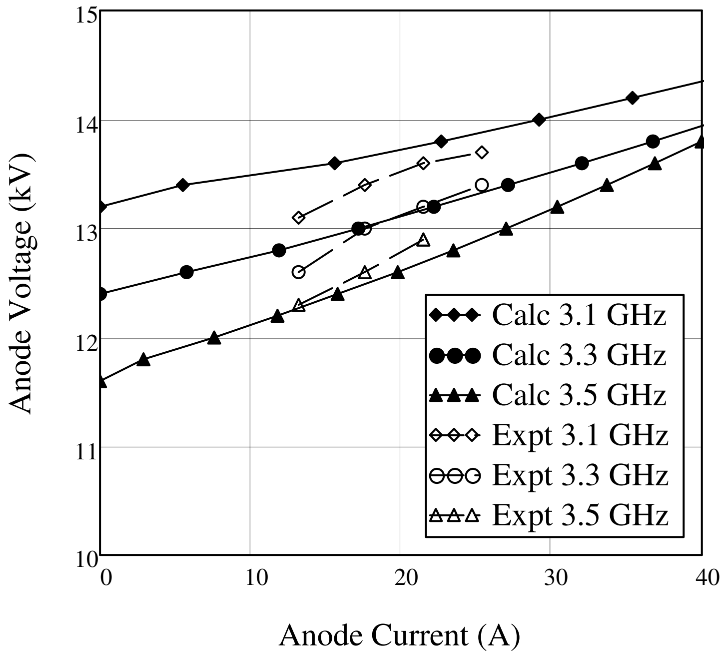


Figure 16.17

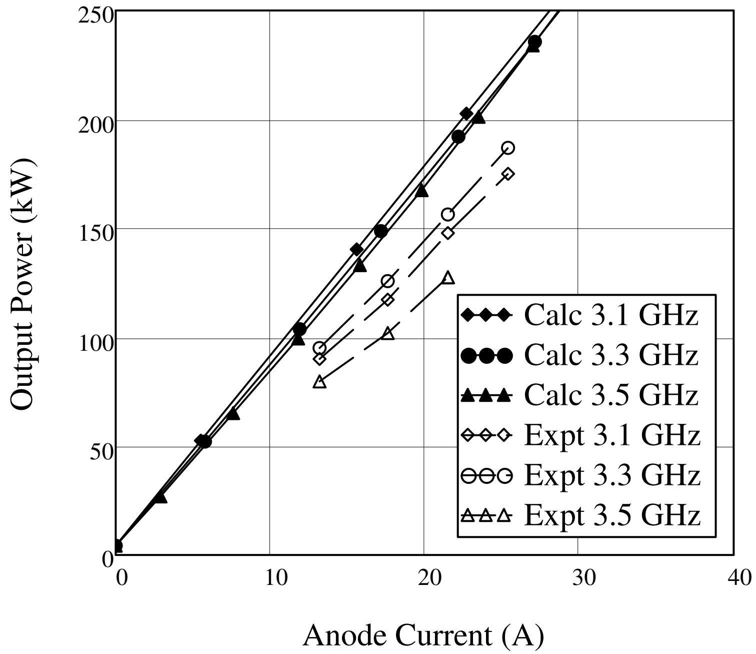


Figure 16.18

Performance chart at 3.3 GHz and $P_{in} = 7 \cdot kW$

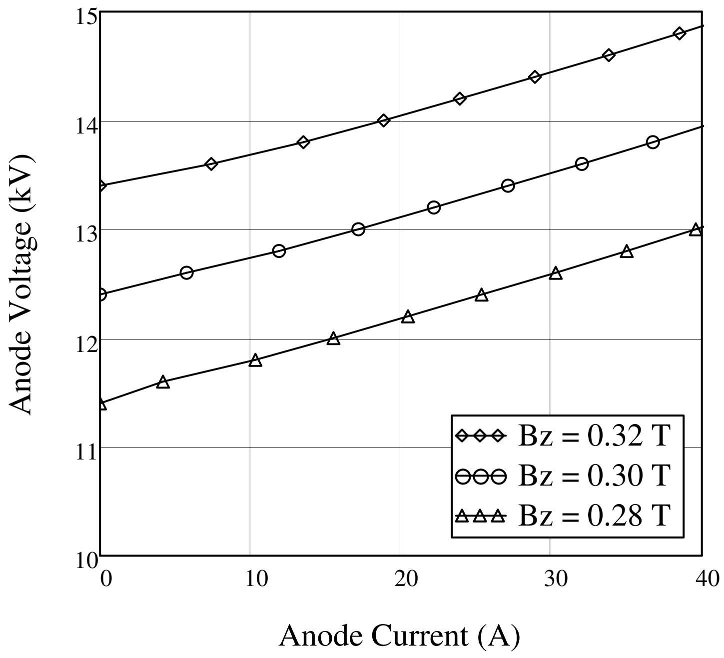


Figure 16.16

Transfer curves at 3.3 GHz and $B_z = 0.3T$ showing