SOLUTIONS for 2nd Edition of Mechanical Behavior

Chapter 1

1. Consider an aluminum single crystal under a stress state, σ_x = 250 psi, σ_y = - 50 psi, $\sigma_z = \tau_{yz} = \tau_{zx} = \tau_{xy}$ = 0, where x = [100], y = [010] and z = [001].

A. What is the resolved shear stress, τ_{nd}, on the (111) plane in the $[1\bar{1}0]$ direction? i.e. with n = [111], d = $[1\bar{1}0]$

B. What is the resolved shear stress on the $(11\bar{1})$ plane in the [101] direction?

Solution A. $\tau_{nd} = \sigma_x \ell_{xn} \ell_{xd} + \sigma_y \ell_{yn} \ell_{yd}$ = 250(1/√3)(1/√2) + (-50)(1/√3)(-1/√2) = 122.5 psi

B. $\tau_{nd} = \sigma_x \ell_{xn} \ell_{xd} + \sigma_y \ell_{yn} \ell_{yd}$ = 250(1/√3)(1/√2) + (-50)(1/√3)(0/√2) = 102. psi

2. Consider the single crystal in problem 1. Now suppose that slip does occur on the (111) plane in the $[10\bar{1}]$direction and only on that slip system. Also assume that the resulting strains are small. Calculate the ratios of the resulting strains $\varepsilon_x/\varepsilon_x$ and $\varepsilon_z/\varepsilon_x$.

$\varepsilon_x = \gamma_{nd}(\ell_{xn} \ell_{xd}) = \gamma_{nd}(1/\sqrt{6})$. $\varepsilon_x = \gamma_{nd}(\ell_{yn} \ell_{yd}) = -\gamma_{nd}(1/\sqrt{6})$,

$\varepsilon_z = \gamma_{nd}(\ell_{zn} \ell_{zd}) = -\gamma_{nd}(0)$. Therefore $\varepsilon_z/\varepsilon_x$ = -1 and $\varepsilon_y/\varepsilon_x$ = 0.

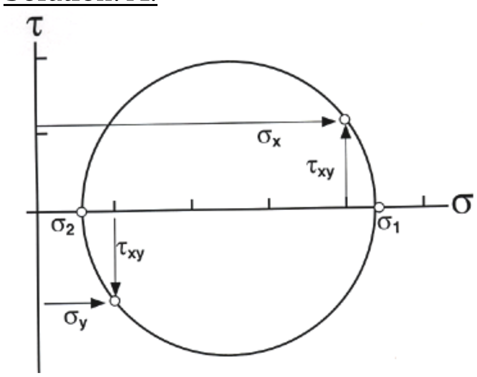
3. A body is loaded under a stress state, σ_x = 400, σ_y = 100, τ_{xy} = 120, $\tau_{yz} = \tau_{zx} = \sigma_z$ =0.

A. Sketch the Mohr's circle diagram.

B. Calculate the principal stresses

C. What is the largest shear stress in the body? (Do not neglect the z direction.)

Solution: A.



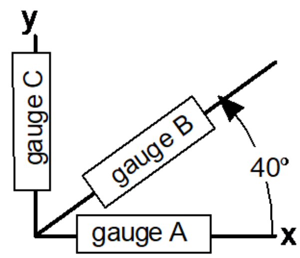
B. $\sigma_1, \sigma_2 = (\sigma_x + \sigma_y)/2 \pm \{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}{}^2\}^{1/2} = 250 \pm [150^2 + 120^2]^{1/2}$ = 442, 58, $\sigma_3 = \sigma_z$ = 0.

C. $\tau_{max} = (\sigma_1 - \sigma_3)/2$ = 221.

4. Three stain gauges have been pasted on the surface of a piece of steel in the pattern shown below. While the steel is under load, these gauges indicate the strains parallel to their axes:

Gauge A 450x10^{-6}: Gauge B 300x10^{-6}: Gauge C - 150x10^{-6}

Figure 1.21. Arrangement of strain gauges.



A. Calculate the principal strains ε_1 and ε_2.

B. Find the angle between the 1 axis and the x-axis, where 1 is the axis of the largest principal strain? [Hint: Let the direction of gauge B be x', write the strain transformation equation expressing the strain $e_{x'}$ in terms of the strains along the x-y axes, solve for γ_{xy} and finally use the Mohr's circle equations.]

Solution: A. $\varepsilon_B = \varepsilon_A \cos^2 40 + \varepsilon_c \cos^2 50 + \square\gamma_{AB} \cos 40 \cos 50 =$

$300 = 450 \times 0.587 - 150 \times 0.413 + \gamma_{AB} 0.766 \times 0.623$

$\gamma_{AB} = 198,$

$\varepsilon_1, \varepsilon_2 = (\varepsilon_A + \varepsilon_C)/2 \pm \{[(\varepsilon_A - \varepsilon_C)/2]^2 + (\gamma_{xy}/2)^2)\}^{1/2} = 150 \pm [300^2 + 198^2]^{1/2} = 509, -209 \times 10^{-6}$

B. $\tan(2\theta) = \gamma_{AB}/(e_A - \square\varepsilon_B) = 198/600 = 0.33$, $\theta = \text{atan}(0.33)/2 = 9.1°$

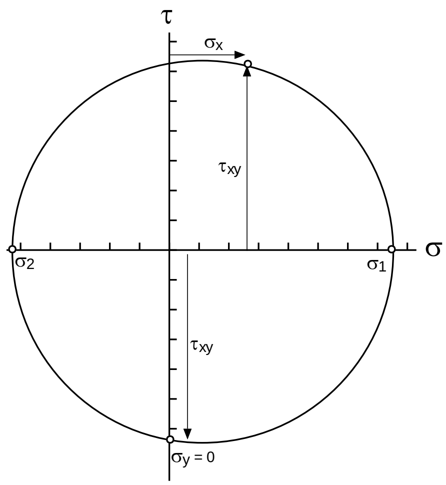
5. Consider a thin-wall tube that is 1 inch in diameter and has a 0.010-inch wall thickness. Let x, y, and z be the axial, tangential (hoop) and radial directions respectively.

A. The tube is subjected to an axial tensile force of 80 lbs. and a torque of 100 in-lbs.
   i. Sketch the Mohr's circle diagram showing stresses in the x-y plane.
   ii. What is the magnitude of the largest principal stress?
   iii. At what angles are the principal stress axes, 1 and 2, to the x and y directions?

B. Now let the tube be capped and let it be subject to an internal pressure of 120 psi and a torque of 100 in-lbs.
   i. Sketch the Mohr's circle diagram showing stresses in the x-y plane.
   ii. What is the magnitude of the largest principal stress?
   iii. At what angles are the principal stress axes, 1 and 2, to the x and y directions?

Solution: A. A torque balance gives 100 in-lb = $\tau_{xy}(\pi \times 1\text{in})(0.010\text{in})(0.5\text{in})$, $\tau_{xy} = 100/(0.010\pi \times 0.5) =$ 6366 psi. The axial stress, $\sigma_x = 80/[(\pi \times 1\text{in})(0.010\text{in})] = 2546$ psi

i.

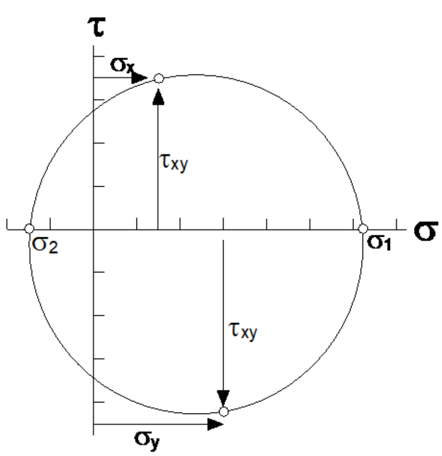


ii. $\sigma_1 = (\sigma_x + \sigma_y)/2 \pm \{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}{}^2\}^{1/2} = 2546/2 + [1273^2 + 6366^2]^{1/2} = 7765$ psi

iii. $\tan(2\theta) = 6366/(2546/2) = 5.0$, $\theta = (0.5)\mathrm{atan}(5.0) = 39.3°, 129°$

B. Now there is a hoop stress, $\sigma_y = Pd/(2t) = 120x1/(2x.01) = 6000$ psi, an axial stress of $\sigma_x = Pd/(4t) = 3000$ psi and a shear stress of 6366 psi.

i.



ii $\sigma_1 = (\sigma_x + \sigma_y)/2 + \{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}{}^2\}^{1/2} = 4500 + [1500^2 + 6366^2]^{1/2} = 11{,}040$ psi

iii $\tan(2\theta) = 6366/1500 = 4.244$, $\theta = (.5)\mathrm{atan}(4.244) = 38.4°, 128.4°$

6. A solid is deformed under plane-strain conditions ($\varepsilon_z = 0$). The strains in the x-y plane are $\varepsilon_x = 0.010$, $\varepsilon_y = 0.005$, and $\gamma_{xy} = 0.007$.

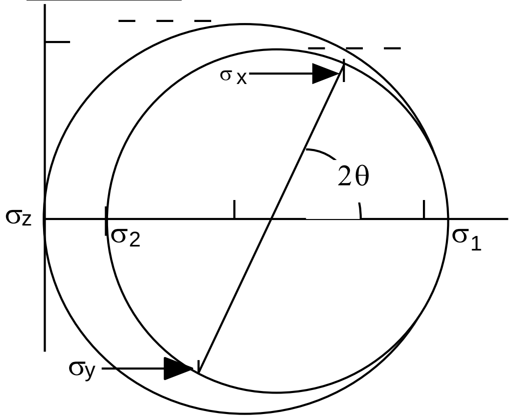
A. Sketch the Mohr's circle diagram.

B. Find the magnitude of ε_1 and ε_2.

C. What is the angle between the 1 and x axes?

C. What is the largest shear strain in the body? (Do not neglect the z direction.)

Solution: A



B. $\varepsilon_1, \varepsilon_2 = (0.010 + 0.005)/2 \pm \{[(0.010 - 0.005)/2]^2 + (0.007/2)^2)\}^{1/2} = 0.0118, 0.00320$

C. $\tan(2\theta/) = \gamma_{xy}/(\varepsilon_x - \varepsilon_y) = 0.007/0.005 = 1.4$, $\theta = (1/2)\mathrm{atan}(0.28) = 27.2°$

D $\gamma_{max} = (\varepsilon_1 - \varepsilon_3) = 0.018$

7. A grid of circles, each 10.00 mm diameter, was etched on the surface of a sheet of steel. When the sheet was deformed the grid circles were distorted into ellipses. Measurement of one indicated that the major and minor diameters were 11.54 and 10.70 mm respectively

A. What are the principal strains, ε_1 and ε_2?

B. If the axis of the major diameter of the of the ellipse makes an angle of 34° to the x-direction, what is the shear strain, γ_{xy}?

C. Draw the Mohr's strain circle showing ε_1, ε_2, ε_x, and ε_y.

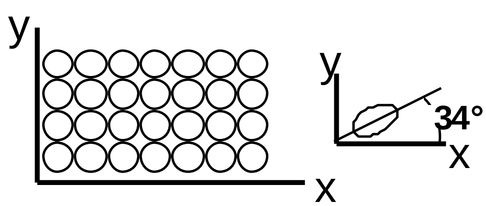


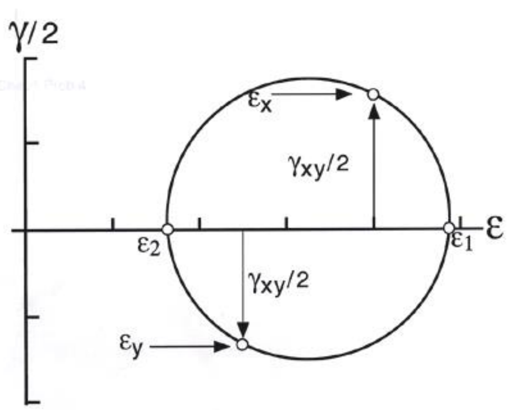
Figure 20. Circle grids printed on a metal Sheet.

