Cooperative Communications and Networking

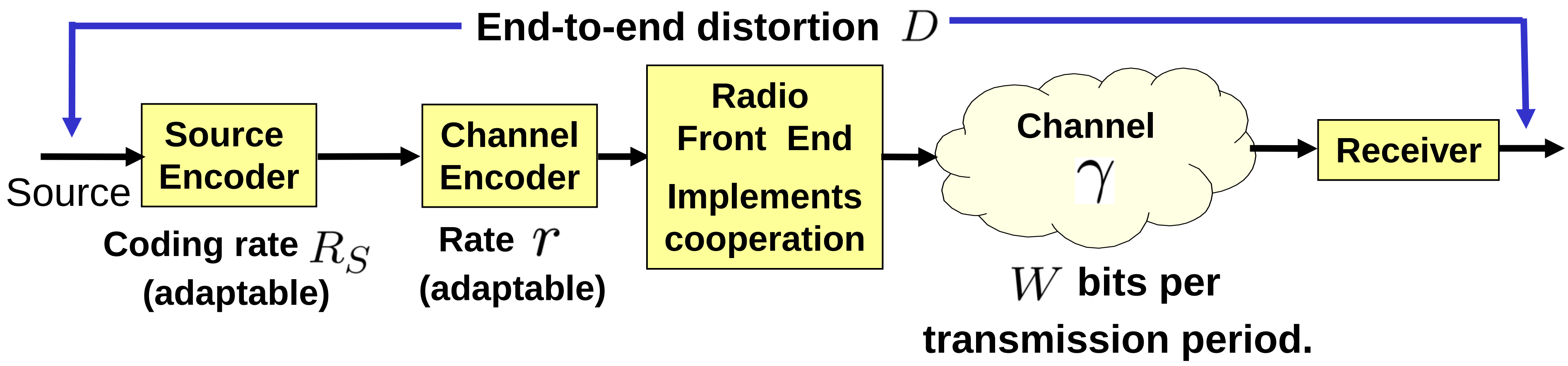
Chapter 14

Source-Channel Coding with Cooperation

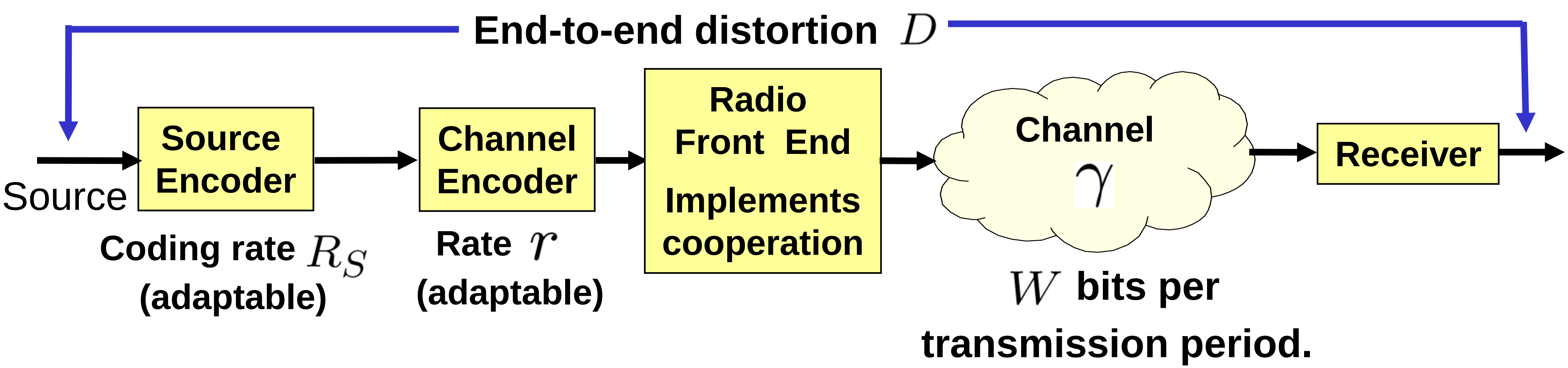
Outline

- Joint Source-Channel Coding Bit Rate Allocation
- Joint Source-Channel Coding with User Cooperation
- The Source-Channel-Cooperation Tradeoff Problem
- Source Codec
  - Practical Source Codecs
- Channel Codec
- Analysis of Source-Channel-Cooperation Performance
  - Amplify-and-Forward Cooperation
  - Decode-and-Forward Cooperation
  - No Use of Cooperation
- Effects of Source-Channel-Cooperation Tradeoffs

Joint Source-Channel Coding Bit Rate Allocation

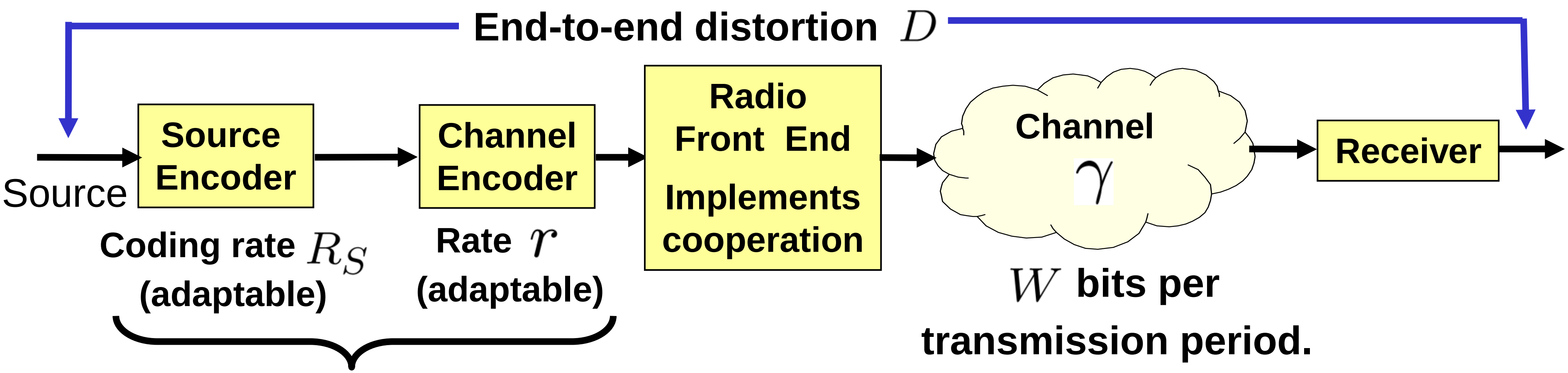


Joint Source-Channel Coding Bit Rate Allocation



$$D = \underbrace{D_F}_{\text{Channel induced distortion}} P(f) + \underbrace{D_S}_{\text{Distortion due to source quantization and coding}} (1 - \underbrace{P(f)}_{\text{Probability that the transmitted frame has channel errors after channel decoding and has to be discarded}}))$$

Joint Source-Channel Coding Bit Rate Allocation

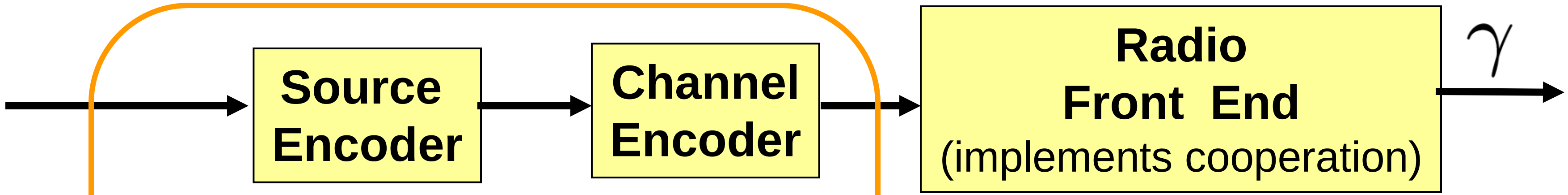


- Want to find best choice of R_S, r and use or not of cooperation to <u>minimize end-to-end distortion</u> D.

Criteria used to obtain best end-to-end performance in conversational multimedia systems.

$$D = \underbrace{D_F}_{\text{Channel induced distortion}} P(f) + \underbrace{D_S}_{\text{Distortion due to source quantization and coding}} (1 - \underbrace{P(f)}_{\text{Probability that the transmitted frame has channel errors after channel decoding and has to be discarded}})$$

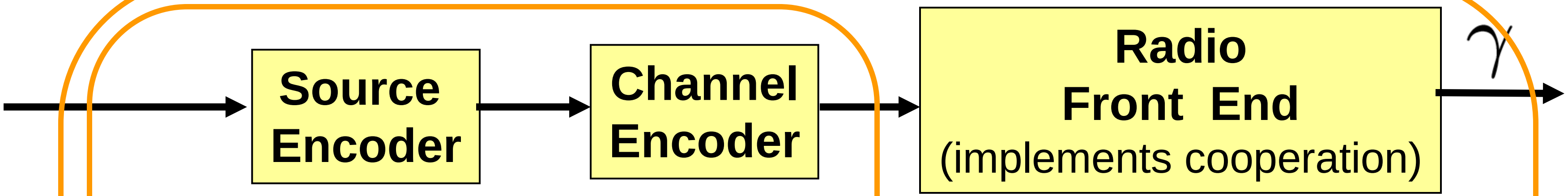
Joint Source-Channel Coding with User Cooperation



- **Joint Source-Channel Coding**
  - **Allocation:** R_S, r.
  - W **fixed.**

$$\uparrow R_S \quad \uparrow r \quad \downarrow D_S \quad \uparrow \widetilde{P}_{\Omega_i}(\gamma)$$