the limiting input power for spoke formation calculated using the trajectory model below.

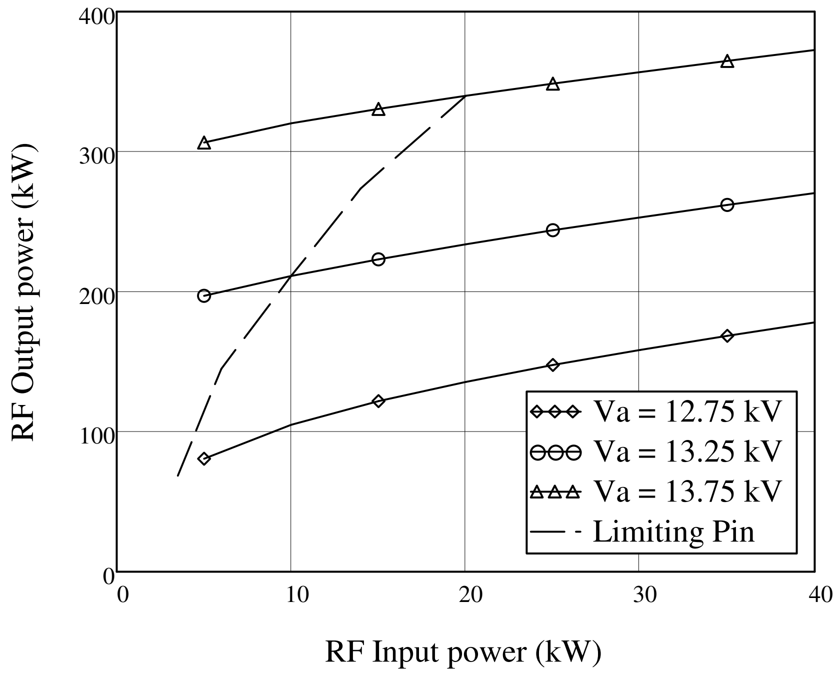


Figure 16.19

Power growth along the slow-wave structure at 3.3 GHz, 13.25 kV and 0.3 T
with results from McDowell (2002) fig.14 for comparison