Solution:

A. $\varepsilon_1 = \ln(1.154) = 0.143$ $\varepsilon_2 = \ln(1.070) = 0.0676$ (or $e_1 = 0.154$, $e_2 = .07$)

B. $\tan(2\theta) = \gamma_{xy}/(\varepsilon_1 - \varepsilon_2)$, $\gamma_{xy} = (0.143 - .067)\tan(68°) = 0.188$

C.

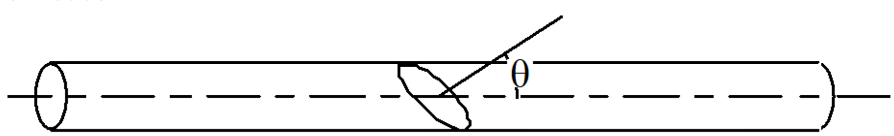


8. Consider an aluminum single crystal under a stress state, $\sigma_x = +75$ psi, $\sigma_y = +25$ psi $\sigma_z = \tau_{yz} = \tau_{zx} = \tau_{xy} = 0$, where x = [100], y = [010], and z = [001].
What is the resolved shear stress, τ_{nd}, on the (111) plane and $[10\bar{1}]$ direction?
Solution: $\tau_{nd} = \sigma_x \ell_{xn} \ell_{xd} + \sigma_y \ell_{yn} \ell_{yd} = 75(1/\sqrt{6}) + 25(0) = 30.6$ psi.

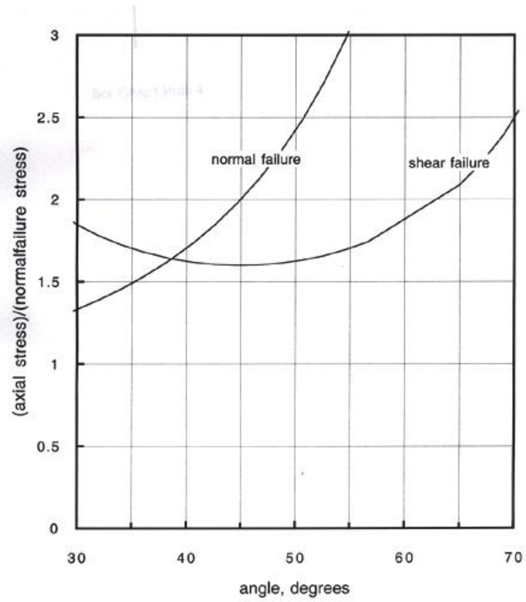
9. Consider the torsion of a rod that is 1 meter long and 50 mm in diameter.
A. If one end of the rod is twisted by 1.2° relative to the other end, what would be the largest principal strain on the surface?
B. If the rod were extended by 1.2% and its diameter decreased by 0.4% at the same time it was being twisted, what would be the largest principal strain?
Solution; A. $\gamma_{xy} = \tan(1.2°) = 0.0209$, $\varepsilon_1 = \gamma_{xy}/2 = .0105$
B. $\varepsilon_1 = (\varepsilon_x + \varepsilon_y)/2 + \{[(\varepsilon_x - \varepsilon_y)/2]^2 + (\gamma_{xy}/2)^2]\}^{1/2} = (.012 - .004)/2 + \{[(.012 + .004)/2]^2 + .0209^2\}^{1/2} = .0253$

10. Two pieces of rod are glued together along a joint whose normal makes an angle, θ, with the rod axis, x. The joint will fail if the shear stress on the joint exceeds its shear strength, τ_{max}. It will also fail if the normal stress across the joint exceeds its normal strength, σ_{max}. The shear strength, τ_{max}, is 80% of the normal strength σ_{max}. The rod will be loaded in uniaxial tension along its axis, and it is desired that the rod carry as high a tensile force, F_x as possible. The angle, θ, cannot exceed 65°.
A. At what angle, θ, should the joint be made so that a maximum force can be carried?
B. If θ_{max} were limited to 45°, instead of 65°, how would your answer be altered? [Hint: plot σ_x/σ_{max} vs. θ for both failure modes.

Figure 1.23. Glued rod.



Solution:
Let the axial stress be σ, the max normal stress be $\sigma_{n(max)}$ the max shear stress be $\tau_{(max)}$.
$\tau_{(max)} = 0.8\sigma_{n(max)}$
$\sigma_n = \sigma\cos^2\theta$. Normal failure will occur when $\sigma = \sigma_{n(max)}/\cos^2\theta$.
$\tau = \sigma\cos\theta\sin\theta$. Shear failure will occur when $\sigma = \tau_{(max)}/\cos\theta\sin\theta = 0.8\,\sigma_{n(max)}/\cos^2\theta$.
Now plotting $\sigma/\sigma_{n(max)}$ vs. θ, it can be seen

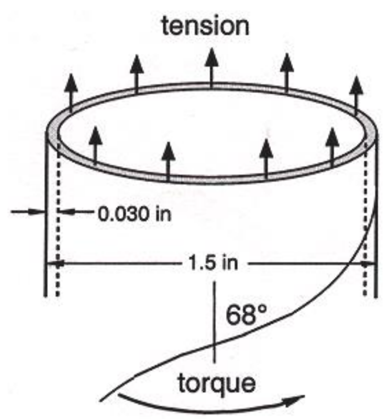


A. that if $\theta \leq 65°$, the max value of $\sigma_{max} = 2.089_{n(max)}$ at $\theta = 65°$

B. hat if $\theta \leq 45°$, the max value of σ_{max} corresponds to the intersection of the two curves and $\sigma_{max} = 1.64\ \sigma_{n(max)}$ at $\theta = 38.65°$

11. Consider a tube made by coiling and gluing a strip as show below. The diameter is 1.5 in., the length is 6 in. and the wall thickness is 0.030 in. If a tensile force of 80 lbs. and a torque of 30 in-lbs. are applied in the direction shown, what is the stress normal to the glued joint? [Hint: Set up a coordinate system]

Figure 22. Tube formed from a coiled and glued strip.



Solution: The torque, 30 in-lb = π(1.5 in)(.030in)(0.75in)τ_{xy}.

τ_{xy}= 30/[π(1.5 in)(.030in)(0.75in)] = 283 psi , σ_x = 80lbs/[π(1.5 in)(.030 in) = 566

$\sigma_n = \sigma_x \cos^2 22 - \tau_{xy} \cos 22 \cos 68 = 486$. Note that for the sketch shown, the torque contributes a compressive stress on the joint.

Chapter 2

1. Reconsider problem 4 in chapter 1 and assume that E = 205 GPa and $\upsilon = 0.29$.

A. Calculate the principal stresses under load

B. Calculate the strain, ε_z.

Solution: A. With $\sigma_3 = 0$, $\varepsilon_1 = (1/E)[\sigma_1 - \upsilon\sigma_2]$ and $\varepsilon_2 = (1/E)[\sigma_2 - \upsilon\sigma_1]$ so $\sigma_2 = E\varepsilon_2 + \upsilon\sigma_1$.
Substituting into $\varepsilon_1 = (1/E)[\sigma_1 - \upsilon\sigma_2] = (1/E)[\sigma_1 - \upsilon^2\sigma_1 - \upsilon E\varepsilon_2]$.
$\sigma_1 = (\varepsilon_1 + \upsilon\varepsilon_2)E/(1-\upsilon^2)$. Taking ε_1 and ε_2 as 509 $\times 10^{-6}$ and -209 $\times 10^{-6}$, σ_1= 100.2 MPa,
$\sigma_2 = (\varepsilon_2 + \upsilon\varepsilon_1)E/(1-\upsilon^2)$ = -13.7 MPa.
B. $\varepsilon_3 = (1/E)[-\upsilon(\sigma_1 + \sigma_2)] = -122 \times 10^{-6}$

2. Consider a thin-wall tube, capped at each end and loaded under internal pressure. Calculate the ratio of the axial strain to the hoop strain, assuming that the deformation is elastic. Assume $E = 10^7$ psi and $\upsilon = 1/3$. Does the length of the tube increase, decrease or remain constant?
Solution: Let the axial direction be x and the hoop direction y. Then from example problem 1.10, $\sigma_x = (1/2)\sigma_y$, $\sigma_z = 0$. $\varepsilon_x/\varepsilon_y = \{(1/E)[\sigma_x - \upsilon\sigma_y]\}/\{(1/E)[\sigma_y - \upsilon\sigma_x]\} = [\sigma_x - \upsilon\sigma_y]/[\sigma_y - \upsilon\sigma_x]\} = (1/2 - 1/3)/(1-1/6) = (1/6)/(5/6) = 1/5$. The tube elongates.

3. A sheet of metal was deformed elastically under balanced biaxial tension ($\sigma_x = \sigma_y$, $\sigma_z = 0$)

A. Derive an expression for the ratio of elastic strains, e_z/e_x, in terms of the elastic constants.

B If E = 70 GPa and $\upsilon = 0.30$, and e_y is measured as 1.00×10^{-3}, what is the value of e_z?

Solution: A. $e_z = (1/E)[-\upsilon(\sigma_x + \sigma_y)] = (1/E)[-2\upsilon\sigma_x]$, $e_x = (1/E)[\sigma_x - \upsilon(\sigma_y + \sigma_z)] = (1/E)[\sigma_x(1-\upsilon)]$; $e_z/e_x = -2\upsilon/(1-\upsilon)$
B. $e_z = -2\upsilon/(1-\upsilon)e_y = [-2(0.3)/(1-0.3)]1.00 \times 10^{-3} = -0.857 \times 10^{-3}$

4. A cylindrical plug of a gummite* is placed in a cylindrical hole in a rigid block of stiffite*. Then the plug is compressed axially (parallel to the axis of the hole). Assume that plug exactly fits the hole and that the stiffite does not deform at all. Assume elastic deformation and that Hooke's law holds. Derive an expression for the ratio of the axial strain to the axial stress, ε_a/σ_a, in the gummite in terms of the Young's modulus, E, and Poisson's ratio, υ, of the gummite.
*"gummite" and "stiffite" are fictitious names.
Solution: Let the axial direction be a, the hoop direction y and the radial direction z.
$e_z = e_y = 0 = (1/E)[\sigma_z - \upsilon(\sigma_y + \sigma_a)]$. By symmetry of the Hooke's laws, $\sigma_y = \sigma_z$, so
$0 = (1/E)[\sigma_y - \upsilon\sigma_y - \upsilon\sigma_a)]$, $\sigma_y = \sigma_z = [\upsilon/(1-\upsilon)]\sigma_a$. Substituting into Hooke's law
$e_a = \; = (1/E)[\sigma_a - \upsilon(\sigma_y + \sigma_z)] = (\sigma_a/E)\,[1 - 2\upsilon/(1-\upsilon)]$.
$e_a/\sigma_a = (1/E)[1 - 2\upsilon/(1-\upsilon)] = (1/E)[(1 - 3\upsilon/)/(1-\upsilon)]$

5. Strain gauges mounted on a free surface of a piece of steel (E = 205 GPa, $\upsilon = 0.29$) indicate strains of $\varepsilon_x = -0.00042$, $\varepsilon_y = 0.00071$, and $\gamma_{xy} = 0.00037$.

A. Calculate the principal strains.

B. Use Hooke's laws to find the principal stresses from the principal stresses.
C. Calculate σ_x, σ_y, and τ_{xy} directly from ε_x, ε_y, and γ_{xy}.
D Calculate the principal stresses directly from σ_x, σ_y, and τ_{xy} and compare your answers with the answers to B.

