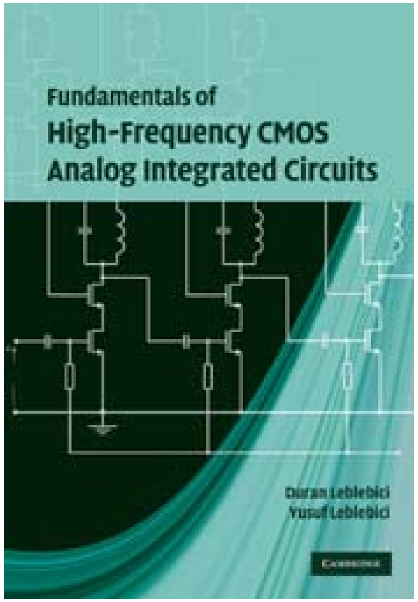

Fundamentals of High-Frequency CMOS Analog Integrated Circuits

Duran Leblebici
Yusuf Leblebici

Chapter 5

RF Oscillators

Classification of oscillators

a) Non sinusoidal oscillators

- Relaxation oscillators (charge-discharge osc.)
- Astable flip-flops
- Ring oscillators

b) Sinusoidal oscillators

- R-C oscillators
- L-C oscillators
  - Lumped or distributed L-C oscillators
  - Crystal oscillators

Principles of oscillation:

a) Negative resistance oscillators
b) Feedback oscillators

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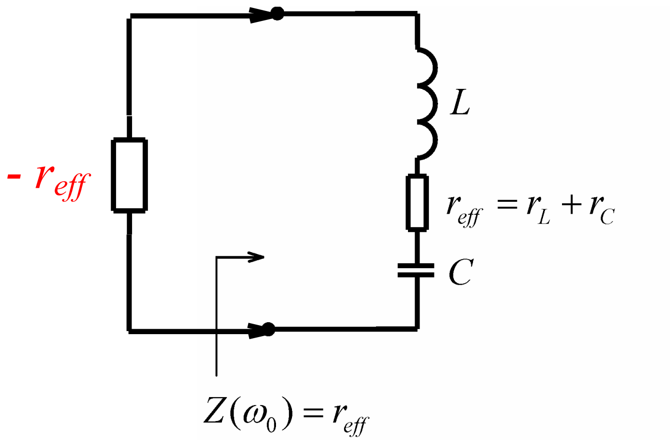
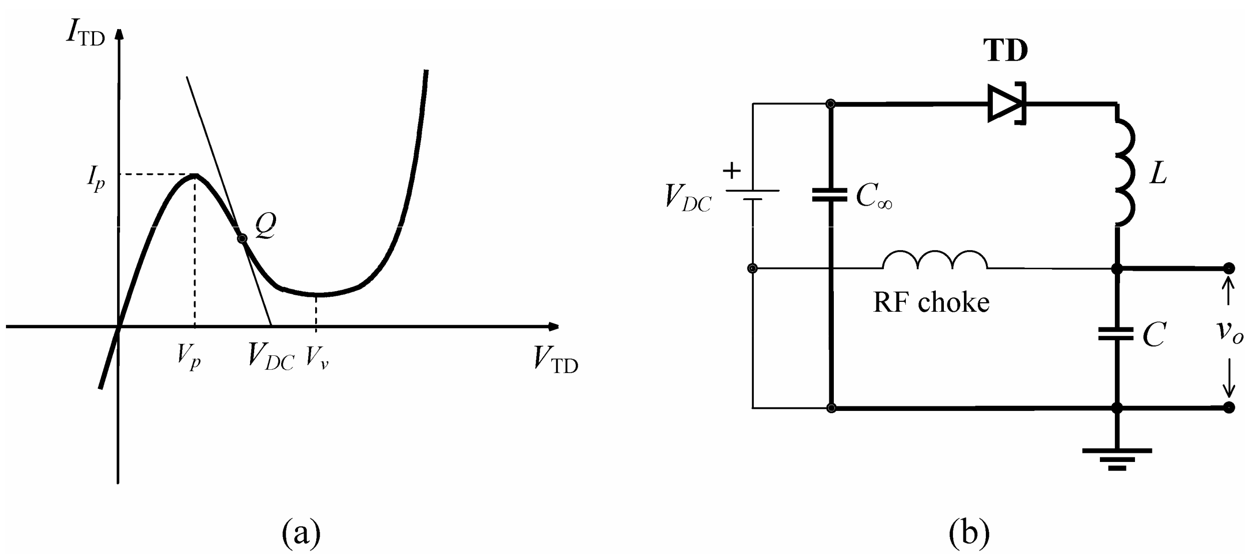
Negative resistance oscillators

1. Oscillators operating at series resonance:



$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Example: Tunnel diode oscillator



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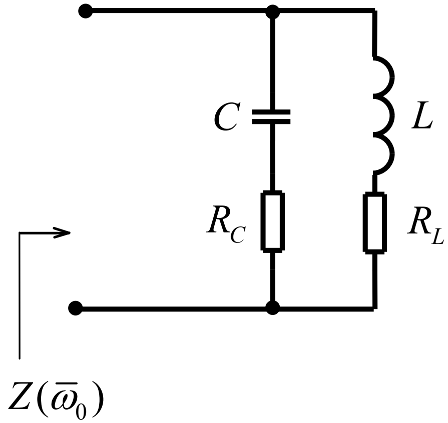
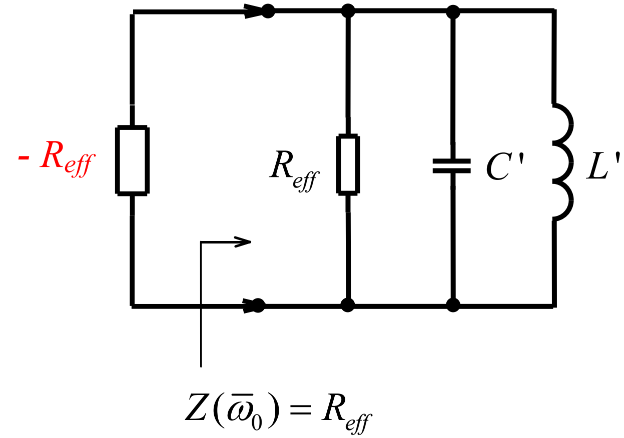
2. Oscillators operating at parallel resonance:





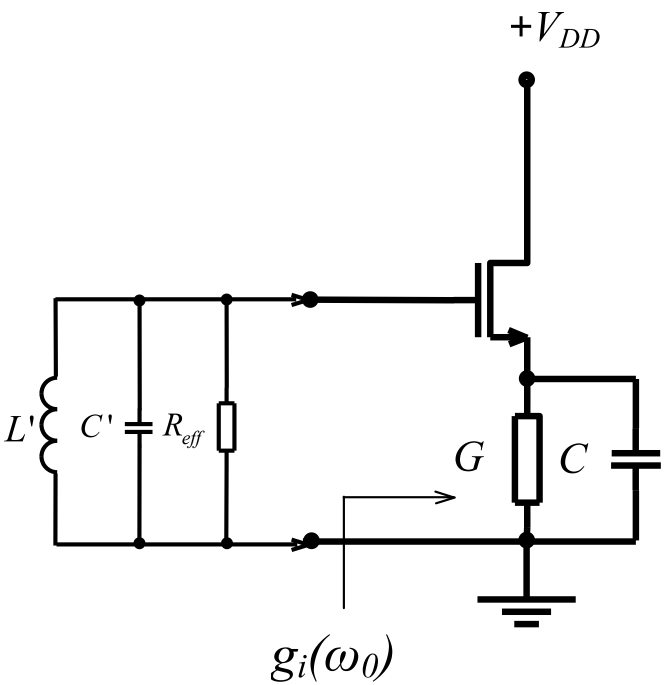
$$\omega_{(\mathrm{Re})} = \sqrt{\frac{1}{L'C'}} = \sqrt{\frac{1}{LC}} \times \sqrt{\frac{L - Cr_L^2}{L - Cr_C^2}}$$

$$C' = \frac{C}{1 + \omega_{(\mathrm{Re})}^2 C^2 r_C^2} \qquad L' = L + \frac{r_L^2}{\omega_{(\mathrm{Re})}^2 L}$$

$$G_{eff} = \frac{1}{r_C(Q_C^2 + 1)} + \frac{1}{r_L(Q_L^2 + 1)} = \frac{1}{R_{eff}}$$

