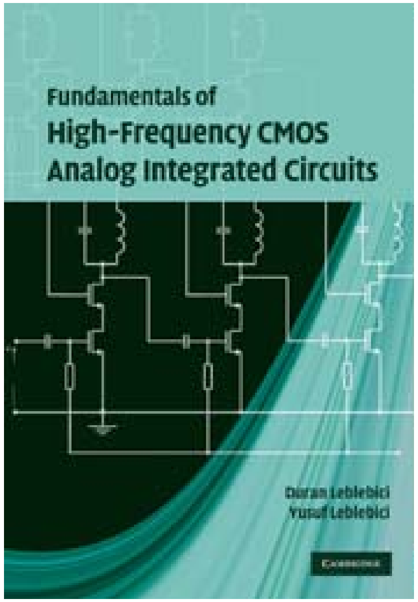


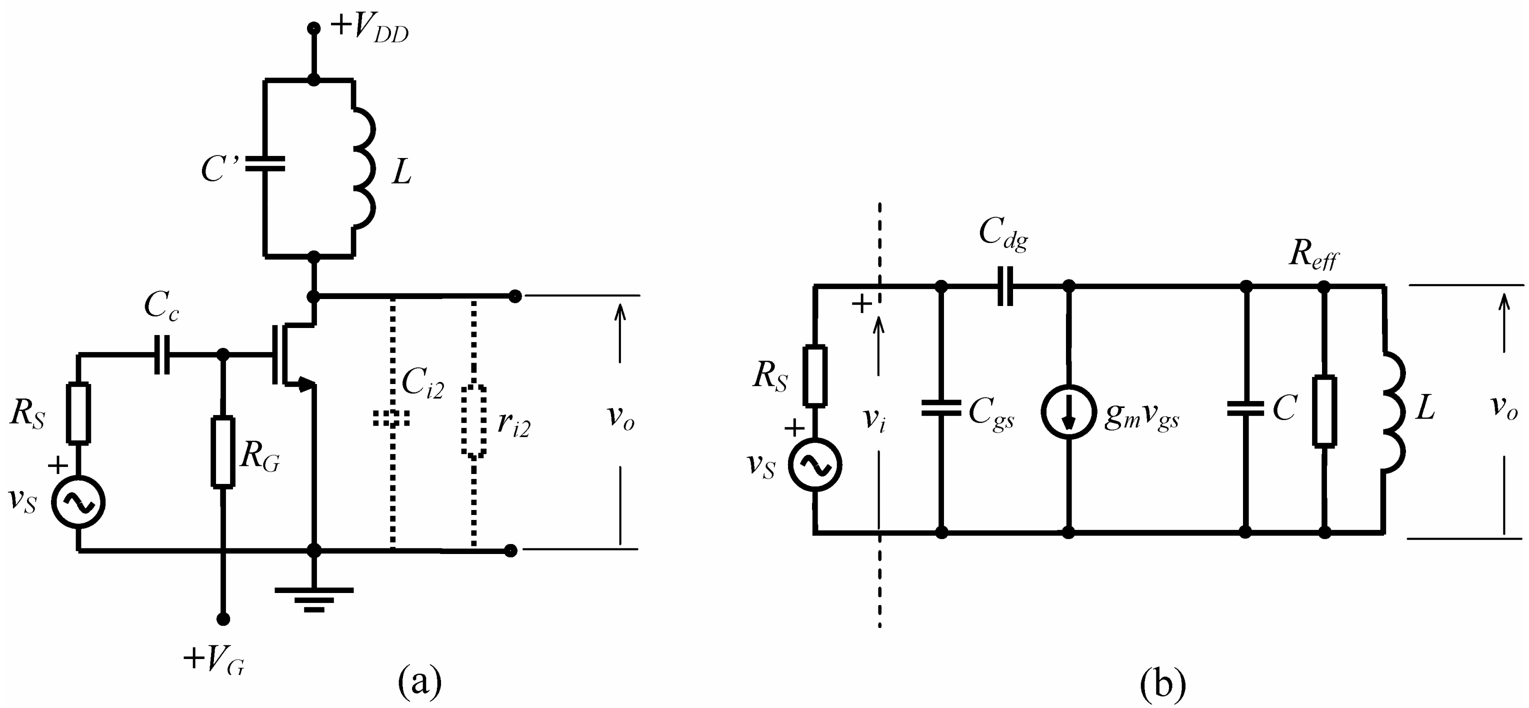

# Fundamentals of High-Frequency CMOS Analog Integrated Circuits

**Duran Leblebici**
**Yusuf Leblebici**

# Chapter 4

## Frequency-Selective RF Circuits (Tuned Amplifiers)

# a) Common source tuned amplifier:




$$A_v = -\frac{g_m - sC_{dg}}{Y_o} \quad \rightarrow \quad A_v = -\frac{g_m - sC_{dg}}{sC + \frac{1}{sL} + G_{eff}}$$

There are two ways to investigate the frequency characteristics:

a) Calculate the magnitude and the phase of the gain as functions of the frequency.

b) Use the pole-zero diagram of the gain function.