<u>Solution:</u> A. $\varepsilon_1, \varepsilon_2 = (\varepsilon_x + \varepsilon_y)/2 \pm \{[(\varepsilon_x - \varepsilon_y)/2]^2 + (\gamma_{xy}/2)^2)\}^{1/2} =$
$(-0.00042 + 0.00071)/2 \pm \{[(-0.00042 - 0.00071)/2]^2 + (0.00037/2)^2)\}^{1/2} =$
$0.000145 \pm [(-.00113)^2 + (0.000185^2)\}^{1/2} = 0.000145 \pm .000594 = 0.000740, -0.000450.$

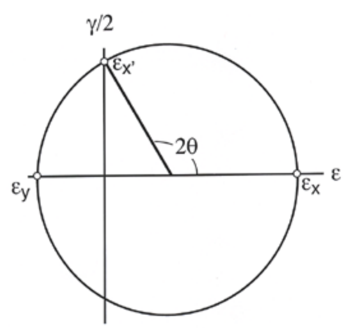
B. With $\sigma_3 = 0$, $_1 = (1/E)[\sigma_1 - \upsilon\sigma_2]$, and $e_2 = (1/E)[\sigma_2 - \upsilon\sigma_1]$, $\sigma_2 = Ee_2 + \upsilon\sigma_1$. Substituting, $e_1 = (1/E)[\sigma_1(1-\upsilon^2)] - \upsilon e_2$. $\sigma_1 = E(e_1 + \upsilon e_2)/(1-\upsilon^2) =$
$[205(1-.29^2)][0.000740 + 0.29(-0.000450)] = 136$ MPa
$\sigma_2 = E(e_2 + \upsilon e_1)/(1-\upsilon^2) = -52.7$ MPa

C. $\sigma_x = E(e_x + \upsilon e_y)/(1-\upsilon^2) = -47.9$ MPa
$\sigma_y = E(e_y + \upsilon e_x)/(1-\upsilon^2) = 131.6$ MPa
From equation 2.3, $G = E/[2(1+\upsilon)] = 79.5$ GPa, $\tau_{xy} = G\gamma_{xy} = 29.4$ MPa
$\sigma_1, \sigma_2 = (\sigma_x + \sigma_y)/2 \pm \{[(\sigma_x - \sigma_y)/2]^2 + (\tau_{xy})^2)\}^{1/2} = 136, -52.6$ MPa

D. They should correspond exactly.

6. A steel block ($E = 30\text{x}10^6$ psi and $\upsilon = 0.29$ is loaded under uniaxial compression along x.
A. Draw the Mohrs circle diagram.
B. There is an axis, x', along which the strain $\varepsilon_{x'} = 0$. What is the angle between x and x'?

<u>Solution: A.</u>



B. $e_{x'} = 0 = e_x\cos^2\theta + e_y\sin^2\theta$. $e_x/e_y = 1/0.29 = \tan^2\theta$. $\theta = \arctan(1\sqrt{0.29}) = 61.7°$
(2θ = 123° which agrees with the Mohr's circle diagram.

7. Poisson's ratio for rubber is 1/2. What does this imply about the bulk modulus?
<u>solution:</u> Substituting $\upsilon = 0$ into equation 2.6d, $B = E/[3(1-2\upsilon)]$ results in $B = \infty$, which means that rubber is incompressible.

8. A sandwich is made of a plate of glass surrounded by two plates of polymethyl - methacrylate as shown in Figure 2.11. Assume that the composite is free of stresses at 40 °C. Find the stresses when the sandwich is cooled to 20 °C? The properties of the glass and the

polymethylmethacrylate are given below. The total thicknesses of the glass and the PMMA are equal. Assume each to be isotropic and assume that creep is negligible.

glass

PMMA

thermal expansion coefficient (K^{-1})	$9.x10^{-6}$	$90.x10^{-6}$
Young's modulus (GPa)	69.	3.45
Poisson's ratio	0.28	.38

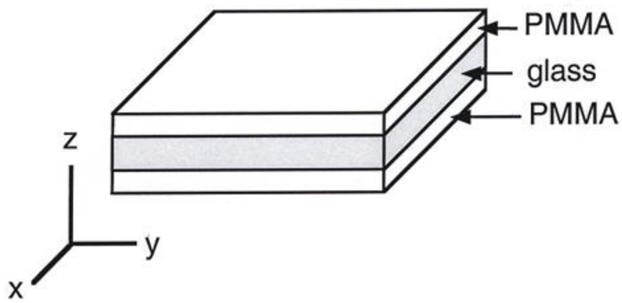


Figure 2.11. Laminated sheets of PMMA and glass.

Solution: From symmetry, $e_{xA} = e_{yA}$, $e_{xB} = e_{yB}$, $\sigma_{yA} = \sigma_{xA}$, $\sigma_{yB} = \sigma_{xB}$. The boundary conditions are that $\sigma_{zA} = \sigma_{zB} = 0$. From compatability, $e_{xA} = e_{xB}$, $e_{yA} = e_{yB}$,

From a force balance, $\sigma_{xA} = -\sigma_{xB}$, $\sigma_{yA} = -\sigma_{yB}$. Let PMMA = A and glass = B.

$e_{xA} = (1/E_A)[\sigma_{xA}(1 - \upsilon_A)] + \alpha_A\Delta T = (1/E_B)[-\sigma_{xA}(1 - \upsilon_B)] + \alpha_B\Delta T,$

$\sigma_{xA}[(1 - \upsilon_A)/E_A) + (1 - \upsilon_B)/E_B)] = (\alpha_B - \alpha_A)\,\Delta T.$

$\sigma_{xA} = (\alpha_B - \alpha_A)\Delta T[(1 - \upsilon_A)/E_A) + (1 - \upsilon_B)/E_B)] =$

$(9 – 90)\,x10^{-6}(– 40)/[\,(0.62)/\,3.45\,x10^9/(0.62) + (0.72)]\,/69x10^9] = 190$ MPa (tension)

9. A bronze sleeve, 0.040 in. thick, was mounted on a 2.000 in diameter steel shaft by heating it to 100°C, while the temperature of the shaft was maintained at 20°C Under these conditions, the sleeve just fit on the shaft with zero clearance. Find the principal stresses in the sleeve after it cools to 20°C. Assume that friction between the shaft and the sleeve prevented any sliding at the interface during cooling, and assume that the shaft is so massive and stiff that strains in the shaft itself are negligible. For this bronze, $E = 16x10^6$psi, $\upsilon = -\,0.30$, and $\alpha = 18.4x10^{-6}(°C)^{-1}$.

Solution: Let the axial, hoop and radial directions be x, y and z. $e_x = e_y = 0 =$

$(1/E)[\sigma_x - \upsilon(\sigma_y + \sigma_z)] + \alpha\Delta T = 0$. Substituting $\sigma_x = \sigma_y$ and $\sigma_z = 0$, $(1/E)\sigma_x(1 - \upsilon) + \alpha\Delta T = 0$.

$\sigma_x = -\,\alpha E\Delta T/(1 - \upsilon) = [18.4x10^{-6}(°C)^{-1}](16x10^6\text{psi})(-80°C)/0.7 = 33{,}600$ psi.

10 Calculate Young's modulus for an iron crystal when tension is applied along a <122> direction.

Solution: Using equation 2.19, $1/E_{122} = s_{11} + (-2s_{11} + 2s_{12} + s_{44})(4 + 4 + 16)/(4 + 4 + 1)^2 =$

$s_{11} + (-2s_{11} + 2s_{12} + s_{44})(24/81) = 0.407s_{11} + 0.296(2s_{12} + s_{44}) =$

$0.407(7.56) + 1.296(-2x2.78 + 8.59) = 7.00x10^{-12}\ Pa^{-1}$, $E_{122} = 143$ GPa

11. Zinc has the following elastic constants:

$s_{11} = 0.84x10^{-11}\ Pa^{-1}$; $s_{33} = 2.87x10^{-11}\ Pa^{-11}$; $s_{12} = 0.11x10^{-11}\ Pa^{-1}$

$s_{13} = -0.78x10^{-11}\ Pa^{-1}$; $s_{44} = 2.64x10^{-11}\ Pa^{-11}$; $s_{66} = 2(s_{11} - s_{12})$

Find the bulk modulus of zinc?

Solution: $B = (1/3)(\sigma_1 + \sigma_2 + \sigma_3)/(e_1 + e_2 + e_3)$; $\sigma_1 = \sigma_2 = \sigma_3$ so
$B = \sigma/(e_1 + e_2 + e_3) = \sigma/\{[(s_{11} + s_{12} + s_{13}) + (s_{12} + s_{11} + s_{13}) + (s_{13} + s_{13} + s_{33})]\sigma\} =$
$1/(2s_{11} + 2s_{12} + 4s_{13} + s_{33}) = 1/\{[2(0.84) + 2(0.11) + 4(-.78) + 2.64] \times 10^{-11}\ Pa^{-1}\} = 0.704 \times 10^{11}$ Pa = 70.4 GPa

12 Calculate the effective Young's modulus for a cubic crystal loaded in the [110] direction in terms of the constants, s_{11}, s_{12}, and s_{44}. Do this by assuming uniaxial tension along [110] and expressing $\sigma_1, \sigma_2, \ldots\tau_{12}$ in terms of $\sigma_{[110]}$. Then use the matrix of elastic constants to find $e_1, e_2, \ldots\gamma_{12}$, and finally resolve theses strains onto the [110] axis to find $e_{[110]}$.
Solution: For uniaxial tension along [110], $\sigma_1 = \sigma_2 = \tau_{12} = (1/2)\sigma_{[110]}$, $\sigma_3 = \tau_{23} = \tau_{31} = 0$,
$e_1 = e_2 = \sigma_1 s_{11} + \sigma_2 s_{12} + \sigma_3 s_{12} = (1/2)\sigma_{[110]}(s_{11} + s_{12})$
$\gamma_{12} = \tau_{12} s_{44} = (1/2)\sigma_{[110]} s_{44}$
$e_{[110]} = (1/2)e_1 + (1/2)e_2 + (1/2)$; $\gamma_{12} = (1/4)(2s_{11} + 2s_{12} + s_{44})\sigma_{[110]}$
$E_{[110]} = \sigma_{[110]}/e_{[110]} = 4/(2s_{11} + 2s_{12} + s_{44})$.

13. When a polycrystal is elastically strained in tension, it is reasonable to assume that the strains in all grains are the same. According to this assumption, calculate for iron, the ratio of the stress in grains oriented with <111> parallel to the tensile axis to the average stress, σ_{111}/σ_{av}. Then calculate σ_{100}/σ_{av}.
Solution: $E_{100} = 1/s_{11} = 1/7.56(\text{TPa})^{-1} = 132$ GPa, From Table II, $E_{111} = 2.14\, E_{100} = 283$ GPa, and From Table III, $E_{av} =$ 208.2 GPa so
$\sigma_{111}/\sigma_{av.} = E_{111}/E_{av.} = 283/208 = 1.36$; $\sigma_{100}/\sigma_{av.} = E_{100}/E_{av.} = 132/208 = 0.58$

14. Take the cardboard back of a pad of paper and cut it so that is a square. Then support the cardboard horizontally with blocks at each end apply a weight in the middle and note the deflection. Next rotate the cardboard 90° and repeat the experiment with the same weight. By what factor do the two measured deflections differ? By what factor does the elastic modulus, *E*, vary with direction? Why was the cardboard used for the backing of the pad placed in the orientation that it was?
Solution: The stiffness ratio varies with the cardboard, but deflection ratio is in the range of 5. The modulus ratio must be the same. The stiffer direction id aligned with the longer dimension of the pad to minimize deflection.

