**Fig. 1.5** (a) Comparison of various reanalysis fields for January 1

2006, 0Z, ECMWF interim. This shows the specific humidity (top left),

temperature (top right), zonal, meridional wind (middle left and right),

and vertical wind and geopotential height (bottom left and right).

All fields are at 700 mb. Reproduced from Lovejoy and Schertzer

(2011). (b) Comparisons of the spectra of different atmospheric fields

from the ECMWF interim reanalysis. Top (red) is the geopotential (β = 3.35),

second from the top (green) is the zonal wind (β= 2.40), third from the top(cyan) is the meridional wind (β = 2.40), fourth from the top (blue) is the temperature (β= 2.40), fifth fromthe top (orange) is the vertical wind (β= 0.4), at the bottom (purple) is the specific humidity (β = 1.6). All are at 700mb and between ±45o latitude, everyday in2006 at 0GMT. The scale at the far left corresponds to 20 000 km in the east–west direction, at the far right to 660 km. Note that for these 2D spectra, Gaussian white noise would yield β = –1 (i.e. a positive slope = +1). Reproduced from Lovejoy and Schertzer (2011).

**Fig. 1.6** (a) Aircraft temperature spectra. Red slopes are 1.9,

black 1.7. The bottom three curves are averages of 10 samples and

each curve is taken at roughly a one-year interval; the top curve is the

overall ensemble average. The curves are displaced in the vertical for

clarity. Adapted from Chigirinskaya et al. (1994).

**Fig. 1.8** (a) An example of a 3D drop reconstruction. For clarity only

the 10% largest drops are shown, only the relative sizes and positions of

the drops are correct, the colours code the size of the drops. The

boundaries are defined by the flash lamps used for lighting the drops

and by the depth of field of the photographs. Adapted from Lovejoy

and Schertzer (2008). See colour plate section. (b) The angle averaged

drop spectra for five storms, 18 image triplets and, for reference,

Corssin–Obukhov passive scalar theory lines (rain has statistics like

tracer). This shows the 3D isotropic (angle-integrated) spectrum of the

19 stereophotographic drop reconstructions for , the particle mass

density. Each of the five storms had 3–7 “scenes” (from matched

stereographic triplets) with 5000 - 40 000 drops, each taken over a

15–30 minute period. (orange = f207, yellow = f295, green = f229, blue-green = f142 and cyan = f145; the numbers refer to the different storms). The data were taken from regions roughly 4.4 x4.4 x 9.2 m3 in extent (slight changes in the geometry were made between storms). The region was broken into 1283 cells (3.4 x 3.4 x 7.2 cm3, geometric mean = 4.4 cm); we used the approximation that the extreme low wavenumber (log10k = 0) corresponds to the geometric mean (i.e. 5.6 m, the minimum in the plot corresponds to

about 40–70 cm). The single lowest wavenumbers (k = 1) are not

shown since the largest scales are nonuniform due to poor lighting and

focus on the edges. The reference lines have slopes -5/3, +2, i.e. the

theoretical values for the Corrsin–Obukhov (l1/3) law and white noise,

respectively. Adapted from Lovejoy and Schertzer (2008).

**Fig. 1.9** (a) The scaling of hourly surface temperatures from four

stations in the northwest USA, for four years (2005–2008) from the

US Climate Reference Network. The data are discussed more fully

in Section 8.1.2. To reduce noise the data were divided into sections

of 112 days and the 48 spectra averaged (the rise at the extreme

low frequency is connected with the annual cycle; see Fig. 8.3c for

the full four-year spectra and discussion of detrending). One can see

that in spite of the strong diurnal cycle (and harmonics) the basic

scaling extends to about 7 days. The reference lines (with absolute

slopes 0.2, 2) are theoretically motivated: see Chapter 10. (b) 1D

spectra from the thermal infrared over the Pacific Ocean (MTSAT).

Analyzed in time (with diurnal peak, blue), in the east–west direction

(bottom at right, pink), and in the north–south direction (orange). Units are such that the highest wavenumber is (60 km)-1 and highest frequency is

(2 hours)-1 (i.e. the Nyquist wavenumber and frequency of data at

30 km and 1 hour resolutions). In Chapter 8 we show that the lowfrequency/

wavenumber curvature is an artefact of the finite geometry of the MTSAT scene coupled with some horizontal and space-time anisotropy. The reference line has slope =1.5. Reproduced from Pinel (2012).

**Fig. 2.11** A comparison of the GASP and MOZAIC spectra from

commercial aircraft flying on isobars, adapted from Skamarock

(2004), reproduced from Lovejoy et al. (2010). The thick (red) lines show

the behaviour predicted if the atmosphere has a perfect k-5/3

horizontal spectrum but estimated from an aircraft following

roughly horizontal trajectories until about 100 km (indicated by

the arrows) and then following gradually sloping trajectories

(either on isobars or gradual changes in altitude due to fuel

consumption).

**Fig. 2.15** (a) A contour plot of the mean squared transverse (top) and longitudinal (bottom) components of the wind, as estimated by a year’s (≈14500) TAMDAR flights, 484 000 wind difference measurements. All the fluctuations were from a single aircraft at different parts of its trajectory, and only trajectories between 5 and 5.5 km were used. Black shows the empirical contours, purple the theoretical contours assuming scaling stratification and the functional form indicated in the text. The numbers next to the contours are the values of the contours (italics is theory, bold is empirical, to improve the statistics, reflection symmetries were used). Reproduced from Pinel et al. (2012).

**Fig. 2.20** Sample spectra from WRF forecasts of zonal wind

averaged over the isobaric surfaces covering roughly the range 3–9

km in altitude, adapted from Skamarock (2004). Although they

claimed that this shows a “clear k-3 regime” for the solid (oceanic)

spectrum it only spans a range of factor 2–3 in scale, and this at the

relatively unreliable extreme low wavenumbers (between the

downpointing arrows, upper left). Except for the extremes, the

spectra again follow the isobaric predictions k-2.4 (red) very well over

most of the range. Reproduced from Lovejoy et al. (2010).

**Fig. 2.21a**: Third order structure functions (diagonal contributions) adapted from Plate 1 of (Cho and Lindborg, 2001). Red indicates a negative sign, black, positive indicating large to small and small to large scale transfers respectively. The theoretical reference lines (green) were added. Their slopes correspond to the predictions of the sloping isobaric trajectory model presented here (section 2.6.2 and ch. 6) with the third order vertical structure function *v* (*q*) = *qH* - *K*(*q*) =1.82 (using *q* =3, *H* = 0.77, *K*(3) ≈ 0.49) and horizontal structure function with *h* (*q*) =*v* (*q*) *Hz* = 0.83 (*Hz* = 0.46; parameters from ch. 6 and (Lovejoy et al., 2010)). The transition is not far from the 40 *km* found in the Gulfstream 4 analyses (ch. 6).

**Fig. 2.23** (a) The angular integral of T(k) for each year: 2005 (green), 2006 (magenta, dashed), 2007 (blue, stippled). The average of the three is orange, thick, dashed which is only clearly distinguishable to the right of the 0.8 position (it’s the line that barely drops below the axis at log10k= 0.9). The thick darker lines that roughly define the envelope are the isotropic energy spectrum E(k) (top) and its negative (bottom). (b) The corresponding plots of P(k),which is the integral of T(k), over wavenumbers higher than k for each year: 2005, 2006, 2007 (same colours). The average of the three (orange, thick, dashed) is again only clearly distinguishable when its drop below the upper envelope at about log10k≈ 1.0. The thick red lines that define the envelope are the same as in Fig. 2.23a.

