**CHAPTER 15 SOLUTIONS**

1. Let ** the correlation between reading comprehension and intellectual ability in the population. Then H0: ** 0 and H1: ** 0
2. There is a statistically significant, moderate, positive linear relationship between reading comprehension and intellectual ability among public school children in the urban area who have been diagnosed with learning disabilities, *r*(*N* = 76) = .29, *p* = .01. That is, such children who have relatively low intellectual ability tend to have relatively low reading comprehension and those with relatively high intellectual ability tend to have relatively high reading comprehension. The strength of the linear relationship is moderate, according to Cohen’s rule of thumb guidelines.
3. There is a statistically significant moderate negative linear relationship between reading comprehension grade level among public school children in the urban area who have been diagnosed with learning disabilities, *r*(*N* = 76) = -.322, *p* = .004. That is, such children who have relatively low reading comprehension tend to be in the higher grades, while those with relatively high reading comprehension tend to be in the lower grades. Because the reading comprehension scores are relative to all students in the urban area, not just those with learning disabilities, these results indicate that the students with learning disabilities fall further behind their peers as they get older. The strength of the linear relationship is moderate, according to Cohen’s rule of thumb guidelines.
4. There is a statistically significant moderate correlation between reading comprehension and classroom placement among public school children in the urban area who have been diagnosed with learning disabilities, *r*(*N* = 76) = -.44, *p* < .0005. Students with the part-time resource placement have higher reading comprehension scores, on average, than those with the self-contained classroom placement. The strength of the linear relationship is moderate, according to Cohen’s rule of thumb guidelines.
5. The elimination of the 29 students without reading comprehension scores could bias obtained results if their missing data were related systematically to reading comprehension or any of the variables studied in relation to reading comprehension. For example, if the 29 students who did not have reading comprehension scores missed the reading comprehension test because they were doing especially poorly in reading and were being tutored at the time the reading test was given, the obtained correlations of reading comprehension with intellectual ability, grade level, and class placement would likely be biased.
6. (.056, .435)
7. (.067, .56)
8. Yes. Because 0 is not contained in the 95 percent confidence interval for *b*, it is not a plausible value for *b*, so that the model is statistically significant.
9. Because the range of values for both the confidence interval of ** and the population *b* are positive, the linear relationship between math comprehension and intellectual ability is positive. That is, public school children in the urban area who have been diagnosed with learning disabilities who have low math comprehension tend also to have low intellectual ability while those with high math comprehension tend also to have high reading comprehension. Because the slope in the population is likely to be between .067 and .56, a one point increase in intellectual ability is likely to be associated with a .067 to .56 point increase in math comprehension in the population..
10. Because the value of ** is thought to be between .06 and .44, we conclude that the strength of the correlation is anywhere from very weak to moderate.
11. Because the points of the scatterplot do not follow a curve, and therefore, the relationship between these two variables appears to be linear, a correlation analysis is appropriate in this case.



