WS 13.2 Simplified small signal model of a klystron

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

Define the beam parameters - SLAC 50 MW tube data

Anode voltage	Beam current	Centre frequency	Input power	Magnetic field
$V_a := 315 \cdot kV$	$I_0 := 354 \cdot A$	$f_0 := 2856 \cdot MHz$	$P_{in} := 5 \cdot W$	$B_0 := 0.11 \cdot T$
Tunnel radius	Beam radius	Number of cavities	Field profile parameter (Equation 3.90)	
$a := 15.9 \cdot mm$	$b := 11.0 \cdot mm$	$NCAV := 6$	$kgap := 4$	

kgap = 0 for a uniform field in the gap
kgap = 4 for approximation to a knife edge field

Define the cavity parameters

NB. The first element of each vector is not used. The cavity count starts from 1. A cavity is unloaded if Qe >= 95000.

Cavity frequency

$$fc := \begin{pmatrix} 0 \\ 2860 \\ 2870 \\ 2890 \\ 2910 \\ 2970 \\ 2853 \end{pmatrix} \cdot \mathrm{MHz}$$

Cavity harmonic

$$nh := \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

External Q

$$Qe := \begin{pmatrix} 0 \\ 200 \\ 95000 \\ 95000 \\ 95000 \\ 95000 \\ 21 \end{pmatrix}$$

Unloaded Q

$$Q0 := \begin{pmatrix} 0 \\ 2000 \\ 2000 \\ 2000 \\ 2000 \\ 2000 \\ 2000 \end{pmatrix}$$

R/Q

$$R\_Q := \begin{pmatrix} 0 \\ 80 \\ 75 \\ 87 \\ 96 \\ 96 \\ 85 \end{pmatrix} \cdot \Omega$$

Gap length

$$gap := \begin{pmatrix} 0 \\ 0.0068 \\ 0.0072 \\ 0.0082 \\ 0.011 \\ 0.0116 \\ 0.0162 \end{pmatrix} \cdot \mathrm{m}$$

Gap position

$$zg := \begin{pmatrix} 0 \\ 0 \\ 0.056 \\ 0.111 \\ 0.166 \\ 0.444 \\ 0.555 \end{pmatrix} \cdot \mathrm{m}$$

The detailed calculations can be hidden to allow the data and results to be viewed on the screen simultaneously

Define the charge/mass ratio of the electron. Note that the primary electric constant and the velocity of light are already defined in Mathcad.

$\eta := 1.759 \cdot 10^{11} \cdot \frac{C}{kg}$ $\varepsilon_0 = 8.854 \times 10^{-12} \cdot \frac{F}{m}$ $c = 2.998 \times 10^{8} \frac{m}{s}$ $\mu Perv := \mu A \cdot V^{-1.5}$ $V_R := \frac{c^2}{\eta}$

$\omega_0 := 2 \cdot \pi \cdot f_0$ $dB := 1$

Calculate tube constants and small-signal parameters

Calculate the beam voltage and velocity allowing for space-charge potential depression and relativity

$$V_0 := \left| \begin{array}{l} V_0 \leftarrow V_a \\ \text{for } n \in 0..3 \\ \quad \left| \begin{array}{l} u_n \leftarrow c \cdot \left[1 - \dfrac{1}{\left(1 + \dfrac{V_n}{V_R}\right)^2}\right]^{0.5} \\ V_{n+1} \leftarrow V_0 - \dfrac{I_0}{2 \cdot \pi \cdot \varepsilon_0 \cdot u_n} \cdot \left(\dfrac{1}{2} - \ln\left(\dfrac{b}{a}\right)\right) \end{array} \right. \\ \text{return } V_{n+1} \end{array} \right.$$

Equation 1.4

Equation 7.8

$$V_0 = 291.1 \cdot kV$$

$$u_0 := c \cdot \left[1 - \frac{1}{\left(1 + \dfrac{V_0}{V_R}\right)^2}\right]^{0.5}$$

Equation 1.4

$$u_0 = 2.311 \times 10^8 \frac{m}{s}$$

$$\gamma_{rel} := 1 + \frac{V_0}{V_R}$$

Equation 7.12

$$\gamma_{rel} = 1.57$$

Electronic propagation constant

$$\beta_e(\omega) := \frac{\omega}{u_0} \qquad \lambda_e(\omega) := \frac{2 \cdot \pi}{\beta_e(\omega)} \qquad \gamma_e(\omega) := \frac{\beta_e(\omega)}{\gamma_{rel}}$$

$$\beta_e(\omega_0) = 77.655 \frac{1}{m} \qquad \beta_e(\omega_0) \cdot b = 0.854 \qquad \gamma_e(\omega_0) = 49.471 \frac{1}{m}$$

$$\beta_e(\omega_0) \cdot a = 1.235 \qquad \gamma_e(\omega_0) \cdot a = 0.787$$

Calculate the plasma frequency and the reduced plasma frequency

$$\omega_{p0} := \sqrt{\frac{\eta}{\varepsilon_0}\cdot\frac{I_0}{\pi\cdot b^2\cdot u_0}}$$