Examples to negative resistance oscillators:

a) With a single ended negative resistance circuit

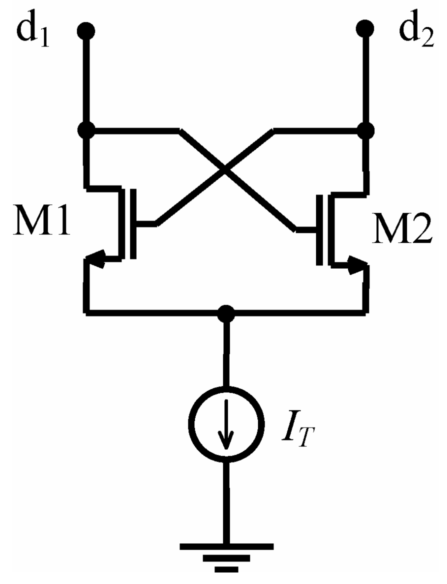


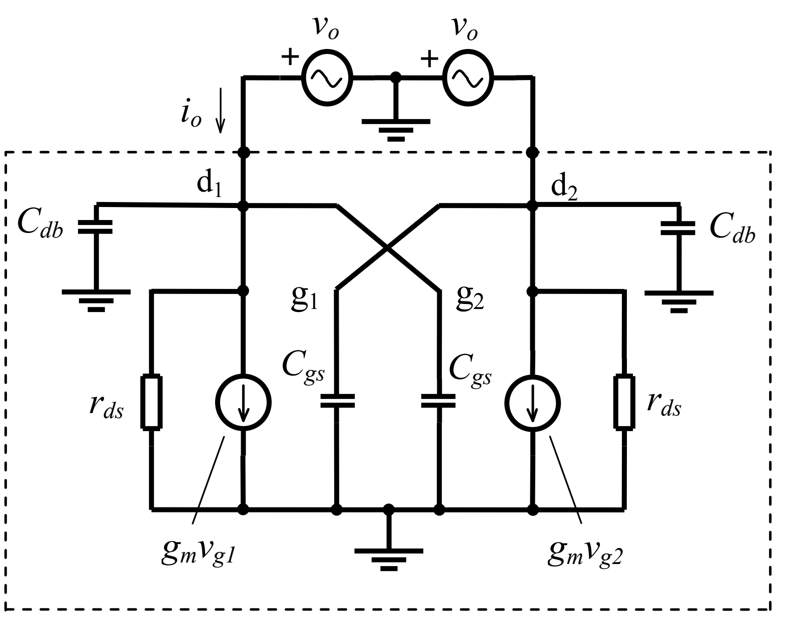
$$g_i(\omega) = \frac{C_{gs}}{(C_{gs}+C)^2}(GC_{gs} - g_m C)\frac{1}{1+(\omega_p/\omega)^2}$$

$$\omega_p = \frac{(g_m + G)}{(C_{gs}+C)}$$

$$g_i(\omega_0) = -\frac{1}{R_{eff}}$$

b) With a differential negative resistance circuit:

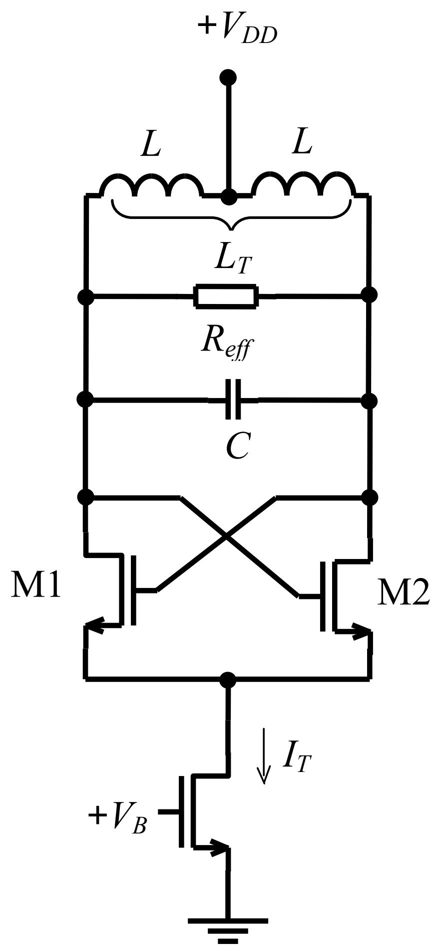




(Small signal eq. circuit to calculate d_1 - d_2 output resistance)

$$y_o = -\frac{1}{2}(g_m - g_{ds}) + s\frac{1}{2}(C_{gs} + C_{db})$$

$$r_o = -\frac{2}{(g_m - g_{ds})} \cong -\frac{2}{g_m} \quad \Rightarrow \quad g_o \cong -\frac{g_m}{2}$$



$$\omega_0 = \frac{1}{\sqrt{C_T L_T}}$$

$$C_T = C + \frac{1}{2}(C_{gs} + C_{db})$$

$$L_T = 2L(1+k)$$

The condition of oscillation:

$$|g_o| \geq \frac{1}{R_{eff}} \quad \Rightarrow \quad g_m \geq \frac{2}{R_{eff}}$$

Note that:

- If M1 and M2 operate without velocity saturation ;

$g_m = \sqrt{2\mu C_{ox}(W/L)I_D}$ (can be controlled with I_T !)

- If M1 and M2 operate with velocity saturation ;

$g_m = kWC_{ox}v_{sat}$ (can not be controlled with I_T !)

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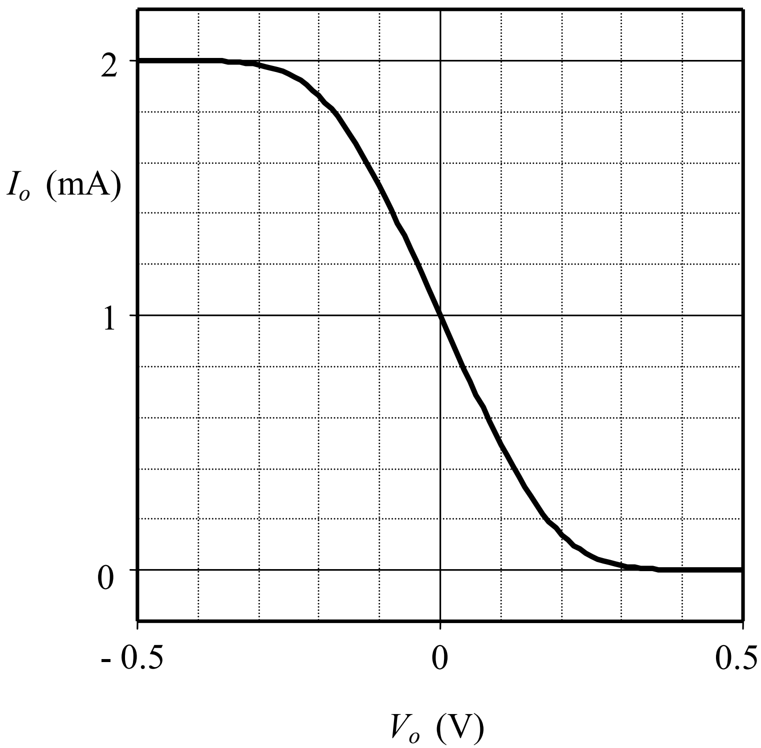
Effects of the non-linearity:



- Slope is maximum and equal to g_o at mid-point.
- Therefore to guarantee the oscillation g_m must be 1..10% higher then the calculated value.
- The saturation of the current at both ends results in flattening of the current waveform at peaks.