(useful to investigate multi-stage, stagger-tuned and double-tuned circuits)

$$A_v = \frac{C_{dg}}{C} \frac{s - \frac{g_m}{C_{dg}}}{s^2 + s\frac{G_{eff}}{C} + \frac{1}{LC}} = \frac{C_{dg}}{C} \frac{s(s - s_0)}{(s - s_{p1})(s - s_{p2})}$$

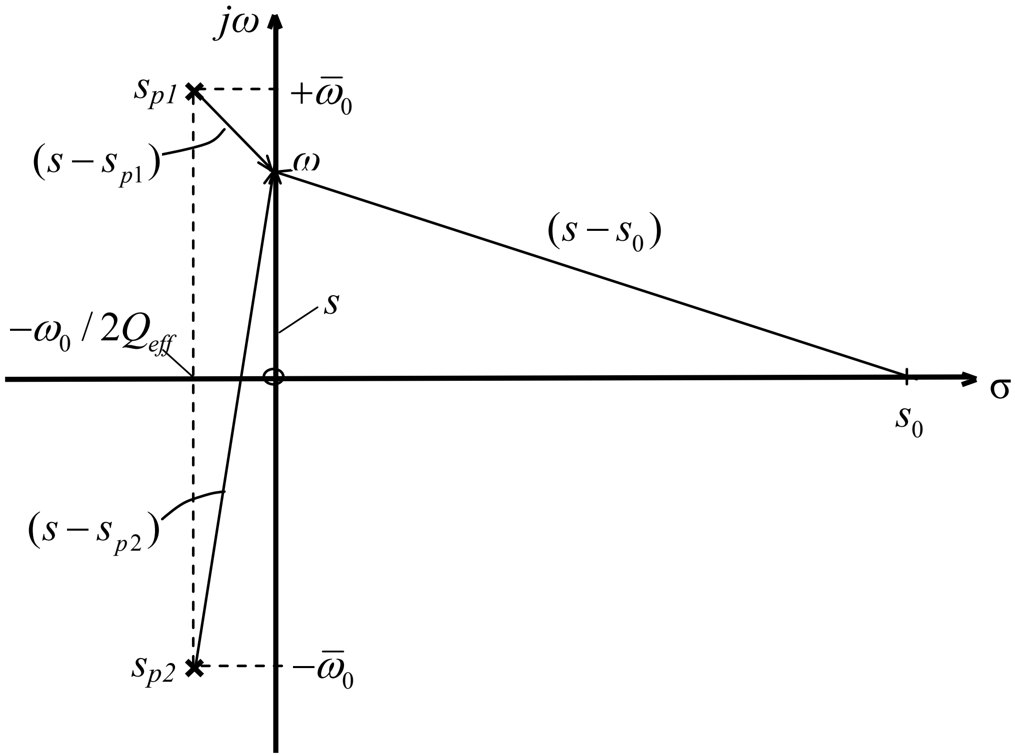
$$s_0 = +\frac{g_m}{C_{dg}} \quad , \quad s_{p1,p2} = -\frac{G_{eff}}{2C} \mp j\sqrt{\frac{1}{LC} - \left(\frac{G_{eff}}{2C}\right)^2}$$

with $\omega_0^2 = \frac{1}{LC}$ , $\sigma = -\frac{G_{eff}}{2C} = -\frac{\omega_0}{2Q_{eff}}$

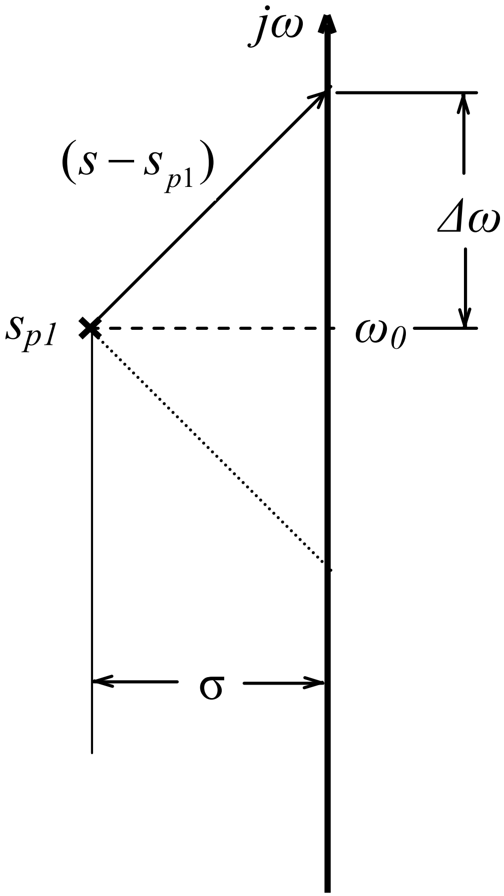
$$s_0 = +\frac{g_m}{C_{dg}} \quad , \quad s_{p1,p2} = \sigma \mp j\sqrt{\omega_0^2 - \sigma^2}$$

$$\left|A\right| = \frac{C_{dg}}{C} \frac{\left|s\right|\left|s - s_0\right|}{\left|s - s_{p1}\right|\left|s - s_{p2}\right|}$$

$$\Phi = \Phi_s + \Phi_{(s-s_0)} - \Phi_{(s-s_{p1})} - \Phi_{(s-s_{p2})}$$



For $|s_0| \gg \omega_0$ and $\omega_0 \gg \sigma$ in the vicinity of $\omega_0$;




$$|A| \cong \frac{C_{dg}}{C} \frac{|\omega||s_0|}{|s - s_{p1}| 2\omega_0} = \frac{g_m}{2C|s - s_{p1}|}$$

$$\Phi \cong \frac{\pi}{2} + \pi - \Phi_{(s - s_{p1})} - \frac{\pi}{2} = \pi - \Phi_{(s - s_{p1})}$$

$$\Delta\omega = \sigma = \frac{\omega_0}{2Q_{ff}} \quad \rightarrow \quad \Delta f = \frac{f_0}{2Q_{eff}}$$

$$f_{(-3dB)} = f_0 \pm \Delta f = f_0 \pm \frac{f_0}{2Q_{eff}}$$

$$B = 2\Delta f = \frac{f_0}{Q_{eff}}$$

Gain at resonance frequency:

With $\left| s - s_{p1} \right| = \sigma = \omega_0 / 2Q_{eff}$ ;

$$\left| A(\omega_0) \right| = g_m \frac{Q_{eff}}{\omega_0 C} = g_m R_{eff} \;, \quad \Phi\left(\omega_0\right) = \pi$$

