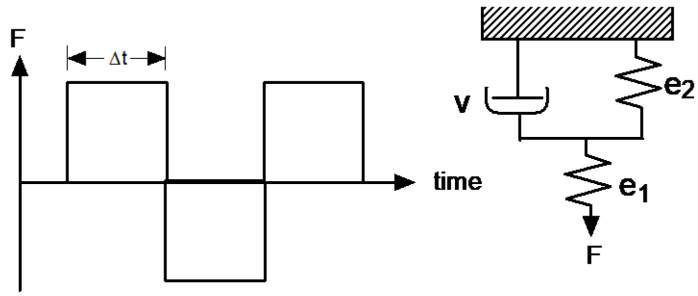
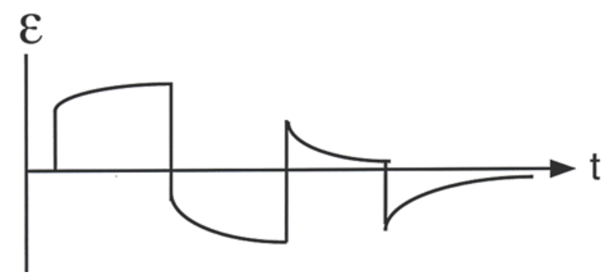
Chapter 16

1. Consider a visco-elastic material whose behavior is adequately described by the combined series-parallel model. Let it be subjected to the force vs. time history shown in Figure 16.21. There is a period of tension followed by compression and tension again. After that, the stress is 0. Let the time interval, $\Delta t = K_v/K_{e2} = t_e$, and assume $K_{e1} = K_{e2}$. Sketch as carefully as possible the corresponding variation of strain with time.

Figure 16.21. Loading of a viscoelastic material.

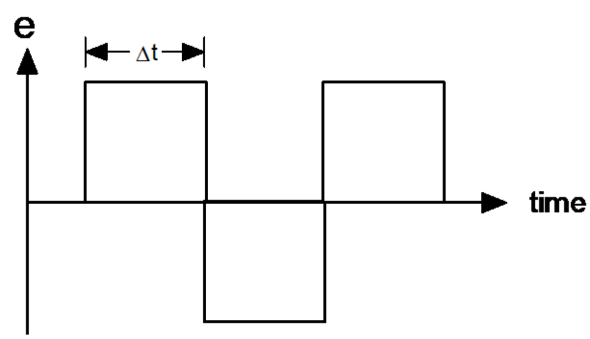


Solution:

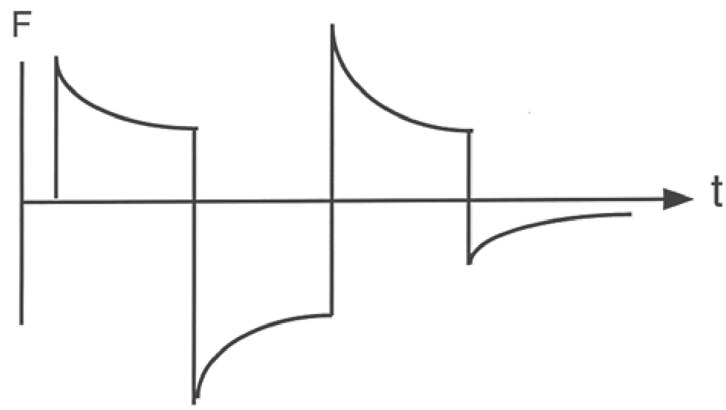


2. Consider a visco-elastic material whose behavior is adequately described by the combined series-parallel model. Let it be subjected to the strain vs. time history shown in Figure 15.22. There is a period of tension followed by compression and tension again. After that, the stress is returns to zero. Let the time interval, $\Delta t = K_v/K_{e2} = \tau_e$, and assume $K_{e1} = K_{e2}$. Sketch in the space provided as carefully as possible the corresponding variation of force with time.

Figure 15.22. Strain cycles imposed on a viscoelastic material.



Solution:



3. An elastomer was suddenly stretched in tension and the elongation was held constant. After 10 minutes, the tensile stress in the polymer dropped by 12%. After an extremely long time the stress dropped to 48% of its original value.

A. Find the relaxation time, τ, for stress relaxation.

B. How long will it take the stress to drop to 75% of its initial value?

Solution: Substituting $F_\infty/F_o = 0.48$ into equation 15.8, $F = F_o 0.48 - 0.52F_o\exp(-t/\tau_\sigma)$;

Substituting $F = 0.88F_o$ and t = 10 min, $(0.88 - 0.48)/0.52 = .\exp(-10/\tau_\sigma)$

$10/\tau_\sigma = -\ln(.40/.52)$; $\tau_\sigma = -10/\ln(.40/.452) = 38$ min.

B. Substituting $F = 0.75F_o$, $0.75F_o = F_o 0.48 + 0.52F_o\exp(-t/38)$;

$t/38 = -\ln[(.75-.48)/.52)]$, t = 25min.

4. A certain bronze bell is tuned to middle C (256 Hz). It is noted that the intensity of the sound drops by one decibel (i.e. 20.56%) every 5 seconds. What is the phase angle δ in degrees?

Solution: From equation 15.16, $\delta = (\Delta U/U)/(2\pi) = 0.2056/(2\pi) = 0.0327$ radians = 1.9°

5. A piece of aluminum is subjected to a cyclic stress of ±120 MPa. After 5000 cycles, it is noted that the temperature of the aluminum has risen by 1.8°C. Calculate Λ and the phase angle, δ, assuming that there has been no transfer of heat to the surroundings and all the energy loss/cycle is converted to heat.

Misc. data for aluminum:

crystal structure	fcc.	lattice parameter	0.4050 nm
density:	2.70 Mg/m^3	Young's modulus	62. GPa
heat capacity	900. J/kg·K	melting point	660. °C

[Hint: ΔU can be found from the temperature rise, and U can be found from the applied stress and Young's modulus.]

Solution: $\Delta U = \Delta T\rho C/5000 = 1.8°C(900 J/kg\text{-}°C)(2700 kg/m^3/5000 = 875 J/m^3$.

$U = (1/2)\sigma\varepsilon = (1/2)\sigma^2/E = (1/2)(120\times10^6)^2/62\times10^9 = 116\times10^3\ J/m^3$

$\Delta U/U = 7.53\times10^{-3}$, $\delta = (\Delta U/U)/(2\pi) = 1.2\times10^{-3}$radians = 0.068° $\Lambda = \pi\delta = 3.8\times10^{-3}$.

6. Measurements of the amplitude of vibration of a freely vibrating beam are:

cycle	amplitude
0	250
25	206
50	170
100	115
200	53

A. Calculate the log decrement, Λ.

B. Is Λ dependent on the amplitude for this material in the amplitude range studied? (Justify your answer.)

C. What is the phase angle δ?

[Hint: Assume Λ is constant so $e_n/e_{n+1} = e_{n+1}/e_{n+2}$ = etc. Then $e_n/e_{n+m} = [e_n/e_{n+1}]^m$].

<u>Solution</u>: $\Lambda = (1/25)\ln(250/206) = 0.0077$, $\delta = (1/25)\ln(207/170) = 0.0079$,
$\Lambda = (1/50)\ln(170/115) = 0.0078$, $\delta = (1/100)\ln(115/53) = 0.0077$,

B. There is no significant variation of Λ with amplitude.

C. $\delta = \Lambda/\pi = 0.0078/\pi = 0.00248$ radians or 0.142°

7A. A high strength steel can be loaded up to 100,000 psi in tension before any plastic deformation occurs. What is the largest amount of thermo-elastic cooling that can be observed in this steel at 20°C?

B. Find the ratio of adiabatic Young's modulus to isothermal Young's modulus for this steel at 20°C.

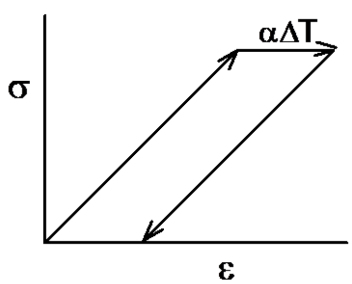
C. A piece of this steel is adiabatically strained elastically to 10^{-3} and then allowed to reach thermal equilibrium with the surroundings (20°C) at constant stress. It is then unloaded adiabatically, and again allowed to reach thermal equilibrium with its surroundings. What fraction of the initial mechanical energy is lost in this cycle? [Hint: Sketch the σ–ε path.]