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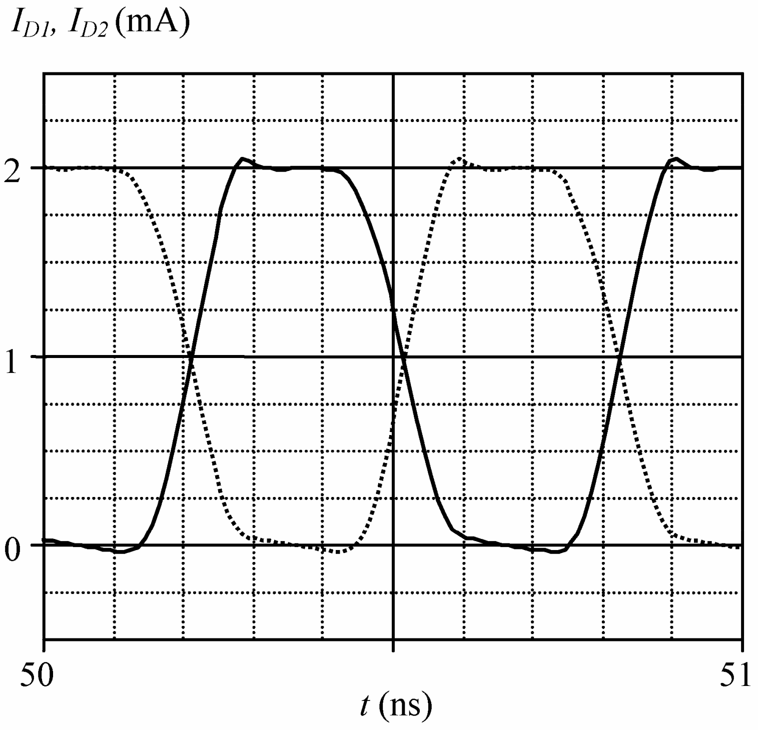
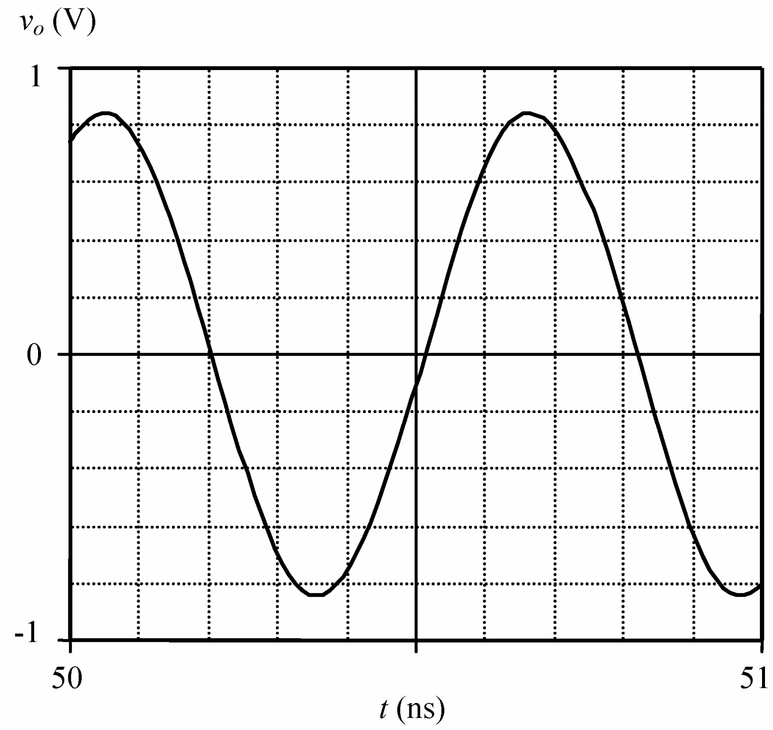
Simulation results :

$$(f_0 = 1,6\ \text{GHz},\ L = 5\ \text{nH},\ Q_L = 7,\ I_T = 2\ \text{mA},\ (W/L) = 108)$$

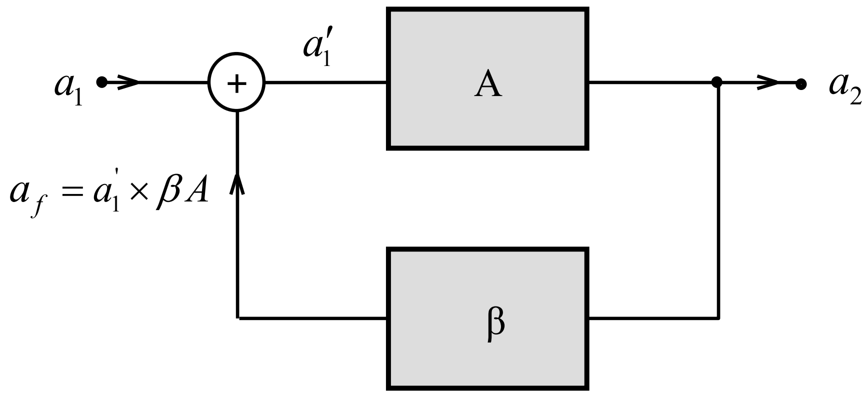


(a)



(b)

Feedback Oscillators (H. Georg Barkhausen, 1921)



$$A = \frac{a_2}{a_1'} \quad , \quad \beta = \frac{a_f}{a_2} \quad , \quad a_1' = a_1 + a_f \quad , \quad A_f = \frac{a_2}{a_1}$$

