Chapter 10

1. Typical values for the dislocation density in annealed and heavily deformed copper are 10^7 and 10^{11} cm^{-2}.
A. Calculate the average distance between dislocations for both cases. For simplicity, assume all of the dislocations are parallel and in a square pattern.
B. Use equation 9.11 to calculate the energy/length of dislocation line for both cases assuming the dislocations are screws.
For copper: lattice parameter a = 0.361 nm, E = 110 GPa, $\upsilon = 0.30$, $\rho = 8.96$ Mg/m^3.
Solution: A. $d = 1/\sqrt{\rho}$; For $\rho = 10^7$, d = 316µm. For $\rho = 10^{11}$, d = 3.16µm.
B. $E_L = [Gb^2/(4\pi)]\ln(r_1/r_o)$
Taking G = E/[2(1+υ)] = 110GPa/(2x1.3) = 42.3G Pa,
r_o = b/4 = 0.090 nm and r_1= d/2 = so
$E_L = [(42.3\times10^9)(0.361\times10^{-9})^2/(4\pi)]\ln(d/0.180\times10^{-9}) =$
$4.39\times10^{-10}\ln(d/0.180\times10^{-9})$
For $\rho = 10^7$, $E_L = 4.39\times10^{-10}\ln(316\times10^{-6}/0.180\times10^{-9}) = 6.31\times10^{-9}$ J/m
For $\rho = 10^{11}$, $E_L = 4.39\times10^{-10}\ln(3.16\times10^{-6}/0.180\times10^{-9}) = 4.29\times10^{-9}$ J/m.

2. For a typical annealed metal, the yield stress in shear is $10^{-4}G$. Using a typical value for b, deduce the typical spacing of a Frank-Read source.
Solution: $\tau = 2Gb/d$, $d = 2Gb/\tau$. Taking b = 2×10^{-9}m,
$d = 2G(0.2\times10^{-9}\text{m})/(10^{-4}G)$ = 4 mm

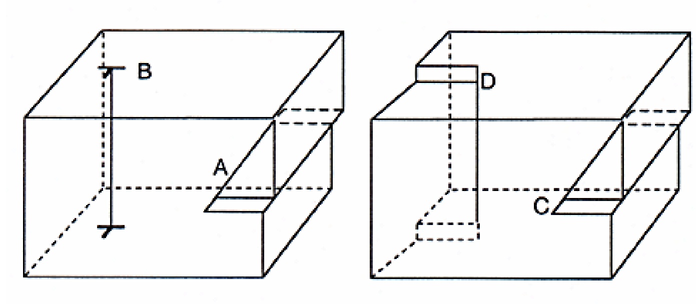
3. Several theoretical models predict that the dislocation density, ρ, should increase parabolically with strain, $\rho = Ce^{1/2}$. Assuming this and the dependence of t on r shown in Figure 10.4, predict the exponent n in power law approximation of the true stress-true strain curve, $\sigma = K\varepsilon^n$.
Solution: From Figure 10.4, $\tau = A\sqrt{\rho}$ (where A is a constant) so $\tau = CA\varepsilon^{1/4}$. Since σ is proportional to τ , $\sigma = C'\varepsilon^{0.25}$, or n = 0.25.

4. Orowan showed that the shear strain rate $\dot{\gamma} = d\gamma/dt = \rho b\bar{v}$ where ρ is the dislocation density, and $\bar{v}$ is the average velocity of the dislocation. In a tension test, the tensile strain rate, $\dot{\varepsilon}$, is approximately half of the shear strain rate, $\dot{\gamma}$. (The half assumes shear the Schmid factor is 1/2 which is a bit too high.) In a typical tension test, the crosshead rate is 0.2 in/min and the gauge section is 2 in. in length. Estimate $\bar{v}$ in a typical tension test for a typical metal with an initial dislocation density of 10^{10}/cm^2.
Solution: Taking $\dot{\gamma}$ = 2(0.2/60 in/s)/2 in = .033s^{-1}, $\bar{v} = \dot{\gamma}/(b\rho)$ =
$.033/[(10^{10}/\text{cm}^2)(10^4\text{cm}^2/\text{m}^2)(0.25\times10^{-9})] = 1.32\times10^{-7}$ m/s = 1.32×10^{-7} µm/s