Joint Source-Channel Coding with User Cooperation

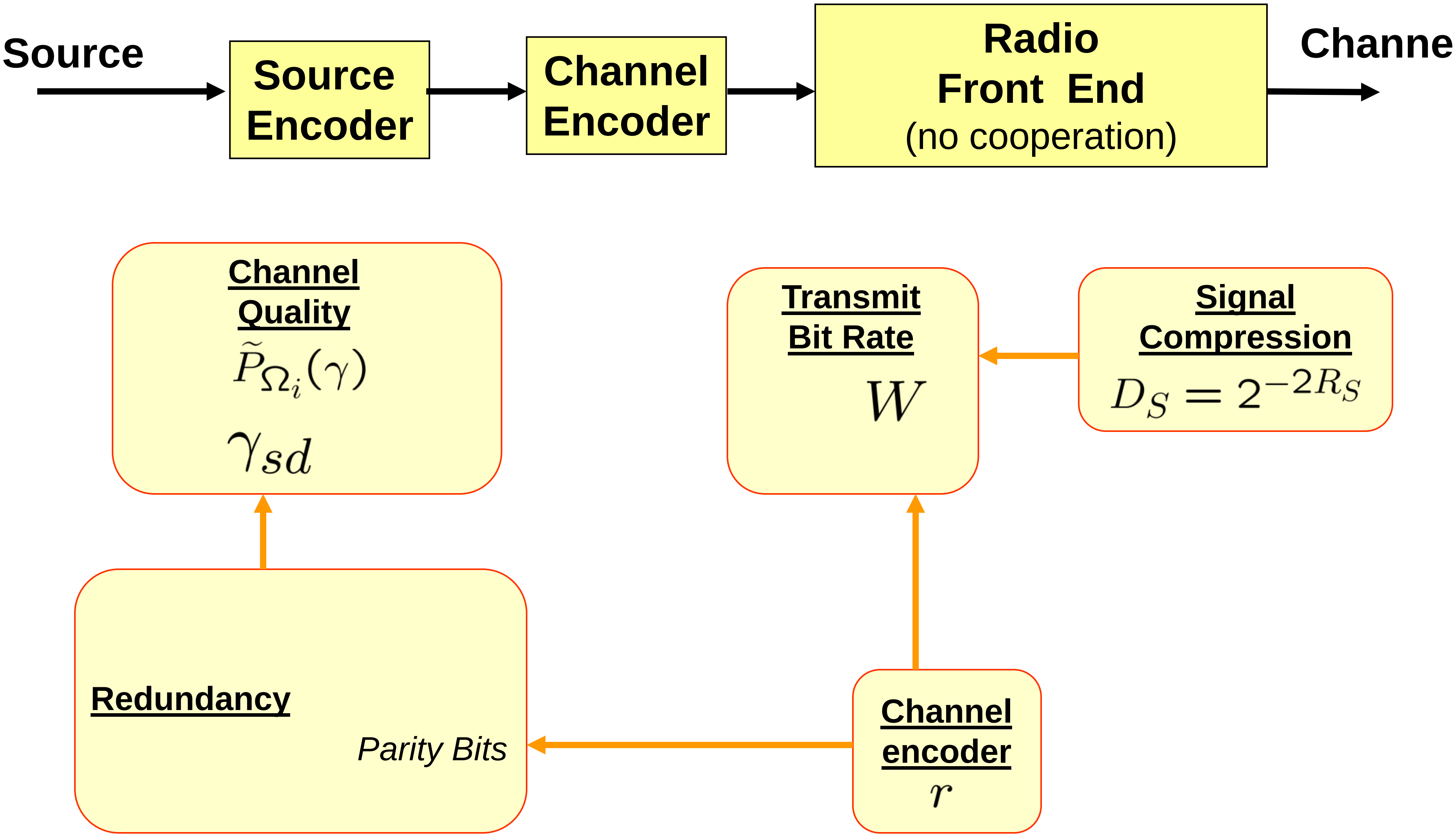


- **Joint Source-Channel Coding**
  - **Allocation:** R_S, r.
  - W **fixed.**

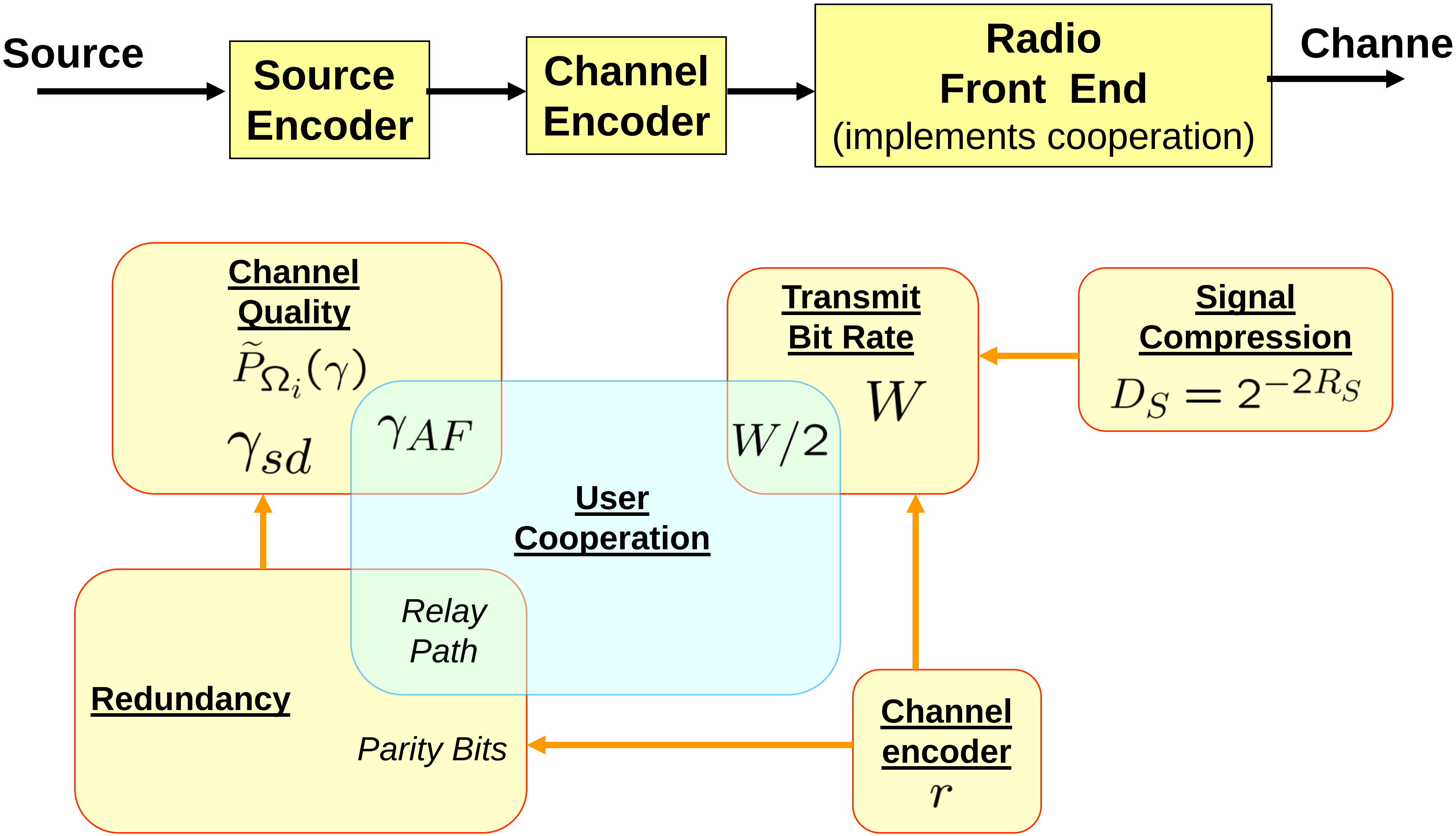
  $\uparrow R_S \quad \uparrow r \quad \downarrow D_S \quad \uparrow \tilde{P}_{\Omega_i}(\gamma)$

- **No cooperation:** W , γ_{sd}
- **Cooperation:** $W/2$, γ_{AF} / γ_{DF}

Interdependencies in Joint Source-Channel Coding

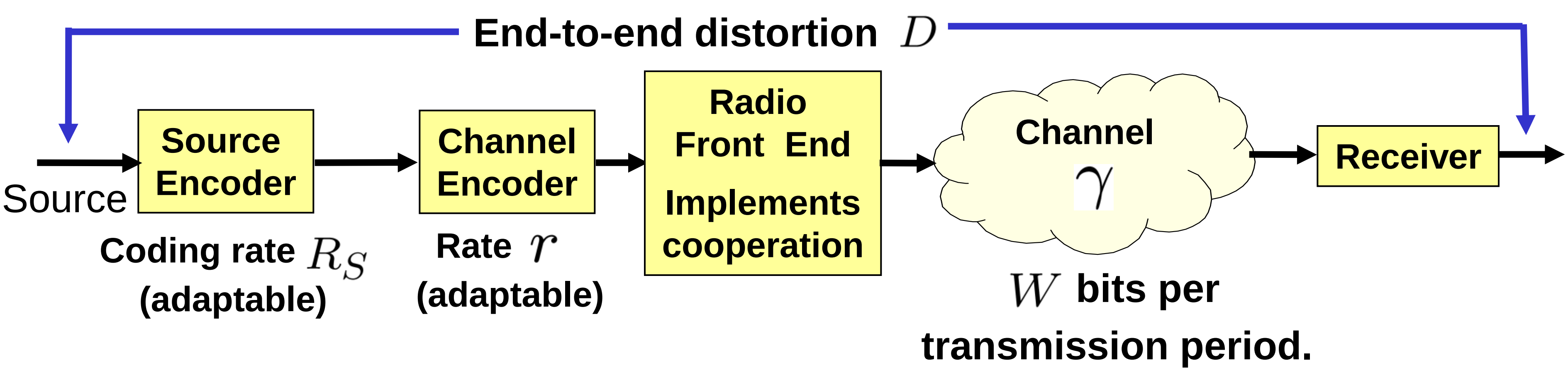


Interdependencies in Joint Source-Channel-Cooperation

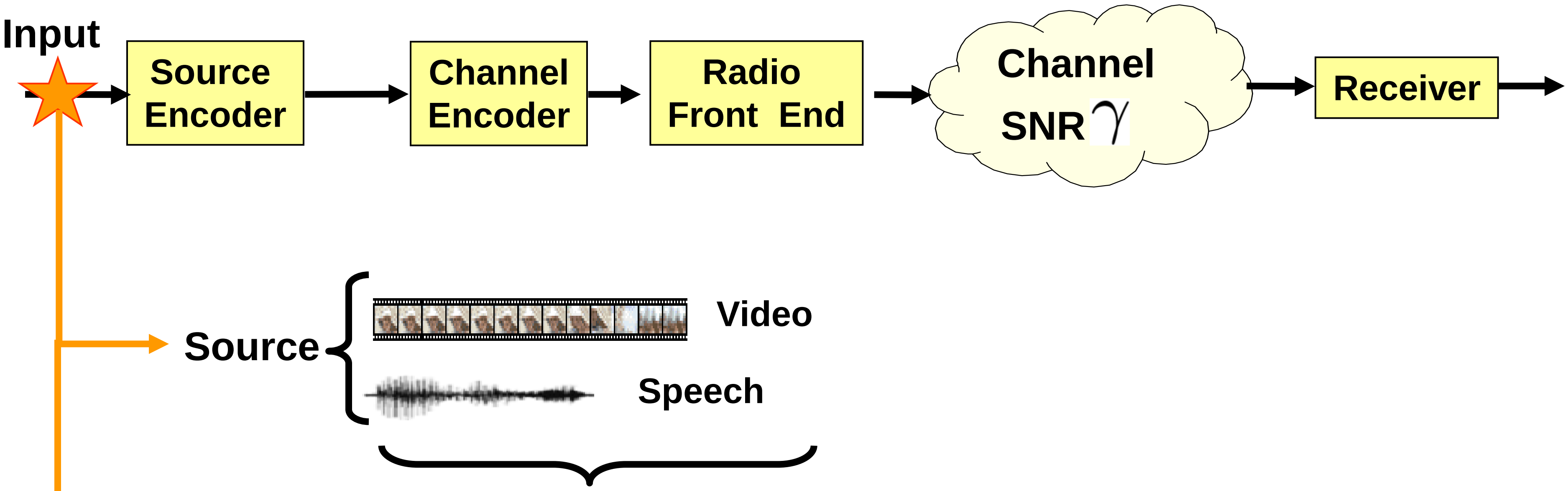


The Source-Channel-Cooperation Tradeoff Problem

- **Assumption: Signal is conversational; i.e. there is an strict delay constraint that prevents use of ARQ. Packets with errors need to be dropped.**
- **Assumption: Each call uses a channel that delivers W bits per transmission period (constant number of bits).**
- **Assumption: Low mobility scenario, the channel is quasi-static fading.**
- **Assumption: Perfect knowledge of the channels (based on quasi-static channel assumption and also as study approach to decouple channel estimation errors from results).**
- **System setup overview:**



