WS 5.4 Planar diode with initial velocity

This Mathcad 14 worksheet is designed to accompany the author's book "Microwave and RF Vacuum Electronic Power Sources", Cambridge University Press (2018). The section, equation, and figure numbers refer to the corresponding sections, equations, and figures in the book. Data input fields are highlighted in yellow and output fields are highlighted in green.

This sheet is based on:
Jaffe, G. (1944). "On the currents carried by electrons of uniform initial velocity." Physical Review **65**(3-4): 91.

$$U = \frac{V}{V_k}$$

$$Y(U) := \sqrt{1 + U}$$

The symbol Y is used in place of W to avoid redefining a Mathcad built-in variable.

Branch I: Potential increasing monotonically

$$\alpha_2(\alpha_1) := \frac{4}{3}\cdot(1-2\cdot\alpha_1)\cdot(1+\alpha_1)^{\frac{1}{2}}$$

Equation 5.104

$$X1(\alpha_1, U) := \frac{4}{3}\cdot(Y(U)-2\cdot\alpha_1)\cdot(Y(U)+\alpha_1)^{\frac{1}{2}} - \alpha_2(\alpha_1)$$

Equation 5.105

Branch II: Potential minimum but no reflection of electrons

$$X2a(\alpha_1, U) := \alpha_2(\alpha_1) - \frac{4}{3}(Y(U)-2\cdot\alpha_1)\cdot(Y(U)+\alpha_1)^{\frac{1}{2}}$$

Equation 5.106

$$X2b(\alpha_1, U) := \frac{4}{3}\cdot(Y(U)-2\cdot\alpha_1)\cdot(Y(U)+\alpha_1)^{\frac{1}{2}} + \alpha_2(\alpha_1)$$

Equation 5.107

Note: Setting W = 1 and X = 0 in (5.106) and (5.107) we find that α3 and α4 both satisfy (5.104). thus the can be replaced by α2 in (5.106) and (5.107).

At the minimum of potential

$$X2_{min}(\alpha_1) := \frac{4}{3}\cdot(1-2\cdot\alpha_1)\cdot(1+\alpha_1)^{\frac{1}{2}}$$

Equation 5.109

$$U2_{min}(\alpha_1) := {\alpha_1}^2 - 1$$

Since $Y = -\alpha_1$ when $X = X_m$ (line before equation (5.108))

The current is space-charge limited at the maximum of the curve of constant U_a

$$\alpha_{max}(U_a) := \frac{-\sqrt{1+U_a}}{1+\sqrt{1+U_a}}$$

Equation 5.111

$$X_{max}(U) := \frac{4}{3}\cdot\left(1+\sqrt{1+U}\right)^{\frac{3}{2}}$$

Equation 5.112

$\alpha_1 := -1, -0.99 .. 0.2$ $\alpha_{11} := -1, -0.99 .. 4$

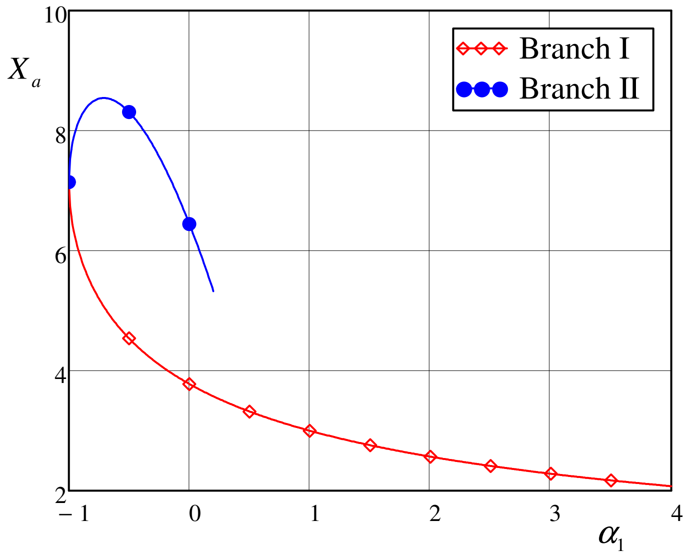


Figure 5.13

The normalised anode potential is $U_a := 5$

Note:

