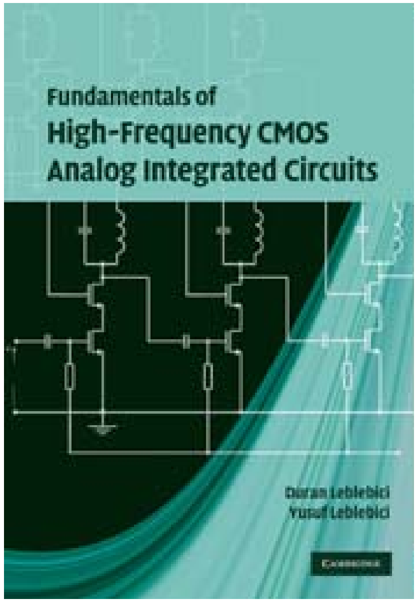


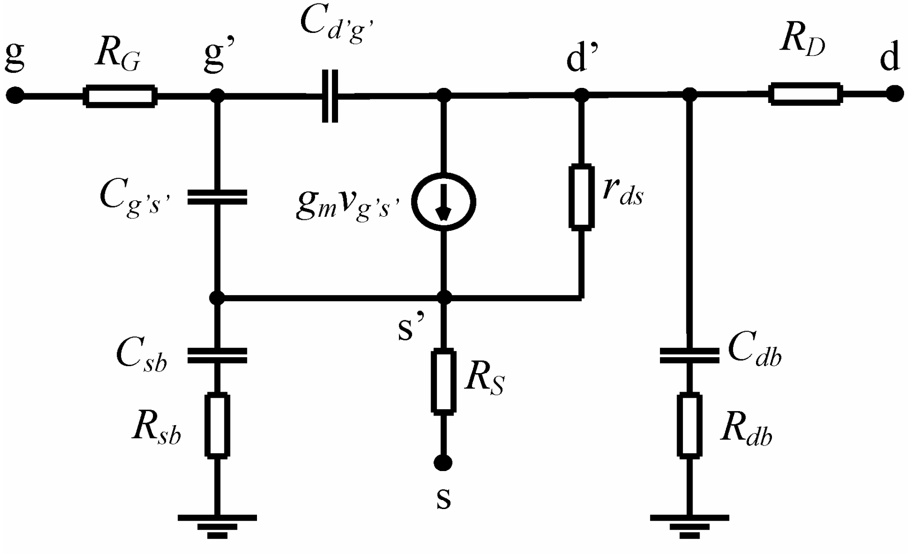

# Fundamentals of High-Frequency CMOS Analog Integrated Circuits

Duran Leblebici
Yusuf Leblebici

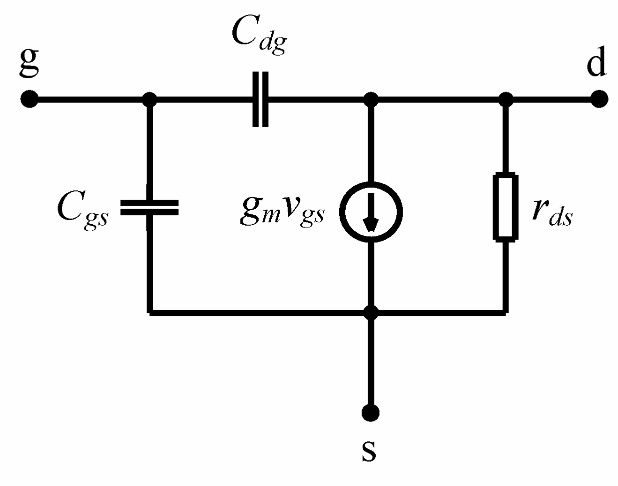
# Chapter 3

## High-Frequency Behavior of Basic Amplifiers

# High frequency small signal equivalent circuit of a MOS transistor



(a)



(b)



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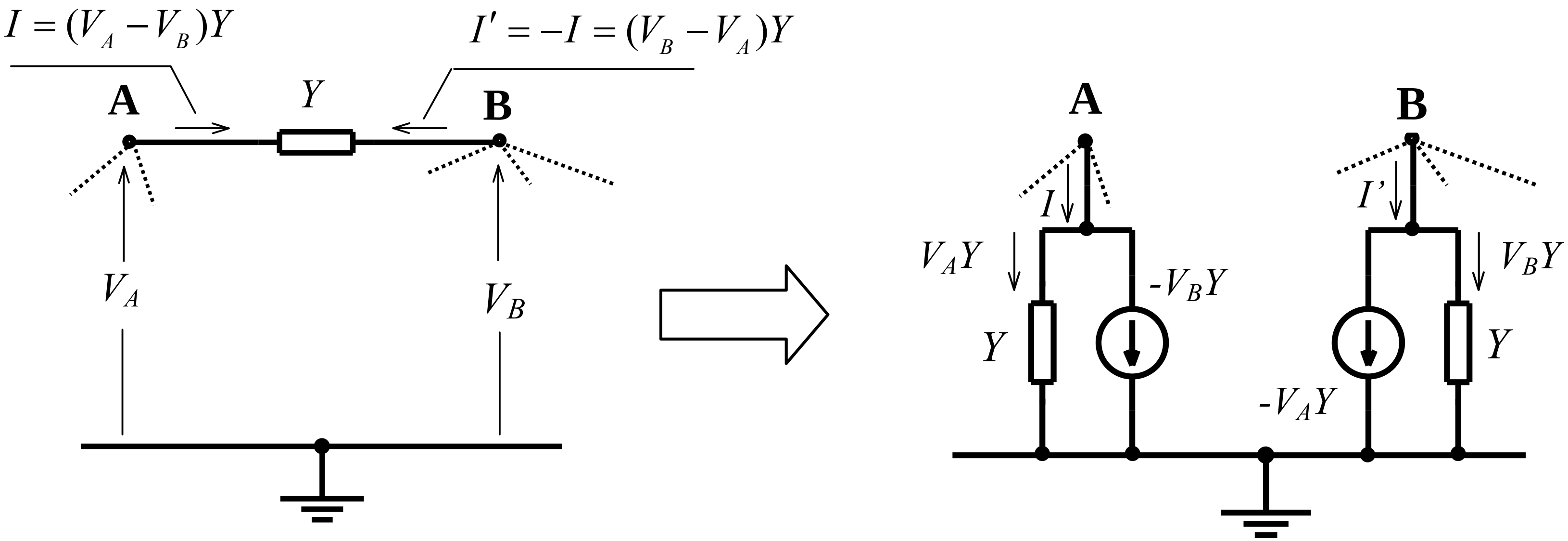
# Modified Miller conversion



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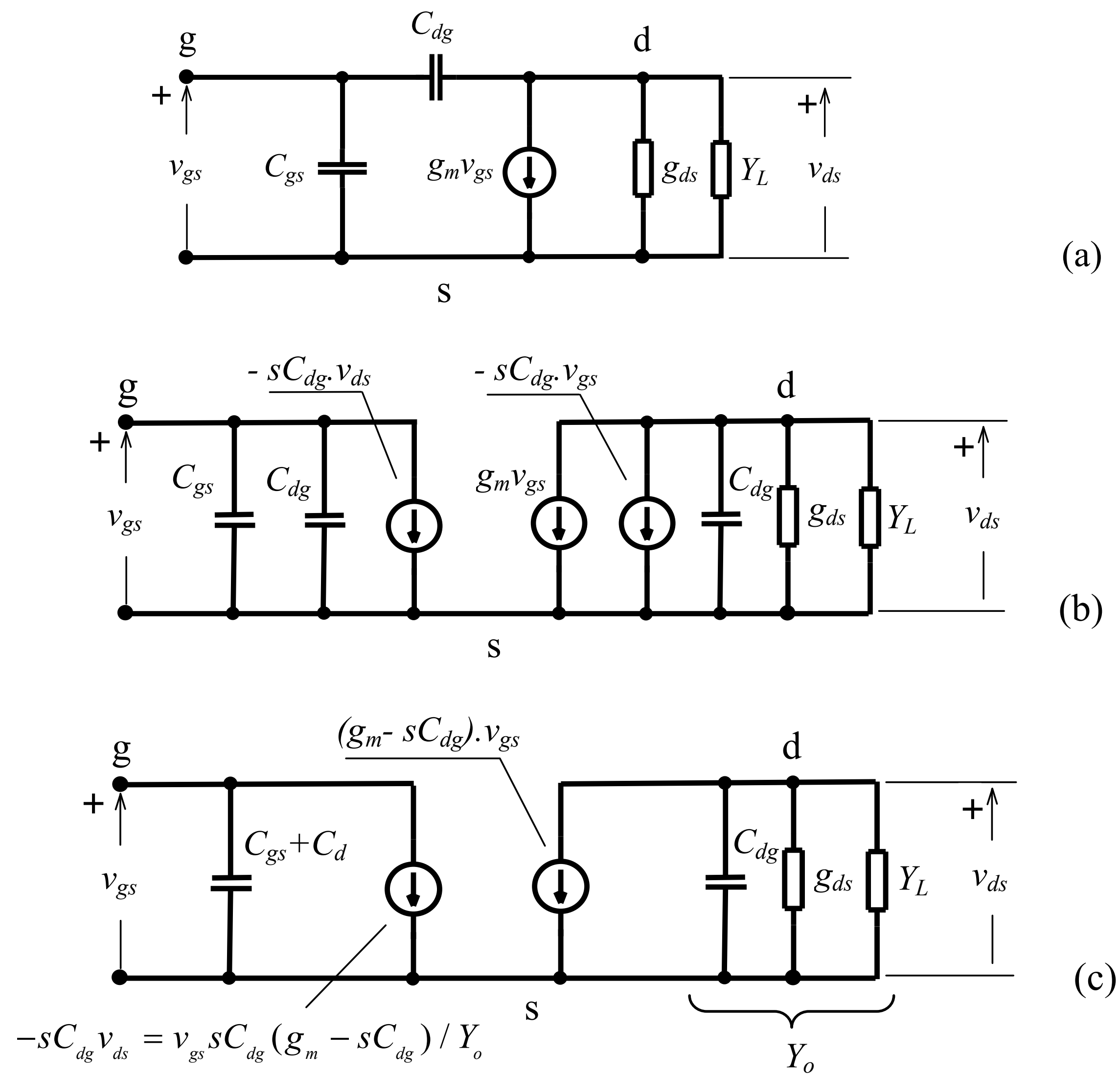
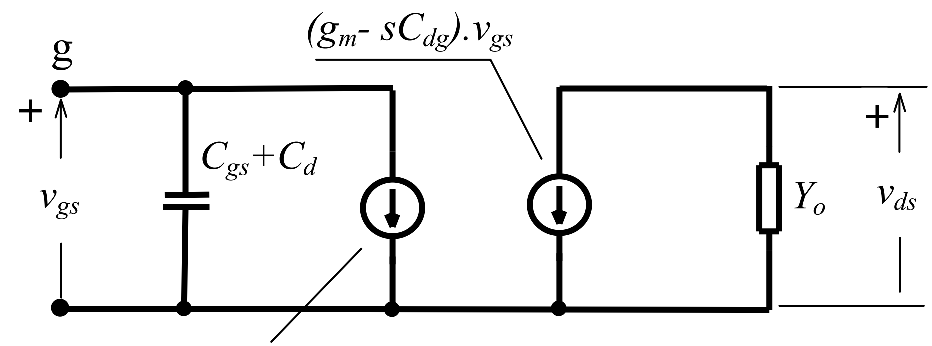
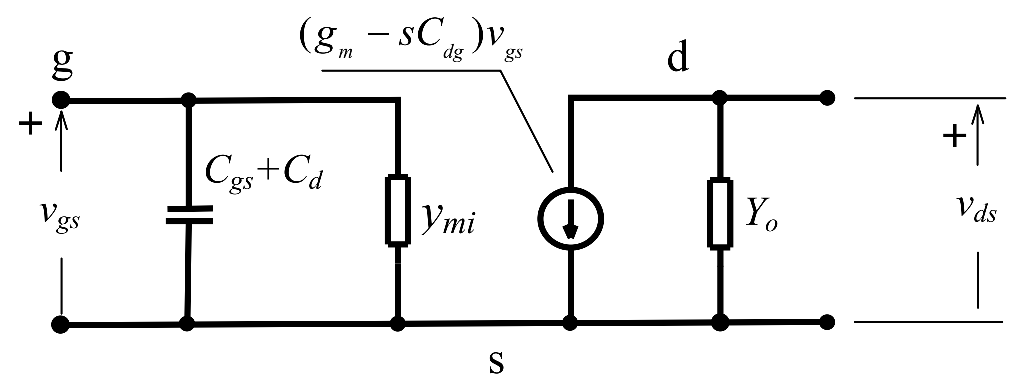
# High frequency behavior of common source amplifier



$$v_{ds} = -\frac{g_m - sC_{dg}}{Y_o} v_{gs} \qquad A_v = -\frac{g_m - sC_{dg}}{Y_o}$$




$$v_{gs} \frac{(g_m - sC_{dg})}{Y_o} sC_{dg} = -v_{gs} A_v . sC_{dg}$$




$$y_{mi} = sC_{dg} \frac{g_m - sC_{dg}}{Y_o} = sC_{dg}(-A_v)$$



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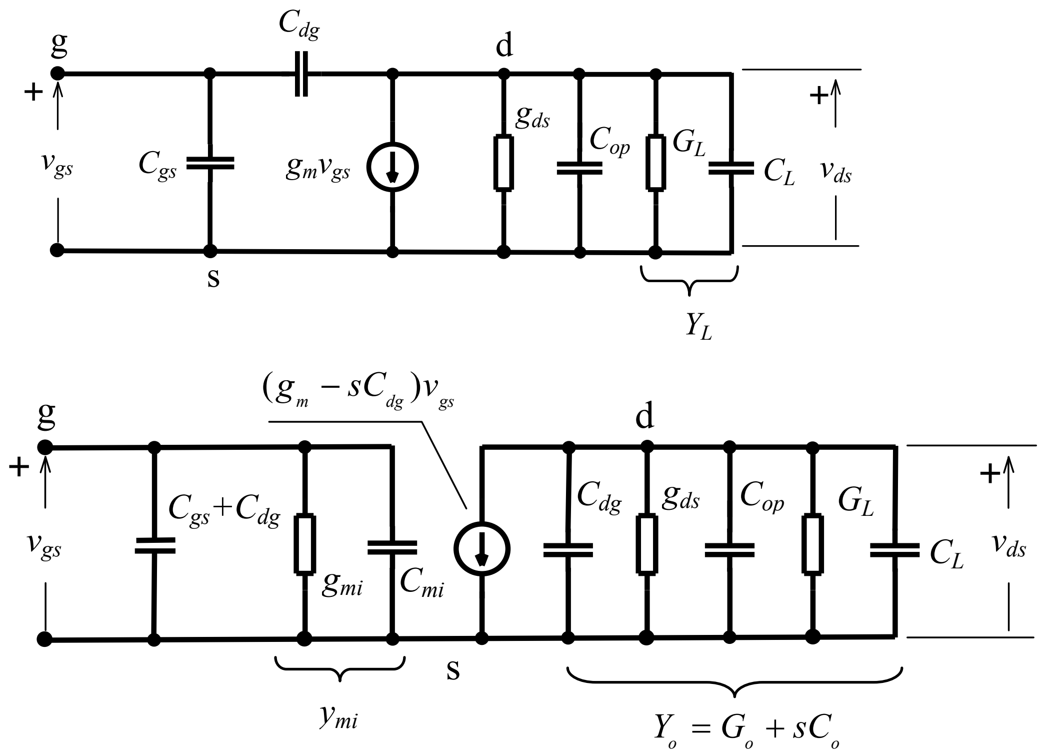
# R-C Loaded amplifier



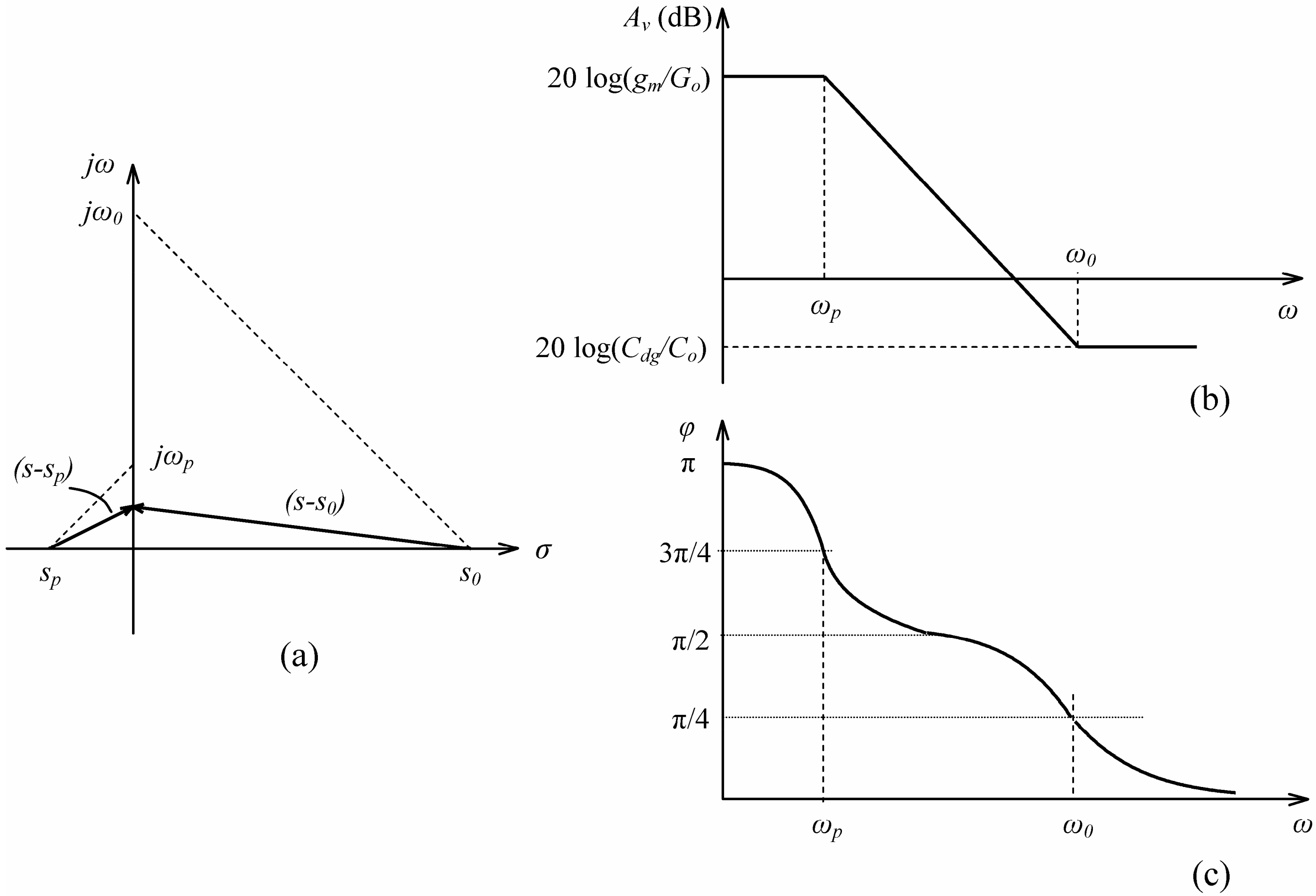

$$A_v = -\frac{g_m - sC_{dg}}{G_o + sC_o} = \frac{C_{dg}}{C_o}\frac{(s - s_o)}{(s - s_p)}$$

$$A_v = \frac{C_{dg}}{C_o}\frac{(s - s_o)}{(s - s_p)}$$

$$s_o = +\frac{g_m}{C_{dg}}$$

$$s_p = -\frac{G_o}{C_o} = -\frac{(g_{ds} + G_L)}{(C_{op} + C_{dg} + C_L)}$$

$$A_v(0) = -\frac{g_m}{G_o} = -\frac{g_m}{(G_L + g_{ds})}$$




Magnitude of gain at pole frequency: $\left|A_v(\omega_p)\right| = \frac{1}{\sqrt{2}}\left|A_v(0)\right|$

The gain-bandwidth product: $GBW = \left|A_v\right| \times \frac{1}{2\pi}\left|s_p\right| \cong \frac{g_m}{2\pi C_o}$

