Solutions to the Tutorial Problems in the book "Magnetohydrodynamics of the Sun" by ER Priest (2014)
CHAPTER 1

PROBLEM 1.1. The Red-Giant Phase of the Sun
What will happen to the Earth and other planets when the Sun expands to a red giant?

SOLUTION. (adapted from an article by John Debes in the Astronomy magazine).
In about 7.5 billion years the supply of hydrogen in the Sun's core will be exhausted and it will switch to fusing hydrogen in a shell outside the core. As a result, the Sun will become a red giant, namely, a star with a radius tens to hundreds of times larger than the current solar radius. Main sequence stars of spectral types A through K with masses between 0.5 and 5 solar masses are believed to become red giants.

In the Sun's case it will swell to more than 200 times its present size and therefore expand its surface to beyond 1 AU. It will have a lower surface temperature than at present and will blow away much of its mass as a solar wind. Present calculations suggest it will have lost a third of its mass when it reaches its largest size.

At the same time the planets will slowly expand their orbits as the Sun's mass and therefore gravitational attraction decreases. The question therefore is whether a planet's orbit increases beyond the expanding radius of the red giant. At the Sun's largest size the planets will have expanded their orbits by about 50% beyond their current values of 0.38 AU (Mercury), 0.72 AU (Venus), 1 AU (Earth) and 1.52 AU (Mars). Thus Mercury and Venus will definitely lose the race, but Earth may well remain just outside the surface. However, the effect of tidal interaction between the Sun and Earth is to extract energy from the Earth's orbit and so lower its orbit so that it could instead be engulfed by the Sun.

Well before the red giant phase, the Earth's biosphere will have been destroyed due to the increase in solar brightness as the hydrogen supply dwindles and the solar core contracts. After a billion or so years the oceans will evaporate and the hydrogen from the water will be permanently lost to space. A total loss of water will have occurred by 3 billion years. Earth's atmosphere will be similar to that of Venus, but after another billion years most of the atmosphere will have been lost, leaving the Earth as a desiccated dead planet with a surface of molten rock.

As for the giant planets, they are likely to survive even beyond when the Sun becomes a white dwarf.

PROBLEM 1.2. Sunlight Duration During an Equinox.
Why are the durations of sunlight and night-time not the same at an equinox?

SOLUTION. At two days in the year around March 20/21 and September 22/23, the vernal and autumnal equinoxes, the Sun rises due east at the equator and sets due west, so that geometrically day and night are equal since the axis of the Earth is then tilted neither away from nor towards the Sun. However, the daytime is about 14 mins longer than night-time at the equator on the day of the equinox (longer still near the poles), for three main reasons.

Firstly, the calculation of the equinox is for a point at the centre of the Sun, but the top of the Sun rises at sunrise a couple of minutes before the centre of the Sun and sets at sunset a couple of minutes later. Secondly, the refraction or bending of sunlight by the Earth's atmosphere implies that we can still see the Sun when it is just below the horizon. This accounts for another couple of minutes. Thirdly, the Earth's orbit around the Sun is elliptical and so the Earth moves fastest when it is closest to the Sun (around January 3) and slowest when it is furthest away (around July 4), which causes variations in the length of the solar day and the times of sunset and sunrise.

The times when the daytime and night-time are equal (called equiluxes) occur a few days before the vernal equinox and after the autumnal equinox.

PROBLEM 1.3. Association of the Corona with the Sun.
If you had lived in the 1840's or 1850's, how would you have shown that the corona and prominences seen during an eclipse are associated with the Sun rather than the Moon?

SOLUTION. From early times until the mid-nineteenth century, it was not clear whether the solar corona viewed during solar eclipses is associated with the Sun or the Moon. For example, Kepler gives a clear mention of the corona, but considers it a property of the Moon. As discussed in Chap. 2 of

Golub and Pasachoff (1997, The Solar Corona, Cambridge University Press), he suggests that the Moon's "blemishes" are due to the fact that different parts of the Moon reflect solar rays differently.

In 1836, Francis Baily travelled to Scotland to observe an eclipse and in 1852 to another in Italy. Since these produced a range of different accounts and interpretations, he suggested that future eclipses be observed by teams of observers. Some felt that prominences and the corona are solar features and others that they are due to a lunar atmosphere.

At the next eclipse on the Baltic coast, a hundred astronomers were stationed along its path. Clear descriptions of red prominences at the eastern and western limbs being covered and revealed by the advancing Moon suggested that these phenomena belong to the Sun.

Later, at the eclipse of 1860, 150 astronomers observed it from Oregon through Canada to Spain and North Africa. One detailed set of photographs was taken by De la Rue in Spain and another set by Angelo Secchi 400 km to the southeast. Both sets agreed point by point and so clinched the case for prominences belonging to the Sun.