Band width:

$$B = 2\Delta f = \frac{f_0}{Q_{eff}}$$

# The Miller effect on the input admittance

$$y_i = s\left(C_{gs} + C_{dg}\right) + y_{mi} = s\left(C_{gs} + C_{dg}\right) + s\,C_{dg}\,\frac{g_m - sC_{dg}}{Y_o}$$

$$y_{mi}(\omega) = j\omega C_{dg}\,\frac{g_m - j\omega C_{dg}}{G + j\omega C + \dfrac{1}{j\omega L}} = \omega^2 L C_{dg}\,\frac{-g_m + j\omega C_{dg}}{(1-\omega^2 LC) + j\omega LG}$$

$$\mathrm{Re}\left\{y_{mi}\right\} = \frac{\left(\omega/\omega_0\right)^2 \dfrac{C_{dg}}{C}}{\left[1-\left(\omega/\omega_0\right)^2\right]^2 + \omega^2 L^2 G^2}\left(-g_m\left[1-\left(\omega/\omega_0\right)^2\right] + \left(\omega/\omega_0\right)^2 \frac{C_{dg}}{C} G\right)$$

$$\mathrm{Im}\left\{y_{mi}\right\} = \frac{\left(\omega/\omega_0\right)^2 \dfrac{C_{dg}}{C}}{\left[1-\left(\omega/\omega_0\right)^2\right]^2 + \omega^2 L^2 G^2}\,\omega\left\{C_{dg}\left[1-\left(\omega/\omega_0\right)^2\right] + LGg_m\right\}$$

- The input conductance at resonance: $\mathrm{Re}\{y_{mi}(\omega_0)\} = \dfrac{\omega_0^2 C_{dg}^2}{G}$

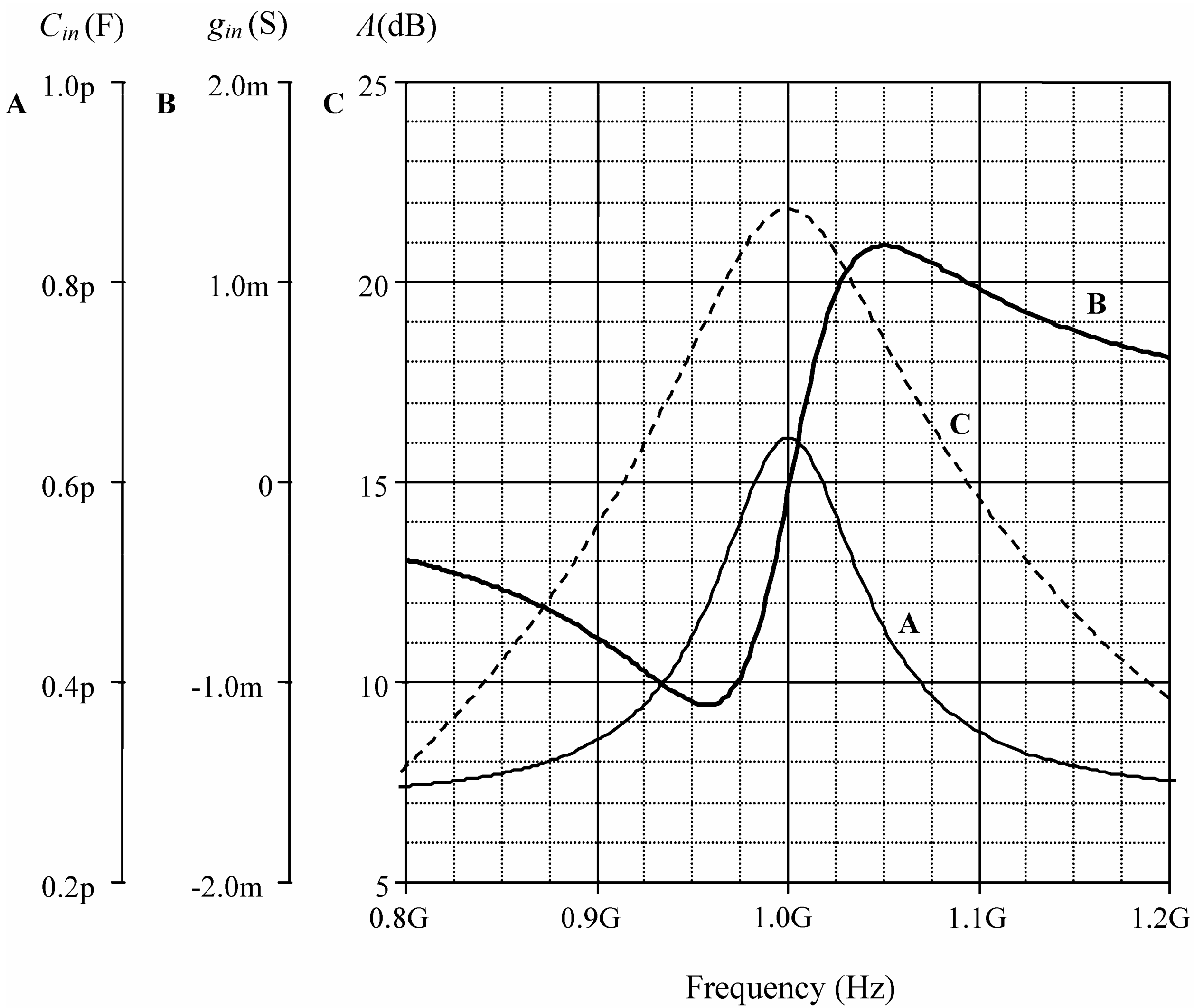
- The input conductance above resonance is always positive.

- Below the resonance frequency, the input conductance has a frequency dependent negative component.

- The input susceptance is capacitive and frequency dependent.