$\boxed{\text{Equation 7.47}}$

$$\omega_p := \omega_{p0}\cdot\sqrt{\frac{1}{\gamma_{rel}^{\ 3}}}$$

Calculate the Brillouin field and the ratio mB = B_0 / B_B

$$B_B := \sqrt{\frac{2}{\gamma_{rel}}}\cdot\frac{\omega_{p0}}{\eta}$$

$\boxed{\text{Equation 7.55}}$

$B_B = 0.0574\,T$

$$mB := \frac{B_0}{B_B}$$

$mB = 1.916$

Calculate the plasma frequency reduction factor $p = \omega_q / \omega_p$

$$\tau b(\beta b, m, p) := \beta b\cdot\left[\frac{\frac{1}{p^2} - 1}{\frac{1}{p^2 - 2\cdot\left(m^2 - 1\right)} - 1}\right]^{\frac{1}{2}}$$

$$fn1(\beta b, A) := \frac{1}{\beta b}\cdot\frac{I1(\beta b)\cdot K0(A\cdot\beta b) + I0(A\cdot\beta b)\cdot K1(\beta b)}{I0(\beta b)\cdot K0(A\cdot\beta b) - I0(A\cdot\beta b)\cdot K0(\beta b)}$$

$$fn2(\beta b, m, p) := \frac{1 - \frac{1}{p^2}}{\tau b(\beta b, m, p)}\cdot\frac{I1(\tau b(\beta b, m, p))}{I0(\tau b(\beta b, m, p))}$$

$$fn(\beta b, A, m, p) := \frac{1}{fn1(\beta b, A)} - \frac{1}{fn2(\beta b, m, p)}$$

$p := 0.9$

$P(\beta b, A, m) := root(fn(\beta b, A, m, p), p)$

$$\omega_q(\omega) := P\left[\left(\gamma_e(\omega)\cdot b\right), \frac{a}{b}, mB\right]\cdot\omega_p$$

$$\beta_q(\omega) := \frac{\omega_q(\omega)}{u_0}$$

Fast and slow wave propagation constants

$$\beta_f(\omega) := \beta_e(\omega) - \beta_q(\omega) \qquad \beta_s(\omega) := \beta_e(\omega) + \beta_q(\omega)$$

$$\beta_f(\omega_0) = 72.021\,\frac{1}{m} \qquad \beta_s(\omega_0) = 83.289\,\frac{1}{m}$$

$$\gamma_f(\omega) := \sqrt{\beta_f(\omega)^2 - \frac{\omega^2}{c^2}} \qquad \gamma_s(\omega) := \sqrt{\beta_s(\omega)^2 - \frac{\omega^2}{c^2}}$$

Electronic admittance

$$Y_e(\omega) := \frac{I_0}{\gamma_{rel}\cdot(\gamma_{rel}+1)\cdot V_a}\cdot\frac{\omega}{\omega_q(\omega)}$$

$$Y_e(\omega_0) = 3.84\times 10^{-3}\,\frac{1}{\Omega}$$

Axial gap coupling factor

$$\mu d(\beta, gap, k) := \begin{cases} \operatorname{sinc}\left(\dfrac{\beta\cdot gap}{2}\right) & \text{if } k = 0 \\[2ex] \dfrac{k\cdot\left(\beta\cdot\cosh\left(\dfrac{gap\cdot k}{2}\right)\cdot\sin\left(\dfrac{\beta\cdot gap}{2}\right) + k\cdot\sinh\left(\dfrac{gap\cdot k}{2}\right)\cdot\cos\left(\dfrac{\beta\cdot gap}{2}\right)\right)}{\beta^2\cdot\sinh\left(\dfrac{gap\cdot k}{2}\right) + k^2\cdot\sinh\left(\dfrac{gap\cdot k}{2}\right)} & \text{otherwise} \end{cases}$$

Axial gap coupling factor for field shape parameter *k*. See equations 3.90 and 11.36