Problem Setup

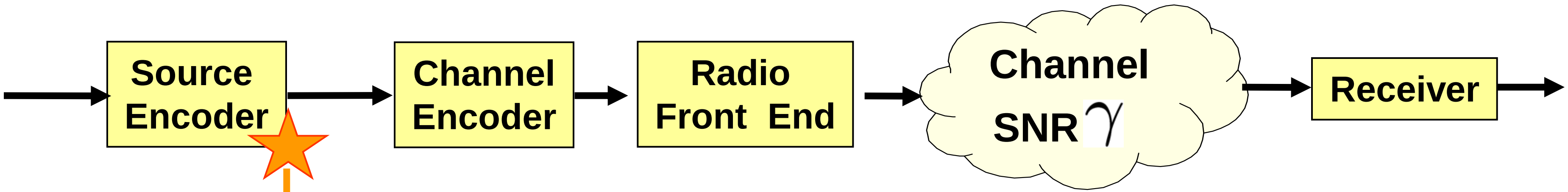


Modeled in a generic way:

N **Samples ~ Standard Gaussian distribution** $\mathcal{N}(0,1)$

Conversational / interactive traffic → strict delay constraint

<u>Source Codec</u>

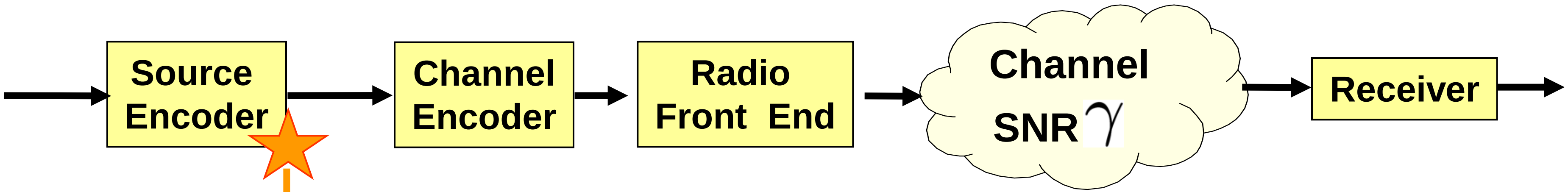


- Encodes source: <u>quantization and compression.</u>
- Coding rate: R_S (bits/sample)
  - adaptable
- Distortion: Added by compression
  - measured as mean-squared error
  - frequently modeled as:

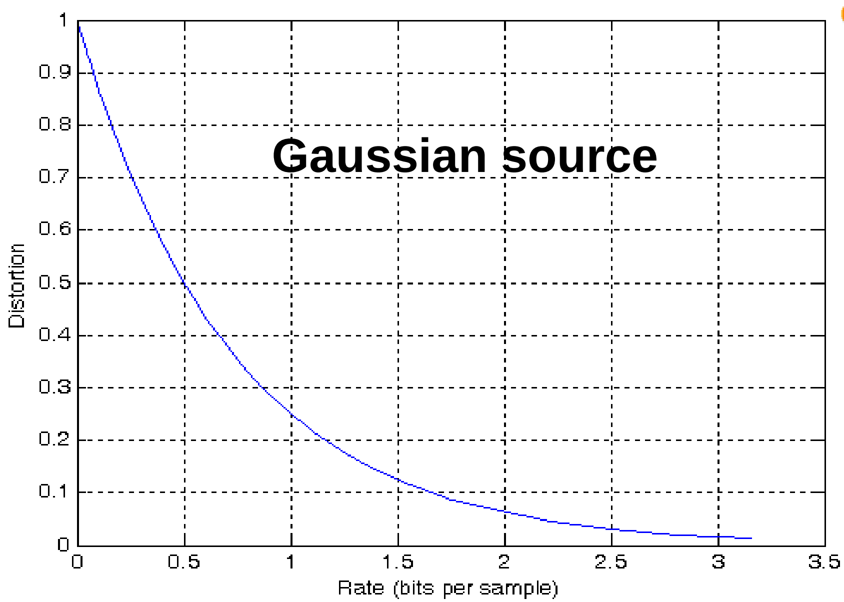
$$D_S(R_S) = c_1 2^{-c_2 R_S}.$$

- This function can be used to approximate or bound many practical, well-designed codecs.

Source Codec



- For a generic source:
- Assume that the input samples are memoryless, with a zero-mean, unit-variance Gaussian distribution and are coded with a long block source code.

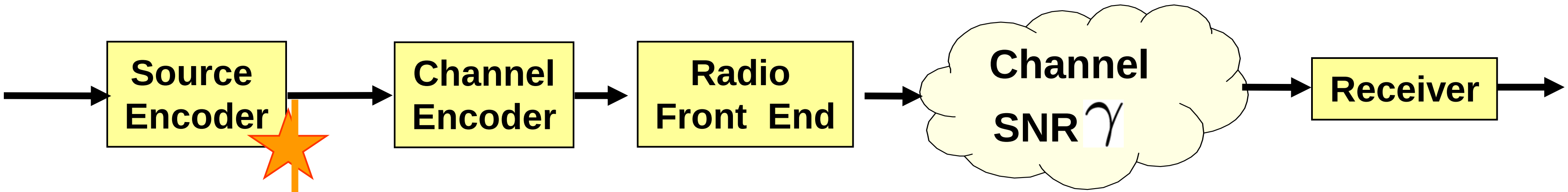


- Distortion:

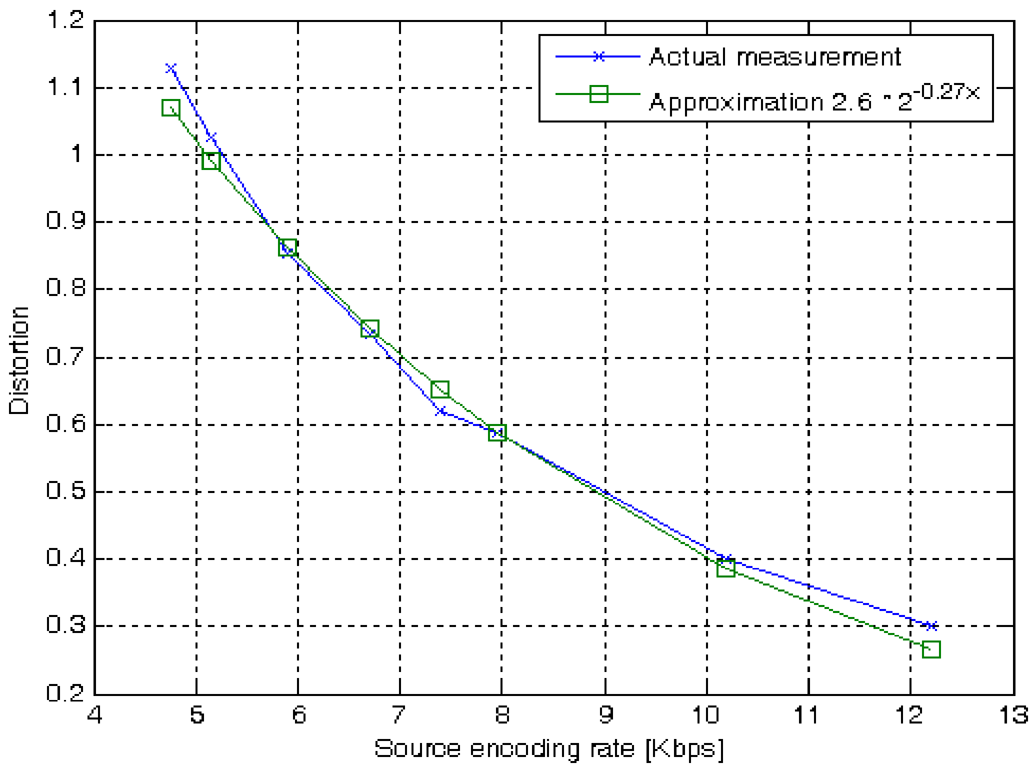
If transmission succeeds $D_S = 2^{-2R_S}$.

If transmission fails $D_S = D_F = 1$.