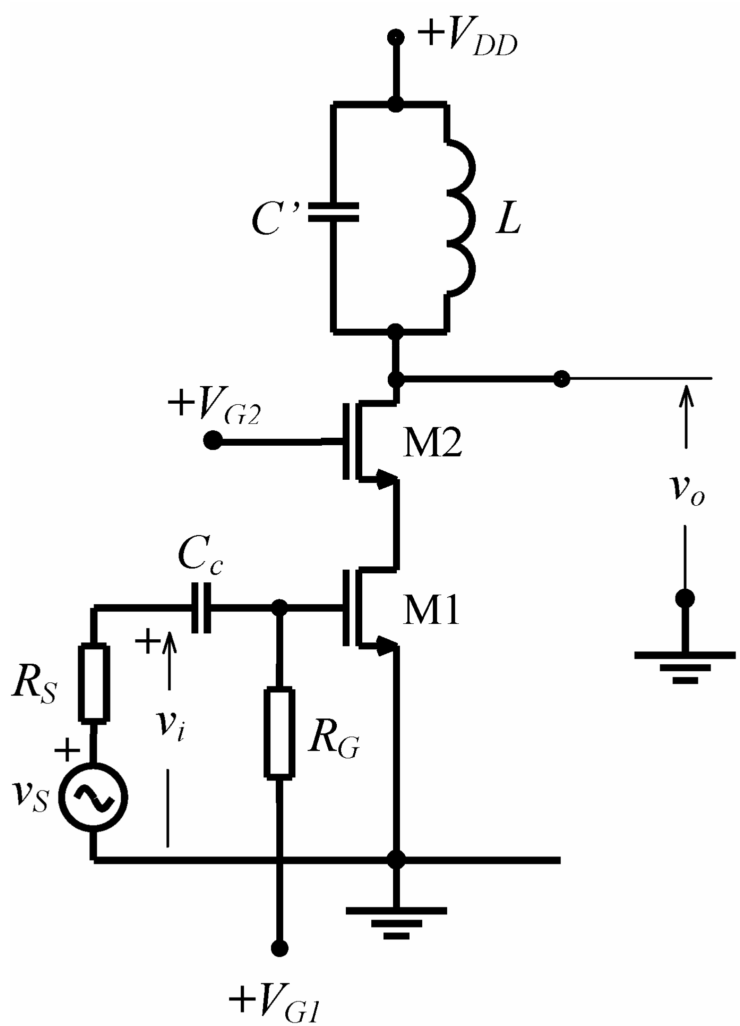
- If the amplifier is driven by another tuned amplifier;
  - Its frequency characteristic skews (becomes asymmetric).
  - There is a risk of oscillation due to the negative conductance.

Example: AMS 035, 200/0.35, $V_{DD}$ = 3V, $V_{GS}$ =0.8V, L = 10 nH, C = 2.34 pF, $Q_{eff}$ = 15.9

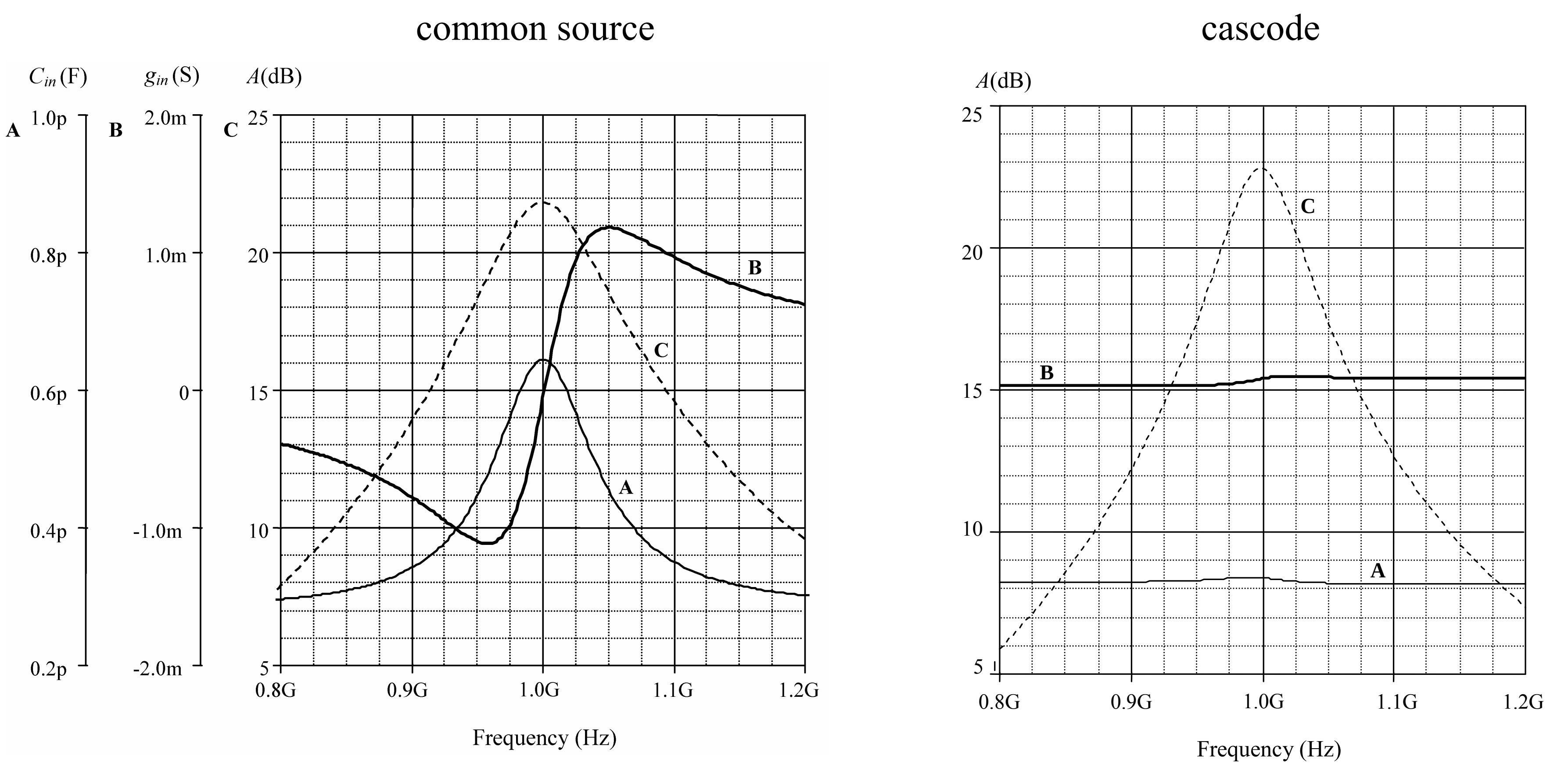



(A) Input capacitance, (B) Input conductance, (C) Voltage gain.

