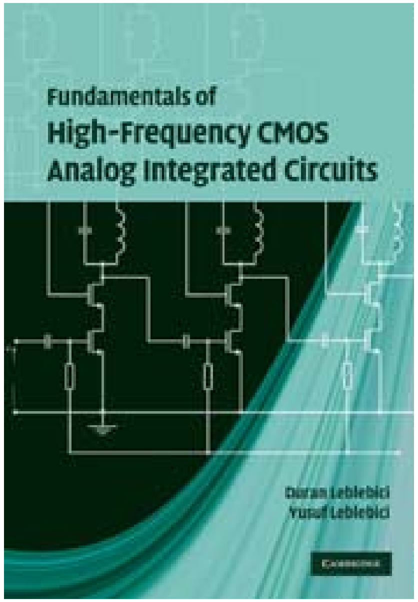

Fundamentals of High-Frequency CMOS Analog Integrated Circuits

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Chapter 4

Frequency-Selective RF Circuits (Noise)

- All signal components at the output of an amplifier, not related to the input signal are called as "noise".

- Noise types that can be eliminated or reduced by necessary precautions are mostly originated from external sources.

- The noise generated in the components of the amplifier, originating from the random movements of the carriers due to the thermal agitation is called as the "thermal noise".

- The major part of the noise originating from thermal agitation of charge carriers exhibits a flat power density spectrum and is called as the "white noise".

- The another type of noise, mostly related to the material imperfections and is higher at low frequencies, is called as the "pink noise" or "1/f noise".

The metrics to characterize the noise of an amplifier:

The "Noise Factor" and the "Noise Figure"

The noise factor: $$F = \frac{(S/N)_{input}}{(S/N)_{output}}$$

$$F = \frac{S_{in}}{S_{out}} \frac{N_{out}}{N_{in}} = \frac{S_{in}}{A_p \times S_{in}} \frac{(A_p \times N_{in}) + N_{amp}}{N_{in}}$$

Where;

- A_p is the power gain of the amplifier,
- N_{amp} is the noise power at the output,

  originating from amplifier itself.

$$\Downarrow$$

The "Noise Factor": $F = 1 + \dfrac{N_{amp}}{A_p \times N_{in}}$

The "Noise Figure": $NF(dB) = 10 \times \log F$

The thermal noise generated in a conductor:

- Observed and measured by J. B. Johnson in 1928.
- Theoretically derived expression by H. Nyquist (1928):

$$P_n = 4kTB$$

P_n : The noise power

k : The Boltzmann constant (1.36×10^{-23} joules / K)

T : The temperature in K

B : The bandwidth for which P_n is calculated or measured.

$$P_n = 4kTB$$

Indicates that the noise power generated in a conductor due to the random movements of the charge carriers:

- Depends only on the temperature of the conductor and the interested bandwidth, regardless of the position of this bandwidth on the frequency axis.

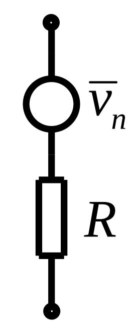
- Is independent of the material and the shape of the conductor.

- The noise power in terms of the mean-square voltage of the random noise: $P_n = \overline{v}_n^2 \times \dfrac{1}{R}$

$$\Downarrow$$

- Root-mean square noise voltage: $\overline{v}_n = \sqrt{4kTB \times R}$

- The noise voltage equivalent of a resistor:



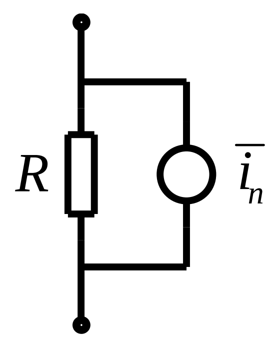
$$\overline{v}_n = \sqrt{4kTB \times R}$$

- The noise power in terms of the mean-square current of the random noise: $P_n = \overline{i_n}^2 \times R$

$$\Downarrow$$

- Root-mean square noise current: $\overline{i_n} = \sqrt{4kTB\frac{1}{R}}$

- The noise current equivalent of a resistor:



$$\overline{i_n} = \sqrt{4kTB/R}$$

The spectral density; noise for 1 Hz band-width:

- Spectral density of the noise power:

$$S_p = P_n\Big|_{B=1Hz} = 4kT$$

- Spectral density of the mean square noise voltage:

$$S_v = \overline{v_n}^2\Big|_{B=1Hz} = 4kT \times R$$

- Spectral density of the mean square noise current:

$$S_i = \overline{i_n}^2\Big|_{B=1Hz} = \frac{4kT}{R}$$

Noise of a MOS Transistor

(First investigated by A. Van der Ziel, 1986)

Noise generating resistors in a MOS transistor:

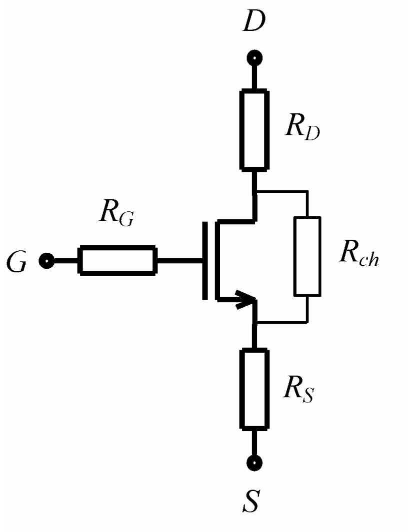
- Parasitic series resistances (R_S, R_D, R_G)

- The resistance of the channel:

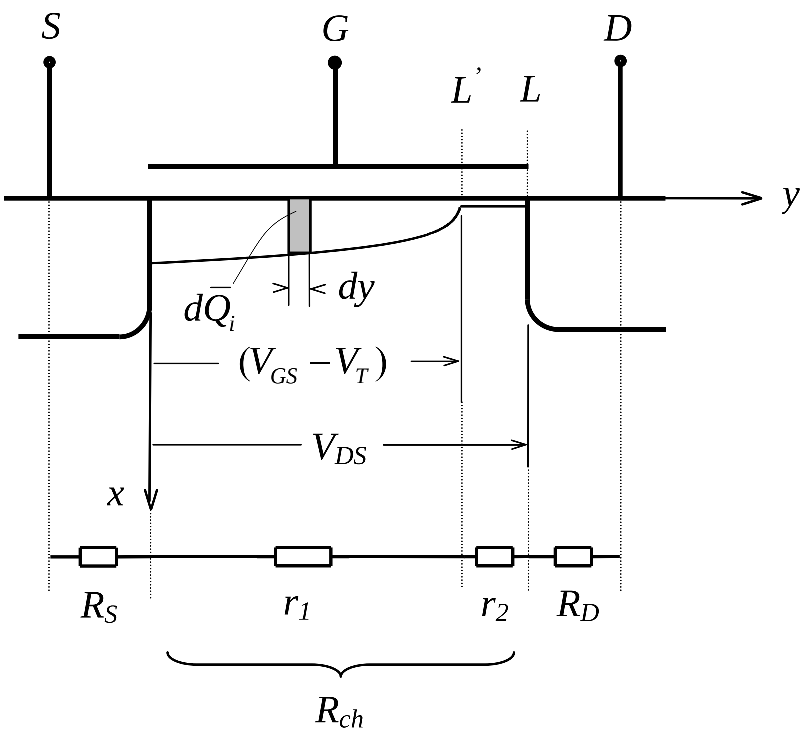
  (a) The resistance of the non velocity saturated

  (pre-pinch off) part of the channel (the dominating noise source).

  (b) The resistance of the pinched-off part of the channel.



The value of the inversion channel resistance:



$$dr(y) = \frac{dV_c}{I_D}$$

$$I_D = \frac{d\bar{Q}_i(y)}{dt}$$

$$d\bar{Q}_i(y) = -C_{ox}W(V_{GS} - V_T)\sqrt{1 - \frac{y}{L'}}\ .\ dy$$

$$dt = \frac{dy}{v} = \frac{dy}{\mu E(y)} = \frac{dy}{-\mu(dV_c/dy)}$$

$$dr(y) = \frac{dy}{\mu C_{ox} W (V_{GS} - V_T)\sqrt{1 - \frac{y}{L'}}}$$

$$R_{ch} = \int_0^{L'} dr(y) = \frac{2}{\mu C_{ox} \frac{W}{L'} (V_{GS} - V_T)}$$

$$R_{ch} = \frac{(V_{GS} - V_T)}{I_D} \qquad \text{(as expected!)}$$

From basic $I_D = f(V_{DS})$ relation:

$$R_{ch} = \sqrt{\frac{2}{I_D \mu C_{ox} \frac{W}{L}}} \quad \Rightarrow \quad R_{ch} = \frac{2}{g_m}$$

The mean-square noise current of the inversion channel resistance:

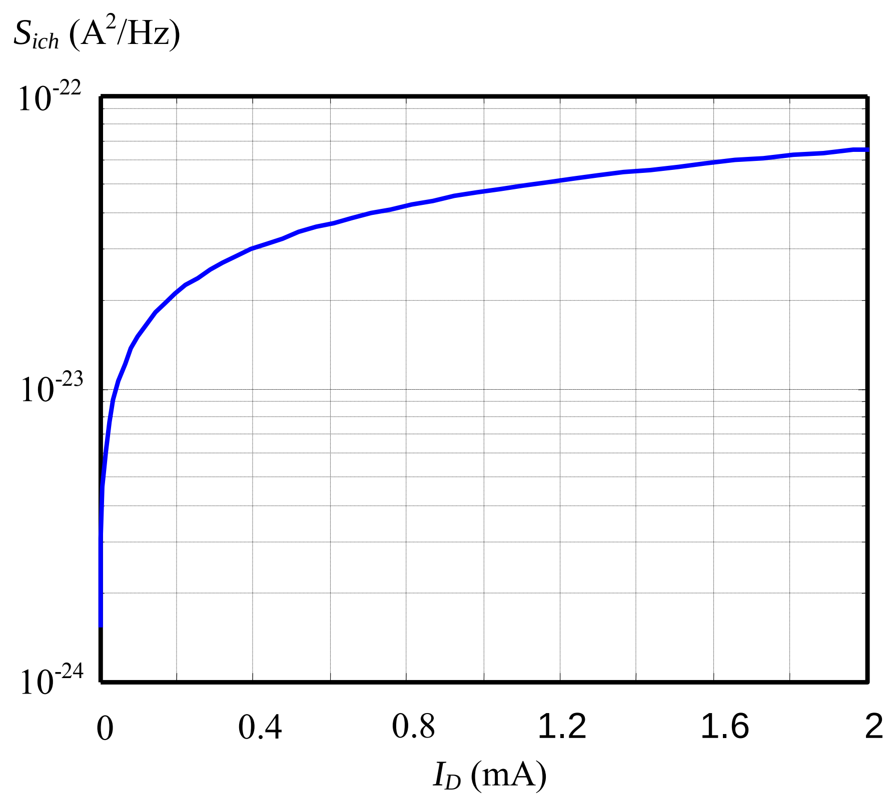
$$\overline{i_{nd}^{2}} = \frac{4kTB}{R_{ch}} = 2kTB \times g_m = 4kTB\sqrt{\frac{1}{2}\mu C_{ox}\frac{W}{L}I_D}$$

- Increases with g_m.
- Increases with the drain DC current.
- Increases with (W / L).
- Is higher for small geometries, due to the higher C_{ox} values.
- Is smaller for PMOS transistors, related to mobility.

The spectral density of the mean square channel noise:

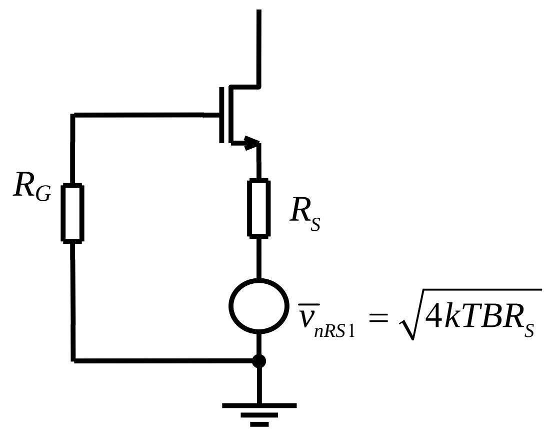
$$S_{ich} = \overline{i_{nd}^2}\Big|_{B=1Hz} = \frac{4kT}{R_{ch}} = 2kTg_m \quad [\mathrm{A}^2/\mathrm{Hz}]$$

Simulation result for a 35µ / 0.35µ AMS NMOS transistor

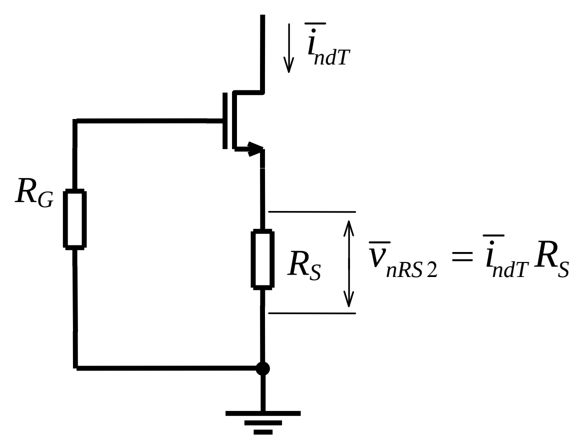


The contribution of the source series resistance on noise

(a) The contribution of the noise generated in R_S itself:

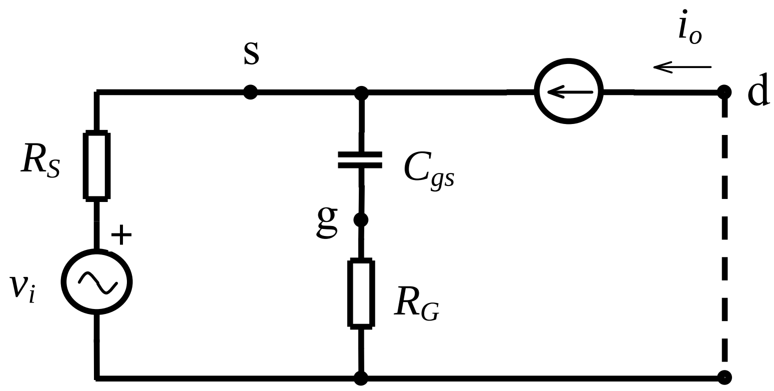


(b) The contribution of the noise voltage drop on R_S due to the channel noise current:



Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

To investigate the contribution (a), it is necessary to calculate the transconductance from $\overline{v}_{nRS1}$ to $\overline{i}_{ndT}$:



$$g_{mSD(eff)} = -\frac{g_m}{R_S(g_m + j\omega C_{gs}) + (1 + j\omega C_{gs} R_G)}$$

$$\left| g_{mSD(eff)} \right| = \frac{g_m}{\sqrt{(1 + g_m R_S)^2 + \omega^2 C_{gs}{}^2 (R_S + R_G)^2}}$$

$$\overline{i}_{ndS1} = \left| g_{mSD(eff)} \right| \overline{v}_{nRS1}$$

Similarly, to investigate the contribution (b), the transconductance from $\overline{v}_{nRS2}$ to $\overline{i}_{ndT}$, that corresponds to $R_S = 0$, must be calculated :

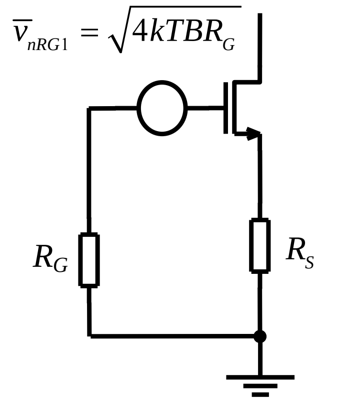
$$g'_{mSD(eff)} = -\frac{g_m}{(1 + j\omega C_{gs} R_G)}$$

$$\left| g'_{mSD(eff)} \right| = \frac{g_m}{\sqrt{1 + (\omega C_{gs} R_G)^2}}$$

$$\overline{i}_{ndS2} = \left| g'_{mSD(eff)} \right| \overline{i}_{ndT} R_S$$

The contribution of the gate series resistance on noise

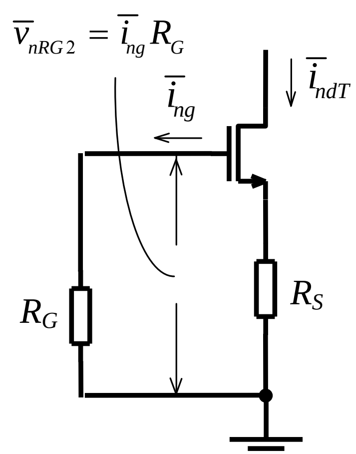
(a) The contribution of the noise generated in R_G:



$$\overline{v}_{nRG1} = \sqrt{4kTBR_G}$$

$$\overline{i}_{ndG1} = \left| g_{m(eff)} \right| \overline{v}_{nR_G}$$

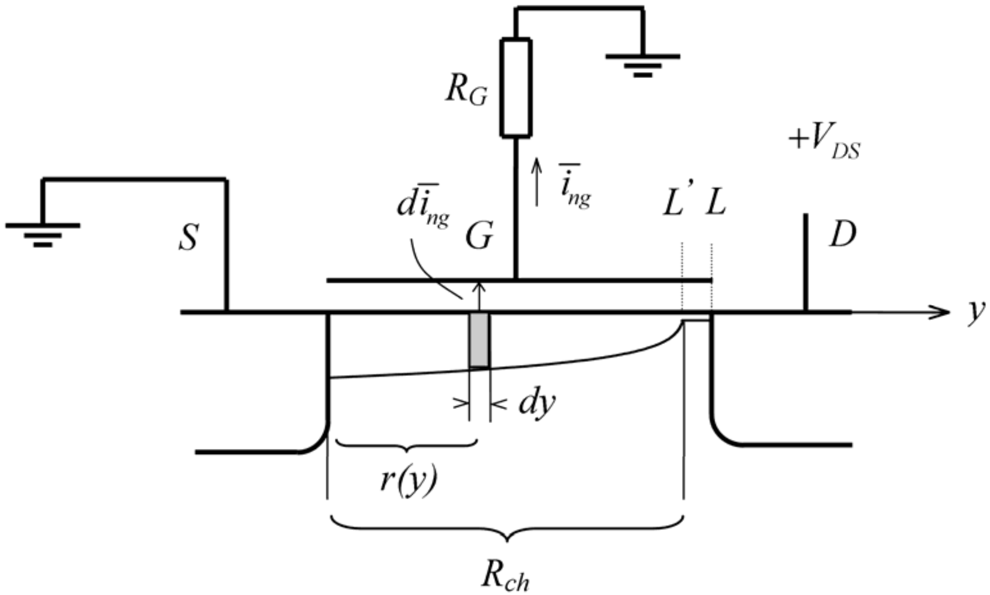
(b) The contribution of the noise current flowing over C_{dg} at high frequencies:



$$\overline{v}_{nRG2} = \overline{i}_{ng} R_G$$

$$\overline{i}_{ndG2} = \left| g_{m(eff)} \right| \overline{v}_{nRG2}$$

Calculation of the non-white gate noise current, $\overline{i_{ng}}$:



$$dC_g = C_{ox} W dy$$

$$r(y) = \frac{2}{\mu C_{ox} \frac{W}{L'}(V_{GS} - V_T)} \left(1 - \sqrt{1 - \frac{y}{L'}}\right)$$

$$\overline{v}_n(y) = \overline{i}_{nd} . r(y)$$

$$d\overline{i}_{ng} = j\omega(dC_g) \times \overline{v}_n(y) = j\omega(C_{ox}Wdy) \times r(y)\overline{i}_{nd}$$

$$\overline{i}_{ng} = j\omega C_{ox}W \frac{2}{\mu C_{ox}\frac{W}{L'}(V_{GS} - V_T)} \int_0^{L'} \left(1 - \sqrt{1 - \frac{y}{L'}}\right) dy \times \overline{i}_{nd}$$