1. Among college-bound students who are always at grade level from urban areas, there is little or no linear relationship between family size and self-concept in eighth grade, *r*(*N* = 123) = -.11, *p* = .23.
2. The 95 percent confidence interval for ** is (-.59, .43). Because 0 is contained in the confidence interval, we know that the correlation between family size and self-concept in eighth grade is not statistically significant.
3. Although, in this text, we have adopted the practice of not reporting effect sizes for results that are not statistically significant, it is worthwhile to note that the confidence interval can be more useful than the sample correlation for estimating the size of the effect in the population. In this case, the sample correlation value is *r* = -.11, which is a small effect size. The confidence interval shows just how imprecise the estimate is, in this case, because according to the confidence interval, the correlation in the population could be anything from a strong negative correlation to a moderate to strong positive correlation. That it includes zero we know most importantly, in this case, that zero is a plausible value for the null hypothesis and, therefore, the null hypothesis cannot be rejected.
4. The regression model is statistically significant. The results of the ANOVA *F*(1,498) = 58.10, *p* < .0005 and the test of the significance of the *b*-weight, *t*(498) = 7.62, *p* < .0005 both indicate equivalently that the model is significant.
5.  = .368 (X) + 50.123
6. The value of the slope of the regression equation indicates that for each one point increase is socioeconomic status there is, on average, a .368 point increase in twelfth grade math achievement.
7. Given that an SES value of 0 is meaningful on this scale used to measure SES in the NELS study, the value of the y-intercept indicates that a person with an SES score of 0 is predicted to have a twelfth grade math achievement score of 50.123.
8. The value of *R*2 = adjusted *R*2 = .10, which is an effect size measure indicating the proportion of twelfth grade math achievement variance that can be explained by socioeconomic status, a moderate effect.
9. The regression model is not statistically significant. The result of the significance test associated with the *b*-weight, *t(103)* = -.11, *p* = .92 indicates that the model is not significant.
10. Because the regression model is not statistically significant, gender is not useful for predicting intellectual ability in the population.
11. The regression model is statistically significant. The result of the significance test associated with the *b*-weight, *t*(74) = -4.27, *p* < .0005 indicates that the model is significant.
12. = -12.564 (X) + 81.434
13. The slope is the difference between the average reading comprehension score of students in the full-time self-contained placement (PLACEMEN = 1) and those in the resource room part-time (PLACEMEN = 0). That is, according to this analysis, those in the self-contained classroom score 12.56 points lower, on average, than those in the resource room.
14. Because the value of 0 on the placement variable is meaningful (PLACEMEN = 0 represents students in the part-time resource room), we may interpret the y-intercept as the average reading comprehension score of resource room students
15. The predicted reading comprehension score for a student with a resource room placement is = -12.564 (0) + 81.434 = 81.434. The predicted value is equal to the mean reading comprehension score for all students with a resource room placement.
16. The predicted reading comprehension score for a student with a self-contained classroom placement is = -12.564 (1) + 81.434 = 68.87.
17. A good approximation is given by *R*2, which is equal, in the case of simple linear regression, to Beta2. Thus the proportion of the variance in reading comprehension scores that is explained by placement type is (-.444)2 = .20, which may be considered a moderate effect.
18. Equivalently, an independent samples *t*-test or one-way ANOVA could have been used to determine whether reading comprehension may be predicted from type of placement.
19. The regression model is statistically significant. The results of the ANOVA *F*(1,498) = 15.79, *p* < .0005 and the test of the significance of the *b*-weight, *t*(498) = 3.97, *p* < .0005 both indicate equivalently that the model is significant.
20.  = 2.765 (X) + 55.596
21. Students whose families owned a computer when they were in eighth grade scored 2.765 points higher, on average, in twelfth grade math achievement, than those whose families did not own a computer.
22. The value of the y-intercept is obtained when the variable, computer ownership, equals zero, which represents those students whose families did not own a computer in eighth grade. Accordingly, we may interpret the y-intercept of 55.596 as the average twelfth grade math achievement of those students whose families did not own a computer in eighth grade.
23. The value *R*2 = .03 (or equivalently, in this case, *adj R2*) equals the proportion of twelfth grade math achievement scores variance that can be explained by computer ownership, which may be described as a small to moderate effect. Because the relationship between twelfth grade math achievement and computer ownership is a comparison of those who did and did not own computers in eighth grade in terms of average twelfth grade math achievement, Cohen’s *d* may be calculated as another measure of effect size. To perform the calculation, we need means and standard deviations of twelfth grade math achievement for the two computer ownership groups, which may be obtained through the SPSS Means procedure. According to this calculation,

students who owned a computer in eighth grade performed approximately .36 standard deviations higher in twelfth grade math achievement than those who did not own a computer in eighth grade, which may be considered a small to moderate effect.

1. There is a statistically significant, moderate, positive correlation between intelligence and brain size, *r*(*N* = 40) = .36, *p* = .02. That is, among students with extremely high or extremely low intelligence, those with relatively low intelligence tend to have relatively small brain size and those with relatively high intelligence tend to have relatively large brain size.
2. The relationship between intelligence and gender among students with extremely high or extremely low intelligence, *r*(*N* = 40) = -.07, *p* = .69 is not statistically significantly different from zero.
3. Recall that in Chapter 5 it was mentioned that a correlation (calculated on a sample in which, at least, one of the variables contains only extreme values) tends to be inflated relative to the correlation value calculated on the entire distribution of values (i.e., including the middle values as well). Accordingly, one must take care not to generalize the finding in part (a) to all college students (those with low, middle, and high intelligence) as the correlation in that group is likely to be weaker than what was calculated in part (a).



1. The two clouds of points represent the two extreme groups that form this sample of data, those with extremely low intelligence and those with extremely high intelligence. There are no points in the middle because there is no one in this sample with moderate intelligence.
2. The regression model, = .0001192 (X) + 5.168, is statistically significant. The results of the ANOVA *F*(1,38) = 5.57, *p* = .02 and the test of the significance of the *b*-weight, *t*(38) = 2.36, *p* = .02 both indicate equivalently that the model is significant. The model fit as measured by *R*2 equals .13, indicating that 13 percent of FSIQ variance is explained by brain size.
3. *R*2single model = .13. *R*2low = .28. *R*2high = .30

The separate regression models provide a better fit to the data overall than the single regression equation fit to the entire sample. In short, we were able to take advantage of the existence of the two separate clouds of points in the scatterplot to produce two models which fit the data set better overall.