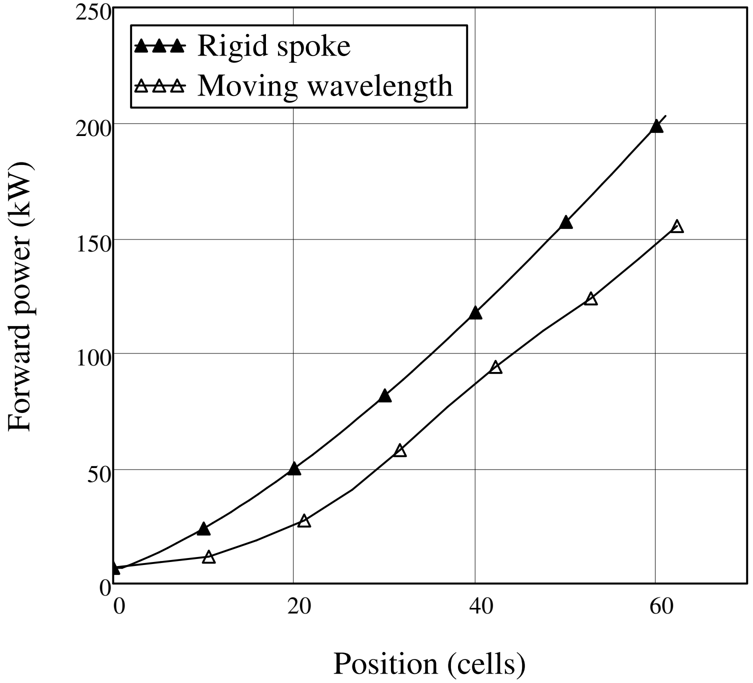


Figure 16.21

CALCULATION OF THE PROPERTIES OF THE EQUIVALENT MAGNETRON

using the guiding centre and electron trajectory models. Double symbols are used to indicate the specific parameter values being used. The anode current of the equivalent magnetron is calculated using the SPOKE selected above. The RF voltage is determined by the forward power is specified here. The results show the development of the spokes as the forward power is adjusted. The conditions under which spokes can just form can be determined. It is found that some trajectories circulate endlessly.

$VV_a := V_a$ $\quad BB_z := B_z$ $\quad VV_a = 13.25 \cdot kV$

$P_f := 11 \cdot kW$ $\quad VV_1 := \sqrt{2 \cdot Z_c \cdot P_f}$ $\quad VV_1 = 0.69 \cdot kV$

$SPOKE = 0$ $\quad$ SPOKE= 0 for rigid spoke
= 1 for guiding centres $\quad II_0 := I_{dc}(VV_a, VV_1, BB_z)$ $\quad II_0 = 12.2\,A$

Specify the number of trajectories to be traced per spoke, the angle of the magnetron to be simulated, the maximum solution time normalised to the synchronous frequency and the number of output data points, and define the initial conditions

$N_s := 48$ $\quad plotangle := 60 \cdot deg$ $\quad \tau max := 10$ $\quad nmax := 500$

Determine the dc and rf voltages and the cylotron frequency from the model selected

$\omega c\_\omega s := \dfrac{\eta \cdot BB_z}{\omega_s}$ $\quad \omega c\_\omega s = 48.4$ $\quad \omega b := \omega_b(VV_a, BB_z)$ $\quad rr_b := r_b(VV_a, BB_z)$ $\quad RR_b := \dfrac{rr_b}{r_c}$ $\quad n_e := N_s \cdot M_s \cdot \dfrac{plotangle}{2 \cdot \pi}$ $\quad n_e = 152$

Calculate the guiding centre trajectories for plotting

$$f(r, \theta_0) := \frac{1}{VV1(VV_1, r)} \cdot \left[V0(r_b(VV_a, BB_z), VV_a, BB_z) - V0(r, VV_a, BB_z) \ldots \atop + VV1(VV_1, r_b(VV_a, BB_z)) \cdot \cos(n \cdot \theta_0) + \frac{\omega_s \cdot \eta \cdot B_z}{2 \cdot \eta} \cdot \left[r^2 - (r_b(VV_a, BB_z))^2 \right] \right]$$

$$\theta0(r) := \frac{1}{deg}\cdot acos(f(r,0)) \qquad \theta1(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{\pi}{6n}\right)\right) \qquad \theta2(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{\pi}{3n}\right)\right) \qquad \theta3(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{\pi}{2n}\right)\right)$$

$$\theta4(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{2\pi}{3n}\right)\right) \qquad \theta5(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{5\pi}{6n}\right)\right) \qquad \theta6(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\frac{\pi}{n}\right)\right) \qquad \theta7(r) := \frac{1}{deg}\cdot acos\left(f\left(r,\theta_{c1}\left(VV_a, VV_1, BB_z\right)\right)\right)$$