# The input admittance of a R-C loaded amplifier

- The total input admittance: $y_i = y_{mi} + s(C_{gs} + C_{dg})$

- The Miller admittance: $y_{mi} = sC_{dg} \dfrac{g_m - sC_{dg}}{G_o + sC_o}$

- The Miller admittance in $\omega$ domain: $y_{mi}(\omega) = j\omega C_{dg} \dfrac{g_m - j\omega C_{dg}}{G_o + j\omega C_o}$

- Real part of $y_{mi}$ (input conductance): $g_{mi}(\omega) = \dfrac{C_{dg}}{C_o}\left(g_m + G_o \dfrac{C_{dg}}{C_o}\right)\dfrac{1}{(\omega_p / \omega) + 1}$

- Imaginary part of $y_{mi}$: $b_{mi}(\omega) = \omega \dfrac{C_{dg}^2}{C_o} \dfrac{\omega_o \omega_p - \omega^2}{\omega_p^2 + \omega^2} = \omega C_{mi}$



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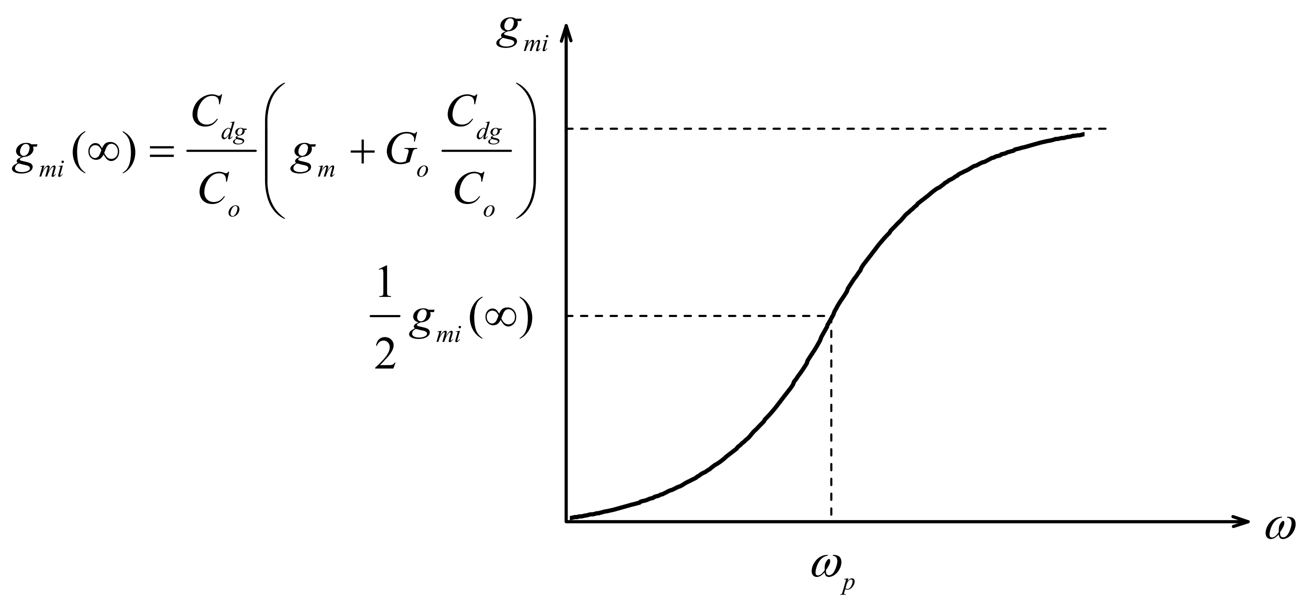
The input conductance as a function of $\omega$: $(g_i \equiv g_{mi})$

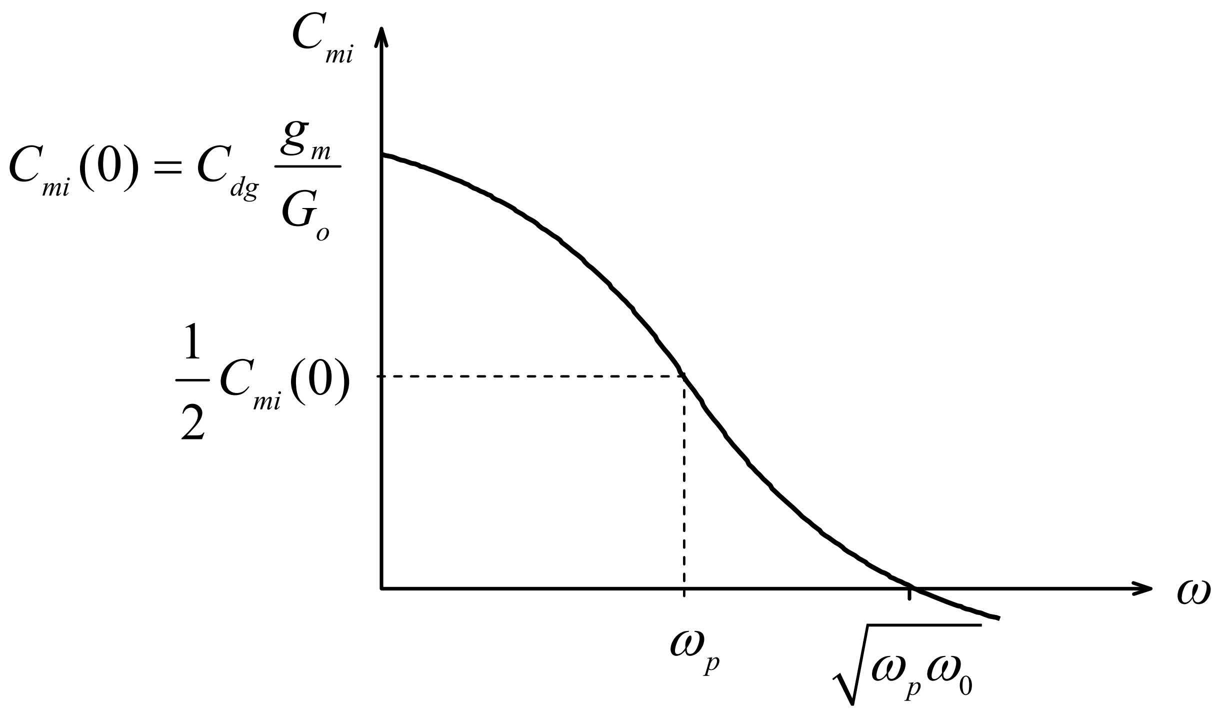
$$g_{mi}(\omega \to \infty) = \frac{C_{dg}}{C_o}\left( g_m + G_o \frac{C_{dg}}{C_o} \right)$$

$$g_{mi}(\omega_p) = \frac{g_{mi}(\omega \to \infty)}{2} = \frac{1}{2}\frac{C_{dg}}{C_o}\left( g_m + G_o \frac{C_{dg}}{C_o} \right)$$

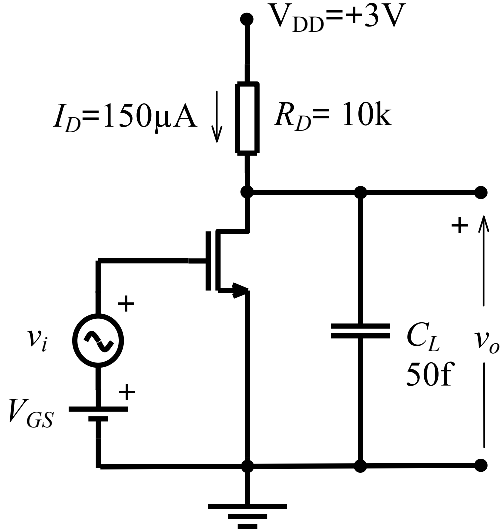


Miller capacitance as a function of $\omega$: $(C_i = C_{mi} + C_{gs} + C_{dg})$

$$C_{mi} = \omega \frac{C_{dg}^2}{C_o} \frac{\omega_o \omega_p - \omega^2}{\omega_p^2 + \omega^2}$$



# Example:




$$I_D = 150\ \mu\text{A}\ \ (V_{DS} = 1.5\ \text{V}),$$
$$g_m = 2\ \text{mS},$$
$$C_{gs} = 90\ \text{fF},\ C_{dg} = 20\ \text{fF}$$
$$g_{ds} = 30\ \mu\text{S},\ C_{op} = 10\ \text{fF}$$

$$A_v(0) = -\frac{2\times 10^{-3}}{(30\times 10^{-6} + 10^{-4})} = -15.38 \quad \Rightarrow \quad 23.74\ \text{dB}$$

$$f_p = \frac{1}{2\pi}\frac{(30\times 10^{-6} + 10^{-4})}{(10\times 10^{-15} + 20\times 10^{-15} + 50\times 10^{-15})} = 258.7\ \text{MHz}$$

Input conductance at pole frequency:

$$g_i(f_p) = g_{mi}(f_p) = \frac{1}{2}\frac{20\times10^{-15}}{80\times10^{-15}}\left(2\times10^{-3} + 10^{-4}\frac{20\times10^{-15}}{80\times10^{-15}}\right)$$

$$= 0.253\times10^{-3}\ \text{siemens} \quad \Rightarrow \quad r_i(f_p) = 3.95\ \text{k ohm !}$$

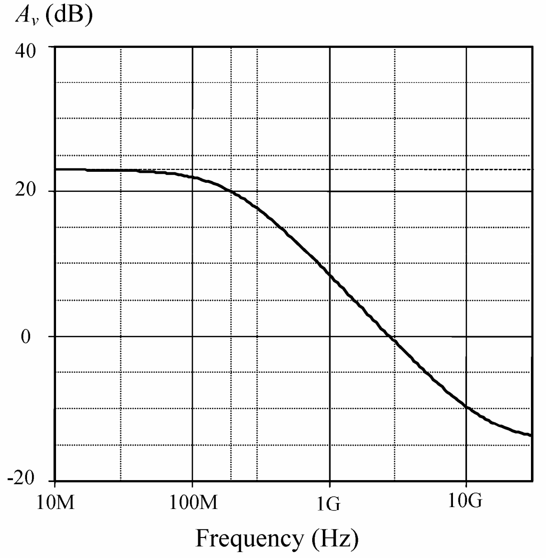
Input conductance at $(f_p/10)$: $4.95\ \mu\text{S} \;\Rightarrow\; r_i(f_p/10) \approx 200\ \text{k ohm}$

Maximum value of the Miller capacitance:

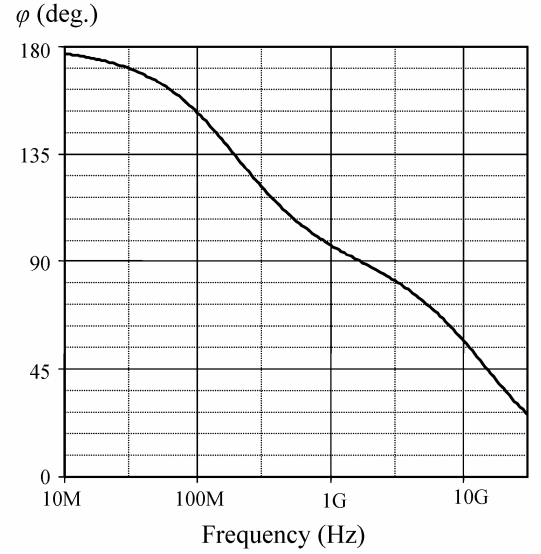
$$C_{mi}(0) = C_{dg}\frac{g_m}{G_o} = 20\times10^{-15}\frac{2\times10^{-3}}{130\times10^{-6}} = 307.7\, fF$$

The total input capacitance:

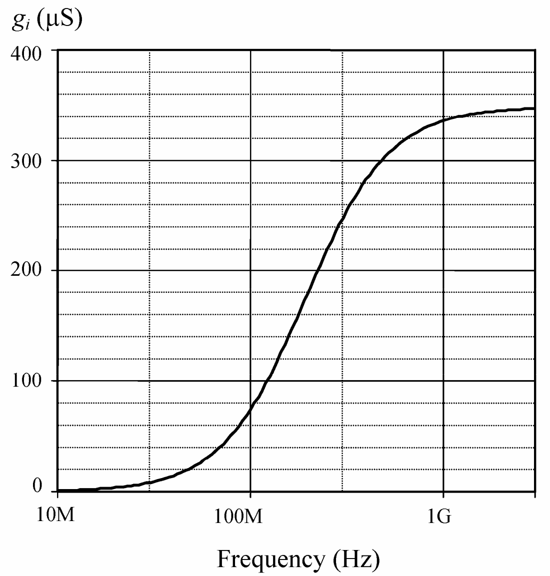
$$C_i = C_{gs} + C_{dg} + C_{mi} = 90 + 20 + 307.7 = 417.7\, fF$$



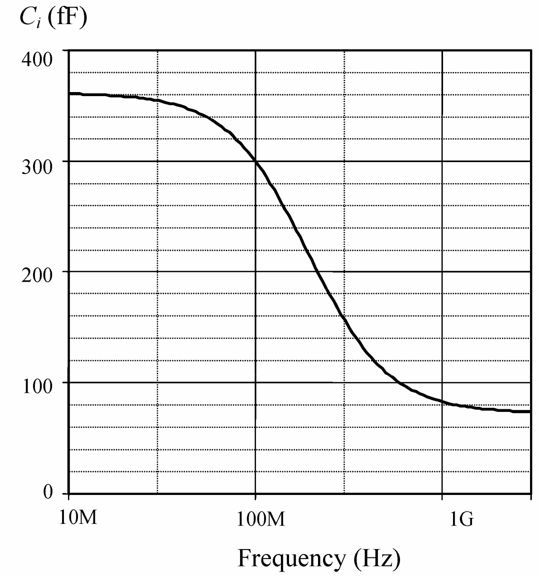

(a)




(b)



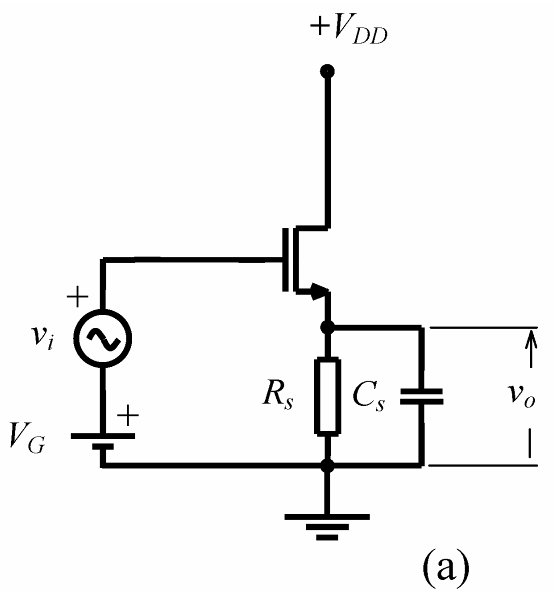

(c)

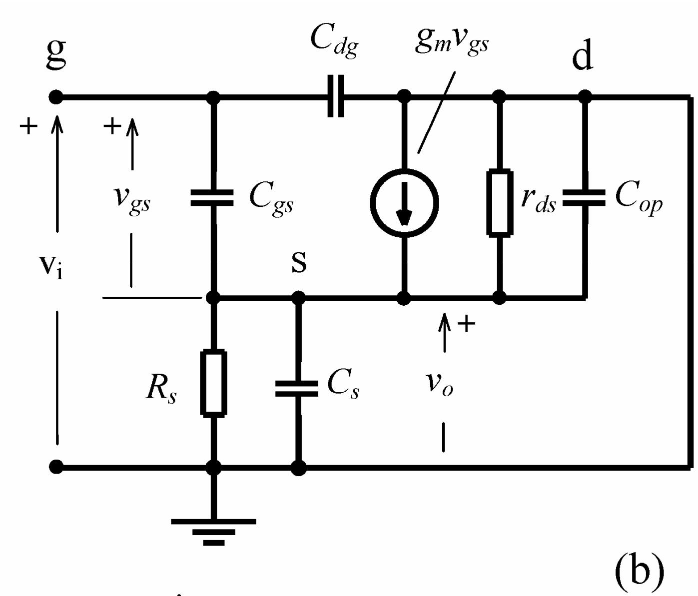



(d)

PSpice results: $A_v(0) = 23$ dB, $f_p = 192$ MHz, $g_i(f_p) = 230\ \mu$S, $C_i(0) = 360$ fF

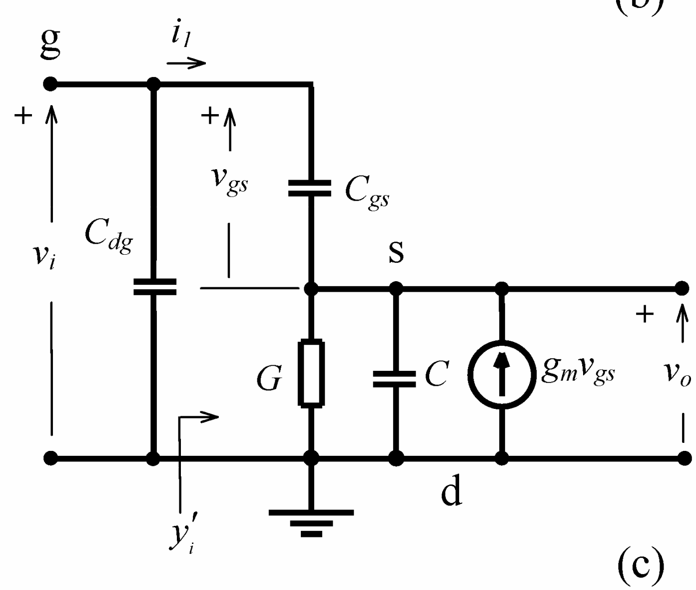
# High frequency behavior of source follower




$$v_o = \frac{i_i + g_m v_{gs}}{Y}$$

$$v_i = v_{gs} + v_o$$





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The voltage gain:

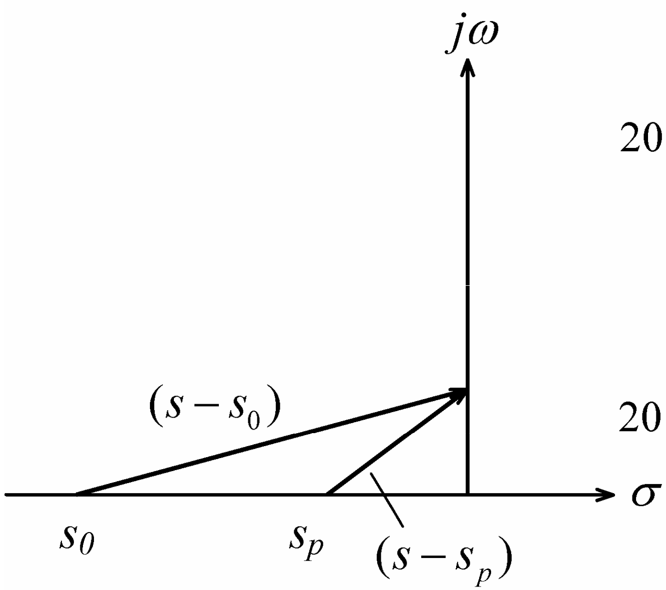
$$A_v = \frac{v_o}{v_i} = \frac{g_m + s\,C_{gs}}{\left(g_m + G\right) + s\left(C_{gs} + C\right)}$$

$$A_v = \frac{C_{gs}}{\left(C_{gs} + C\right)} \frac{\left(s - s_0\right)}{\left(s - s_p\right)}$$

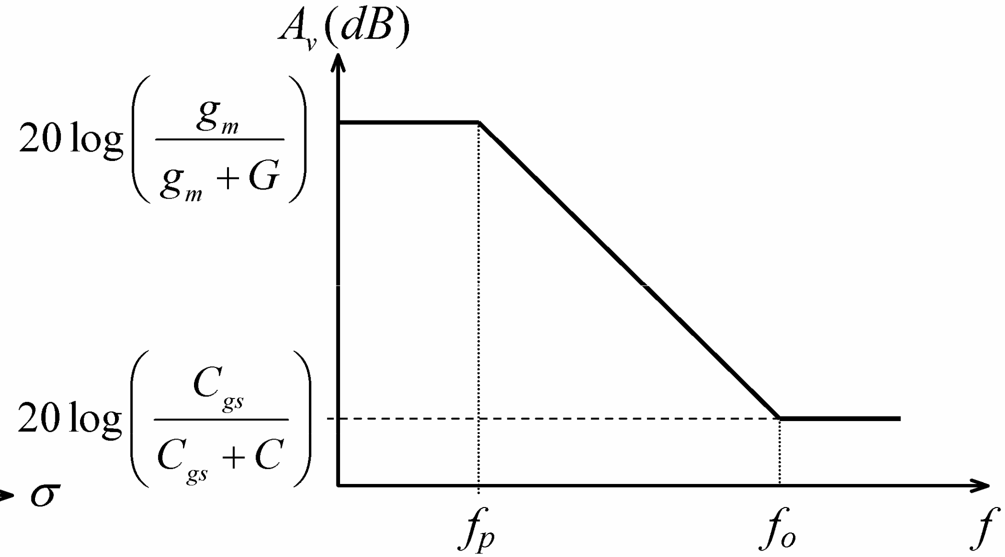
where $s_0 = -\dfrac{g_m}{C_{gs}}$ , $s_p = -\dfrac{\left(g_m + G\right)}{\left(C_{gs} + C\right)}$

$$A_v(0) = \frac{g_m}{\left(g_m + G\right)}$$

Always ( $\prec$ 1) !