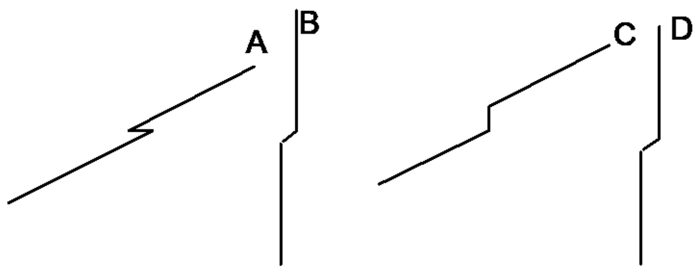
5. Consider the possible intersections of dislocations sketched in Figure 10.18. In the sketches, dislocations B and D are moving to the right and A and C are moving to the left.
A. Sketch the nature of the jogs or kinks in each dislocation after they intersect one-another.

B. Which, if any, would leave a trail of vacancies or interstitials?

Figure 10.18. Sketch for problem 5.



Solution: A



B. Neither

6. The sketches Figure 10.19 show pairs of dislocations that are about to intersect. The arrows show the directions of motion (Dislocations A, C and E are moving into the paper, dislocations B and F are moving out of the paper and dislocation D is moving to the left.)
A. After intersection, which of these dislocations (A, B, C, D, E, or F) would have jogs that would produce point defects if the dislocations continued to move in the direction shown?
B. For each answer to A, indicate whether the point defects would be vacancies or interstitials.

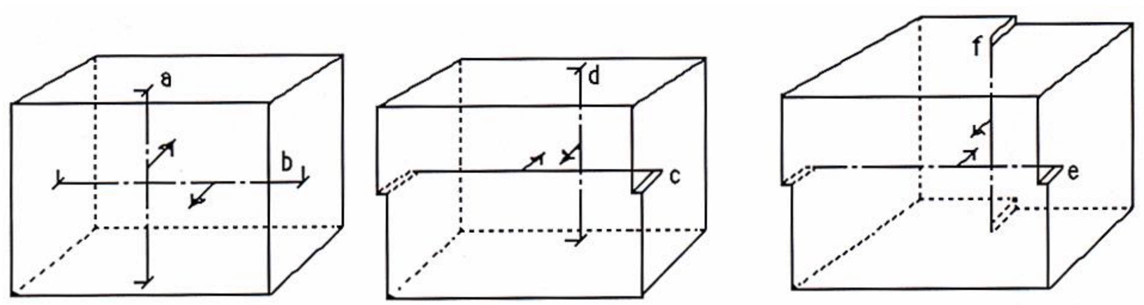


Figure 10.19. Sketch for problem 6

Solution: A. e and f
B. interstitials

7. Consider how dislocation intersections affect the stress necessary to continue slip. Let the density of screw dislocations be ρ(meters of dislocations/m^3) and for simplicity assume that these dislocations are equally divided into three sets of dislocations, each set being parallel to one of the three orthogonal axes.
A. Assume each set of dislocations is arranged in a square pattern. Express the distance between the dislocations in a set in terms of ρ.
B. Now consider the jogs formed as a screw dislocation moves. What will be the distance between jogs?

C. Assume the energy of forming a row of interstitials is so high that the jogs essentially pin the dislocation. Express the shear stress be necessary to continue the motion in terms of G, b, and ρ.

D. Now assume that the dislocation density is proportional to $\sqrt{\varepsilon}$. What would this model predict as the value of n in $\sigma = K\varepsilon^n$.

<u>Solution:</u> A. $d = 1/\sqrt{(\rho/3)}$

B $= d = 1/\sqrt{(\rho/3)}$

C. $\tau = 2Gb/d = 2Gb\sqrt{(\rho/3)}$

D. σ is proportional to τ which is proportional to $\varepsilon^{1/4}$ so n = .25

8A. Derive an equation for estimating the shear stress, τ, necessary to move a dislocation with vacancy-producing jogs in terms of the Burgers vector, b, the distance between jogs, d, and the energy to create a vacancy, E_v. Note that there must be a vacancy produced each time a segment of dislocation of length, d, advances a distance, b. The work done by the shear stress would be bdf_L where the force per length of dislocation is $f_L = \tau b$.