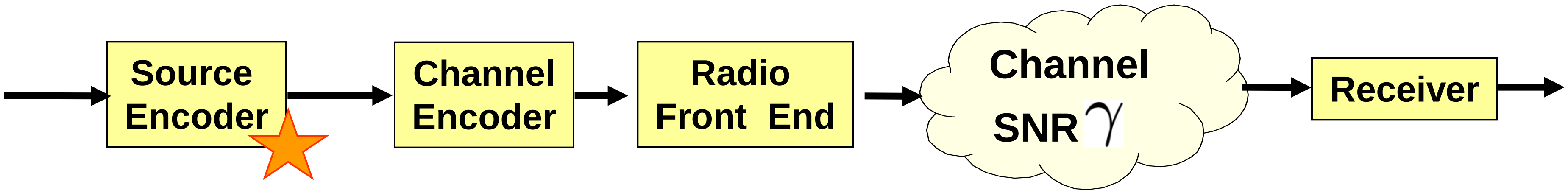
Speech Codec



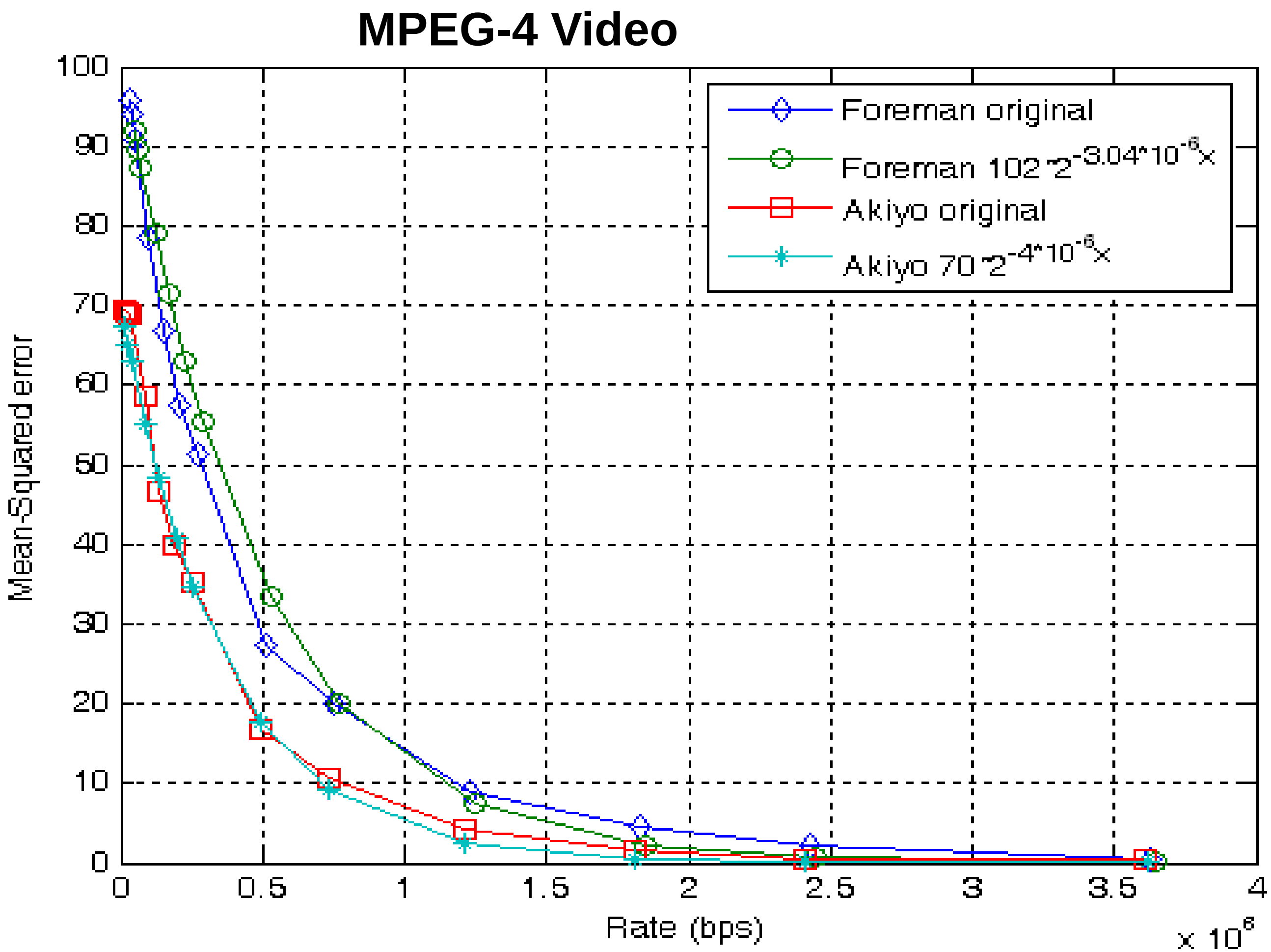
- Distortion more meaningfully measured using subjective measures



Video Codec

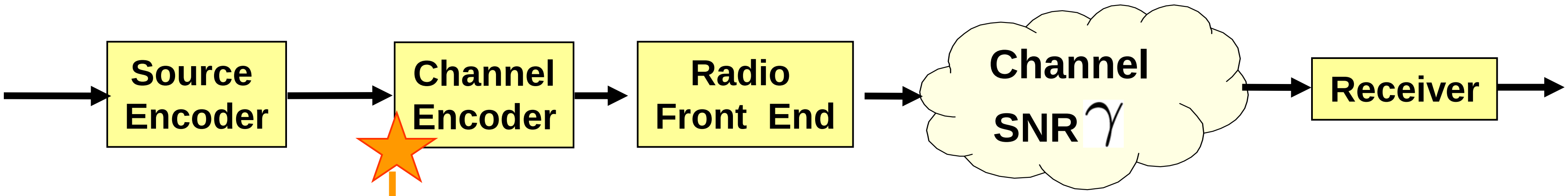


Source Encoder
Channel Encoder
Radio Front End
Channel
SNR γ
Receiver



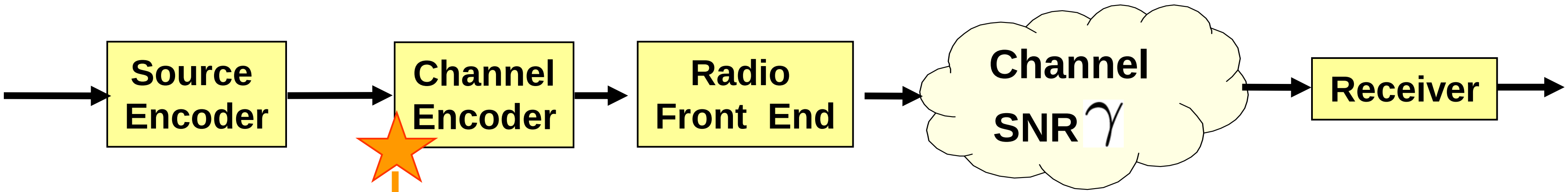
MPEG-4 Video
Foreman original
Foreman $102*2^{-3.04*10^{-6}x}$
Akiyo original
Akiyo $70*2^{-4*10^{-6}x}$
Mean-Squared error
Rate (bps)
$\times 10^6$

Channel Codec



- Adds redundancy to combat channel impairments
- Rate: $r = \dfrac{\text{input bits}}{\text{output bits}}$ (adaptable)
- Rate-Compatible Punctured Convolutional (RCPC) codes

Problem Setup



- Adds redundancy to combat channel impairments
- Rate: r (adaptable)
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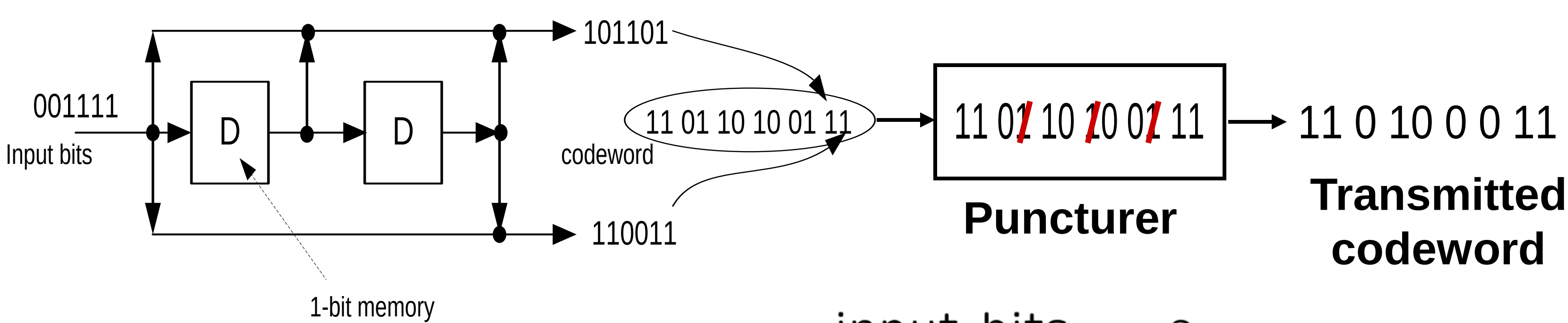
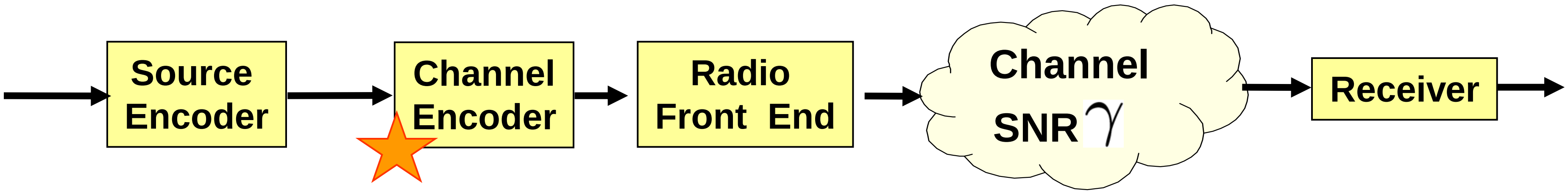
Convolutional code



$$r = \frac{\text{input bits}}{\text{output bits}} = \frac{2}{3}$$

Problem Setup



- **Probability of a decoding error:** $P_e(d|\gamma) = .5 \ \mathrm{erfc}\sqrt{d\gamma}$

Number of bits with errors

- **Probability of having a source frame with errors after channel decoding :**

$$P(\gamma) \leq 1 - \left[1 - \sum_{d=d_f}^{W} a(d) P_e(d|\gamma)\right]^{NR_s}$$

Number of error events with Hamming distance d

Free distance: smallest Hamming distance between two different codewords.

Analysis of Source-Channel-Cooperation Performance

- Approach:
  - First characterize the end-to-end distortion as a function of SNR: the *D-SNR curve*.
  - Use D-SNR curve to compare the performance of different schemes.
- End-to-end distortion is formed by:
  - Source encoder distortion: depends on the source encoding rate)
  - Channel-induced distortion: depends on the channel SNR, use of cooperation, channel coding rate and error concealment operations.
- Best adaptation to the known channels SNRs is achieved by jointly setting both the source and channel coding rates so as to minimize the end-to-end distortion (while not exceeding W).

Operating Mode

- Define *Operating Mode*:

Triplet $$\Omega_i = \{R_S, r, \text{cooperation yes/no}\}$$

- Since $\dfrac{W}{2} = \dfrac{NR_S}{r}$ (with cooperation)

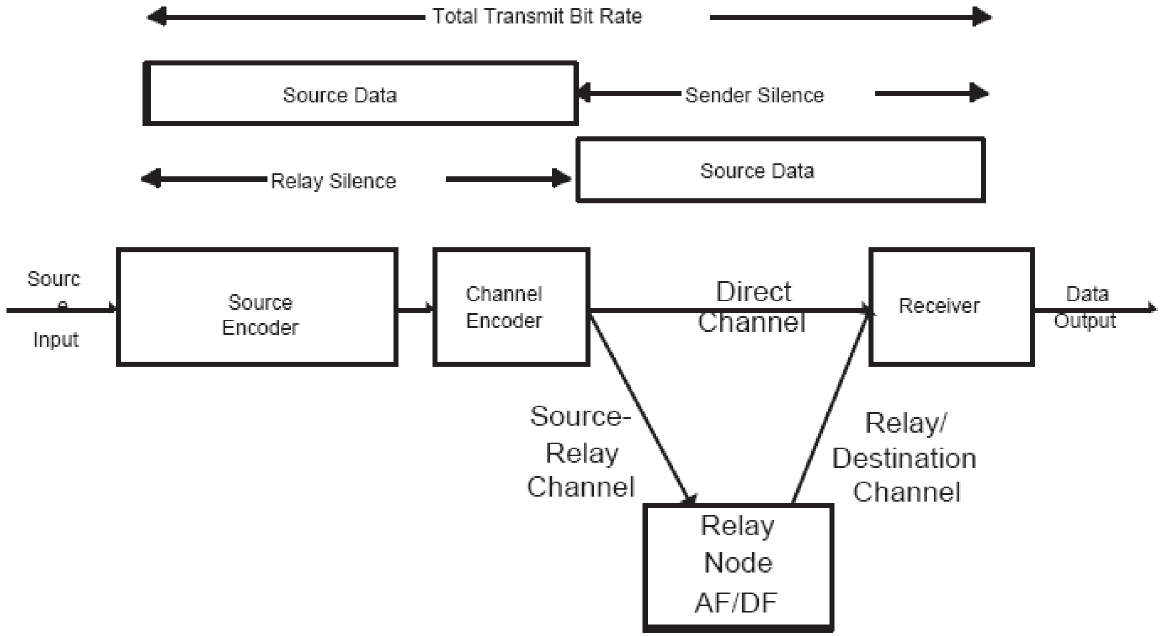
$W = \dfrac{NR_S}{r}$ (without cooperation)