1. With the exception perhaps of the homoscedasticity assumption, the scatterplot does not appear to suggest a violation of any of these underlying assumptions. Although there appears to be a non-constant variance of reading comprehension scores by grade (notice how tightly clustered the grade 1 distribution is relative to the grade 3 or grade 4 distribution, for example), the sample size is relatively small, and these variations simply may be due to chance. Nonetheless, reasons ought to be explored as to why grade 1 scores appear to cluster so much more tightly relative to the other grades. Once the regression model is developed, sensitivity analyses should be carried out to measure the extent to which the outliers in grade 4 influence results.
2. For case number 32 (in grade 4), the residual is negative and the case is over-predicted.
3. An extreme point in either the first or fifth grade would have relatively large leverage. One example is case number 19 in grade 5.
4. Case number 43 is farthest (in terms of vertical distance) from the regression line.
5. Case number 43 is farthest (in terms of vertical distance) from the regression line and because the person is in fourth grade, some distance from the mean of grade. Case number 70 is also relatively influential.



1. With the possible exception of homoscedasticity, all assumptions appear to be met. Reasons should be explored to explain the apparent non-constant variance of reading comprehension residuals across grades, and, in particular, the clustering of the residuals of grade 1 relative to the other grades. In general, with the exception of the low residual in grade 4, the residuals form a rectangular distribution, suggesting that once the linear association between grade and reading comprehension has been accounted for, there appears to be no systematic association (e.g., quadratic) between grade and reading comprehension, and the assumption of linearity is met. The low residual in grade 4 should be examined more closely.
2. For case number 32, the residual is negative and the case is over-predicted.
3. An extreme point in either the first or fifth grade would have relatively large leverage. One example is case number 19 in grade 5.
4. Case number 43 has the largest standardized residual.
5. Case number 43 is farthest (in terms of vertical distance) from the regression line and because the person is in fourth grade, some distance from the mean of grade. Case number 70 is also relatively influential.
6. While, in this case, both convey the same information, because two of the key assumptions underlying a regression analysis (normality and heteroscedasticity) involve the error (residual) term, the residual plot is, in general, preferred to the original scatterplot for analyzing violations of regression assumptions. Furthermore, by removing the linear association between the dependent and independent variables, the residual plot offers a more easily interpreted view of whether underlying assumptions appear to be met.



1. The residual plot does not depict an array of points scattered randomly in a rectangular band above and below the regression line that would characterize an absence of violations of assumptions. There is a suggestion of some curvilinearity, and it may be useful to test whether the square of reading comprehension improves the fit of the model. That is, to center reading comprehension and to include it in the equation along with a variable that equals the square of the centered reading comprehension variable. For more detail, the interested reader should reference (Cohen, Cohen, West, & Aiken; 2003).
2. The residual for case number 2 appears to have the largest magnitude. The residual is positive and the case is under-predicted by the model.
3. One possible answer is case number 43. Although the magnitude of its residual is not the most extreme in the model, given its position in the scatterplot, it has a lot of leverage.
4. For the model without case number 43, =.616(X) + 38.258

For the model with all cases, =.549(X) + 43.691

Given where case number 43 sits in the scatterplot, one would expect that by excluding it, the slope would increase, as it does, and yet the intercept would decrease, as it does. The differences in both slope and intercept that result when this case is excluded from the analysis are not dramatic, but further indicate (beyond what is suggested by the residual plot) the extent to which this case is influential.

1. The regression model with case 43 is statistically significant (for the overall model, *F*(1,72) = 23.30, *p* < .0005; and equivalently in the case of simple regression, for the b-weight, *t*(72) = 4.83, *p* < .0005)

The regression model without case 43 is also statistically significant (for the overall model,F(1,71) = 22.01, *p* < .0005; and equivalently in the case of simple regression, for the *b*-weight, *t*(71) = 4.69, *p* < .0005).

1. With case 43, according to the adjusted *R*2, the model explains 23.4 percent of math comprehension variance.

Without case 43, according to the adjusted *R*2, the model explains 22.6 percent of math comprehension variance, a difference of less than one percent.

1. No, because the lowest reading comprehension score in the data set is 22, and not zero.
2. According to the model with case 43, a one point increase in reading comprehension is associated, on average, with a .55 point increase in math comprehension.

According to the model without case 43, a one point increase in reading comprehension is associated, on average, with a .62 increase in math comprehension.