**Fig. 4.5** (a) The cascade exponents C1 from the spatial (zonal) analysis. From bottom to top this shows the zonal wind (u, dashed blue), the meridional wind (v, dashed blue), the temperature (T, black), the geopotential height (z, red), the vertical velocity (w, dashed, purple), the specific humidity (hs, top black). The extreme latitude bands (±75–90o) were not used, since the mean map factor is very large and the results were considered unreliable. Reproduced from Lovejoy and Schertzer (2011). (b) Same as Fig. 4.5a except for . From bottom to top this shows the specific humidity (hs black), the meridional wind (v,blue), the zonal wind (u, black, second from top left), the temperature (T, black, third from top, left), the vertical velocity (w, purple), the geopotential height (z, red). Since theoretically multifractal indices  must be ≤2, the estimates  > 2 are presumably unreliable, although error estimates are hard to obtain. Reproduced from Lovejoy and Schertzer (2011). (c) The effective external scales Leff as functions of latitude (in units of km) from the zonal cascade analyses. The dashed line is a convenient reference line, corresponding to the largest great-circle distance on the earth, 20 000 km. From bottom to top (at the extreme left) this shows the zonal wind (u, blue, bottom), the meridional wind (v, blue slightly above), the geopotential height (z, red), the temperature (T, black, third from the top, left), the specific humidity (hs,black, second from top, left) the vertical velocity (w, purple). Reproduced from Lovejoy and Schertzer (2011).

**Fig. 4A.1**: The “universal” trace moments for quasi-gaussian processes obtained from 100 realizations of an OU process with 0 = (128)-1 (grid points)-1 using 100 realizations of process 213 points long. This shows the convergence of the moments up to order 2.9 (at increments of 0.1) to the small  asymptote Log*M* =1. Although this is for an OU process, the result is essentially identical to that of a pure Gaussian white noise (and to other quasi-Gaussian processes with <4). The thick red curves are for *q* = 2, 2.9. The straight thin lines correspond to *C*1 = 0.082,  =1.79 and an outer scale of ≈10 pixels. The thick black lines compare the log-log linear fit for moments *q* =1.7 for quasi Gaussian processes (bottom) and Levy processes ( =1.8, top; moments for *q*> are infinite). Even though the probability distribution is extreme, the outer scale only increases to 20 grid points and the nondimensional *M* for the *q* =1.7 moment at the smallest scales is only about 50% larger. In order to obtain significantly stronger variability, strong long range statistical dependencies are needed.

**Fig. 5.10** (a) c(γ) estimated from the PDMS method, c(γ) ≈ –logPr/logλ (described in the text), shown for resolution degraded by factors of 2 from 280 m to ~36 km (longest to shortest curves). The figure shows aircraft altitude, pressure (upper left, right), longitudinal and transverse wind speed (lower left, right) for 24 flight legs, each 4000 points long, 280 m resolution (i.e. 1120 km). For reference, lines of slope 3 (top row) and 5 (bottom row) are given corresponding to power-law probability distributions with the given exponents. Since the bisectrix touches the curve at the point (C1, C1), we can see graphically that C1’s are typically ≈0.1 (see Chapter 4 for more precise estimates). (b) Same as Fig. 5.10a except for the thermodynamic variables temperature, potential temperature (upper left, right), humidity and equivalent potential temperature (lower left and right). The reference lines all have slopes of 5. (c) c(γ) estimated from hourly rain gauge data at Nîme, France, from 1972–1975 with the resolution degraded by factors of 2–32 hours (red 1 hour, purple 2 hours, yellow, 4 hours, green, 8 hours, blue, 16 hours, grey, 32 hours). The reference line (added) has slope 3. Adapted from Schertzer et al. (2010).

**Fig. 5.20** (a) Probability distributions of raindrop volumes nondimensionalized by dividing by the mean mass (each curve is for a different storm, brown = 207, dark green =295, blue =226, purple =142, red =145, the numbers indicate the different storms). The reference line has absolute slope qD = 5. Note that for the same data, the slightly different statistic, the total liquid water in a given volume in the scaling regime (typically about 30 cm: see Fig. 1.8b), has qD ≈ 3, and this value is theoretically predicted from a compound Poisson cascade process obeying Corrsin–Obukhov statistics (Lovejoy and Schertzer, 2006b). The figure is reproduced from Lovejoy and Schertzer (2008), where there are more details. (b) This figure shows the log10 of the probability distribution Pr(R > s) of the absolute rain rate difference DR at times one hour apart for the 13 x 21 = 273 CPC series, each 257 000 hours long (a total of 7.0 x 107 points) as a function of the log10 threshold s. The reference line is the theoretically predicted qD ≈ 3 behaviour. This is the most convincing evidence to date for power-law probability tails in rain. This is probably due to the following: (a) the hourly resolution is long enough that many of the high-frequency gauge saturation problems are not so important; (b) the gridded data involve averages from several gauges per grid point, again making the results robust; (c) the dataset is extremely large, so the power law holds over a wide range of scales and the result is clear. The units of s are hundredths of inches per hour. Adapted from Lovejoy et al. (2012).

**Fig. 5.35** (a) The popular Mexican hat wavelet (the second derivative of the Gaussian (smooth curve, red) valid for –1 < H < 2) compared with the (negative) second finite difference wavelet (solid bars representing the relative weights of δ functions, valid for 0 < H < 2)), and \_the second-order “quadratic” Haar wavelet (rectangles, red) obtained from the third difference of the running sum (i.e. where s(x) is the running sum valid for –1 < H < 2). (b) The “poor man’s” wavelet, with solid bars representing the amplitudes of Dirac δ functions (the basis of the usual difference structure function, valid for 0< H< 1), uniform dark shading showing the Haar wavelet (the basis of the Haar structure function, uniform blue shading, the second difference of the running sum, valid for –1< H< 1), and stippled (red) shading representing the wavelet used for the “tendency” structure function valid for –1< H< 0. Reproduced from Lovejoy and Schertzer (2012a).

**Fig. 5B.1:** The compensated power spectrum for 200 realizations of the  = 2, *C*1 = 0.2 process (with  = 214). The compensation is using the theoretical power law form *k*- with  = *K*(2) = (*C*1/(-1))(2-2). The bottom (brown) is the result for the pure singularity, the middle (red) *x*-1/ terms and the top (dark blue) shows the “*x*-1” method (Lovejoy and Schertzer, 2010a).

**Fig. 5B.2** (a) Compensated spectra for the d = 1 causal and acausal cases for α = 2, C1 = 0.2 averaged over 200 realizations each 214 long (cf. the comparable Fig. 5B.1) both with and without Δx\_1/α corrections discussed in the text; using the code in Appendix 5C. From top to bottom: dark green is the corrected causal spectrum, blue, the uncorrected causal, red, the corrected acausal and brown, the uncorrected acausal simulations. Reproduced from Lovejoy and Schertzer (2010a).

(b) 1D/2D (acausal) comparison for α = 2, C1 = 0.2 showing compensated spectra. Numbering the curves 1–4 bottom to top on the far right, nos. 2 (pink) and 1 (brown) are 1D acausal, compensated spectra (respectively corrected and uncorrected, 200 realizations 214 points). Nos. 4 (green) and 3 (blue) are respectively corrected and uncorrected 2D cases (each 10 realizations, 29 x 29 points). They have been shifted horizontally so that for one and two dimensions the highest wavenumbers are the same, and they have been shifted in the vertical so that the corresponding low-wavenumber parts of the spectra roughly overlap (i.e. the corrected with the corrected, and the uncorrected with the uncorrected). The vertical scale is arbitrary and the thick horizontal line is the theoretical pure power law spectrum. Reproduced from Lovejoy and Schertzer (2010a).