For iron, $\alpha = 11.76\text{x}10^{-6}$/°C, E = 29x$10^6$ psi, C_v = 0.46 J/g°C, and ρ = 7.1 g/cm^3.

<u>Solution</u>: A. $\Delta T = -\sigma\alpha T/(\rho C)$ =
$-(10^5 \text{ psi})(6.9\text{x}10^3 \text{ Pa/psi})(11.76\text{x}10^{-6}\text{/K})(293\text{K})/(0.46\text{J/kg°C}\text{x}7.1\text{x}10^6 \text{ g/m}^3) = -0.73°\text{C}$

B. $\Delta E/E = ET\alpha^2/(\rho C) = (205\text{x}10^9\text{Pa})293(11.76\text{x}10^{-6}\text{/°C})^2/(0.46\text{J/kg°C}\text{x}7.1\text{x}10^6 \text{ g/m}^3)$ = 0.0025, $E_a/E_i = 1.0023$.

C



$\Delta U = \sigma\alpha\Delta T = \sigma\alpha[\sigma\alpha T/(\rho C)]$, $U = (1/2)\sigma\varepsilon = (1/2)\sigma^2/E$.;
$\Delta U/U = 2\alpha^2 ET/(\rho C) = (2(11.76\text{x}10^{-6})^2(205\text{x}10^9)(293)/(0.46\text{x}7.1\text{x}10^6) = 0.5\text{x}10^{-3}$

8. For iron the adiabatic Young's modulus is $(1 + 2.3x10^{-3})$ times the isothermal modulus at room temperature. If the anelastic behavior of iron is modeled by a series-parallel model, what is the ratio of K_1 to K_2?

<u>Solution:</u> $F_0/F_\infty = (1 + 2.3x10^{-3}) = (K_1 + K_2)/K_2 = 1 + K_1/K_2$: $K_1/K_2 = 2.3x10^{-3}$

9. Damping experiments on iron were made using a torsion pendulum with a natural frequency of 0.65 cycles/sec. The experiments were run at various temperatures and the maximum log decrement was found at 35°C. The activation energy for diffusion of carbon in α-iron is 78.5 kJ/mole. At what temperature would you expect the damping peak to occur if the pendulum were redesigned so that it had a natural frequency of 10 Hertz?

<u>Solution</u>: $f_1/f_2 = \exp[(-Q/R)(1/T_1-1/T_2)]$; $1/T_1-1/T_2 = -(R/Q)\ln(f_1/f_2)$

$= (-8.314/78{,}500)\ln(10/0.65) = -2.89\ x10^{-4}$;

$1/T_1 = -2.89\ x10^{-4} +1/(273 + 35)$. $T_1 = 338K = 65°C$.

10. A polymer is subjected to a cyclic stress of 5 MPa at a frequency of 1 Hz for 1 minute. The phase angle is 0.05°. Calculate the temperature rise assuming no loss to the surroundings. E = 2GPa, C = 1.0 J/kg·K and ρ = 1.0 Mg/m^3.

<u>Solution:</u> $\Delta U/U = 2\pi\sin\delta = 2\pi\sin(0.05°) = 5.48x10^{-3}$.

$U = (1/2)\sigma^2/E = (1/2)(5x10^6)^2/(2\ x10^9) = 6250$.

$\Delta U = (5.48\ x10^{-3})(6250) = 34.2$ J/cycle

$\Delta T = (34.2$ J/cycle}/[(1 J/kg·K)(1000 kg/m^3)] = 0.034°C/cycle.

In 1 minute the temperature rise would be 2.06°C

Chapter 17

1. Stress vs. rupture life data for a super alloy are listed below. The stresses are given in MPa and rupture life is given in hours.

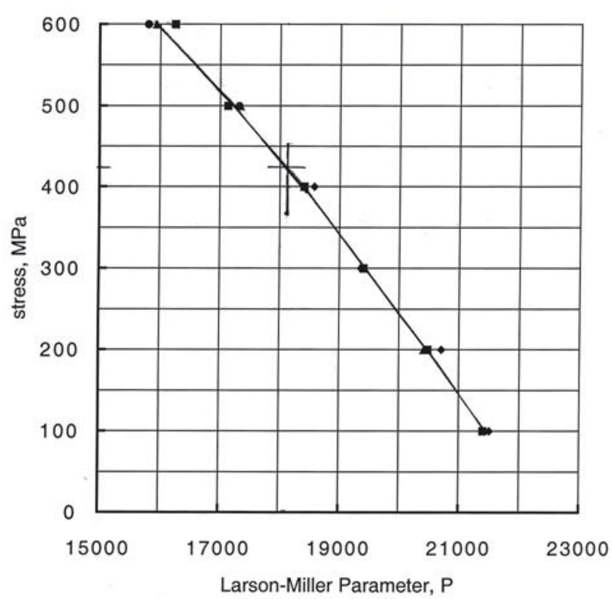
A. Make a plot of stress (log scale) vs. P_{LM} where $P_{LM} = (T)(C + \log_{10}t)$. T is the temperature in Kelvin, t is in hours and C = 20.

B. Predict from that plot what stress would cause rupture in 100,000 hrs at 450°C.

Stress MPa	rupture time (hrs) 500°C	600°C	700°C	800°C
600	2.8	0.018	0.0005	---
500	250.	0.72	0.004	---
400	--	12.1	0.082	0.00205
300	--	180	0.87	0.011
200	--	2412	11.	0.198
100	--	--	98.0	1.10

Solution. Finding the values of P_{LM} for each point, and plotting:

Stress MPa	P_{LM} 500°C	600°C	700°C	800°C
600	15,805	15,937	16,248	---
500	17,313	17,335	17,127	---
400	--	18,405	18,403	18,575
300	--	19,428	19,401	19,358
200	--	20,412	20,473	20,705
100	--	--	21,397	21,504



B. For 100,000 hrs at 450°C (723K), P_{LM} = 18,080. Reading off of the plot, s = 423 MPa

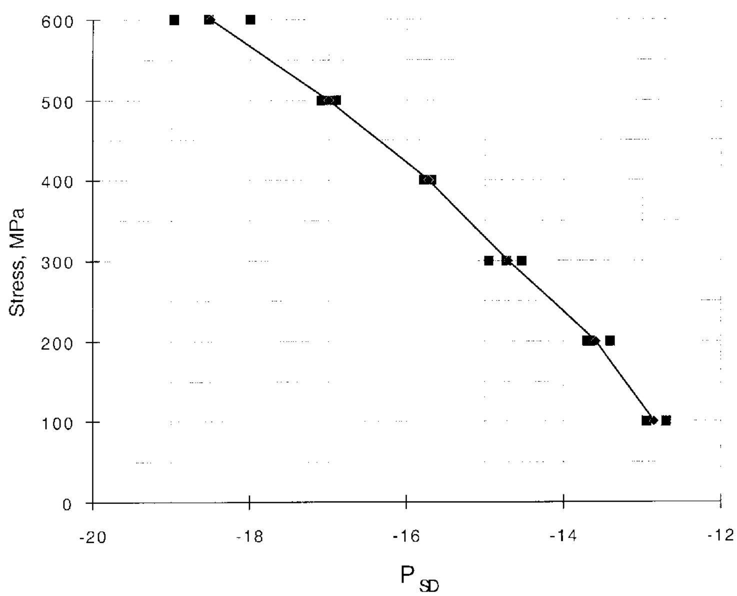
2.A. Using the data in problem 1, plot the Sherby-Dorn parameter, $P_{SD} = \log\theta$ where

θ = texp(-Q/RT) and Q = 340 kJ/mole.

B. Using this plot, predict from that plot what stress would cause rupture in 100,000 hrs at 450^{o}C.

Solution: Calulating θ = texp(-Q/RT) and P_{SD} = logθ for each point and plotting

MPa/T°C	500	600	700	800
600	-18.97	-18.53	-18.00	---
500	-17.02.	-16.90	-17.09	---
400	--	-15.71	-15.78	-15.68
300	--	-14.53	-14.73	-14.95
200	--	-13.41	-13.66	-13.70
100	--	--	-12.70	-12.95



B. For t = 100,000 hrs and T = 450°C (723K), and P_{SD} = -16.01. Reading the plot, σ = 423 MPa.

3. For many materials, the constant C in the Larson-Miller parameter, P = (T + 460)(C + $\log_{10}t$) (where T is in Fahrenheit, and t in hours) is equal to 20. However, the Larson-Miller parameter can also be expressed as P' = T(C' + lnt) with t in seconds and T in Kelvin, using the natural logarithm of time. In these cases, what is the value of C'?