1. For model with case 43: =.549(75) + 43.691 = 84.87

For model without case 43: =.616(75) + 32.258 = 84.46

1. Because both models, from a practical perspective are similar and yield similar interpretations, unless one can make a strong argument as to why case 43 should be eliminated from the analysis, it would be preferable to report and use the results from the model based on all cases.
2. The formula for the total degrees of freedom in the ANOVA summary table is *N* - 1, where *N* is the total number of scores used in the analysis. Because *dftotal* = 73, we see that *N* = 74.
3. The values of *R*2 (.244) and adjusted *R*2 (.234) are so similar because the ratio of the number of cases in the sample size to the number of predictors in the equation is very large at 76:1, way in excess of the 30:1 ratio suggested by the rule of thumb.
	1. In order to evaluate the appropriateness of the fit of the simple linear regression model to the data and to explore possible violations of assumptions, a series of analyses was carried out using both the unstandardized and studentized residuals. Given the difficulty of judging the normality of a dependent variable when the regressor is dichotomous (e.g., computer ownership groups) using a scatterplot (see below), boxplots of the residuals (both standardized and studentized) were constructed. The boxplots of the residuals suggest that even with the outliers in the group that did not own a computer, both distributions appear to be only mildly negatively skewed. In addition to these residual analyses, an analysis of Cook’s influence was carried out. Note that although there are some outliers, all of the Cook’s scores are considerably less than one, suggesting that there were no individual points in the analysis that appeared to unduly influence the results and that the regression analysis was appropriate as reported.





 

1. Yes, the points of the scatterplot do not appear to violate the underlying assumptions of normality, homoscedasticity, and linearity.



1. The residual plot (SES values versus studentized residuals) is a random scatter of points organized in a rectangular fashion about a residual value of zero for each value of SES and suggests an appropriate fit of the model to these data overall. In particular, since the variability of points for each value of SES is similar, the data appear to satisfy homoscedasticity and with no pattern of relationship between the residuals and SES, the assumption of linearity also appears to be met. To look more closely at specific residual values and particular cases that were not fit well by the model, a frequency distribution of studentized residuals was obtained. There are 17 cases with studentized residual values that are greater than 2 or less than -2, indicating the presence of bivariate outliers to the model. Accordingly, we follow-up by testing each point for its level of influence on the model.

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1. According to the boxplot of Cook’s influence scores, all of the values are substantially less than 1, suggesting that no single point unduly influenced the result the regression analysis and that the model is appropriate as obtained.



1. 58.59
2. 1.10
3. The value of the residual with the greatest magnitude is -21.65; it is associated with the person that has ID = 82. But, as shown in the plot of Cook’s influence scores, even this person’s data does not unduly influence the results of the analysis. That is, if this person’s data were removed from the analysis, and a new model were fit to the data (without this person), results would remain not change by very much.
4. The scatterplot was created with the case numbers included for each point. Because there are so few points, it is difficult to assess the normality and homoscedasticity assumptions. The data do appear linear, so the linearity assumption appears viable. Case number 8 is an outlier with a studentized residual value of -2.04. Cook’s influence scores were found for each point. The most influential point is case number 5. Although it is much more influential than the others, its Cook’s influence value is much less than 1 (.51), so that we do not need to be concerned about its undue influence on the analysis.



1. The 95 percent CI for *Z* is (.90, 2.03). The 95 percent CI for ** is (.72, .97). There is a statistically significant, strong, positive correlation between the weight gain of the mother and the birth weight of the infant. That is, such mothers with a relatively high weight gain tended to have infants with relatively high birth weight and mothers with a relatively low weight gain tended to have infants with relatively low birth weight.
2. =.113(X) + 4.278
3. The regression model is statistically significant. The results of the ANOVA *F*(1,13) = 54.96, *p* < .0005 and the test of the significance of the *b*-weight, *t*(13) = 11.24, *p* < .0005 both indicate equivalently that the model is significant.
4. 7.1lbs.
5. .346
6. The accuracy of predicting the weight gain of an infant given a particular weight gain for the mother is captured by the standard of estimate. That is, since the standard error of estimate is the standard deviation of scores about the regression line for given values of X, we know that, if normality and homoscedasticity are assumed to be true, then 68% of the predicted values of Y (the weight of the infant) for given values of X (the weight gain of the mother during pregnancy) will be within approximately one standard deviation (or .346) of the regression line. Likewise, 95% of predicted values of Y will fall approximately within two standard deviations of the regression line (2 x .346 = .692) for all values of X. Said differently, we can predict with 95% accuracy the weight of an infant for a mother who gained 25 pounds during pregnancy to be 7.1 - .692 to 7.1 + .692 pounds, or approximately 6.4 to 7.8 pounds.
	1. c)
	2. a)
	3. e)
	4. b)
	5. c)
	6. c)
	7. a)
	8. b)
	9. c)
	10. a)
	11. b)
	12. c)
	13. e)
	14. e)
	15. b)

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