Liquidity and Asset Pricing

Based on de Jong and Rindi, Chapter 7

Frank de Jong

Tilburg University

Liquidity and Asset Pricing

Literature has identified two reasons why liquidity affects asset prices

1. Expected transaction costs imply lower asset prices
   - and therefore higher discount rates
2. Unexpected fluctuations in market-wide liquidity is an additional priced risk factor
   - higher required returns

Transaction costs and asset prices

- Transaction costs on financial markets reduce the return on investments
- Rational investors will require a compensation for expected transaction costs
- This affects the price an investor is willing to pay for an asset
- In equilibrium, transaction costs lead to lower prices for assets
- as a result, expected returns (before transaction costs) will be higher

An example

- Suppose you buy an asset for \$ 100 now (ask price) and you hold the asset for one year
- After one year, the ask price is \$ 104, the bid price is \$ 102 (bid-ask spread is \$ 2)
- You sell the asset at the bid price, i.e. \$ 102
- Return on the asset (ask-ask) is 4%, but your realized return is only 2%!

A second example

How much to pay for an asset?

- The expected value at the end of the year is \$ 150
- The required risk-adjusted return is 6%
- The transaction cost (b/a spread) for liquidating the position is \$ 3

Valuation:

$$P = \frac{150 - 3}{1.06} = 138.68$$

In terms of expected return:

$$\mathsf{E}[R] = \frac{150 - P}{P} = 8.16\%$$

The expected return equals the risk-adjusted required return, 6%, plus the relative bid-ask spread, 2% (and a small cross-product effect)

A formal model (Amihud and Mendelson, 1986)

Generalization of the Gordon model for asset valuation

- Perpetual per-period dividend d
- Required risk-adjusted return r
- Relative bid-ask spread S
- Expected trading frequency: μ times per period

Amihud and Mendelson (1986) show that under these assumptions, the value of asset is

$$P = \frac{d}{r + \mu S}$$

Compare this to the value without trading frictions, $P^* = d/r$

Example

Let $d = 5$, $r = 5\%$, $S = 1\%$, $\mu = 1.5$

Price without frictions

$$P^* = \frac{5}{0.05} = 100$$

Price with 1.5% transaction costs (1.5 * 1%)

$$P = \frac{10}{0.05 + 0.015} = 76.92$$

A reduction of the spread from 1% to 0.5% would lead to

$$P = \frac{10}{0.05 + 0.0075} = 86.95$$

or a price increase by 13%!

Expected returns

Amihud-Mendelson analysis implies relation between expected returns and liquidity

$$\mathsf{E}[R] = r + \mu S$$

Using the CAPM to determine risk-adjusted required returns we find

$$\mathsf{E}[R] = r_f + \beta\left(\mathsf{E}[R_M] - r_f\right) + \mu S$$

where r_f is the risk free rate and $\mathsf{E}[R_M] - r_f$ is the market risk premium

So, expected returns are a sum of three components

- the risk free rate of return
- a risk premium, determined by the "beta" of the asset
- a liquidity premium, determined by the relative bid-ask spread

Liquidity and expected returns: some data

Excess returns for SIZE / PIN portfolios of US stocks over the period 1984–1998. Source: Easley, Hvidkjaer and O'Hara (2002), Table III

Size/PIN	Low	Medium	High
Small	0.148	0.202	0.474
2	0.462	0.556	0.743
3	0.647	0.695	0.892
4	0.873	0.837	0.928
Large	0.953	1.000	0.643

Formal tests

Amihud and Mendelson (1986) test the relation between expected return and spreads

- data from NYSE stocks, 1960-1979
- controlling for other determinants of expected returns such as risk and firm size

Step 1: for every stock, calculate relative spread and estimate its "beta"

Step 2: sort all assets into portfolio's based on "beta" and spread

Step 3: cross sectional regression of average monthly portfolio return on the spread and portfolio "beta"

$$\bar{R}_p = \gamma_0 + \gamma_1 \beta_p + \gamma_2 S_p + e_p$$

Empirical results

Estimates from Amihud and Mendelson (1986) p.238 (transformed to % returns):

$$\bar{R}_p = 0.36 + 0.672\beta_p + 0.211S_p + u_p$$

The value 0.211 is an estimate of μ; it implies that each stock is traded approximately once every 5 months

Empirical results (2)

Amihud (2002) repeats the experiment from Amihud and Mendelson (1986) with data from 1964–1997

- using an alternative time-varying liquidity measure: $ILLIQ$
- controls for momentum effects

Results:

$$\bar{R}_{pt} = -0.364 + 1.183\beta_{pt} + 0.162 ILLIQ_{pt} + u_{pt}$$

With an additional control for market capitalization ($SIZE$):

$$\bar{R}_{pt} = 1.922 + 0.217\beta_{pt} + 0.112 ILLIQ_{pt} - 0.134 \ln(SIZE)_{pt} + u_{pt}$$

Brennan and Subramahnyam (1996)

Measure of liquidity: price impact of order flow from Glosten-Harris regression

Construct portfolios of assets sorted by estimated price impact and by size

Add dummy for liquidity quintile to the standard Fama and French (1993) three-factor model

$$R_{it} = \alpha + \sum_{i=2}^{5} \gamma_i L_i + \beta_i R_{Mt} + s_i SMB_t + h_i HML_t + e_{it}.$$

where SMB and HML are the returns on the size and value factors.