$$A_f = \frac{A}{(1 - \beta A)}$$

Condition of oscillation: $\beta A = 1 + j.0$

$$\Downarrow$$

$$\mathrm{Re}\{\beta A\} = 1 \text{ and } \mathrm{Im}\{\beta A\} = 0$$

or $$|\beta A| = 1 \text{ and } \varphi(\beta A) = 0$$

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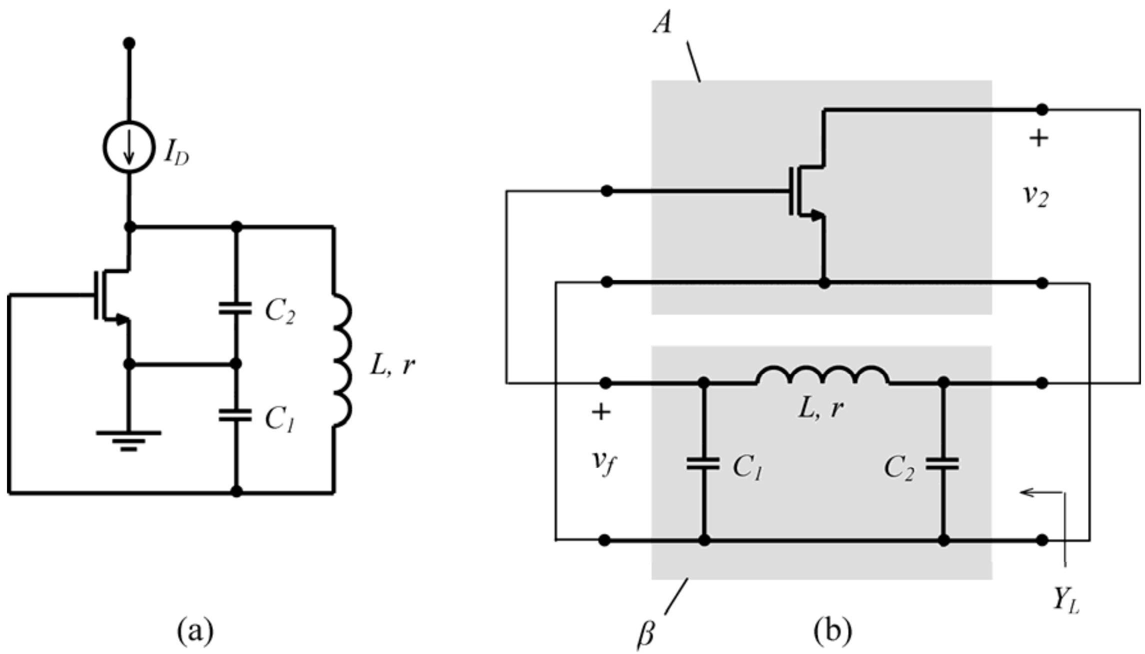
Example: The Colpitts oscillator



$$Y_L = sC_2 + \frac{1}{sL + r + \frac{1}{sC_1}} = \frac{s^3 LC_1C_2 + s^2 C_1C_2 r + s(C_1 + C_2)}{s^2 LC_1 + sC_1 r + 1}$$

$$A_v = -g_m \frac{1}{(g_{ds} + Y_L)} \qquad \beta = \frac{1}{s^2 LC_1 + sC_1 r + 1}$$

$$\beta A_v = -g_m \frac{1}{s^3 L C_1 C_2 + s^2 (C_1 C_2 r + L C_1 g_{ds}) + s\left[(C_1 + C_2) + r C_1 g_{ds}\right] + g_{ds}}$$

$$\mathrm{Im}\{\beta A_v\} = 0 \qquad \Rightarrow \qquad \omega_{osc} = \omega_0 \sqrt{1 + r \cdot g_{ds} \frac{C_1}{C_1 + C_2}}$$

$$\mathrm{Re}\{\beta A_v\} = +1\,,\ \left|\beta A_v\right| = 1 \qquad \Rightarrow g_m = \omega^2 C_1 (C_2 \cdot r + L \cdot g_o) - g_o$$

For $g_m \gg g_o$ and $C_1 \gg C_2$

$$g_m \cong \frac{1}{R_{eff}} \frac{C_1 + C_2}{C_2} \quad \text{or} \quad g_m \cong \frac{1}{rQ^2} \frac{C_1 + C_2}{C}$$

To guarantee the oscillation; $g_m = k_s \dfrac{1}{rQ^2} \dfrac{C_1 + C_2}{C}$

where the safety factor $k_s = 1.01 \ldots 1.1$

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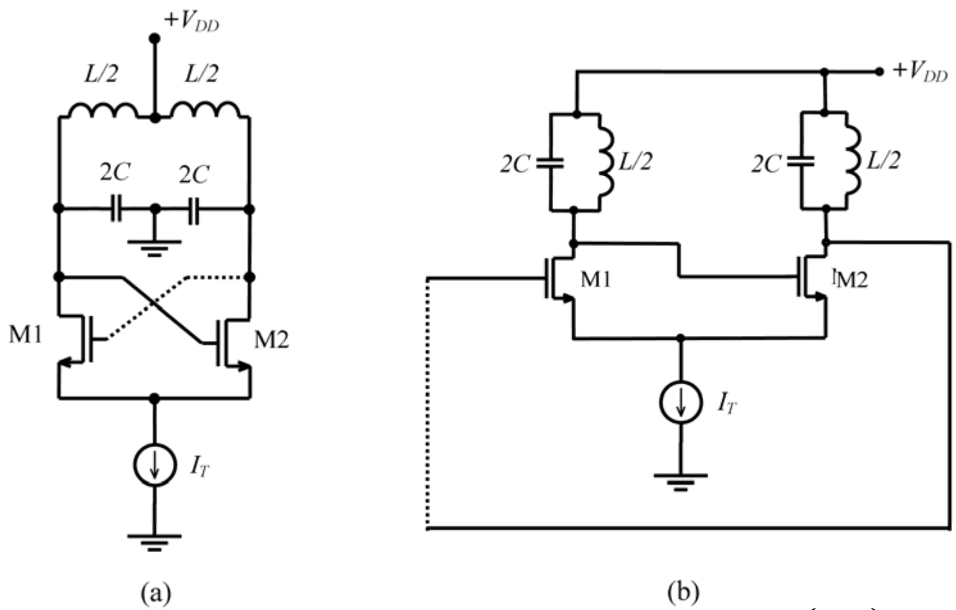
Negative resistance oscillators from the point of view of feedback oscillators:



$$A_{vT}(\omega_0) = (-g_m \cdot R'_{eff})^2 \qquad R'_{eff} = \left(\frac{L}{2}\right)\omega_0 \times Q_{eff}$$

$$A_{vT}(\omega_0) = g_m^2 \frac{1}{4}\left(L\omega_0 \times Q_{eff}\right)^2 = g_m^2 \frac{1}{4} R_{eff}^2$$

$$\beta A_{vT}(\omega_0) = g_m^2 \frac{1}{4} R_{eff}^2 \qquad \Rightarrow \qquad g_m = \frac{2}{R_{eff}}$$

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Frequency stability of L-C oscillators

Example - 1:

Oscillation frequency expression of a Colpitts oscillator can be arranged as:

$$\omega_{osc} = \frac{1}{\sqrt{LC}} \sqrt{1 + k_s \frac{g_{ds}}{g_m} \frac{1}{Q^2} \frac{C_1 + C_2}{C_2}}$$

$$\omega_{osc} = \frac{1}{\sqrt{LC}} \sqrt{1+\varepsilon}$$

This is a general form valid for all L-C oscillators, where ε (the error term) depends on the circuit parameters.

For a stable oscillation frequency ε must be as small as possible.

- In case of a Colpitts oscillator
  - High Q,
  - High g_m,
  - Small g_{ds} is advantageous.

A more general expression:

$$d\omega_{osc} = \frac{\partial \omega_{osc}}{\partial L} dL + \frac{\partial \omega_{osc}}{\partial C_1} dC_1 + \frac{\partial \omega_{osc}}{\partial C_2} dC_2 + \frac{\partial \omega_{osc}}{\partial g_m} dg_m + \frac{\partial \omega_{osc}}{\partial g_{ds}} dg_{ds} + \frac{\partial \omega_{osc}}{\partial Q} dQ$$

provides information about the effects of the tolerances and dependences of several parameters on $d\omega_{osc}$, the shifts of the oscillation frequency.

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Example - 2:

Differential negative resistance oscillator:

$$\omega_{(\mathrm{Re})} = \omega_{(osc)} = \sqrt{\frac{1}{LC}} \times \sqrt{\frac{L - C r_L^2}{L - C r_C^2}}$$

- r_L and r_C must be as low as possible
  (Q_L and Q_C must be as high as possible)

- An interesting possibility:
  For $r_L = r_C$, $\omega_{(osc)}$ depends only on L and C.