Coupling factors of the gaps

$$M(\omega) := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad Mf_n \leftarrow \dfrac{2\cdot I1(\gamma_e(\omega)\cdot b)}{(\gamma_e(\omega)\cdot b)\cdot I0(\gamma_e(\omega)\cdot a)}\cdot \mu d\left(\gamma_e(\omega), gap_n, \dfrac{kgap}{gap_n}\right) \\ \text{return } Mf \end{array}\right.$$

$$M(\omega_0) = \begin{pmatrix} 0 \\ 0.888 \\ 0.887 \\ 0.885 \\ 0.879 \\ 0.877 \\ 0.861 \end{pmatrix}$$

Calculate the beam loading admittance assuming a uniform field in the gap

$$\mathrm{ReI}(\beta,\gamma,a,b,gap) := \frac{\left(\mathrm{I0}(\gamma\cdot b)^2 - \mathrm{I1}(\gamma\cdot b)^2\right)}{\mathrm{I0}(\gamma\cdot a)^2}\cdot \mathrm{sinc}(0.5\cdot\beta\cdot gap)^2$$

Equation 11.101

Beam loading conductance

$$\mathrm{Gb}(gap,\omega) := \frac{Y_e(\omega)}{4}\cdot\left(\mathrm{ReI}\left(\beta_f(\omega),\gamma_f(\omega),a,b,gap\right) - \mathrm{ReI}\left(\beta_s(\omega),\gamma_s(\omega),a,b,gap\right)\right)$$

Equation 11.98

Choose the number of terms in the summations for the gap susceptance $nmax := 2$

$$\lambda := \left|\begin{array}{l} \text{for } n\in 1..nmax \\ \quad \left|\begin{array}{l} x \leftarrow n\cdot\pi \\ \lambda_n \leftarrow \mathrm{root}(\mathrm{J0}(x),x)\end{array}\right. \\ \text{return } \lambda \end{array}\right.$$

Zeroes of J0

$$r(\omega,gap) := \left|\begin{array}{l} \text{for } n\in 1..nmax \\ \quad r_n \leftarrow \frac{\lambda_n\cdot gap}{a}\cdot\sqrt{1-\left(\frac{a\cdot\beta_e(\omega)\cdot b}{\lambda_n\cdot gap}\right)^2\cdot\left(\frac{u_0}{c}\right)^2} \\ \text{return } r \end{array}\right.$$

Equation 11.103

$$\mathrm{N1}(\omega,gap) := \left|\begin{array}{l} \text{for } n\in 1..nmax \\ \quad N_n \leftarrow \left(\frac{\lambda_n}{r(\omega,gap)_n\cdot\frac{a}{gap}}\right)^2 \\ \text{return } N \end{array}\right.$$

$$x(gap) := \left|\begin{array}{l} \text{for } n\in 1..nmax \\ \quad x_n \leftarrow \left(\frac{gap\cdot\mathrm{J0}\left(\frac{b\cdot\lambda_n}{a}\right)}{a\cdot\mathrm{J1}\left(\lambda_n\right)}\right)^2 \\ \text{return } x \end{array}\right.$$

Equation 11.106

$$y(gap) := \left|\begin{array}{l} \text{for } n\in 1..nmax \\ \quad x_n \leftarrow \left(\frac{gap\cdot\mathrm{J1}\left(\frac{b\cdot\lambda_n}{a}\right)}{a\cdot\mathrm{J1}\left(\lambda_n\right)}\right)^2 \\ \text{return } x \end{array}\right.$$

Equation 11.107

$$A1(\omega, gap) := \left| \begin{array}{l} \text{for } n \in 1..nmax \\ \quad A_n \leftarrow 2 \cdot N1(\omega, gap)_n \cdot \left(x(gap)_n + y(gap)_n\right) \cdot \dfrac{\left(r(\omega, gap)_n - 1 + \exp\left(-r(\omega, gap)_n\right)\right)}{r(\omega, gap)_n} \\ \text{return } A \end{array} \right.$$

Equation 11.104

$$B1(\omega, gap) := \left| \begin{array}{l} \text{for } n \in 1..nmax \\ \quad B_n \leftarrow N1(\omega, gap)_n \cdot \left(x(gap)_n + y(gap)_n\right) \cdot \left[\dfrac{3 - 2 \cdot r(\omega, gap)_n - \left(3 + r(\omega, gap)_n\right) \cdot \exp\left(-r(\omega, gap)_n\right)}{\left(r(\omega, gap)_n\right)^3}\right] \\ \text{return } B \end{array} \right.$$