For $L \cong L'$; $\quad \left|\overline{i}_{ng}\right| \cong \frac{\omega C_{gs}}{g_m}\overline{i}_{nd} = \omega \frac{1}{A}\sqrt{\frac{W}{I_D}} \times \overline{i}_{nd}$

where $\quad A = \sqrt{\frac{2\mu}{C_{ox}L^3 k_{ol}^2}}$

Total noise of a MOS transistor

(a) Uncorrelated (independent) components of the channel noise:

$$\overline{i}_{ndU}^{2} = \overline{i}_{nd}^{2} + \overline{i}_{ndS1}^{2} + \overline{i}_{ndG1}^{2}$$

$$\overline{i}_{nd}^{2} = \frac{4kTB}{R_{ch}} = 2kTBg_{m}$$

$$\overline{i}_{ndS1}^{2} = \left| g_{mSD(eff)} \right|^{2} \overline{v}_{nR_S}^{2} = \left| g_{mSD(eff)} \right|^{2} 4kTB\,R_S$$

$$\overline{i}_{ndG1}^{2} = \left(\left| g_{m(eff)} \right| \overline{v}_{nR_G} \right)^{2} = \left| g_{m(eff)} \right|^{2} 4kTB\,R_G$$

(b) Noise components related to the total channel noise:

$$\overline{i}_{ndS2}^{\,2} = \left| g'_{mSD(eff)} \right|^2 R_S^2 \cdot \overline{i}_{ndT}^{\,2} = \alpha_1 \cdot \overline{i}_{ndT}^{\,2} \ ,$$

Where $\alpha_1 = \left| g'_{mSD(eff)} \right|^2 R_S^2$

$$\overline{i}_{ndG2}^{\,2} = \left(\left| g_{m(eff)} \right| \overline{v}_{nRG2} \right)^2 = \left| g_{m(eff)} \right|^2 R_G^2 \cdot \overline{i}_{ng}^{\,2}$$

$$= \left| g_{m(eff)} \right|^2 \left(\omega \frac{1}{A} \sqrt{\frac{W}{I_D}} \right)^2 R_G^2 \cdot \overline{i}_{ndT}^{\,2} = \alpha_2 \cdot \overline{i}_{ndT}^{\,2}$$

Where $\alpha_2 = \left| g_{m(eff)} \right|^2 \left(\omega \frac{1}{A} \sqrt{\frac{W}{I_D}} \right)^2 R_G^2$

The total channel noise:

$$\overline{i}_{ndT}^{2} = \overline{i}_{ndTU}^{2} + \overline{i}_{ndS2}^{2} + \overline{i}_{ndG2}^{2}$$

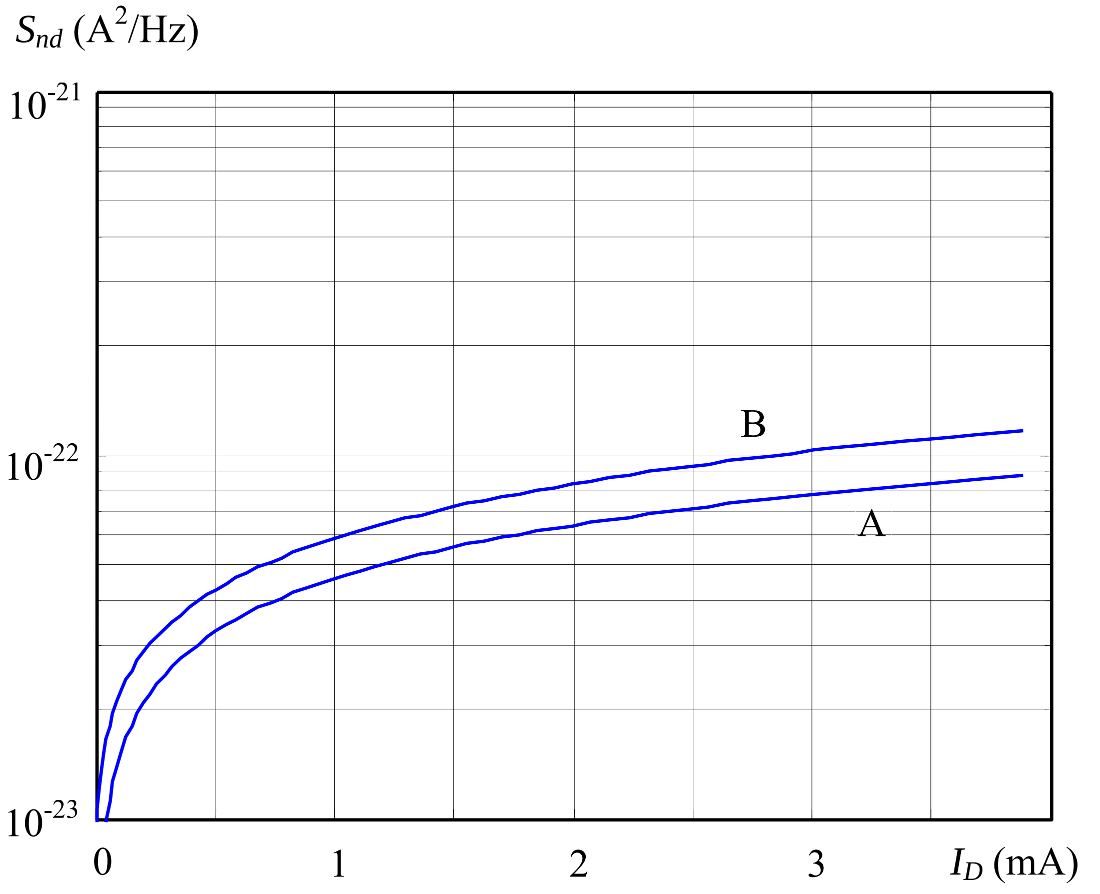
$$= \overline{i}_{ndTU}^{2} + \alpha_1 \cdot \overline{i}_{ndT}^{2} + \alpha_2 \cdot \overline{i}_{ndT}^{2}$$