B. Evaluate τ for $d = 1$ μm, $E_v = 0.7$ eV and $b = 0.3$ nm.

<u>Solution:</u> A. The increase of energy for an advancement of he dislocation of length, d, by b is $bdf_L = E_v$. Since $f_L = \tau b$, $\tau = f_L/b = E_v/b^2d$

B. $\tau = [0.7\text{eV}(1.6\text{x}10^{-18}\text{J/eV})]/[(.3\text{ x}10^{-9})^2(10^{-6})] = 12.4$ MPa

9. Assume that a resolved shear stress of 1.4 MPa is applied to a crystal and this causes dislocations to pile up at a precipitate particle. Assume that the shear strength of the particle is 7.2 MPa. What is the largest number of dislocations that can pile up at the precipitate before it yields?

<u>Solution:</u> $\tau_n = n\tau = 7.2 = 1.4n$, $n = 6.1$ or 7.

Chapter 11

1. The rolling texture of most hcp metals can be roughly described by an alignment of the c-axis with the rolling plane normal. Zinc is an exception. The c-axis tends to be rotated as much as 80° from the rolling plane normal toward the rolling direction. Explain this observation in terms of $\{10\bar{1}2\}<10\bar{1}1>$ twinning and the fact that easy slip occurs only on the basal plane.
Solution: Slip tends to align the (0001) with the rolling plane. The compressive stress then causes $\{101\bar{2}\}<10\bar{1}1>$ twinning which reorients the basal plane by about a little less than 90°

2. For extruded bars of the magnesium alloy AZ61A, the Metals Handbook (ASM v1, 8th ed. p. 1106) reports the tensile yield strength as 35,000 psi and the compressive yield strength as 19,000 psi. Assuming the difference is due to the directionality of twinning, deduce how the c-axis must be oriented relative to the rod axis in these extrusions.
Solution: Assuming that the tests are made with tension or compression parallel to the rod axis, the c-axis is perpendicular to the rod axis.

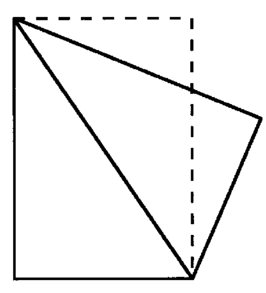
3. When magnesium twins on the $\{101\bar{2}\}$ planes in $<\bar{1}01\bar{1}>$ directions, what is the angle between the c-axes (i.e., the [0001] directions) of the twin and the untwinned material? For Mg the lattice parameters are: a = 0.32088 nm and c = 0.52095 nm.
Solution: The angle of reorientation is $\theta = 2\arctan(c/\sqrt{3}a) = 2\arctan(0.937) = 86.3°$

4. Suppose an investigator has reported $<1\bar{1}0>\{110\}$ in a cubic crystal. Comment on this claim. (Think about the resulting atomic arrangement.)
Solution: This is a silly report. A cubic crystal already has mirror symmetry about {110} so a "twin" couldn't be distinguished from the untwined matrix.

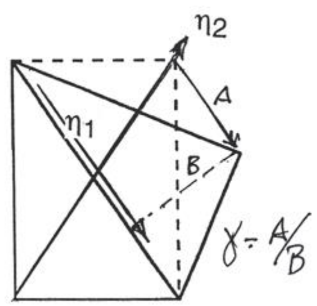
5. How many different $\{111\}<11\bar{2}>$ twinning systems are there in an fcc crystal?
Solution: There are 4{111} planes and each contains 3 $<11\bar{2}>$ directions so there are 12 $\{111\}<11\bar{2}>$ twinning systems in an fcc crystal.

6. Figure 11.24 represents a crystal, which is partially sheared by twinning. The shear direction and the normal to the mirror plane are in the plane of the paper.
A. Indicate the shear strain, γ, associated by twinning in terms of dimensions on the sketch. Put appropriate dimensions on the sketch and indicate, γ, in terms of these dimensions.
On the drawing clearly mark the 1st and 2nd undistorted directions, η_1 and η_2.

Figure 11.24. Partially twinned crystal.

