Chapter 8

```
In[1]:= Needs["DiscreteMath`RSolve`"]
```

à Question 1

(i)

This is a purely theorical question. Given

$$q_t^d = a - b\, p_t$$
$$q_t^s = c + d\, p_{t-1}$$

and $q_t^d = q_t^s = q_t$ in equilibrium, then

$$a - b\, p_t = c + d\, p_{t-1}$$
$$p_t = (a - c) + (\tfrac{d}{-b})\, p_{t-1}$$

Since $b < 1$ then $-b > 1$ and with $d > 0$ then $\frac{d}{-b} > 0$. If $0 < \frac{d}{-b} < 1$ then the system converges on the equilibrium value. On the other hand, if $\frac{d}{-b} > 1$ then the system diverges from equilibrium.

(ii)

Since $\dfrac{d}{-b} > 0$ then the system does not oscillate.

An example of this type of situation is given in Question 2(ii) in which $\frac{d}{-b} = \frac{1}{2}$, which we shall consider next.

à Question 2

(i)

The system is:

$$q_t^d = 100 - 2\, p_t$$
$$q_t^s = -20 + 3\, p_{t-1}$$

with initial condition $p_0 = 10$. First we need to derive the difference equation. This is

$$100 - 2\, p_t = -20 + 3\, p_{t-1}$$
$$p_t = 60 - \tfrac{3}{2}\, p_{t-1}$$

Using the instructions given in the text, we can derive the cobweb. Here we supress all intermediate graphics and just display the final cobweb.

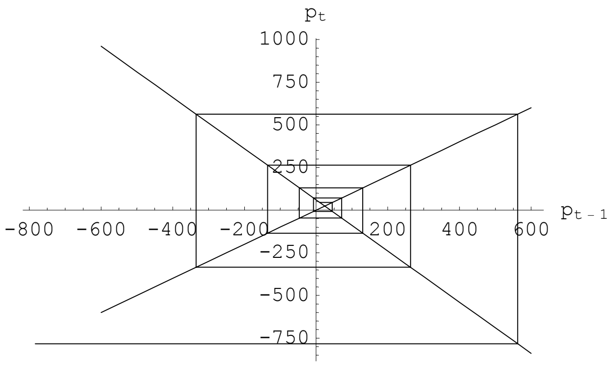
```
In[2]:= Clear[f, p, p0]
```

```
In[3]:= f[p_] := 60 - 3/2 p
```

```
In[4]:= Solve[60 - 3/2 p == p, p]
```

```
Out[4]= {{p → 24}}
```

```
In[5]:= p0 = 10

Out[5]= 10

In[6]:= points = Rest[Partition[Flatten[Transpose[
                    {NestList[f, p0, 10], NestList[f, p0, 10]}
                      ]], 2, 1]];

In[7]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[8]:= lines = Plot[{f[p], p}, {p, -600, 600}, DisplayFunction -> Identity];

In[9]:= cobweb = Show[web, lines,
            AxesLabel -> {"pt-1", "pt"}, DisplayFunction -> $DisplayFunction];
```



This illustrates an oscillatory cobweb which is divergent.

(ii)

The system is:

$$q_t^d = 5 + 2\, p_t$$
$$q_t^s = 35 + p_{t-1}$$

with initial condition $p_0 = 10$.

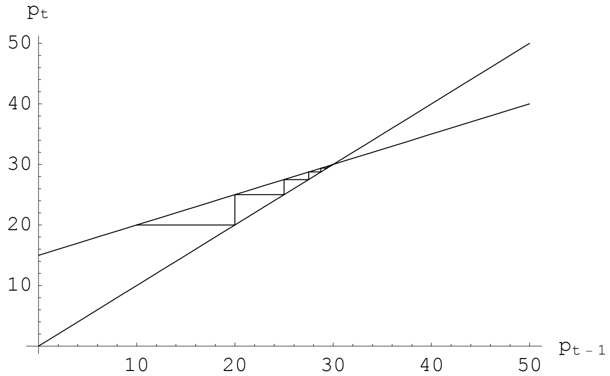
First we need to derive the difference equation. This is:

$$5 + 2\, p_t = 35 + p_t$$
$$p_t = 15 + \tfrac{1}{2}\, p_{t-1}$$

Using the instructions given in the text, we can derive the cobweb. Again we supress all intermediate graphics and just display the final cobweb.

```
In[10]:= Clear[f, p, p0]

In[11]:= f[p_] := 15 + 1/2 p

In[12]:= Solve[15 + 1/2 p == p, p]

Out[12]= {{p -> 30}}

In[13]:= p0 = 10

Out[13]= 10
```

```
In[14]:= points = Rest[Partition[Flatten[Transpose[
                    {NestList[f, p0, 10], NestList[f, p0, 10]}
                     ]], 2, 1]];

In[15]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[16]:= lines = Plot[{f[p], p}, {p, 0, 50}, DisplayFunction -> Identity];

In[17]:= cobweb = Show[web, lines,
           AxesLabel -> {"pt-1", "pt"}, DisplayFunction -> $DisplayFunction];
```



This illustrates an non-oscillatory cobweb which is convergent.