# The cascode tuned amplifier



# The comparison of the common source and the cascode amplifiers:




(A) Input capacitance, (B) Input conductance, (C) Voltage gain.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# The obvious advantages of the cascode circuit:

- The input capacitance is almost constant in the entire frequency band.

- The input conductance is positive and almost constant.
  (no adverse effect on the load resonance circuit of the previous stage)

- The bandwidth is smaller (the selectivity is better)
  thanks to the higher output resistance, consequently higher $Q_{eff}$.

- Higher voltage gain due to the higher $Q_{eff}$.

# Cascaded tuned stages and staggered tuning

$$A_v = \frac{C_{dg}}{C}\frac{s(s-s_0)}{(s-s_{p1})(s-s_{p2})} \cong -\frac{C_{dg}}{2C}\frac{s_0}{(s-s_{p1})}$$

$$A_v(\omega_0) = -g_m R_{eff} \;\;,\quad Q_{eff} = \omega_0 C R_{eff}$$

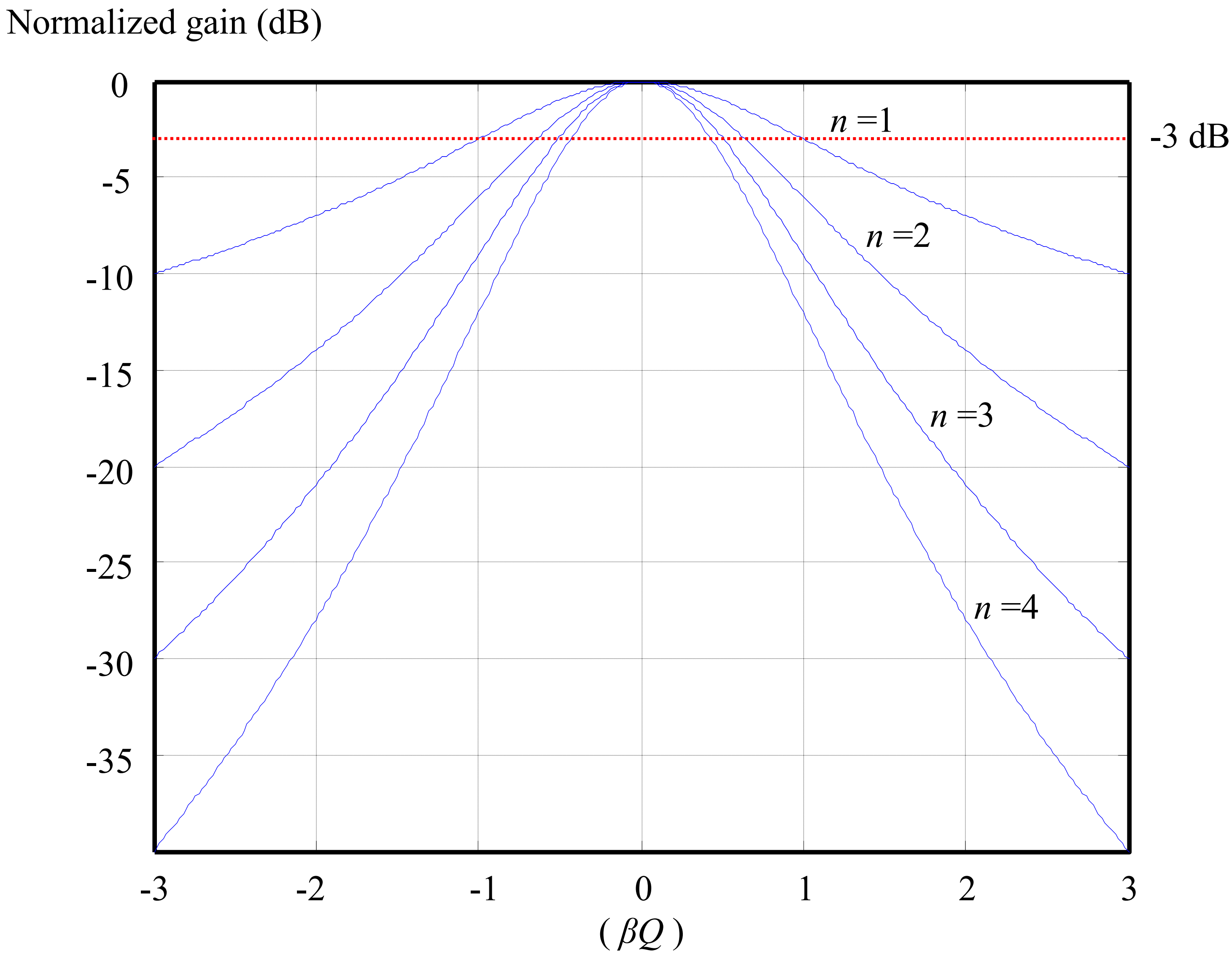
$$A_v \cong A_v(\omega_0)\frac{\omega_0}{2Q_{eff}}\frac{1}{(s-s_{p1})} = A_v(\omega_{p1})\frac{\omega_{p1}}{2Q_{eff}}\frac{1}{(s-s_{p1})}$$