Electron trajectories in the presence of the hub space-charge

Derivations of the working equations in a frame of reference rotating with angular velocity ω_s

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta'}{dt}\right)^2 - 2r\omega_s\frac{d\theta'}{dt} - r\omega_s^2 = -\frac{e}{m_0}E_r - r\frac{d\theta'}{dt}\frac{e}{m_0}B_z - r\omega_s\frac{e}{m_0}B_z$$

$$\frac{d^2(r/r_c)}{d(\omega_s t)^2} = \frac{r}{r_c}\left(\frac{d\theta'}{d(\omega_s t)}\right)^2 + 2\frac{r}{r_c}\frac{d\theta'}{d(\omega_s t)} + \frac{r}{r_c} - \frac{e}{m_0\omega_s^2 r_c}E_r - \frac{r}{r_c}\frac{\omega_c}{\omega_s}\frac{d\theta'}{d(\omega_s t)} - \frac{r}{r_c}\frac{\omega_c}{\omega_s}$$

$$\frac{dR}{d\tau} = \dot{R} \quad where \quad R = r/r_c \quad and \quad \tau = \omega_s t$$

$$\frac{d\dot{R}}{d\tau} = R\left\{\dot{\theta}'^2 + \left(2-\frac{\omega_c}{\omega_s}\right)\dot{\theta}' + 1 - \frac{\omega_c}{\omega_s}\right\} - \frac{e}{m_0\omega_s^2 r_c}E_r$$

$$= R\left(\dot{\theta}'+1\right)\left\{\dot{\theta}' + 1 - \frac{\omega_c}{\omega_s}\right\} - \frac{e}{m_0\omega_s^2 r_c}E_r$$

$$\frac{d}{dt}\left(r^2\frac{d\theta'}{dt} + r^2\omega_s\right) = -\frac{e}{m_0}rE_\theta + r\omega_c\frac{dr}{dt}$$

$$2r\frac{dr}{dt}\frac{d\theta'}{dt} + r^2\frac{d^2\theta'}{dt^2} + 2r\omega_s\frac{dr}{dt} = -\frac{e}{m_0}rE_\theta + r\omega_c\frac{dr}{dt}$$

$$\frac{d^2\theta'}{d(\omega_s t)^2} + = -\frac{1}{r}\left\{\frac{e}{m_0\omega_s^2}E_\theta + \left(2-\frac{\omega_c}{\omega_s}\right)\frac{dr}{d(\omega_s t)} + 2\frac{dr}{d(\omega_s t)}\frac{d\theta'}{d(\omega_s t)}\right\}$$

$$\frac{d\theta'}{d\tau} = \dot{\theta}'$$

$$\frac{d\dot{\theta}'}{d\tau} = -\frac{e}{m_0\omega_s^2 R\, r_c}E_\theta - \frac{1}{R}\left(2 + 2\dot{\theta} - \frac{\omega_c}{\omega_s}\right)\frac{dR}{d\tau}$$

Specify the initial conditions for the trajectories

$$R1 := \left| \begin{array}{l} \text{for } i \in 0,4..4\cdot(n_e - 1) \\ \quad \left| \begin{array}{l} R_i \leftarrow RR_b \\ R_{i+1} \leftarrow 0 \\ R_{i+2} \leftarrow \left(\dfrac{i}{2\cdot n_e} - 1\right)\cdot\dfrac{plotangle}{2} \\ R_{i+3} \leftarrow \dfrac{\omega b}{\omega_s} - 1 \end{array} \right. \\ \text{return } R \end{array} \right.$$

Definitions of the variables

$$R_0 = \frac{r}{r_c} \qquad \tau = \omega_s\cdot t$$

$$R_1 = \frac{d}{d\tau}R_0$$

$$R_2 = \theta$$

$$R_3 = \frac{d}{d\tau}\theta$$

When the forward power is small spokes do not form and no electrons reach then anode. A small increase in the initial radius is sufficient to allow spokes to form.