Solution: 20 + $\log_{10}$[t(hrs)] = x + ln[t(s)]. For 1 hr = 3600 s,

x = 20 + $\log_{10}$[1(hrs)] - ln(3600s) = 11.81.

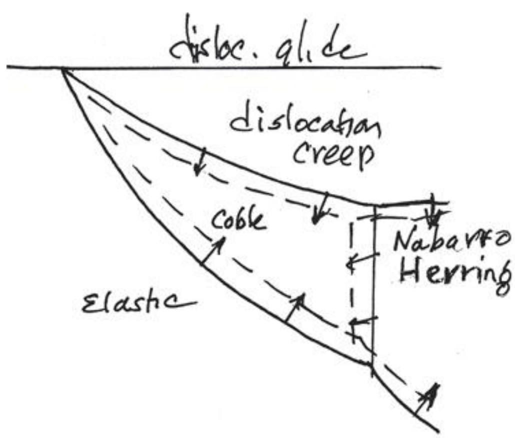
4. Stress rupture data is sometimes correlated with the Dorn parameter, θ = $t$exp[-Q/(RT)], where t is the rupture time, T is absolute temperature and q is assumed to depend only on stress. If this parameter correctly describes a set of data, then a plot of log(t) vs.1/T for data at a single level of stress would be a straight line. If the Larson-Miller parameter correctly correlates data, a plot of data at constant stress (therefore constant P) of log(t) vs.1/T also would be a straight line.

A. If both parameters predict straight lines on log(t) vs. 1/T plots, are they really the same thing?

B. If not, how do they differ? How could you tell from a plot of log(t) vs. (1/T) which parameter better correlates a set of stress rupture data?

Solution: A. No.
B. According to the Larson-Miller parameter, a plot of log(t) vs. (1/T) for different stresses would intercept (1/T) = 0 at the same value (-C).
According to the Sherby-Dorn parameter, a plot of log(t) vs. (1/T) for different stresses would have the same slopes but intercept (1/T) = 0 at different values (P_{SD}).

5. Sketch how the boundaries in Figure 16.7 for the creep mechanisms in nickel would change if the grain size were 1 mm instead of 32 μm.
Solution: Both Coble and N-H creep would decrease. Boundary between N-H and coble would move to left.



6. Data for the steady-state creep of a carbon steel are plotted in Figure 16.19.
A. Using the linear portions of the plot, determine the exponent m in $\dot{\varepsilon}_{SC} = B\sigma^m$ at 538°C and 649°C.

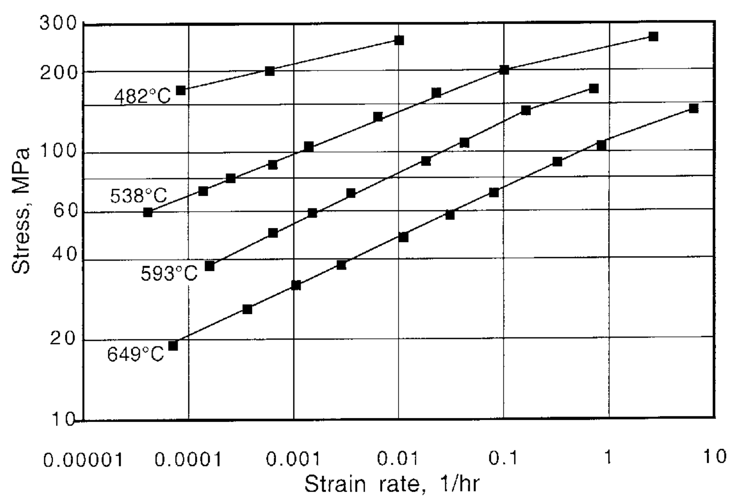


Figure 16.19. Creep data for a carbon steel. Data from P. N. Randall, *Proc, ASTM*, v. 57 (1957).

B. Determine the activation energy, Q, in the equation $\dot{\varepsilon} = f(\sigma)\exp[-Q/(RT)]$.
Solution:A. $m = \ln(\dot{\varepsilon}_2/\dot{\varepsilon}_1)/\ln(\sigma_2/\sigma_1)$. At 538°C, m = $\ln(10^{-1}/10^{-4})/\ln(200/70) = 6.58$
At 649°C, $m = \ln(1/10^{-4})/\ln(105/20) = 5.55$. (Note that m decreases with increasing temperature.)

B. Comparing data at the same stress ((100 MPa $\dot{\varepsilon}_2 = 0.9$ @ $T = 649 + 273 = 922$K and $\dot{\varepsilon}_1 = 0.001$ @ $T = 538 + 273 = 811$K. $Q = -R\ln(\dot{\varepsilon}_2/\dot{\varepsilon}_1)/(1/T_1 - 1/T_2) =$ $-8.314\ln(.9/.001)/(1/922-1/811) = 381$ kJ/mole.

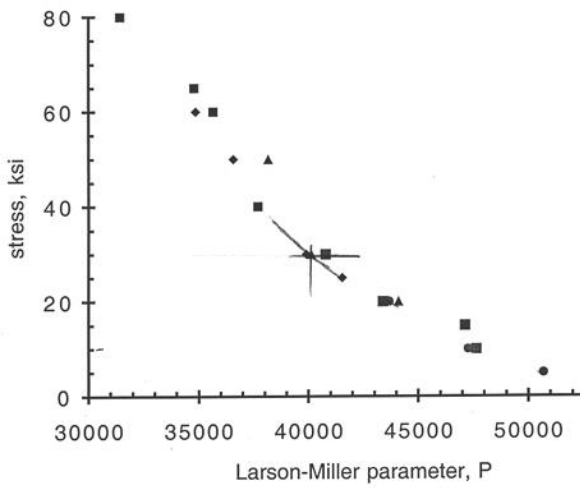
7. The following data were obtained in a series of stress rupture tests on a material being considered for high temperature service.

at 650°C	stress (ksi)	80	65	60	40
	rupture life (hrs)	0.08	8.5	28	483
at 730°C	stress (ksi)	60	50	30	25
	rupture life (hrs)	0.20	1.8	127	1023
at 815°C	stress (ksi	50	30	20	
	rupture life (hrs)	0.30	3.1	332	
at 925°C	stress (ksi)	30	20	15	10
	rupture life (hrs)	0.08	1.3	71	123
at 1040°C	stress (ksi)	20	10	5	
	rupture life (hrs)	0.03	1.0	28	211

A. Make a Larson-Miller plot of the data.

B. Predict the life for an applied stress of 30 ksi at 600°C.

Solution: A.Taking C = 20



B. For 30 ksi, P = 40,000. For 600°C T°F = 490 = 1571. so 40,000 = 1571[20 + log$t$], log(t) = (40,000/1571) – 20 = 5.46, t = 290,000 hr = 33 years

8. Figure 16.15 shows how service temperature affects the usable stress levels for various metals.

A. Tungsten has a melting point of 3400°C. Why is it not considered for use in jet engines?

B. What advantages do aluminum alloys have over more refractory materials at operating temperatures of 400°F?

Solution: Light weight and oxidation resistance.

9. Consider the creep rate vs. stress curves for an aluminum alloy plotted in Figure 16.20. Calculate the stress exponent, m at 755 K.

Chapter 18

1. For steels, the endurance limit is approximately 1/2 of the tensile strength, and the fatigue strength at 10^3 cycles is approximately 90% of the tensile strength. The S-N curves can be approximated by a straight lines between 10^3 and 10^6 cycles when plotted as log(S) vs. log(N). Beyond 10^6 cycles the curves are horizontal.

A. Write a mathematical expression for S as a function of N for the sloping part of the S-N curve, evaluating the constants in terms of the approximations above.

B. A steel part fails in 12,000 cycles. Use the above expression to find what % decrease of applied (cyclic) stress would be necessary to increase in the life of the part by a factor of 2.5 (to 30,000 cycles).

C. Alternatively, what % increase in tensile strength would achieve the same increase in life without decreasing the stress?