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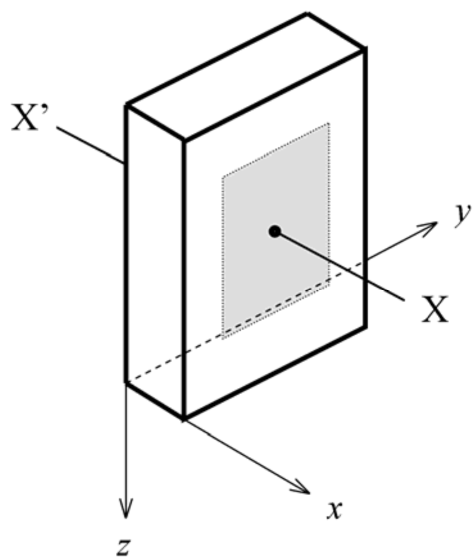
Crystal oscillators

To reach better frequency stability, a piezoelectric crystal (a quartz crystal as the best piezoelectric material) having appropriate dimensions, can be used as the resonator of an electronic oscillator circuit.



x : Electrical axis

y : Mechanical axis

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- If a sinusoidal voltage is applied to X - X' the crystal vibrates in y dimension.

- Amplitude is maximum for the mechanical resonance frequency of the crystal in y direction, for which the losses are very small, consequently Q very high.

- This mechanical vibration provokes a voltage on X - X'; therefore a piezoelectric crystal can be used as a very high Q resonance circuit from its X - X' port.

Mechanical resonance frequencies of a prism shaped body:

$$f = \frac{1}{2}\sqrt{\frac{y}{\rho}}\left[\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2\right]^{1/2}$$

(y: Young modulus, ρ: density)

The fundamental resonance frequency in y direction: $f_0 = \dfrac{1}{2l_y}\sqrt{\dfrac{y}{\rho}}$

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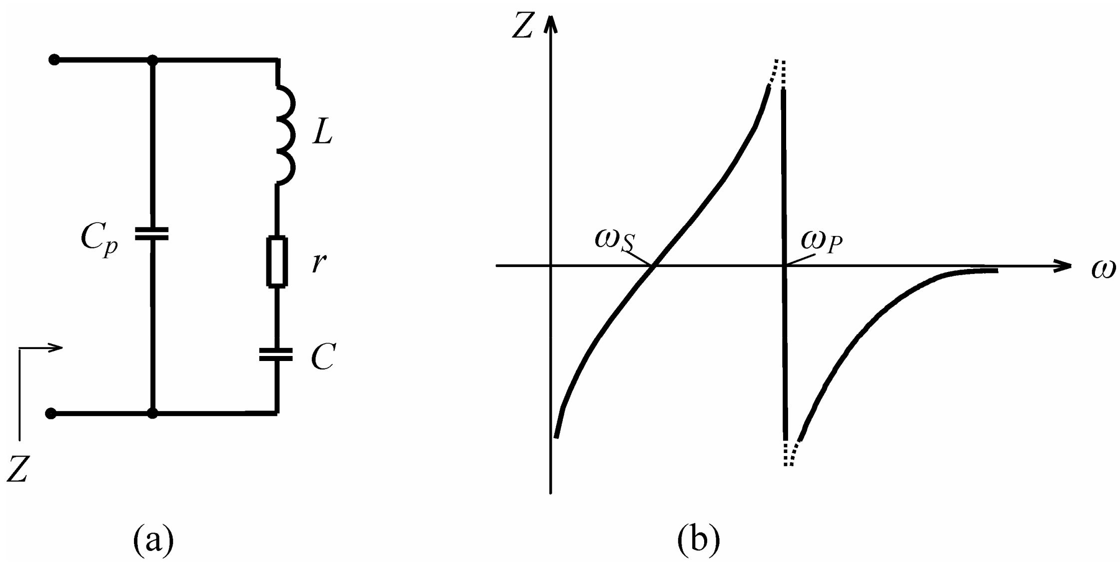
Electrical equivalent circuit of a piezoelectric crystal:



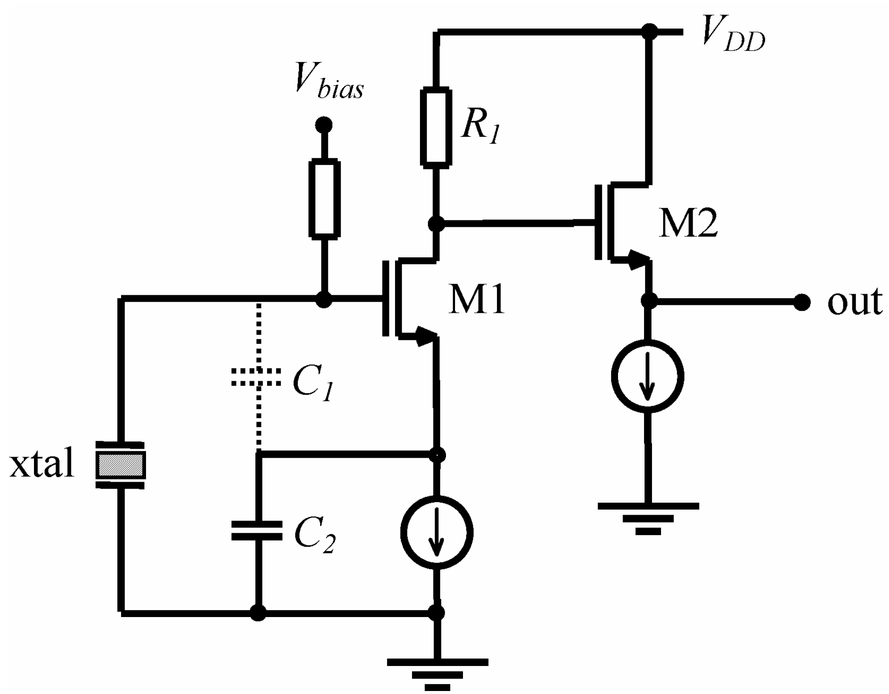
Example:

$l_x = 0.636$ cm, $l_y = 2.75$ cm, $l_z = 3.33$ cm; $f_0 = 430$ kHz

$L = 3.3$ H, $R = 4500$ ohm, $C = 42$ fF, $C_p = 5.8$ pF $\Rightarrow$ $Q \approx 2000$

Example:

An oscillator operating at the parallel resonance frequency of the crystal



(A negative resistance oscillator or a Colpitts oscillator?)

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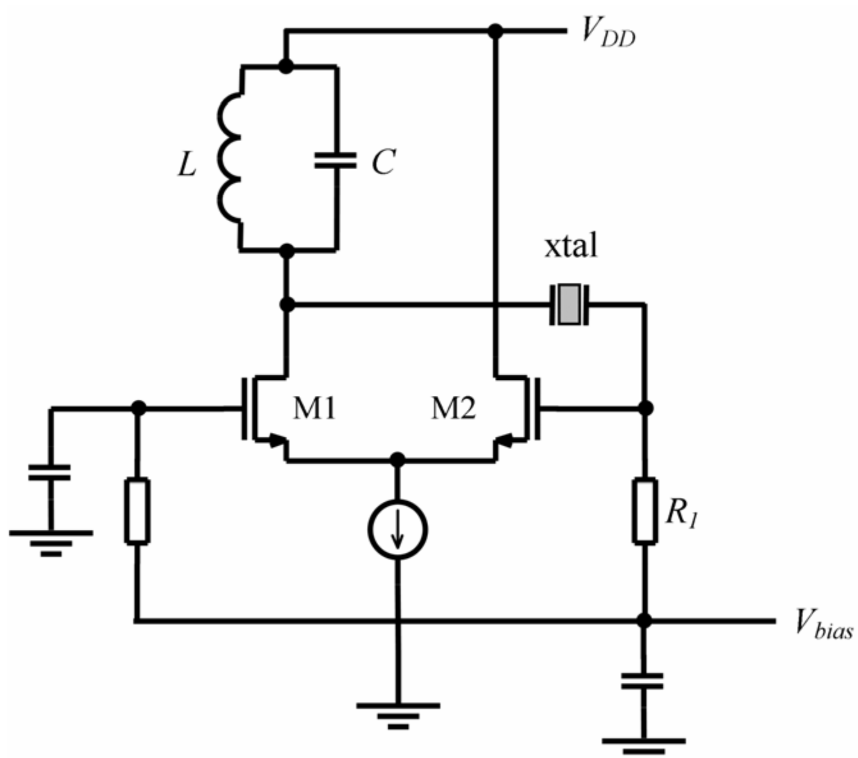
Example:

An oscillator operating at the series resonance frequency of the crystal



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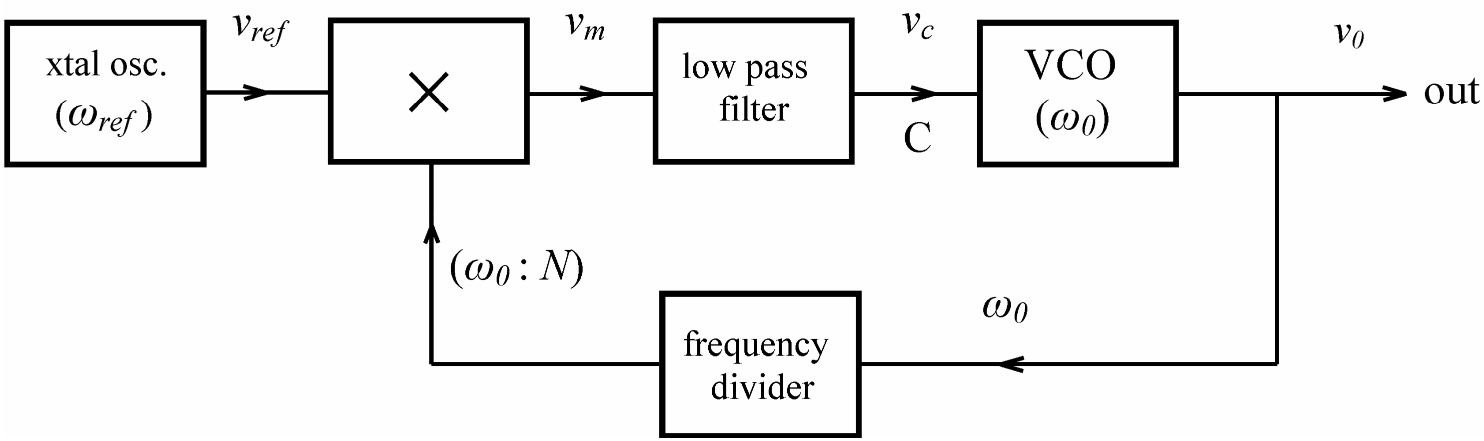
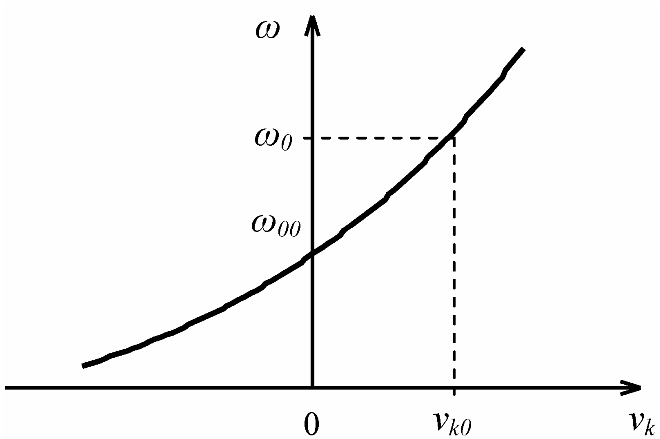
Phase-lock technique





$$v_{ref} = V_{ref}\sin(\omega_{ref}t)$$

$$v_0 = V_0\sin(\omega_{00}t + \varphi_0)$$

$$v_x = V_x\sin\left(\frac{\omega_{00}}{N} + \varphi_x\right)$$

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When the loop is open at C:

$$v_c = k\left[V_{ref}\sin(\omega_{ref}t)\right] \times \left[V_x \sin\left(\frac{\omega_{00}}{N} + \varphi_x\right)\right]$$

$$v_m = \frac{1}{2}kV_{ref}V_x\cos\left[\left(\omega_{ref} - \frac{\omega_{00}}{N}\right)t - \varphi_x\right] - \frac{1}{2}kV_{ref}V_x\cos\left[\left(\omega_{ref} + \frac{\omega_{00}}{N}\right)t + \varphi_x\right]$$

$$v_c = \frac{1}{2}kV_{ref}V_x\cos\left[\left(\omega_{ref} - \frac{\omega_{00}}{N}\right)t - \varphi_x\right]$$

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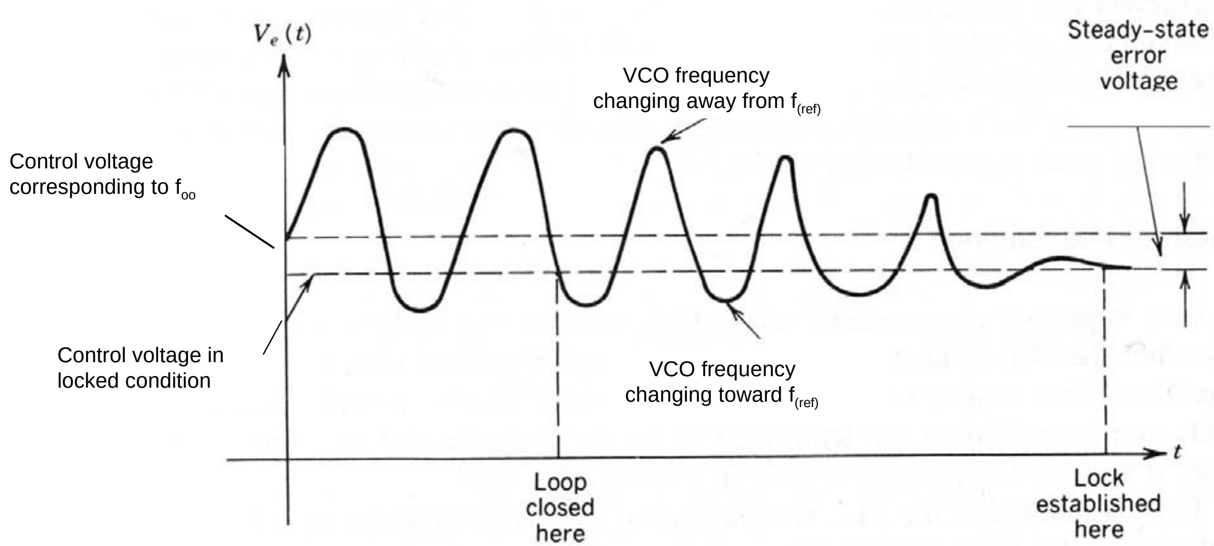
The on-set of locking upon the closing of the loop:



$\Downarrow$

At the on-set of locking; $f_0 = f_{ref}$

(After Alan B. Grebene)