(iii)

The system is:

$q_t^d = 100 - 2\,p_t$

$q_t^s = -20 + 2\,p_{t-1}$

with initial condition $p_0 = 24$. First we need to derive the difference equation. This is

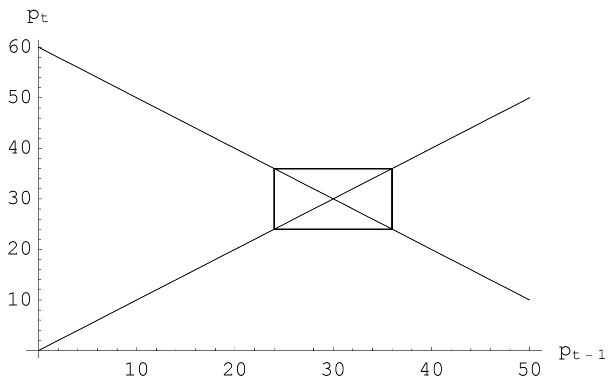
$100 - 2\,p_t = -20 + 2\,p_{t-1}$

$p_t = 60 - p_{t-1}$

Using the instructions given in the text, we can derive the cobweb. We supress all intermediate graphics and just display the final cobweb.

```
In[18]:= Clear[f, p, p0]

In[19]:= f[p_] := 60 - p

In[20]:= Solve[60 - p == p, p]

Out[20]= {{p → 30}}

In[21]:= p0 = 24

Out[21]= 24

In[22]:= points = Rest[Partition[Flatten[Transpose[
                    {NestList[f, p0, 10], NestList[f, p0, 10]}
                     ]], 2, 1]];

In[23]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[24]:= lines = Plot[{f[p], p}, {p, 0, 50}, DisplayFunction -> Identity];
```

```
In[25]:= cobweb = Show[web, lines,
           AxesLabel -> {"p_{t-1}", "p_t"}, DisplayFunction -> $DisplayFunction];
```



This illustrates an oscillatory cobweb which is cyclical.

(iv)

The system is:

$$q_t^d = 18 - 3\,p_t$$
$$q_t^s = -10 + 4\,p_{t-1}$$

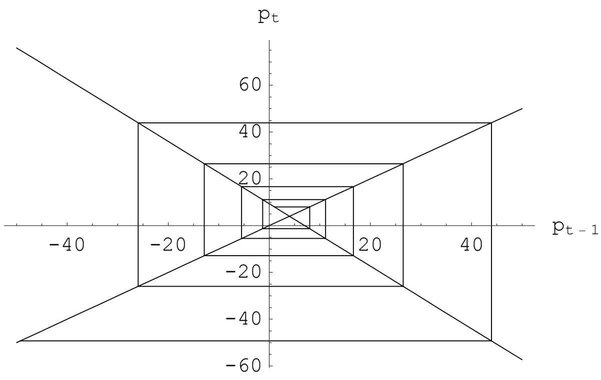
with initial condition $p_0 = 1$. First we need to derive the difference equation. This is

$$18 - 3\,p_t = -10 + 4\,p_{t-1}$$
$$p_t = \frac{28}{3} - \frac{4}{3}\,p_{t-1}$$

Using the instructions given in the text, we can derive the cobweb. We supress all intermediate graphics and just display the final cobweb.

```
In[26]:= Clear[f, p, p0]

In[27]:= f[p_] := 28/3 - 4/3 p

In[28]:= Solve[28/3 - 4/3 p == p, p]

Out[28]= {{p → 4}}

In[29]:= p0 = 1

Out[29]= 1

In[30]:= points = Rest[Partition[Flatten[Transpose[
                    {NestList[f, p0, 10], NestList[f, p0, 10]}
                    ]], 2, 1]];

In[31]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[32]:= lines = Plot[{f[p], p}, {p, -50, 50}, DisplayFunction -> Identity];
```

```
In[33]:= cobweb = Show[web, lines,
           AxesLabel -> {"pt-1", "pt"}, DisplayFunction -> $DisplayFunction];
```



This illustrates an oscillatory cobweb which is divergent.

à Question 3

For the general model

$$q_t^d = a - b\,p_t \quad b > 0$$
$$q_t^s = c + d\,p_t^e \quad d > 0$$
$$q_t^d = q_t^s = q$$
$$p_t^e = p_{t-1}^e - \lambda(p_{t-1}^e - p_{t-1}) \quad 0 \leqslant \lambda \leqslant 1$$

we have shown in the text that the solution to this system is

$$p_t = \lambda\left(\frac{a-c}{b}\right) + \left[1 - \lambda - \left(\frac{\lambda d}{b}\right)\right] p_{t-1}$$

We can deal with each problem by substituting the parameter values and then solving the difference equations.

(i)

```
In[34]:= Clear[f, p, p0]

In[35]:= const = λ ((a - c)/b)

Out[35]= (a - c) λ / b

In[36]:= coef = (1 - λ - (λ d / b))

Out[36]= 1 - λ - d λ / b

In[37]:= const1 = const /. {a -> 100, b -> 2, c -> -20, d -> 3, λ -> 0.5}

Out[37]= 30.

In[38]:= coef1 = coef /. {a -> 100, b -> 2, c -> -20, d -> 3, λ -> 0.5}

Out[38]= -0.25

In[39]:= RSolve[{p[t] == 30 - 0.25 p[t - 1], p[0] == 10}, p[t], t]

Out[39]= {{p[t] → -4. (3.5 (-0.25)^t - 6. 1.^t)}}
```

In other words,

$$p(t) = 24 - 14\,(-0.25)^t$$

(ii)

```
In[40]:= const2 = const /. {a -> 5, b -> -2, c -> 35, d -> 1, λ -> 0.2}

Out[40]= 3.

In[41]:= coef2 = coef /. {a -> 5, b -> -2, c -> 35, d -> 1, λ -> 0.2}

Out[41]= 0.9

In[42]:= RSolve[{p[t] == 3 + 0.9 p[t - 1], p[0] == 10}, p[t], t]

Out[42]= {{p[t] → 1.11111 (-18. 0.9^t + 27. 1.^t)}}
```

In other words,

$$p(t) = 30 - 20\,(0.9)^t$$

à Question 4

The stability of the systems in this question can be obtained immediately by considering the ratio $(-d/b)$, where d is the slope of the supply curve and b is the slope of the demand curve. Thus,

(i) $\frac{-d}{b} = \frac{-25}{50} = -\frac{1}{2}$ so the system is oscillatory and convergent

(ii) $\frac{-d}{b} = \frac{0.1}{0.5} = 0.2$ so the system is non-oscillatory and convergent

We can, however, investigate these systems in more detail.