Gain of the cascaded $n$ identical stages:

$$A_{vT} \cong \left[A_v(\omega_{p1})\right]^n \left(\frac{\omega_0}{2Q_{eff}}\right)^n \frac{1}{(s-s_{p1})^n}$$

Band-width of the cascaded $n$ identical stages:

$$B_T = B\sqrt{2^{1/n}-1}$$




Frequency characteristics of *n* cascaded identical stages

(Normalized to the gain at resonance frequency)



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# Gain for cascaded $n$ non-identical stages

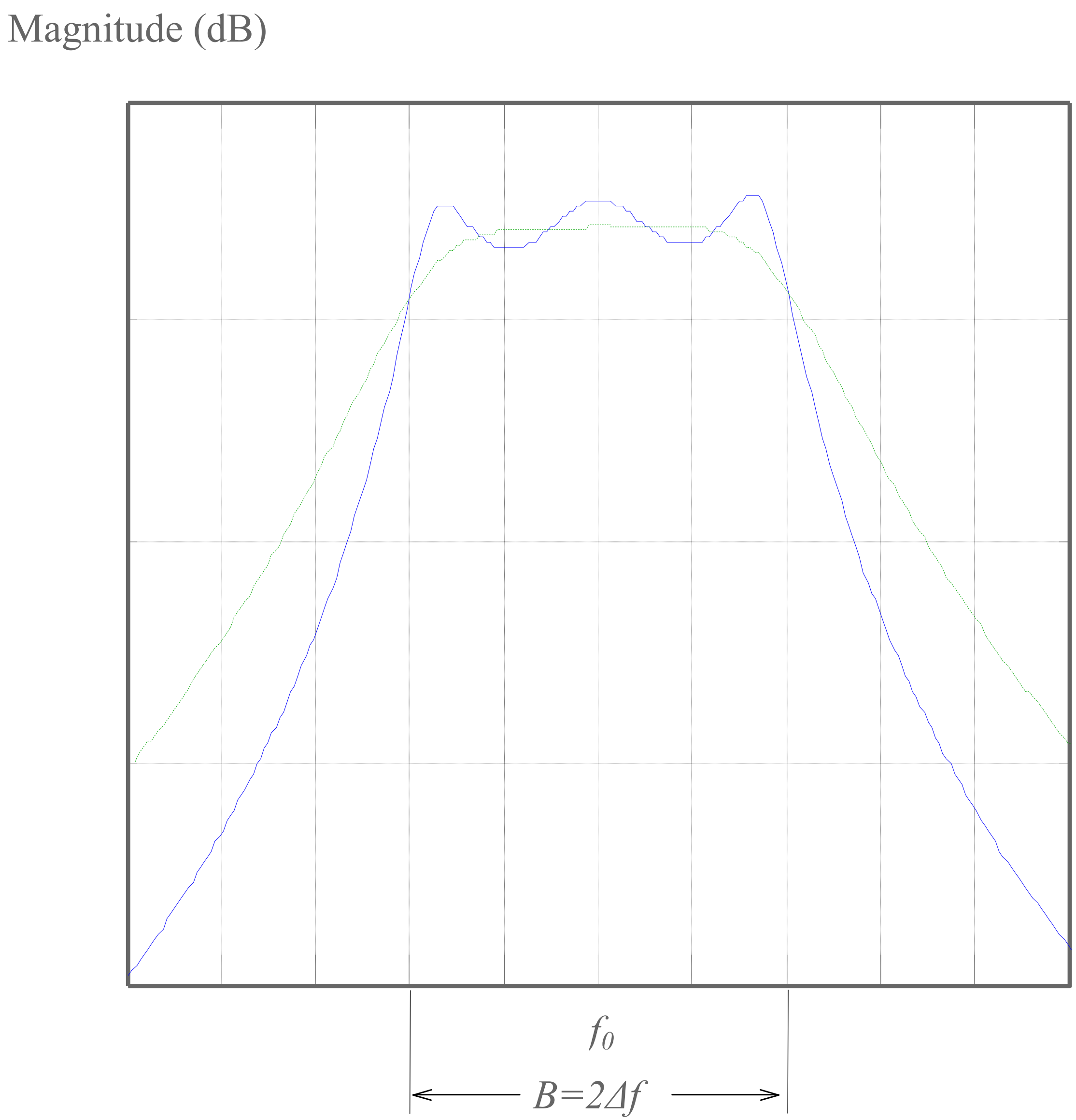
$$A_{vT} \cong A_{v1}(\omega_{p1}) \cdots A_{vn}(\omega_{pn}) \frac{\omega_{p1}}{2Q_{eff1}} \cdots \frac{\omega_{pn}}{2Q_{effn}} \frac{1}{(s - s_{p1}) \cdots (s - s_{pn})}$$

Important typical pole distributions:

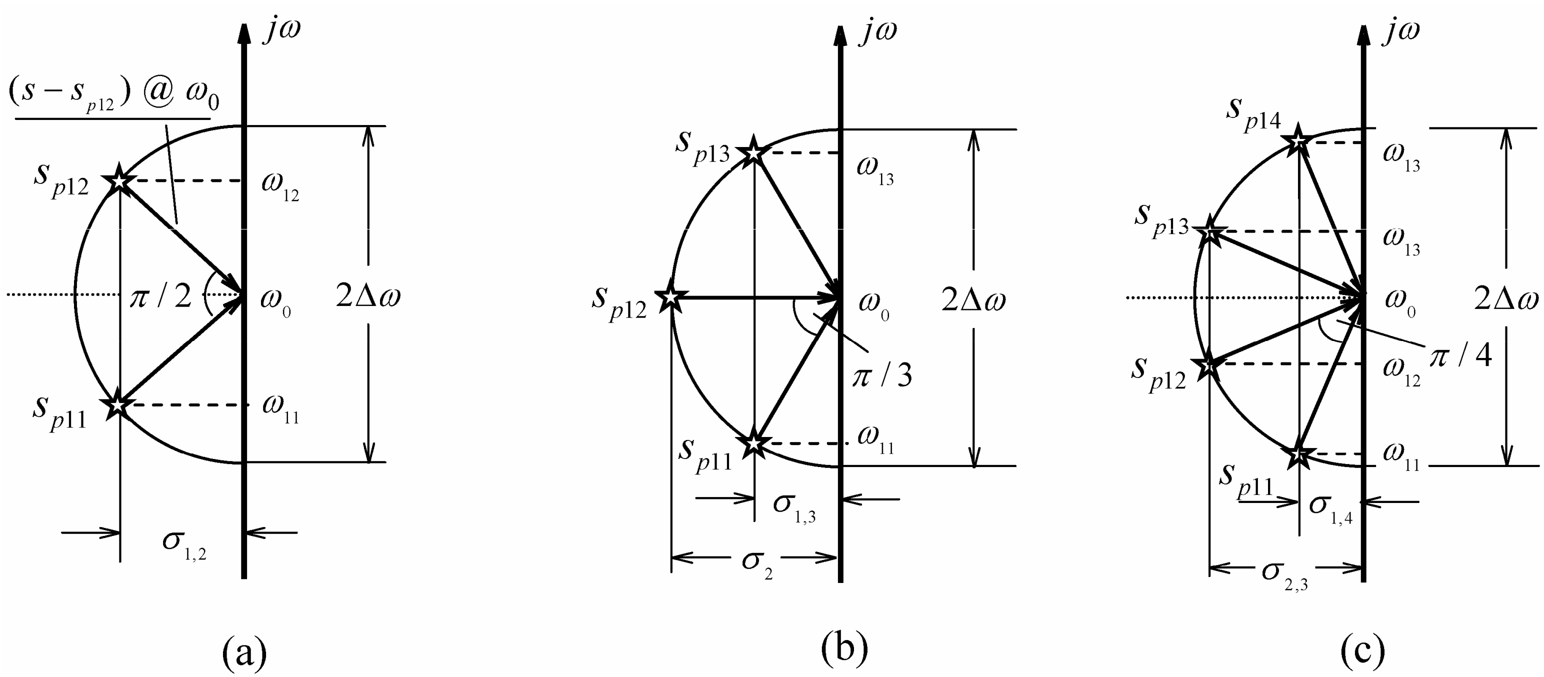
a) Butterworth type pole positioning for maximally flat frequency response.

b) Chebyshev type pole positioning for equi-ripple response with steeper side-walls

# Typical Butterworth and Chebyshev type characteristics having the same relative bandwidth



# 2, 3 and 4 pole Butterworth type pole distributions:




$$\sigma_{1,2} = -\frac{\omega_{11}}{2Q_1} = -\frac{\omega_{12}}{2Q_2}$$

$$\sigma_2 = -\frac{\omega_0}{2Q_2}$$

$$\sigma_{1,3} = -\frac{\omega_{11}}{2Q_1} = -\frac{\omega_{13}}{2Q_3}$$

$$\sigma_{1,4} = -\frac{\omega_{11}}{2Q_1} = -\frac{\omega_{14}}{2Q_4}$$

$$\sigma_{2,3} = -\frac{\omega_{12}}{2Q_2} = -\frac{\omega_{13}}{2Q_3}$$

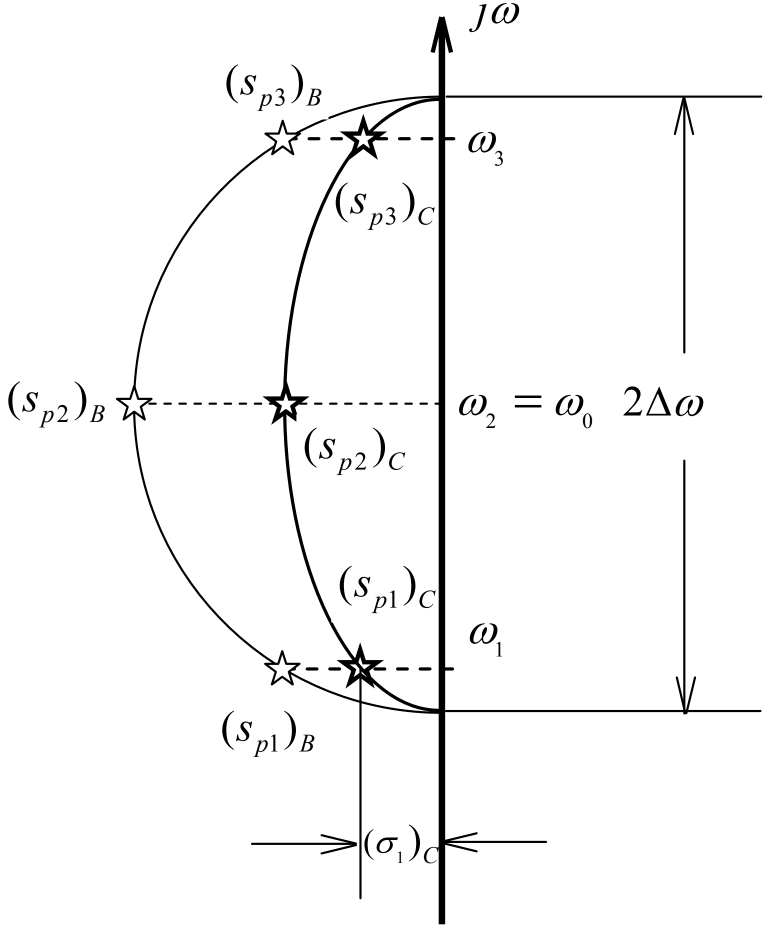
Approximate expressions for $2\Delta\omega \ll \omega_0$:

$$\sigma_{1,2} \cong -\frac{\omega_0}{2Q_1} = -\frac{\omega_0}{2Q_2} \;\Rightarrow\; Q_1 \cong Q_2$$

$$\sigma_{1,3} \cong -\frac{\omega_0}{2Q_1} = -\frac{\omega_0}{2Q_3} \;\Rightarrow\; Q_1 \cong Q_3$$

$$\sigma_{1,4} \cong -\frac{\omega_0}{2Q_1} = -\frac{\omega_0}{2Q_4} \;,\; \sigma_{2,3} \cong -\frac{\omega_0}{2Q_2} = -\frac{\omega_0}{2Q_3} \;\Rightarrow\; Q_1 \cong Q_4 \;,\; Q_2 \cong Q_3$$