Chapter 3

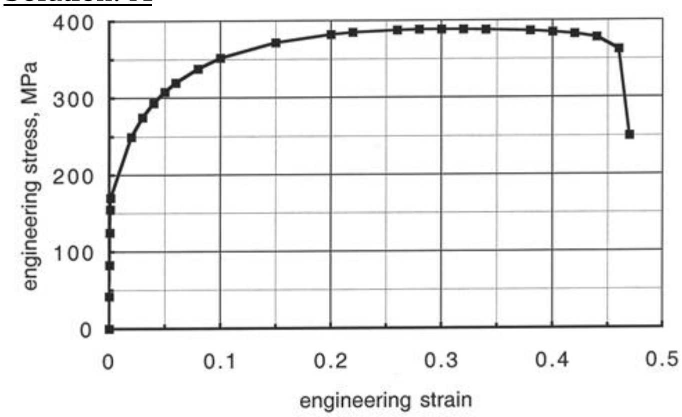
1. The results of a tensile test on a steel test bar are given below. The initial gauge length was 25.0 mm and the initial diameter was 5.00 mm. The diameter at the fracture was 2.6 mm. The engineering strain and engineering stress in MPa are:

Strain	stress	strain	stress	strain	stress
0.0	0.0	0.06	319.8	0.32	388.4
0.0002	42.	0.08	337.9	0.34	388.0.
0.0004	83.	0.10	351.1	0.38	386.5
0.0006	125	0.15	371.7	0.40	384.5
0.0009	155	0.20	382.2	0.42	382.5
0.0015	170.	0.22	384.7	0.44	378.
0.02	249.7	0.24	386.4	0.46	362
0.03	274.9	0.26	387.6	0.47	250
0.04	293.5	0.28	388.3	0.05	308.
0.30	388.5				

A. Plot the engineering stress strain curve.

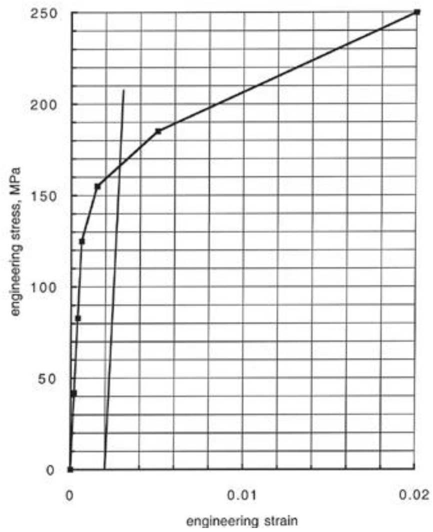
B. Determine i. Young's modulus ii. the 0.2% offset yield strength iii. the tensile strength. iv. the percent elongation iv. the percent reduction of area.

<u>Solution: A</u>



D. i. 83/.0004 = 207 GPa

ii. expanding the low strain region of the graph and constructing a 0.002 offset, YS = 167 MPa

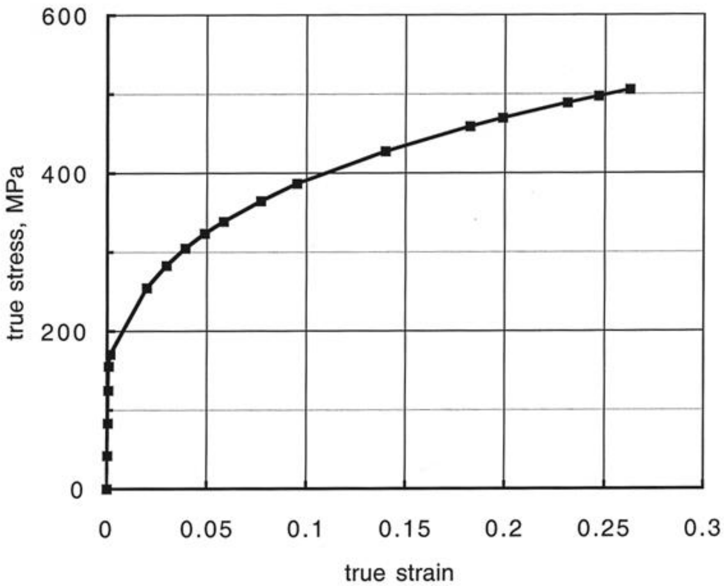


iii. The TS = max engineering stress is 388.5 MPa

iv. RA = 1- $(2.6/5)^2$ = 0.73 or 73%

2. Construct the true stress - true strain curve for the material in problem 1. Note that necking starts at maximum load, so the construction should be stopped at this point.

<u>Solution:</u>



3. Determine the engineering strains, e, and the true strains, ε, for each of the following:

A. Extension from L = 1.0 to L = 1.1

B. Compression from h = 1 to h = 0.9

C. Extension from L = 1 to L = 2

D Compression from h = 1 to h = 1/2

<u>Solution:</u> A. e = 0.1, ε = ln(1.1/1) = 0.095

B. e = -0.1, ε = ln(0.95/1) = -1.05

C e = 1, ε = ln(2/1) =0.693

D. e = -0.5 , ε =ln(.5/1) = -0.693

4. The ASM *Metals Handbook* (Vol. 1, 8th ed., p 1008) gives the % elongation in a 2 in. gauge section for annealed electrolytic tough-pitch copper as

55% for a 0.505 in. diameter bar,
45% for a 0.030 in. thick sheet and
38.5% for a 0.010 in diameter wire.

Suggest a reason for the differences.

Solution: The total % elongation is the sum of the uniform % elongation and the % necking elongation. The later depends on the ratio of the specimen diameter (or thickness) to the gauge length. For wire and sheet this is very small.

5. The tensile strength of iron-carbon alloys increases as the % carbon increases up to contents of about 1.5 to 2%. Above this it drops rapidly with increased % carbon. Speculate about the nature of this abrupt change.

Solution: Up to about 1.5 to 2% the tensile strength corresponds to the stress at which necking occurs. With more carbon, the flow stress increases, so the tensile strength increases. Beyond about 1.5 to 2% the tensile strength corresponds to the fracture stress. Increased carbon content lowers the fracture strength. It is present as graphite flakes.

6. The area under an engineering stress-strain curve up to fracture is the energy/volume. The area under a true stress-strain curve up to fracture is also the energy/volume. If the specimen necks, these two areas are not equal. What is the difference? Explain.

Solution: The area under the true stress strain curve is the energy per volume at the location of the maximum strain (in the neck). The area under the engineering stress strain curve is the total energy per total volume. They are the same as long as the deformation is uniform, but after necking starts, the energy is concentrated in the neck.

7. Suppose it is impossible to use an extensometer on the gauge section of a test specimen. Instead a button-head specimen (Figure 3.2c) is used and the strain is computed from the cross head movement. There are two possible sources of error with this procedure. One is that the gripping system may deform elastically and the other is that the button head may be drawn partly through the collar. How would each of these errors affect the calculated true stress and true strain?

Solution: In both cases the measured elongation would be too large, so the calculated true strain would be too great. This causes the calculated cross sectional area to be too small so calculated true stress would be too high.

8. Equation 3.10 relates the average axial stress in the neck, σ, to the effective stress, »σ. The variation of the local stresses with distance, r, from the center is given by

$$\sigma_z = \bar{\sigma}\{1 + \ln[1 + a/(2R) - r^2/(2aR)]\} \quad \text{with} \quad \sigma_x = \sigma_y = \sigma_z - \bar{\sigma}.$$

Derive an expression for the level of hydrostatic stress, $\sigma_H = (\sigma_x + \sigma_y + \sigma_z)/3$ at the center in terms of a, R, and $\bar{\sigma}$.

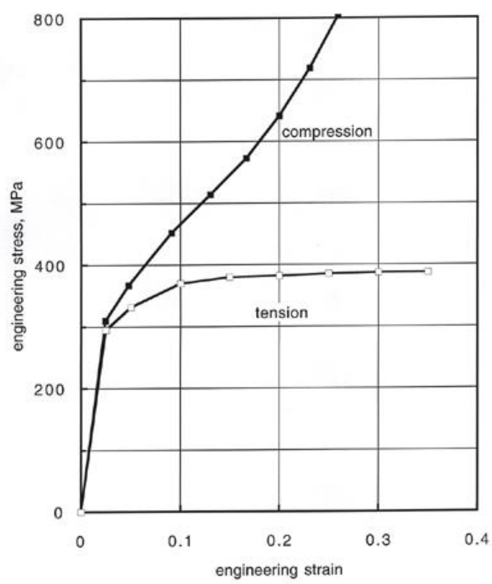
Solution: $\sigma_H = (\sigma_x + \sigma_y + \sigma_z)/3 = (3\sigma_z - 2\bar{\sigma})/3 = \sigma_z - (2/3)\bar{\sigma} =$

At $r = 0$, $\sigma_H/\bar{\sigma} = (1/3) + \ln[1 + a/(2R)]$

9. The tensile strengths of brazed joints between two pieces of steel are often considerably higher than the tensile strength of the braze material itself. Furthermore, the strength of thin joints is higher than that of a thick joint. Explain.
Solution: The tensile strength of the braze itself is determined by necking. However in the joint, as the softer braze material tends to deform under tension, the steel prevents it from contracting laterally. This builds up a high state of hydrostatic tension raising the stresses to a level that either the braze or the braze-steel interface fractures.

10. Two strain gauges were mounted on opposite sides of a tensile specimen. Strains were measured as the bar was pulled in tension and used to compute Young's modulus. Readings from one gauge gave a modulus much higher than those from the other gauge. What was the probable cause of this discrepancy?
Solution: Bending of the specimen. If the specimen were initially bent, the inside of the bend will elongate more, leading to a lower calculated modulus.

11 The engineering stress-strain curve from a tension test on a low-carbon steel is plotted in Figure 3.34. From this construct the engineering stress-strain curve in compression, neglecting friction.
solution:



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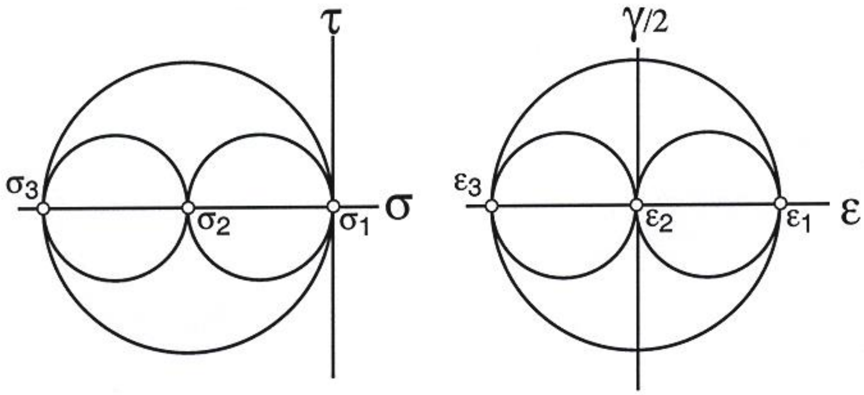
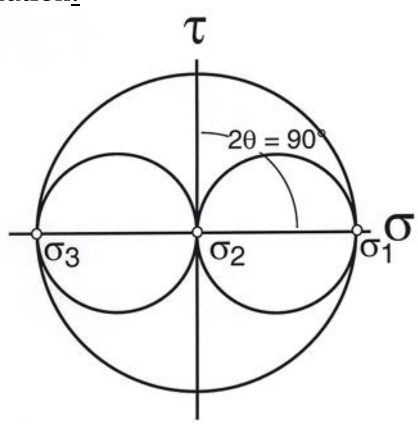
12 Discuss the how friction and inhomogeneous deformation affect the results from
The two types of plane-strain compression tests illustrated in Figure 3.19.
The plane-strain tension tests illustrated in Figure 3.20.
Solution: A. In channel compression there is friction on the sides as well as the top and bottom.
In the wide sheet compression there is no side-wall friction, but the deformation at the extreme edges is in uniaxial compression and there is a small region of lateral spreading.
There is no friction in any of the plane-strain tension tests. In all of them, however, there is a region at the edges where the stress state is uniaxial tension and there is lateral contraction.