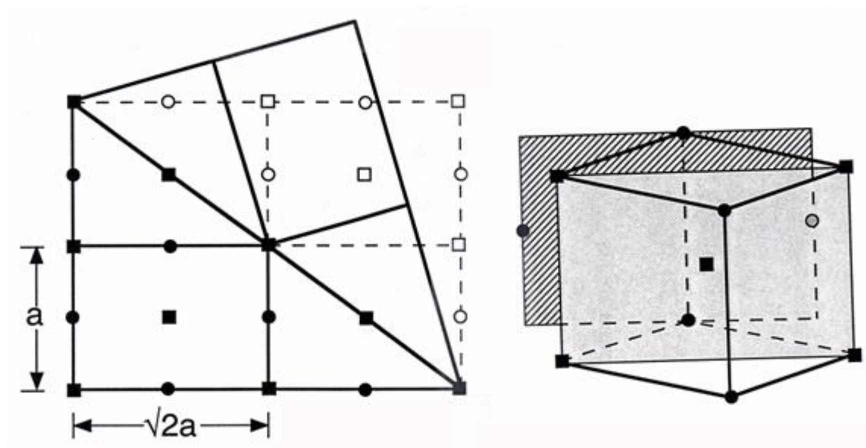


Solution: A and B

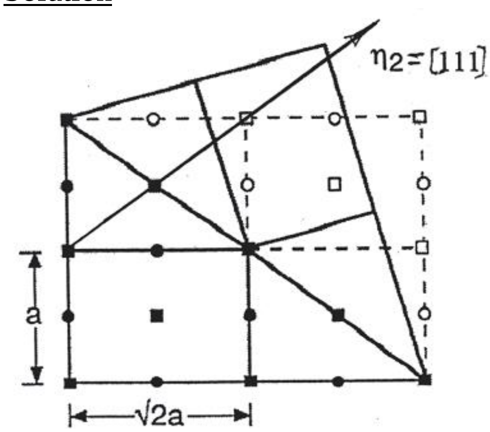


7. Figure 11.25 illustrates twinning in a bcc crystal. Indicate on the sketch the undistorted direction, η_2, and give its indices.

Figure 11.25. Twinning shear in a bcc crystal.



Solution



8. Consider an ordered bcc alloy, ordered into a B1 structure. (One species of atoms occupies the body-centering positions and the other the corner positions, so each atom is surrounded by 8 atoms of the other species.) Suppose this crystal is subjected to exactly the same $\langle 11\bar{2}\rangle\{111\}$ shear as would produce a twin in a disordered bcc alloy. By drawing a $\{1\bar{1}0\}$ plan view, deduce the atomic arrangement after the shear. How many near-neighbors of an atom are of the opposite specie? Is this a true twin, or is the crystal structure changed?

Solution: After the shear, only 4 of the next-nearest neighbors are correct. This is a different crystal structure, so it isn't a true twin.

9. The shear strain associated with the martensitic transformation of a shape-memory material is 0.18. What is the maximum tensile strain associated with the shape-memory effect? ?

<u>Solution:</u> When one half of the material reverses the direction of shear, it will undergo a reverse shear strain of 36%. However only half of the material undergoes this strain, so the average shear strain is 18%. The maximum tensile strain is half of this or 9%.

Chapter 12

1 Measurements of negative strain-rate sensitivites underestimate how negative the rate sensitivity is. Explain why.

Solution: With a negative strain-rate sensitivity, deformation tends to localize thereby causing the actual strain rate to be much higher than the apparent strain rate.

2 If strain-aging occurs in aluminum alloys containing magnesium deformed at a strain rate of 100/s below 150°C, predict the temperature below which dynamic strain-aging will occur during deformation at a strain rate of 1/s. The diffusivity of magnesium in aluminum is given by $1.2 \times 10^{-4} \exp[-131{,}000/(RT)]\ m^2/s$.

Solution: If $\dot{\varepsilon} = A\exp(-Q/RT)$, $\dot{\varepsilon}_2/\dot{\varepsilon}_1 = \exp[(-Q/R)(1/T_2 - 1/T_1)$ so $(1/T_2 - 1/T_1) = -(R/Q)\ln(2) = -4.4 \times 10^{-5}$. With $T_1 = 150 + 273 = 423K$, $1/T_2 = 1/423 - 4.4 \times 10^{-5} = 2.32 \times 10^{-3}$. $T_2 = 431K = 158\ °C$

3 For the stick-slip model in Figure 12.23, predict the frequency of load-drops if the weight of block A is 10 N, the sticking and sliding coefficients of friction are 0.20 and 0.10 respectively, the spring constant is 20N/m and the speed of travel of C is 10 cm/s.