**Fig. 5B.3** Compensated spectra for the d = 2 causal simulations for α = 2, C1 = 0.2 averaged over 10 realizations each 29x29 both with and without the Δx-d/ corrections; using the code in Appendix 5C. In top to bottom order the curves are: the 1D spectrum of Δx-d/α corrected simulations in the t direction (green); the corresponding spectra of uncorrected simulations (blue); the 1D spectrum of Δx-d/α corrected simulations in the x direction (red); the corresponding 1D spectrum of uncorrected simulations (brown). Reproduced from Lovejoy and Schertzer (2010a).

**Fig. 5E.4** Comparison of the compensated Haar structure function (thick), the difference structure function (thin, below the axis) and the tendency structure function (thin, above the axis). The pairs of curves, top to bottom, have H = –3/10, H = –1/10, H = 1/10, H = 3/10 (green, red, orange, blue). It can be seen that the standard difference structure function has poor scaling for nearly two orders of magnitude when H = 1/10, and one order of magnitude for H = 3/10 (see Fig. 5.E.2 for quantitative estimates). Reproduced from Lovejoy and Schertzer (2012a).

**Fig. 5E.2** Regression estimates of the compensated exponents for the spectra (red), Haar structure function (q = 2, green), quadratic, q = 2 MFDFA (dashed, red), the usual difference (poor man’s) structure function (blue, q = 2, for H > 0) and the tendency structure function (q = 2, same line, for H< 0). Reproduced from Lovejoy and Schertzer (2012a).

**Fig. 6.1** (a) The vertical spectra of seven sondes showing roughly Bolgiano–Obukhov spectra (β = 11/5; dashed reference lines added); offset in the vertical for clarity. Adapted from Endlich et al. (1969).

(b) Mean absolute vertical gradients of horizontal wind (first-order structure functions) for layers of thickness increasing logarithmically, with regression lines added. The three coloured reference lines have slopes Hv = 1/3 (Kolmogorov, K), Hv = 3/5 (Bolgiano–Obukhov, BO), Hv = 1 (gravity waves, GW). The regression Hv estimates are given next to the

lines. The data for each level are offset by one order of magnitude for clarity, units m/s. Reproduced from Lovejoy et al. (2007).

**Fig. 6.2** Top left: the means and standard deviations of the H values calculated from the moduli of the vector differences in horizontal winds. The blue curves to the left of each graph in the top row are from the H values in Fig. 6.1b, i.e. from regressions over all pairs of points below the altitude indicated, estimated over the entire range of scales available (i.e. up to 12.6 km at the highest altitudes). The points are fits from individual sondes, as indicated in Fig. 6.1b. The error bars indicate the sonde-to-sonde variability. Top right: the same, but for the corresponding spectral exponents β: (nonintermittent) Kolmorogov theory yields β = 5/3, Bolgiano–Obukhov β = 11/5. The blue lines are somewhat to the left since they are weighted to be near the indicated altitude whereas the points are from data within a kilometre of the indicated altitude. Bottom left: the C1 values. Bottom right: the corresponding  values. Reproduced from Lovejoy et al. (2007).

**Fig. 6.10** The Brunt–Väisälä frequency squared (N2) as a function of altitude. The two sondes (red and blue) were released at an interval of 0.3 s, and most of the time their two traces are indistinguishable, indicating that the error in the measurement is less than the width of the lines. Reproduced from Lovejoy et al. (2008).

**Fig. 6.12** The stability of the atmosphere as determined by two dropsondes dropped about 30 m apart (indicated by pink and red transitions), using the stability criterion  where N2 is estimated using layers at 5 m thickness. The transitions from unstable (left) to stable (right) are shown as a function of altitude from the ocean (bottom) to 12 km altitude (top). Nearly the same fractal structure is found in both, showing that the fractality is not an artefact of noise. Reproduced from Lovejoy et al. (2008).

**Fig. 6.15** Comparison of vertical sampling intervals of two near-simultaneous sondes (red, blue). Notice the strong (and typical) clustering of the outages. The mean Δz is larger at high altitudes due to the lower air resistance. Reproduced from Lovejoy et al. (2009a).

**Fig. 6.25** The empirical space-space relations for the zonal wind (u, bottom, black), and meridional wind (v, top, brown) calculated using the implicit relation Eqn. (6.68). The reference lines have slopes =1/ Hy = 0.8, 1, 1/0.8. The sphero-wavenumber is where the bisectrix (middle) intersects the space-space line. Reproduced from Lovejoy and Schertzer (2011).

**Fig. 6B.1** Coherences (C, right axis) and phases (θ, left axis) of the longitudinal wind with pressure (blue) and altitude (red; the bottom oscillating curves, indicated by “p” and “z” respectively). The solid lines are coherences; those greater than 0.2–0.3 are statistically significant, and they are highly significant over most of the range. The dashed lines are phases (confidence intervals were suppressed for clarity; they are significant over most of the range). A positive phase means that the wind leads (pressure or altitude), a negative phase that it lags behind. Between about 4 and 40 km, the altitude leads the wind but the pressure lags behind: the situation is reversed at larger scales (smaller wavenumbers). The direct interpretation is that for the higher wavenumbers ((4 km)-1 > k > (40 km)-1, corresponding to time scales of 10–150 s) the aircraft autopilot and inertia cause the change in altitude, with the pressure then following the altitude. For the smaller wavenumbers (k < (40 km) -1), the situation is reversed, with the pressure changes causing the change in wind and altitude; this is presumably the regime where the aircraft tightly follows the isobars. Comparing with Fig. 2.14, we see that the spectra of the longitudinal and transverse curves, which have slopes –5/3 and –2.4, have transitions at wavenumbers approximately at the phase change scale. Adapted from Lovejoy et al. (2009b).

**Fig. 7.5** (a) A schematic diagram showing the balls associated with the canonical system with ls = 8 and: d = 1, c = –0.1, f = –0.2, e = 0.1.  = 0.2 is the smallest scale which is completely resolved by the 1x1 pixel grid,  = 6.3 is the largest scale completely resolved by the 64x256 pixel simulation region;  = 74 is the largest scale that influences the simulation region. (b) This shows the contributions from the fully resolved band (scales  to ) and the partially resolved band  to  to the total simulation;  = 1.6, C1 = 0.1 (same G as in Fig. 7.5a), the colours from blue to white indicate values from low to high.

**Fig. 7.10** (a) OK as is

**Fig. 8.1** OK as is

8.3 (d) The temperature spectrum from the average of 370 continental daily temperature series from the National Climatic Data Center (NCDC) as a function of frequency (in year-1), adapted from Pelletier and Turcotte (1997), having thick red reference lines with absolute slopes β = 0.3, 1.8; the regression values given in the original paper had β = 0.37 (the thin black line, left) and 1.37 (right) but depend somewhat on the exact frequency range chosen.

Fig. 8.4 (c) Spectra for the wind speed (ν), the maximum wind (“gusts,” vmax) and the inverse visibility (the effective extinction coefficient, k) for the same database as in Figs. 8.4a, 8.4b. The reference lines have absolute slopes β = 1, 0.2, 5/3 (left to right; the middle plateau value is theory, the latter is

close to the value in Table 4.4). Adapted from Lovejoy and Schertzer (2010). (d) A comparison of the temporal spectra of the CPC data (red, thin background, noisy curve) and the ECMWF 3-hourly dataset (black, dashed). The solid blue curve is the CPC spectrum averaged over logarithmically spaced frequency bins (10 per order of magnitude). The long thick black curve is from the 20CR at 45o N, from the full 3-hourresolution data (from 1871–2008). The transition scale from the high-frequency weather regime

and low-frequency macroweather regime is indicated by the dashed line at periods of 5 days. The axis is in units such that ω = 1 is (29 years)-1; i.e. the full length of the CPC series. There are three reference lines with absolute

slopes indicated; the value 0.08 is from the Haar structure function analysis (Fig. 10.14) from 3 months to 29 years. Reproduced from Lovejoy et al. (2012).