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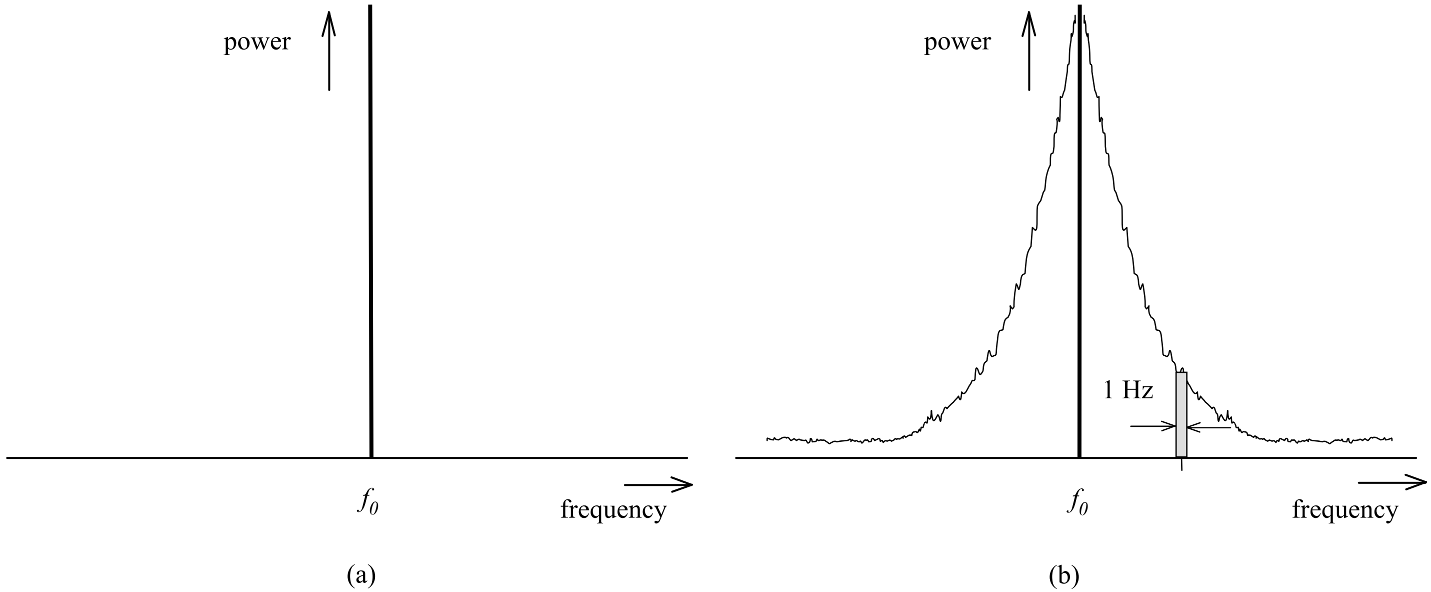
Phase noise in oscillators



(a) Frequency spectrum of an ideal oscillator.

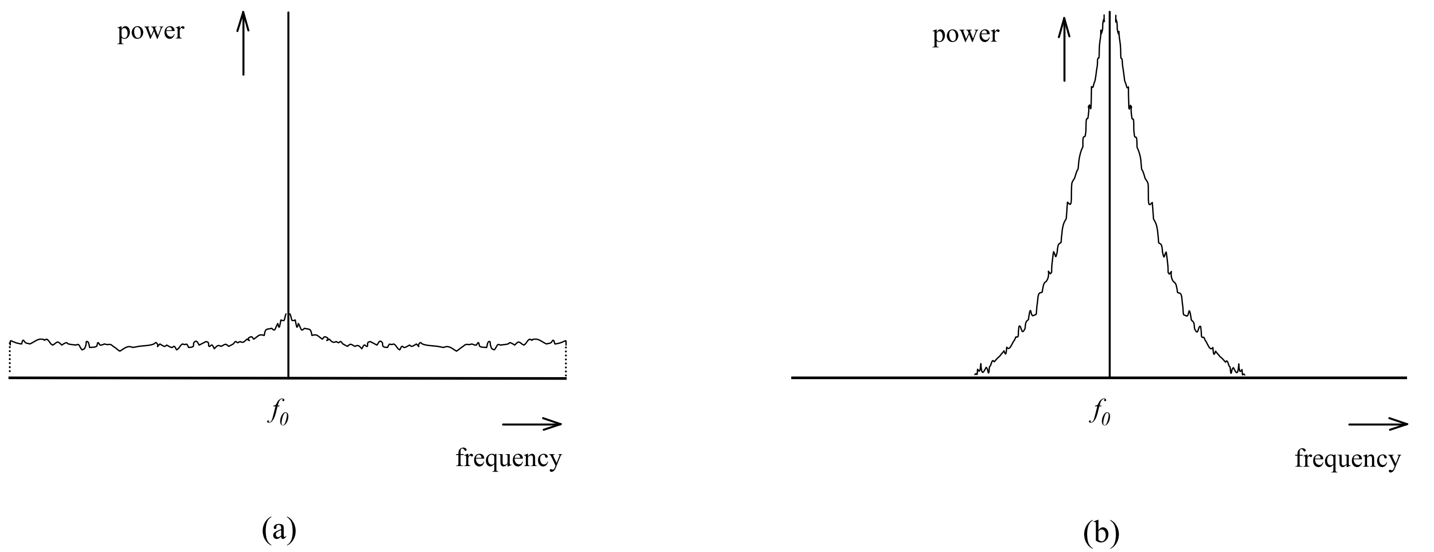
(b) Frequency spectrum of a non-ideal oscillator.

$$\text{Phase noise: } L\left(\Delta\omega\right) = 10\log\left(\frac{P(f_0+\Delta f)_{B=1Hz}}{P(f_0)}\right)$$

There are several mechanisms contributing to the phase noise:

- Random changes of the frequency due to the low stability,
- The thermal noise on the voltages and currents in the circuit,
- Shifts of the zero-crossing point of the output signal,
- Frequency modulation of f_0 with thermal noise.

Frequency modulation of f_0 with thermal noise

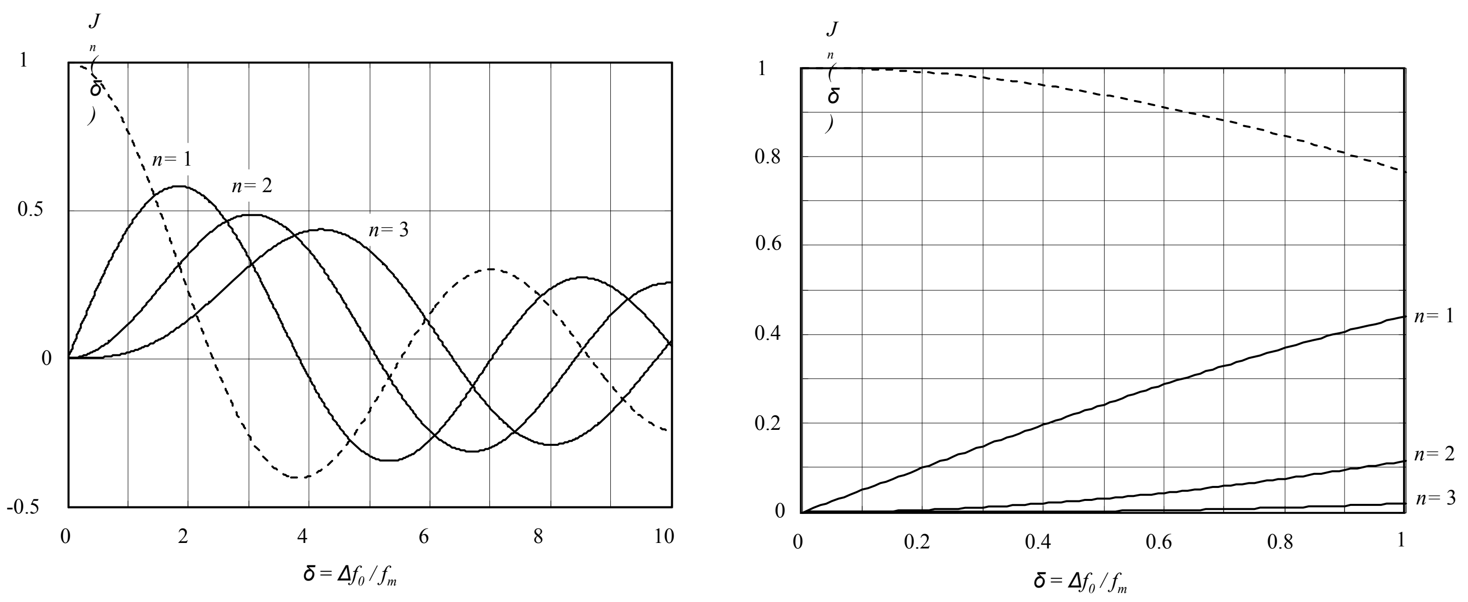


(a) Spectrum of an oscillator, amplitude modulated with white noise,

(b) Spectrum of an oscillator, frequency modulated with white noise.