Solution: The sticking and sliding forces are respectively, 0.2(10) = 2 N and 0.1(10 = 1 N. The spring extension corresponding to these two forces are 2N/20N/m = 10 cm and 5 cm respectively. At 10 cm/s the interval between peeks is 0.5 s so the frequency is 2/s.

4 From the slopes in Figure 12.7 of the temperature and strain-rate dependence find the activation energies for the onset of dynamic strain aging in steels containing 1.4% titanium.

Solution: $\dot{\varepsilon}_1/\dot{\varepsilon}_2 = \exp\{(-Q/R)(1/T_1 - 1/T_2)]$, so $Q = -R\ln(\dot{\varepsilon}_1/\dot{\varepsilon}_2)/(1/T_1 - 1/T_2)]$.

For the onset of strain aging, $1/T_1 = 1.28 \times 10\text{-}3$ @ $\dot{\varepsilon}_1 = 0.1$ and $1/T_2 = 1.38 \times 10\text{-}3$ @ $\dot{\varepsilon}_2 = 0.001$.

Substituting, Q =

Chapter 13

1 Write equations to describe the strain hardening of iron shown in Figure 13.11.

Solution: $\sigma = 4 + (140-4)/11 =$ kgf/mm2

2 For a unit elongation along a <111> direction in a bcc metal, determine the ratio of the amount of slip required for axially symmetric flow to that required for plane-strain.

3 Predict the ratio of the flow stresses for copper wire with a <111> texture to that with a <100> texture. Assume power-law hardening with n = 0.3.

1. Derive the relation between % El and % RA for a material that fractures before it necks. (Assume constant volume.)

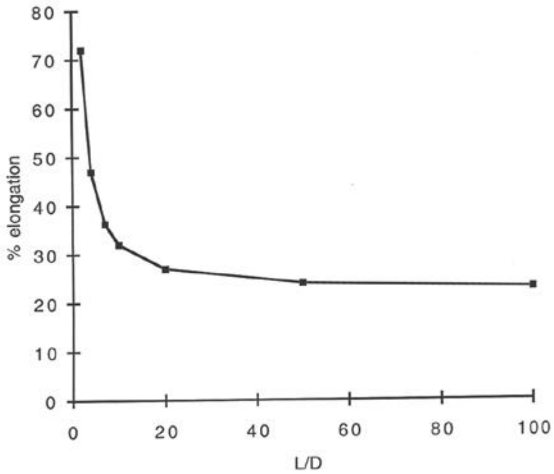
solution: $A_oL_o = A_fL_f$; $L_f/L_o = A_o/A_f$, $1+\Delta L/L_o = 1/(1- \Delta A/A_o)$;

$\Delta L/L_o = 1/(1-\Delta A/A_o) - 1$; $\%EL = 100\Delta L/L_o = 100/(1- \Delta A/A_o) - 100$; $\Delta A/A_o = \%RA/100$ so

$\%EL = 100/(1- \%RA/100) - 100$

2. Consider a very ductile material that begins to neck in tension at a true strain of 0.20. Necking causes an additional elongation that approximately equal to the bar diameter. Calculate the % elongation of this material if the ratio of the gauge length to bar diameter is 2, 4, 10 and 100. Plot % elongation vs. L_o/D_o.

Solution: Uniform % EL = exp(.2) – 1 = 22%, If L/D = 2, %EL = 22% + .5 = 72%,
If L/D = 4, %EL = 22% + .25 = 47%, If L/D = 10, %EL = 22% + .1 = 32%,
If L/D = 100, %EL = 22% + .0.1 = 23%,



3. For a material with a tensile yield strength, Y, determine the ratio of the mean stress $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$ to Y, at yielding in
A) a tension test,
B) a torsion test, and
C) a compression test.

Solution: A. $\sigma_m/Y = 1/3$, B. $\sigma_m/Y = 0$, C. $\sigma_m/Y = -1/3$

4. The cleavage planes in sodium chloride are the {100} planes. Assuming there is a critical normal stress, σ_c, for cleavage, what are the highest and lowest ratios of applied tensile stress, σ, to σ_c for cleavage? (In looking for the maximum, realize that if the angle between the tensile axis and the [100] direction gets too high, the angles between the tensile axis and the [010] or [001] directions will be smaller. You must decide at what of orientation the tensile axis, is the angle to the nearest <100> the greatest. It may help to refer to Figure AII.2

solution: The lowest ratio of $\sigma_a/\ \sigma_c = 1$ for tension along <100>; The highest ratio of $\sigma_a/\ \sigma_c$ occurs for tension along <111>. $\cos\phi = 1/\sqrt{3}$ so $\sigma_a/\ \sigma_c = 3$.