**Fig. 8.5** (c) The variation of w as a function of latitude as estimated from the 138-year-long 20CR reanalyses. The estimates were made by performing bilinear regressions on spectra from 180-day-long series averaged over 280 segments per grid point. The top wide and thick curve (red) shows the mean over all the longitudes, the dashed red lines a bit above and below are the corresponding one-standard-deviation spreads. The bottom wide green curves are the corresponding results for the surface precipitation rates. Also shown (thick black line between ± 45o) is the similarly averaged theoretically predicted eddy turnover times estimated from the tropospherically averaged zonal wind from the year 2006 using the ECMWF reanalysis data (Fig. 8.6a).

**Fig. 8.6** (a) Estimates of ε using gradients of the vector wind at resolution 3 o (330 km at equator, dashed blue using the EW gradients, red using NS gradients and blue, the isotropic gradient), all averages over the troposphere (i.e. p > 200 mb), as functions of latitude starting at 45o N, the contribution from the different pressure levels have been weighted by the air density. The straight lines are the corresponding latitude averages (isotropic: ε = 0.00093m2/s3. EW: ε = 0.00075m2/s3, NS: ε = 0.0014m2/s3 and the lowest line is the back-of-the-envelope calculation discussed in the text yielding ε ≈ 0.00093 m2/s3). Adapted from Lovejoy and Schertzer (2010). (b) The predicted hemispheric antipode large-scale velocity difference obtained from the isotropic estimate of ε (20.7± 7.37 m/s) and Kolmogorov’s law (blue, solid line), and the actual hemispheric difference (orange, dashed line: the cube root of the mean cube hemispheric difference, 17.3± 5.7 m/s). Adapted from Lovejoy and Schertzer (2010). (c) The distribution of the fluctuation ocean currents at 15mdepth as estimated by drifters at 3 o x 2 o resolution, 5-day average (remapped here on a 6 ox 2 o grid), reproduced from Niiler (2001). The mean seems to be about 20 cm/s, although near equator it may be closer to 40cm/s. (d) Ocean and atmospheric plateaus superposed, showing their great similarity. Left: A comparison of the monthly sea surface temperature (SST, blue) spectrum and monthly atmospheric temperatures over land (purple) for monthly temperature series from 1911 to 2010 on a 5ox 5 o grid (the NOAA NCDC data: see Table 8.2 for details). Only near-complete series (missing less than 20 months out of 1200) were considered: 465 for the SST, 319 for the land series; the missing data were filled using interpolation. The reference slopes correspond to β=0.2 (top), 0.6 (bottom left) and 1.8 (bottom right). A transition at 1 year corresponds to a mean ocean εo≈1x 10-8 m2s-3. The dashed orange lines are Ornstein–Uhlenbeck processes (of the form ): see Appendix 10B; ω0 is a characteristic transition frequency) used as the basis for stochastic linear forcing models. Right: The average of five spectra from 6 year-long sections of a 30- year series from daily temperatures at a station in France (black, taken from Lovejoy and Schertzer, 1986). The reference line has a slope 1.8 (there is also a faint slope β = 0 reference line). The relative up/down placement of this daily spectrum with respect to the monthly spectra (corresponding to a constant factor) was determined by aligning the atmospheric spectral plateaus. Reproduced from Lovejoy and Schertzer (2012). (e) Comparison of the cascade analysis of the monthly SST (blue, left column, HadCRUT data) and monthly land series (right column, CRUTEM3 data) for space (zonal), top row ( = 1 corresponds to 20000 km), and time, bottom row ( = 1 corresponds to 100 years) for the data discussed in Fig. 8.6d. The parameters and sampling details are given in Table 8.2. It is particularly noteworthy that although the land and ocean cascade structures are nearly identical, the corresponding spectra (Fig. 8.6d) are very different, indicating that the intermittency of the land temperatures is controlled by the ocean “weather” variability. The longest “pure” land scale accessible was = 165 o at mid-latitudes, hence the smallest accessible scale ratio was  = 100.2 in the upper right graph. Since the cascade ranges are not so large, we have superposed (thick red lines) the “universal” quasi-Gaussian curve for the envelope of the curves for moments of order q = 2 (Appendix 4A). It may be seen that in all cases the empirical variability is much stronger than would be possible for quasi-Gaussian processes.

**Fig. 8.7 (a)** The trace moments of the ECMWF interim reanalyses from daily data for 2006: the same as Figs. 4.1a and 4.1b but for the temporal analyses. λ = 1 corresponds to 1 year. The effective outer temporal cascade scales (τeff) are indicated with arrows. Also shown (superposed in a thick red line) are the q = 2 envelopes for the universal quasi- Gaussian processes; even the u, ν fields are significantly more variable. Adapted from Lovejoy and Schertzer (2011). (b) The trace moments of the spatial Laplacians of Twentieth Century Reanalysis (20CR) products for the band 44–46o\_N for the zonal wind (upper left), meridional wind (upper right), the temperature (lower left) and specific humidity (lower right) from series at 6-hour resolution. This is the temporal analysis corresponding to the zonal analysis in Fig. 4.2c; the largest scale, λ = 1, corresponds to 138 years; the parameters of the fits are given in Tables 8.3a, 8.3b, 8.3c, 8.3d. Notice the “bulge” in the hs moments up to scales of ~1 year, possibly a reflection of the ocean cascade. Also shown (superposed in thick red lines) are the q = 2 envelopes for the universal quasi-Gaussian processes; all the fields are significantly more variable.

**Fig. 8.7 (c)** The temporal cascades estimated from the GEM data, every 6 hours, temporal (second derivative) flux estimates. Also shown (superposed in solid red lines) are the q = 2.9 envelopes for the universal quasi-Gaussian processes; all the fields are significantly more variable. Reproduced from Stolle et al. (2012). (d) Analysis of the t = 0 GFS meteorological model output: zonal wind at 1000 mb, (top left), and 700 mb (bottom left), and the corresponding plots for the temperature 1000 mb (top right) and 700 mb (bottom right). λ = 1 corresponds to 1 year. Also shown (superposed in solid red lines) are the q = 2.9 envelopes for the universal quasi-Gaussian processes; all the fields are significantly more variable. Reproduced from Stolle et al. (2012). (e) Temporal analyses of precipitation products. The second time flux for the 100 x 100 km gridded (4-day resolution) TRMM radar satellite rain rate estimates (upper left), for the 3 months of the 3-hourly ECMWF interim stratiform rain product (upper right) and 29 years of NOAA’s CPC hourly gridded surface raingauge network (lower left). We have included (lower right) the unique very long 20CR product analyzed at 45 o N at 2 o resolution in space and 6 hours in time from 1871 to 2008; in it, there is a hint of a second lower intermittency cascade from about 10 days to 1 year. The regressions were performed over the range of scales 8 days to 1 year (TRMM), 6 hours to 10 days (ECMWF), 1 hour to 10 days (CPC) and 6 hours to 4 days (20CR). These are the temporal analyses corresponding to the spatial analyses presented in Figs. 4.8a and 4.8b. Reproduced from Lovejoy et al. (2012).