- Operating mode can be specified as

$$\Omega_i = \{r, \text{cooperation yes/no}\}$$

$$\Omega_i = \{R_S, \text{cooperation yes/no}\}$$

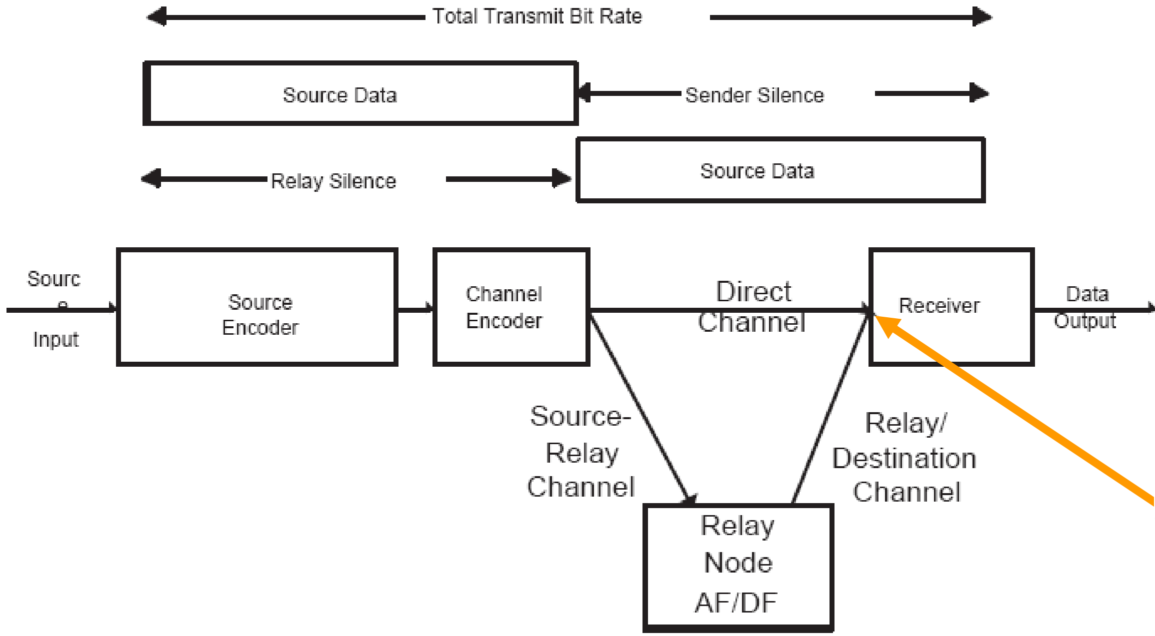
Problem Setup

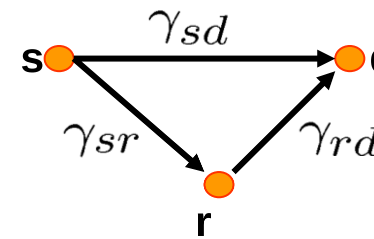


- The D-SNR function for AF cooperation is the solutions to:

$$D_{CAF} = \min_{\Omega_i \in \Omega^{(c)}} \{D_F P_{AF_{\Omega_i}}(\gamma_{AF}) + 2^{-2R_{SC}}(1 - P_{AF_{\Omega_i}}(\gamma_{AF}))\}$$

Problem Setup - AF Cooperation





$$\gamma_{AF} = \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{1 + \gamma_{sr} + \gamma_{rd}}$$

- The D-SNR function for AF cooperation is the solutions to:

$$D_{CAF} = \min_{\Omega_i \in \Omega^{(c)}} \{D_F P_{AF_{\Omega_i}}(\gamma_{AF}) + 2^{-2R_{SC}}(1 - P_{AF_{\Omega_i}}(\gamma_{AF}))\}$$

Operating modes using user cooperation

Source encoding rate when using AF user cooperation

probability of having a source frame with errors after channel decoding when using AF user cooperation

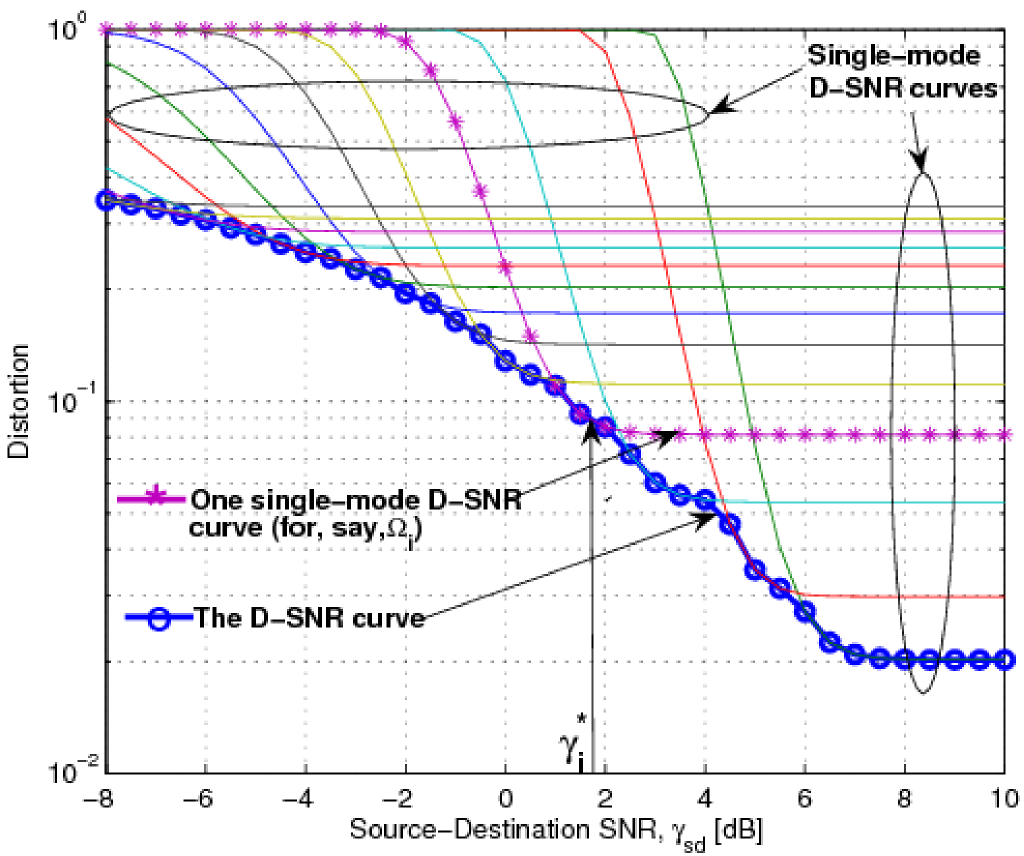
The D-SNR Curve

- Next: want to find a closed form expression for the D-SNR curve.
- Using Information Theory is not the best way if we want to consider delay constraint and limited processing power.

The D-SNR Curve – AF Cooperation (WLOG)

- **Examining the problem in more detail:**

$$\underbrace{D_{CAF}}_{\text{D-SNR curve}} = \min_{\Omega_i \in \Omega^{(c)}} \{\underbrace{D_F P_{AF_{\Omega_i}}(\gamma_{AF}) + 2^{-2R_{SC}}(1 - P_{AF_{\Omega_i}}(\gamma_{AF}))}_{\text{Single-mode D-SNR curves}}\}$$



The D-SNR Curve

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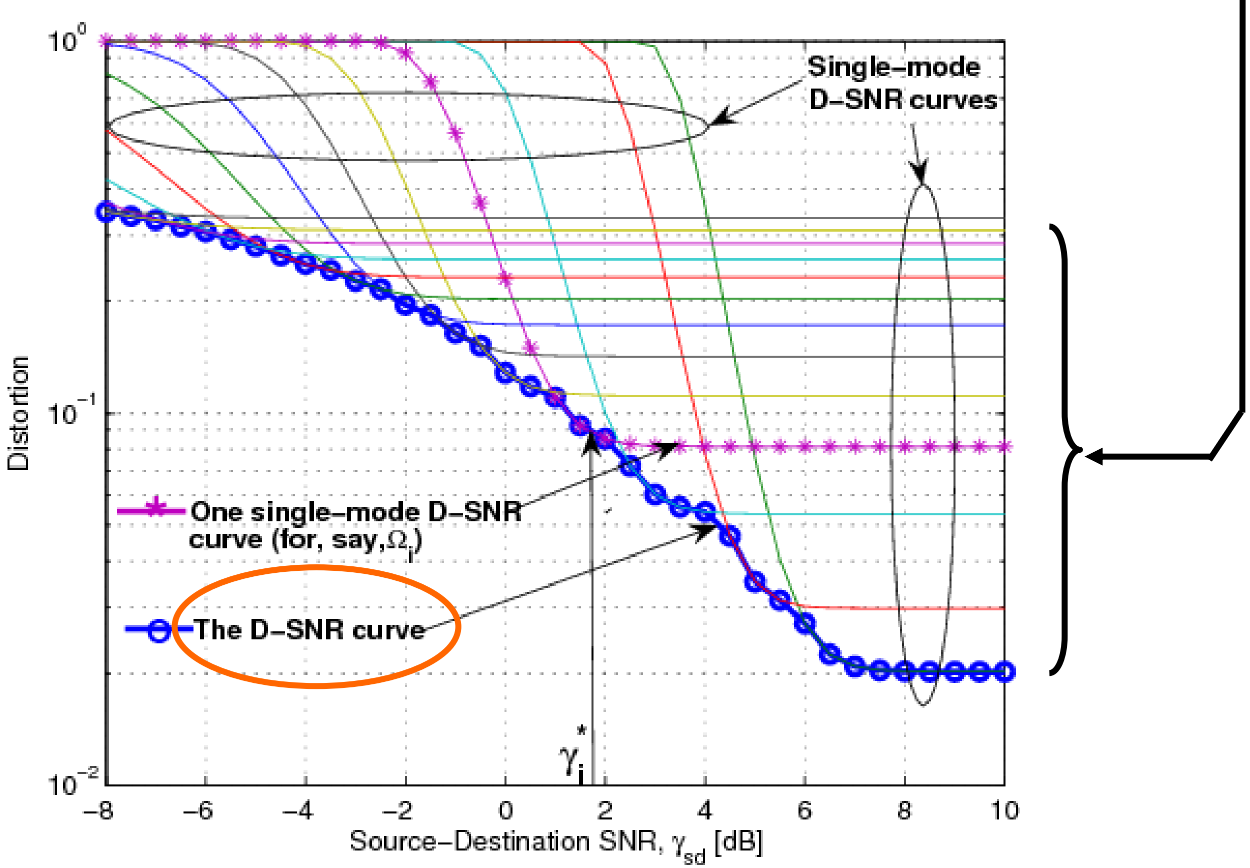