5. Cleavage in bcc metals occurs more frequently as the temperature is lowered and as the strain rate is increased. Explain this observation.

Solution: The stress to cause cleavage doesn't change as much with temperature as the stress required for deformation. Hence as temperature is lowered, higher stresses can be applied, and if high enough, cleavage can occur.

6. It has been argued that the growth of internal voids in a material that is being deformed is given by $dr = f(\sigma_H)d»\varepsilon$, where r is the radius of the void and $f(\sigma_H)$ is a function of the

hydrostatic stress. Explain, using this hypothesis, why ductile fracture occurs at higher effective strains in torsion than in tension.

Solution: The level of σ_H in torsion is very low (zero) so the void growth dr/d»ε is also low.

7. Explain why voids often form at or near hard inclusions, both in tension and in compression.

Solution: Because the hard inclusions do not deform, a high tensile stress is developed at the interface between the matrix and the inclusion at the locations where the matrix is elongating. For a material under tension this is along the tensile axis. For a material under compression it is at 90° to the compression axis.

8. Is it safe to say that brittle fracture can be avoided in steel structures if the steel is chosen so that its Charpy V-notch transition temperature is below the service temperature? If not, of what is the value of specifying Charpy V-notch test data in engineering design?

Solution: No! The transition temperature in service will depend on notch severity and rate of loading. The value of Charpy testing a steel is in comparing it with service history in similar designs and as a check on quality.

9. Hold a piece of newspaper, the upper left corner with one hand and the upper right corner with the other, and tear it. Take another piece of newspaper, rotate it 90° and repeat. One of the tears will be much straighter than the other. Why?

Solution: The tear parallel to the direction that the paper pulp was rolled will be much straighter because the tear can propagate between the fibers rather than cut through them.

Chapter 14

1. Using the two theoretical predictions of fracture strength, A) $\sigma_t = E/\pi$ (equation 14.4) and B) $\sigma_t = \sqrt{(\gamma E/a_o)}$ (equation 14.8), calculate the theoretical fracture strength of a) iron and b) MgO. For iron take a_o as its atomic diameter (0.124 nm) and for MgO as the average of the ionic diameters of Mg^{+2} and O^{-2} (0.105 nm). The surface energies of iron and MgO are about 2.0 J/m^2 and 1.2 J/m^2 respectively. Young's moduli of iron and MgO are about 270 GPA and 300 GPa respectively.
How do these answers compare with each other?
How do these answers compare with the actual fracture strengths of each?

Solution: For iron, A) σ_t = 270GPa/π = 86GPa, B) $\sigma_t = [2.0\text{x}270\text{x}10^9/0.124\ \text{x}10^{-9}]^{1/2}$ = 66 GPa
For MgO, A)) σ_t = 300GPa/π = 95 GPa, B) $\sigma_t = [1.2\text{x}300\text{x}10^9/0.105\ \text{x}10^{-9}]^{1/2}$ = 59 GPa
The two methods agree to within a factor of two which is good considering the assumptions made. They are very much higher than actual values (< 2GPa for Fe and < 0.4 GP for MgO)

2. Class 20 and class 60 gray cast irons have tensile strengths of about 20 ksi and 60 ksi repectively. Assuming that the fractures start from graphite flakes and that the flakes act as pre-existing cracks, use the concepts of the Griffith analysis to predict the ratio of the average graphite flake sizes of the two cast irons.

Solution: The Griffith criterion predicts that with two different crack sizes, the ratio of the fracture strengths would be $\sigma_2/\sigma_1 = \sqrt{(a_1/a_2)}$ so $a_1/a_2 = (\sigma_2/\sigma_1)^2$. Assuming that the graphite flakes act as cracks, the ratio of flake sizes is $(60/20)^2 = 9$.