(i)

Given

$$q_t^d = 250 - 50\,p_t$$
$$q_t^s = 25 + 25\,p_{t-1}$$
$$q_t^d = q_t^s = q_t$$

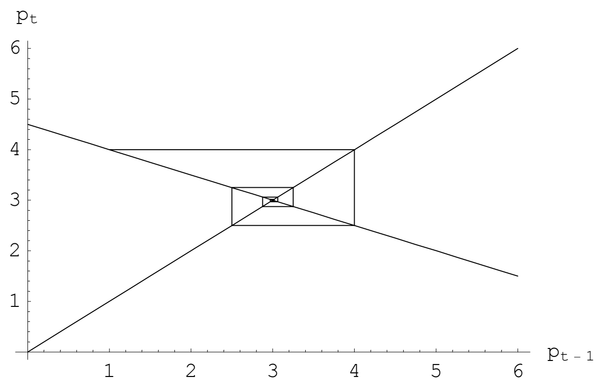
then

$$250 - 50\,p_t = 25 + 25\,p_{t-1}$$
$$p_t = 4.5 - 0.5\,p_t$$

So we can investigate the stability by deriving the cobweb for the initial price $p_0 = 1$.

```
In[43]:= Clear[f, p, p0]

In[44]:= f[p_] := 4.5 - 0.5 p

In[45]:= Solve[4.5 - 0.5 p == p, p]

Out[45]= {{p → 3.}}

In[46]:= p0 = 1

Out[46]= 1
```

```
In[47]:= points = Rest[Partition[Flatten[Transpose[
                              {NestList[f, p0, 10], NestList[f, p0, 10]}
                               ]], 2, 1]];

In[48]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[49]:= lines = Plot[{f[p], p}, {p, 0, 6}, DisplayFunction -> Identity];

In[50]:= cobweb = Show[web, lines,
            AxesLabel -> {"pt-1", "pt"}, DisplayFunction -> $DisplayFunction];
```



As pointed out above, this system is oscillatory and stable.

(ii)

Given

$q_t^d = 100 - 0.5\, p_t$

$q_t^s = 50 - 0.1\, p_{t-1}$

$q_t^d = q_t^s = q_t$

then

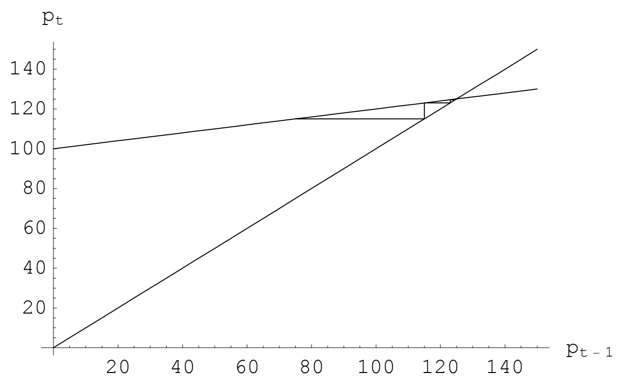
$100 - 0.5\, p_t = 50 - 0.1\, p_{t-1}$

$p_t = 100 + 0.2\, p_t$

So we can investigate the stability by deriving the cobweb for the initial price $p_0 = 75$.

```
In[51]:= Clear[f, p, p0]

In[52]:= f[p_] := 100 + 0.2 p

In[53]:= Solve[100 + 0.2 p == p, p]

Out[53]= {{p → 125.}}

In[54]:= p0 = 75

Out[54]= 75

In[55]:= points = Rest[Partition[Flatten[Transpose[
                              {NestList[f, p0, 10], NestList[f, p0, 10]}
                               ]], 2, 1]];

In[56]:= web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[57]:= lines = Plot[{f[p], p}, {p, 0, 150}, DisplayFunction -> Identity];
```

```
In[58]:= cobweb = Show[web, lines,
            AxesLabel -> {"pt-1", "pt"}, DisplayFunction -> $DisplayFunction];
```



As pointed out above, this system is non-oscillatory and convergent.

à Question 5

(i)

Given the equations

$$q_t^d = 50 - 4\, p_t$$
$$q_t^s = 10 + 10\, p_{t-1} - p_{t-1}^2$$
$$q_t^d = q_t^s = q_t$$

substituting we have

$$50 - 4\, p_t = 10 + 10\, p_{t-1} - p_{t-1}^2$$
$$p_t = 10 - \tfrac{5}{2}\, p_{t-1} + \tfrac{1}{4}\, p_{t-1}^2$$

This is a non-linear difference equation of the first order. *Mathematica* cannot solve this difference equation with the **RSolve** command.

First establish the fixed points.

```
In[59]:= Solve[p == 10 - (5 / 2) p + (1 / 4) p^2, p]

Out[59]= {{p → 4}, {p → 10}}
```

The difference equation can be linearized in the neighbourhood of the fixed point by using

$$p_t = f(p^*) + f'(p^*)(p - p^*)$$

where p^* denotes the fixed point and $f(p) = 10 - (5/2)\, p + (1/4)\, p^2$.

```
In[60]:= f[p_] := 10 - (5 / 2) p + (1 / 4) p^2

In[61]:= f'[p] /. p -> 4

Out[61]= -1/2

In[62]:= f'[p] /. p -> 10

Out[62]= 5/2
```

Thus

$p_t = 4 - \frac{1}{2}(p - 4)$ which is stable since $|f'(p^*)| < 1$

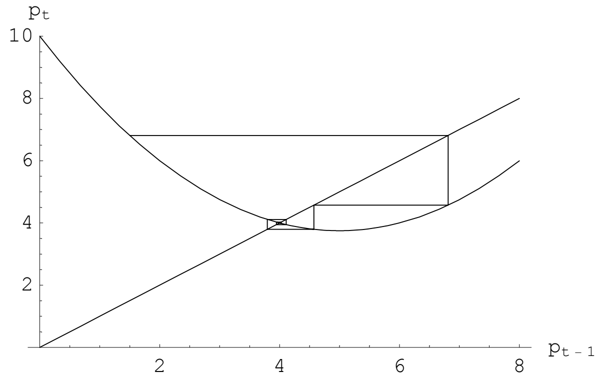
$p_t = 10 + \frac{5}{2}(p - 10)$ which is unstable since $|f'(p^*)| > 1$

Although the linear approximations can give us some insight about stability and instability, there is no guarantee that the non-linear system may lead to a cyclical solution. To see what might be taking place around each equilibria, we shall utilise the procedure for constructing non-linear cobwebs. In doing this we shall suppress all intermediate output.