**Fig. 8.7** (f) Analysis of MTSAT hourly resolution thermal IR imagery over the Pacific. The temporal analysis of the spatial Laplacian (at 30 km resolution) geostationary MTSAT thermal IR imagery over the Pacific for 2 months. This is the temporal counterpart of Fig. 4.10. The external scale is 48 days. Reproduced from Pinel et al. (2012). (g) MTSAT estimates of K(q), showing the near-perfect superposition of horizontal space and time (hence isotropy). Lower line at right is time (C1 = 0.073), upper line is NS/EW (C1 = 0.074). Reproduced from Pinel et al. (2012). (h) The fluxes using the diurnally detrended hourly surface temperature series whose spectra are shown in Fig. 8.3c. On the left is the analysis of the absolute Whitman–Harrison temperature difference (≈ 170 km), and on the right is the mean of the two fluxes defined as the Lander– Harrison and Whitman–Lincoln absolute differences (≈ 400, 450 km respectively). The cascade outer scales are indicated as well as the scales where the scaling becomes poor due to the somewhat excessive smoothing introduced by the overly large spatial scales. The regression lines correspond to C1 = 0.069, α = 1.95, C1 = 0.072, α = 2.00 (left, right). Also shown (superposed in thick lines) are the q = 2 envelopes for the universal quasi-Gaussian processes; both fields are significantly more variable.

**8.8b OK as is since the new figure is B+W:**

**Fig. 8.11** (a) The 1D spectra calculated from the 3D (P(kx, ky, ω)) spectral density. The slightly shorter curve with the small but noticeable diurnal peak (blue) is E(ω), the orange curve just below the arrow (1000 km)-1 is E(kx), the other (red) curve is E(ky) and the reference line has absolute slope β = 1.5. (b) Contours of LogP(kx, ky): the spatial spectral density. (c) Contours of Log P (kx, ω), the zonal wavenumber/frequency subspace. The orientation is a consequence of the mean zonal wind, –3.4 m/s. (d) Contours of Log P(ky, ω), the meridional wavenumber/frequency subspace: there is very little if any “tilting” of structures since the mean meridional wind was small: 1.1 m/s. All reproduced from Pinel et al. (2012).

**Fig. 8.13** Space-time and spacespace plots using the q = 2 moments and using λ = ref /Δt and λ = Lref /Δx for time and space respectively (east– west and time upper left, north–south and time, upper right, north–south and east–west, lower left). Dashed black (u), blue (v), purple (w), dashed blue (hs), dashed black (T), red (z). In all cases, the black reference lines have slopes 1; in the space-time diagrams, it corresponds to a speed of ≈225 km/ day; the spread in the lines indicates a variation over a factor of about 1.6 in speed. In the space-space diagram, the bottom reference line corresponds to isotropy; the top to an aspect ratio of a ≈1.6 difference as discussed in the text. Adapted from Lovejoy and Schertzer (2011).

**Fig. 8.14** A space-time (vertical/time) diagram obtained from the first-order structure functions of 3 lidar time series at 1 s (top, red) and 2 s (lower two, blue, green) resolutions. At the largest scales, the statistics are poor, potentially accounting for the small deviations. We see that the troposphere thickness (which corresponds roughly to planetary sizes in the horizontal) has a time scale of several weeks to a month (see Section 4.1.2). Assuming that ls = 1 m, the top line corresponds to v = 60 m/s, the bottom line to 5 m/s. If instead ls = 10 cm, the top line implies 400 m/s, the bottom one to 30 m/s. This is estimated using the formula: . Reproduced from Lovejoy and Schertzer (2010b).

**Fig. 8.15** (a) A comparison of log10Mq(Δt) for east–west radiance fluxes (shorter and to the right, purple) and time (longer and to the left, pink) for q = 0.4, 1.2, 2, 2.8. λ is defined with respect to a time scale of 2 months for the temporal analyses and 20 000 km for the spatial analyses. The spatial log10Mq(Δx) has been shifted so as to superpose as closely as possible on the log10Mq(Δt) curves. The corresponding speed is ~900 km/day (10 m/s) and the outer cascade scale is ~40 days in time, ≈ 35 000 km in space. The deviations from scaling become important at ~5000 km or ~6 days. Compare this with the nearly perfectly scaling Fig. 4.10, which is the geometric mean of the east–west above with the north– south analysis. Reproduced from Pinel et al. (2012). (b) The horizontal space-time diagram constructed from Fig. 8.15a (upper curve and straight line) and the corresponding diagram from the north–south Mq (lower). Reproduced from Pinel et al. (2012).

**Fig. 8.16** (a) The normalized moments of the TRMM thermal IR data averaged over 100 x 100 km pixels at 12-hour resolution from 5300 orbits (1 year corresponding to λ = 1). The long time variability has been fitted to a cascade with outer scale at 1100 days, which could be a consequence of the ocean cascade. Adapted from Lovejoy and Schertzer (2010). (b) The same normalized moments of the TRMM thermal IR data as Fig. 8.16a but with temporal (pink) and spatial (blue) moments superposed, corresponding to a velocity of 400 km/day. The longer series of dots to the left is the temporal analysis (from Fig. 8.16a), the shorter series of dots to the right is the east–west spatial analysis (corresponding to Fig. 4.9b except that the analysis is not along orbit, and is at lower resolution). Reproduced from Lovejoy and Schertzer (2010).

**Fig. 9.17** (a) The evolution of a pair of one-dimensional multifractals with α=1.8, C1 =0.1, H=0.333. The subgenerators are identical up until t0 =210 – 27 (indicated by the arrow) after which they are independent. The insert shows a blow-up. (b) The “error” E defined in Eqn. (9.64) for the average of 1000 realizations with the parameters of Fig. 9.17a indicating a power-law function of the time Δt from divergence t0. The top curve (pink) is for I1 and I2 statistically independent, the bottom (blue; power law with reference slope =H=0.333) is for subgenerators identical until t=t0, independent thereafter (as in Fig. 9.17a). As expected, by extrapolation we see that the two are the same at Δt=29, which is half the length of the series (the simulated series is periodic so that at longer times the dependent series are actually less dependent).

The new 10.2b no longer has colour so this caption is OK as is.

**Fig. 10.3** A comparison of the spectrum of the same data as in Fig. 10.2a and 10.2b (bottom, blue points), with the simulation (top, red points); both spectra were averaged over logarithmic bins, 10 per order of magnitude. The reference lines have the theoretical slopes βw, βmw, as indicated in Eqn. (10.7).

**Fig. 10.4** The cascade analysis of the data and simulation shown in Figs. 10.1 and 10.2. Notice that the data are significantly more intermittent in the plateau region than the simulation (which has virtually no intermittency (near zero flux moments) for long time periods (to the left, lower figure). The low intermittency of the simulation can be gauged by the comparison with the superposed q = 2 envelopes for the universal quasi-Gaussian processes (thick red curving lines: see Appendix 4A). The simulation is only a bit more variable, whereas the data are much more variable.

**Fig. 10.9** (a) Twentieth-century Lévy collapses (time) of Laplacians of 700 mb, 6-hourly data at 45o N of u, v, T, hs (upper left, upper right, lower left, lower right, and using α = 1.9, 1.9, 1.9, 1.8, respectively). These are the collapses of the cascades shown in Fig. 8.7 b. The C1 values corresponding to the linear (cascade regime) are: 0.083, 0.082, 0.090, 0.083, respectively. The outer scale is 138 years so that 1 year corresponds to log10= 2.15. The lower (red) curves are the envelopes of the corresponding “collapse” curves for quasi-Gaussian processes (collapsed with α = 1.8; see Appendix 4A).

