Appendix C MLEM notation

This appendix from www.cambridge.org/Jacobsen is an online extension of the book *X-ray Microscopy*. Maximum likelihood and estimation maximum (MLEM) is an approach to reconstructing tomographic data as described in Section 8.2.2 in the printed book, as well as an approach to deconvolution of the probe or point spread function from an image so as to obtain something closer to the "true" image. As was noted in Section 8.2.2, this approach originates from a statistical method called expectation maximization [Dempster 1977], which was conceived without connection to image processing or recovery—in fact, Dempster, Laird, and Rubin were associated in part with the company that produced college entrance exams in the USA! It is therefore amusing (exasperating?) to take a short detour here to see how the basic iterative update rule of Eq. 8.18 of

$$f_{j+1}(x) = f_j(x) \left\{ \left[\frac{p(x)}{f_j(x) * g(x)} \right] * p(-x) \right\}.$$

is written in various papers, given their various different conceptual starting points and target research communities. Put Table C.1 in your pipe and smoke it!

Table C.1 Notation used for the MLEM approach to image recovery and tomogram reconstruction, from various papers in the literature (plus two papers on maximum entropy [Frieden 1972, Meinel 1986]). This represents only a few samples from the literature, but one can see that it's enough to make one's head swim!

Source	**Observed image, + noise**	**Object, blur**	**Recovery equation**
[Richardson 1972]	H	W, S	$W_{i,r+1} = W_{i,r} \sum_{k=i}^{c} \frac{S_{k-i+1} H_k}{\sum_{j=a}^{b} S_{k-j+1} W_{j,r}}$
[Lucy 1974]	$\tilde{\phi}(x), \phi(x)$	$\psi(\xi), P(x\|\xi)$	$\psi_j^{r+1} = \psi_j^r \sum_{i=1}^{I} \frac{\tilde{\phi}_i}{\phi_i} P_{ij}$
[Carasso 1999]	$g(x)$, $g(x) = g_e(x) + n(x)$	$f(x), h(x)$	$f^{n+1} = f^n H^* \left(\frac{g}{Hf^n}\right)$
[Fish 1995]	$c(x)$	$f(x), g(x)$	$f_{i+1}(x) = f_i(x) \left\{ \left[\frac{c(x)}{f_i(x) \otimes g(x)} \right] \otimes g(-x) \right\}$
[Mu 2006]	$p, f * h + n$	f, h	$\hat{f}^{(k+1)} = \hat{f}^{(k)} \times \left(h \otimes \frac{p_c}{\hat{f}^{(k)} * h} \right)$
[Frieden 1972]	$I(x), I(x) + N$	$O(x), S(x)$	$O_j = \exp[-1 - \mu - \sum_{m=1}^{M} \lambda_m S(y_m, x_j)]$, $\hat{N}_m = \exp[-1 - \lambda_m/\rho]$
[Meinel 1986]	$i_m, i_m + n_m$	o_m, s_{mh}	$o_m = K Q_m \exp\left[\sum_k s_{mk} \frac{i_k}{\sum_j s_{mj} o_j} - 1 \right]$