13 Sketch the 3-dimensional Mohr's stress and strain diagrams for a plane-strain compression test.

Solution:



13. Draw a Mohr's circle diagram for the surface stresses in a torsion test, showing all three principal stresses. At what angle to the axis of the bar are the tensile stresses the greatest?
Solution:



The stresses are greatest at 45° to the bar axis

15. For a torsion test, derive equations relating the angle, ψ, between the axis of the largest principal stress and the axial direction and the angle, ψ□, between the axis of the largest principal strain and the axial direction in terms of L, r, and the twist angle, θ. Note that for finite strains, these two angles are not the same.
Solution: The principal stress axes are at ψ, = 45° to L and θ directions.
The shear strain, $\gamma = r\theta/L$ so the axis of the largest principal strain axes is at $\psi = \arctan[(r\theta/L) + 1]$ and the other principal axis is at 90° to it.

16. Derive an expression relating the torque, T, in a tension test to the shear stress at the surface, τ_S, in terms of the bar diameter, D, assuming that the bar is
A. entirely elastic so τ varies linearly with the shear strain, γ.
B. entirely plastic and does not work harden so the shear stress, τ, is constant.
Solution: In a differentially thick annulus, $dT = \tau(2\pi r)(r)dr$
A. $\tau = (r/R)\tau_S$, so $T = \int(r/R)\tau_S(2\pi r)(r)dr = (2\pi\tau_S)/R)\int r^3 dr = (1/2)\pi\tau_S R^3$
B. $\tau = \tau_o$, so $T = \tau_o 2\pi\int r^2 dr = (2/3)\pi/\tau_S R^3$

17. The principal strains in a circular bulge test are the thickness strain, ε_t, the circumferential (hoop) strain, ε_c, and the radial strain, ε_r. Describe how the ratio, $\varepsilon_c/\varepsilon_r$, varies over the surface of the bulge. Assume that the sheet is locked at the opening.
Solution: At the periphery of the circle, next to where it is clamped the circumferential (hoop) strain, ε_c, must be zero so $\varepsilon_c/\varepsilon_r$, = 0. At the center of the dome, the hoop and the radial strains are equal so $\varepsilon_c/\varepsilon_r = 1$.

8. Derive an expression for the fracture stress, S_f, in bending as a function of F_f, L, w, and t for three point bending of a specimen having a rectangular cross section, where F_f is the force at fracture, L is the distance between supports, w is the specimen width and t is the specimen thickness. Assume the deformation is elastic, the deflection, y, in bending is given by $y = \alpha FL^3/(EI)$ where α is a constant and E is Young's modulus. How would you expect the value of α to depend the ratio of t/w?
Solution: From equation 4.16, $\sigma_s = 6M/(wt^2)$ where $M = FL/4$. so $\sigma_s = (3/2)FL/(wt^2)$ so
$S_f = (3/2)F_fL/(wt^2)$
The stress in a wide specimen corresponds to plane strain conditions, whereas in a very narrow specimen it corresponds to plane stress. Therefore the deflection in the very wide specimen will be less for the same value of $FL^3/(EI)$. α increases with t/w.

19. Equation 3.16 gives the stresses at the surface of bend specimens. The derivation of this equations is based on the assumption of elastic behavior. If there is plastic deformation during the bend test, will the stress predicted by these equations be a) too low, b) too high, c) either too high or too low depending on where the plastic deformation occurs or d) correct.
Solution: For the same bending moment, the surface stress is (a) lower if the specimen deforms plastically than if it remains elastic.

20. By convention, Brinell, Meyer, Vickers and Knoop hardness numbers are stresses expressed in units of kg/mm^2, which is not an SI unit. To what stress, in MPa, does a Vickers hardness of 100 correspond?
Solution: The force corresponding to 1 kg is $1kg x 10 m^2/s = 10$ N, so
$100\ kg/mm^2 = (100\ kg/mm^2)(10\ N/kg)(10^6 mm^2/m^2) = 1000$ MPa (1 GPa)

21. In making Rockwell hardness tests, it is important that bottom of the specimen is flat so that the load doesn't cause any bending of the specimen. On the other hand, this is not important in making Vickers or Brinell hardness tests. Explain
Solution: The Rockwell machine senses the depth of the penetrator, so specimen deflection will affect the reading. Brinell and Vickers hardnesses are direct measures of the amount of plastic deformation, which depends on the load, not the deflection.

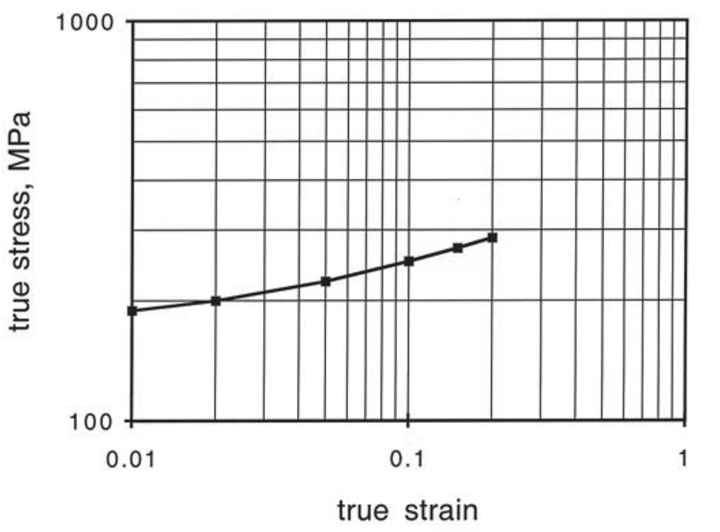
Chapter 4

1. What are the values of K and n in Figure 4.3?
Solution: By extrapolating to $\varepsilon=1$, $K = 580$
$\sigma_1/\sigma_2 = (\varepsilon_1/\varepsilon_2)^n$, $n = \ln(\sigma_1/\sigma_2)/\ln(\varepsilon_1/\varepsilon_2)^n$. Substituting $\sigma_1 = 580$ at $\varepsilon_1 = 1$ and $\sigma_2 = 170$ at $\varepsilon_1 = 0.01$, $n = \ln(580/170)/\ln(1/0.01) = 0.27$

2. The following true stress - true strain data were obtained from a tension test.

strain	stress (MPa)	strain	stress (MPa)
0.00	0.00	0.10	250.7
0.01	188.8	0.15	270.6
0.02	199.9	0.20	286.5
0.05	223.5		

A. Plot true stress vs. true strain on a logarithmic plot
B. What does your plot suggest about n in equation 4.3?
C. What does your plot suggest about a better approximation?
Solution: A.



B. n increases with strain
$\sigma = K(\varepsilon_0+\varepsilon)^n$ where ε_0 is about 0.01

3. The true stress strain curve of a material obeys the power hardening law with $n = 0.18$. A piece of this material was given a tensile strain of $\varepsilon = 0.03$ before being sent to a laboratory for

tension testing. The lab workers were unaware of the prestrain and tried to fit their data to equation 4.3.

A. What value of n would they report if they determined n from the elongation at maximum load?

B. What value of n would they report if they determined n from the loads at $\varepsilon = 0.05$ and 0.15?

Solution: A. They would observe a maximum load at of $\varepsilon = 0.015$, so they would report $n = 0.15$.

B. At what they measured as $\varepsilon = 0.05$, the stress would be $K0.8^n$ and at what they measured as $\varepsilon = 0.15$, the stress would be $K0.18^n$, so they would deduce $n = \ln(\sigma_1/\sigma_2)/\ln(\varepsilon_1/\varepsilon_2)^n = \ln(0.05/.18)^{.18}/\ln(.03/.15) = .14$

4. In a tension test the following values of engineering stress and strain were found: $s = 133.3$ MPa at $e = 0.05$, $s = 155.2$ MPa at $e = 0.10$ and $s = 166.3$ MPa at $e = 0.15$.

A. Determine whether the data fit equation 4.3.

B. Predict the strain at necking.

Solution: For $s = 133.3$ MPa at $e = 0.05$, $\sigma_1 = 1.05\text{x}133.3 = 140.0$ MPa at $\varepsilon_1 = \ln(1.05) = 0.0488$.

For $s = 155.2$ MPa at $e = 0.10$, $\sigma_2 = 1.10\text{x}155.2 = 170.7$ MPa at $\varepsilon_2 = \ln(1.10) = 0.0953$

For $s = 166.3$ MPa at $e = 0.15$, $\sigma_3 = 1.15\text{x}166.3 = 191.2$ MPa at $\varepsilon_2 = \ln(1.15) = 0.1398$

Comparing 1 and 2, $n = \ln(170.7/139.7)/\ln(0.0953/.0488) = 0.29$

Comparing 3 and 2, $n = \ln(170.7/191.2)/\ln(0.0953/.1398) = 0.296$

Comparing 1 and 3, $n = \ln(191.2/139.7)/\ln(0.1398/.0488) = 0.296$

This does fit equation 4.3 within experimental error

5. Two points on a stress strain curve for a material are:

$\sigma = 278$ MPa at $\varepsilon = 0.08$ and $\sigma = 322$ MPa at $\varepsilon = 0.16$.

A. Find K and n in the power-law approximation and predict σ at $\varepsilon = 0.20$.

B. For the approximation $\sigma = K(\varepsilon_o + \varepsilon)^n$ (equation 5.4) with $\varepsilon_o = 0.01$, find K and n and predict σ at $\varepsilon = 0.20$.

Solution: A. $n = \ln(278/322)/\ln(.08/.16) = 0.212$, $K = 322/.16^{.212} = 475$, at $\varepsilon = 0.20$, $\sigma = 337.6$

B. $n = \ln(278/322)/\ln(.09/.17) = 0.231$, $K = 322/.17^{.231} = 484.9$, at $\varepsilon = 0.20$, $\sigma = 338$.

6. The tensile stress - strain curve of a certain material is best represented by a saturation model, $\sigma = \sigma_o[1 - \exp(-A\varepsilon)]$.

A. Derive an expression for the true strain at maximum load in terms of the constants A and σ_o.

B. In a tension test, the maximum load occurred at an engineering strain of e = 21% and the tensile strength was 350 MPa.

Determine the values of the constants A and σ_o for the material.

[Remember that the tensile strength is the maximum <u>engineering</u> stress.]

<u>Solution:</u> A. Maximum load corresponds to $\sigma = d\sigma/d\varepsilon$ or $\sigma_o[1 - \exp(-A\varepsilon)] = A\sigma_o\exp(-A\varepsilon)$,

$(A + 1)\exp(-A\varepsilon) = 1$, $\exp(-A\varepsilon) = 1/(A+1)$; $(-A\varepsilon) = \ln(1/A+1)$; $\varepsilon = \ln(A+1)/A$

B.The <u>true</u> stress at maximum load is then $\sigma_o[1 - \exp(-A\varepsilon)] = \sigma_o\langle 1 - \exp\{-A[\ln(A+1)/A]\}\rangle$

$= \sigma_o[1/(A+1)] = 1 - \sigma_oA/(A+1)$.

The engineering stress $= \sigma/(1+e) = \sigma_oA/(A+1)\exp[\ln(A+1)/A] = \sigma_oA/\exp A$.

$\ln(1.21) = .1906 = \ln(A+1)/A$. Solving numerically for A, A =14.32.