3. A wing panel of a supersonic aircraft is made from a titanium alloy that has a yield strength 1035 MPa and toughness of K_{Ic} = 55 MPa$\sqrt{m}$. It is 3.0 mm thick, 2.40 m long and 2.40 m wide. In service it is subjected to a cyclic stress of ± 700 MPa which is not enough to cause yielding but does cause gradual crack growth of a pre-existing crack normal to the loading direction at the edge of the panel. Assume that the crack is initially 0.5 mm long and grows at a rate of da/dN = 120 nm/cycle
Calculate the number of cycles to catastrophic failure.

Solution: $\sigma_f = K_{Ic}/(f\sqrt{(\pi a)}$, $a = (K_{Ic}/\sigma_f)^2/(\pi f^2)$. Substituting σ_f = 700 MPa, K_{Ic} = 55 MPa$\sqrt{m}$, and f = 1.15, a = $(55/700)^2/(\pi 1.15^2) = 1.486\text{x}10^{-3}$ m = 1.486 mm. The crack must grow by 1.486 – 0.5 = 0.986 mm = $0.986\text{x}10^6$ nm. If da/dN = 120nm,
N = Δa/120nm = $0.986\text{x}10^6$ nm/120nm = 8,200 cycles

4. The support in Figure 14.21a is to be constructed from a 4340 steel plate tempered at 800F. The yield strength of the steel is 228 ksi and its value of K_{Ic} is 51 ksiœŸiŸn. The width of the support, w, is 4 in., the length, L, is 36 in. and the thickness, t, is 0.25 in. Figure 21b gives f(a/w) in the equation σ = K_{Ic}/[f(a/w)œŸπŸa].

A. If the crack length, a, is small enough, the support will yield before it fractures. What is the size of the largest crack for which this is true? (i.e., what is the largest value of a for which general yielding will precede fracture?). Assume that any fracture would be in mode I (plane strain). Discuss critically the assumption of mode

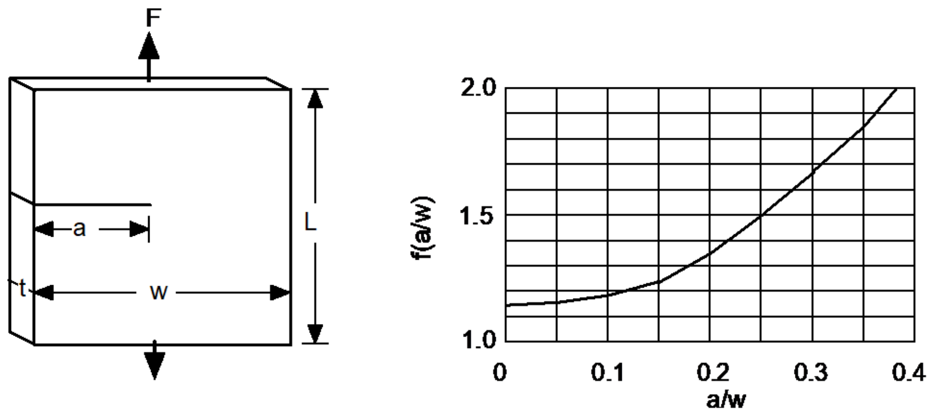


Figure 14.21. A Support shape B. Variation of f with a/w.

B. If it is guaranteed that there are no cracks longer than a length equal to 80% does this assure that failure will occur by yielding, rather than fracture? Explain briefly.

Solution: A. The critical crack size, $a = (K_{Ic}/\sigma)^2/\pi$. At yielding σ = 228 ksi. Assume that $f(a/w) = 1.15$, $a = (51/228)^2/\pi = 0.016$ in. Then a/w = 0.016 in/4 which justifies the assumption that $f(a/w) = 1.15$. The assumption of mode I is valid because $t = 0.25 > 2.5(K_{Ic}/\sigma)^2 = 0.125$.

B. No. Smaller pre-existing cracks may grow by fatigue.

5. For 4340 steel, the fracture toughness and yield strength depend on the prior heat treatment, as shown in Figure 14.14. As yield strength is increased, the fracture toughness decreases. A pipeline is to be built of this steel and to minimize the wall thickness of the pipe, the stress in the wall should be as high as possible without either fracture or yielding. Inspection techniques ensure that there are no cracks longer than 2 mm (a = 1 mm). What level of yield strength should be specified? Assume a geometric constant of f = 1.15

solution: For fracture, $\sigma = K_{Ic}$/[fœŸπŸa]. Substituting f =1.15 and $a = 10^{-3}$m, $\sigma = 15.5K_{Ic}$. Plotting $K_{Ic} = 0.64\sigma$, on Figure 14.14, the intersection gives the stress (1450MPa) at which both yielding and fracture will occur. The level of yield strength should be specified as 1450 MPa.