Set up the differential equations of motion for the electrons

$$D(\tau,R) := \left| \begin{array}{l} \text{for } n \in 0,4..4(n_e - 1) \\ \quad \left| \begin{array}{l} D_n \leftarrow R_{n+1} \\ D_{n+1} \leftarrow R_n\cdot(R_{n+3} + 1)\cdot(R_{n+3} + 1 - \omega c\_\omega s) - \dfrac{\eta}{\omega_s^{\,2}\cdot r_c}\cdot E_r(R_n, R_{n+2}, VV_a, VV_1, BB_z) \\ D_{n+2} \leftarrow R_{n+3} \\ D_{n+3} \leftarrow \dfrac{-1}{R_n}\cdot\dfrac{\eta}{\omega_s^{\,2}\cdot r_c}\cdot E_\theta(R_n, R_{n+2}, VV_1) - \dfrac{1}{R_n}\cdot(2 + 2\cdot R_{n+3} - \omega c\_\omega s)\cdot R_{n+1} \end{array} \right. \\ \text{return } D \end{array} \right.$$

Solve the differential equations $\quad Y := \text{rkfixed}(R1, 0, \tau max, nmax, D)$

The first column of the solution matrix Y is the normalised time. The following columns in groups of 4 are the radius, radial velocity, angle and angular velocity of the electrons being tracked.

When a trajectory is intercepted by either the cathode or the anode set all subsequent values of the matrix to the values at the point of interception

```
YY := | for n ∈ 0,4..4·(n_e − 1)
      |   | intc ← 0
      |   | inta ← 0
      |   | for i ∈ 1..nmax
      |   |   | ii ← i
      |   |   | break if Y_{i,n+1} ≤ 1 ∨ Y_{i,n+1} ≥ 0.99R_a
      |   | for i ∈ ii..nmax
      |   |   | Y_{i,n+1} ← 1 if Y_{ii,n+1} ≤ 1
      |   |   | Y_{i,n+1} ← R_a if Y_{ii,n+1} ≥ 0.99R_a
      |   |   | Y_{i,n+2} ← Y_{ii,n+2}
      |   |   | Y_{i,n+3} ← Y_{ii,n+3}
      |   |   | Y_{i,n+4} ← Y_{ii,n+4}
      | return Y
```

Unpack the solution matrix to give matrices of the variables at each time step

```
R_e := | for n ∈ 0,4..4·(n_e − 1)
       |   | nn ← n/4
       |   | for i ∈ 0..nmax
       |   |   R_{i,nn} ← YY_{i,n+1}
       | return R

dR := | for n ∈ 0,4..4·(n_e − 1)
      |   | nn ← n/4
      |   | for i ∈ 0..nmax
      |   |   dR_{i,nn} ← YY_{i,n+2}
      | return dR

θ_e := | for n ∈ 0,4..4·(n_e − 1)
       |   | nn ← n/4
       |   | for i ∈ 0..nmax
       |   |   θ_{i,nn} ← YY_{i,n+3}
       | return θ

dθ := | for n ∈ 0,4..4·(n_e − 1)
      |   | nn ← n/4
      |   | for i ∈ 0..nmax
      |   |   dθ_{i,nn} ← YY_{i,n+4}
      | return dθ
```