- One of the reasons behind the random frequency modulation of ω_0 is the uncertainty of the oscillation frequency, due to the three frequencies, ω_0, $\omega_{(\mathrm{Re})}$ and $\omega_{\max}$, different but very close to each other.

- The amplitudes of the side frequencies of a carrier f_0, frequency modulated with a signal, f_m are given in terms of Bessel functions of the modulation index, $\delta = \Delta f_0 / f_m$, where Δf_0 is the frequency deviation that is very small and random in our case.



(a) (b)

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- From these data, for small values of δ, the rate of decrease of the r.m.s. values of the side frequencies can be calculated as approximately 20 dB/decade, in good agreement with the measured data.

- Another important issue related to the phase noise is the "jitter", the RMS amplitude of the variations of the zero cross point of the signal in time axis, that is related to the phase noise in f_1 to f_2 interval as

$$J_{RMS}\Big|_{(f_1 \text{ to } f_2)} = \frac{1}{2\pi f_0}\sqrt{2\int_{f_1}^{f_2} 10^{\frac{L(f)}{10}} df}$$

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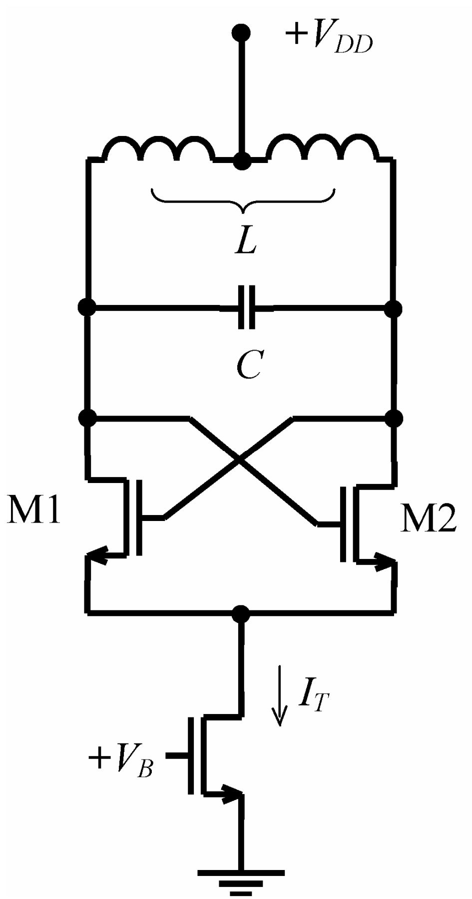
Example:



W/L = 100, $L=0.18um$

I_T = 1.2 mA

L = 10 nH

C = 1 pF

f_o = 1.6 GHz

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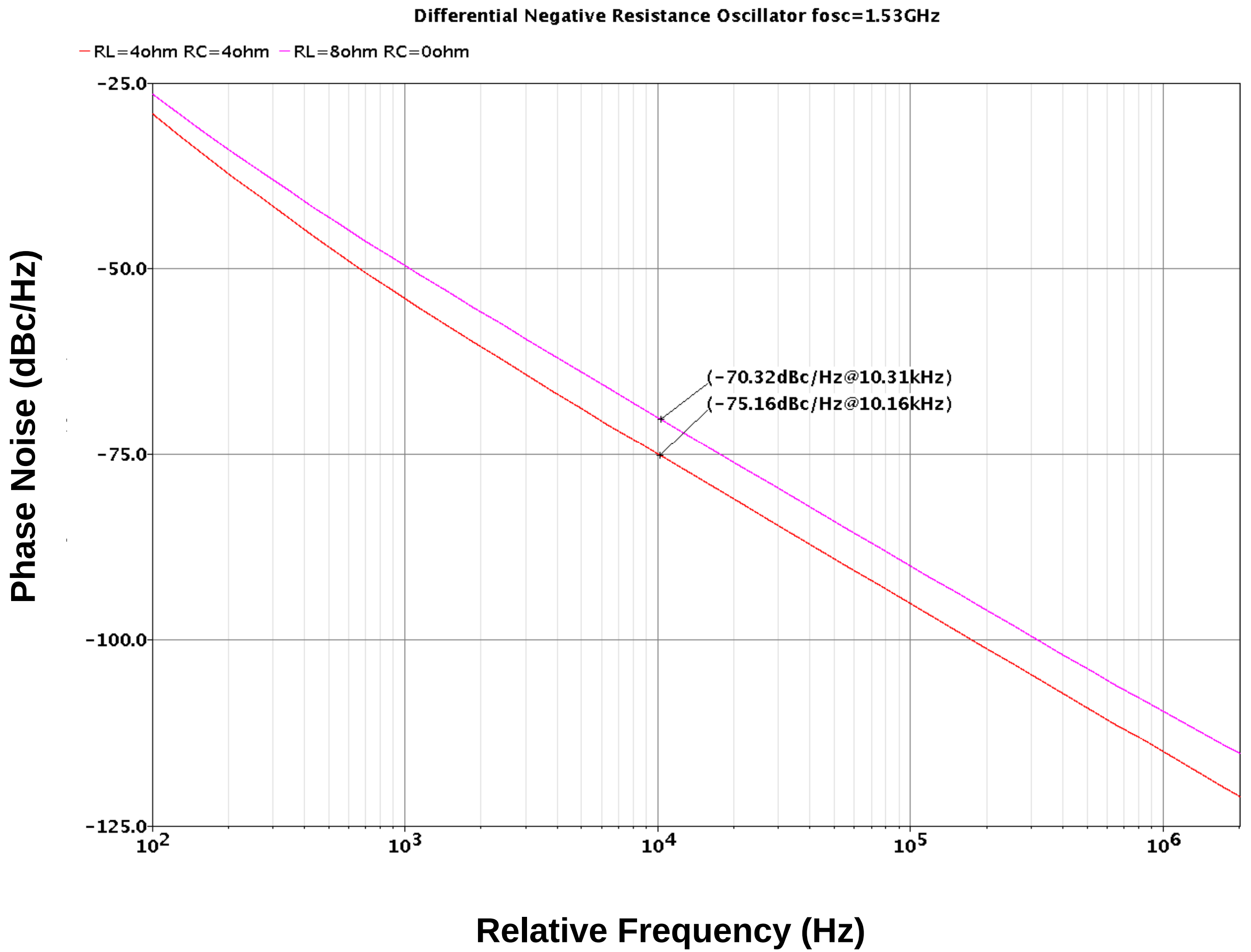

Jitter = 1.12 ps

Jitter = 515 fs