Worksheet 10.4 Multi-element depressed collector model (TWT)

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This worksheet calculates the RF efficiency of a TWT with a multi-element depressed collector using an approximate trapezoidal spent-beam curve defined by the maximum and minimum normalised electron velocities at saturation. (see Figures 10.9 and 14.27).

$V_{max} := 1.1$

$V_{min} := 0.5$

Saturated efficiency

$$\eta_{sat} := 1 - \frac{V_{max} + V_{min}}{2}$$

$\eta_{sat} = 20\,\%$

$N\_stages := 5$

$Vc :=$

	0
0	0
1	0.4
2	0.5
3	0.65
4	0.8
5	1
6	

Probability distribution for electron collection

probability :=

- Constant
- Linear
- Sine
- Sine squared

This section can be collapsed if required to allow the data and the results to be viewed on the same screen.

$$p(x) := \begin{cases} 1 & \text{if } probability = 1 \\ x & \text{if } probability = 2 \\ \sin\left(\frac{\pi \cdot x}{2}\right) & \text{if } probability = 3 \\ \sin\left(\frac{\pi \cdot x}{2}\right)^2 & \text{if } probability = 4 \end{cases}$$

Constant probability

$$\begin{aligned} pc0(Ve) := \; & Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ & \text{for } n \in N\_stages..0 \\ & \quad pc_n \leftarrow \begin{cases} 1 & \text{if } Ve \ge Vc_n \wedge Ve < Vc_{n+1} \\ 1 & \text{if } n = N\_stages \wedge Ve \ge Vc_{n+1} \\ 1 & \text{if } n = 0 \wedge Ve < Vc_1 \\ 0 & \text{otherwise} \end{cases} \\ & \text{return } pc \end{aligned}$$

Variable probability

$$\begin{aligned} pc(Ve) := \; & Vc_{N\_stages+1} \leftarrow 2 \cdot Vc_{N\_stages} - Vc_{N\_stages-1} \\ & \text{for } n \in N\_stages..0 \\ & \quad pc_n \leftarrow \begin{cases} p\left(\frac{Ve - Vc_n}{Vc_{n+1} - Vc_n}\right) & \text{if } Ve \ge Vc_n \wedge Ve < Vc_{n+1} \\ 1 & \text{if } n = N\_stages \wedge Ve \ge Vc_{n+1} \\ 1 & \text{if } n = 0 \wedge Ve < Vc_1 \\ \left(1 - pc_{n+1}\right) & \text{if } n < N\_stages \wedge \left(Ve \ge Vc_{n+1} \wedge Ve < Vc_{n+2}\right) \\ 0 & \text{otherwise} \end{cases} \\ & \text{return } pc \end{aligned}$$

Calculate the DC collector efficiency as a function of electron energy

$$\eta_{DC}(Ve) := \frac{1}{Ve} \cdot \sum_{n=0}^{N\_stages} \left(pc(Ve)_n \cdot Vc_n\right)$$

Equation 10.26

Plot the probability of collection by each electrode as a function of the electron energy with rectangles showing constant probability for comparison.