7. A material has a stress strain relation that can be approximated by $\sigma = 150 + 185\varepsilon$. For such a material

A. What percent uniform elongation should be expected in a tension test?

B. What is the material's tensile strength?

<u>Solution:</u> A. At necking, $\sigma = d\sigma/d\varepsilon$ or $150 + 185\varepsilon = 185$, $\varepsilon = 35/185 = 0.189$. $e = \exp(0.189) - 1 = 0.208$ so % elongation at max load is 20.8%

B. $\sigma_{max\ load} = 150 + 185(0.189) = 185$, $S_{max} = 185/(1.208) = 153$

8. A. Derive expressions for the true strain at the onset of necking if the stress strain curve is:
$\sigma = K(\varepsilon_o + \varepsilon)^n$

B. Write an expression for the tensile strength.

<u>Solution:</u> A. At necking, $\sigma = d\sigma/d\varepsilon$ or $K(\varepsilon_o + \varepsilon)^n = n'K'(\varepsilon_o + \varepsilon)^{n'-1}$. $\varepsilon_o + \varepsilon = n$, $\varepsilon = n - \varepsilon_o$

B. $\sigma_{max\ load} = K(\varepsilon_o + n - \varepsilon_o)^n = K(n)^n$, $S_{max} = K(n)^n/\exp(n - \varepsilon_o)$.

9. Consider a tensile specimen made from a material that obeys the power hardening law with K = 400 MPa and n = 0.20. Assume K is not sensitive to strain rate. One part of the gauge section has an initial cross-sectional area that is 0.99 times that of the rest of the gauge section. What will be the true strain in the larger area after the smaller area necks and reduces to 50% of its original area?

<u>Solution:</u> Using equation 4.15 with $\varepsilon_b = n = 0.2$, $\varepsilon_a{}^{.2}\exp(-\varepsilon_a) = 0.99(.2^{.2})\exp(-.2) = 0.5875$

Solving numerically, $\varepsilon_a = 0.1433$

10. Consider a tensile bar that was machined so that most of the gauge section was 1.00 cm in diameter. One short region in the gauge section has a diameter 1/2 % less (0.995 cm). Assume the stress strain curve of the material is described by the power law with K = 330 MPa and n = 0.23 and the flow stress is not sensitive to strain rate. The bar was pulled in tension well beyond maximum load and it necked in the reduced section.

A. Calculate the diameter away from the reduced section.

B. Suppose that an investigator had not known that the bar initially had a reduced section and had assumed that the bar had a uniform initial diameter of 1.00 cm. Suppose that she measured the diameter away from the neck and had used that to calculate n. What value of n would she have calculated?

Solution: Using equation 6.15 with $f = .995^2 = .99$, $\varepsilon_a^{.23}\exp(-\varepsilon_a) = 0.99(.23^{.23})\exp(-.23) = 0.5610$. Solving numerically, $\varepsilon_a = 0.1686$. She would have concluded that $n = 0.1686$ instead of 0.23.

11. A tensile bar was machined so that most of the gauge section had a diameter of 0.500 cm. One small part of the gauge section had a diameter 1% smaller (0.495 cm). Assume power law hardening with $n = 0.17$. The bar was pulled until necking occurred.

A. Calculate the uniform elongation (%) away from the neck.

B. Compare this with the uniform elongation that would have been found if no reduced section was initially present.

Solution: A. Using equation 6.15 with $f = (0.495/0.5)^2 = 0.98$,
$\varepsilon_a^{.17}\exp(-e_a) = 0.98(.17^{.17})\exp(-.17) = 0.61175$, Solving numerically, $\varepsilon_a = 0.0999$ or $e_a = 10.5$ %

B. If there were no reduced section, $\varepsilon_a = 0.17$ or $e_a = 18.5\%$. This is much larger than $\varepsilon_a = 0.0999$

10. Repeated cycles of freezing of water and thawing of ice will cause copper pipes to burst. Water expands about 8.3% when it freezes.

A. Consider a copper tube as a capped cylinder that cannot lengthen or shorten. If it were filled with water, what would be the circumferential strain in the wall when the water freezes?

B. How many cycles of freezing/thawing would it take to cause the tube walls to neck? Assume $n = 0.55$.

Solution: A. $e = 8.3\%$, $\varepsilon = \ln(1.086) = 0.0797$

B. Each freezing would cause an additional strain of 0.0797. It can stand 0.55/0.797 = 6.89 cycles so it would neck on the 7th cycle.

<u>Chapter 5</u>

1. For the Mises yield criterion with $\sigma_z = 0$, calculate the values of σ_x/Y at yielding if

   a. $\alpha = 1/2$ b. $\alpha = 1$ c. $\alpha = -1$ d. $\alpha = 0$ where $\alpha = \sigma_y/\sigma_x$.

<u>Solution</u>: $\sigma_x/Y = 1/(1 - \alpha + \alpha^2)^{1/2}$.

a. If $\alpha = 1/2$, $\sigma_x/Y = 1.154$; b. If $\alpha = 1$, $\sigma_x/Y = 1$: c. If $\alpha = -1$, $\sigma_x/Y = 0.577$: d. If $\alpha = 0$, $\sigma_x/Y = 1$.

2. For each of the values of α in problem 1, calculate the ratio $\rho = d\varepsilon_y d\varepsilon_x$.

<u>Solution</u>: Using equation 6.15, $\rho = (\alpha - 1/2)/(1 - \alpha/2)$. Now substituting -

a. If $\alpha = 1/2$, $\rho = 0$: b. If $\alpha = 1$, $\rho = 1$: c. If $\alpha = -1$, $\rho = -1$: d. If $\alpha = 0$, $\rho = -1/2$.

3. Repeat problems 1 and 2 assuming the Tresca criterion instead of the Mises criterion.

<u>Solution</u>: For ,σ_x/Y a. If $\alpha = 1/2$, $\sigma_x/Y = 1$; b. If $\alpha = 1$, $\sigma_x/Y = 1$: c. If $\alpha = -1$, $\sigma_x/Y = 0.5$,: d. If $\alpha = 0$, $\sigma_x/Y = 1$.

For ρ, a. If $\alpha = 1/2$, $\rho = 0$; b. If $\alpha = 1$, $0 \le \rho \le \infty$: c. If $\alpha = -1$, $\rho = -1$: d. If $\alpha = 0$, $-1 \le \rho \le 0$:

4. Repeat problems 1 and 2 assuming the following yield criterion:

   $(\sigma_2 - \sigma_3)^a + (\sigma_3 - \sigma_1)^a + (\sigma_1 - \sigma_2)^a = 2Y^a$ where a = 8.

<u>Solution</u>: $\sigma_x/Y = \{2/[\alpha^8 + (1-\alpha)^8 + 1]^{1/8}$; for □ = 1/2, $\sigma_x/Y = 1.089$. For $\alpha = 1$, $\sigma_x/Y = 1$. For $\alpha = -1$, $\sigma_x/Y = 0.5447$. For $\alpha = 0$, $\sigma_x/Y = 1$.

$\varepsilon_y/\varepsilon_x = [(\alpha - 1)^7 + \alpha^7]/[1 + (1-\alpha)^7]$. For $\alpha = 1/2$, $\varepsilon_y/\varepsilon_x = 0$, For $\alpha = 1$, $\varepsilon_y/\varepsilon_x = 1$,

For $\alpha = -1$, $\varepsilon_y/\varepsilon_x = -1$, For $\alpha = 0$, $\varepsilon_y/\varepsilon_x = -1/2$

5. Consider a plane-strain tension test in which the tensile stress is applied along the x-direction. The strain, ε_y, is zero along the transverse direction and the stress in the z-direction vanishes. Assuming the von Mises criterion, write expressions for:

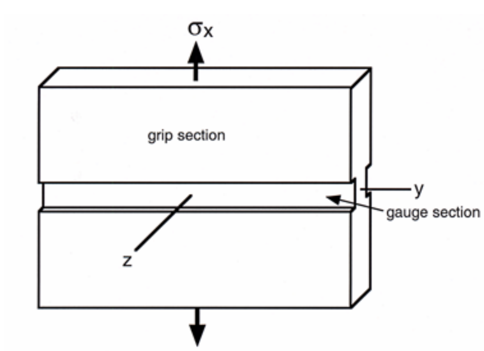
A. $\bar{\sigma}$ as a function of σ_x, and

B. $d\bar{\varepsilon}$ as a function of $d\varepsilon_x$.

C. Using the results of parts A and B, write an expression for the incremental work per volume, dw, in terms of σ_x and $d\varepsilon_x$.

D. Derive an expression for σ_x as a function of ε_x in such a test, assuming that the strain hardening can be expressed by $\bar{\sigma} = K$»ε^n.

Figure 6.12. Plane-strain tensile specimen. Lateral contraction of material in the groove is constrained by material outside the groove.



Solution: A. If $\varepsilon_y = 0$ and if $\sigma_z = 0$, $\sigma_y = (1/2)\,\sigma_x$. Substituting into the von Mises yield criterion, $\bar{\sigma} = \sqrt{(3/4)}\sigma_x$, B. $d\bar{\varepsilon} = \sqrt{(4/3)}d\varepsilon_x$. C. $dw = \bar{\sigma}d\bar{\varepsilon} = \sigma_x d\varepsilon_x$. D. Substituting, $\bar{\sigma} = \sqrt{(3/4)}\sigma_x$ and for monotonous loading $\bar{\varepsilon} = \sqrt{(4/3\varepsilon_x)}$; $\sqrt{(3/4)}\sigma_x = K(\sqrt{(4/3\varepsilon_x)})^n$, $\sigma_x = K(4/3)^{n+1}\varepsilon_x{}^n$.

6. A 1.00 cm diameter circle was printed onto the surface of a sheet of steel before forming. After forming the circle was found to be an ellipse with major and minor diameters of 1.18 cm. and 1.03 cm respectively. Assume that both sets of measurements were made when the sheet was unloaded, that during forming the stress perpendicular to the sheet surface was zero, and that the ratio, α, of the stresses in the plane of the sheet remained constant during forming. The tensile stress-strain curve for this steel is shown in Figure 6.13. Assume the von Mises yield criterion.

A. What were the principal strains, ε_1, ε_2 and ε_3?

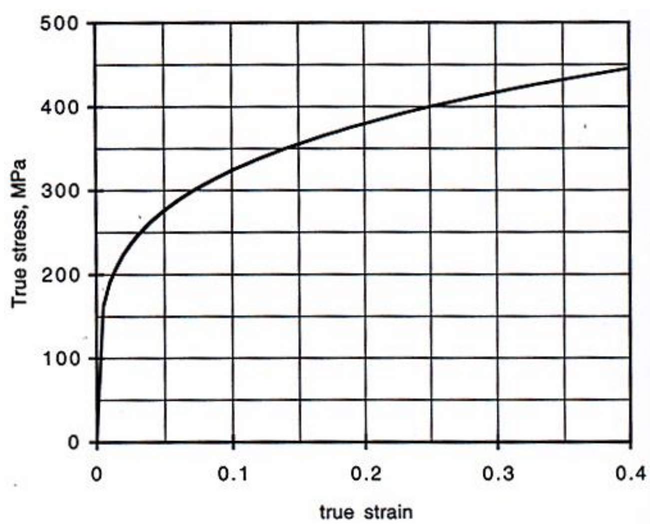
B. What was the effective strain, $\bar{\varepsilon}$?

C. What was the effective stress, $\bar{\sigma}$?

D. Calculate the ratio, $\rho = \varepsilon_2/\varepsilon_1$ and use this to find the ratio, $\alpha = \sigma_2/\sigma_1$. (Take σ_1 and σ_2 respectively as the larger and smaller of the principal stresses in the plane of the sheet.)

E. What was the level of σ_1?

Figure 6.13. True tensile stress strain curve for the steel in problem 6.