Solution: A. logs = A' + blogN or lonS = A + blnN. $S_f = AN_f^b$, $(S_{f1}/ S_{f2}) = (N_1/ N_2)^b$; $b = \ln(S_{f1}/ S_{f2}))/\ln(N_1/ N_2) = \ln(.9/.5)/\ln(10^{-3}) = -0.085$, $A = S_f /N_f^b = 0.9TS/(10^3)^b = 1.62TS$, $S_f = 1.62TS(N)^{-.085}$

B $S_{f3}/S_{f1}= (N_3/ N_1)^b = (2.5)^{-.085} = 0.925$. The stress would have to be lowered by 7.25%

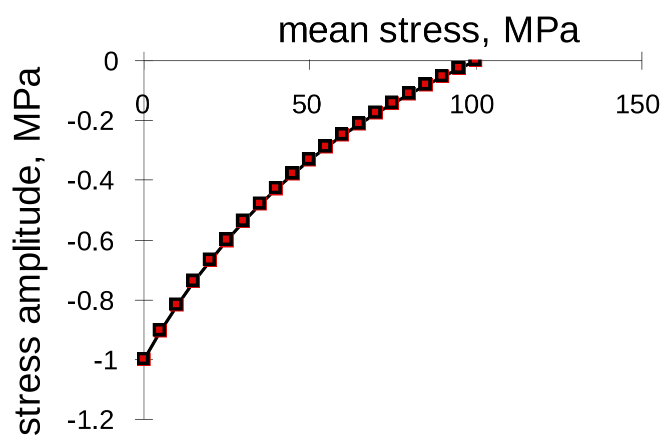
C. 1/0.925 = 1.081, an 8% increase of TS.

2.A. Derive an expression relating the stress ratio, R, to the ratio of cyclic stress amplitude, σ_a, to the mean stress, σ_m.

B. For σ_a = 100 MPa, plot R as a function of σ_m over the range $0 \le \sigma_m \le 100$ MPa.

Solution: A. $R = \sigma_{min}/\sigma_{max} = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a) = (1- \sigma_a/\sigma_m)/(1+ \sigma_a/\sigma_m)$

$R = (\sigma_m - 100)/(\sigma_m + 100)$.



3. A steel has the following properties:

Tensile Strength	460 MPa
Yield Strength	300 MPa
Endurance limit	230 MPa

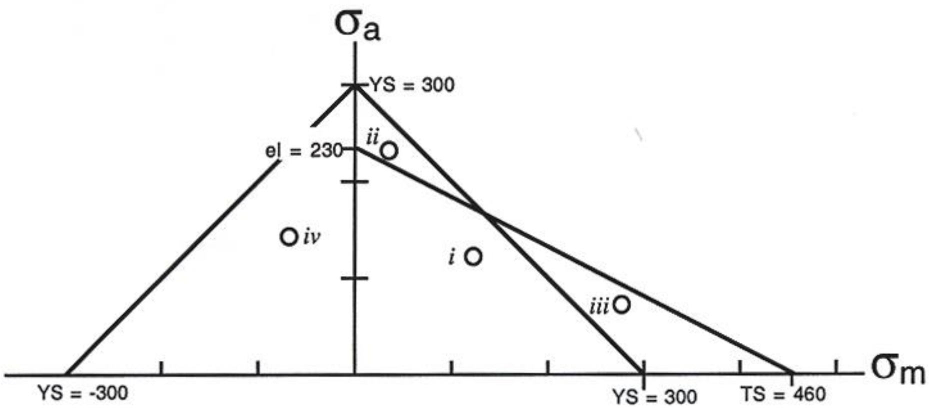
A. Plot a modified Goodman diagram for this steel showing the lines for yielding as well as fatigue failure.

B. For each of the cyclic loading given below determine whether yielding, infinite fatigue life, or finite fatigue life is expected.

i. σ_{max} = 250 MPa, σ_{min} = 0 ii. σ_{max} = 280 MPa, σ_{min} = -200 MPa

iii. σ_{mean} = 280 MPa, σ_a = 70 MPa iv. σ_{mean} = -70 MPa, σ_a = 140 MPa

Solution:



i. (σ_a = 125, σ_m = 125) infinite life, no yielding, ii. (σ_a = 240, σ_m = 40finite life, no yielding,
iii. infinite life, yielding, iv. infinite life, no yielding0

4. The notch sensitivity factor, q, gray cast iron is very low. Offer an explanation in terms of the microstructure.

Solution: The graphite flakes act as notches much more severe than machined notches, so machined notches have little effect.

5. A 1040 steel has been heat-treated to a yield strength of 900 MPa and a tensile yield strength of 1330 MPa. The endurance limit (at 10^6 cycles for cyclical loading about a zero mean stress) is quoted as 620 MPa. Your company is considering using this steel, with the same heat treatment for an application in which fatigue may occur during cycling with R = 0. Your boss is considering shot peening the steel to induce residual compressive stresses in the surface. Can the endurance limit be raised this way? If so, by how much? Discuss this problem with reference to the Goodman diagram using any relevant calculation(s).

Solution: Yes.

The largest increase in fatigue strength corresponds to a negative mean stress, σ_m, which would cause yielding in compression. $\sigma_a = 900 + \sigma_m$.

For fatigue failure, $\sigma_a = 620(1 - \sigma_m/1330)$. Solving these simultaneously,

$900 + \sigma_m = 620 – 0.466\sigma_m$. $–1.466\sigma_m = 280$, $\sigma_m = -191$. $\sigma_a = 900 –0191 = 709$ MPa

The endurance limit can be raised to 709 MPa if a residual compressive stress of –191 MPa can be induced.

6. Low cycle fatigue was the cause of the Comet failures. Estimate how many pressurization-depressurization cycles the planes may have experienced in the two years of operations. An exact answer is not possible, but by making a reasonable guess of the number of landings per day and the number of days of service a rough estimate is possible.

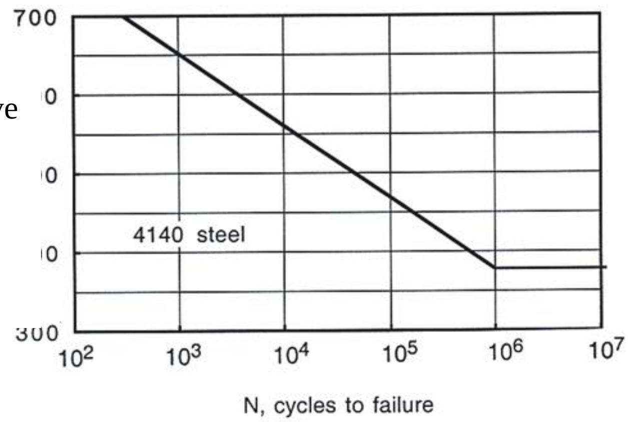
Solution: If the aircraft landed and took off 5 times per day for 320 days per year, the number of cycles would be 2x320x5 = 3200.

7. Frequently the S-N curves for steel can be approximated by a straight line between $N = 10^2$ and $N = 10^6$ cycles when the data are plotted on a log-log scale as shown in the figure for SAE 4140 steel. This implies $S = AN^b$, where A and b are constants.

A. Find b for the 4140 steel for a certain part made from 4140 steel (Figure 17.26).

B. Fatigue failures occur after 5 years. By what factor would the cyclic stress amplitude have to be reduced to increase the life to 10 years? Assume the number of cycles of importance is proportional to time of service.

Figure 17.26. S-N curve for a SAE 4140 steel.

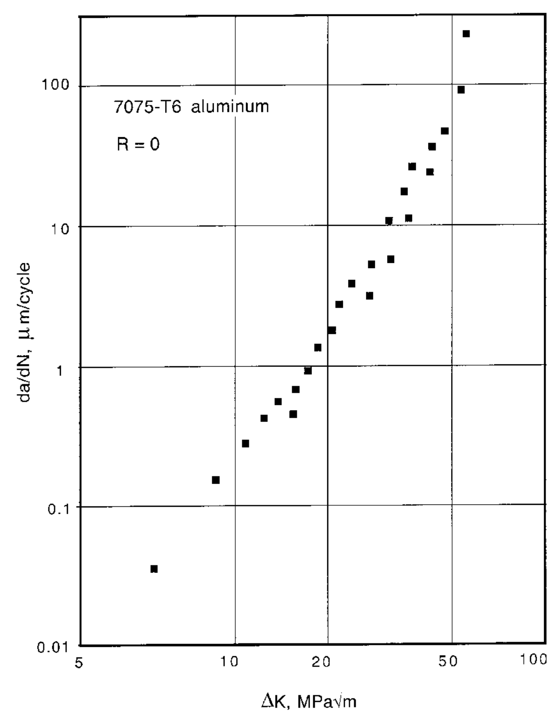


Solution: A. $b = \ln(S_2/S_1)/\ln(N_2/N_1) = \ln(375/650)/\ln(10^6/10^3) = -.0796$

B. $(S_3/S_1) = (N_2/N_1)^{-0.796} = (2)^{-0.0796} = 0.946$.

8. Figure 17.27 shows the crack growth rate in aluminum alloy 7075-T6 as a function of DK for R = 0. Find the values of the constants C and m, in equation 17.19, that describes the straight-line portion of the data. Give units.