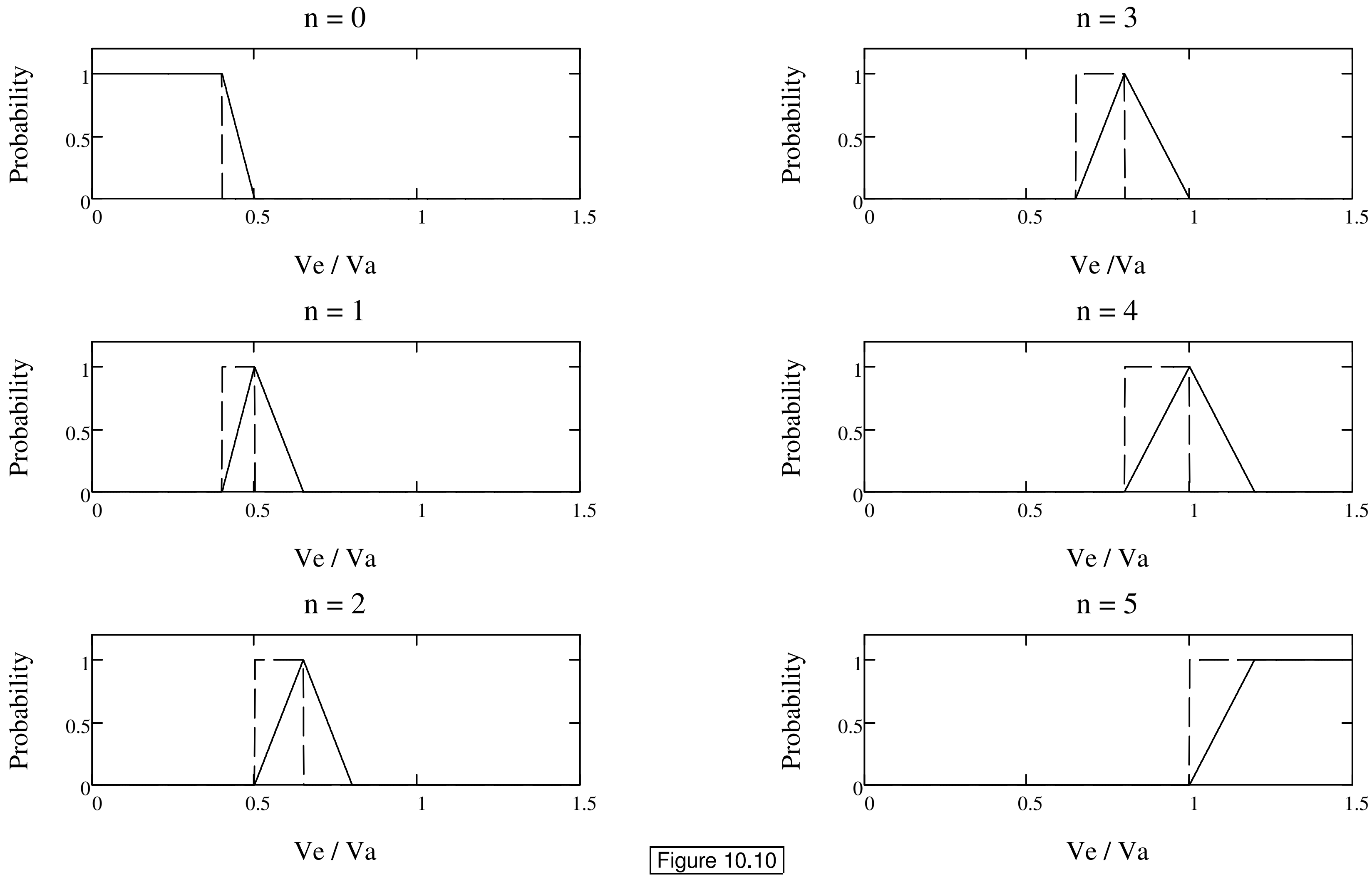


Figure 10.10

Define an approximate trapezoidal spent-beam distribution

Vmax is constant, Vmin is defined by the saturated output power. X is the normalised output power

$$V_1(X) := (2 - V_{max})\cdot(1 - X) + X\cdot V_{min}$$

$$V_s(X,\theta) := V_1(X) + (V_{max} - V_1(X))\cdot\frac{\theta}{\pi}$$

$$\eta_e(X) := 1 - \frac{1}{2}(V_{max} + V_1(X))$$

$$I_s(\theta) := 1$$

$$II_s(\theta) := \frac{1}{\pi}\cdot\int_\theta^\pi I_s(\theta)\, d\theta$$

Calculate the total power entering the collector entering the collector normalised to the maximum RF output power

$$P_{ent}(X) := \frac{1}{\pi}\cdot\int_0^\pi I_s(\theta)\cdot V_s(X,\theta)\, d\theta$$

Check $1 - P_{ent}(0.5) = 0.1$ $\eta_e(0.5) = 0.1$

Calculate the rf efficiency of the collector as a function of the RF grid voltage

$$P_{rec}(X) := \frac{1}{\pi}\cdot\int_0^\pi I_s(\theta)\cdot V_s(X,\theta)\cdot\eta_{DC}(V_s(X,\theta))\, d\theta$$