Solution: A. $\varepsilon_1 = \ln 1.18 = 0.1655$, $\varepsilon_2 = \ln 1.03 = 0.02956$, $\varepsilon_3 = -\varepsilon_2 - \varepsilon_1 = -0.1951$

B. $\bar{\varepsilon} = [(2/3)(\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2)]^{1/2} = 0.2103$

[note that $0.1951 < 0.2103 < (1.15 \text{x} 0.1951)$

C. from the graph, $\bar{\sigma} = 385$ MPa

D. $\rho = 0.2956/0.16550.1786$, $\alpha = (\rho + 1/2)/(\rho/2 + 1) = 0.623$

E. $\sigma_y = \bar{\sigma}/\sqrt{(1 - \alpha + \alpha^2)} = (1.143)(385) = 440$ MPa.

7. Measurements on the surface of a deformed sheet after unloading indicate that $e_1 = 0.154$ and $e_2 = 0.070$. Assume that the von Mises criterion is appropriate and that the loading was proportional (i.e. the ratio, $\alpha = \sigma_y/\sigma_x$ remained constant during loading.) It has been found that the tensile stress - strain relationship for this alloy can be approximated by $\sigma = 150 + 185\varepsilon$ where σ is the true stress in MPa and ε is the true strain.

A. What was the effective strain?

B. What was the effective stress?

C. What was the value of the largest principal stress?

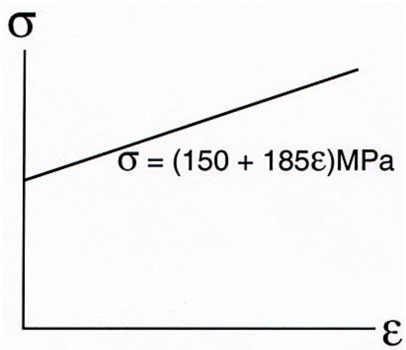


Figure 6.14. True tensile stress strain curve for the steel in problem 7.

<u>Solution:</u> A. $\varepsilon_1 = \ln(1.154) = 0.143$, $\varepsilon_2 = \ln(1.070) = 0.0677$, $\varepsilon_3 = -\varepsilon_1 - \varepsilon_2 = 0.211$.

$\bar{\varepsilon} = [(2/3)(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)]^{1/2} = [(2/3)(0.143^2) + (0.068)^2 + (-0.211)^2)]^{1/2} = 0.215$

B. $\bar{\sigma} = 150 + 185(0.215) = 190$ MPa

C. $\rho = 0.0677/0.143 = 0.473$, $\alpha = (\rho + 1/2)/(\rho/2 + 1) = 0.787$.

$\sigma_y = \bar{\sigma}/\sqrt{(1 - \alpha + \alpha^2)} = 208$ MPa

8. The following yield criterion has been proposed for an isotropic material: "Yielding will occur when the sum of the diameters of the largest and second largest Mohr's circles reaches a critical value. Defining $\sigma_1 \geq \sigma_2 \geq \sigma_3$, this can be expressed mathematically this can be expressed as:

If $(\sigma_1 - \sigma_2) \geq (\sigma_2 - \sigma_3)$, $(\sigma_1 - \sigma_2) + (\sigma_1 - \sigma_3) = C$ or $2\sigma_1 - \sigma_2 - \sigma_3 = C$, (1)

but if $(\sigma_2 - \sigma_3) \geq (\sigma_1 - \sigma_2)$, $(\sigma_1 - \sigma_3) + (\sigma_2 - \sigma_3) = C$ or $\sigma_1 + \sigma_2 - 2\sigma_3 = C$. (2)

A. Evaluate C in terms of the yield strength, Y, in uniaxial tension or the yield strength, -Y, in compression

B. Plot the yield locus as σ_x vs. σ_y for $\sigma_z = 0$ where σ_x, σ_y, and σ_z are principal stresses.

[Hint: For each regions in σ_x vs. σ_y stress space determine whether (1) or (2) applies.]

<u>Solution</u>: A. Consider an x-direction tension test. At yielding, $\sigma_x = \sigma_1 = Y$, $\sigma_y = \sigma_z = \sigma_2 = \sigma_3 = 0$. Therefore $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$ so criterion (1) applies, and $C = (\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2) = 2Y$ Therefore $C = 2Y$.

We can also think about an x-direction compression test. At yielding, $\sigma_x = \sigma_3 = -Y$, $\sigma_y = \sigma_z = \sigma_2 = \sigma_3 = 0$. Therefore $(\sigma_2 - \sigma_3) > (\sigma_1 - \sigma_2)$ so criterion II applies, and $C = (\sigma_1 - \sigma_3) + (\sigma_2 - \sigma_3) = -(-2Y)$ or again $C = 2Y$.

B. Now consider several loading paths:

In region A, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\sigma_z = \sigma_3 = 0$ and $\sigma_x > 2\sigma_y$ so $(\sigma_1 - \sigma_3) > (\sigma_1 - \sigma_2)$

Therefore criterion (1), $(\sigma_x - 0) + (\sigma_x - \sigma_y) = 2Y$, or $\sigma_x = Y + \sigma_y/2$

In region B, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\sigma_z = \sigma_3 = 0$ but $\sigma_x < 2\sigma_y$ so $(\sigma_1 - \sigma_3) < (\sigma_1 - \sigma_2)$

Therefore criterion (2), $(\sigma_x - 0) + (\sigma_y - 0) = 2Y$, or $\sigma_x = 2Y - \sigma_y$

In region C, $\sigma_y = \sigma_1$, $\sigma_x = \sigma_2$, $\sigma_z = \sigma_3 = 0$ but $\sigma_y < 2\sigma_x$ so $(\sigma_1 - \sigma_3) < (\sigma_1 - \sigma_2)$

Therefore criterion (2), $(\sigma_y - 0) + (\sigma_x - 0) = 2Y$, or $\sigma_y = 2Y - \sigma_x$

In region D, $\sigma_y = \sigma_1$, $\sigma_x = \sigma_2$, $\sigma_z = \sigma_3 = 0$ and $\sigma_y > 2\sigma_x$ so $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$

Therefore criterion (1), $(\sigma_y - 0) + (\sigma_y - \sigma_x) = 2Y$, or $\sigma_y = Y + \sigma_x/2$

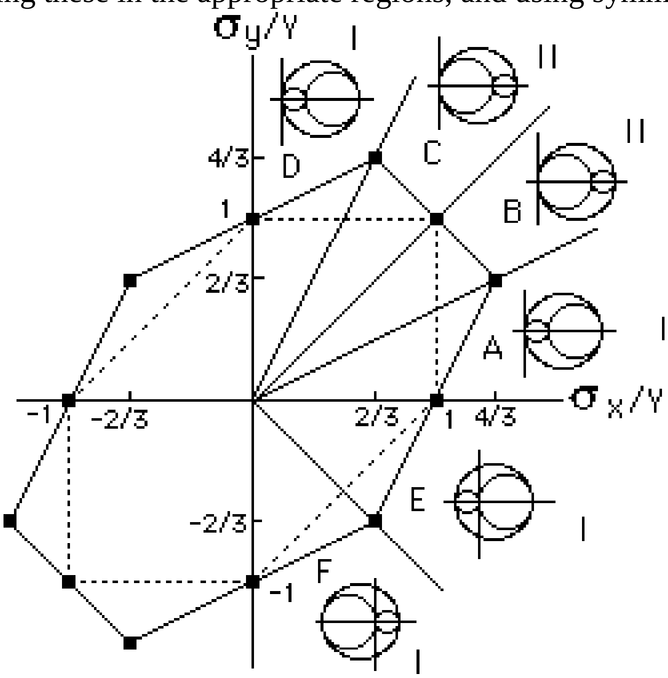
In region E, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_3$, $\sigma_z = \sigma_2 = 0$ and $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$

Therefore criterion (1), $(\sigma_x - 0) + (\sigma_x - \sigma_y) = 2Y$, or $\sigma_x = Y + \sigma_y/2$

In region F, $\sigma_x = \sigma_1$, $\sigma_y = \sigma_3$, $\sigma_z = \sigma_2 = 0$ so $(\sigma_1 - \sigma_2) > (\sigma_2 - \sigma_3)$

Therefore criterion (1), $(\sigma_x - 0) + (\sigma_x - \sigma_y) = 2Y$, or $\sigma_x = Y + \sigma_y/2$

Plotting these in the appropriate regions, and using symmetry to construct the left hand half:



9. Consider a long thin-wall tube, capped at both ends. It is made from a steel with a yield strength of 40,000 psi. Its length is 60 in., its diameter is 2.0 in. and the wall thickness is 0.015 in. The tube is loaded under an internal pressure, *P*, and a torque of 1500 in-lbs is applied.

A. What internal pressure can it withstand without yielding according to Tresca?

B. What internal pressure can it withstand without yielding according to von Mises?

<u>Solution:</u> $d/t = 60$ so this can be regarded as a thin wall tube. For this solution, stresses will be expressed in ksi.Let x = hoop dir, and y be the axizl dir.

$T = \tau(\pi dt)(d/2)$; $\tau = 2T/(\pi d^2 t) = 2\text{x}1.5./(\pi 2^2 0.015) = 15.91$ ksi

$\sigma_x = Pd/(2t)$, $\sigma_y = Pd/(4t)$, $\sigma_y = \sigma_x/2$

A. For Mises, substituting $\sigma_z = \tau_{yz} = t_{zx} = 0$ into the yield criterion, (equation 6.10)

$2Y^2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6\tau_{xy}^2$

$2(40)^2 = (\sigma_x/2)^2 + \sigma_x^2 + (\sigma_x/2)^2 + 6(15.9)^2 = (3/2)\sigma_x^2 + 6(15.9)^2$

$\sigma_x^2 = (2/3)[2(40)^2 - 6(15.9)^2] = 1{,}122$, $\sigma_x = 33.49$, $P = 33.49(2x0.015)/2$ 0.50 ksi or 500 psi

B. For Tresca, yielding will occur when $\sigma_1 = Y$

$\sigma_1 = (\sigma_x + \sigma_y)/2 + (1/2)[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2]^{1/2}$

Substituting $\sigma_y = .5\sigma_x$, $\sigma_1 = Y$

$Y = (3/4)\sigma_x + [\sigma_x^2/16 + \tau_{xy}^2]^{1/2}$

$[Y - (3/4)\sigma_x]^2 = \sigma_x^2/16 + \tau_{xy}^2$

$(9/16)\sigma_x^2 - (3/2)\sigma_x Y - \sigma_x^2/16 + \tau_{xy}^2 = 0$

$0.5\sigma_x^2 - (3/2)\sigma_x Y + Y^2 - \tau_{xy}^2 = 0$, Substituting $\tau_{xy} = 15.91$, and $Y = 40$

$\sigma_x = (3/2)Y + [60^2 - 2x(40^2 - 15.91^2)]^{1/2} = 60 + 30.1 = 90.1$

$P = 2t\sigma_x/d = 1.35$ ksi or 1350 psi

10. In flat rolling of a sheet or plate, the width does not appreciably change. A sheet of aluminum is rolled from 0.050 in. to 0.025 in. thickness. Assume the von Mises criterion.

A. What is the effective strain, »ε, caused by the rolling?

B. What strain in a tension test (if it were possible) would cause the same amount of strain hardening?

<u>Solution:</u> A. $\varepsilon_1 = \ln(.5) = -.693$, $\varepsilon_3 = \ln(2\,) ,= .693$, $\varepsilon_2 = 0$,

$\bar{\varepsilon} = (4/3)^{1/2}\,(0.693)] = (1.154)(.693) = 0.800$

B. the tensile strain would be 0.800

11. A piece of ontarium (which has a tensile yield strength of $Y = 700$ MPa) was loaded in such away that the principal stresses, σ_x, σ_y and σ_z were in the ratio of 1: 0: -0.25. The stresses were increased until plastic deformation occurred.

A. Predict the ratio of the principal strains, $\rho = \varepsilon_y/\varepsilon_x$, resulting from yielding according to von Mises.