Much later, in 1930, Bernard Lyot developed the first coronagraph and used it to observe a prominence, to detect the corona's green line and to produce first photographs of the coronal spectrum outside eclipse.

PROBLEM 1.4. Variation of Distance on Sun Measured as 1 arcsec at the Earth.

(i) If the mean Sun-Earth distance is 1.496×10^{11} m and the solar diameter is 1391 Mm, show that the Sun has an angular diameter of 1915 arcsec at the Earth and that 1 arcsec is equivalent to 726 km on the Sun.

(ii) If the distance to the Sun varies during the Earth's orbit between 1.471×10^{11} m at perihelion and 1.521×10^{11} m at aphelion, show that the distance on the Sun subtended by an angle of 1 arcsec from the Earth varies by ± 12 km.

SOLUTION.

(i) The angle (θ) in radians subtended at the Earth by the Sun is given by

$$\tan \tfrac{1}{2}\theta = \frac{d}{2r},$$

where $d = 1391$ Mm is the solar diameter (corresponding to a radius of 695.5 Mm) and r is the distance from the Sun to the Earth. Since $\theta \ll 1$, may be well approximated by

$$\theta = \frac{d}{r}.$$

In arcsec, the angle becomes

$$\theta = \frac{648,000\ d}{\pi\ r}, \tag{1}$$

and so for the mean Sun-Earth distance (r =1 AU = 1.496×10^{11} m), this gives

$$\theta = 1915 \quad \text{arcsec.}$$

This angle subtends a distance 1391 Mm at the Sun and so the distance at the Sun subtended by 1 arcsec from the Earth is

$$\theta = \frac{1391}{1915}\ \text{Mm} = 726\ \text{km},$$

as required.

(ii) As the Earth orbits the Sun, the Sun-Earth distance (r) varies between a minimum of 1.471×10^{11} m at perihelion and a maximum of 1.521×10^{11} m at aphelion. From Eq.**??**, this variation (δr) produces a variation ($\delta\theta$) in angle at Earth subtended by the solar disc given by

$$\frac{\delta\theta}{\theta} \approx \frac{\delta r}{r}.$$

Thus, a fractional variation of 25/1496=0.0167 in Sun-Earth distance implies a ±1.67% variation in θ and therefore a ±1.67% variation from the distance 726 km the solar surface that translates to 1 arcsec at the Earth. Thus, the equivalence at the Sun of 1 arcsec varies by $\pm1.67\times726/100$ km = 12 km, as required.

Working this out directly, by inserting the values 1.471×10^{11} m and 1.521×10^{11} m for r in Eq.**??** gives 1950 and 1886 arcsec, respectively, for θ, and therefore 1391/1950 Mm = 713 km at perihelion and 1391/1886 Mm = 738 km at aphelion.

PROBLEM 1.5. Effects of Scattering of Different Colours.

Why are

(i) clouds white,

(ii) the sky blue,

(iii) sunsets red and
(iv) the green flash green?

SOLUTION. (Adapted from Pasachoff's *The Complete Idiot's Guide to the Sun, 2003*)

(i) When the scattering particles, such as droplets of water (typically 10^{-6} – 10^{-4} m in size), are larger than the wavelength of light, they scatter the light by the same amount, regardless of the colour or wavelength of the light. Thus, a cloud of water vapour scatters the incoming white light evenly across the spectrum, and so we see the scattered light as white.

(ii) When the scattering particles, such as atoms of oxygen and nitrogen (typically 10^{-10}m in size), are smaller than the wavelength of light, they scatter the different colours of light differently. Thus, blue light is scattered more effectively than red, a process known as *Rayleigh scattering*, in which the scattering of light of wavelength λ is proportional to λ^{-4}. Thus red light at, for example, 650 nanometres (6.5×10^{-7} m) would scatter worse than blue light at, say, 400 nanometres by a factor of $(650/400)^4 = 6$.

When white light from the Sun meets the Earth's atmosphere, the blue light is therefore scattered more effectively, while the red light goes straight through with only little scattering. Therefore, if we look at the sky away from the Sun, we see blue light that has been scattered towards us, and so the sky appears blue.

(iii) On the other hand, when we look straight at the Sun near the horizon, we see the red light that is coming straight through, but none of the blue light that has been scattered, and so the Sun appears red. By contrast, if you were to stand on the Moon or another planet or a satellite with very little atmosphere and so very little scattering of sunlight, the Sun would appear white, while the sky would appear black.

(iv) Light from the Sun near the horizon is bent (refracted) by an amount that depends on wavelength – blue light is bent the most and red the least. In principle then, you may expect to see a series of overlapping images of different colours, with the blue image being the highest and the red one the lowest.

However, very little of the blue light reaches us since it is scattered to give the blue sky seen from other locations. Furthermore, the water vapour in the atmosphere absorbs orange and yellow light very effectively, leaving only two overlapping images, a higher green one and a lower red one. Thus, when the red one sets, only the green one remains, lasting only a few seconds and appearing as a green rim at the top of the setting Sun. This effect is seen best when viewing the Sun over water when the horizon is clear.