The new 10.9b has no colours, caption OK as is:

**Fig. 10.12** A comparison of the different structure function analyses (root mean square, RMS) applied to the ensemble of three monthly surface series discussed in Appendix 10C (NASA GISS, NOAA CDC, HadCRUT3), each globally and annually averaged, from 1881 to 2008 (1548 dicpoints each). The usual (difference, poor man’s) structure function is shown (dotted, black, lower left), the tendency structure function (dot-dash, lower right), the maximum of the two (“Hybrid”, red), and the Haar in blue medium thickness (as indicated); it has been increased by a factor C = 100.35 = 2.2 and the RMS deviation with respect to the hybrid is ± 14%. Reference slopes with exponents (2)/2 ≈ 0.4, –0.1 are also shown (corresponding to spectral exponents β = 1+(2) = 1.8, 0.8, respectively). In terms of difference fluctuations, we can use the global root mean square annual structure functions (fitted for 129 years > Δt > 10 years), obtaining ≈0.08Δt0.33 for the ensemble. In comparison, Lovejoy and Schertzer (1986) found the very similar ≈0.077Δt0.4 using northern hemisphere data (these correspond to βc = 1.66, 1.8 respectively). Reproduced from Lovejoy and Schertzer (2012b).

**Fig. 10.13** (a) The daily averaged, annually detrended RMS Haar temperature structure functions averaged for various latitudes (indicated at left), northern hemisphere (thin), southern (thick) for the period 1871–2008 (all at resolution 2ox 2o). The reference lines, slope = (2)/2, correspond to β = 1 + (2) = 0.2, 0.4, 0.8 (top to bottom, respectively; the absolute slopes are indicated in the figure itself). The “global” curve is the average over all the pixels, weighted by the map factors. The rise at the left starting at Δt = 2 days (the smallest lag for daily data for Haar structure functions) is the meteorological regime (the maximum is at tw ≈ 10 days), the middle is the macroweather regime with minimum at tc ≈10 years, and at the right we see the beginning of the climate regime. This has been “calibrated” by boosting the fluctuations by a factor of 2.2 so that the large Δt part is close to the tendency structure functions shown in Fig. 10.11b; the corresponding temperatures are indicated in degrees K. (b) Haar, q=1, 20CR (green, thick) with 29-year hourly CPC gauges (red, dots: see Section 4.4.2). For the CPC we also show the corresponding grid point to grid point one-standard-deviation limits (black, thin) with reference lines slopes H = –0.42 (solid) and –0.5 (dashed) corresponding to a Gaussian white noise process. Reproduced from Lovejoy and Schertzer (2012b). The short purple line is the spatial CPC analysis converted to time using 280 km/day.

**Fig. 10A.1** A realization of a bare climate cascade with  = 210 (thin blue line) and the corresponding dressed process (thick red line, offset by one unit for clarity) at the same resolution obtained by continuing the cascade to Λc = 216 and then averaging over a scale ratio of 26.

**Fig.10A.2** The log of the compensated by Δt-1, normalized autocorrelation function in two spatial dimensions for C1 = 0.4. The thick blue lines are when Rn is estimated using, the thin red lines are for  (important for the power spectrum over a finite range) Λc = 216, showing the extreme sensitivity of the large scales to the length of the series (Λc). The dashed black lines are flat reference lines corresponding to the asymptotic Δt-1 behaviour. Each curve is for a different α value, increasing from 2 (bottom) to 1.1 (top) at intervals of 0.1.

**Fig. 10C.1** (a) The monthly series over the common part of their domains: 1880–2008 (129 years), bottom to top: NOAA NCDC (green), NASA GISS (blue), HadCRUT3 (purple), 20CR (red). Each series had its mean removed and then was displaced by 0.6 K for clarity; the dashed lines are the displaced axes. (b) Annual averages from Fig 10C.1a. The grey line of variable thickness indicates the mean of the monthly resolution one-standard-deviation spreads of the three surface series; the thin dark line is the 20CR series.

**Fig. 10C.2** The spectra (averaged over logarithmically spaced bins, 10 per order of magnitude, same colours as fig. 10C.1b). The units are such that ω = 1 corresponds to (129 yrs)–1; note the annual spike, (1 year)–1 is at 2.11 on the log10ω axis).

**Fig. 10C.3** (a) Ensemble spectra for the three surface series (top, orange); the ensemble spectra of the same series of the three (signed) differences (middle, green) and the ensemble spectra of the same series using the three absolute differences (bottom, red). Notice that the differences are not white noise (flat), but are themselves scaling with exponents only a little lower than for the series themselves. (b) Analogous to Fig. 10C.3a, but for the 20CR spectrum (top) and the spectrum of the ensemble of differences with respect to the three surface spectra (middle), and absolute differences (bottom). The basic behaviours are the same as for the three surface series although the differences are larger.

**Fig. 10C.4** Trace moments based on the absolute second differences of the globally averaged monthly series. The corresponding parameters are in Table 10C.2. The thick red curves indicate the q = 2 envelopes of quasi-Gaussian processes (Appendix 4A). We can see that the 20CR data are close to quasi-Gaussian (although the scaling seems quite good at high ), the HadCRUT3 data are reasonably more variable, and the other datasets are in between.

**Fig. 10D.1** A comparison of SST and land surface temperatures (from the monthly NOAA NCDC data discussed in Section 8.1.4) with multifractal simulations in time and in one horizontal spatial dimension. The far right spectrum (blue) is the result of 10 simulations of an atmospheric model with w = 11 days, α = 1.8, C1 = 0.1, H = 0.5 simulated over the range 4 years/211 = 1 day and Lw/24= 1000 km (more precisely, this is the a = 5 land model described in the text; i.e. the top curve from Fig. 10D.2a). The red spectrum (lower left) is from 10 realizations of the coupled ocean–atmosphere model described in the text with the same exponents, and with o = 1 year, simulated over grids with temporal resolution 64 years/213= 3 days and spatial resolution Lw/22 =5000 km. The high-frequency reference slopes are roughly the theory value: β = 1 + 2H – K(2) = 1.82. The far left black spectra are the empirical land and SST spectra from fig. 8.6d.

**Fig. 10D.2** (a) Spectra from 10 realizations of the linear combination of ocean and atmosphere models, with the latter weighted by the coefficient a indicated next to the spectra. a≈ 5/6 is close to the land, SST observations in Fig. 8.6 d. (b) Comparison of the 20CR spectra of the 700 mb temperature field (daily resolution to 138 years) at a= 0.6 (green, bottom right) and a = 0.9, (orange, top right).

**Fig. 11.1** The top part shows four successive 10 kyr sections of the 5.2-year resolution GRIP data, the most recent to the oldest (red) from bottom to top. Each series is separated by 10 mils in the vertical for clarity (vertical units: mils – i.e. parts per thousand of isotope excess). For reference, a 5 K corresponding temperature spread is also shown using a calibration constant of 0.5 K/mil. We see that the bottom Holocene GRIP series is indeed relatively devoid of low-frequency variability compared to the previous 10 kyr sections, a fact confirmed by statistical analysis discussed in the text and shown in Fig. 11.2. In contrast, the bottom curve shows the (much lower resolution but on the same scale) paleo-SST curve from ocean core LO09–14 (Berner et al., 2008), taken from a location only 1500 km distant and displaying far larger variability: see Fig. 11.2. Adapted from Lovejoy and Schertzer (2012a).