1. There is a mistake in Figure 5.13 in the book. The upper curve corresponds to $U_a = 5$
2. Jaffe's parameter u_0 in his Figure 1 is given by

$$u_0 = \sqrt{1 + U_a}$$

Position of the maximum of the Branch II curve

$X_{max}(U_a) = 8.542$

$\alpha_{max}(U_a) = -0.7101$

Branch III: Virtual cathode which reflects some of the electrons

$$X3a(R_3, U) := \frac{4}{3} \cdot \frac{1}{\sqrt{2 \cdot R_3 - 1}} \cdot \left(1 - Y(U)^{\frac{3}{2}}\right)$$

Equation 5.119

X3a refers to the region 0 < X < Xmin;
X3b refers to the region Xmin < X < Xa.
R_3 is the ratio of the injected current to the anode current

$$X3b(R_3, U) := \frac{4}{3} \cdot Y(U)^{\frac{3}{2}} + \frac{4}{3} \cdot \frac{1}{\sqrt{2 \cdot R_3 - 1}}$$

Equation 5.121

$$X3_{min}(R_3) := \frac{4}{3} \cdot \frac{1}{\sqrt{2 \cdot R_3 - 1}}$$

Equation 5.118

Note: in the book Y = 1 on the previous line should be W = 1

$$X3(R_3, U) := R_3 \cdot X2b(0, U)$$

X3 defines X in terms of the X in Branch II when $\alpha_1 = 0$

Normalise the potentials to U_{max}

$$X_1(\alpha_1, U) := \frac{X1(\alpha_1, U)}{X1(\alpha_1, U_{max})} \qquad X_{2a}(\alpha_1, U) := \frac{X2a(\alpha_1, U)}{X2b(\alpha_1, U_{max})} \qquad X_{3a}(R_3, U) := \frac{X3a(R_3, U)}{X3b(R_3, U_{max})}$$

$$X_{2b}(\alpha_1, U) := \frac{X2b(\alpha_1, U)}{X2b(\alpha_1, U_{max})} \qquad X_{3b}(R_3, U) := \frac{X3b(R_3, U)}{X3b(R_3, U_{max})}$$

$$X_{2min}(\alpha_1) := \frac{X2_{min}(\alpha_1)}{X2b(\alpha_1, U_{max})} \qquad X_{3min}(R_3) := \frac{X3_{min}(R_3)}{X3b(R_3, U_{max})}$$

Plot normalised potential against normalised position

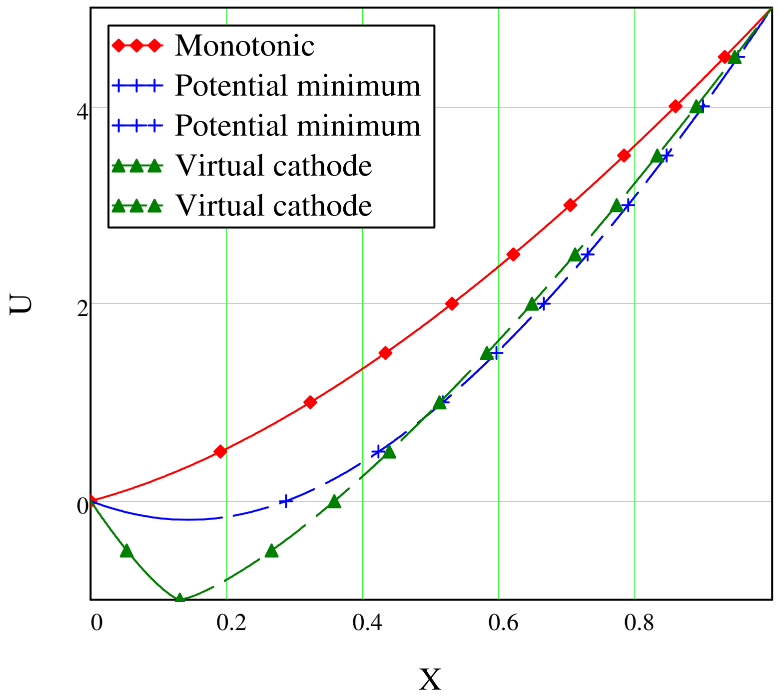
Normalise the potentials to $U_{max} \equiv 5$

Monotonic increase (red) $\alpha 1 := -0.9$

Potential minimum (blue) $\alpha 2 := -0.9$

Virtual cathode (green) $R_3 := 2$

$u := -1, -0.995 .. 5$



When α = -1 the space-charge minimum is at X = 0 and the potential curves for branches I and II are identical.