Equation 11.105

$$ImI(\omega, \beta, \gamma, gap) := -\left[\frac{(\beta \cdot gap - \sin(\beta \cdot gap))\left(I0(\gamma \cdot b)^2 - I1(\gamma \cdot b)^2\right)}{(\beta \cdot gap)^2 \cdot I0(\gamma \cdot a)^2} \ldots \atop + 2 \cdot \beta \cdot gap \cdot \left[\sum_{n=1}^{nmax} \left[\frac{A1(\omega, gap)_n}{\left[(\beta \cdot gap)^2 + \left(r(\omega, gap)_n\right)^2\right]^2} - \frac{B1(\omega, gap)_n}{(\beta \cdot gap)^2 + \left(r(\omega, gap)_n\right)^2}\right]\right]\right]$$

Equation 11.102

Beam loading susceptance

$$Bb(gap, \omega) := \frac{Y_e(\omega)}{2} \cdot \left(ImI\left(\omega, \beta_f(\omega), \gamma_f(\omega), gap\right) - ImI\left(\omega, \beta_s(\omega), \gamma_s(\omega), gap\right)\right)$$

Beam loading admittance

$$Y_b(\omega) := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad Yb_n \leftarrow Gb\left(gap_n, \omega\right) + j \cdot Bb\left(gap_n, \omega\right) \\ \text{return } Yb \end{array} \right.$$

External conductances of the cavities

$$Ge := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad \left| \begin{array}{l} Ge_n \leftarrow \left(R\_Q_n \cdot Qe_n\right)^{-1} \text{ if } Qe_n < 95000 \\ Ge_n \leftarrow 0 \text{ otherwise} \end{array} \right. \\ \text{return } Ge \end{array} \right.$$

Admittances of the cavities

$$Yc(\omega) := \left| \begin{array}{l} f \leftarrow \dfrac{\omega}{2\cdot\pi} \\ \text{for } n \in 1..NCAV \\ \quad Yc_n \leftarrow \dfrac{1}{R\_Q_n}\cdot\left[\dfrac{1}{Q0_n} + j\cdot\left(\dfrac{f}{fc_n} - \dfrac{fc_n}{f}\right)\right] \\ \text{return } Yc \end{array} \right.$$

Total admittances of the gaps

$$Y(\omega) := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad Y_n \leftarrow Y_b(\omega)_n + Yc(\omega)_n + Ge_n \\ \text{return } Y \end{array} \right.$$

$$Z(\omega) := \left| \begin{array}{l} \text{for } n \in 1..NCAV \\ \quad Z_n \leftarrow \dfrac{1}{Y(\omega)_n} \\ \text{return } Z \end{array} \right.$$

Calculate the gap voltages

$$Vg(\omega) := \left| \begin{array}{l} \beta_e \leftarrow \beta_e(\omega) \\ \beta_q \leftarrow \beta_q(\omega) \\ \text{for } n \in 1..NCAV \\ \quad \left| \begin{array}{l} MM_n \leftarrow M(\omega)_1 \\ YY_n \leftarrow Y(\omega)_n \end{array} \right. \\ Vg_1 \leftarrow \sqrt{\dfrac{8\cdot Ge_1}{\left(\left|YY_1\right|\right)^2}\cdot P_{in}} \\ \text{for } n \in 2..NCAV \\ \quad Vg_n \leftarrow \left| \begin{array}{l} Vg_n \leftarrow 0 \\ \text{for } m \in 1..(n-1) \\ \quad Vg_n \leftarrow Vg_n - j\cdot\dfrac{Y_e(\omega)}{YY_n}\cdot MM_m\cdot MM_n\cdot Vg_m\cdot\sin\left[\beta_q\cdot\left(zg_n - zg_m\right)\right]\cdot\exp\left[-j\cdot\beta_e\cdot\left(\left(zg_n - zg_m\right)\right)\right] \\ Vg_n \end{array} \right. \\ \text{return } Vg \end{array} \right.$$

$$Vg\left(\omega_0\right) = \begin{pmatrix} 0 \\ 0.265 \\ 0.738 + 1.317i \\ -5.984 + 2.128i \\ -2.087 - 22.742i \\ -23.855 + 111.015i \\ -211.771 - 324.359i \end{pmatrix}\cdot kV$$

$$P_{out}(\omega) := \frac{1}{2}\cdot\left(\left|Vg(\omega)_6\right|\right)^2\cdot Ge_6 \qquad Gain(\omega) := 10\cdot\log\left(\frac{P_{out}(\omega)}{P_{in}}\right) \qquad Gain(\omega_0) = 69.2\cdot dB$$

$$f := 2.83\cdot GHz, 2.835\cdot GHz .. 2.91\cdot GHz$$