Figure 17.27. Crack growth rate of 7075-T6 aluminum for $\Delta K = 0$. Data from C. M. Hudson, NASA TN D-5300, 1969.



Solution: $da/dN = C(\Delta K_I)^m$, $m = \ln[(da/dN)_1/[(da/dN)_2]/\ln[(\Delta K_I)_1/(\Delta K_I)_2] =$

$\ln[(10^{-1})/(10^{-5})]\ln[(70)/(5)] = 3.5$

$C = (da/dN)/(\Delta K_I)^m = 10^{-1}/70^{3.5} = 3.5x10^{-8}$. The units of C are (mm/cycle)(MPa$\sqrt{m}$)$^{3.5}$

9. Find the number of cycles required for a crack to grow from 1 mm to 1 cm in 7075-T6 (problem 8.) if f = 1 and $\Delta\sigma$ = 10 MPa. Remember that $\Delta K = f\Delta\sigma\sqrt{(\pi a)}$.

Solution: Using equation 17.22, $N = [a^{(1-m/2)} - a_o^{(1-m/2)}]/[Cf^m(\pi\Delta\sigma)^{m/2}] =$

$[10^{-2(1-3.5/2)} - 10^{-1(1-3.5/2)}]/[3.5x10^{-8}(\pi 10)^{3.5/2}] = 1.78x10^6$ cycles

10. For a certain steel, it was found that the fatigue life at ± 70 MPa was 10^4 cycles and it was 10^5 cycles at ± 50 MPa. A part made from this steel was given 10^4 cycles at ±61 MPa. If the part were then cycled at ±54 MPa what would be its expected life?

Solution: . $b = \ln(S_2/S_1)/\ln(N_2/N_1) = \ln(70/50)/\ln(.1) = -.1461$

The life @ S_3 = ±61 MPa, $N_3 = N_1(S_3/S_1)^{1/b} 10^4(61/70)^{1/-.146} = 2.57x10^4$.

The life @ ±54 MPa, $N = 10^4(54/70)^{1/-.146} = 5.91x10^4$.

$n_1/N_1 + n_2/N_2 + \ldots = 1$, $10^4/2.57x10^4 + n/5.91x10^4 = 1$;

$n = 5.91 x10^4 (1 - 10^4/2.57x10^4) = 3.61x10^4$

11. For a certain steel, the fatigue limits are 10,000 cycles at 100 ksi, 50,000 cycles at 75 ksi and 200,000 cycles at 62 ksi. If a component of this steel had been subjected to 5000 cycles at 100 ksi and 10,000 cycles at 75 ksi, how many additional cycles at 62 ksi would cause failure?

Solution: $n_{62}/N_{62} = 1 – n_{100}/N_{100} - n_{75}/N_{75} = 1 - .5 - .25 = .25$; $n_{62} = .25x200{,}000 = 50{,}000$ cycles

12. In fatigue tests on a certain steel, the endurance limit was found to be 1000 MPa for R = 0 (tensile-release) ($\sigma_a = \sigma_m$ = 500 MPa) and 1000 MPa for R = -1 (fully reversed cycling). Calculate whether a bar of this steel would fail by fatigue if it were subjected to a steady stress of 600 MPa and a cyclic stress of 500 MPa.

Solution: .Drawing a Goodman diagram, it is concluded that the part will fail.

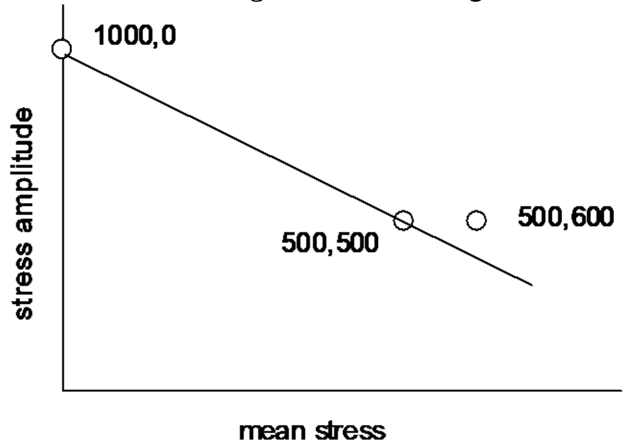
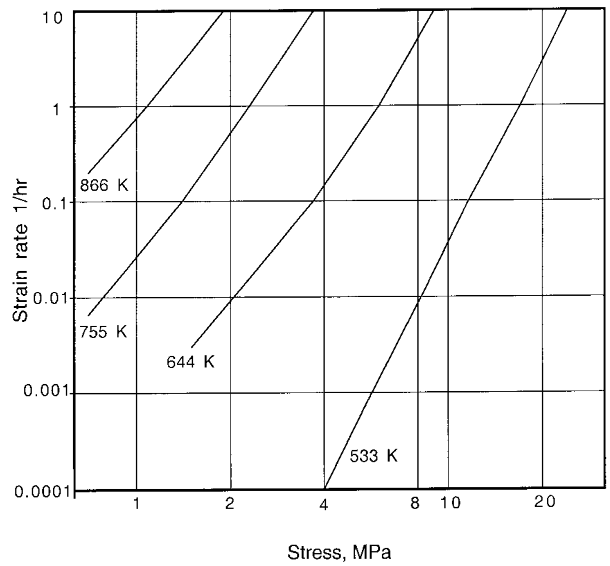


Figure 16.20. Steady state creep rate of an aluminum alloy at several temperatures. Data from O. D. Sherby and P. M . Burke, in *Prog.. Mater. Sci*. v. 13, 1968.



Solution: $m = \ln(\dot{\varepsilon}_1/\dot{\varepsilon}_2)/\ln(\sigma_1/\sigma_2) = \ln(0.6/.02)/\ln 2/1) = 4.9$ or
$m = \ln(20/.004)/\ln 4/0.8) = 5.3$ Hence $m \approx 5$.

Chapter 17

1. For steels, the endurance limit is approximately 1/2 of the tensile strength, and the fatigue strength at 10^3 cycles is approximately 90% of the tensile strength. The S-N curves can be approximated by a straight lines between 10^3 and 10^6 cycles when plotted as log(S) vs. log(N). Beyond 10^6 cycles the curves are horizontal.

A. Write a mathematical expression for S as a function of N for the sloping part of the S-N curve, evaluating the constants in terms of the approximations above.

B. A steel part fails in 12,000 cycles. Use the above expression to find what % decrease of applied (cyclic) stress would be necessary to increase in the life of the part by a factor of 2.5 (to 30,000 cycles).

C. Alternatively, what % increase in tensile strength would achieve the same increase in life without decreasing the stress?

Solution: A. logs = A' + blogN or lonS = A + blnN. $S_f = AN_f^b$, $(S_{f1}/ S_{f2}) = (N_1/ N_2)^b$; $b = \ln(S_{f1}/ S_{f2}))/\ln(N_1/ N_2) = \ln(.9/.5)/\ln(10^{-3}) = -0.085$, $A = S_f /N_f^b = 0.9TS/(10^3)^b = 1.62TS$, $S_f = 1.62TS(N)^{-.085}$

B $S_{f3}/S_{f1}= (N_3/ N_1)^b = (2.5)^{-.085} = 0.925$. The stress would have to be lowered by 7.25%

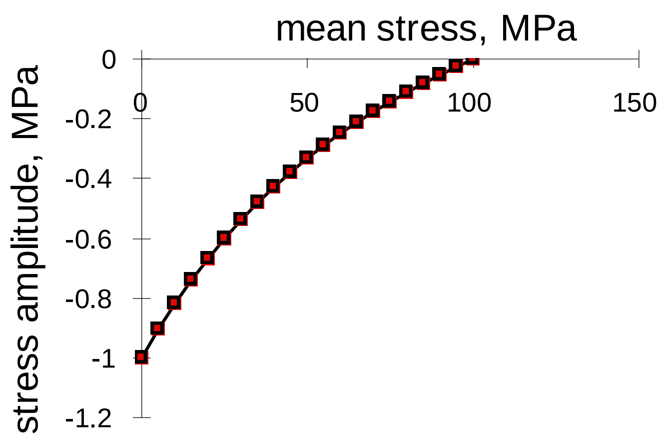
C 1/0.925 = 1.081, an 8% increase of TS.