$$\Downarrow$$

$$\overline{i}_{ndT}^{2} = \frac{\overline{i}_{ndU}^{2}}{\left[1-(\alpha_1+\alpha_2)\right]} = \frac{\overline{i}_{nd}^{2} + \overline{i}_{ndS1}^{2} + \overline{i}_{ndG1}^{2}}{\left[1-(\alpha_1+\alpha_2)\right]}$$

$$\overline{i}_{ndT}^{2} = 4kTB\,\frac{(g_m/2) + \left|g_{mSD(eff)}\right|^2 R_S + \left|g_{m(eff)}\right|^2 R_G}{\left[1-\left(\alpha_1+\alpha_2\right)\right]}$$

Simulation to show the contributions of R_S and R_G



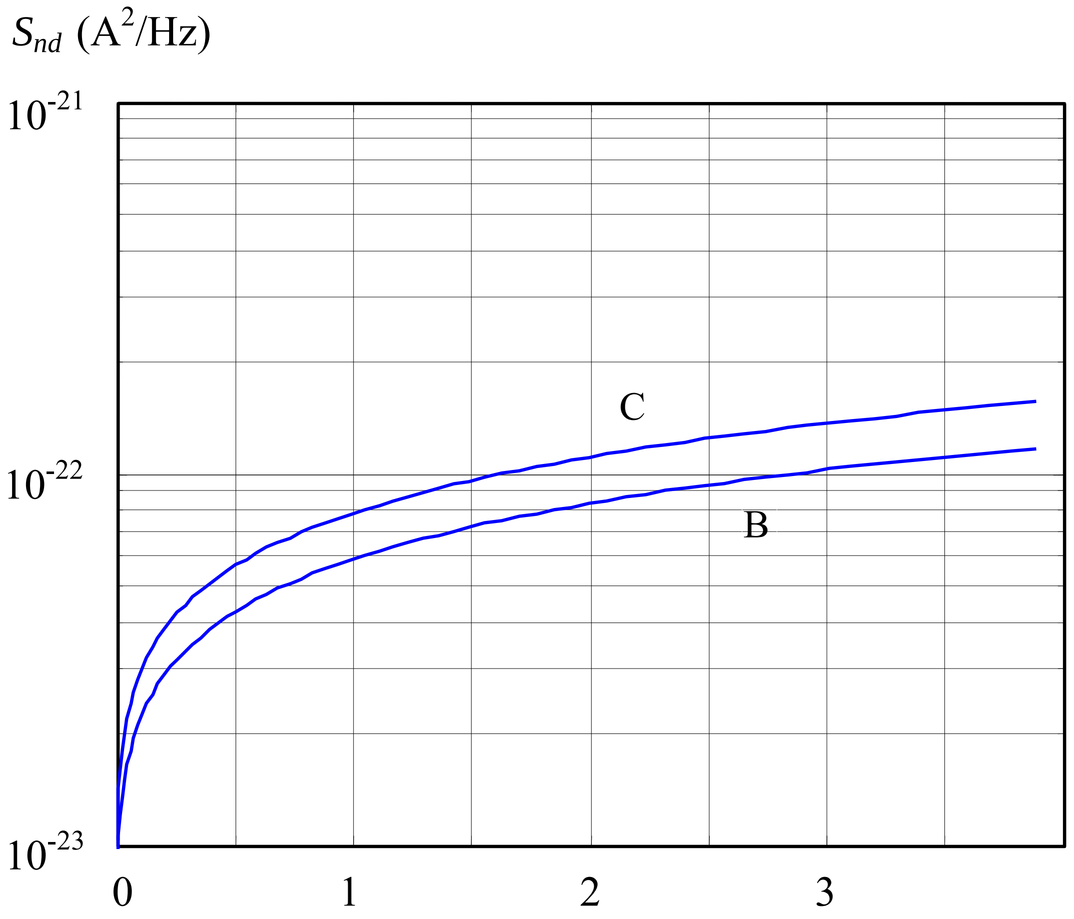
Transistor: $35\mu\text{m}\ (7\times 5\mu\text{m})\ /\ 0.35\mu\text{m}$ AMS NMOS.