Equation 10.30

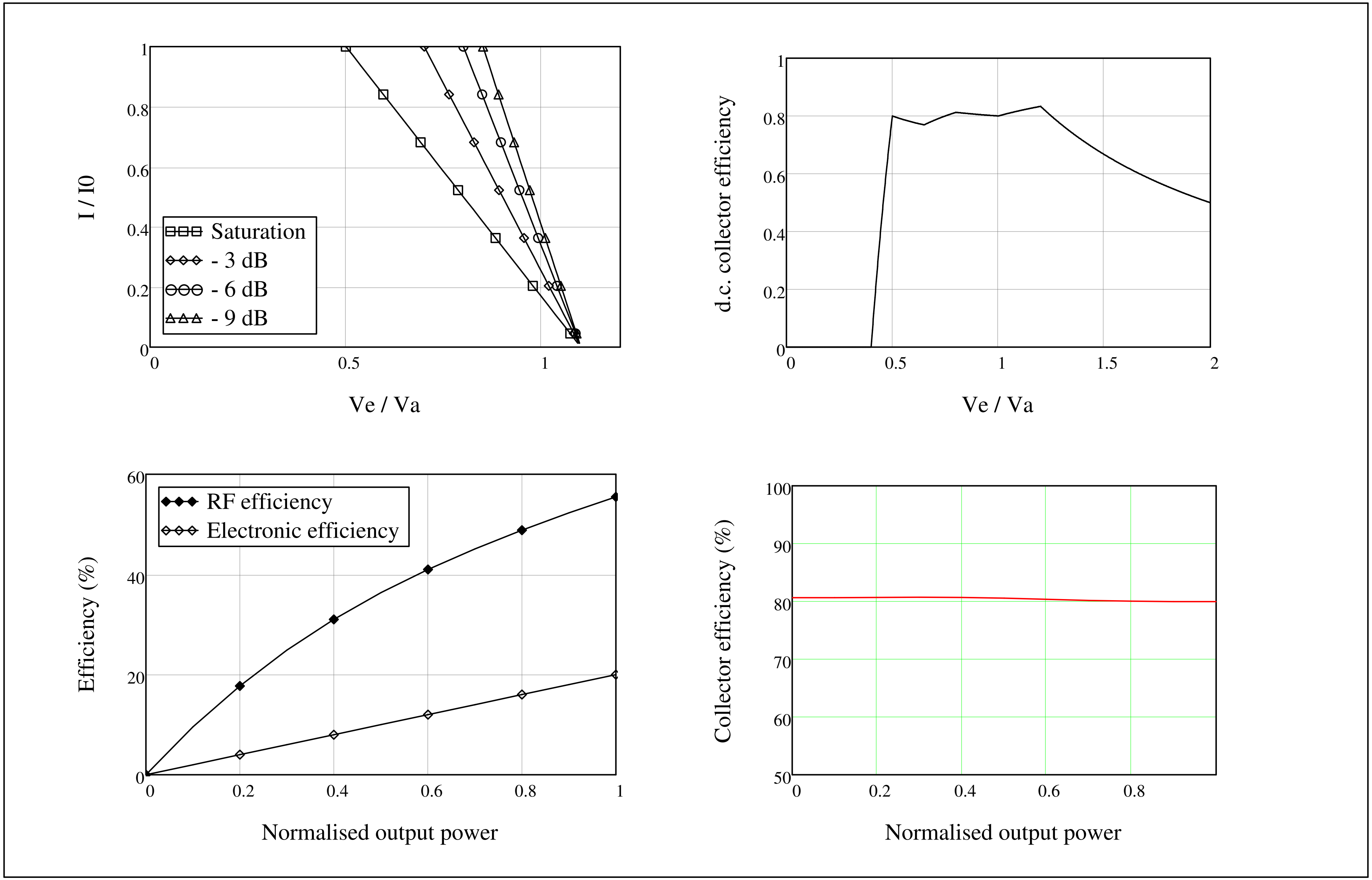
$$\eta_c(X) := \frac{P_{rec}(X)}{P_{ent}(X)}$$

Compute the RF efficiency as function of the normalsied output power

$$\eta_{rf}(X) := \frac{\eta_e(X)}{1 - \eta_c(X)\cdot(1 - \eta_e(X))}$$

Equation 10.13

Plotting ranges $\theta := 0, 0.1 .. \pi$ $X := 0, 0.1 .. 1$


I / I0
Saturation
- 3 dB
- 6 dB
- 9 dB
Ve / Va
d.c. collector efficiency
Ve / Va
Efficiency (%)
RF efficiency
Electronic efficiency
Normalised output power
Collector efficiency (%)
Normalised output power