Find the fraction of trajectories intercepted on the anode and the cathode

$$F_a := \left| \begin{array}{l} n_a \leftarrow 0 \\ \text{for } n \in 0..n_e - 1 \\ \quad n_a \leftarrow n_a + 1 \ \text{ if } R_{e_{nmax,n}} = R_a \\ \text{return } \dfrac{n_a}{n_e} \end{array} \right.$$

$$F_c := \left| \begin{array}{l} n_a \leftarrow 0 \\ \text{for } n \in 0..n_e - 1 \\ \quad n_a \leftarrow n_a + 1 \ \text{ if } R_{e_{nmax,n}} = 1 \\ \text{return } \dfrac{n_a}{n_e} \end{array} \right.$$

Calculate the total impact energies of the electrons striking the anode and the cathode divided by the number of electrons striking the anode. This procedure means that the power dissipated on the anode and the cathode is obtained by multiplying the energies in electron volts by the dc anode current.

$$VVia := \left| \begin{array}{l} V_i \leftarrow 0 \\ \text{for } n \in 0..n_e - 1 \\ \quad V_i \leftarrow V_i + \dfrac{1}{2\cdot\eta}\cdot\left[\left(r_c\cdot\omega_s\cdot dR_{nmax,n}\right)^2 + \left[r_a\cdot\omega_s\cdot\left(d\theta_{nmax,n} + 1\right)\right]^2\right] \ \text{ if } R_{e_{nmax,n}} = R_a \\ \text{return } \dfrac{V_i}{n_e\cdot F_a} \end{array} \right.$$

$$VVic := \left| \begin{array}{l} V_i \leftarrow 0 \\ \text{for } n \in 0..n_e - 1 \\ \quad V_i \leftarrow V_i + \dfrac{1}{2\cdot\eta}\cdot\left[\left(r_c\cdot\omega_s\cdot dR_{nmax,n}\right)^2 + \left[r_c\cdot\omega_s\cdot\left(d\theta_{nmax,n} + 1\right)\right]^2\right] \ \text{ if } R_{e_{nmax,n}} = 1 \\ \text{return } \dfrac{V_i}{n_e\cdot F_a} \end{array} \right.$$

Impact energies

$$V\_ia(\theta_s) := \frac{1}{2\cdot\eta}\cdot\left(v_r(R_a,\theta_s,VV_1,BB_z)^2 + v_\theta(R_a,\theta_s,VV_a,VV_1,BB_z)^2\right) \qquad V\_ic(\theta_s) := \frac{0.04\cdot VV_a}{\sin(n\cdot\theta_s)} \qquad \alpha(V_a,V_1,B_z,\theta_c) := 1 - \frac{n\,\theta_c}{2\pi}$$

Define quantities for plotting

$$VVa1(\theta) := \left(1 + \frac{VV_1}{VV_a}\cdot\cos(n\cdot\theta)\right)\cdot 2 \qquad R_a = 1.11 \qquad RR_b = 1.024 \qquad \theta := -\pi, -0.99\cdot\pi .. \pi \qquad VT := \frac{V_T(BB_z)}{V_a}\cdot 2$$