**Fig. 11.2** A comparison of the RMS Haar structure function (S(Δt)) for both Vostok and GRIP high-resolution cores (resolutions 5.2 and 50 years respectively over the last 90 kyr). The Haar fluctuations were calibrated and are accurate to ≈ ± 20%. For Vostok we used the Petit et al. (1999) calibration, for GRIP, 0.5 K/mil. These series were broken into 10 kyr sections. The thick dashed lines show the most recent of these (roughly the Holocene). The top thick dashed blue line is the (Berner et al., 2008) paleo-SST series; the middle thick dashed line is from Vostok; the bottom thick dashed blue line is from GRIP. The thick continuous lines are the S(Δt) of the ensemble of eight 10–90 kyr GRIP (longest, blue) and Vostok (shorter, green). The one-standard-deviation variations about the mean are indicated by dashed lines (blue and green respectively). Also shown are reference lines with slopes ξ(2)/2 = –0.3, 0.2, 0.4, corresponding to β = 0.4, 1.4, 1.8 respectively. Although the Holocene is exceptional for the GRIP and Vostok series, for GRIP it is exceptional by many standard deviations. However, the paleo-SST curve (from only \_ 1500 km away) is quite different and is very close to the pre- Holocene GRIP results, presumably a consequence of the strong spatial intermittency (see Table 11.4 for a comparison). For the Holocene we can see that τc ≈1 kyr for Vostok, and ≈ 2 kyr for GRIP, although for the previous 80 kyr we find τc ≈ 100 years for both. Adapted from Lovejoy and Schertzer (2012a).

**Fig. 11.3** GRIP high resolution. The first four sections of 10 kyr high resolution (same axes); the upper left is the most recent (Holocene) 10 kyr section. Moving left to right, top to bottom we display the moments q = 0 to q = 2 (intervals 0.2) for the next 10 kyr periods, up to 30–40 kyr (bottom right). The reference scale (corresponding to  = 1 in the figure) is 10 kyr, the mean outerscale eff = 380 ± 140 years. The superposed red curves are the envelopes of the trace moments of the quasi-Gaussian processes as discussed in Appendix 4A. It can be seen that the data are not far from quasi-Gaussian, so the cascade parameter estimates are not too reliable, although the C1 estimates are supported by the fluctuation analyses in Section 11.2.3.

**Fig. 11.5** (a) Comparison of the latitude dependence of the monthly averaged pixel-scale reanalysis spectral exponents β (top) with the corresponding exponents of a control run of the IPSL GCM, discussed in Section 11.3.2 (bottom). The top curve (20CR, blue) and bottom curve (IPSL, red) show the low-frequency estimates (ω < 25 (years)-1; for the reanalysis this is over the lowest available five frequencies), and the bottom curve (20CR) and top curve (IPSL) show the estimates over the high frequencies ((3 years) -1 < ω < (3 months) -1). The error bars indicate the one-standard deviation spreads over all the estimates at the given longitude; the high-frequency spreads are about ± 0.3 whereas the low frequency spreads are about ± 0.6, ± 0.2 (20CR, IPSL, respectively, reflecting the greater length of the IPSL estimates, 500 years). A key point to note is that the relative positions of the of high- and low-frequency curves are inverted: whereas the 20CR low-frequency β’s are much larger than the corresponding IPSL β’s, the opposite is true for the high frequency β’s. For reference, we have also shown on the graph as lines the values of the exponents of the globally averaged temperatures from Fig. 10.12. The Greenland paleo temperatures are at roughly 75o N, where we note particularly low β values. (b) Latitudinal dependence (mean thick, one standard deviation thin dashed) of the critical climate transition time τc for 20CR 700 mb temperature (bottom, red) and surface precipitation (top, green). This is estimated from bilinear regression of slopes ξ(1) = H = –0.4, +0.4 on the q = 1 Haar structure functions. One can see that the precipitation τc’s are generally somewhat larger and roughly north– south symmetric whereas the temperature τc’s are somewhat asymmetric, qualitatively the same as the variation in low-frequency β’s (top row, Fig. 11.5a). This figure can be compared with Fig. 8.5c, which shows the corresponding 20CR estimates of τw.

**Fig. 11.6** The spectral exponent as a function of latitude for the 138-year monthly averaged 20CR temperature at 700 mb and various other proxy estimates discussed in Section 11.1.4. The thick (blue) and thin (brown) continuous lines are the mean exponents for frequencies ω < (25 years)-1, and for (3 months) -1 < ω < (3 years) -1 respectively (from Fig. 11.5a). The dashed lines indicate the one-standard-deviation longitude-to-longitude variations (corresponding to the map in Fig. 11.4). The vertical bars marked “NH, SH” are the exponents of the mean northern and mean southern hemisphere. Each of the three surface series (1880–2008) was used; the centre of the bar is the mean exponent for ω < (25 years) -1 and the length of the bar indicates the series-to-series variation. The dashed line indicating “global” is the mean over all the latitudes of the pixel by- pixel exponents (spread ± 1.28, not shown). On the right are indicated the regression estimates for the eight annual reconstructions discussed in the text and Table 11.3. The regressions for the reconstructions are for (480 years) -1 < ω < (25 years) -1 (the period 1500–1979). The rough range of latitudes where almost all the proxies are situated is indicated. Also indicated by circles are the Holocene (last 10 kyr) GRIP (at 5.2-year resolution, 72.57 o N) and Vostok (at 300-year resolution, 78.45o S) β’s; the vertical bars are the one-standard-deviation variations of β for the 8 x 10 kyr periods 10–90 kyr (GRIP) and the previous 41 x 10 kyr period 10 – 420 kyr (Vostok). Finally, the multiproxy estimates of South American annual temperatures (at 0.5o resolution; Neukom et al., 2010) are shown, both high and low frequency (respectively the short bottom green and top gold curves between 20 o and 60o S), as well as the overall average values (horizontal dashed lines). The high-frequency 20CR β distribution with latitude is almost the same as that of Huybers and Curry (2006), who fit NCEP reanalysis β’s from 2 months to 30 years. The near convergence of the high- and low-frequency β’s over part of the northern hemisphere is probably mostly due to a larger and highly variable transition scale τc

**Fig. 11.9** The RMS Haar fluctuation for the mean of the pre- and post-2003 series from 1500 to 1979 (bottom blue and middle brown solid lines respectively and excluding the Crowley series because of its poor resolution), along with the mean of the globally averaged monthly resolution surface series (NOAA CDC, NASA GISS, HadCRUT3) (solid, red, top). In order to assess the effect of the twentieth-century warming, the structure functions for the multiproxy data were recalculated from 1500–1900 only (the dashed lines that join the solid lines at small lags) and for the instrumental surface series with their linear trends from 1880–2008 removed (the data from 1880–1899 are too short to yield a meaningful S(Δt) estimate for the lower frequencies of interest). Although in all cases the large Δt variability is reduced, the basic power-law trend seems to remain, although the transition scale τc increases (especially for the post-2003 reconstructions). Note that the decrease in S(Δt) for the linearly detrended surface series over the last factor of 2 or so in lag Δt is a pure artefact of the detrending. Reference lines corresponding to β = 0.8 and 1.8 have been added. Reproduced from Lovejoy and Schertzer (2012a).

**Fig. 11.10** RMS Haar fluctuations for the mean monthly global surface series (left, brown), the mean pre-2003 and mean post-2003 proxies (bottom, red and middle left, blue, respectively) as well as the mean Vostok S(Δt) function over the last 420 kyr interpolated to 300-year resolution and using the Petit et al. (1999) calibration (upper right, blue). Also shown is the “interglacial window,” the probable typical range of fluctuations and quasi periods of the glacial/interglacials. Reproduced from Lovejoy and Schertzer (2012a).

**Fig. 11.15** Trace moment analysis of the temporal resolution dependence of the Vostok 1 m resolution core (length 3312 m). The variable analysed is the sequence of inter-layer time intervals Δt. The parameters are: C1 = 0.026, α = 2, the outer scale = 4170 m. Note that the scaling is reasonable for scales above ≈ 10 m. Also shown (thick red curve) is the envelope of the q = 2 quasi-Gaussian processes (Appendix 4A); the data are quite far from this.