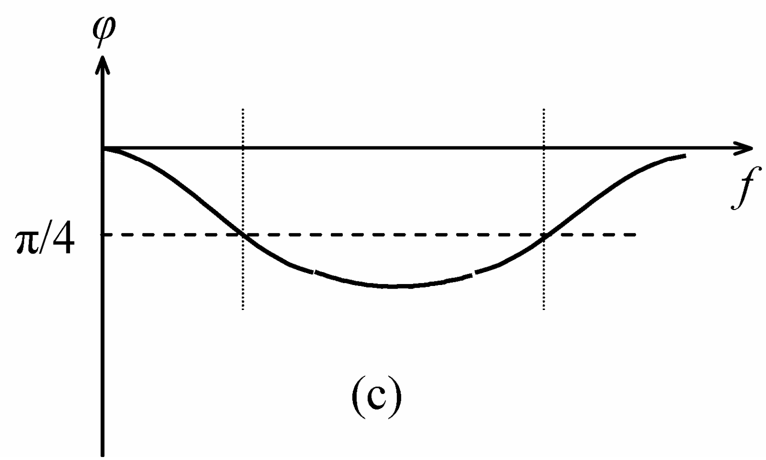



(a)




(b)

$$f_p = \frac{\omega_p}{2\pi} = \frac{|s_p|}{2\pi} \cong \frac{(g_m + G)}{(C_{gs} + C)}$$




(c)

The input admittance of a source follower:

$$i_i = (v_i - v_o)\ sC_{gs} = v_i\left(1 - A_v\right)\ sC_{gs}$$

$$y_i^{'}(\omega) = \frac{i_i}{v_i} = \left(1 - \frac{(g_m + j\omega C_{gs})}{(g_m + G) + j\omega(C_{gs} + C)}\right) j\omega C_{gs} = g_i + j\omega C_i$$

($C_{dg}$ excluded!)

The input conductance:

$$g_i(\omega) = \frac{C_{gs}}{(C_{gs} + C)^2}(GC_{gs} - g_m C)\frac{1}{1 + (\omega_p / \omega)^2}$$

$$g_i(\omega) = g_i(\infty)\frac{1}{1 + (\omega_p / \omega)^2}\ ,\quad \frac{C_{gs}}{(C_{gs} + C)^2}(GC_{gs} - g_m C)$$

The input conductance

is positive for $GC_{gs} \succ g_m C$

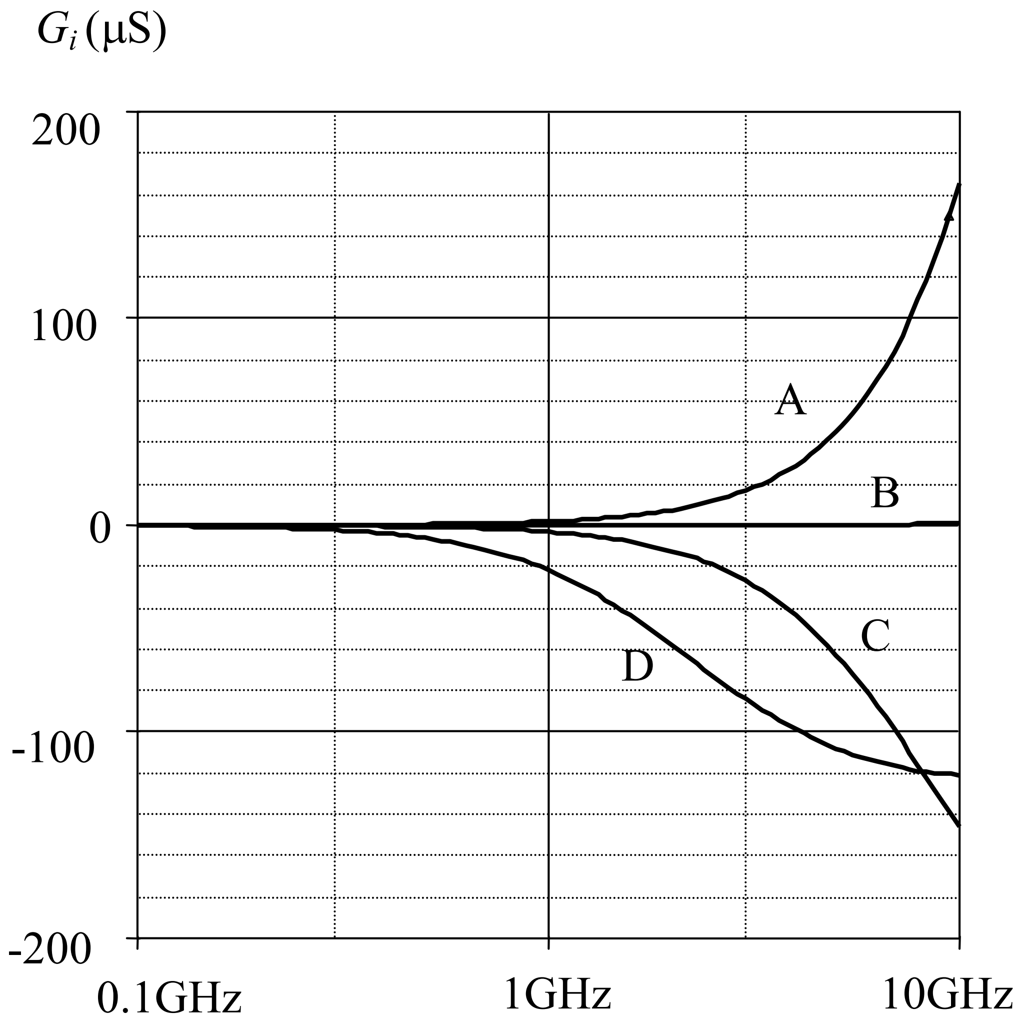
is zero for $GC_{gs} = g_m C$

is negative for $GC_{gs} \prec g_m C$

A negative input conductance

- Can lead to excessive ringing, even to oscillation,
- Can be used

  whenever a negative resistance is necessary.




Simulated input conductance of a source follower for

(A) $C = 20$ fF, (B) $C = 80$ fF, (C) $C = 200$ fF, $C = 1$ pF



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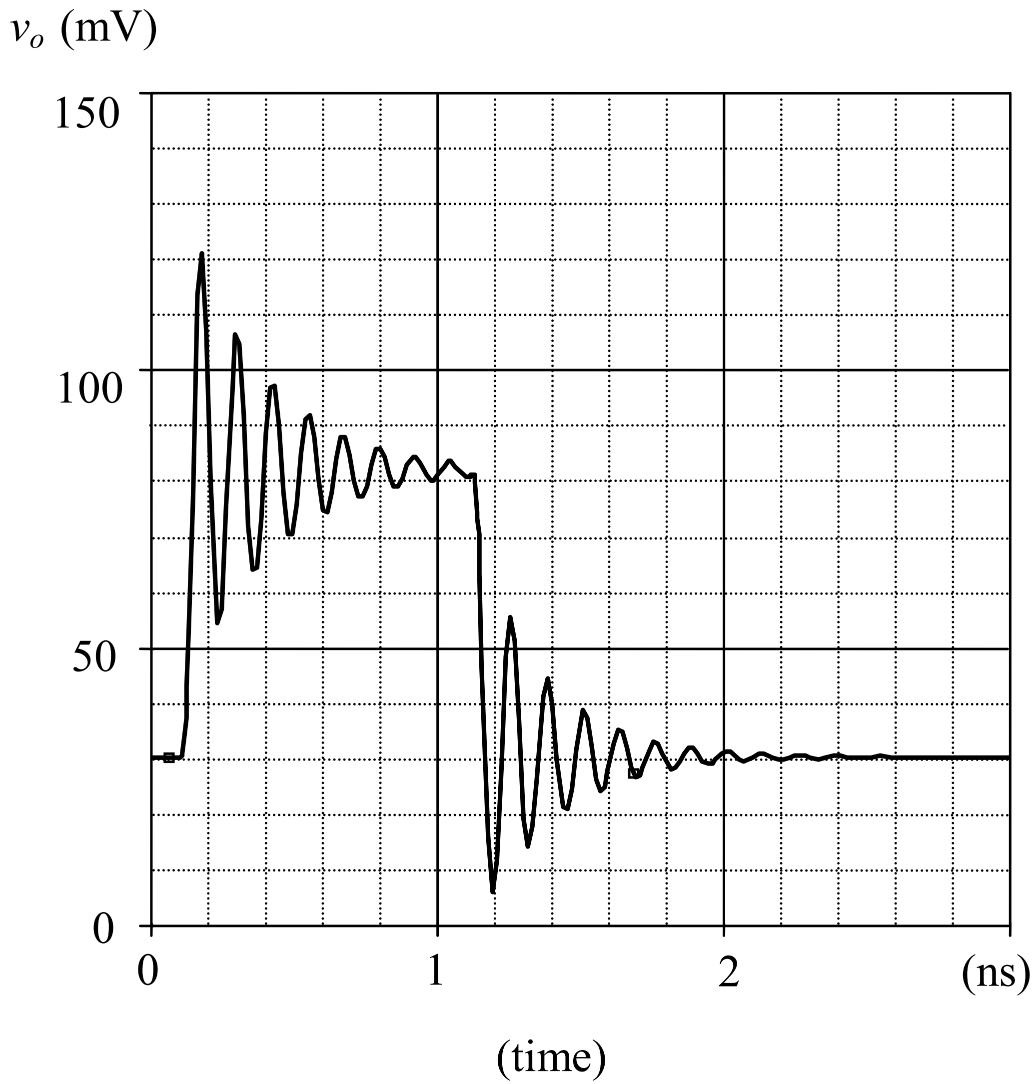



Square wave response of a source follower operating in the negative conductance mode with a 5 nH inductance series to the gate.

The input capacitance:

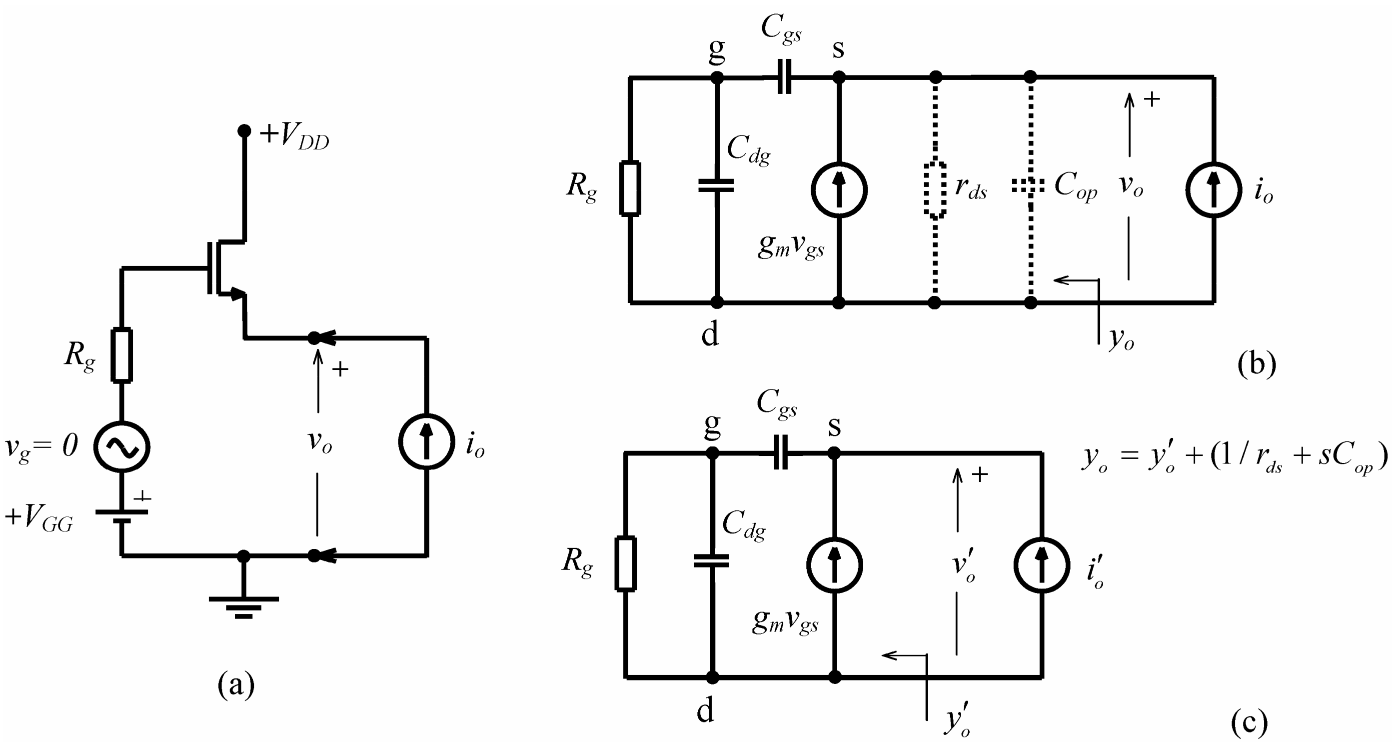
$$C_{iT} = C_{dg} + C\,\frac{\omega_0'\omega_p + \omega^2}{\omega_p^2 + \omega^2}$$

$$\omega_p = \frac{\left(g_m + G\right)}{\left(C_{gs} + C\right)}\ , \quad \omega_0' = \frac{G}{C}$$

$$C_{iT}(0) = C_{dg} + (C_{gs} + C)\frac{G}{(g_m + G)}$$

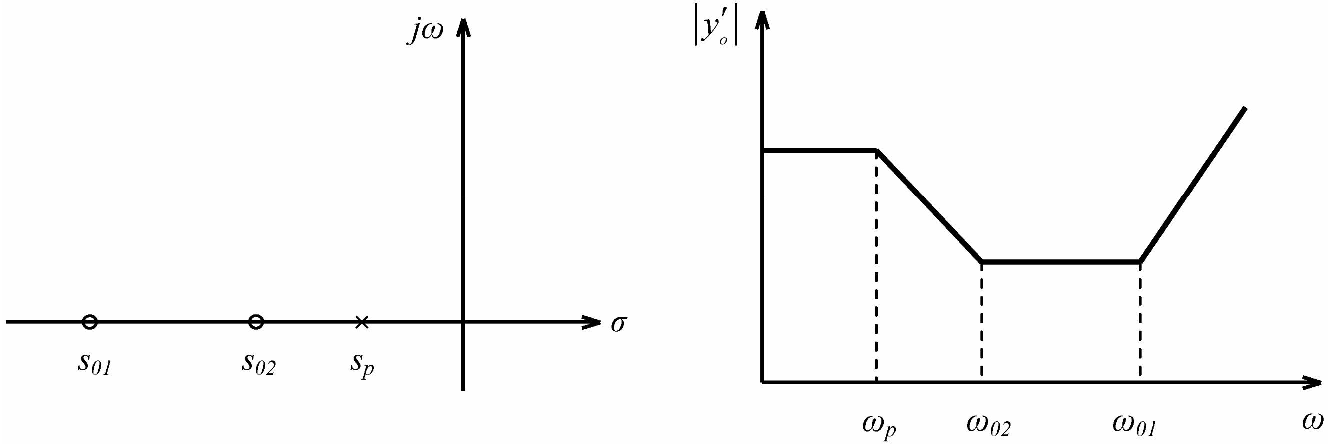
$$C_{iT}(\infty) = C_{dg} + C$$

# The output admittance of a source follower:

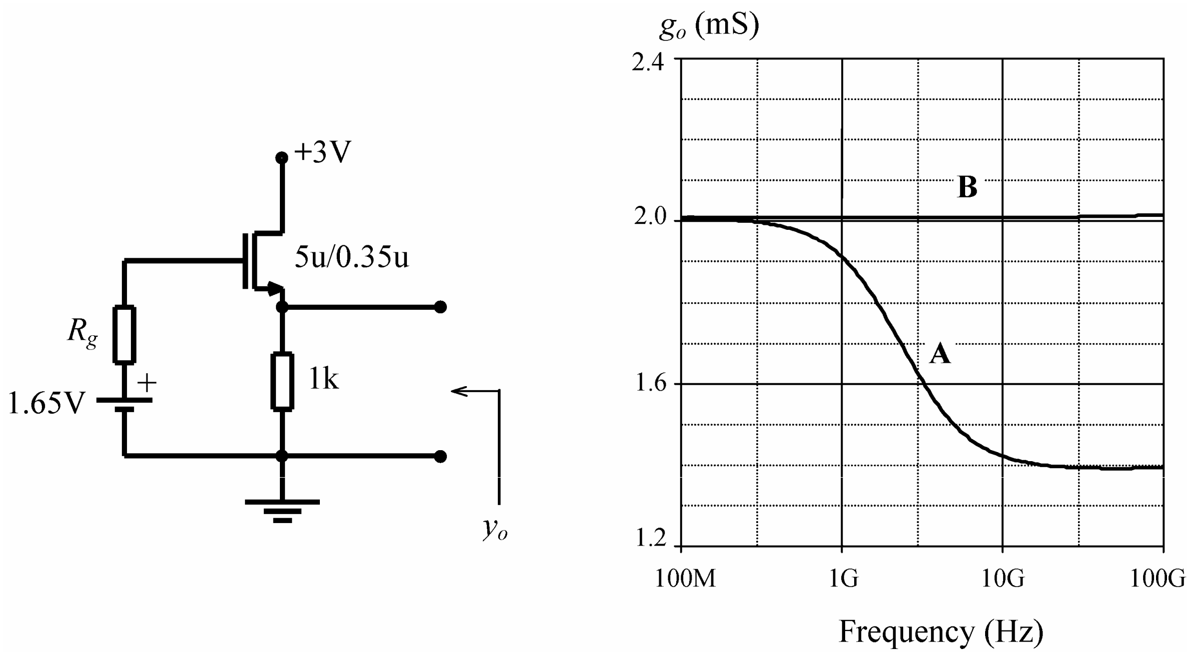



$$y_o' = \frac{i_o'}{v_o'} = \frac{(G_g + sC_{dg})(g_m + sC_{gs})}{G_g + s(C_{dg} + C_{gs})} \quad , \quad y_o'(0) = g_m$$

$$s_{01} = -\frac{G_g}{C_{dg}} \quad , \quad s_{02} = -\frac{g_m}{C_{gs}} \quad , \quad s_p = -\frac{G_g}{(C_{dg} + C_{gs})}$$




(a) (b)