Rigid spoke model trajectories

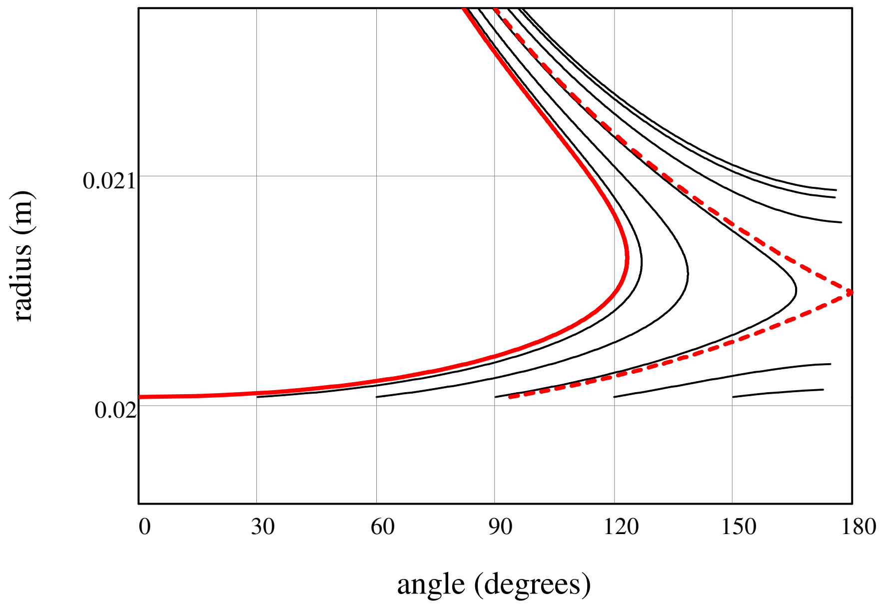
$$r := rr_b, 1.0002\cdot rr_b .. r_a$$

$$RR(r) := \frac{r}{r_c} \qquad \phi s := \begin{pmatrix} 0 \\ n\cdot\theta_s(VV_a,VV_1,BB_z)\cdot deg^{-1} \\ n\cdot\theta_c(VV_a,VV_1,BB_z)\cdot deg^{-1} \end{pmatrix} \qquad Rs := \begin{pmatrix} \frac{rr_b}{r_c} \\ R_a \\ \frac{rr_b}{r_c} \end{pmatrix} \qquad \Delta\tau(\theta) := \int_{r_b(VV_a,BB_z)}^{r_a} \frac{\omega_s\cdot BB_z}{E_\theta\left(\frac{r}{r_c},\theta,VV_1\right)}\,dr$$

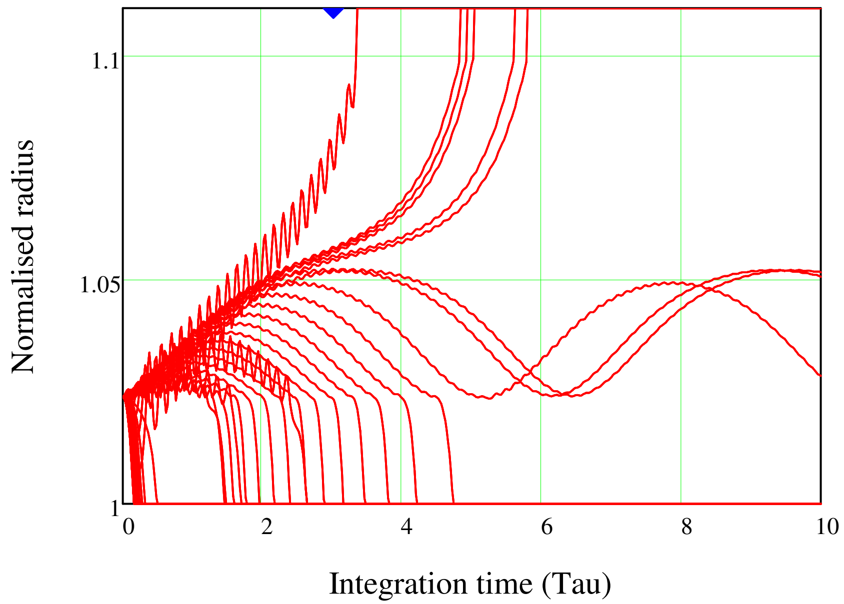
Fractions of electrons landing on the anode and the cathode. If the sum is not unity then see the trajectory plots below for further information. The integration time τ_{max} can be adjusted if necessary.