6. A steel plate, 10 ft long, 0.25 in thick and 6 in wide is loaded under a stress of 50 ksi. The steel has a yield strength of 95 ksi and a fracture toughness of 112 ksiœŸiŸn. There is a central crack perpendicular to the 10 ft dimension.

A. How long would the crack have to be for failure of the plate under the stress?

B. If there is an accidental overload (i.e., the stress rises above the 50 ksi specified) the plate might fail by either yielding or by fracture depending on the crack size. If the designer wants to be sure that an accidental overload would result in yielding rather than fracture, what limitations must be placed on the crack size?

Solution: A. The critical crack size, $a = [K_{Ic}/(1.15\square)]^2/\pi = (112/1.15x50)^2/\pi = 1.2$ in.

B. The critical crack size for yield before fractue is $a = (112/1.15x95)^2/\pi = 0.33$ in.

7. A structural member is made from a steel that has a K_{Ic} = 180 MPaœm and a yield strength of 1050 MPa. In service it should neither break nor deform plastically, since either would be considered a failure. Assume f = 1.0. If there is a pre-existing surface crack of a = 4 mm, at what stress will the structural member fail? Will it fail by yielding or fracture?

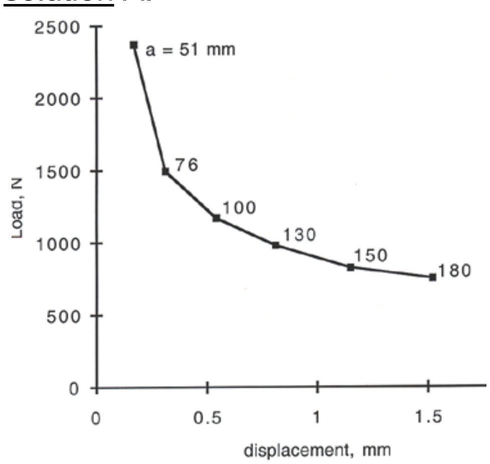
Solution: The critical stress for fracture is $\sigma = K_{Ic}/[f$œŸπŸ$a]$. Assuming f = 1.15,
$\sigma = 180\text{x}10^6$Pa/[1.15œ ($\pi(4\text{x}10^{-3}$m)]   = 776 MPa. It will fracture before yielding.

8. An estimate of the effective strain, ε, in a plane-strain fracture surface can be made in the following way. Assume that the material is not work-hardening and the effective strain is constant throughout the plastic zone so the plastic work per volume is Yε. Assume the depth of the plastic zone is given by equation $r_p = (K_I/Y)^2/(6\pi)$ so the strained volume is $2r_pA$. Derive an expression for the plastic strain, ε, associated with running of a plane-strain fracture. (Realize that G_c is the plastic work per crack area and that K_{Ic} and G_c are related by equation 14.22.)
solution: Total work/area = $G_c = (\varepsilon)2(K_I/Y)^2/(6\pi)$. Substituting $K_{Ic}^2 = G_cE$,
$G_c = (\varepsilon/Y)G_cE/2\pi$, $\varepsilon = 2\pi(Y/E)$.

9. The data below were taken on tests a material to determine its fracture toughness. The specimens were 25 mm thick.
A. Make a plot of P vs. u.
B. Using this plot with Gurney's method, determine the value of G for several combinations of points.

crack length, a mm	displacement, u mm	load, P N
51	.17	2370
76	.31	1495
100	.54	1170
125	.81	980
155	1.15	825
180	1.52	750

solution A.



A. $G = (1/2)(P_1u_2 - P_2u_1)/[0.025)(a_2 - a_1)]$
G = 2[(2370)(.31) – (1495)(0.17)]/(76 – 51) = 38.4 J/m^3
G = 2[(1495)(.54) – (1170)(0.31)]/(100 – 76) = 35.6
G = 2[(1170)(.81) – (980)(0.54)]/(125 – 100) = 33.5
G = 2[(980)(1.15) – (825)(0.81)]/(150 – 125) = 30.6
G = 2[(825)(1.52) – (750)(1.15)]/(180 – 155) = 31.3