

Figure 1: Sketch of the stratification profile for problem 6.

## 6: An unstable layer in an inviscid fluid

Knowing that  $\operatorname{sech}^2(0) = 1$  and  $\operatorname{sech}^2(\pm \infty) = 0$ , we can tell that  $B_z$  goes to  $B_{z0}$  far from z = 0 and  $-B_{z0}$  at z = 0 (figure 1). In a layer surrounding z = 0 (specifically -0.88 < z < 0.88),  $B_z$  is negative.

We seek a solution of (2.29) for the special case  $\tilde{k} = \alpha$ :

$$\sigma^2 \Big( \frac{\mathrm{d}^2}{\mathrm{d}z^2} - \alpha^2 \Big) \hat{w} = B_z \alpha^2 \hat{w}.$$

Try the suggested function  $\hat{w} = \operatorname{sech}^2 \beta z$ , and seek a value of  $\beta$  for which the equation is satisfied. Guessing  $\beta = \alpha$ , we find that the equation is satisfied and the growth rate is

$$\sigma=\sqrt{B_{z0}/3}.$$