Consider first the fixed point $p^* = 4$.

```
In[63]:= Clear[f, p, p0, points, web, lines]
```

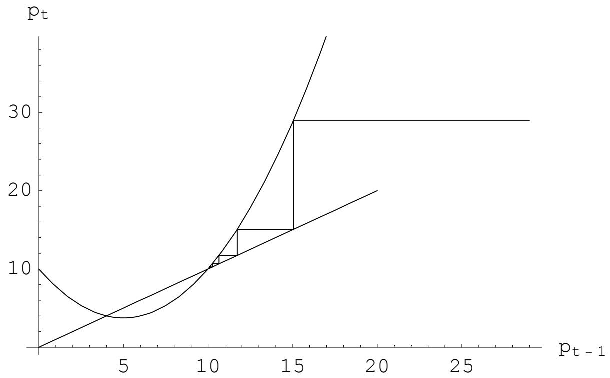
```
In[64]:= f[p_] := 10 - (5 / 2) p + (1 / 4) p^2
         p0 = 1.5;
         points = Rest[Partition[Flatten[Transpose[
                                     {NestList[f, p0, 20], NestList[f, p0, 20]}]],
                       2, 1]];
         web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];
         lines = Plot[{f[p], p}, {p, 0, 8}, DisplayFunction -> Identity];
         cobweb = Show[web, lines,
                  AxesLabel -> {"p_{t-1}", "p_t"},
                  PlotRange -> {0, 10},
                  DisplayFunction -> $DisplayFunction];
```



Consider next the fixed point $p^* = 10$.

```
In[70]:= Clear[f, p, p0, points, web, lines]
```

```
In[71]:= f[p_] := 10 - (5 / 2) p + (1 / 4) p^2
         p0 = 10.1;
         points = Rest[Partition[Flatten[Transpose[
                               {NestList[f, p0, 5], NestList[f, p0, 5]}]],
                    2, 1]];
         web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];
         lines = Plot[{f[p], p}, {p, 0, 20}, DisplayFunction -> Identity];
         cobweb = Show[web, lines,
                 AxesLabel -> {"p_(t-1)", "p_t"},
                 DisplayFunction -> $DisplayFunction];
```



(ii)

Given the equations

$$q_t^d = 50 - 4\, p_t$$
$$q_t^s = 2 + 10\, p_{t-1} - p_{t-1}^2$$
$$q_t^d = q_t^s = q_t$$

substituting we have

$$50 - 4\, p_t = 2 + 10\, p_{t-1} - p_{t-1}^2$$
$$p_t = 12 - \tfrac{5}{2}\, p_{t-1} + \tfrac{1}{4}\, p_{t-1}^2$$

This is a non-linear difference equation of the first order. *Mathematica* cannot solve this difference equation with the **RSolve** command.

First establish the fixed points.

```
In[77]:= Solve[p == 12 - (5 / 2) p + (1 / 4) p^2, p]

Out[77]= {{p → 6}, {p → 8}}

In[78]:= Clear[f]

In[79]:= f[p_] := 12 - (5 / 2) p + (1 / 4) p^2

In[80]:= f'[p] /. p -> 6

Out[80]= 1/2

In[81]:= f'[p] /. p -> 8

Out[81]= 3/2
```

Thus

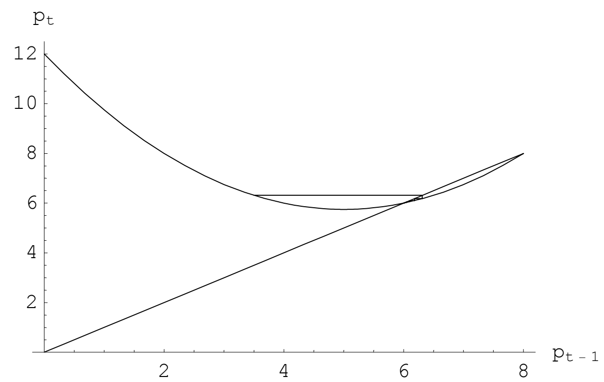
$p_t = 6 + \frac{1}{2}(p - 6)$ which is stable since $|f'(p^*)| < 1$

$p_t = 8 + \frac{3}{2}(p - 8)$ which is unstable since $|f'(p^*)| > 1$

Again, let us investigate the non-linear system in the neighbourhood of each equilibrium value. Take first $p^* = 6$.

```
In[82]:= Clear[f, p, p0, points, web, lines]
```

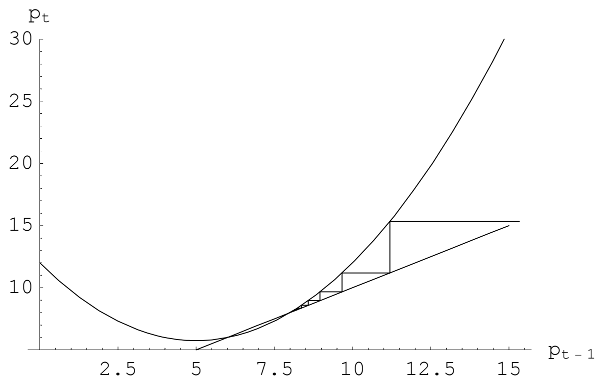
```
In[83]:= f[p_] := 12 - (5 / 2) p + (1 / 4) p^2
         p0 = 3.5;
         points = Rest[Partition[Flatten[Transpose[
                                {NestList[f, p0, 20], NestList[f, p0, 20]}]],
                   2, 1]];
         web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];
         lines = Plot[{f[p], p}, {p, 0, 8}, DisplayFunction -> Identity];
         cobweb = Show[web, lines,
                 AxesLabel -> {"p_{t-1}", "p_t"},
                 PlotRange -> {0, 12.5},
                 DisplayFunction -> $DisplayFunction];
```



Take next the fixed point $p^* = 8$.

```
In[89]:= Clear[f, p, p0, points, web, lines]
```

```
In[90]:= f[p_] := 12 - (5 / 2) p + (1 / 4) p^2
         p0 = 8.1;
         points = Rest[Partition[Flatten[Transpose[
                                 {NestList[f, p0, 8], NestList[f, p0, 8]}]],
                     2, 1]];
         web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];
         lines = Plot[{f[p], p}, {p, 0, 15}, DisplayFunction -> Identity];
         cobweb = Show[web, lines,
                 AxesLabel -> {"p_{t-1}", "p_t"},
                 PlotRange -> {5, 30},
                 DisplayFunction -> $DisplayFunction];
```



Question 6

Although this, and the next few questions suggest using a spreadsheet, we shall employ *Mathematica* to do the same job.