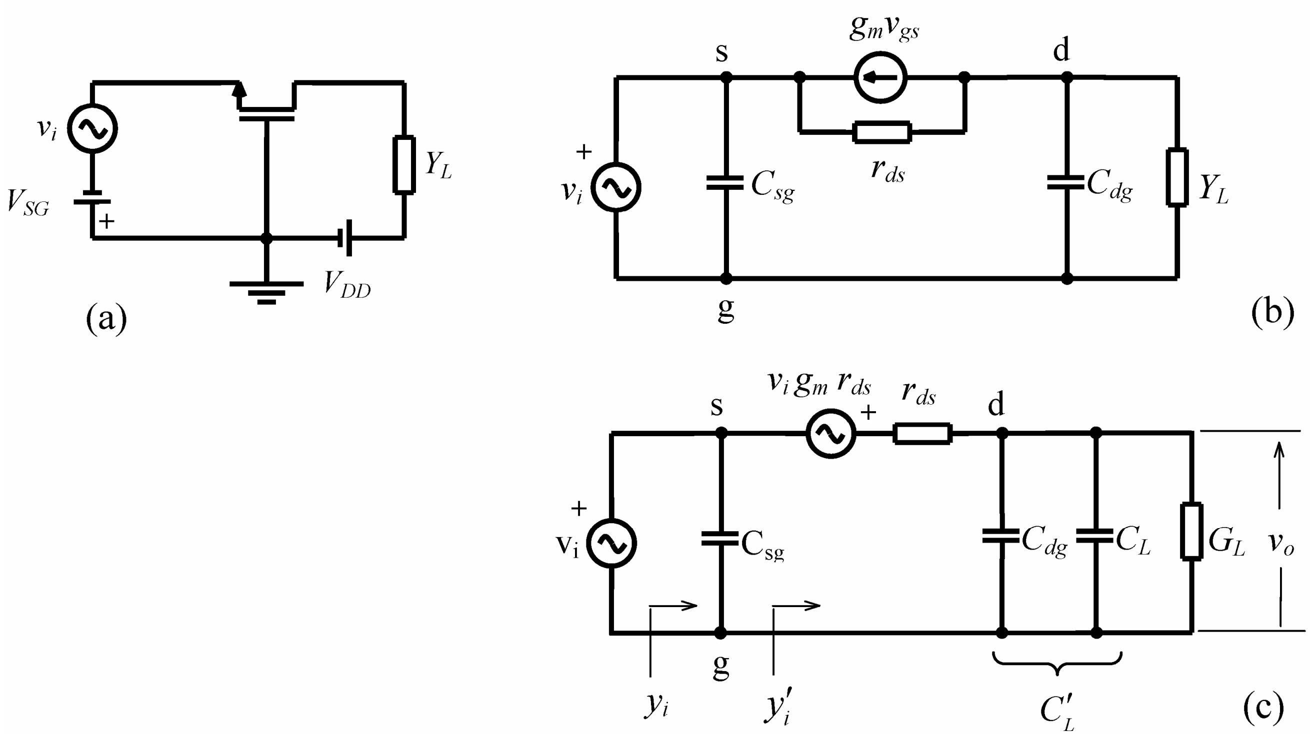



(A) $\omega_{02} \succ \omega_p$

(B) $\omega_{02} = \omega_p$

(a) (b)

# High frequency behavior of a common-gate amplifier



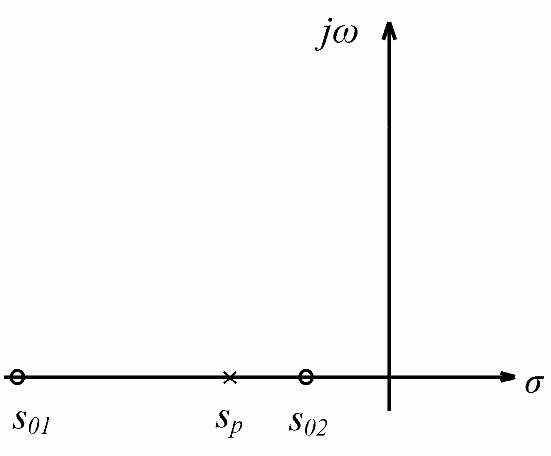

$$A_v = \frac{v_o}{v_i} = \frac{g_m + g_{ds}}{C_L'} \frac{1}{(s - s_p)}, \qquad s_p = -\frac{G_L + g_{ds}}{C_L'}$$

$$A_v(0) = \frac{g_m + g_{ds}}{G_L + g_{ds}} \cong \frac{g_m}{G_L + g_{ds}} \quad , \quad GBW \cong \frac{1}{2\pi} \frac{g_m}{C_L'}$$

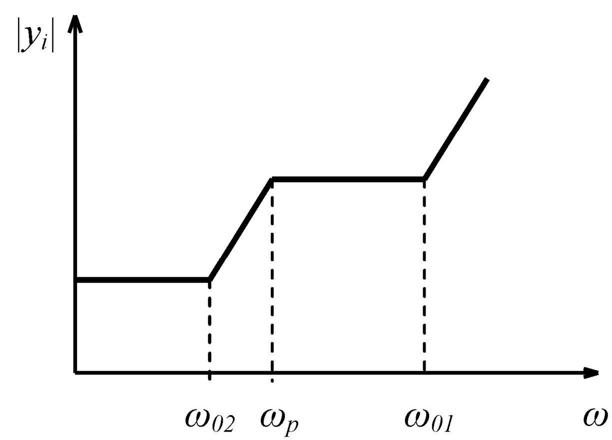
## The input admittance:

$$y_i \cong \frac{s^2 C_{gs} C_L' + s g_m C_L' + G_L g_m}{(g_{ds} + G_L) + s C_L'} = C_{gs} \frac{(s - s_{01})(s - s_{02})}{(s - s_p)}$$

$$s_p = -\frac{G_L + g_{ds}}{C_L'} \ , \ s_{01} \cong -\frac{g_m}{C_{gs}} \ , \ s_{02} \cong -\frac{G_L}{C_L'} \qquad y_i(0) = g_m \frac{1}{1 + \frac{g_{ds}}{G_L}} \cong g_m$$



(a)



(b)



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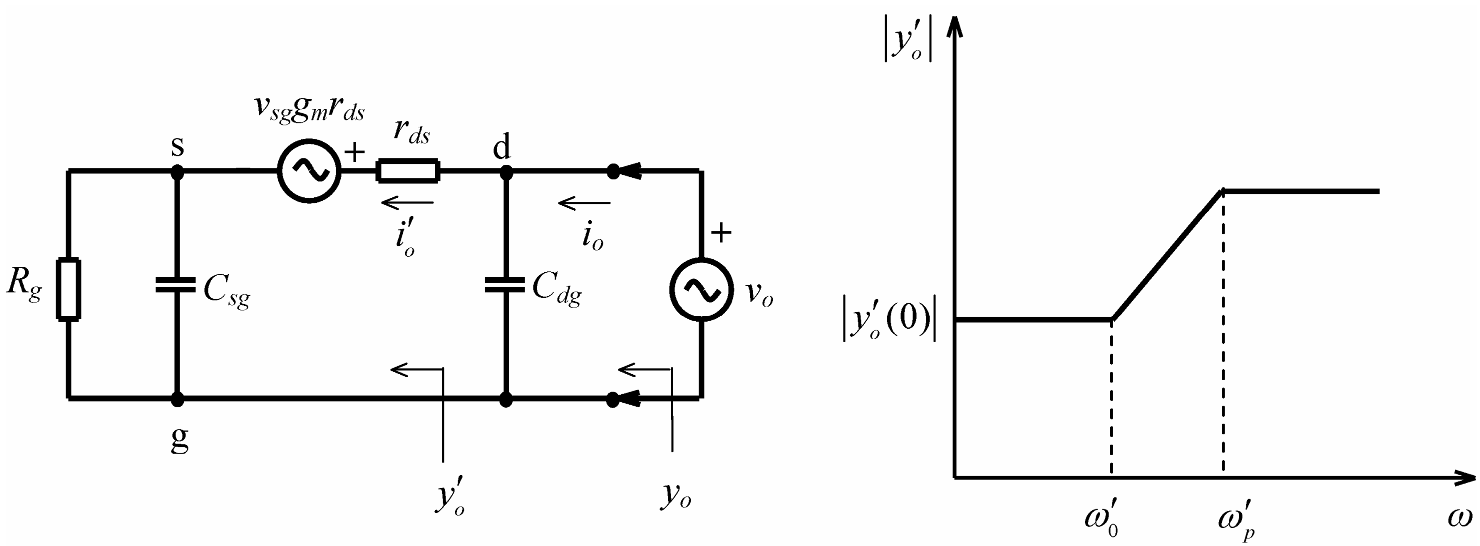
# The output admittance:

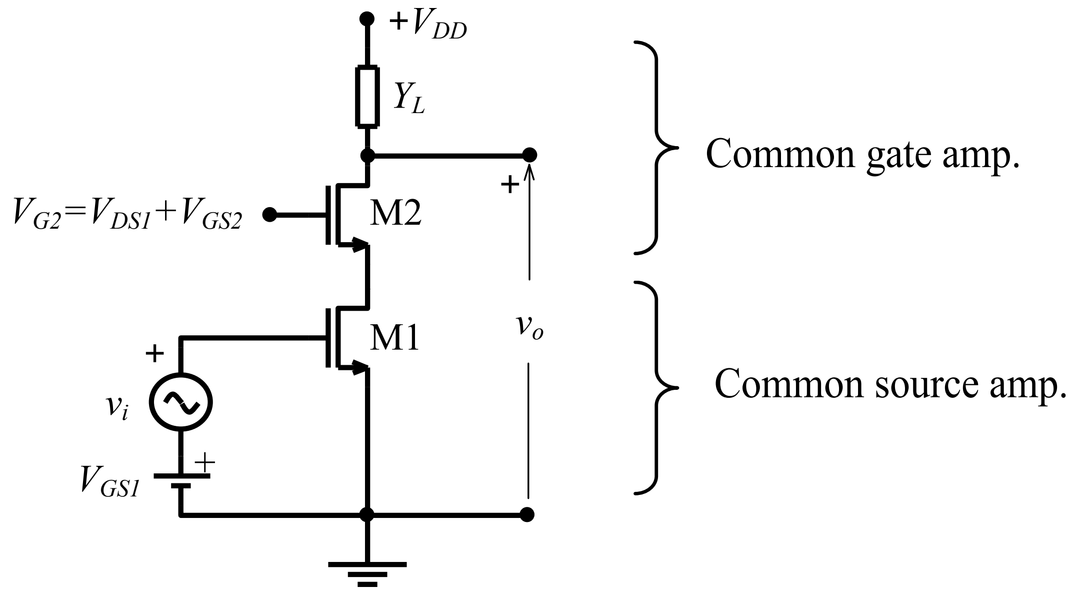


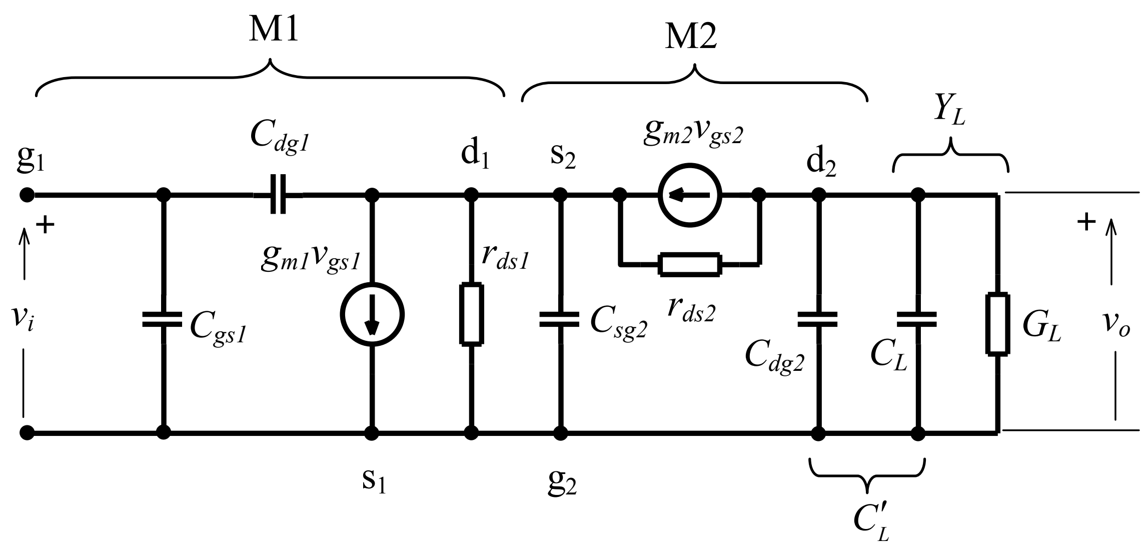

(a) (b)

$$y'_o = \frac{i'_o}{v_o} = \frac{1}{r_{ds}}\frac{s - s'_o}{s - s'_p} \qquad s'_o = -\frac{1}{R_g C_{gs}} \quad , \quad s'_p = -\frac{1 + \dfrac{R_g}{r_{ds}}(1 + g_m r_{ds})}{R_g C_{gs}}$$

$$y'_o(0) = \frac{1}{r_{ds}(1 + g_m R_g) + R_g} \approx \frac{1}{r_{ds}}$$

# Cascode Amplifier



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$$A_v = A_{v1} \cdot A_{v2}$$

$$A_{v1} = -\frac{g_{m1} - sC_{dg1}}{y_{i2}} = -C_{dg1}\frac{(s - s_{011})}{y_{i2}}$$

$$y_{i2} = C_{gs2}\frac{(s - s_{01})(s - s_{02})}{(s - s_p)}$$

$$s_p = -\frac{G_L + g_{ds2}}{C_L'}, \quad s_{01} \cong -\frac{g_{m2}}{C_{gs2}}, \quad s_{02} \cong -\frac{G_L}{C_L'}$$

$$A_{v1} = -\frac{C_{dg1}}{C_{gs2}}\frac{\left(s - s_{011}\right)\left(s - s_p\right)}{\left(s - s_{01}\right)\left(s - s_{02}\right)} \quad \Rightarrow -\frac{C_{dg1}}{C_{gs2}}\frac{(s - s_{01}')(s - s_{02}')}{(s - s_{p1}')(s - s_{p2}')}$$

$$s_{01}' = s_{011} = -\frac{g_{m1}}{C_{dg1}}, \; s_{02}' = s_p = -\frac{G_L + g_{ds2}}{C_L'}, \; s_{p1}' = s_{01} \cong -\frac{g_{m2}}{C_{gs2}}, \; s_{p2}' = s_{02} \cong -\frac{G_L}{C_L'}$$

$$A_{v2} = \frac{v_o}{v_{g2}} = \frac{g_{m2} + g_{ds2}}{C'_L} \frac{1}{(s - s'_{p3})} \cong \frac{g_{m2}}{C'_L} \frac{1}{(s - s'_{p3})} \quad , \quad s'_{p3} = -\frac{G_L + g_{ds2}}{C'_L}$$

$$A_v = A_{v1} \cdot A_{v2} = -\frac{C_{dg1}}{C_{gs2}} \frac{g_{m2}}{C'_L} \frac{(s - s'_{01})(s - s'_{02})}{(s - s'_{p1})(s - s'_{p2})(s - s'_{p3})}$$

Note that; $s'_{02} = s'_{p3} \quad \Rightarrow \quad A_v = -\frac{C_{dg1}}{C_{gs2}} \frac{g_{m2}}{C'_L} \frac{(s - s'_{01})}{(s - s'_{p1})(s - s'_{p2})}$

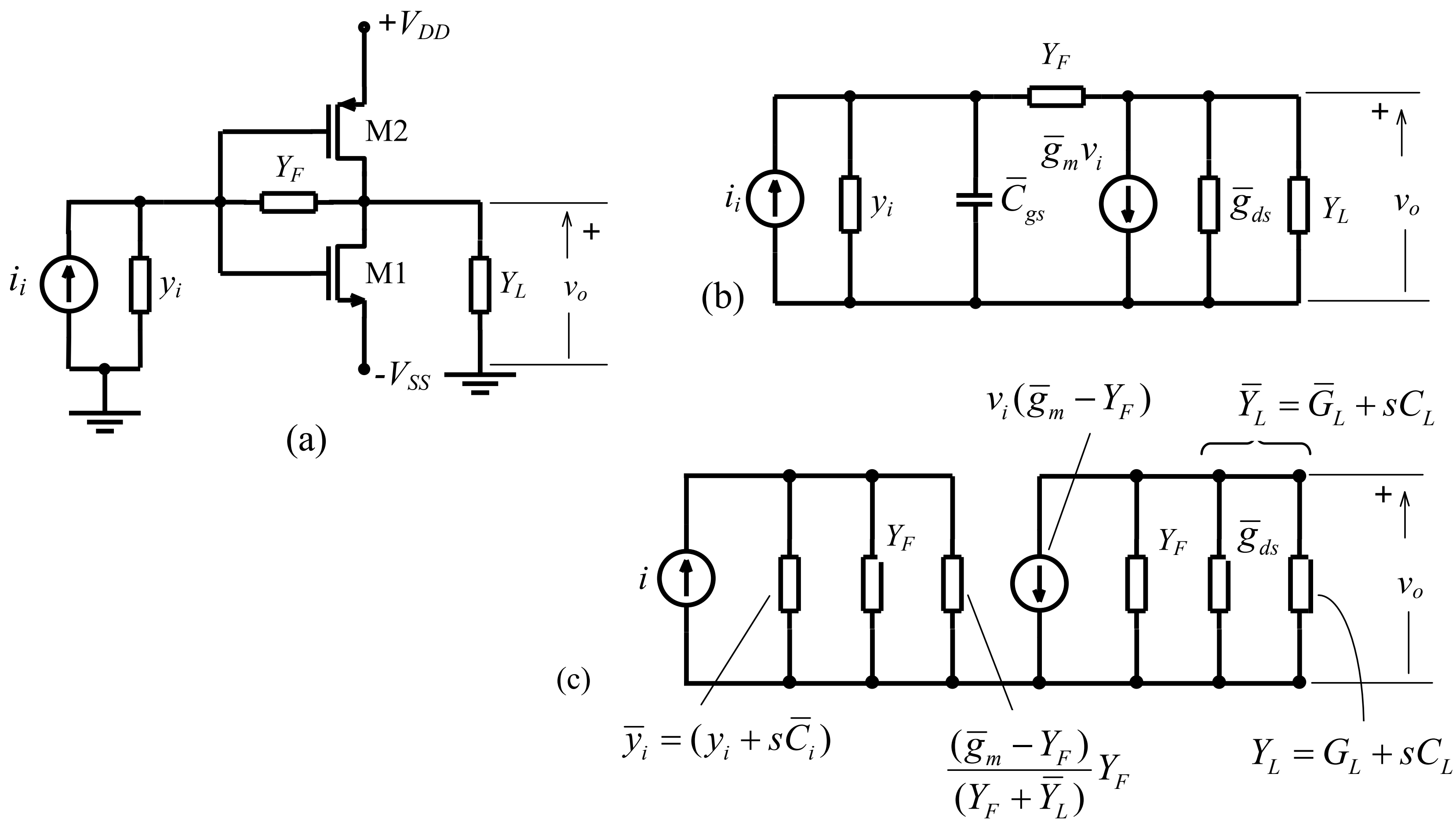
$$A_v(0) = -\frac{C_{dg1}}{C_{gs2}} \frac{g_{m2}}{C'_L} \frac{(-s'_{01})}{(-s'_{p1})(-s'_{p2})} = -\frac{g_{m1}}{G_L}$$

The frequency characteristic and the -3 dB frequency depend on the relative positions of $s'_{p1}$ and $s'_{p2}$. For $|s'_{p1}| \gg |s'_{p2}|$; $f_{3dB} \cong \frac{G_L}{2\pi C'_L}$, $GBW \cong \frac{g_{m1}}{2\pi C'_L}$

The voltage gain, the -3dB frequency and the GBW product of a cascode amplifier is approx. equal to that of M1 if it were loaded with the same load!

Advantage: Lower Miller effect $\rightarrow$ smaller input admittance!