$I_D = 2$ mA, $V_{DS} = 3$V, $T = 300K$, $f = 3$ GHz,

(A) The inversion resistance only,

(B) Contributions of R_S and R_G included.

Simulation to show the contribution of temperature



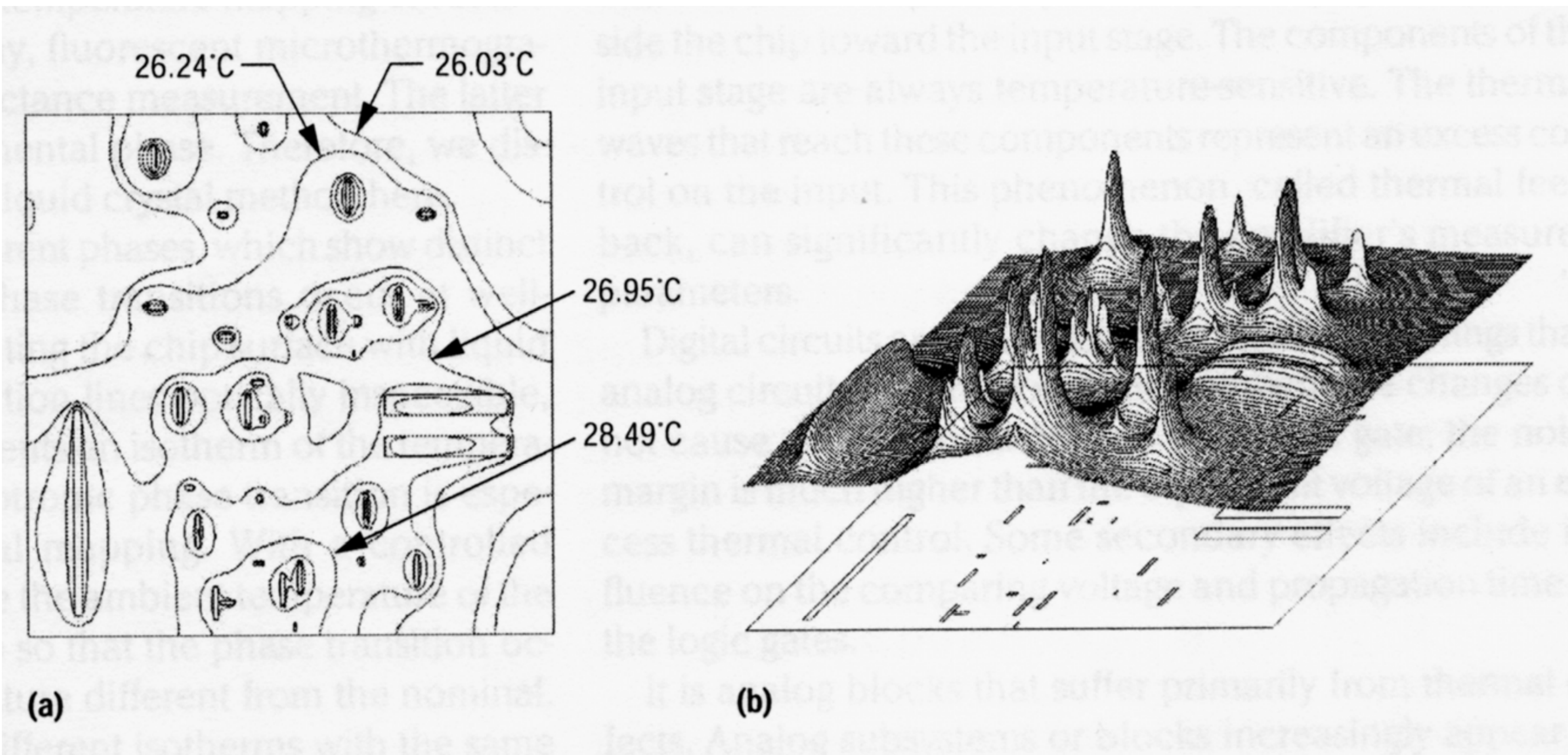
The total noise for the same transistor and the same operating conditions:

(B) 300K, (C) 400K

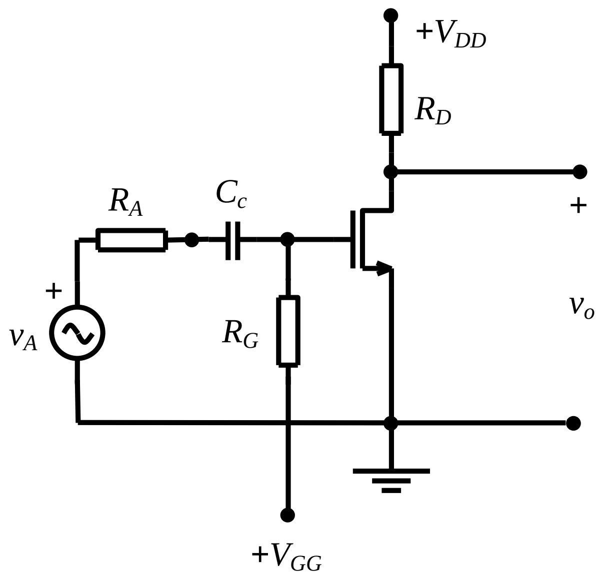
It must be noted that

the temperatures of the channels of various transistors on a chip can be significantly different!

Example (after V. Szekely and B. Courtois):



Example: The noise figure of a CS amplifier:



$g_m = 20$ mS

$R_A = 50$ ohm

$R_D = 200$ ohm

(a) $R_G = 50$ ohm

(b) $R_G = 1000$ ohm

Parasitics neglected

Spectral density of the channel noise: $\overline{i}_{nd}^{2} = 2kTB \times g_m$

Spectral density of the noise current of R_D: $\overline{i}_{nRD}^{2} = 4kTB / R_D$

Noise voltage generated on R_G: $\overline{v}_{nRG} = \sqrt{4kTB \times R_G}$

Noise voltage generated on R_A: $\overline{v}_{nRA} = \sqrt{4kTB \times R_A}$

Noise current at the output owing to $\overline{v}_{nRA}$: $\overline{i}_{noRA} = g_m \overline{v}_{nRA}$

Noise current at the output owing to $\overline{v}_{nRG}$: $\overline{i}_{noRG} = g_m \overline{v}_{nRG}$

Total output noise power

generated in the amplifier : $P_{no} = (\overline{i}_{nd}^{\,2} + \overline{i}_{nRD}^{\,2} + \overline{i}_{noRG}^{\,2}) R_D$

$$P_{no} = 4kTB\left(\frac{1}{2} g_m + \frac{1}{R_D} + R_G g_m^2\right) R_D$$

Noise power at the output owing to R_A : $P_{noRA} = \overline{i}_{noRA}^{\,2} R_D = 4kTBR_A g_m^2 R_D$

The Noise factor: $F = 1 + \dfrac{P_{no}}{P_{noRA}} = 1 + \dfrac{(4kTB)\left(\frac{1}{2} g_m + \frac{1}{R_D} + R_G g_m^2\right) R_D}{(4kTB) R_A g_m^2 R_D}$

$$F = 1 + \frac{\frac{1}{2} g_m + \frac{1}{R_D}}{R_A g_m^2} + \frac{R_G}{R_A}$$

(a) For $R_G = 50$ ohm:

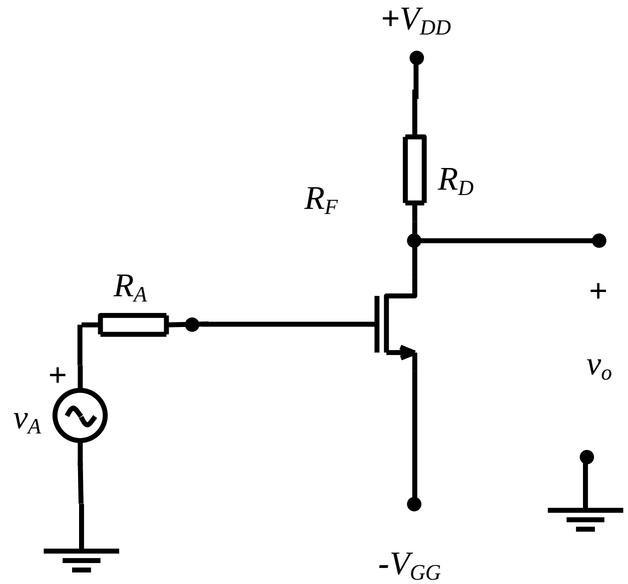
$$F = 1 + \frac{\frac{1}{2}(20.10^{-3}) + \frac{1}{200}}{50(20.10^{-3})^2} + \frac{50}{50} = 2.75 \quad \Rightarrow \quad NF = 4.39 \text{ dB (high!)}$$

(b) For $R_G = 1000$ ohm:

$$F = 1 + \frac{\frac{1}{2}(20.10^{-3}) + \frac{1}{200}}{50(20.10^{-3})^2} + \frac{1000}{50} = 21.75 \quad \Rightarrow \quad NF = 13.37 \text{ dB (very high!)}$$

Note that the dominant component of the noise factor is related to R_G , a resistor parallel to the input of the amplifier!

A possible solution to eliminate R_G:



The noise figure without R_G :

$$F = 1 + \frac{\frac{1}{2}(20.10^{-3}) + \frac{1}{200}}{50(20.10^{-3})^2} = 1.75 \quad \Rightarrow \quad NF = 2.43 \text{ dB}$$