(i)

First we need to solve the system,

$$q_t^d = 52 - 9\,p_t$$
$$q_t^s = 3 + 5\,p_{t-1}$$
$$q_t^d = q_t^s = q_t$$

for the equilibrium price and quantity. Substituting we obtain the following first-order difference equation:

$$52 - 9\,p_t = 3 + 5\,p_{t-1}$$
$$p_t = \frac{49}{9} - \frac{5}{9}\,p_{t-1}$$

```
In[96]:= Solve[p == (49 / 9) - (5 / 9) p, p]
```

Out[96]= $\left\{\left\{p \to \frac{7}{2}\right\}\right\}$

```
In[97]:= qstar = 52 - 9 p /. p -> 7 / 2
```

Out[97]= $\frac{41}{2}$

The equilibrium price and quantity are $p^* = 3.5$ and $q^* = 20.5$.

(ii)

```
In[98]:= p0 = 3.5 - 0.1 * (3.5)

Out[98]= 3.15
```

This starting price was used to compile the following data. The data file is called "ch08q06.txt" which was saved from the spreadsheet using the text format.We need to inform *Mathematica* that the data is a series of numbers, also we want to preserve the list structure of the data. We do this by using the command RecordLists->True. Finally we call the input list dataqu06. Thus,

```
In[99]:= dataqu06 = ReadList[
           "e:\EconDynamics 2nd ed Manuals\Mathematica 4\Notebooks\ch08q06.txt",
           Number, RecordLists -> True];

In[100]:= TableForm[dataqu06, TableHeadings -> {{}, {"t", "p(t)", "% dev"}}]

Out[100]//TableForm=
```

t	p(t)	% dev
0	3.15	-10.
1	3.69	5.56
2	3.39	-3.09
3	3.56	1.71
4	3.47	-0.95
5	3.52	0.53
6	3.49	-0.29
7	3.51	0.16
8	3.5	-0.09
9	3.5	0.05
10	3.5	-0.03
11	3.5	0.02
12	3.5	-0.01
13	3.5	0.
14	3.5	0.
15	3.5	0.
16	3.5	0.
17	3.5	0.
18	3.5	0.
19	3.5	0.
20	3.5	0.
21	3.5	0.
22	3.5	0.
23	3.5	0.
24	3.5	0.
25	3.5	0.

It is readily seen from this table that by period 4 the price is within 1% of the equilibrium price.

à Question 7

Equation 8.16 of the text, p.336 gave the general result

$$p_t = \lambda(\frac{a-c}{b}) + [1 - \lambda - (\frac{\lambda d}{b})]\, p_{t-1}$$

Substituting the values $a = 52$, $b = 9$, $c = 3$ and $d = 5$, then

$$p_t = \lambda(\frac{52-3}{9}) + (1 - \lambda - \frac{5\lambda}{9})\, p_{t-1}$$
$$= \frac{49\lambda}{9} + (1 - \frac{14\lambda}{9})\, p_{t-1}$$

We can first check this result by obtaining the equilibrium and showing that it is independent of λ.

```
In[101]:= Solve[p == 49 λ/9 + (1 - 14 λ/9) p, p]

Out[101]= {{p → 7/2}}
```

(i)-(iv)

Using the equation for p_t and the initial value $p_0 = 3.15$ we obtain the following data from a spreadsheet (where we concentrate only on the deviations from equilibrium):

```
In[102]:= dataqu07 =
           ReadList[
            "e:\EconDynamics 2nd ed Manuals\Mathematica 4\Notebooks\ch08q07.txt",
            Number, RecordLists -> True];

In[103]:= TableForm[dataqu07,
           TableHeadings -> {{}, {"t", "% dev(λ=1)", "% dev(λ=0.75)",
              "% dev (λ=0.5)", "% dev(λ=0.25)"}}]

Out[103]//TableForm=
```

t	% dev(λ=1)	% dev(λ=0.75)	% dev (λ=0.5)	% dev(λ=0.25
0	-10	-10	-10	-10
1	5.56	1.67	-2.22	-6.11
2	-3.09	-0.28	-0.49	-3.73
3	1.71	0.05	-0.11	-2.28
4	-0.95	-0.01	-0.02	-1.39
5	0.53	0	-0.01	-0.85
6	-0.29	0	0	-0.52
7	0.16	0	0	-0.32
8	-0.09	0	0	-0.19
9	0.05	0	0	-0.12
10	-0.03	0	0	-0.07
11	0.02	0	0	-0.04
12	-0.01	0	0	-0.03
13	0	0	0	-0.02
14	0	0	0	-0.01
15	0	0	0	-0.01
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0
19	0	0	0	0
20	0	0	0	0

From the table we have the following periods when the price series comes within 1% of the equilibrium price:

For λ=1, period 4
For λ=0.75, period 2
For λ=0.5, period 2
For λ=0.25, period 5

à Question 8

(i)