# The CMOS Inverter as a Transimpedance Amplifier




Main advantage: Rail-to-rail output voltage swing

$$Z_m = \frac{v_o}{i_i} = -\frac{(\bar{g}_m - Y_F)}{Y_F(\bar{g}_m + \bar{y}_i) + \bar{Y}_L(\bar{y}_i + Y_F)}$$

$$\bar{g}_m = \left(g_{m1} + g_{m2}\right),\ \bar{y}_i = \left(y_{i1} + y_{i2}\right)\ ,\ \bar{g}_{ds} = g_{ds1} + g_{ds2}$$

$$Z_m \cong \frac{sC_F - (\bar{g}_m - G_F)}{s^2 C_o(C_i + C_F) + s[\bar{g}_{ds}(C_F + C_i) + G_F(C_i + C_o) + \bar{g}_m C_F)] + G_F(\bar{g}_{ds} + \bar{g}_m)}$$

$$Z_m(0) = -\frac{(\bar{g}_m - G_F)}{G_F(\bar{g}_{ds} + \bar{g}_m)}$$

$$\Rightarrow \quad Z_m(0) \cong -(1/G_F) \quad \text{for} \quad \bar{g}_{ds}, G_F \ll \bar{g}_m$$

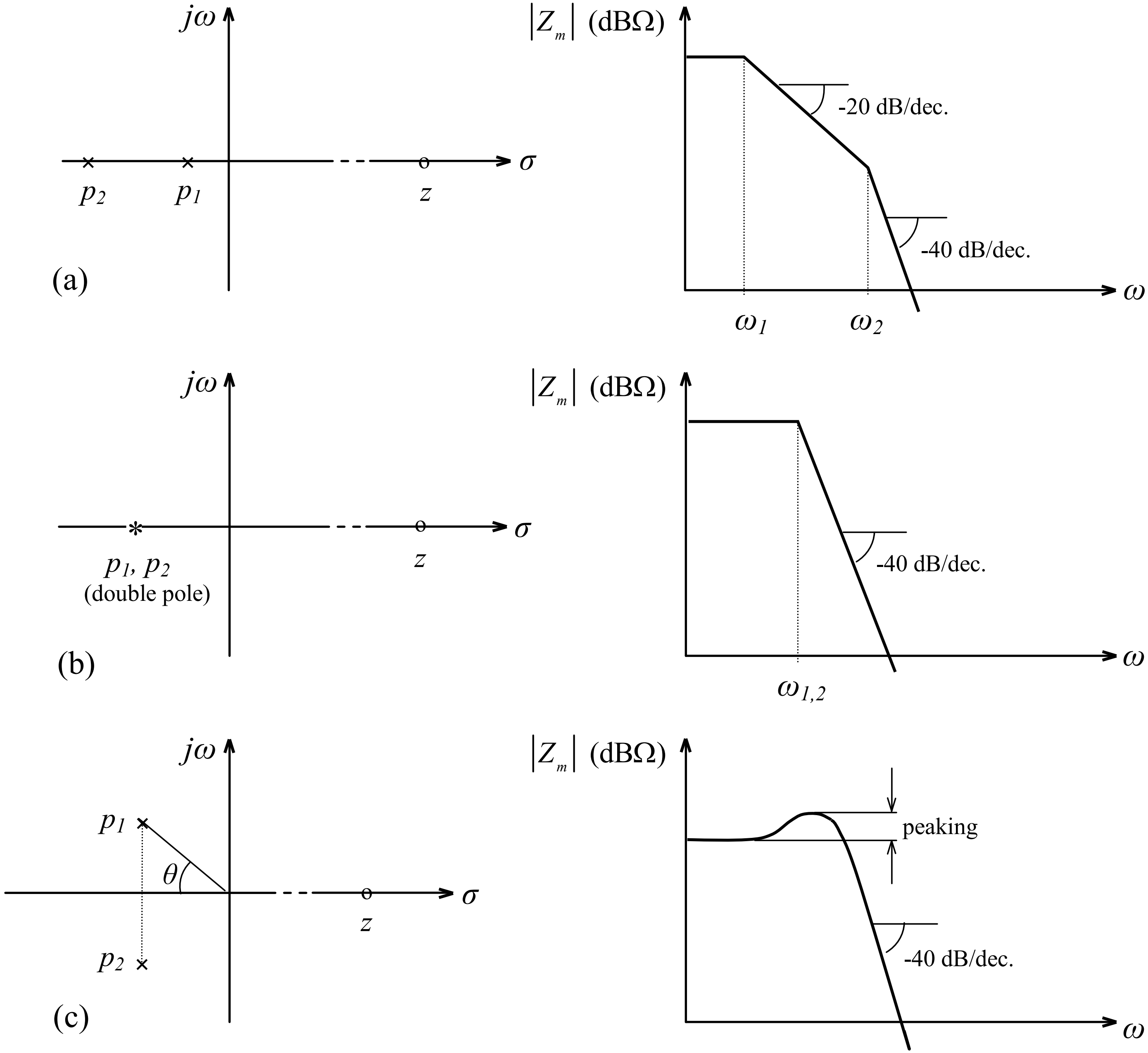
$$Z_m \cong \frac{sD+E}{s^2A+sB+C} \qquad \Rightarrow \quad Z_m = \frac{(s-s_0)}{(s-s_{p1})(s-s_{p2})}$$

$$s_z = -\frac{E}{D} = \frac{(\overline{g}_m - G_F)}{C_F} \qquad s_{p1,p2} = \frac{B}{2A}\left(-1 \mp \sqrt{1-\frac{4AC}{B^2}}\right)$$

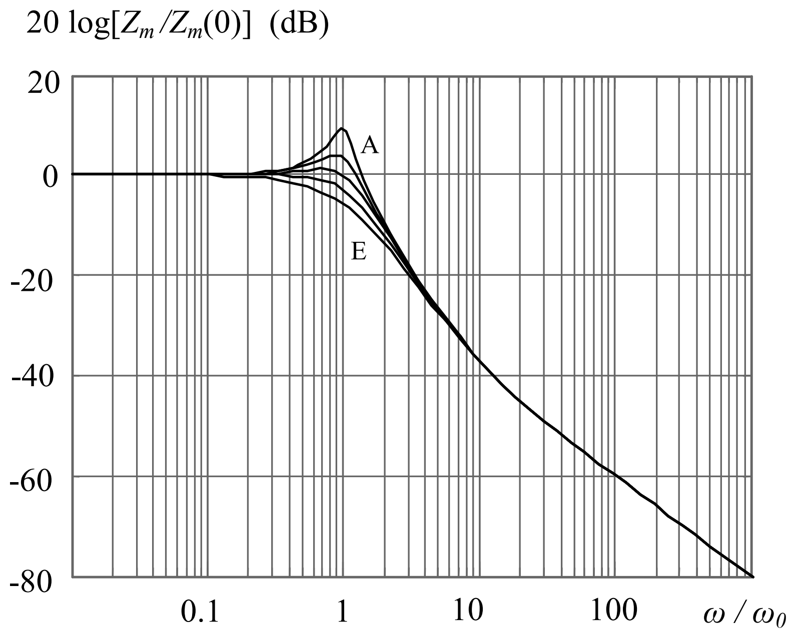
a) two separate negative-real poles for $\frac{4AC}{B^2} \prec 1$

b) two equal negative-real poles for $\frac{4AC}{B^2} = 1$

c) one complex-conjugate pair for $\frac{4AC}{B^2} \succ 1$



jω
σ
p2
p1
z
(a)
|Zm| (dBΩ)
-20 dB/dec.
-40 dB/dec.
ω
ω1
ω2
jω
σ
p1, p2
(double pole)
z
(b)
|Zm| (dBΩ)
-40 dB/dec.
ω
ω1,2
jω
p1
θ
σ
z
p2
(c)
|Zm| (dBΩ)
peaking
-40 dB/dec.
ω




$$\omega_0^2 = \frac{C}{A}$$

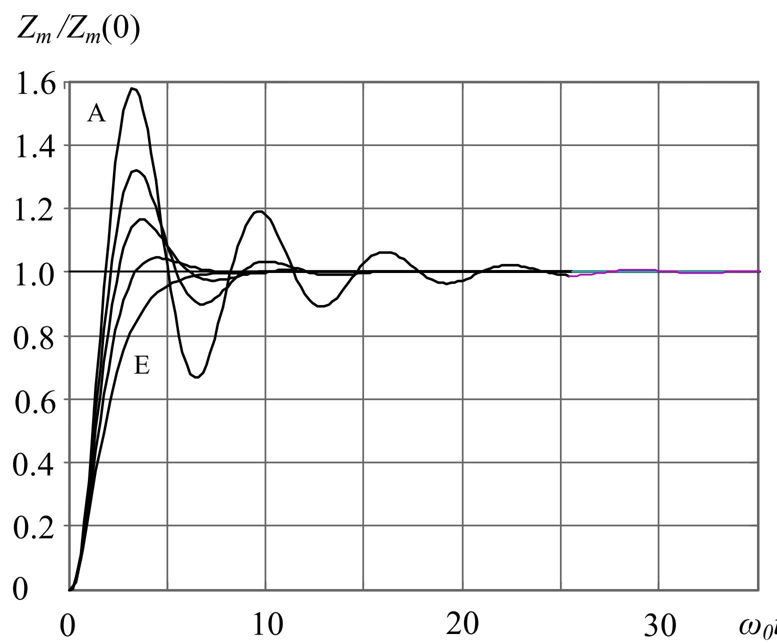
Curve A: $\theta = 80°$

Curve B: $\theta = 70°$

Curve C: $\theta = 60°$

Curve D: $\theta = 45°$

Curve E: $\theta = 0$





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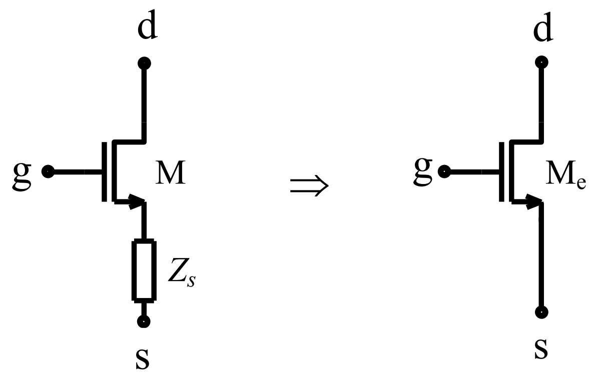
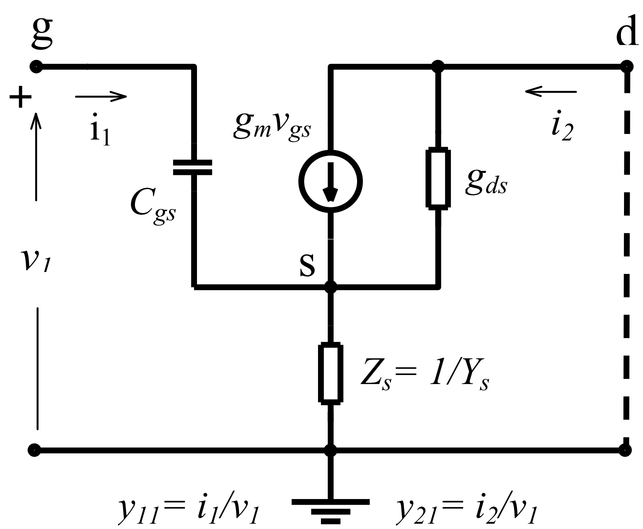
# Source degeneration at high frequencies




MOS transistor with $Z_S$ Equivalent transistor




$$y_{11} \cong \frac{sC_{gs}}{1 + (g_m + sC_{gs})Z_s}$$

$$y_{21} \cong g_m \frac{1}{1 + g_m Z_s}$$



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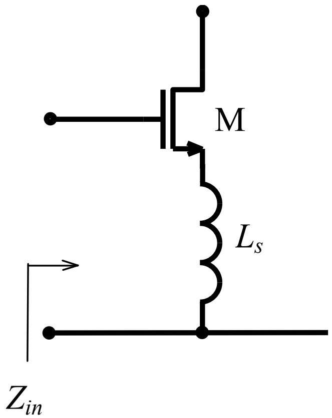
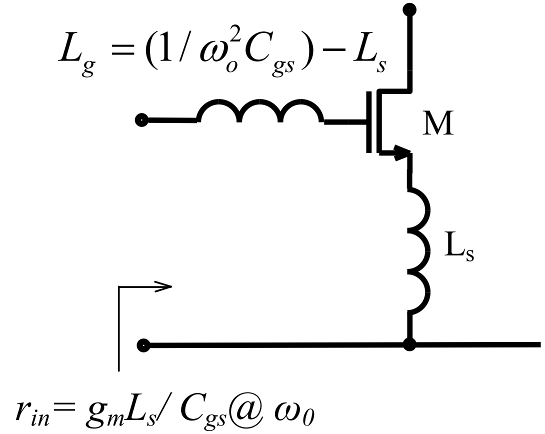
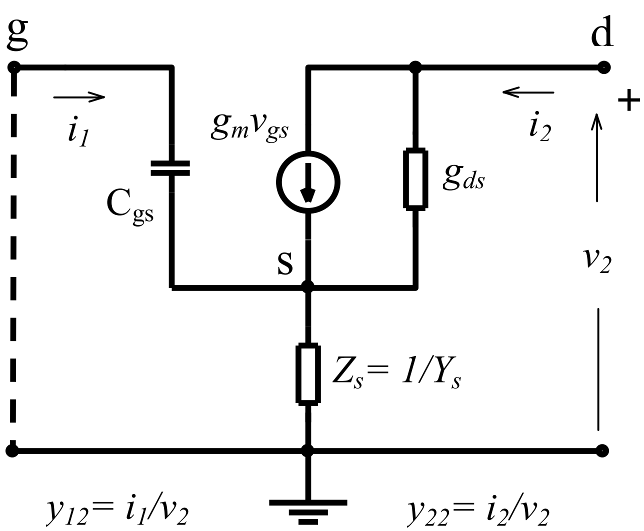
# Special case: $Z_S = sL$




$$y_{in} \cong y_{11} \cong \frac{sC_{gs}}{1+(g_m + sC_{gs})sL_s}$$

$$z_{in} = \frac{1}{y_{in}} = \frac{1}{sC_{gs}} + \frac{g_m L_s}{C_{gs}} + sL_s$$



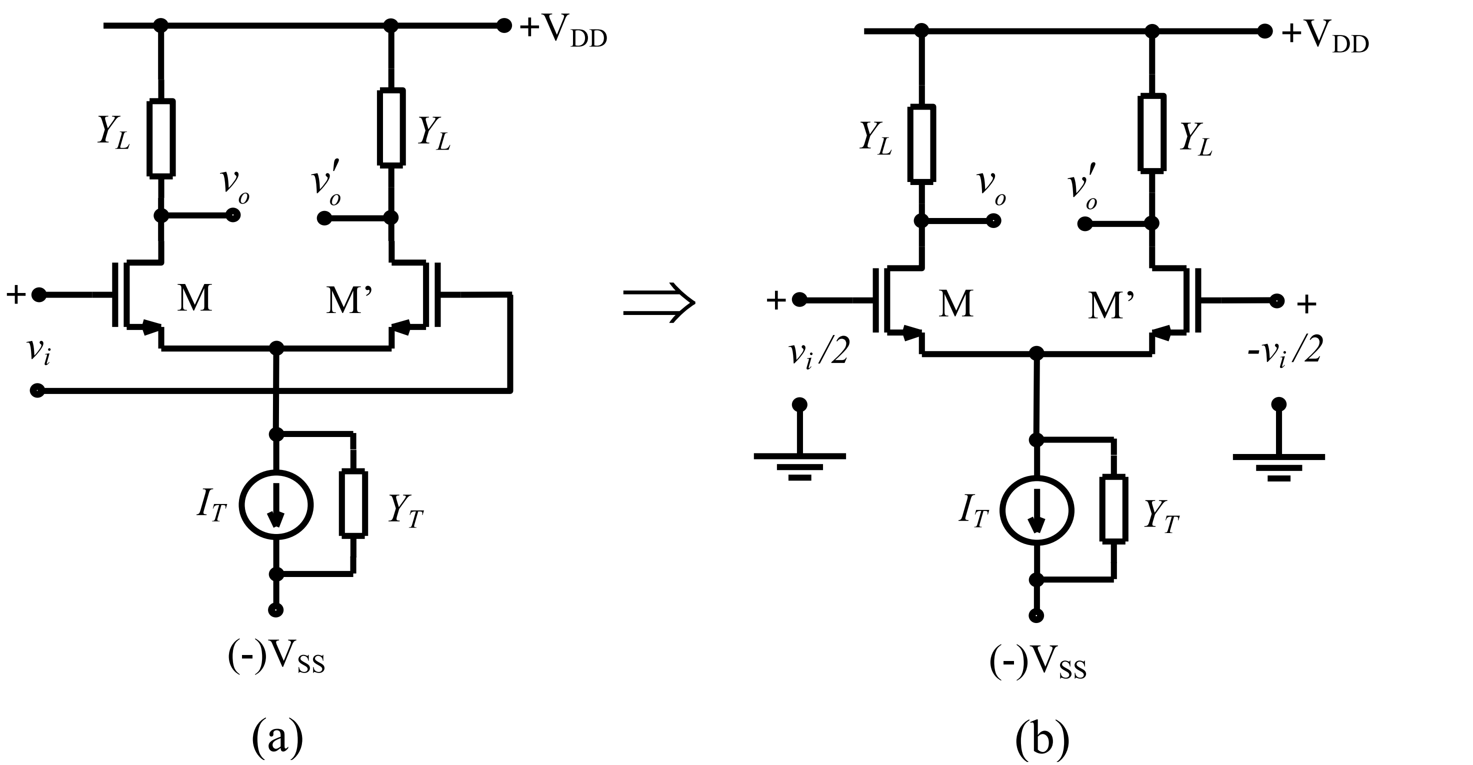




$$y_{12} \cong -\frac{sC_{gs} g_{ds}}{sC_{gs} + Y_s + g_m}$$

$$y_{22} \cong \frac{g_{ds}}{1 + \dfrac{g_m}{(sC_{gs} + Y_s)}}$$

# High frequency behavior of differential amplifiers

a) R- C loaded long-tailed pair:



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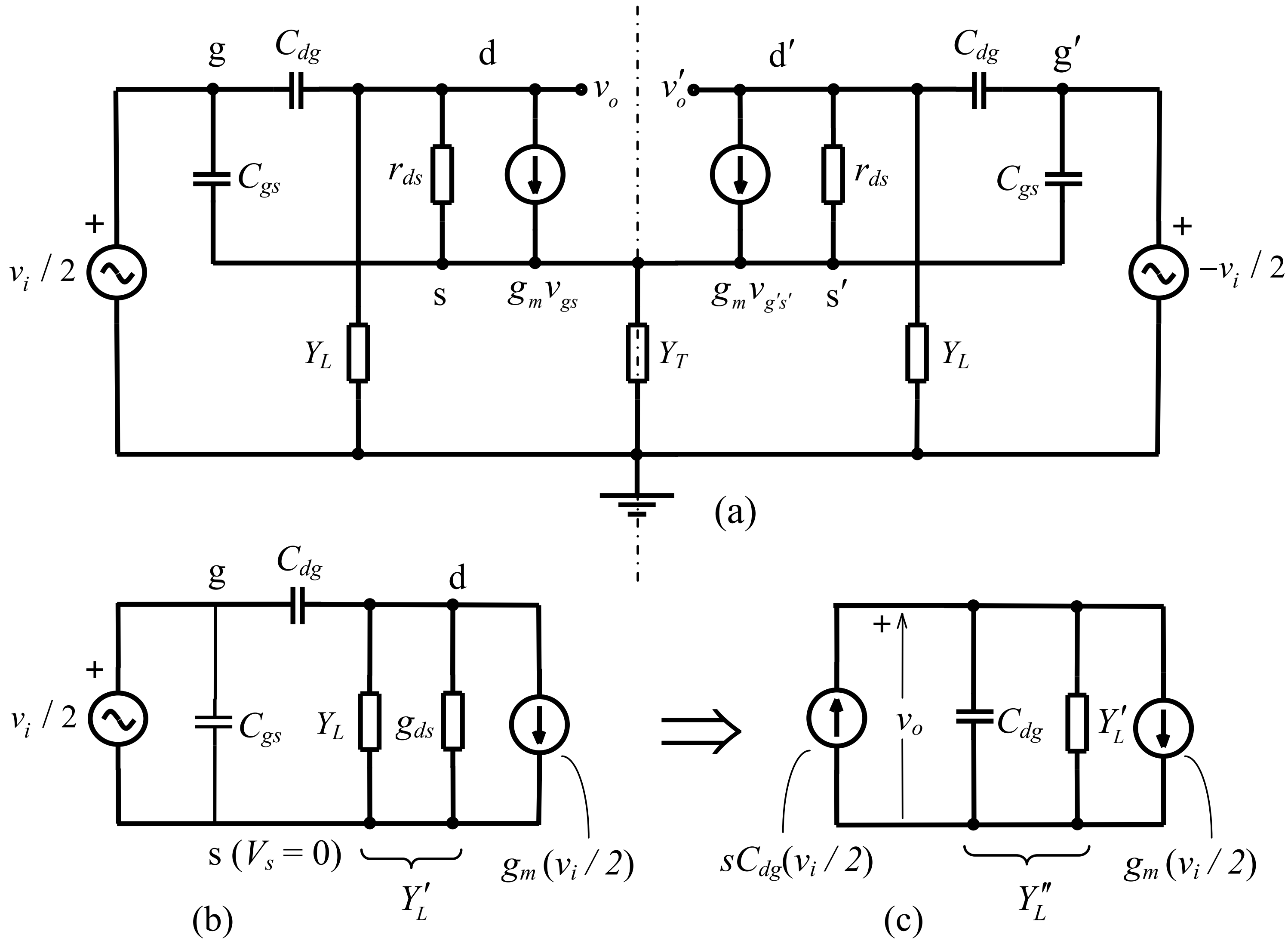

g
$C_{dg}$
d
$v_o$
$v'_o$
d′
$C_{dg}$
g′
$C_{gs}$
$r_{ds}$
$r_{ds}$
$C_{gs}$
+
$v_i / 2$
s
$g_m v_{gs}$
$g_m v_{g's'}$
s′
+
$-v_i / 2$
$Y_L$
$Y_T$
$Y_L$
(a)
g
$C_{dg}$
d
+
$v_i / 2$
$C_{gs}$
$Y_L$
$g_{ds}$
s ($V_s = 0$)
$Y'_L$
$g_m (v_i / 2)$
(b)
+
$v_o$
$C_{dg}$
$Y'_L$
$sC_{dg}(v_i / 2)$
$Y''_L$
$g_m (v_i / 2)$
(c)