2.A. Derive an expression relating the stress ratio, R, to the ratio of cyclic stress amplitude, σ_a, to the mean stress, σ_m.

B. For $\sigma_a = 100$ MPa, plot R as a function of σ_m over the range $0 \leq \sigma_m \leq 100$ MPa.

Solution: A. $R = \sigma_{min}/\sigma_{max} = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a) = (1- \sigma_a/\sigma_m)/(1+ \sigma_a/\sigma_m)$

$R = (\sigma_m - 100)/(\sigma_m + 100)$.



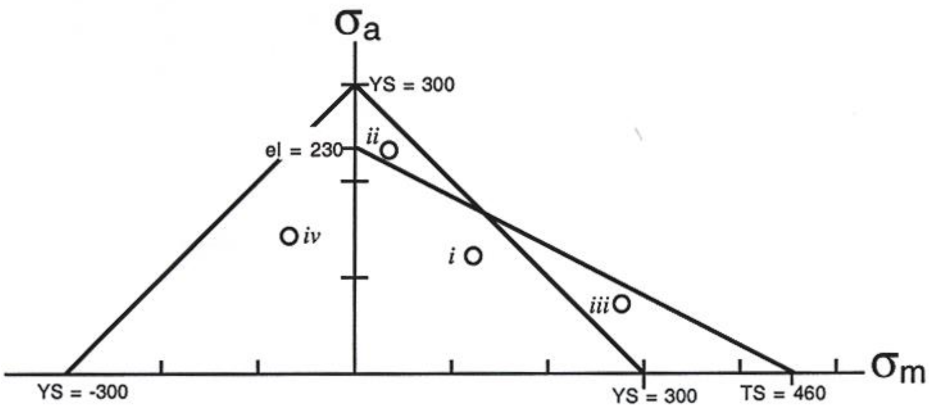
3. A steel has the following properties:

Tensile Strength	460 MPa
Yield Strength	300 MPa
Endurance limit	230 MPa

A. Plot a modified Goodman diagram for this steel showing the lines for yielding as well as fatigue failure.
B. For each of the cyclic loading given below determine whether yielding, infinite fatigue life, or finite fatigue life is expected.

i. σ_{max} = 250 MPa, σ_{min} = 0 ii. σ_{max} = 280 MPa, σ_{min} = -200 MPa
iii. σ_{mean} = 280 MPa, σ_a = 70 MPa iv. σ_{mean} = -70 MPa, σ_a = 140 MPa

Solution:



i. (σ_a = 125, σ_m = 125) infinite life, no yielding, ii. (σ_a = 240, σ_m = 40finite life, no yielding,
iii. infinite life, yielding, iv. infinite life, no yielding0

5. The notch sensitivity factor, q, gray cast iron is very low. Offer an explanation in terms of the micros: The graphite flakes act as notches much more severe than machined notches, so machinedtructure.

Solution: notches have little effect.

5. A 1040 steel has been heat-treated to a yield strength of 900 MPa and a tensile yield strength of 1330 MPa. The endurance limit (at 10^6 cycles for cyclical loading about a zero mean stress) is quoted as 620 MPa. Your company is considering using this steel, with the same heat treatment for an application in which fatigue may occur during cycling with R = 0. Your boss is considering shot peening the steel to induce residual compressive stresses in the surface. Can the endurance limit be raised this way? If so, by how much? Discuss this problem with reference to the Goodman diagram using any relevant calculation(s).

Solution: Yes.

The largest increase in fatigue strength corresponds to a negative mean stress, σ_m, which would cause yielding in compression. $\sigma_a = 900 + \sigma_m$.

For fatigue failure, $\sigma_a = 620(1 - \sigma_m/1330)$. Solving these simultaneously,

$900 + \sigma_m = 620 - 0.466\sigma_m$. $-1.466\sigma_m = 280$, $\sigma_m = -191$. $\sigma_a = 900 - 0191 = 709$ MPa

The endurance limit can be raised to 709 MPa if a residual compressive stress of –191 MPa can be induced.

6. Low cycle fatigue was the cause of the Comet failures. Estimate how many pressurization-depressurization cycles the planes may have experienced in the two years of operations. An exact answer is not possible, but by making a reasonable guess of the number of landings per day and the number of days of service a rough estimate is possible.

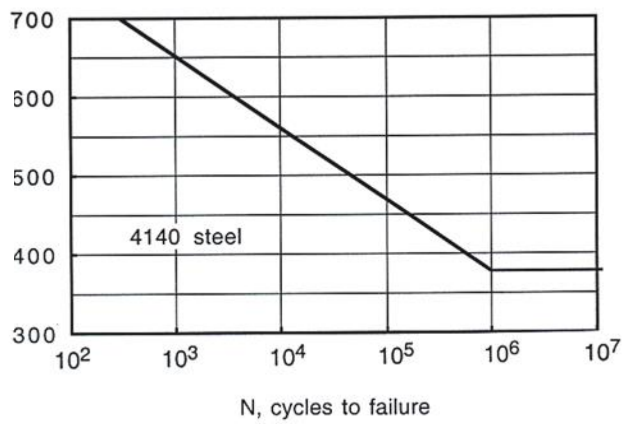
Solution: If the aircraft landed and took off 5 times per day for 320 days per year, the number of cycles would be 2x320x5 = 3200.

7. Frequently the S-N curves for steel can be approximated by a straight line between N = 10^2 and N = 10^6 cycles when the data are plotted on a log-log scale as shown in the figure for SAE 4140 steel. This implies $S = AN^b$, where A and b are constants.

C. Find b for the 4140 steel for a certain part made from 4140 steel (Figure 17.26).

D. Fatigue failures occur after 5 years. By what factor would the cyclic stress amplitude have to be reduced to increase the life to 10 years? Assume the number of cycles of importance is proportional to time of service.

Figure 17.26. S-N curve for a SAE 4140 steel.

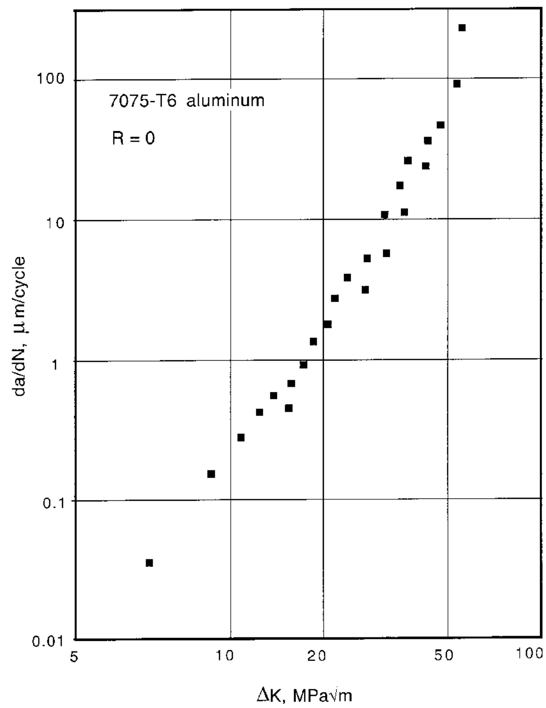


Solution: A. $b = \ln(S_2/S_1)/\ln(N_2/N_1) = \ln(375/650)/\ln(10^6/10^3) = -.0796$

B. $(S_3/S_1) = (N_2/N_1)^{-0.796} = (2)^{-0.0796} = 0.946.$

8. Figure 17.27 shows the crack growth rate in aluminum alloy 7075-T6 as a function of DK for R = 0. Find the values of the constants C and m, in equation 17.19, that describes the straight-line portion of the data. Give units.