Given

$$q^d(t) = 250 - 50\,p(t) - 2\,p'(t)$$

$$q^s(t) = 25 + 25\,p(t)$$

From these equations we can solve for $p'(t)$ as follows:

```
In[104]:= Solve[250 - 50 p[t] - 2 p'[t] == 25 + 25 p[t], p'[t]]

Out[104]= {{p'[t] → -75/2 (-3 + p[t])}}
```

and hence solve for $p(t)$.

```
In[105]:= DSolve[p'[t] == -75/2 (-3 + p[t]), p[t], t]

Out[105]= {{p[t] → 3 + e^(-75 t/2) C[1]}}
```

(ii)

We can establish the stability by considering the limit of this equation.

```
In[106]:= Limit[3 + E^(-75 t/2) C[1], t -> ∞]

Out[106]= 3
```

Hence the market is stable

à Question 9

(i)

Equating demand and supply and solving for w_t we get

```
In[107]:= Solve[42 - 4 w[t] == 2 + 6 w[t - 1], w[t]]

Out[107]= {{w[t] → 1/2 (20 - 3 w[-1 + t])}}

In[108]:= Solve[w == (1 / 2) (20 - 3 w), w]

Out[108]= {{w → 4}}
```

(ii)

We have just established that

$$w_t = 10 - 1.5\,w_{t-1}$$

and at the minimum wage of w=3, we have

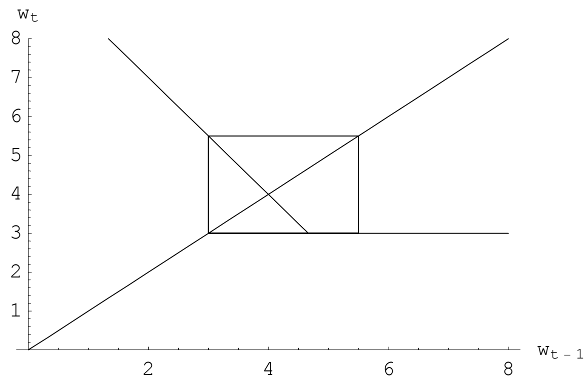
```
In[109]:= Solve[3 == 10 - (3 / 2) w, w]

Out[109]= {{w → 14/3}}
```

Hence we have the equation

$$f(w) = 10 - 1.5\,w \qquad w \le 14/3$$

$$= 3 \qquad w > 14/3$$

```
In[110]:= Clear[f]

In[111]:= f[w_] := If[w > 14 / 3, 3, 10 - 1.5 w]

In[112]:= w0 = 5

Out[112]= 5

In[113]:= points = Rest[Partition[Flatten[Transpose[
                                 {NestList[f, w0, 8], NestList[f, w0, 8]}]],
                   2, 1]];
          web = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];
          lines = Plot[{f[w], w}, {w, 0, 8}, DisplayFunction -> Identity];
          cobweb = Show[web, lines,
                 AxesLabel -> {"w_{t-1}", "w_t"},
                 PlotRange -> {0, 8},
                 DisplayFunction -> $DisplayFunction];
```



à Question 10

This is a purely theoretical question, but we can approach it in the following manner. Let the demand and supply system be given by:

$$q_t^d = a - b\,p_t$$
$$q_t^s = c + d\,p_{t-1}$$
$$q_t^d = q_t^s = q_t$$

in equilibrium

$$a - b\,p_t = c + d\,p_{t-1}$$
$$p_t = (a-c) + (\tfrac{d}{-b})\,p_{t-1}$$

with solution

```
In[117]:= Clear[p, p0]

In[118]:= RSolve[{p[t] == (a - c) + (-d / b) p[t - 1], p[0] == p0}, p[t], t]
```

Out[118]= $\left\{\left\{p[t] \to \frac{b^{-t}\,(-b^{1+t}\,c + a\,b\,(b^t - (-1)^t\,d^t) + (-1)^t\,d^{1+t}\,p0 + (-1)^t\,b\,d^t\,(c+p0))}{b+d}\right\}\right\}$

In[119]:= solp = $\frac{1}{b+d}\,(b^{-t}\,(-b^{1+t}\,c + a\,b\,(b^t - (-1)^t\,d^t) + (-1)^t\,d^{1+t}\,p0 + (-1)^t\,b\,d^t\,(c+p0)))$

Out[119]= $\frac{b^{-t}\,(-b^{1+t}\,c + a\,b\,(b^t - (-1)^t\,d^t) + (-1)^t\,d^{1+t}\,p0 + (-1)^t\,b\,d^t\,(c+p0))}{b+d}$

In[120]:= **Apart[solp]**

Out[120]= $-\frac{b\,(-a+c)}{b+d}-\frac{(-1)^t\,b^{-t}\,d^t\,(a\,b-b\,c-b\,p0-d\,p0)}{b+d}$

So the solution for $p(t)$ can be written

$$p(t)=\frac{a\,b-b\,c}{b+d}+\left(-\frac{d}{b}\right)^t\left(p_0-\frac{a\,b-b\,c}{b+d}\right)$$

If $|b|>|d|$ then $|\frac{d}{b}|<1$and $0<\frac{-d}{b}<1$ so the system converges on the equilibrium in an oscillatory fashion. The greater the absolute value of the demand curve relative to the supply curve, the larger the parameter b and the smaller $(-b/d)$, hence the smaller the oscillations around the equilibrium and so the sooner equilibrium is reached.

à Question 11

In[121]:= **Clear[p, p0]**

In[122]:= **Solve[10 - 2 p[t] == 4 + 2 p[t - 1], p[t]]**

Out[122]= {{p[t] → 3 - p[-1 + t]}}

In[123]:= **Solve[pstar == 3 - pstar, pstar]**

```
General::spell1 :
 Possible spelling error: new symbol name "pstar" is similar to existing symbol "qstar".
```

Out[123]= $\{\{\text{pstar}\to\frac{3}{2}\}\}$

With difference equation,

$$p_t=1.5+(-1)^t\,(p-1.5)$$

The periodicity and the periodic values are readily established using *Mathematica*'s RSolve command.