PROBLEM 1.6. The Limb of the Sun.
(i) Why is the limb of the Sun so sharp?

(ii) What is the cause of limb darkening (i.e., the fact that in, e.g., white light, as we approach the limb the image becomes darker)?

SOLUTION. (i) A gas's opacity is a measure of how opaque it is. Thus, if we look through some gas with an optical thickness less than about 1/2, it appears transparent to us, but, if the optical thickness exceeds about 2, it looks opaque.

Thus, the reason why the Sun has such a sharp edge is that if we look at the Sun from the Earth, the change in optical thickness is very large over a small change in viewing angles. And this in turn is due to the very rapid decrease in density with height. The pressure (and therefore the density) decreases exponentially over an e-folding length of the pressure scale-height, and in the photosphere this is only about 150 km. By comparison with a solar radius of about 700 Mm, such a scale-height is extremely small, which explains why the edge of the solar disk looks so sharp.

(ii) If we look at the centre of the solar disc, we see into the Sun to a distance at which it becomes opaque (i.e., the opacity is large enough). However, looking near the limb of the Sun we are viewing the Sun diagonally. The fact that the Sun appears darker near the limb suggests that the slightly higher regions that are being sampled near the limb are somewhat cooler than those that are being observed when we look straight down to deeper regions near disc centre.

The surface temperature of the Sun is about 5800^o K, but this is a mean temperature averaged over the disc. The surface brightness of a perfect emitter (a so-called *black body*) depends sensitively on its temperature, and the temperature estimated from its brightness is called the *brightness temperature.* Since the centre of the solar disc is more than 50% brighter than the limb, the brightness temperature near the limb is more than 10% lower. Indeed, the brightness temperature near disc centre is about 6390^o K, while near the limb it is only 5000^o K.

PROBLEM 1.7. Duration of a Solar Eclipse
What causes the duration to vary from one solar eclipse to the next? Why

does the duration vary along the eclipse path?

SOLUTION. (adapted from an article by Fred Espinak in the Astronomy magazine).
Two main factors cause changes in the duration of solar eclipses. Firstly, the Moon's distance from the Earth varies between 226,000 miles (363,000 km) at perigee and 252,000 miles (406,000 km) at apogee. Such an 11.6% range causes the Moon's apparent diameter (i.e., the angular diameter measured from Earth) to vary by the same amount. When the Moon appears larger, it hides the Sun for longer during the eclipse.

Secondly, the Earth's elliptical orbit about the Sun makes the Sun's distance and therefore apparent diameter vary by roughly 3% from closest approach (perihelion) to farthest approach (aphelion). This affects the eclipse duration because a smaller apparent Sun would be eclipsed for longer than a larger one.

Thus, the longest total eclipses take place when the Moon is near perigee and the Earth near aphelion. Indeed, this is what caused the unusually long totality at the solar eclipse on July 22, 2009. Other factors also affect eclipse duration in a minor way, namely: the season (due to the axial tilt of the Earth with respect to the Sun); the inclination of the Moon's orbit; the latitude of the eclipse path; and the shape of the Earth's surface.

The variation in duration along the eclipse path is due to the nature of the path across the Earth's surface swept by the Moon's shadow, starting at the sunrise terminator, passing through the mid-day meridian and finishing at the sunset terminator.

Two main factors are involved. Firstly, near the terminators the Moon's shadow hits the surface of the Earth obliquely, which causes the shadow to move quickly and shorten the eclipse duration. But near the meridian the angle made by the shadow is more normal to the surface and so the shadow moves more slowly and the duration is longer. Secondly, the Earth's surface rotates in the same general direction as the motion of the shadow and so lengthens the duration, but the speed of the surface is greater near the equator than near the poles. Thus, the location of the shadow on the Earth's surface affects the eclipse duration.

PROBLEM 1.8. Intensity of Sunspot.
Suppose that the intensity (I) of part of the photosphere at temperature (T) obeys the *Stefan-Boltzmann law* for black-body radiation, namely, $I =$

$\sigma_{SB}T^4$, where σ_{SB} is constant. Then, if the temperatures of a sunspot umbra and the ambient photosphere are 3700 K and 5800 K, respectively, by what factor is the umbra less bright than the photosphere?

SOLUTION.

The ratio of the intensity (I_{umbra}) of the sunspot umbra to the intensity (I_{photo}) of the photosphere is, according to the Stefan-Boltzmann law

$$\frac{I_{umbra}}{I_{photo}} = \frac{T_{umbra}^4}{T_{photo}^4} = \frac{3700^4}{5800^4} = 0.17,$$

so that a sunspot is roughly one-fifth as bright as the surrounding photosphere.