Brennan and Subramahnyam (1996) report an additional return of 6.6% per year for the lowest as against the highest liquidity portfolio

Easley, Hvidkjaer and O'Hara (2002) and Duarte and Youn

Fama-MacBeth regressions of the form

$$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_p + \gamma_{2t} SIZE_{i,t-1} + \gamma_{3t} BM_{i,t-1} + \gamma_{4t} PIN_{i,t-1} + \gamma_{5t} ILLIQ_{i,t-1} + \eta_{it}$$

	Beta	*SIZE*	*BM*	*PIN*	*ILLIQ*
EHO	–0.175	0.161	0.051	1.800	
	(–0.48)	(2.81)	(0.48)	(2.50)	
DY	0.175	0.043	0.268	1.004	
	(0.49)	(0.63)	(2.96)	(1.91)	
DY	0.149	0.088	0.254	0.648	0.0003
	(0.42)	(1.38)	(2.82)	(1.17)	(2.99)

Liquidity risk

- Liquidity fluctuates over time
  - periods of financial market stress often associated with low liquidity, e.g. Asia crisis and LTCM crisis
- Several studies have documented commonality in liquidity fluctuations
  - common factor affecting all stock's liquidity, see Chordia, Roll and Subrahmanyam (2000) and Hasbrouck and Seppi (2001)
- Such a common liquidity shock cannot be diversified and may be a priced risk factor

Liquidity as a priced risk factor

Unexpected shocks to illiquidity

$$U_t = L_t - E_{t-1}(L_t) \tag{1}$$

obtained e.g from an AR(p) model for L_t

First step: exposures of stock returns on the market return and the unexpected changes in liquidity:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \delta_i U_t + e_{it}$$

Second step: expected returns as function of exposures

$$E(r_i) = \beta_i E(r_m) + \delta_i \lambda_U$$

λ_U is the liquidity risk premium

Pastor and Stambaugh (2003) and Sadka (2006) combine first and second step and use GMM for estimation

$$r_{it} = \beta_i r_{mt} + \delta_i \left(U_t + \lambda_U \right) + e_{it}$$

Alternative is two-step estimation: second step cross-sectional regression of average excess returns on the estimated factor loadings

$$\bar{r}_i = \lambda_m \widehat{\beta}_i + \lambda_U \widehat{\delta}_i + u_i, \quad i = 1, .., N$$

λ_m and λ_U are the market and liquidity risk premium parameters

Liquidity CAPM

Acharya and Pedersen (2005) specify a model with expected liquidity and liquidity risk exposure

$$E(r_i) = \alpha + \mu E(c_i) + \lambda \beta_i^{net}$$

with 'beta' defined on the net-of-transaction cost returns

$$\begin{aligned}
\beta_i^{net} &= \frac{Cov(r_{it} - c_{it}, r_{mt} - c_{mt})}{Var(r_{mt} - c_{mt})} \\
&= \frac{Cov(r_{it}, r_{mt})}{Var(r_{mt} - c_{mt})} + \frac{Cov(c_{it}, c_{mt})}{Var(r_{mt} - c_{mt})} - \frac{Cov(r_{it}, c_{mt})}{Var(r_{mt} - c_{mt})} - \frac{Cov(c_{it}, r_{mt})}{Var(r_{mt} - c_{mt})} \\
&= \beta_{1i} + \beta_{2i} - \beta_{3i} - \beta_{4i}
\end{aligned}$$

β_{1i} is the traditional CAPM beta

the other betas measure different aspects of liquidity risk

Liquidity CAPM: empirical evidence

Acharya-Pedersen classify stocks in 25 groups, from low liquidity to high liquidity

Estimates of the liquidity and betas of the highest and lowest liquidity portfolio

portfolio	$E(r_i)$	$E(c_i)$	β_{1i}	β_{2i}	β_{3i}	β_{4i}	β_i^{net}	β_i^{liq}
1	0.48%	0.25%	0.551	0.000	–0.008	–0.000	0.543	0.008
25	1.10%	8.83%	0.845	0.004	–0.017	–0.045	0.911	0.066

Acharya and Pedersen's preferred specification is

$$E(r_i) = -0.333 + 0.034E(c_i) + 1.153\beta_{1i} + 4.334\beta_i^{liq}$$

where β_{1i} is proportional to the standard CAPM beta and

$$\beta_i^{liq} = \beta_{2i} - \beta_{3i} - \beta_{4i}$$

collects all the terms of the net beta that involve transaction costs

Low liquidity stocks have 4.6% higher expected returns than high liquidity stocks

- 3.5% is due to differences in expected liquidity
- 1.1% is due to the liquidity risk premium