In[124]:= **RSolve[{p[t] == 3 - p[t - 1], p[0] == p0}, p[t], t]**

Out[124]= {{p[t] → If[Even[t], p0, 3 - p0]}}

à Question 12

First let us establish the fixed points of the system. Equating demand and supply and solving for equilibrium p we find,

In[125]:= **Solve[4 - 3 p == p^2, p]**

Out[125]= {{p → -4}, {p → 1}}

Since a negative price is ruled out, we consider only $p^*=1$.

Before considering the cobweb version of this non-linear model, consider the stability of the linear approximation. First we need to obtain the difference equation for p.

In[126]:= **Solve[4 - 3 p[t] == p^2[t - 1], p[t]]**

Out[126]= $\{\{p[t]\to\frac{1}{3}\,(4-p^2[-1+t])\}\}$

Thus, $p_t = f(p_{t-1}) = \frac{4}{3} - \frac{1}{3}\, p_{t-1}^2$ and $f'(p) = -\frac{2}{3}\, p$, with $f'(p^* = 1) = -\frac{2}{3}$. Since $|f'(p^*)| < 1$, then the system is stable. The linear approximation is given by:

$$p_t = 1 - \frac{2}{3}\,(p_{t-1} - 1)$$

which can be expressed,

$$p_t = \frac{5}{3} - \frac{2}{3}\, p_{t-1}.$$

```
In[127]:= RSolve[{p[t] == 5/3 - 2/3 p[t - 1], p[0] == p0}, p[t], t]

Out[127]= {{p[t] → 1 - (-2/3)^t + (2/3)^t e^(i π t) p0}}
```

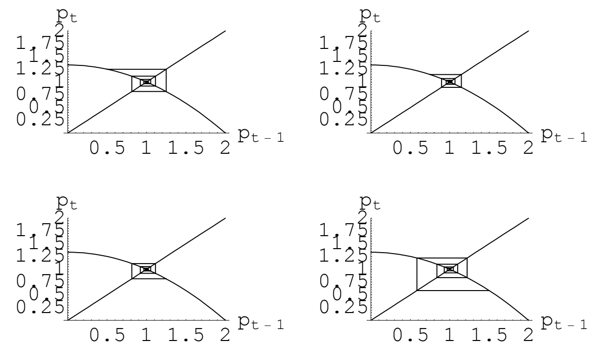
This is one of those occasions when *Mathematica*'s output is more complex than it need be, and it reduces down to the simple result

$$p_t = 1 + \left(-\frac{2}{3}\right)^t (\mathrm{p0} - 1)$$

```
In[128]:= Clear[f, p0]

In[129]:= f[p_] := 4/3 - 1/3 p^2
          p0 = 0.5;
          points1 = Rest[Partition[Flatten[Transpose[
                                   {NestList[f, p0, 8], NestList[f, p0, 8]}]],
                      2, 1]];
          web1 = ListPlot[points1, PlotJoined -> True, DisplayFunction -> Identity];
          lines1 = Plot[{f[p], p}, {p, 0, 2}, DisplayFunction -> Identity];
          cobweb1 = Show[web1, lines1,
                  AxesLabel -> {"p_(t-1)", "p_t"},
                  PlotRange -> {0, 2},
                  DisplayFunction -> Identity];

In[135]:= p0 = 0.75;
          points2 = Rest[Partition[Flatten[Transpose[
                                   {NestList[f, p0, 8], NestList[f, p0, 8]}]],
                      2, 1]];
          web2 = ListPlot[points2, PlotJoined -> True, DisplayFunction -> Identity];
          lines2 = Plot[{f[p], p}, {p, 0, 2}, DisplayFunction -> Identity];
          cobweb2 = Show[web2, lines2,
                  AxesLabel -> {"p_(t-1)", "p_t"},
                  PlotRange -> {0, 2},
                  DisplayFunction -> Identity];

In[140]:= p0 = 1.25;
          points3 = Rest[Partition[Flatten[Transpose[
                                   {NestList[f, p0, 8], NestList[f, p0, 8]}]],
                      2, 1]];
          web3 = ListPlot[points3, PlotJoined -> True, DisplayFunction -> Identity];
          lines3 = Plot[{f[p], p}, {p, 0, 2}, DisplayFunction -> Identity];
          cobweb3 = Show[web3, lines3,
                  AxesLabel -> {"p_(t-1)", "p_t"},
                  PlotRange -> {0, 2},
                  DisplayFunction -> Identity];
```

In[145]:=
```
p0 = 1.5;
points4 = Rest[Partition[Flatten[Transpose[
                    {NestList[f, p0, 8], NestList[f, p0, 8]}]],
          2, 1]];
web4 = ListPlot[points4, PlotJoined -> True, DisplayFunction -> Identity];
lines4 = Plot[{f[p], p}, {p, 0, 2}, DisplayFunction -> Identity];
cobweb4 = Show[web4, lines4,
      AxesLabel -> {"pt-1", "pt"},
      PlotRange -> {0, 2},
      DisplayFunction -> Identity];
```

In[150]:=
```
Show[GraphicsArray[{{cobweb1, cobweb2}, {cobweb3, cobweb4}}]];
```



à Question 13

Equation 8.16 of the text, p.336 gave the general result

$$p_t = \lambda(\frac{a-c}{b}) + [1 - \lambda - (\frac{\lambda d}{b})]\, p_{t-1}$$

Substituting the values $a = 24$, $b = 5$, $c = -4$ and $d = 2$, then

$$p_t = \lambda(\frac{24+4}{5}) + (1 - \lambda - \frac{2\lambda}{5})\, p_{t-1}$$
$$= \frac{28\lambda}{5} + (1 - \frac{7\lambda}{5})\, p_{t-1}$$

We can first check this result by obtaining the equilibrium and showing that it is independent of λ.