The D-SNR Curve

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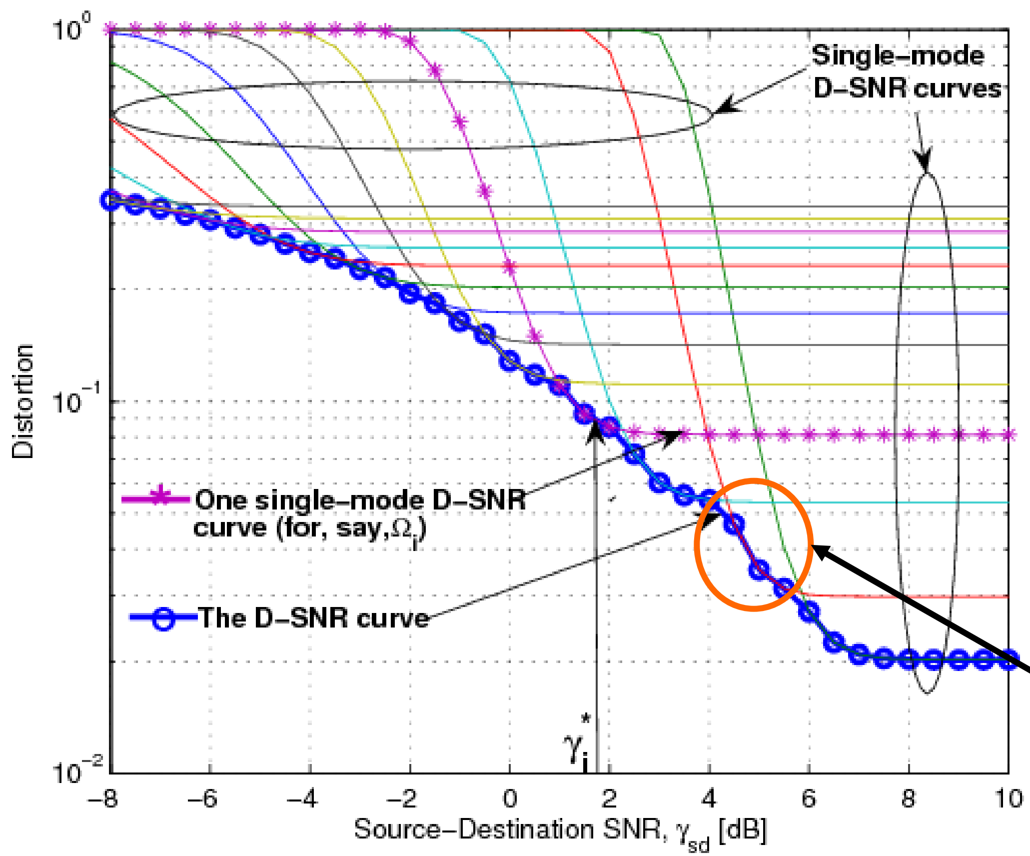
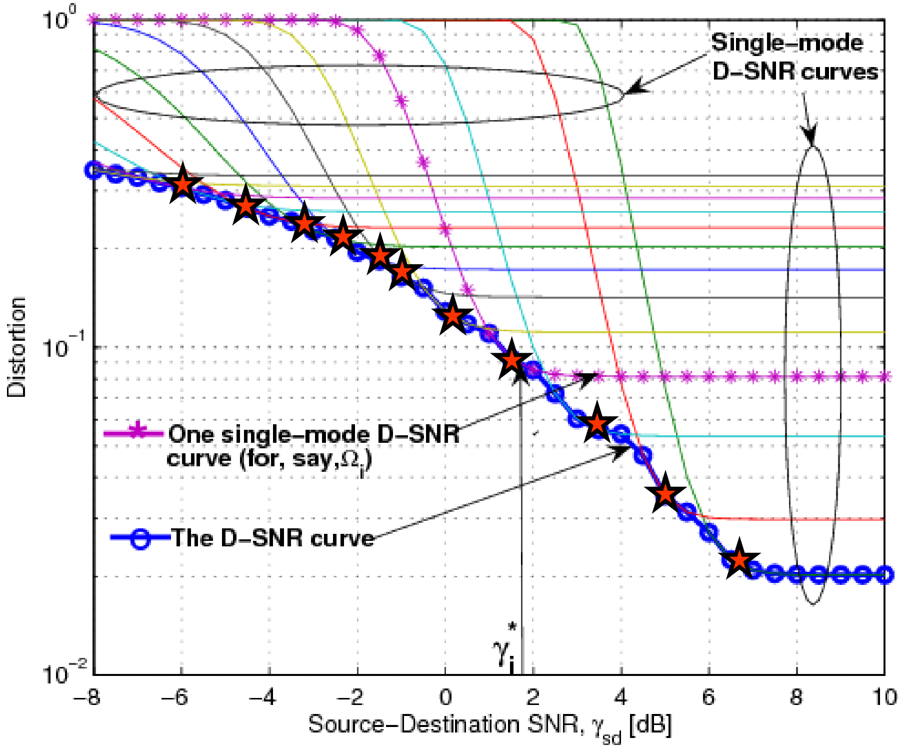


Contribution of one single-mode D-SNR curve to the overall D-SNR curve.

Characterizing the D-SNR Curve

- **Approach ⟶ Focus on selected set of points:**



  - **Those points where the relative contribution of channel errors to end–to–end distortion is fixed to a small value.**

- **Mathematically:**

$$D = (1 + \Delta)D_S(\Omega_i) = D_F P_{AF_{\Omega_i}}(\gamma_{AF}) + D_S(\Omega_i)(1 - P_{AF_{\Omega_i}}(\gamma_{AF}))$$

$$\Longrightarrow P_{AF_{\Omega_i}}(\gamma_{AF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

Characterizing the D-SNR Curve – RCPC Codes Case

$$P_{AF_{\Omega_i}}(\gamma_{AF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

$$P_{AF_{\Omega_i}}(\gamma_{AF}) \approx 1 - \left[1 - \sum_{d=d_f}^{W/2} a(d) P(d|\gamma_{AF})\right]^{NR_{SC}}$$

Characterizing the D-SNR Curve – RCPC Codes Case

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Probability of an error event with Hamming distance d

Number of error events with Hamming distance d

Free distance: smallest Hamming distance between two different codewords.

Characterizing the D-SNR Curve - RCPC Codes Case

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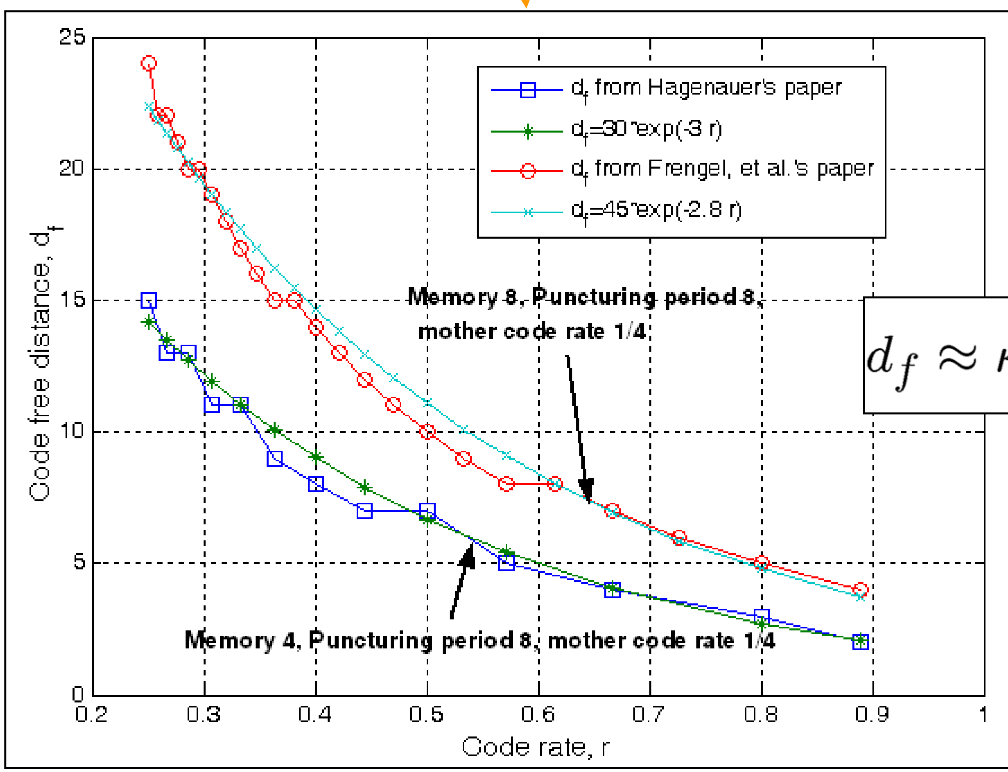


$$d_f \approx \kappa \exp(-cr) \approx \kappa \exp\left(-\frac{2cNR_{SC}}{W}\right)$$

Characterizing the D-SNR Curve - AF Cooperation

$$P_{AF_{\Omega_i}}(\gamma_{AF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

$$P_{AF_{\Omega_i}}(\gamma_{AF}) \approx 1 - \left[1 - \sum_{d=d_f}^{W/2} a(d) P(d|\gamma_{AF})\right]^{NR_{SC}}$$

$$\frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1} \approx 1 - (1 - \frac{\bar{a}}{2}\,\mathrm{erfc}(\sqrt{\kappa e^{-2cNR_{SC}/W}\gamma_{AF}})^{NR_{SC}}$$

$$\gamma_{AF_{dB}} \approx \frac{20cNR_{SC}\log(e)}{W} - 10\log(\kappa) + 10\log(\Psi + 2\bar{R}_{SC}\ln(2)) + 10\frac{\log(4)}{\Psi + 2\bar{R}_{SC}\ln(2)}(R_{SC} - \bar{R}_{SC})$$

$$\downarrow$$

$$\Psi = \ln(\frac{\bar{a}ND_F\bar{R}_{SC}}{2\Delta})$$

$$\Longrightarrow D_{CAF} \approx (G_c\gamma_{AF})^{-10m_c}$$

$$m_c = \frac{\log(4)}{A_1} = \left[20(\frac{cN}{W\ln(4)} + \frac{1}{2\Psi + 2\bar{R}_{SC}\ln(4)})\right]^{-1}, \quad G_c = \frac{\kappa(1+\Delta)^{\frac{-1}{10m}}}{\Psi + \bar{R}_{SC}\ln(4)} 10^{\frac{\log(4)\bar{R}_{SC}}{\Psi + \bar{R}_{SC}\ln(4)}}$$

Characterizing the D-SNR Curve - DF Cooperation

- D-SNR Curve given by:

$$D_{CDF} = \min_{\Omega_i \in \Omega^{(c)}} \{ D_F P_{DF_{\Omega_i}}(\gamma_{DF}) + 2^{-2R_{SC}} (1 - P_{DF_{\Omega_i}}(\gamma_{DF})) \}$$

$$\begin{aligned} P_{DF_{\Omega_i}}(\gamma_{DF}) &= P_{\Omega_i}(\gamma_{DF}|\Psi) P_{\Omega_i}(\Psi) + P_{\Omega_i}(\gamma_{DF}|\bar{\Psi}) P_{\Omega_i}(\bar{\Psi}) \\ &= P_{\Omega_i}(\gamma_{sd} + \gamma_{rd}) \left(1 - P_{\Omega_i}(\gamma_{sr})\right) + P_{\Omega_i}(\gamma_{sd}) P_{\Omega_i}(\gamma_{sr}) \end{aligned}$$