$F_a = 0.125$ $F_c = 0.816$ $F_a + F_c = 0.941$

Guiding centre trajectories



Electron transit times with the theoretical transit time for the rigid spoke centre



$P_f = 11 \cdot kW$ $N_s = 48$

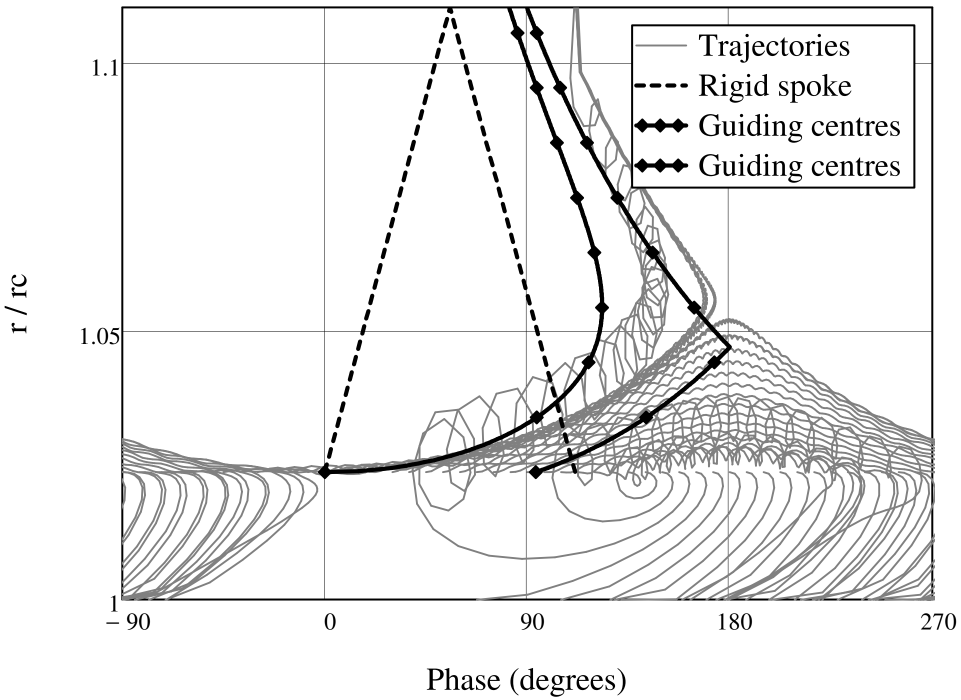


Figure 16.20

Compare the results from the three models

	Trajectory tracking	Rigid spoke model	Guiding centre model
		$\theta s0 := \theta_{s0}(VV_a, VV_1, BB_z)$	$\theta s1 := \theta_{s1}(VV_a, VV_1, BB_z)$
		$\theta c0 := \theta_{c0}(VV_a, VV_1, BB_z)$	$\theta c1 := \theta_{c1}(VV_a, VV_1, BB_z)$
Anode impact energy	$\frac{VVia}{VV_a} = 11.4 \cdot \%$	$\frac{V\_ia(\theta s0)}{V_a} = 11.5 \cdot \%$	$\frac{V\_ia(\theta s1)}{V_a} = 10.5 \cdot \%$
Cathode impact energy	$\frac{VVic}{V_a} = 7.7 \cdot \%$	$\frac{V\_ic(\theta s0)}{V_a} = 4.8 \cdot \%$	$\frac{V\_ic(\theta s1)}{V_a} = 4.0 \cdot \%$
Current fraction returned to the cathode	$F_c = 0.816$	$\alpha(VV_a, VV_1, BB_z, \theta c0) = 0.691$	$\alpha(VV_a, VV_1, BB_z, \theta c1) = 0.740$
	$\eta_{e2} := 1 - \frac{VVia + VVic}{VV_a}$	$\eta_{e0} := 1 - \frac{V\_ia(\theta s0) + V\_ic(\theta s0)}{VV_a}$	$\eta_{e1} := 1 - \frac{V\_ia(\theta s1) + V\_ic(\theta s1)}{VV_a}$
Electronic efficiency	$\eta_{e2} = 80.9 \cdot \%$	$\eta_{e0} = 83.7 \cdot \%$	$\eta_{e1} = 85.4 \cdot \%$

Plot the trajectories of the electrons