In[151]:=
```
Solve[p == 28 λ/5 + (1 - 7 λ/5) p, p]
```

Out[151]=
```
{{p → 4}}
```

In[152]:=
```
Clear[p, path1, path2, path3, path4]
```

In[153]:=
```
path1[p_] := 28 λ/5 + (1 - 7 λ/5) p /. λ -> 0.25
```

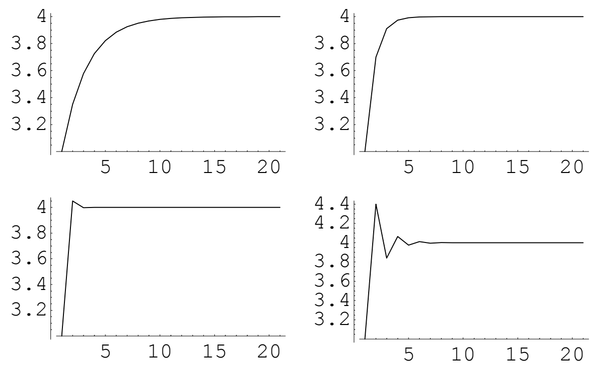
In[154]:=
```
path2[p_] := 28 λ/5 + (1 - 7 λ/5) p /. λ -> 0.5
```

In[155]:=
```
path3[p_] := 28 λ/5 + (1 - 7 λ/5) p /. λ -> 0.75
```

In[156]:=
```
path4[p_] := 28 λ/5 + (1 - 7 λ/5) p /. λ -> 1
```

In[157]:=
```
graph1 = ListPlot[NestList[path1, 3, 20], PlotJoined -> True,
    AxesOrigin -> {0, 3}, PlotRange -> All, DisplayFunction -> Identity];
```

```
In[158]:= graph2 = ListPlot[NestList[path2, 3, 20], PlotJoined -> True,
              AxesOrigin -> {0, 3}, PlotRange -> All, DisplayFunction -> Identity];

In[159]:= graph3 = ListPlot[NestList[path3, 3, 20], PlotJoined -> True,
              PlotRange -> All, AxesOrigin -> {0, 3}, DisplayFunction -> Identity];

In[160]:= graph4 = ListPlot[NestList[path4, 3, 20], PlotJoined -> True,
              PlotRange -> All, AxesOrigin -> {0, 3}, DisplayFunction -> Identity];

In[161]:= Show[GraphicsArray[{{graph1, graph2}, {graph3, graph4}}]];
```



à Question 14

(i)

The linear approximation is given by:

$$\dot{P} = r(P - P^*) - R'(h^*)\,(h - h^*)$$
$$\dot{h} = g'(P^*)\,(P - P^*) - (d + n)\,(h - h^*)$$

(ii)

Given the assumptions, then

$$\dot{P} = 0.05\,(P - P^*) + 0.5\,(h - h^*)$$
$$\dot{h} = (P - P^*) - 0.03\,(h - h^*)$$

Assume $P^* = 1$ and $h^* = 1$, then if $\dot{P} = 0$ and $\dot{h} = 0$,

```
In[162]:= Solve[0.05 (P - 1) + 0.5 (h - 1) == 0, P]

Out[162]= {{P → -20. (-0.05 + 0.5 (-1. + h))}}

In[163]:= ExpandAll[-20.` (-0.05` + 0.5` (-1.` + h))]

Out[163]= 11. - 10. h

In[164]:= Solve[(P - 1) - 0.03 (h - 1) == 0, P]

Out[164]= {{P → 1. + 0.03 (-1. + h)}}

In[165]:= ExpandAll[1.` + 0.03` (-1.` + h)]

Out[165]= 0.97 + 0.03 h
```

(iii)

```
In[166]:= matrixA = (0.05  0.5
                     1    -0.03)

Out[166]= {{0.05, 0.5}, {1, -0.03}}

In[167]:= Eigenvalues[matrixA]

Out[167]= {0.718237, -0.698237}
```

Because eigenvalues r and s are of opposite sign, then we have a saddle point solution. This is also verified by the fact that the determinant is negative, thus

```
In[168]:= Det[matrixA]

Out[168]= -0.5015

In[169]:= Eigenvectors[matrixA]

Out[169]= {{0.599096, 0.800677}, {-0.555604, 0.831447}}
```

à Question 15

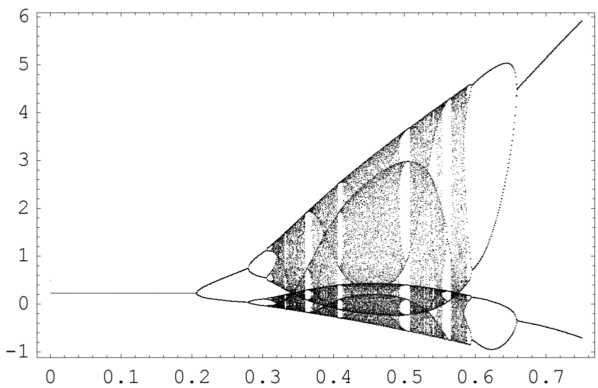
The equation we wish to investigate is

$$p_{t+1}^e = (1-\lambda)\, p_t^e + \frac{\lambda\, a}{b} - \frac{\lambda \arctan(\mu\, p_t^e)}{b}$$

with $a = 0.8$, $b = 0.25$ and $\mu = 4$.

```
In[170]:= Clear[λ]

In[171]:= f[pe_] := (1 - λ) pe + λ 0.8/0.25 - λ ArcTan[4 pe]/0.25

In[172]:= f[pe]

Out[172]= pe (1 - λ) + 3.2 λ - 4. λ ArcTan[4 pe]
```

```
In[173]:= ListPlot[
              Flatten[Table[
                    Transpose[ {
                          Table[λ, {129}],
                          NestList[f, Nest[f, 0.5, 500], 128]
                    }],
                 {λ, 0, 0.75, 0.001}
            ], 1],
            PlotStyle -> PointSize[0.001], Axes -> False, Frame -> True];
```



When λ has a value around 0.15 the model exhibits a stable equilibrium expected price. As λ increases infinitely many doubling bifurcations occur, eventually reaching chaotic behaviour. However, as λ continues to increase there are many period halving bifurcations arising and expected price behaviour takes on a more regular pattern once again. At λ around 0.75 the system settles down to a stable two-period cycle. Also note that as λ increases so does the amplitude of the expected price oscillations.