- Three cases to analyze:
  a. **"Good" source-relay channel the source frame error probability.**
  b. **"Bad" source-relay channel the source frame error probability.**
  c. **Channel states are such that there is no solution**

$$P_{DF_{\Omega_i}}(\gamma_{DF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

Characterizing the D-SNR Curve - DF Cooperation

- "Good" source-relay channel the source frame error probability: $P_{DF_{\Omega_i}}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd} + \gamma_{rd})$ for most operating modes.

$$D = (1 + \Delta)D_S(\Omega_i) = D_F P_{\Omega_i}(\gamma_{sd} + \gamma_{rd}) + D_S(\Omega_i)(1 - P_{\Omega_i}(\gamma_{sd} + \gamma_{rd}))$$

$$\boxed{D_{CDF} \approx (G_c(\gamma_{sd} + \gamma_{rd}))^{-10m_c}}$$

- "Bad" source-relay channel the source frame error probability: $P_{DF_{\Omega_i}}(\gamma_{DF}) \approx P_{\Omega_i}(\gamma_{sd})$ for most operating modes.

$$\boxed{D_{CDF} \approx (G_c \gamma_{sd})^{-10m_c}}$$

Characterizing the D-SNR Curve - DF Cooperation

- Channel states are such that there are no solutions to:

$$P_{DF_{\Omega_i}}(\gamma_{DF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

$$\lim_{\gamma_{sd} \to 0} P_{DF_{\Omega_i}}(\gamma_{DF}) = \Delta\left(\frac{D_F}{D_S(\Omega_i)} - 1\right)^{-1}.$$

$$D_a \approx 2^{-2R_{SC}}$$

$$R_{SC} = \frac{1}{2}\log_2\left(1 + \frac{\Delta}{1 - [1 - \frac{\bar{a}}{2}\mathrm{erfc}(\sqrt{\kappa \exp(-\frac{2cNR_{SC}}{W})\gamma_{rd}})]^{NR_{SC}}}\right).$$

Characterizing the D-SNR Curve - NoCooperation

- Channel states are such that there are no solutions to:

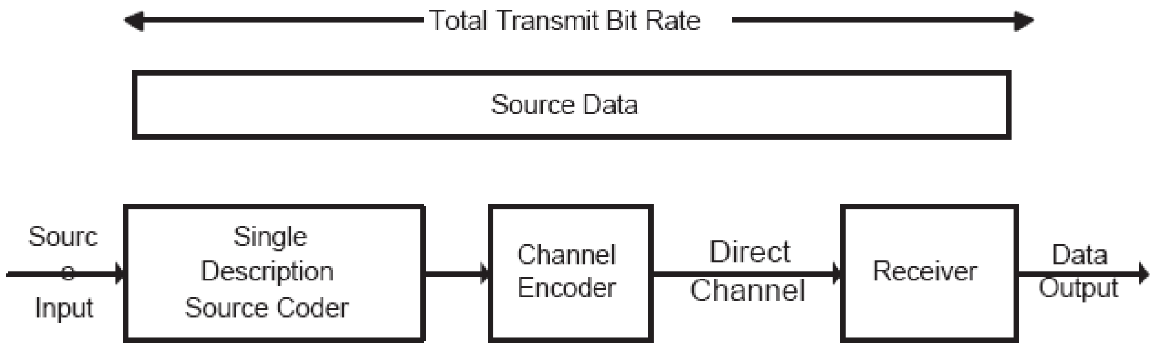
$$P_{DF_{\Omega_i}}(\gamma_{DF}) = \frac{\Delta}{\frac{D_F}{D_S(\Omega_i)} - 1}$$

$$\lim_{\gamma_{sd}\to 0} P_{DF_{\Omega_i}}(\gamma_{DF}) = \Delta\left(\frac{D_F}{D_S(\Omega_i)} - 1\right)^{-1}.$$

$$D_a \approx 2^{-2R_{SC}}$$

$$R_{SC} = \frac{1}{2}\log_2\left(1 + \frac{\Delta}{1 - [1 - \frac{\bar{a}}{2}\text{erfc}(\sqrt{\kappa \exp\left(-\frac{2cNR_{SC}}{W}\right)\gamma_{rd}})]^{NR_{SC}}}\right).$$

Characterizing the D-SNR Curve - No Cooperation



- D-SNR Curve given by:

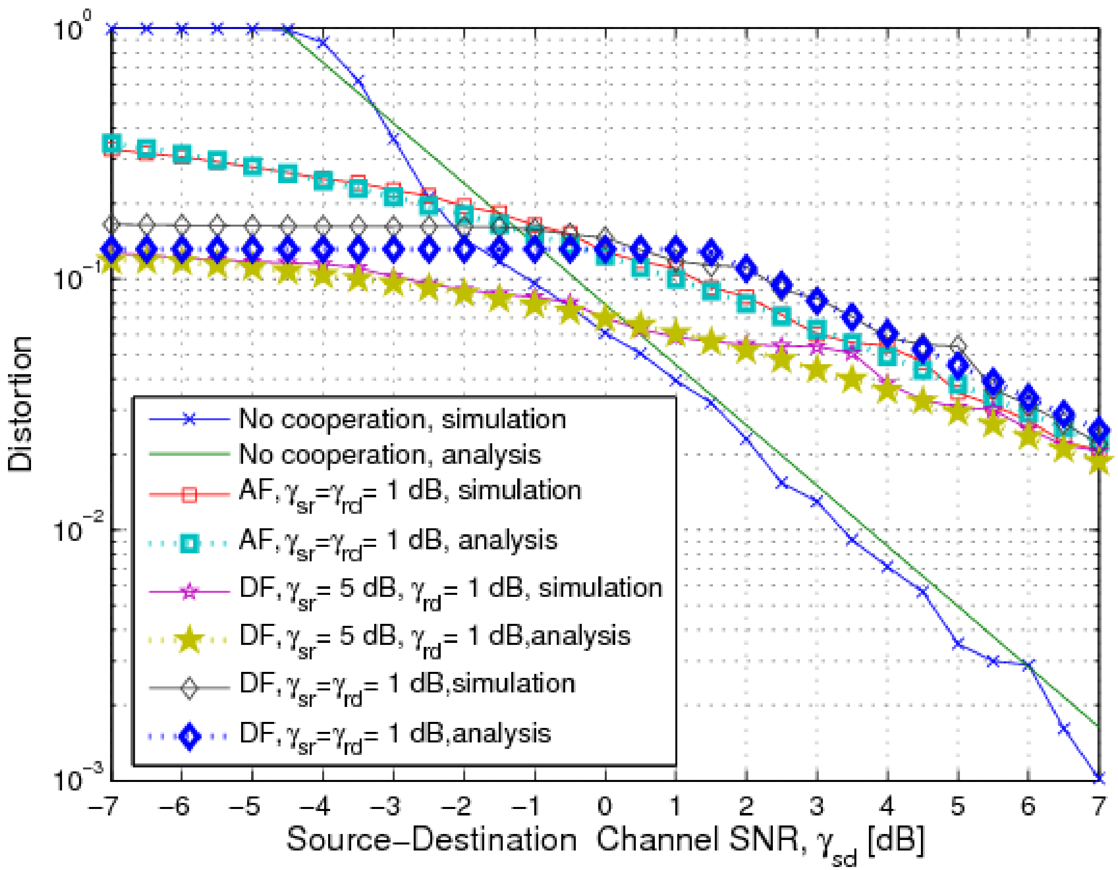
$$D_{SN} = \min_{\Omega_i \in \Omega^{(nc)}} \{ D_F P_{\Omega_i}(\gamma_{sd}) + 2^{-2R_{SN}} (1 - P_{\Omega_i}(\gamma_{sd})) \}$$

$$\Longrightarrow D_N \approx (G_N \gamma_{sd})^{-10 m_n}$$

$$m_n = \left[10 \left(\frac{cN}{W \ln(4)} + \frac{1}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})} \right) \right]^{-1},$$

$$G_N = \frac{\kappa (1 + \Delta)^{-1/(10 m_n)}}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})} 10^{\frac{2 \log(4) \bar{R}_{SC}}{\Psi + \ln(2)(1 + 4\bar{R}_{SC})}}$$

Characterizing the D-SNR Curve



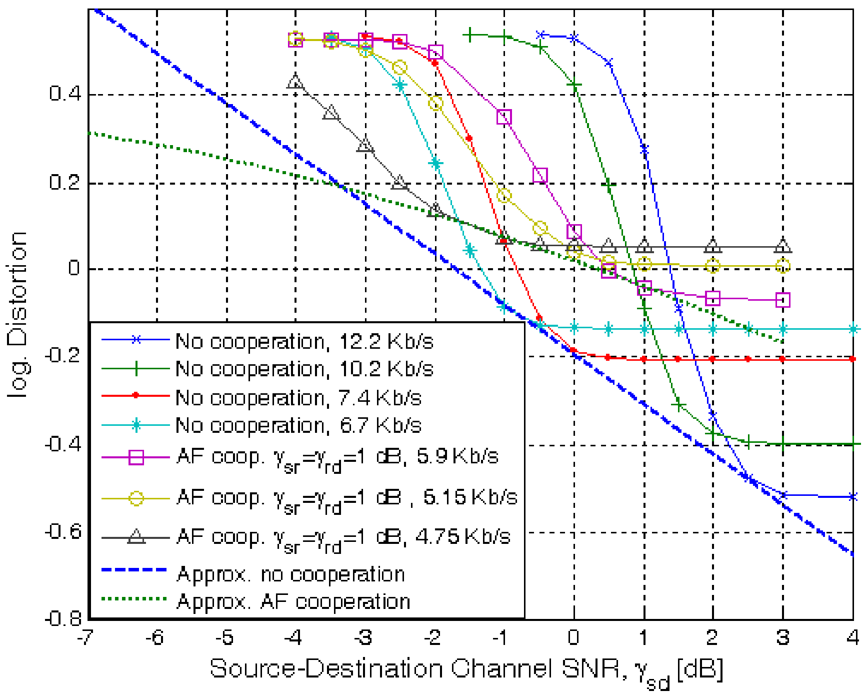
- **Channel encoder:**
  - **RCPC: Memory 4, puncturing period 8, mother code rate ¼.**