Figure 17.27. Crack growth rate of 7075-T6 aluminum for DK = 0. Data from C. M. Hudson, NASA TN D-5300, 1969.



Solution: da/dN = $C(\Delta K_I)^m$, m = $\ln[(da/dN)_1/[(da/dN)_2]/\ln[(\Delta K_I)_1/(\Delta K_I)_2]$ = $\ln[(10^{-1})/(10^{-5})]\ln[(70)/(5)] = 3.5$

$C = (da/dN)/(\Delta K_I)^m = 10^{-1}/70^{3.5} = 3.5\text{x}10^{-8}$. The units of C are $(\text{mm/cycle})(\text{MPa}\sqrt{\text{m}})^{3.5}$

9. Find the number of cycles required for a crack to grow from 1 mm to 1 cm in 7075-T6 (problem 8.) if f = 1 and $\Delta\sigma$ = 10 MPa. Remember that $\Delta K = f\Delta\sigma\sqrt{(\pi a)}$.

Solution: : Using equation 17.22, $N = [a^{(1-m/2)} - a_o^{(1-m/2)}]/[Cf^m(\pi\Delta\sigma)^{m/2}] = [10^{-2(1-3.5/2)} - 10^{-1(1-3.5/2)}]/[3.5\text{x}10^{-8}(\pi 10)^{3.5/2}] = 1.78\text{x}10^6$ cycles

10. For a certain steel, it was found that the fatigue life at ± 70 MPa was 10^4 cycles and it was 10^5 cycles at ± 50 MPa. A part made from this steel was given 10^4 cycles at ±61 MPa. If the part were then cycled at ±54 MPa what would be its expected life?

Solution: $b = \ln(S_2/S_1)/\ln(N_2/N_1) = \ln(70/50)/\ln(.1) = -.1461$

The life @ S_3 = ±61 MPa, $N_3 = N_1(S_3/S_1)^{1/b}10^4(61/70)^{1/-.146} = 2.57\text{x}10^4$.

The life @ ±54 MPa, $N = 10^4(54/70)^{1/-.146} = 5.91\text{x}10^4$.

$n_1/N_1 + n_2/N_2 + \ldots = 1$, $10^4/2.57\text{x}10^4 + n/5.91\text{x}10^4 = 1$;

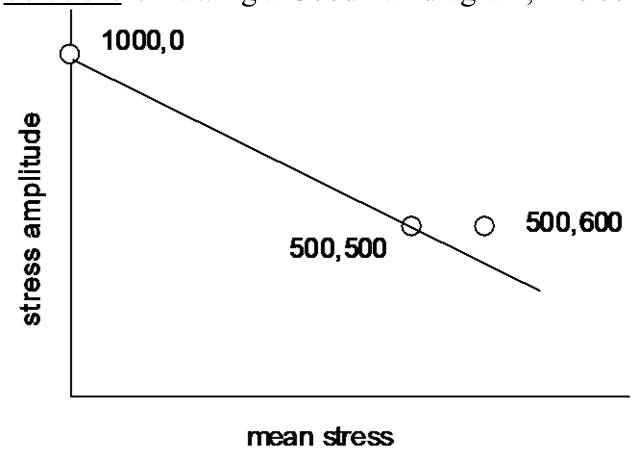
$n = 5.91\text{ x}10^4 (1 - 10^4/2.57\text{x}10^4) = 3.61\text{x}10^4$

13. For a certain steel, the fatigue limits are 10,000 cycles at 100 ksi, 50,000 cycles at 75 ksi and 200,000 cycles at 62 ksi. If a component of this steel had been subjected to 5000 cycles at 100 ksi and 10,000 cycles at 75 ksi, how many additional cycles at 62 ksi would cause failure?

Solution: : $n_{62}/N_{62} = 1 - n_{100}/N_{100} - n_{75}/N_{75} = 1 - .5 - .25 = .25$; $n_{62} = .25\text{x}200{,}000 = 50{,}000$ cycles

14. In fatigue tests on a certain steel, the endurance limit was found to be 1000 MPa for R = 0 (tensile-release) ($\sigma_a = \sigma_m$ = 500 MPa) and 1000 MPa for R = -1 (fully reversed cycling). Calculate whether a bar of this steel would fail by fatigue if it were subjected to a steady stress of 600 MPa and a cyclic stress of 500 MPa.

Solution: .Drawing a Goodman diagram, it is concluded that the part will fail.



<u>Chapter 18</u>

1. Consider a piece of polycrystalline iron that has been plastically deformed in tension under a stress of 220 MPa and then unloaded. Because of the orientation dependence of the Taylor factor, it is reasonable to assume that the stress before unloading was 20 % higher in grains oriented with <111> parallel to the tensile axis than in the average stress. Young's modulus for polycrystalline iron is listed as 208 GPa but for crystals oriented in a <111> direction, it is 283 MPa. Determine the level of residual stress in the <111>-oriented grains.

<u>solution:</u> The stress under load in the [111] grains was 1.2x220 = 264 GPa. On unloading the change of strain was -220MPa/208GPa = $-1.06\text{x}10^{-3}$. The change of stress in the [111] grains was -283 GPa($1.06\text{x}10^{-3}$) = -267 MPa. The final stress in the [111] grains was 264 – 267 = -3 MPa

2. The residual stresses adjacent to a long butt weld between two deck plates of a ship were found by x-rays. The average values of the lattice parameter, a, were a = 2.8619 parallel to the weld bead and a = 2.86106 perpendicular to the weld bead. For unstrained material, a = 2.8610. Find the values of the residual stresses parallel and perpendicular to the weld. Assume E = $30\text{x}10^6$ psi, υ = 0.29 and that the stress normal to the plate is zero.

<u>solution:</u> $e_1 = 0.0009/2.861 = 3.15\text{x}10^{-4}$; $e_2 = 0.00006/2.861 = 2.1\text{x}10^{-5}$;

$e_2 = (1/E)[\sigma_2 - \upsilon\sigma_1]$; $\sigma_2 = Ee_2 + \upsilon\sigma_1$. $e_1 = (1/E)[\sigma_1 - \upsilon\sigma_2] =$

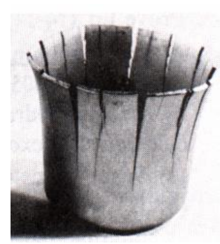
$(1/E)[\sigma_1 - \upsilon \square Ee_2 + \upsilon\sigma_1)]$; $\sigma_1 = E(e_1 - ve_2)/(1-\upsilon^2)$

$\sigma_1 = E(e_1 - \upsilon e_2)/(1-\upsilon^2) = (30\text{x}10^6 \text{ psi})[3.15\text{x}10^{-4} - 0.29(2.1\text{x}10^{-5})]/(1-.29^2) = 10{,}100$ psi,

$\sigma_2 = E(e_2 - \upsilon e_1)/(1-\upsilon^2) = -7{,}700$ psi

3. The stresses in the walls of a deep drawn stainless steel cup are suddenly released when the walls split by stress corrosion cracking as shown in Figure 18.14. After splitting the segments of the wall curved to a radius of curvature of 10 in. Assume the stresses drop to zero. The wall thickness is 0.030 in. and Young's modulus is $30\text{x}10^6$ psi. What were the residual stresses in the wall before stress corrosion cracking? Note that if a narrow strip is bent elastically, the change of stress is given by: $\Delta\sigma = E\Delta\varepsilon$ and $\Delta\varepsilon = z/\rho$ where z is the distance from the neutral axis and ρ is the radius of curvature. Therefore $\Delta\sigma$ varies linearly through the thickness and is a maximum at the surfaces.

Figure 18.14. Stress-corrosion cracks in a drawn cup. From W. F. Hosford and R. M. Caddell, *Metal Forming: Mechanics and Metallurgy,* 2nd ed. Prentice Hall, 1993, p. 305.



<u>solution:</u> On bending, $\Delta\sigma = Ez/\rho$. At the surface where $z = t/2$, $\Delta\sigma = Et/(2\rho)$.

= $(30 \times 10^6)(0.030)/(20)$ = 45,000 psi. If the final surface stress is zero, the residual stress before unbending must have been 45,000 psi.