When α = 0 and R3 = 1 the potential curves for branches II and III are identical.

Monotonic increase	Potential minimum	Virtual cathode
	$X_{2min}(\alpha 2) = 0.143$	$X_{3min}(R_3) = 0.131$
	$U2_{min}(\alpha 2) = -0.19$	
$X1(\alpha 1, U_a) = 5.872$	$X2b(\alpha 2, U_a) = 8.234$	

For space-charge limitation

$X_{max}(U_a) = 8.542$

If d and V_k are fixed then the cathode current is proportional to X^2.
The anode current normalised to the maximum limited by space-charge is

$\left(\frac{X1(\alpha 1, U_a)}{X_{max}(U_a)}\right)^2 = 0.473$ $\left(\frac{X2b(\alpha 2, U_a)}{X_{max}(U_a)}\right)^2 = 0.929$ $\left(\frac{X3b(R_3, U_a)}{X_{max}(U_a)}\right)^2 = 0.474$

Plot anode current against injected current both normalised to the space-charge limited current

$\alpha 11 := -1, -0.99 .. 10$

$\alpha 22 := -1, -0.99 .. 0$

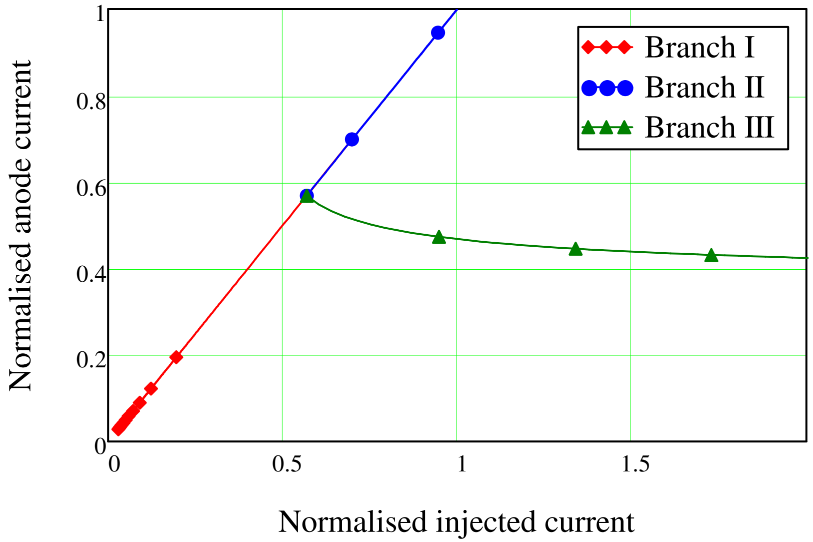
$R3 := 1, 1.1 .. 10$

The definition of X in equation (5.116) differs from that in equation (5.100). to make this clear we can replace X in (5.116) by X' so that

$$X' = \sqrt{\frac{J_a}{J_i}}X$$

Then the injected current and the anode current normalised to the space-charge limited current are given by

$$J1_a(\alpha 1, U) := \left(\frac{X1(\alpha 1, U)}{X_{max}(U)}\right)^2 \qquad J2_a(\alpha 2, U) := \left(\frac{X2b(\alpha 2, U)}{X_{max}(U)}\right)^2 \qquad J3_i(R3, U) := \left(\frac{\sqrt{R3}\cdot X3b(R3, U)}{X_{max}(U)}\right)^2 \qquad J3_a(R3, U) := \left(\frac{X3b(R3, U)}{X_{max}(U)}\right)^2$$



Normalised anode potential

$U_a = 5$