$$\bar{a} = 6.1$$
$$\kappa = 30$$
$$c = 3$$

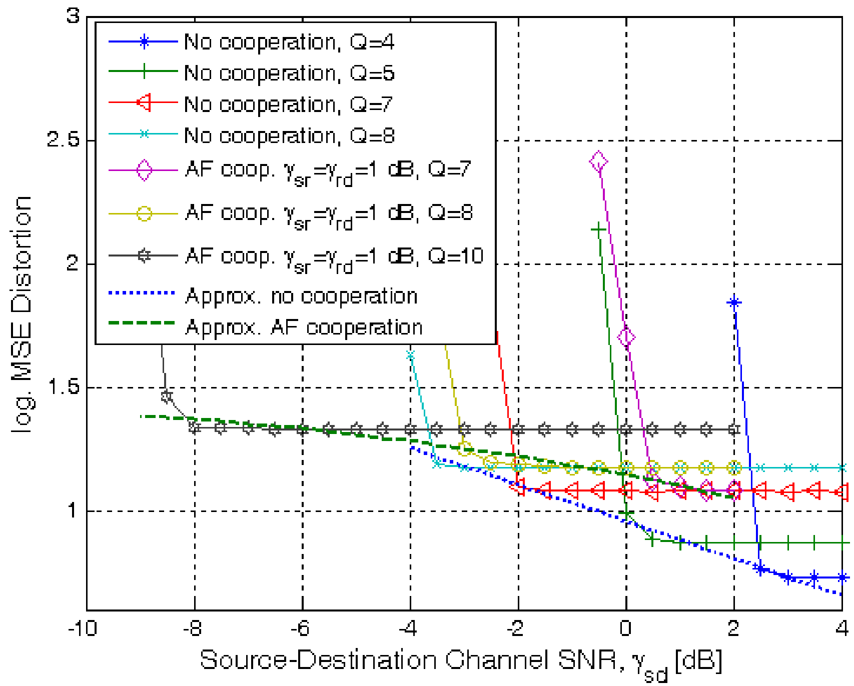
- $N = 150$
- $W = 950$
- $\Delta = .1$

Characterizing the D-SNR Curve

- Source: speech
- Source encoder: GSM-AMR
- Distortion: 4.5-PESQ



- Source: video
- Source encoder: MPEG-4FGS
- Distortion: MSE



Effects of Cooperation on the D-SNR Curve

$$m_N = \left[10\left(\frac{cN}{W\ln(4)} + \frac{1}{\Psi + \ln(2)(1+4\bar{R}_{SC})}\right)\right]^{-1},$$

$$\downarrow$$

$$\boxed{1 < \frac{m_N}{m_c} < 2}$$

$$\uparrow$$

$$m_c = \frac{\log(4)}{A_1} = \left[20\left(\frac{cN}{W\ln(4)} + \frac{1}{2\Psi + 2\bar{R}_{SC}\ln(4)}\right)\right]^{-1},$$

Furthermore:

$$\frac{cN}{W\ln(4)} >> \frac{1}{\Psi + \bar{R}_{SN}\ln(4)}$$

$$\frac{cN}{W\ln(4)} >> \frac{1}{2\Psi + \ln(4)(\bar{R}_{SN} - 1)}$$

$$\boxed{\frac{m_N}{m_C} \approx 2}$$

Effects of Cooperation on the D-SNR Curve

Also:

$$G_N = \frac{\kappa(1+\Delta)^{-1/(10m_n)}}{\Psi + \ln(2)(1+4\bar{R}_{SC})} 10^{\frac{2\log(4)\bar{R}_{SC}}{\Psi+\ln(2)(1+4\bar{R}_{SC})}}$$

$$\downarrow$$

$$\boxed{G_{C_{dB}} > G_{N_{dB}}}$$

$$\uparrow$$

$$G_c = \frac{\kappa(1+\Delta)^{\frac{-1}{10m}}}{\Psi + \bar{R}_{SC}\ln(4)} 10^{\frac{\log(4)\bar{R}_{SC}}{\Psi+\bar{R}_{SC}\ln(4)}}$$

Effects of Cooperation on the D-SNR Curve

$$\left(G_c\left(\gamma_{sd}+\frac{\gamma_{sr}\gamma_{rd}}{1+\gamma_{sr}+\gamma_{rd}}\right)\right)^{-10m_c}=(G_N\gamma_{sd})^{-10m_n}$$

$$\Rightarrow \frac{G_N^{m_n}}{G_c^{m_c}}=\frac{\left(\gamma_{sd}+\frac{\gamma_{sr}\gamma_{rd}}{1+\gamma_{sr}+\gamma_{rd}}\right)^{m_c}}{\gamma_{sd}^{m_n}}$$

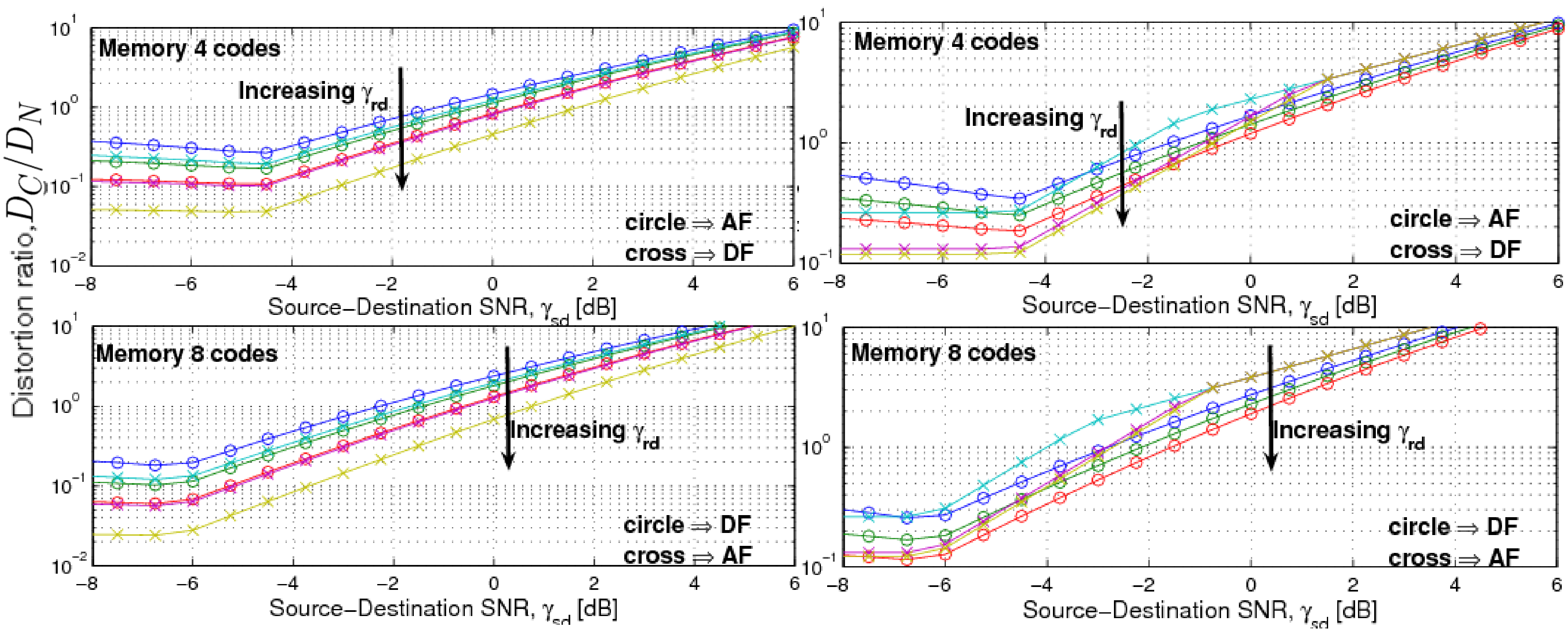
$$\gamma_{sd}\approx\frac{G_c^2}{G_N}\left(1+\sqrt{1+4\left(\frac{\gamma_{sr}\gamma_{rd}}{1+\gamma_{sr}+\gamma_{rd}}\right)\frac{G_N^2}{G_c}}\right)$$

- **There is a range of values of γ_{sd} for which it is better not to use cooperation and a range of values for which it is better to use cooperation.**

Effects of Cooperation on the D-SNR Curve

$$m_N/m_c \approx 2 \qquad G_{C_{dB}} > G_{N_{dB}}$$

- There is a range of values of γ_{sd} for which it is better not to use cooperation and a range of values for which it is better to use cooperation.



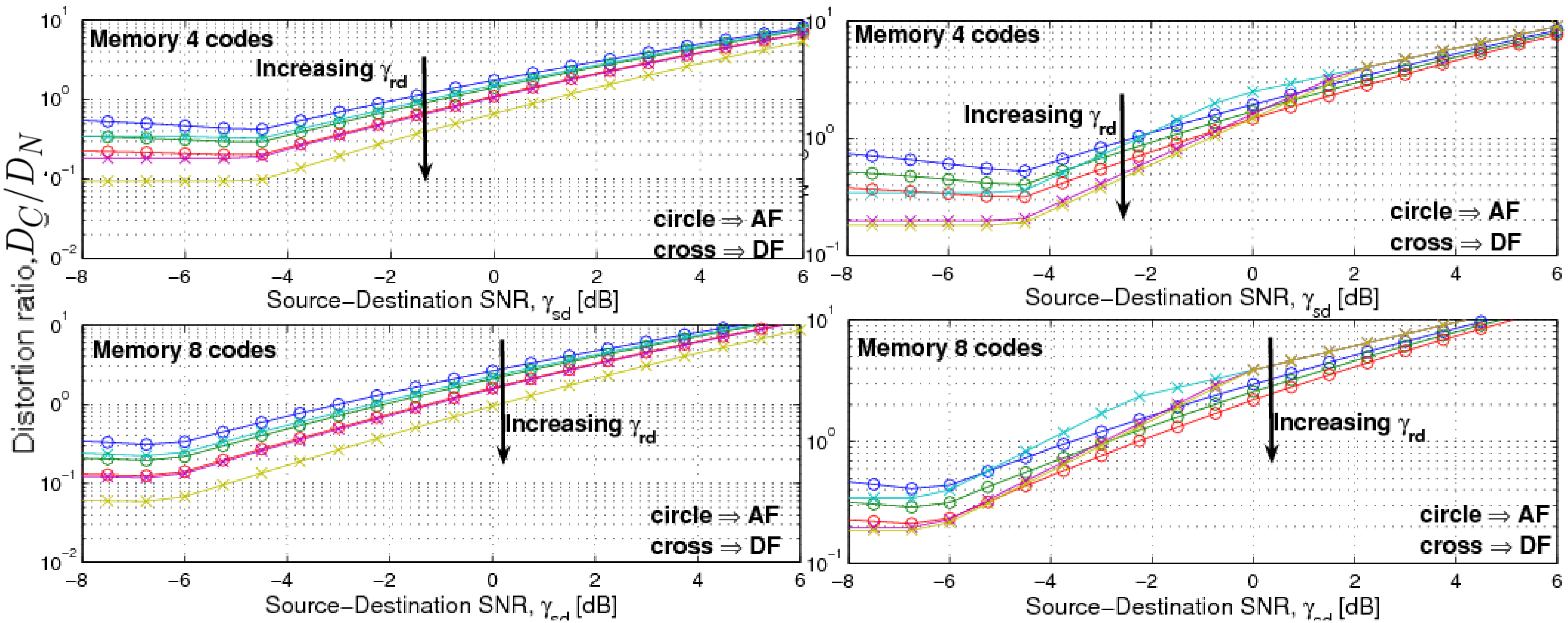
Good source-relay channel

Bad source-relay channel

Effect of Source Codec Efficiency

$$D_S(R_S) = 2^{-2(1-\lambda)R_S} = 2^{-\hat{\lambda}R_S} \qquad \hat{\lambda} = 2(1-\lambda)$$

- m_N and m_C change with $\hat{\lambda}$ approximately in a linear fashion.



Good source-relay channel

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