Voltage gain from gate-to-drain of M1:

$$v_o = \frac{v_i}{2}\left(sC_{dg} - g_m\right)\frac{1}{Y''}$$

$$A_v = \frac{v_o}{(v_i/2)} = \left(sC_{dg} - g_m\right)\frac{1}{Y''}$$

For an R-C load; $Y_L = G_L + sC_L$

$$Y_L' = (G_L + sC_L) + g_{ds}$$

$$Y_L'' = Y_L' + sC_{dg} = (G_L + g_{ds}) + s(C_L + C_{dg})$$

$$A_v = \frac{\left(sC_{dg} - g_m\right)}{(G_L + g_{ds}) + s(C_L + C_{dg})} = \frac{C_{dg}}{(C_L + C_{dg})}\frac{(s - s_0)}{(s - s_p)}$$

$$s_p = -\frac{(G_L + g_{ds})}{(C_L + C_{dg})} \quad , \quad s_0 = +\frac{g_m}{C_{dg}}$$

Similarly; gain from gate-to-drain of M2:

$$A'_v = \frac{v'_o}{-(v_i/2)} = \frac{C_{dg}}{(C_L + C_{dg})} \frac{(s - s_0)}{(s - s_p)}$$

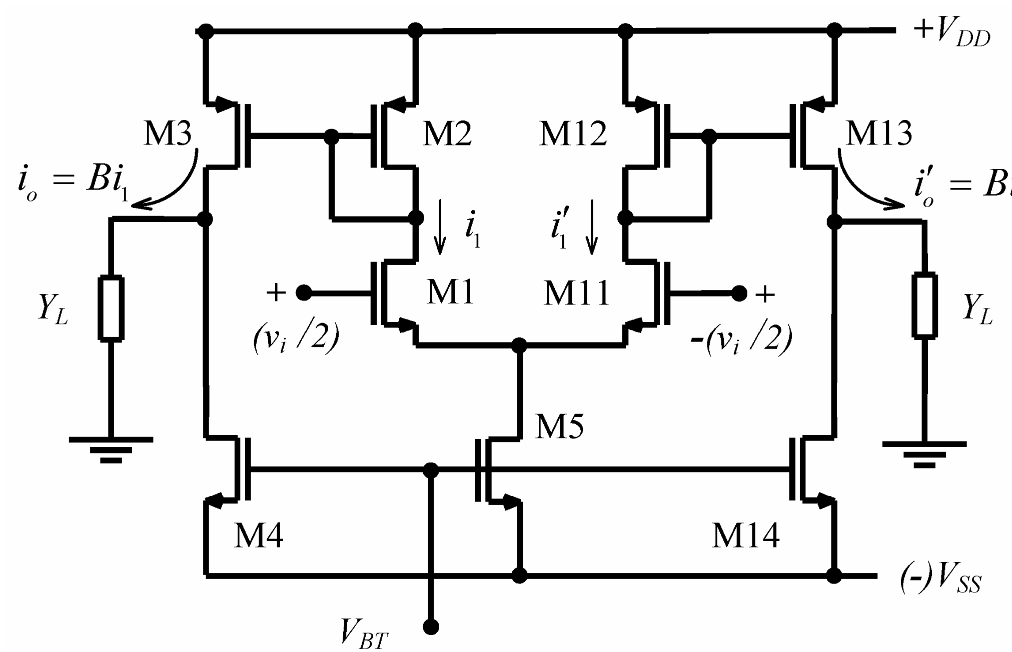
The differential input to differential output voltage gain:

$$A_{vdd} = \frac{(v_o - v'_o)}{(v_i/2) - (-v_i/2)} = \frac{v_{od}}{v_i} = \frac{C_{dg}}{(C_L + C_{dg})} \frac{(s - s_0)}{(s - s_p)}$$

–3 dB frequency corresponding to the pole of the gain:

$$f_{3dB} = \frac{1}{2\pi} \frac{(G_L + g_{ds})}{(C_L + C_{dg})}$$

# b) Fully differential, current mirror loaded amplifier (Differential OTA)




$$i_1 = \frac{v_1}{2} g_{m1} \qquad i_3 = i_1 B$$

The current transfer ratio of a current mirror (see Appendix D): $B = B_o \dfrac{s_p}{s_o} \dfrac{(s - s_o)}{(s - s_p)}$

$$s_0 = +\frac{g_{m3}}{C_{dg3}} \;, \quad s_p = -\frac{g_{m2}}{(C_{gs2} + C_{gs3}) + C_{xp}} \;, \;\; C_{gs3} = B_0 \cdot C_{gs2}$$

$$i_3 = i_1 B = i_1 B_0 \frac{s_p}{s_0} \frac{(s - s_0)}{(s - s_p)}$$

The trans-admittance from $v_i$ to $i_o$ :

$$y_m = \frac{i_o}{v_1} = \frac{1}{2} g_{m1} B_0 \frac{s_p}{s_0} \frac{(s - s_0)}{(s - s_p)}$$

Since $|s_0| \gg |s_p|$; $\quad y_m \cong \frac{1}{2} g_{m1} B_0 s_p \frac{1}{(s - s_p)} \cong g_m s_p \frac{1}{(s - s_p)}$

The -3 dB frequency:

$$f_{3dB} = \frac{1}{2\pi} \frac{g_{m2}}{(1 + B_0) C_{gs2} + C_{xp}} = \frac{1}{2\pi} \frac{\mu_p C_{ox} (W_2 / L) |V_{GS2} - V_{TP}|}{\frac{2}{3}(1 + B_0) W_2 L C_{ox} + C_{xp}}$$

If $\quad C_{xp} \ll (1 + B_0) C_{gs2}$ ; $\quad f_{3dB} \approx \frac{1}{2\pi} \frac{\mu_p}{(1 + B_0) L^2} (|V_{GS2} - V_{TP}|)$

The output voltages on equal loads $(Y_L)$:

$$v_o = \frac{i_o}{Y_L} = v_i \frac{y_m}{Y_L} \ , \quad v'_o = \frac{i'_o}{Y_L} = \frac{-i_o}{Y_L} = -v_1 \frac{y_m}{Y_L}$$

The differential output voltage:

$$v_{odd} = (v_o - v'_o) = 2v_i \frac{y_m}{Y_L} = v_i \frac{1}{Y_L} g_{m1} B_o s_p \frac{1}{(s - s_p)}$$

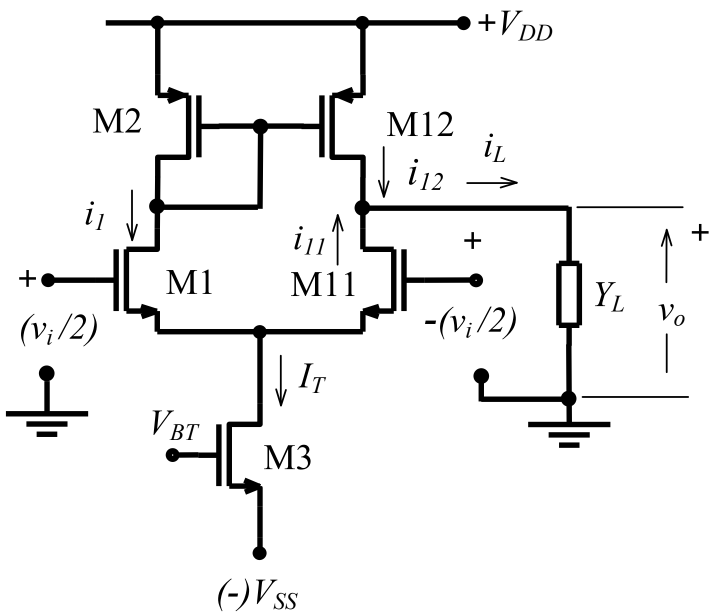
The differential voltage gain:

$$A_{vdd} = \frac{1}{Y_L} g_{m1} B_o s_p \frac{1}{(s - s_p)}$$

For an $R$-$C$ load $(Y_L = G_L + sC_L)$:

$$A_{vdd} = \frac{1}{C_L} g_{m1} B_o s_p \frac{1}{(s - s_p)(s - s_{pL})} \ , \quad s_{pL} = -\frac{G_L}{C_L}$$

c) Single-ended output long tailed pair:




$$i_L = i_{12} + i_{11} = i_{12} + i_1 \ , \ i_{12} = B \cdot i_1$$

$$i_L = i_1(B+1) = \frac{1}{2} v_i g_{m1} (B+1)$$

$$v_o = \frac{i_L}{Y_L} \qquad \Rightarrow \quad A_v = \frac{v_o}{v_i} = \frac{1}{2} \frac{g_{m1}}{Y_L} (B+1)$$



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The current transfer ratio of a current mirror:

$$B = B_o \frac{s_p}{s_o} \frac{(s - s_o)}{(s - s_p)}$$

Since $\mathrm{B}_0 = 1$ for this case;

$$B = \frac{s_p}{s_o} \frac{(s - s_o)}{(s - s_p)}$$

$$s_o = + \frac{g_{m12}}{C_{dg12}} = \frac{g_{m2}}{C_{dg2}} \qquad s_p = - \frac{g_{m1}}{(C_{gs1} + C_{gs12} + C_{pT})} = \frac{g_{m1}}{(2C_{gs1} + C_{pT})}$$

$$(B+1) = 1 + \frac{s_p}{s_o} \frac{(s - s_o)}{(s - s_p)} = \frac{s_o(s - s_p) + s_p(s - s_o)}{s_o(s - s_p)}$$

Since $s_0 \gg \left| s_p \right|$ ; $\quad (B+1) \cong \dfrac{(s - 2s_p)}{(s - s_p)}$



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$$A_v \cong \frac{1}{2}\frac{g_{m1}}{Y_L}\frac{(s-2s_p)}{(s-s_p)}$$

For a capacitive load $(Y_L = g_L + sC_L)$;

$$A_v \cong \frac{1}{2}\frac{g_{m1}}{C_L}\frac{(s-2s_p)}{(s-s_p)(s-s_L)} \ , \quad s_L = -\frac{g_L}{C_L}$$

$$A_v(0) \cong \frac{g_{m1}}{g_L}$$



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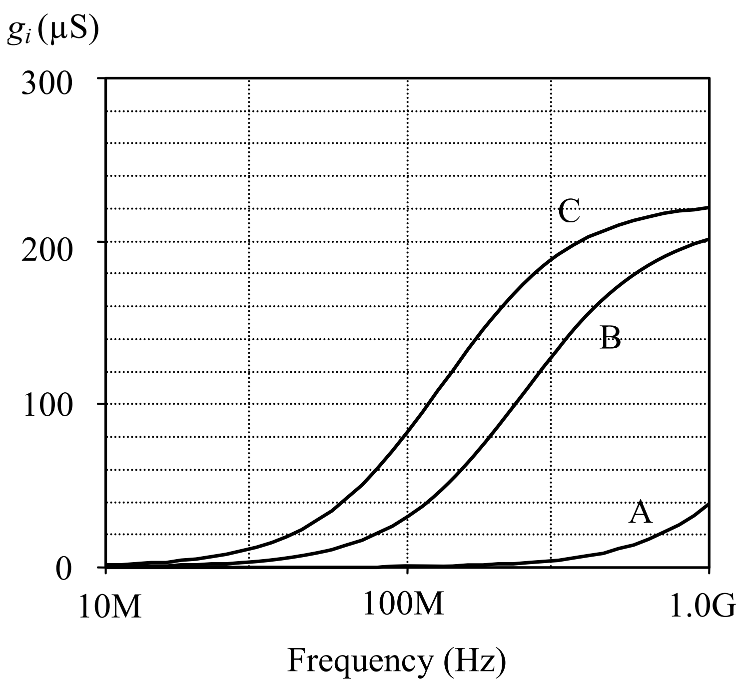
# The input admittances of a long-tailed pair

## a) A differentially driven symmetrical amplifier:

Since the sources are at zero signal potential, both inputs exhibit

equal admittances similar to that of a CS circuit, equal to the sum of :

- The gate source capacitance of M1 or M11,
- Corresponding Miller components with their capacitive and conductive part




A) $R_L = 500$ ohm,

B) $R_L = 5$ k ohm,

C) $R_L = 10$ k ohm





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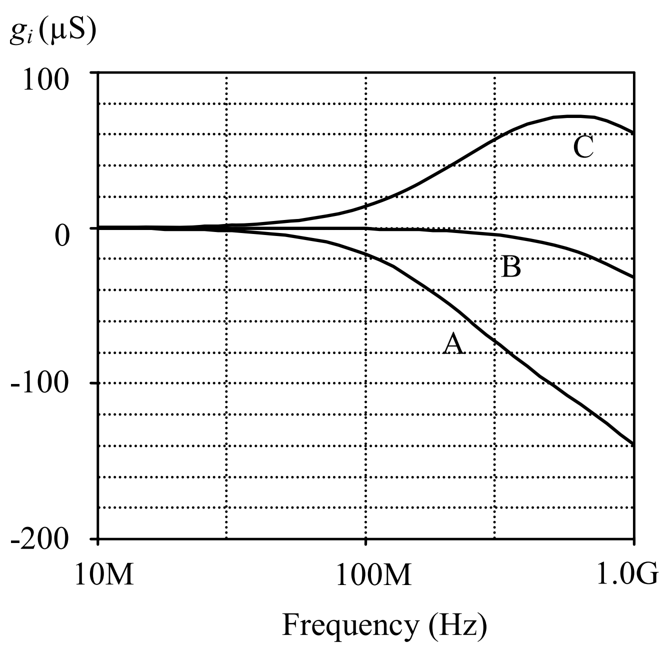
## b) Driven from one input (other input grounded)

- The source of the driven transistor is loaded with the input of the other transistor operating as a CG amplifier and the parasitics.
- Since this load is capacitive, the input of the driven transistor acquires a negative conductance component.
- In addition there is the Miller component.
- The total input admittance is the sum of these components.




A) $R_L = 500$ ohm,
B) $R_L = 5$ k ohm,
C) $R_L = 10$ k ohm



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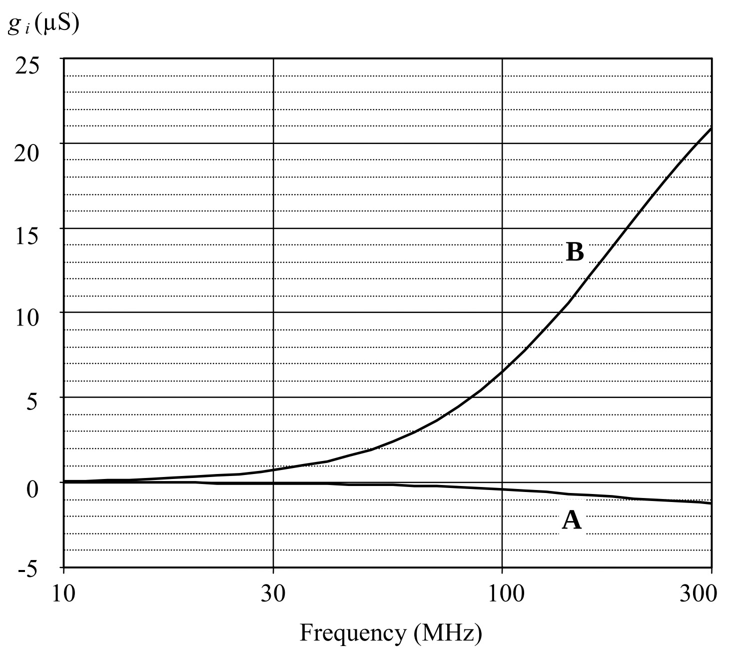
## c) The input admittances of a single-ended output DA

- M1 has a diode connected (low impedance) load.
- M11 has a current source (high impedance) load.
- Therefore gate-to-drain voltage gains, consequently Miller components are not same.
- The input conductance of M1 is smaller than that of M11.




A) Input conductance of M1

B) Input conductance of M11



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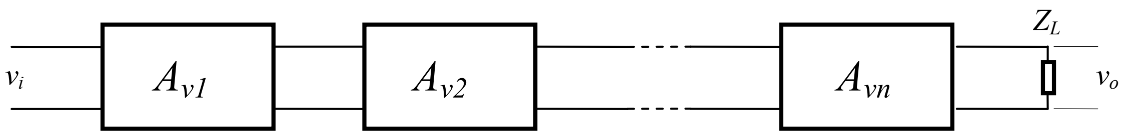
# Gain Enhancement Techniques for HF Amplifiers

a) Additive Amplifiers:

- The conventional way to increase the gain is to cascade the stages:

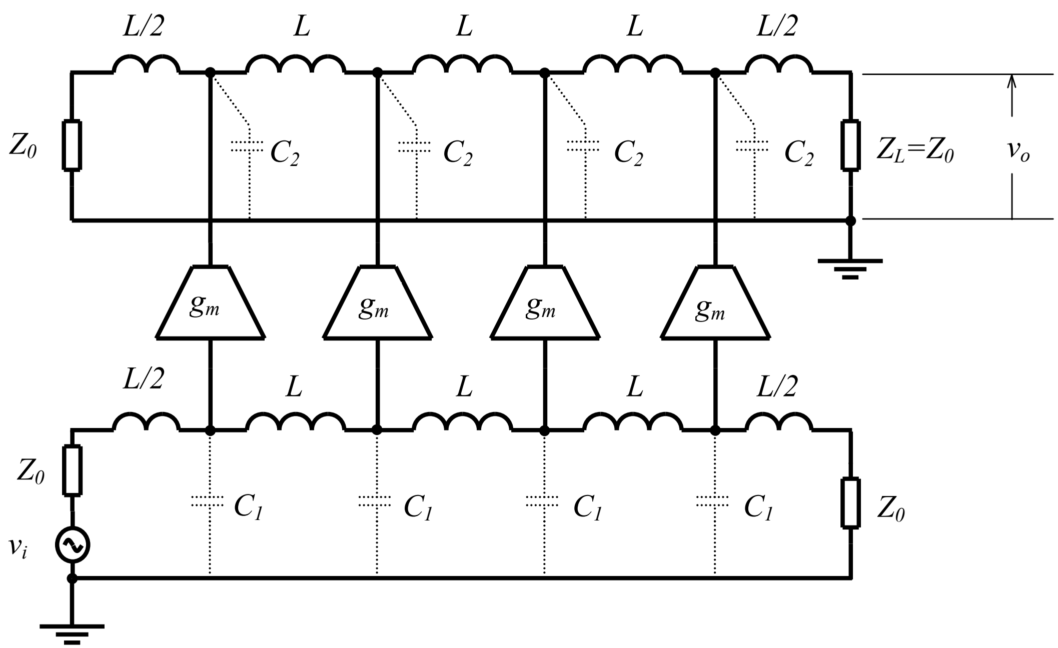



$$A_{vT} = A_{v1} \, . \, A_{v2} \ldots\ldots A_{vn}$$

- For any stage GBW is constant; BW□ , G□
- For $A_v \leq 1$ cascading does not help to increase the gain.

- Solution is to add the gains of stages instead of to multiply.