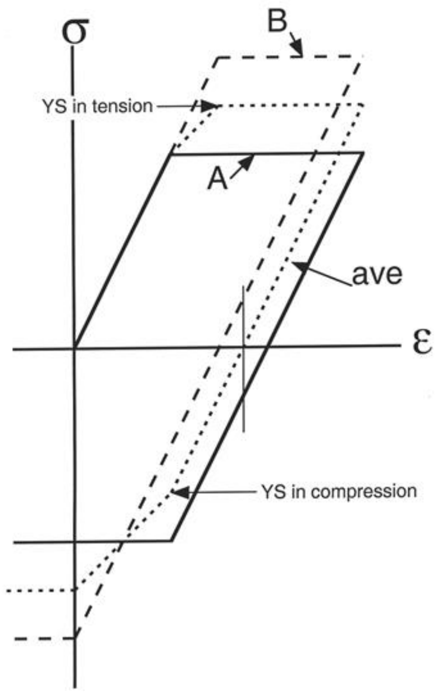
4. Consider a material composed of equal volumes of having different yield strengths, Y_A, and $Y_B = 1.5Y_A$, but the same elastic moduli ($E_A = E_B$). Assume that during straining both regions undergo the same strains ($\varepsilon_A = \varepsilon_B$) and that there is no strain hardening in either region.

A) Let the material be subjected to a tensile strain $\varepsilon_A = \varepsilon_B = 2Y_B/E$. Sketch the individual and overall stress strain curves, σ_A, σ_B, and $\sigma_{av} = (\sigma_A + \sigma_B)/2$. What is the overall yield strength of the material (i.e., value of σ_{av} corresponding to the first deviation from linearity)?

B) Now consider the behavior on unloading to $\sigma_{av} = 0$. Add plots of σ_A, σ_B, and σ_{av} to your sketch. What is the level of residual stress in each region?

C) After loading in tension and unloading, consider the behavior on loading under compression until the entire material yields. Assume that the tensile and compressive yield strength of each region is the same. ($|Y_{Acomp}| = Y_{Atens}$) and ($|Y_{Bcomp}| = Y_{Btens}$). Add the compressive behaviors to your plot. What is the new overall yield strength in compression and how does this compare with what the overall yield strength would be if it hadn't been first loaded in tension?

<u>solution:</u> A. Averaging at the same strain, $Y_{comp} = (Y_A + Y_B)/2 = 1.25Y_A$



B. $\sigma_A = -0.25Y_A$, $\sigma_B = +0.25Y_A$,

E. $YS_{ave} = -0.75Y_A$. If it hadn't been first deformed in tension theyield strength in compression would have been $-1.25Y_A$ so the compression is less than it would have been had if it never been loaded in tension

5. A 1 cm diameter steel ball is cooled after austenitization. At one point during the cooling, a 0.5 mm thick layer at the surface transforms to martensite while the center is still austenite. For simplicity, assume that the interior is at an average temperature of 200°C and the surface is at 20°C

and that there are no stresses in the material at this point. When the center cools to 20°C, it transforms to martensite. For this steel, the austenite to martensite transformation is accompanied by a 1.2% volume expansion. Assume this expansion occurs equally in all directions. For steel, the coefficient of linear thermal expansion is $\alpha = 6x10^{-6}/°C$. and Young's modulus is $30x10^6$ psi and $\upsilon = 0.29$. Assume these apply to both martensite and austenite. Find the stress state at the surface.

<u>solution:</u> The martensite transformation will cause a linear expansion of 1.2%/3 = .4% = 0.004. The thermal contraction will be $-180°C(6x10^{-6}/°C) = 0.00108$ so the net expansion is e = .004-0.00108 =0.0029. The surface is under biaxial tension so $e = (1/E)\sigma(1-\upsilon)$ and $\sigma = eE(1-\upsilon) = 0.0029(30x10^6)/(1-.29) = 122,000$ psi.

6. If a metal sheet is bent plastically under a bending moment without applied tension, there will be springback after the moment is released and there will be residual stresses in the sheet as shown in Figure 18.9. If the surface layers of such a sheet are removed by etching or corrosion, how will the bend change? Will the radius of the bend increase, decrease, or remain unchanged? Explain.

<u>Solution:</u> The outside of the bend is under compression. Removal of this layer will cause unbending to preserve a net moment of zero. The inside of the bend is under residual tension. Removal of this layer will cause unbending to preserve a net moment of zero. Conclusion – it will unbend.

7. A surface layer, 0.001 in. thick was removed from an aluminum sheet, 0.015 in. thick. On removal of the layer, the sheet curled to form a dish with a radius of curvature of 30 in. What was the stress in the surface layer before it was removed. For aluminum, $E = 10x10^6$ psi and $\upsilon = 0.30$.

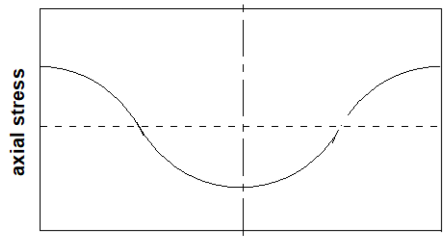
<u>Solution:</u> According to equation 18.21, $\sigma = E(1+\upsilon)t^2/(6\rho\Delta t) =$
$(10x10^6)(1.3)(0.014)^2/[6(30)(0.001)] = 14,200$ psi

8. For polycrystalline magnesium the coefficient of thermal expansion is $25.2x10^{-6}/°C$ and Young's modulus is 68.4 GPa. Parallel to the c-axis of a single crystal the coefficient of thermal expansion is $24.3x10^{-6}/°$ and Young's modulus is 80.9 GPa. Estimate the stress parallel to the c-axes of grains in a randomly oriented polycrystal when the temperature is changed by 100°C. Assume that the dimensional change parallel to the c-axis of the grain is the same as that in the polycrystal. For simplicity, assume also that the stress in the polycrystal is negligible.

<u>Solution:</u> $\sigma = (\alpha_1 - \alpha_2)E\Delta T = (25.2-24.3)(10^{-6})(80.9 \text{ GPa})(100) = 7.3$ MPa

9. A typical residual stress pattern in an extruded bar is shown in the accompanying sketch. To find the residual stresses in an extruded bar of brass, 1.000 in. diameter and 10.000 in. long, the bar was put in a lathe and machined to a diameter of 0.900 in. After machining, the length was found to be 10.004 in. What was the average residual stress in the layer that was machined away? For the brass, E = 110 GPa. For simplicity, neglect the Poisson effects.

Figure 18.15. Residual stress pattern in an extruded bar.



Solution: A force balance gives

$\pi(1)(0.1)\sigma = \pi(.9/2)^2(110\text{ GPa})(0.0004/10)$;

$\sigma = (.9/2)^2(110\text{ GPa})(0.0004) = 8.9\text{ MPa}$

10. During a stress relief anneal, creep converts elastic strains (and therefore stresses) into plastic strains. The decrease in stress should be $\Delta\sigma = -E\varepsilon$, or $d\sigma/dt = -Ed\varepsilon/dt = -E\dot{\varepsilon}$. Assume that the stress relief anneal is done in a temperature range where the basic creep rate is given by $d\varepsilon/dt = \dot{\varepsilon} = C\sigma^m$ and that the residual stress before annealing was σ_o.

A) If $m = 5$, as is typical of creep at the low-temperatures used for stress relieving, $\sigma_o = 5000$ psi and $E = 10\text{x}10^6$ psi, and $C = 6\text{x}10^{-22}(\text{psi})^{-5}$/hr at the annealing temperature, how long would it take for the stress to drop to 2,500 psi?

B) For the same temperature, how long would it take the stress to drop to 1000 psi?

Solution: $d\sigma/dt = -Ed\varepsilon/dt = -EC\sigma^m$,

$\sigma^{-m}\, d\sigma = -ECdt$, Integrating from $\sigma = \sigma_o$ to σ,

$[\sigma^{1-m} - \sigma_o^{1-m}]/(1-m) = -ECt$,

$t = [\sigma^{1-m} - \sigma_o^{1-m}]/[(-EC)(1-m)]$

A. $t = [2500^{(1-5)} - 5000^{(1-5)}]/[(-10\text{x}10^6)(6\text{x}10^{-22})(-4)] = 1.0$ hrs

B. $t = [1000^{(1-5)} - 5000^{(1-5)}]/[(-10\text{x}10^6)(6\text{x}10^{-22})(-4)] = 42$hrs