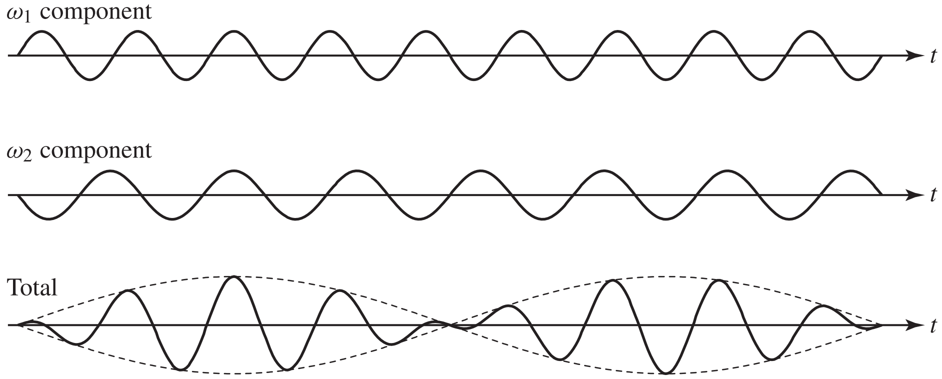

Figure 3.23 Wave packet composed of two frequency components showing the carrier and the envelope. The carrier travels at the phase velocity, whereas the envelope travels at the group velocity.

In practice, a propagating optical wave rarely contains only one frequency. It usually consists of many frequency components that are grouped around some center frequency, ω_0. For the simplicity of illustration, we consider a wave packet traveling in the z direction that is composed of two plane waves of equal real amplitude $\mathcal{E}$. The frequencies and propagation constants of the two components are

$$\begin{aligned} \omega_1 &= \omega_0 + \mathrm{d}\omega, \quad k_1 = k_0 + \mathrm{d}k, \\ \omega_2 &= \omega_0 - \mathrm{d}\omega, \quad k_2 = k_0 - \mathrm{d}k. \end{aligned} \tag{3.163}$$

The space- and time-dependent total real field of the wave packet is then given by

$$\begin{aligned} \boldsymbol{E} &= \mathcal{E} \exp(\mathrm{i}k_1 z - \mathrm{i}\omega_1 t) + \mathrm{c.c.} + \mathcal{E} \exp(\mathrm{i}k_2 z - \mathrm{i}\omega_2 t) + \mathrm{c.c.} \\ &= 2\mathcal{E}\Big\{\cos\big[(k_0 + \mathrm{d}k)z - (\omega_0 + \mathrm{d}\omega)t\big] + \cos\big[(k_0 - \mathrm{d}k)z - (\omega_0 - \mathrm{d}\omega)t\big]\Big\} \\ &= 4\mathcal{E} \cos(\mathrm{d}kz - \mathrm{d}\omega t) \cos(k_0 z - \omega_0 t). \end{aligned} \tag{3.164}$$

As illustrated in Fig. 3.23, the resultant wave packet has a *carrier*, which has a frequency of ω_0 and a propagation constant of k_0, and an *envelope*, which varies in space and time as $\cos(\mathrm{d}kz - \mathrm{d}\omega t)$. Therefore, a fixed point on the envelope is defined by $\mathrm{d}kz - \mathrm{d}\omega t = \text{constant}$, which travels with a velocity of

$$v_\mathrm{g} = \frac{\mathrm{d}\omega}{\mathrm{d}k}. \tag{3.165}$$

This is the velocity of the wave packet and is called the *group velocity*.

Because the energy of a harmonic wave is proportional to the square of its field amplitude, the energy carried by a wave packet that is composed of many frequency components is concentrated in the regions where the amplitude of the envelope is large. Therefore, the energy in a wave packet is transported at the group velocity v_g. Because a wave packet carries an optical signal, thus information, optical signals and optical information are transmitted at the group velocity. *The constant-phase wavefront travels at the phase velocity, but optical energy and information are transmitted at the group velocity.*

In reality, the group velocity is usually a function of the optical frequency. Then,

$$\frac{\mathrm{d}^2 k}{\mathrm{d}\omega^2} = \frac{\mathrm{d}}{\mathrm{d}\omega} v_\mathrm{g}^{-1} \neq 0, \tag{3.166}$$