# Additive (distributed) amplifiers




Properties of an artificial transmission line:

- Characteristic impedance: $Z_0 = \sqrt{L / C}$
- Cut-off frequency: $\omega_0 = 2 / \sqrt{LC}$ , $f_0 = 1 / \pi\sqrt{LC}$
- Signal delay per sector: $\tau = 2 / \omega_0 = \sqrt{LC}$

- Input and output lines have equal delays per sector.
- Input and output lines terminated with their characteristic impedances.

- For each of the transconductors: $i_o(v_i) = g_m v_i$
- Forward components of the output currents: $i_f(v_i) = i_o(v_i)/2$

- The total current flowing over $Z_L$ is $n \times i_f(v_i)$;

$$v_o = n \times \frac{1}{2} g_m v_i Z_o \ , \qquad A_v = \frac{v_o}{v_i} = \frac{1}{2} g_m Z_o n$$

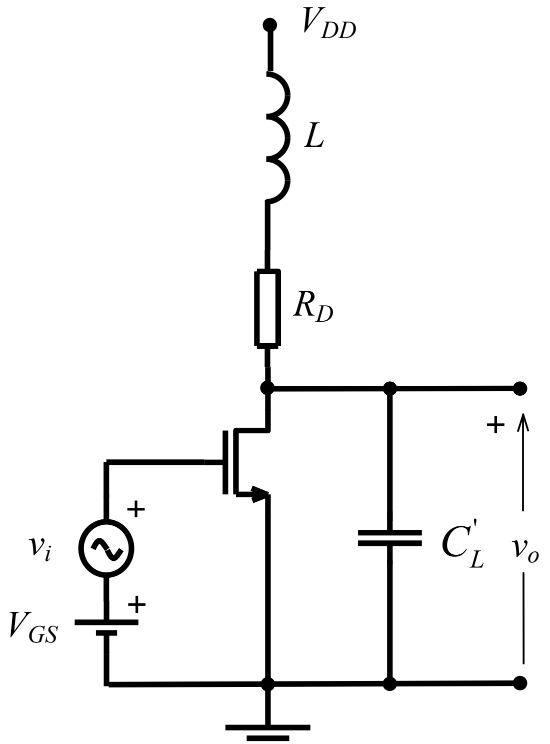
- After reaching $A_v \succ 1$, similar $m$ distributed amplifier stages can be cascaded to increase the gain further:

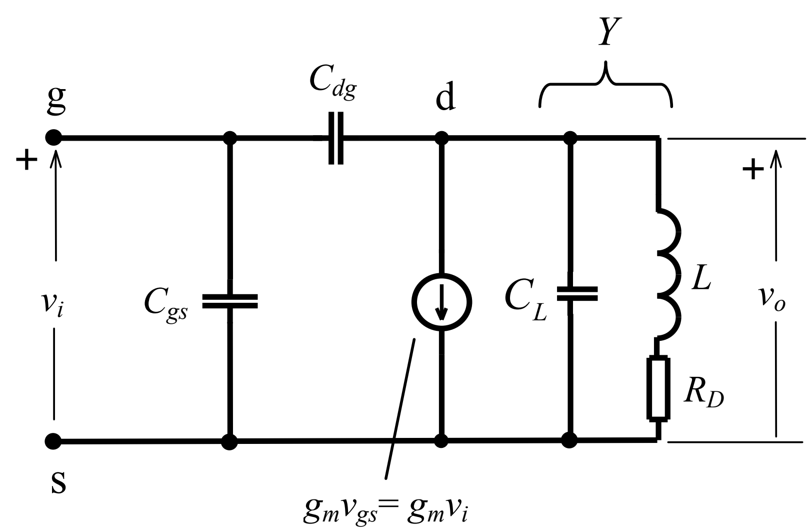
$$A_{vT} = (A_v)^m = \left( \frac{1}{2} g_m Z_o n \right)^m$$

# A Band Widening Technique: Inductive Peaking

- The main factor limiting the BW is the parallel parasitic capacitance to the load.
- To compensate it with an appropriate inductor at the high end of the band helps.

a) Parallel peaking:



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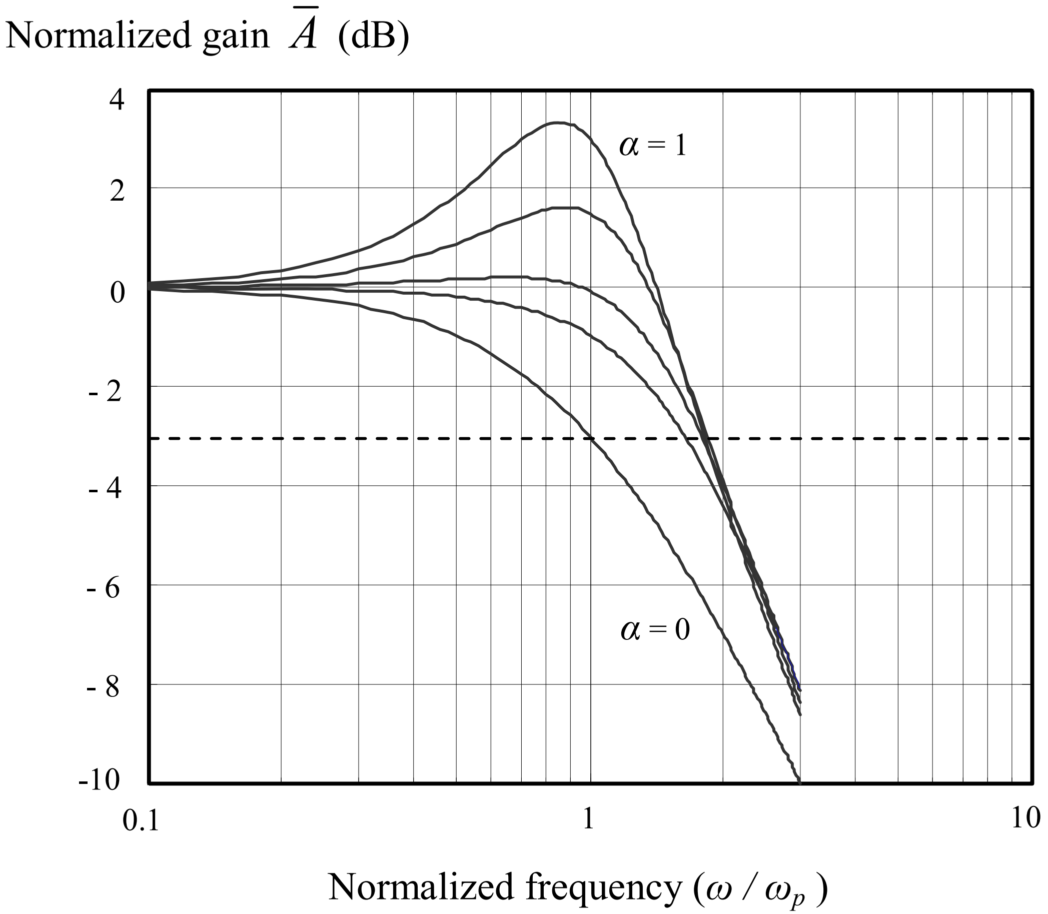
The voltage gain: $A_v = \frac{v_o}{v_i} = -g_m \frac{R_D + sL}{s^2 LC_L + sC_L R_D + 1}$

The normalized gain: $\overline{A} = \frac{|A_v(\omega)|}{|A_v(0)|} = \frac{1 + j\omega \frac{L}{R_D}}{(1-\omega^2 LC_L) + j\omega C_L R_D}$

With $\omega_p = 1/R_D C_L$ , $\omega_o = 1/\sqrt{LC_L}$ and $\alpha = \omega_p / \omega_0$;

$$\overline{A} = \frac{1 + j\left(\omega / \omega_P\right)\alpha^2}{\left[1-\left(\omega / \omega_P\right)^2 \alpha^2\right] + j\left(\omega / \omega_P\right)}$$

$$\alpha = \frac{1}{R_D C_L}\sqrt{LC_L} \quad \rightarrow \quad L = \alpha^2 R_D^2 C_L$$




$\alpha = 0,\ 0.6,\ 0.7,\ 0.85$ and 1

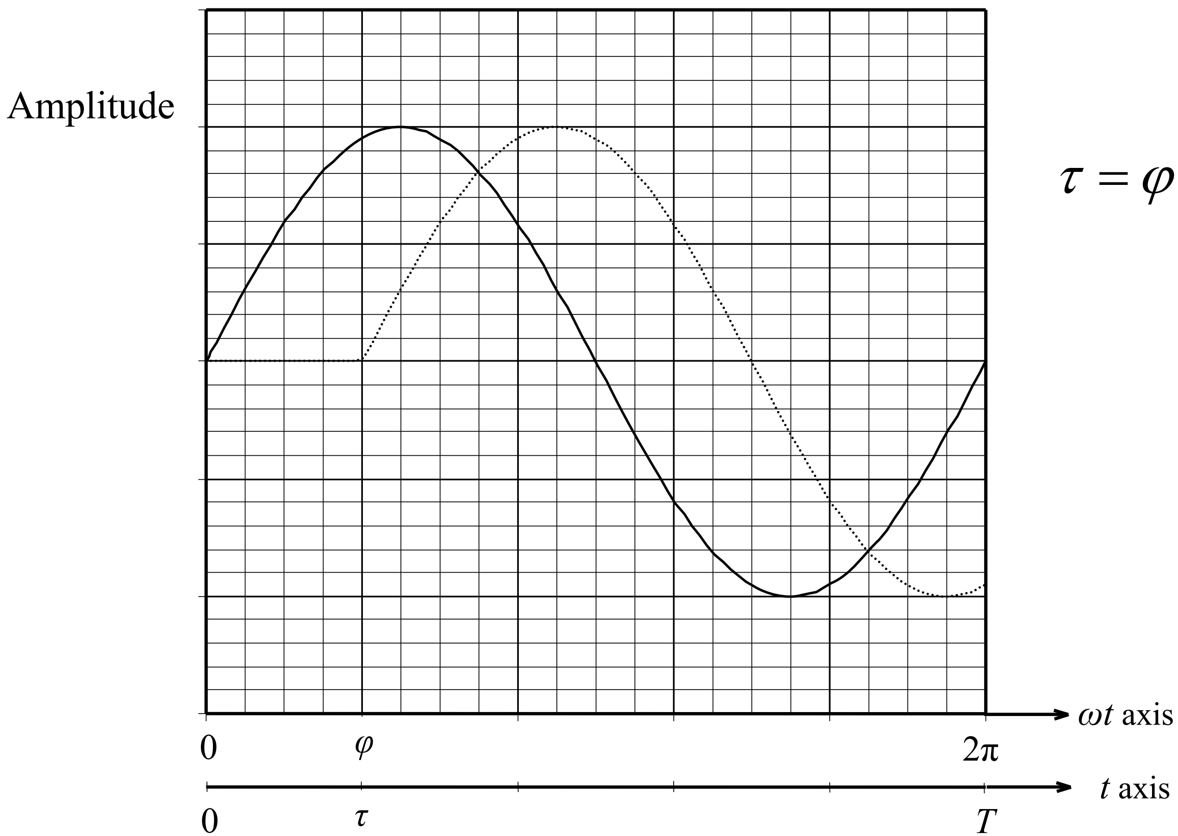
Optimum $\alpha$ value for minimum amplitude distortion: 0.7

For $\alpha = 0.7,$ the -3dB frequency is 87% higher than a non-compensated amplifier

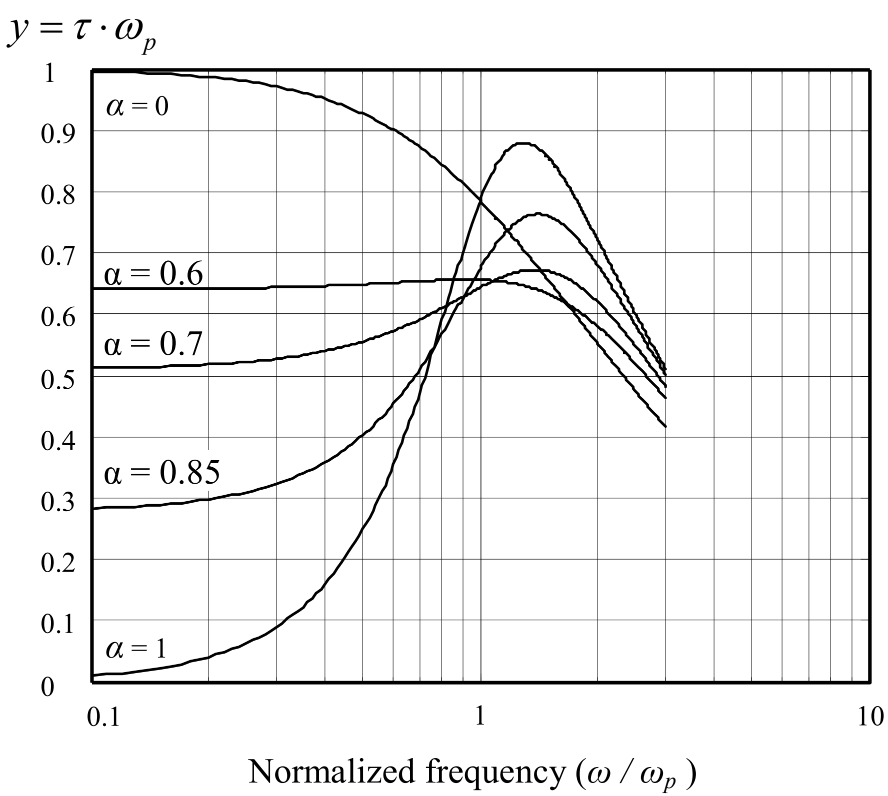
The "delay distortion" is important for complex wave forms!

(From input to output, all Fourier components of the signal must have equal delays)

The relation of the phase shift and the delay of a sinusoidal signal:



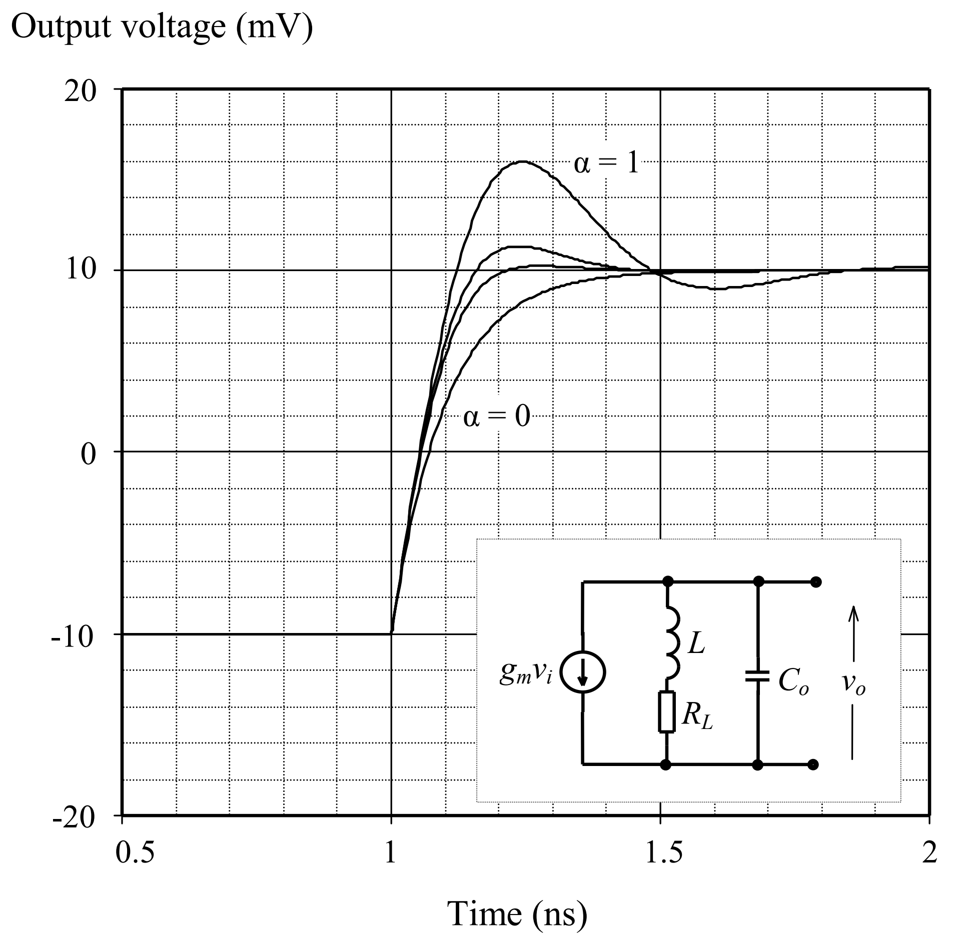

$$\tau = \varphi \frac{T}{2\pi} = \frac{\varphi}{2\pi f} = \frac{\varphi}{\omega}$$




Optimum $\alpha$ value for minimum delay distortion: 0.6

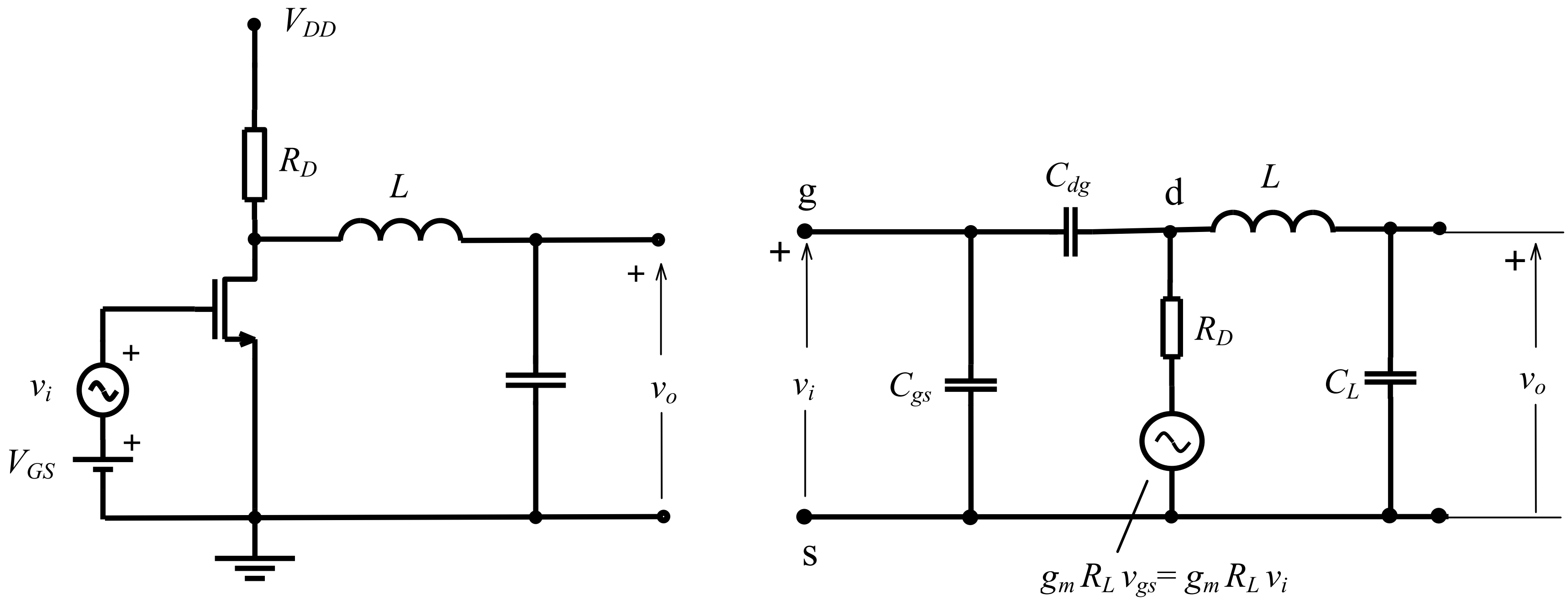
For $\alpha = 0.6,$ the -3dB frequency is 62% higher than a non-compensated amplifier

# Effects of $\alpha$ on the pulse response of an amplifier

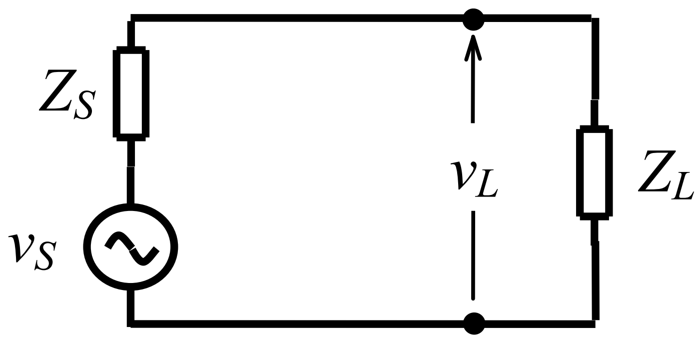



$\alpha = 0,\ 0.6,\ 0.7$ and 1

a) Series peaking:



# Maximum signal transfer from source to load



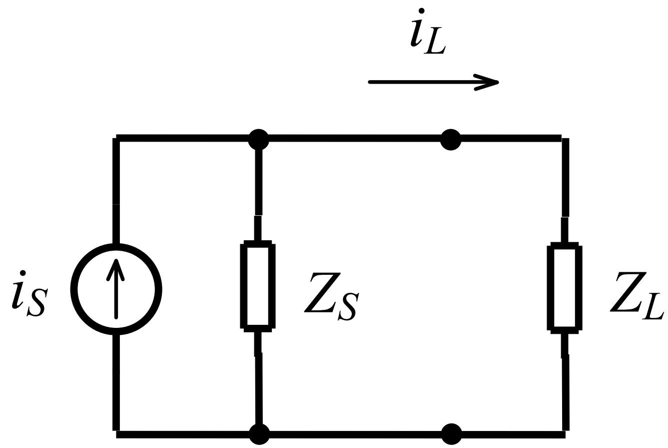

For maximum power transfer: $Z_L = \bar{Z}_S \;;\; Z_L = r_L + jx_L \;,\; Z_S = r_S + jx_S$

$$r_L + jx_L = r_S - jx_S \;,\quad r_L = r_S, \quad x_L = -x_S$$

(Corresponds to the resonance of $x_L$ and $x_S$, can be satisfied only at a certain frequency)

For maximum voltage transfer: $|Z_L| \gg |Z_S|$

(Possible in a wide band)




For maximum current transfer: $|Z_L| \ll |Z_S|$

(Possible in a wide band)

# Cascading strategies for wide-band amplifiers

- For maximum power transfer from the output of the first stage to the input of the following stage:

$$Z_{o1} = r_{o1} + jx_{o1} \ , \quad Z_{i2} = \bar{Z}_{o1} = r_{o1} - jx_{o1}$$

- This means the resonance of the reactive parts; possible only for a certain frequency or in a narrow band.