B. Predict the value of $\rho = \varepsilon_y/\varepsilon_x$ resulting from yielding according to Tresca.

C. Predict the value of σ_x when yielding occurred according to von Mises.

D. Predict the value of σ_x when yielding occurred according to Tresca.

<u>Solution:</u> . The stress state produces the same strains as one in which the ratio of stresses is 1.25: 0.25:0. ($\alpha = 1/5$.). Therefore $\rho = d\varepsilon_y d\varepsilon_x = (1 - 0.2/2)/[0.2-(1/2)] = -0.3$.

B. According to Tresca, $\rho = 0$ (the stress state is on the right hand side of the locus where the normal is horizontal)

C. $\sigma_x = \bar{\sigma}\,/\sqrt{(1 - \alpha + \alpha^2)} = 700/\sqrt{(1 - .2 + .04)} = 764$ MPa.

D. According to Tresca, $\sigma_x = 700$ MPa.

12. A new yield criterion has been proposed for isotropic materials. It states that yielding will occur when the diameter of Mohr's largest circle plus half of the diameter of Mohr's second largest circle equals a critical value. This criterion can be expressed mathematically, following the convention that $\sigma_1 \geq \sigma_2 \geq \sigma_3$, as

$(\sigma_1 - \sigma_3) + 1/2(\sigma_1 - \sigma_2) = C$ if $(\sigma_1 - \sigma_2) \geq (\sigma_2 - \sigma_3)$ and

$(\sigma_1 - \sigma_3) + 1/2(\sigma_2 - \sigma_3) = C$ if $(\sigma_2 - \sigma_3) \geq (\sigma_1 - \sigma_2)$.

A. Evaluate C in terms of the tensile (or compressive) yield strength, Y.

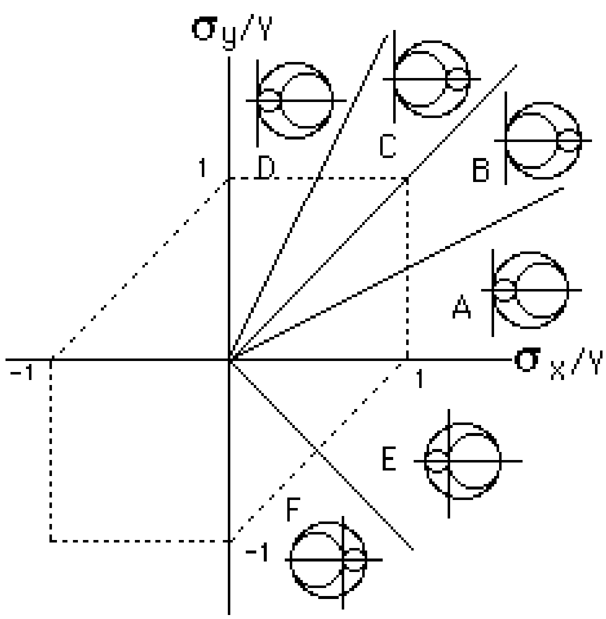
B. Let x, y and z be directions of principal stress, and let $\sigma_z = 0$.

Plot the σ_y vs. σ_x yield locus. (That is, plot the values of σ_y/Y and σ_x/Y that will lead to yielding according to this criterion.)

[Hint: consider different loading paths (ratios of σ_y/σ_x), and for each decide which stress (σ_1, σ_2, or σ_3) corresponds to (σ_x, σ_y or $\sigma_z = 0$), then determine whether $(\sigma_1 - \sigma_2) \geq (\sigma_2 - \sigma_3)$, substitute σ_x, σ_y and 0 into the appropriate expression, solve and finally plot.]

Solution: A. Consider a 1-direction tension test at yielding, $\sigma_1 = Y$, $\sigma_2 = \sigma = 0$, so $C = (3/2)Y$

B. First divide stress space into sectors according to the relative size of the Mohr's circles and apply the proposed yield criterion to each.



A. $(\sigma_x - \sigma_z) + 1/2(\sigma_x - \sigma_y) = 3/2Y$, or with $\sigma_z = 0$, $\sigma_x - 1/3\sigma_y = Y$

B. $(\sigma_x - \sigma_z) + 1/2(\sigma_y - \sigma_z) = 3/2Y$, or with $\sigma_z = 0$, $2/3\sigma_x + 1/3\sigma_y = Y$

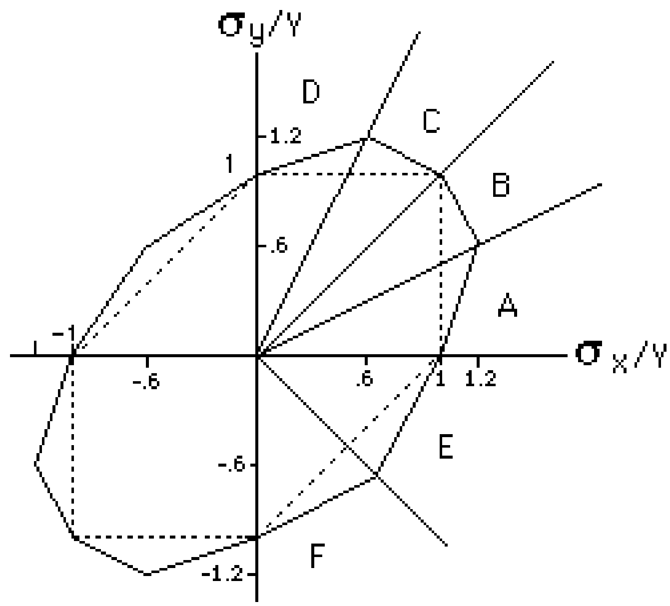
C. $(\sigma_y - \sigma_z) + 1/2(\sigma_x - \sigma_z) = 3/2Y$, or with $\sigma_z = 0$, $2/3\sigma_y + 1/3\sigma_x = Y$

D. $(\sigma_y - \sigma_z) + 1/2(\sigma_y - \sigma_x) = 3/2Y$, or with $\sigma_z = 0$, $\sigma_y - 1/3\sigma_x = Y$

E. $(\sigma_x - \sigma_y) + 1/2(\sigma_x - \sigma_z) = 3/2Y$, or with $\sigma_z = 0$, $\sigma_x - 2/3\sigma_y = Y$

F $(\sigma_x - \sigma_y) + 1/2(\sigma_z - \sigma_y) = 3/2Y$, or with $\sigma_z = 0$, $2/3\sigma_x - \sigma_y = Y$

Now plotting;



13. The tensile yield strength of an aluminum alloy is 14,500 psi. A sheet of this alloy is loaded under plane-stress conditions ($\sigma_3 = 0$) until it yields. On unloading it is observed that $\varepsilon_1 = 2\varepsilon_2$ and both ε_1 and ε_2 are positive.

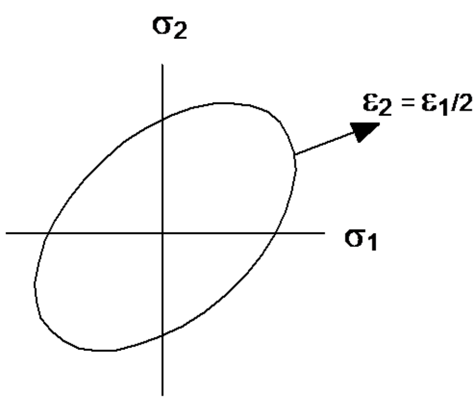
A. Assuming the von Mises yield criterion, determine the values of σ_1 and σ_2 at yielding.

B. Sketch the yield locus and show where the stress state is located on the locus.

<u>Solution</u>: A. $\alpha = (\rho+1/2)/(1+\rho/2) = (/1/2 + 1/2)/(1/4+1/2) = 0.8$.

$\sigma_1 = \bar{\sigma}/\sqrt{(1 - \alpha + \alpha^2)} = 14{,}500/\sqrt{(1-.8 +.64)} = 15{,}820$ psi; $\sigma_2 = (.8)(15{,}820) = 12{,}660$ psi

B.



14. Consider a capped thin-wall cylindrical pressure vessel, made from a material with planar isotropy and loaded to yielding under internal pressure. Predict the ratio of axial to hoop strains, $\rho = \varepsilon_a/\varepsilon_h$, as a function of R, using:

A) The Hill criterion and its flow rules (equations 6.33 and 6.34).

B) The high exponent criterion and its flow rules (equations 6.40 and 6.41).

<u>Solution</u>: $\sigma_a = (1/2)\sigma_h$.

A. For the Hill criterion, □□$\varepsilon_a/\varepsilon_h$ = [(R+1)(1/2) –R]/[R + 1 -(1/2)R] = (1-R)/(R+2)

B. For the high exponent criterion, $\varepsilon_a/\varepsilon_h = [R(-1/2)^{a-1}+(1/2)^{a-1}]/[1 + R(1/2)^{a-1}]$
This can be simplified. a-1 is an odd number so $(-1/2)^{a-1} = -(1/2)^{a-1}$ so
$\varepsilon_a/\varepsilon_h = [(1-R)(1/2)^{a-1}/[1 + R(1/2)^{a-1}]$.

15. In a tension test of an anisotropic sheet, the ratio of the width strain to the thickness strain, $\varepsilon_w/\varepsilon_t$, is R.

A. Express the ratio □□$\varepsilon_2/\varepsilon_1$, of the strains in the plane of the sheet, in terms of R. Take the 1 direction as the rolling direction, the 2-direction as the width direction in the tension test and the 3-direction as the thickness direction.

B. There is a direction, x, in the plane of the sheet along which $e_x = 0$. Find the angle, θ, between x and the tensile axis.

C. How accurately would this angle have to be measured to distinguish between two materials having R-values of 1.6 and 1.4?

<u>Solution:</u> A. $R = \varepsilon_2/\varepsilon_3 = \varepsilon_2/(-\varepsilon_2-\varepsilon_1) = -(\varepsilon_2/\varepsilon_1)/[(\varepsilon_2/\varepsilon_1)+1]$. $(\varepsilon_2/\varepsilon_1) = -R/(R+1)$

B. $\varepsilon_x = 0 = \varepsilon_1\cos^2\theta + \varepsilon_2\sin^2\theta$; $\varepsilon_1/\varepsilon_2 = -\sin^2\theta/\cos^2\theta = -\tan^2\theta$

$\theta = \text{atan}[(R+1)/R]^{1/2}$

C. If R = 1.6, θ = 51.9°, if R = 1.8, θ = 52.3. One would have to measure θ to ± 0.4/2 = 0.2°.

16. Redo problem 6, assuming the Tresca criterion instead of the Mises criterion

<u>Solution:</u> A. $\varepsilon_1 = 0.1655$, $\varepsilon_2 = 0.02926$, $\varepsilon_3 = -0.1951$

B $\bar{\varepsilon} = 0.1951$ (magnitude of the absolutely largest principal stress)

C. From the graph, $\bar{\sigma} = 380$ MPa

$\rho = 0.1786$, $\alpha = 1$ (the stress state must be at the corner)

E. $\sigma_1 = \sigma_2 = \bar{\sigma} = 380$ MPa

17. The total volume of a foamed material decreases when it plastically deforms in tension.

A. What does this imply about the effect of $\sigma_H = (\sigma_1 + \sigma_2 + \sigma_3)/3$ on the shape of the yield surface in $\sigma_1, \sigma_2, \sigma_3$ space?

B. Would the absolute magnitude of the yield stress in compression be greater, smaller or the same as the yield strength in tension?

C. When it yields in compression, would the volume increase, decrease or remain constant?

<u>Solution:</u>

A. If $\Delta v/v < 0$, the yield surface in $\sigma_1, \sigma_2, \sigma_3$ space must expand as σ_H increases.

B. The absolute magnitude of the compressive yield stress must be less than the tensile yield stress.

C. It would contract.