**Note that the corresponding fig, is mislabelled 11.15a:**

**Changes are larger and bold font**

Fig. 11.16 (a) Comparison of the RMS structure function S(Δt) of the high-resolution (5.2-year) GRIP (red), IPSL (blue), 20CR mean surface series (dashed blue)**,** mean of the three post-2003 Northern hemisphere reconstructions for globally averaged temperatures (bottom left set, green) and the mean at Greenland latitudes (upper set, red), all using fluctuations defined as differences (poor man’s wavelet) so that the vertical scale directly indicates typical changes in temperature. In addition, the GRIP data are divided into two groups: the Holocene (taken as the last 10 kyr, lower) and the entire 91 kyr of the highresolution GRIP series (upper). The GRIP δ18O data have been calibrated by lining up the Holocene structure function with the mean 75o N 20CR reanalysis structure function (corresponding to = 0.65 K/mil). When this is done, the 20CR and surface mean global structure functions can be extrapolated with exponent H≈ 0.4 (see the corresponding line) to the “interglacial window” (box at top right) corresponding to half pseudo-periods between 30 and 50 kyr with variations (=S/2) between ±2 and ±3 K. This line corresponds to spectral exponents β = 1.8 (this is exactly the same line as proposed in Lovejoy and Schertzer, 1986). Finally, we show a line with slope ξ(2)/2 = 0.2 corresponding to the GRIP β = 1.4; we can see that extrapolating it to 50 kyr explains the local temperature spectra quite well. (b) The equivalent of Fig. 11.16a except for the RMS Haar structure function rather than the RMS difference structure function and including daily-resolution 20CR data (see Fig. 11.17a for the first-order Haar structure function). At the left top we show grid-point-scale (2ox 2 o) daily-scale fluctuations for both 75oN (orange) and globally averaged (blue)along with reference slope ξ(2)/2 = –0.4 ≈ H (20CR, 700 mb). On the lower left (brown), we see, at daily resolution, the corresponding globally averaged structure function. Also shown are the average of the three in-situ surface series (red**,** Fig. 10.12) as well as the post-2003 multiproxy structure function (Fig. 11.9). At the right we show both the GRIP (55 cm resolution, with calibration constant 0.5 K/mil, dashed, brown) and the Vostok paleotemperature series(purple). Also shown is the interglacial “window.” All reproduced from Lovejoy and Schertzer (2012a).

**Fig. 11.17** (a) A composite showing (calibrated) Haar structure functions for both the mean fluctuation (logarithmic slope ξ(1), bottom of each pair) and the RMS fluctuation (logarithmic slope ξ(2)/2, top of each pair). On the left we use the 20CR daily data at one-grid-point resolution averaged over the longitudes at 75o N (top left, orange), averaged over all longitudes and over the globe (bottom left, blue). In the middle, the mean of the three surface series from Fig. 10.12 and Appendix 10C (brown); at the bottom, the post-2003 northern hemisphere (NH) reconstructions (green), and right, the GRIP 5.2-year resolution series (dashed brown) back to 91 kyr and the Vostok series, interpolated to 50 years, back to 420 kyr (purple). Solid reference slopes –0.4, –0.1, 0.4 correspond (ignoring intermittency) to β = 0.2, 0.8, 1.8 respectively. Also shown is a dashed reference slope –0.5 corresponding to Gaussian white noise. The tendency for the mean and RMS curves toconverge is due to intermittency, the rate of convergence has exponent =K(2)/2≈ C1; see below. The box indicates the “glacial/interglacial window” discussed earlier. (b) The same data (and colours) as in Fig. 11.17a: this figure attempts to isolate the intermittency near the mean by estimating the function with Δq =0.1, in the small Δq limit, *F*≈ *tC*1. This relation exploits the relation Kʹ(1) = C1 = ξ(1) – ξ’(1), see Eqn. (10.11). Reference lines with slopes 0.03 (left pair) and 0.065 (right pair) are shown. Note the relatively low intermittency (roughly flat) lines associated with the surface global average temperatures and the 75o N 20CR series.

Fig. 11.19: Comparison of RMS Haar fluctuations for various solar (red), volcanic (green), orbital and CO2 data (blue) in units of radiative forcing (RF, units W/m2). For the solar radiances, the values of estimated total solar irradiance (TSI) were converted into RF using coalbedo = 0.7 and a geometric factor ¼ (yielding an overall reduction factor of 0.175). The TIMS satellite data is for 8.7 years from 2003 to the present at a 6-hour resolution; the data are from http://eobadmin.gsfc.nasa.gov/Features/SORCE/sorce\_07.php. Note that the Lean (2000) reconstruction includes the 11-year solar cycle

whereas the Wang (2005) curve is only for the background. The Krivova 2007 curve has a 10-year resolution. The Shapiro (2011) curve (the last

8963 years) was degraded to 20-year resolution to average out the solar cycle; the Steinhilber (2009) curve was at a 40-year resolution over the

last 9300 years. The volcanic series were from reconstructions of stratospheric sulfates using ice-core proxies. The Vostok paleo-CO2 series were converted to RF using 3.7 W/m2 per CO2 doubling (IPCC AR4: Solomon et al., 2007), the solar insolation at the north pole on June 15th was divided by 20 but is not a true RF. The orbital variation curve was interpolated to 100-yesar resolution and the low- and high-frequency fall-offs have logarithmic slopes –1, 1, i.e. they are the minimum and maximum possible for the usual (linear) Haar fluctuations. All the structure functions have been increased by a factor of 2 (i.e. without changing their relative amplitudes) so that the temperature fluctuations are roughly “calibrated” with the difference and tendency fluctuations as discussed in Section 10.2.2. Reproduced from Lovejoy and Schertzer (2012b).

**Fig. 11.21** (a) Comparison of the RMS Haar structure functions for temperatures from instrumental (“data”, daily 20CR, blue, monthly surface series, red), multiproxies (post 2003, yearly resolution, green) GCM control runs (thick, monthly, brown) and the FIF stochastic model (thin, brown). The data are averaged over hemispheric or global scales (except for the 20CR 2ox 2 o grid-scale curve added for reference). The surface curve (blue) is the mean of three surface series (NASA GISS, NOAA CDC and HadCRUT3, all 1881–2008); the 20CR curves are from the 700 mb level (1871–2008). The IPSL is a 500-year control run, the EFS is from a 3000-year control run; the “bump” at 2–4 years is a broad model quasi-periodic artefact. The multiproxies are from the three post-2003 reconstructions: two curves are shown, the top from 1500–1980, the bottom from 1500–1900, showing the effect of the twentieth-century data. The reference lines have slopes ξ(2)/2 so that β = 1+ ξ(2) = 0.2, 0.4, 1.8. The amplitude of the Haar structure functions has been calibrated using standard and tendency structure functions and is accurate to within ± 25%. At the upper right we have sketched the Vostok and GRIP paleocurves (see Fig. 11.16b, dashed, red) and haveindicated the likely glacial/interglacial mean temperature contrast (difference) by the arrows.

(b) Comparison of the RMS Haar structure functions of global-scale ECHO-G and EFS GCM simulations (the latter with both 0.1% and 0.25% solar forcing levels), with the models analyzed over their entire ranges including the twentieth century. Also shown are the 20CR, surface and multiproxy curves as in Fig. 11.21a. Although the 0.25% solar forcing curve has very large multidecadal, multicentennial variability, this is due to high simulated twentieth-century temperatures (see Figs. 11.21d, 11.21e, the analyses before 1900). It can be seen that the ECHO-G simulation is quite close to the northern hemisphere multiproxy reconstructions. Again the reference lines show slopes ξ(2)/2 = -0.4.