Not meaningful for wide-band applications.

## For wide-band applications:

The voltage or current from the output of the first stage to the input of the following stage must be efficiently transferred in a wide band.

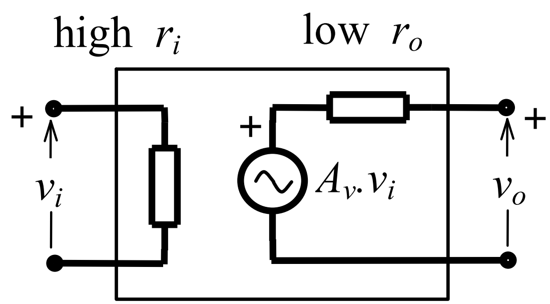
Goal: minimize the adverse effects of the parallel capacitors at the cascading points.

Solution: Minimize the parallel conductance without signal amplitude loss.

- To cascade a low input impedance stage to a high output impedance stage, (To drive the a low input impedance stage from a current source)

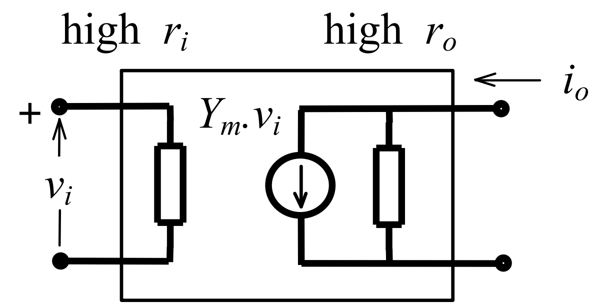
- To cascade a high input impedance stage to a low output impedance stage, (To drive the a high input impedance stage from a voltage source source)

# The four basic amplifier configurations:



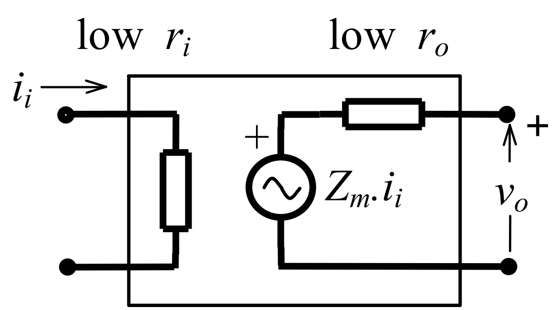

Voltage amplifier

(a)



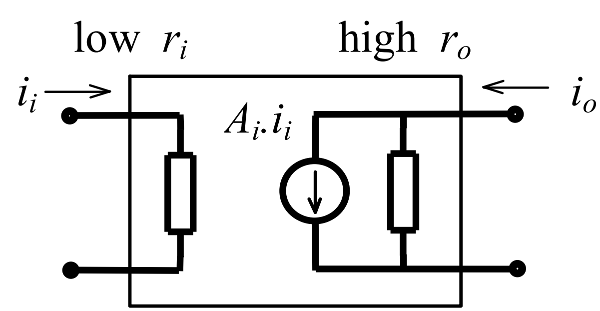

Trans-admittance amplifier

(b)




Trans-impedance amplifier

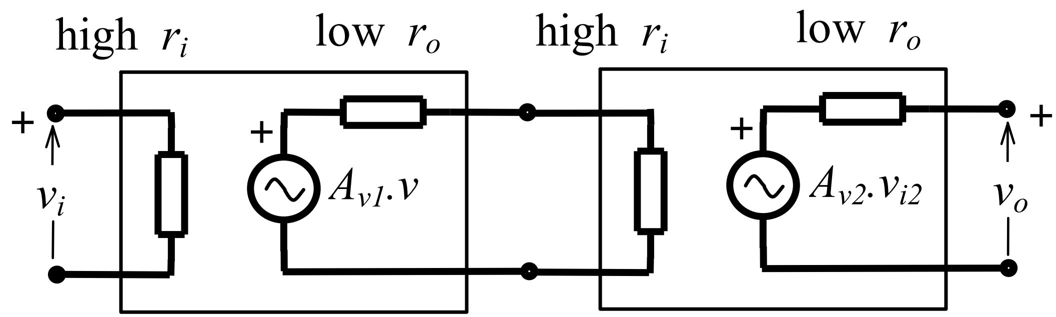
(c)




Current amplifier

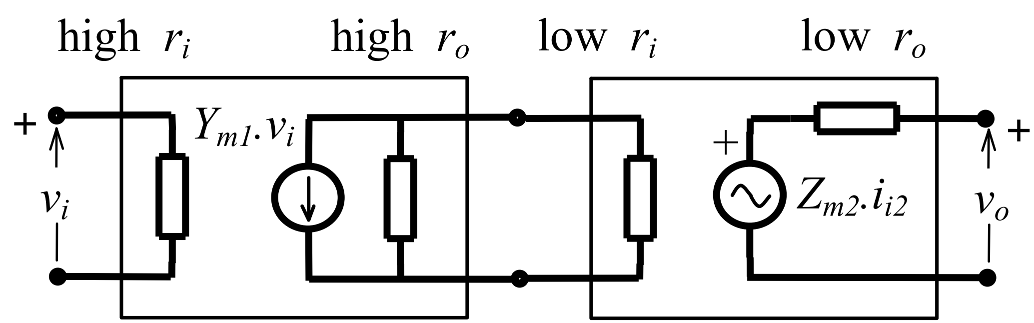
(d)

# Appropriate configurations for two stage voltage amplifiers



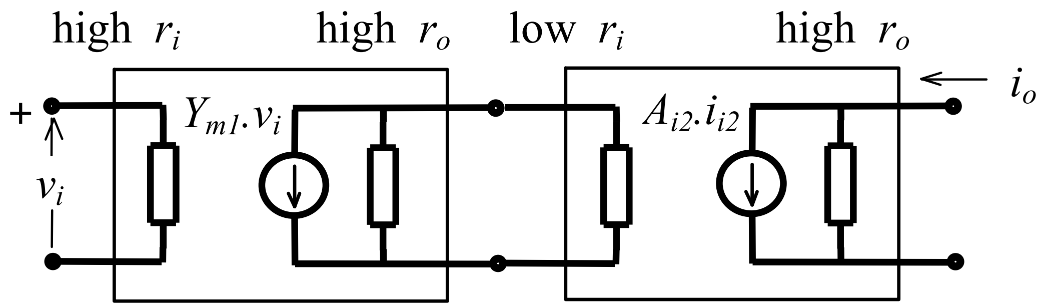

$$A_{vT} \approx A_{v1} \times A_{v2} \qquad \text{for} \quad r_{i2} \gg r_{o1}$$

(a)



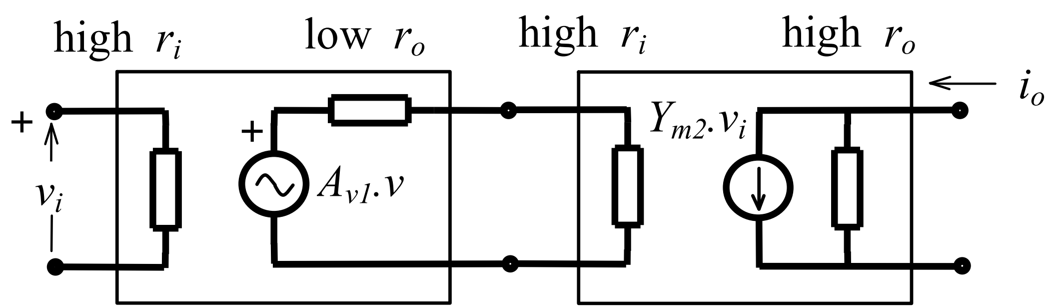

(b)

# Appropriate configurations for two stage trans-admittance amplifiers




$$Y_{mT} \approx -Y_{m1} \times A_{i2} \qquad \text{for} \quad r_{i2} \ll r_{o1}$$

(a)




$$Y_{mT} \approx A_{v1} \times Y_{m2} \qquad \text{for} \quad r_{i2} \gg r_{o1}$$

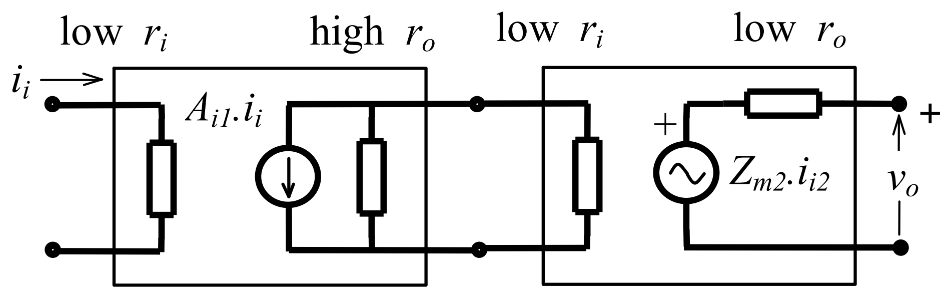
(b)





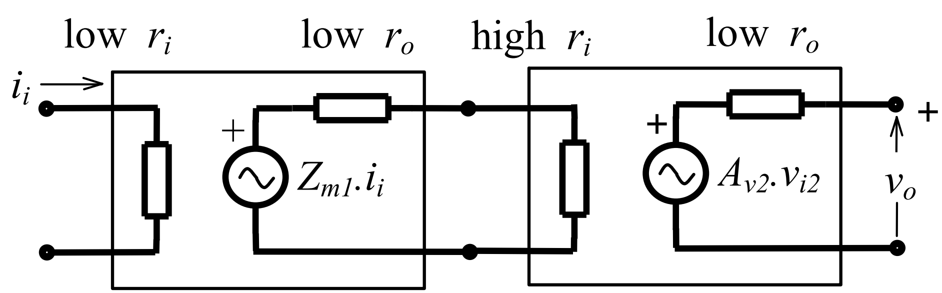
Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

# Appropriate configurations for two stage trans-impedance amplifiers




$$Z_{mT} = -A_{i1} \times Z_{m2} \qquad \text{for} \quad r_{i2} \ll r_{o1}$$

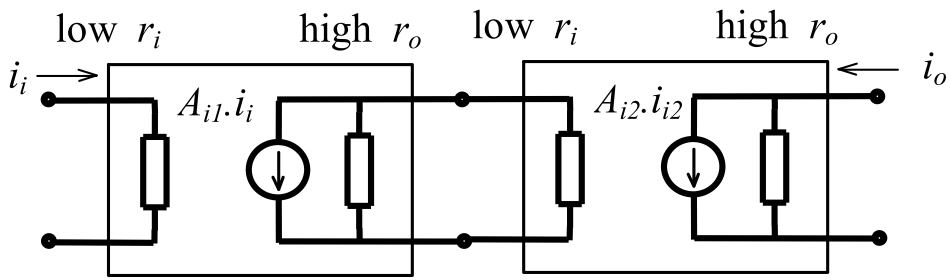
(a)




$$Z_{mT} = Z_{m1} \times A_{v2} \qquad \text{for} \quad r_{i2} \gg r_{o1}$$

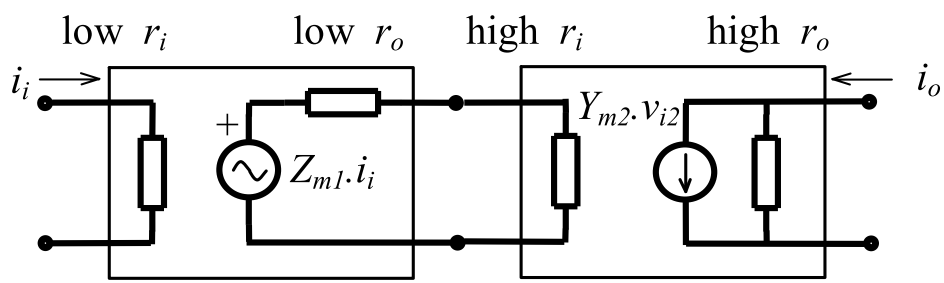
(b)

# Appropriate configurations for two stage current amplifiers




$$A_{iT} = -A_{i1} \times A_{i2} \qquad \text{for} \quad r_{i2} \ll r_{o1}$$

(a)



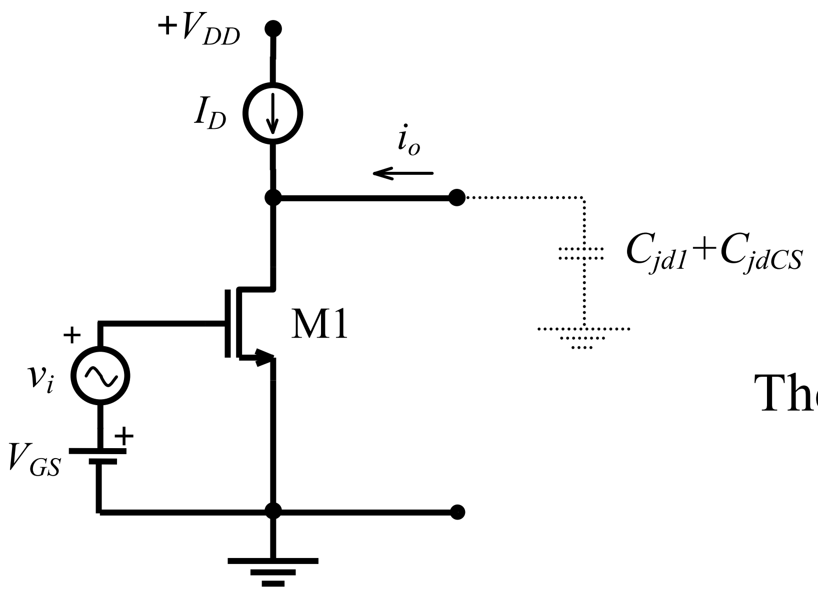

$$A_{iT} = Z_{m1} \times Y_{m2} \qquad \text{for} \quad r_{i2} \gg r_{o1}$$

(b)

# Example: Cherry-Hooper amplifier.

(Trans-admittance amp.+ Trans-impedance amp. = Voltage amp.)

The simple trans-admittance amp.:



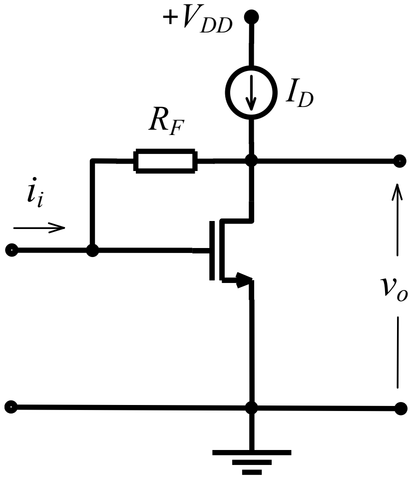

The low frequency trans admittance:

$$Y_m(0) = g_m = \frac{i_o}{v_i}$$

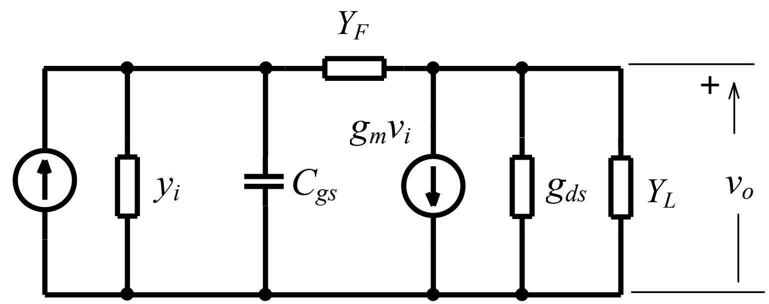


Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

# A simple trans-impedance amplifier:



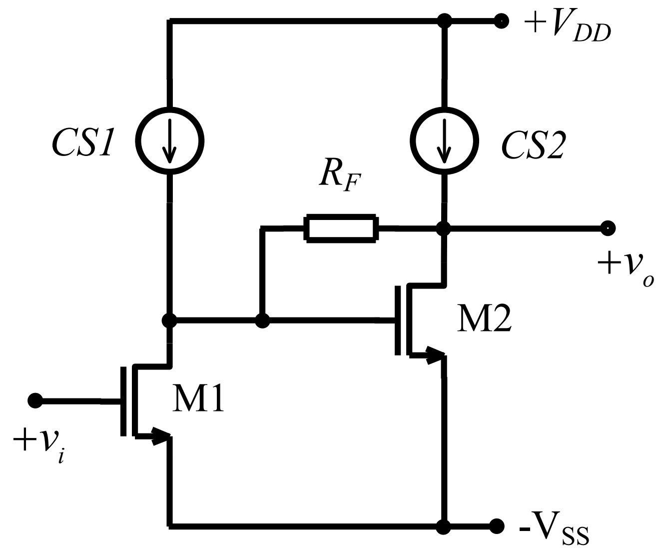

(a)




(b)

The low frequency transimpedance: $Z_m(0) = \dfrac{(g_m - G_F)}{G_F(g_{ds} + g_m)}$

For $g_{ds} \ll g_m$ and $G_F \ll g_m$; $Z_m(0) \cong \dfrac{(g_m - G_F)}{g_m G_F} \cong -\dfrac{1}{G_F} = -R_F$




The low frequency voltage gain: $A_v(0) = Y_{m1}(0) \cdot Z_{m2}(0)$

$$A_v(0) = -g_{m1} \cdot R_F$$

The low frequency input impedance of M2: $Z_i(0) = \dfrac{R_F + r_{ds2}}{1 + g_{m2} r_{ds2}} \cong \dfrac{1}{g_{m2}}$

The cascading node pole freq.: $\omega_{pC} = \dfrac{g_{m2}}{C_{jd1} + C_{jdCS1} + C_{gs2}}$

The low frequency output impedance of M2: $Z_o(0) = \dfrac{r_{ds2}}{1 + g_{m2} r_{ds2}} \cong \dfrac{1}{g_{m2}}$

The output node pole frequency: $\omega_{pL} = \dfrac{g_{m2}}{C_{jd2} + C_{jdCS2} + C_L}$



Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

# The differential Cherry Hooper amplifier