# Converting Butterworth poles to Chebyshev poles




$$(\sigma_i)_C = \tanh\alpha \times (\sigma_i)_B$$

$$\alpha = \frac{1}{n}\sinh^{-1}\frac{1}{\sqrt{\varepsilon}} \; ,$$

$$\varepsilon = \log^{-1}\frac{r(\mathrm{dB})}{10} - 1$$

# $(\tanh \alpha)$ for $n = 2, 3, 4$

# and various ripple values

| $r$ (dB) | $n = 2$ | $n = 3$ | n = 4 |
|---|---|---|---|
| 0.05 | 0.898 | 0.750 | 0.623 |
| 0.1 | 0.859 | 0.696 | 0.567 |
| 0.2 | 0.806 | 0.631 | 0.505 |
| 0.3 | 0.767 | 0.588 | 0.467 |
| 0.4 | 0.736 | 0.556 | 0.439 |
| 0.5 | 0.709 | 0.524 | 0.416 |

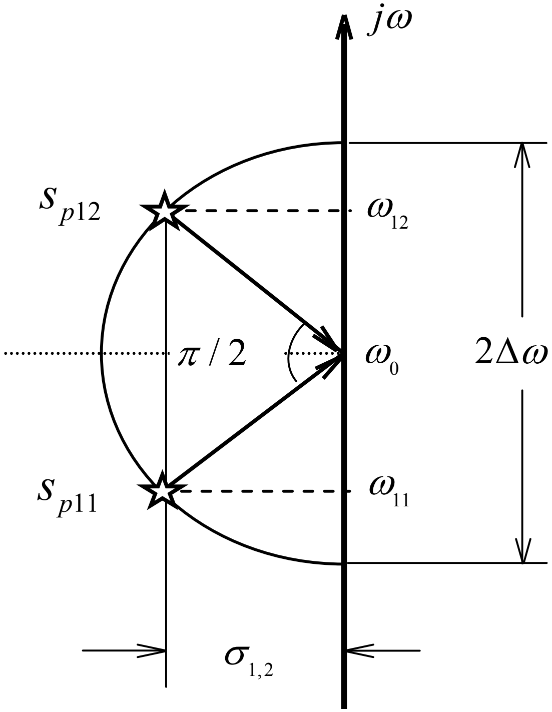
# Example: Two-stage stagger-tuned amplifier

Voltage gain: $\approx 40$ dB

Center frequency: $f_0 = 900$ MHz

Band width: $B = 120$ MHz

Response: Butterworth type




$$f_0 = 900 \text{ MHz}$$

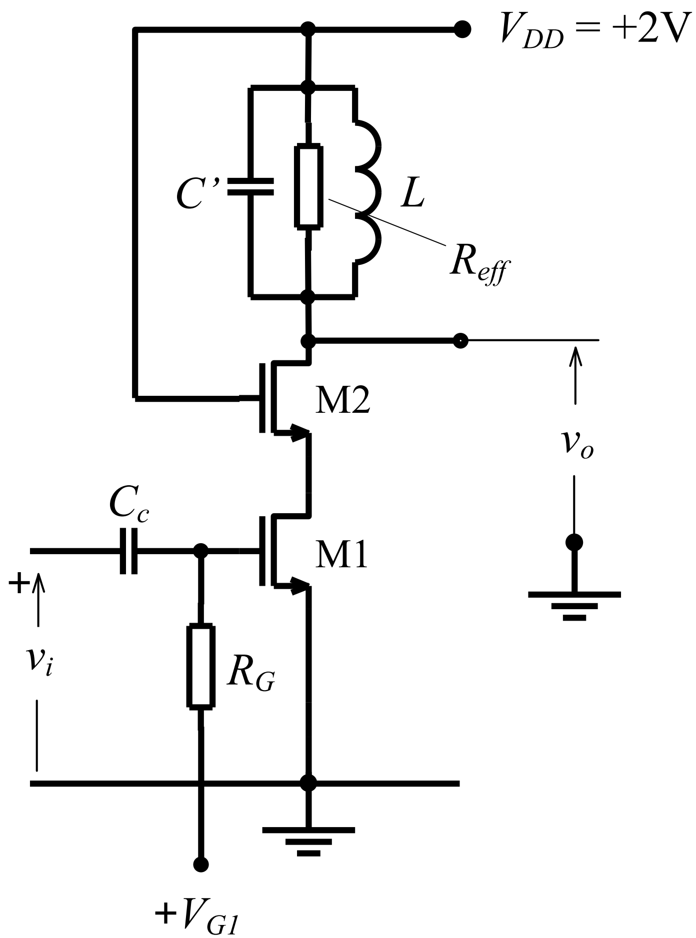
$$\Delta f = 60 \text{ MHz}$$

$$f_1 = 900 - \frac{60}{\sqrt{2}} = 857.6 \text{ MHz}$$

$$f_2 = 900 + \frac{60}{\sqrt{2}} = 942.4 \text{ MHz}$$

$$\sigma_1 = \sigma_2 = \frac{\omega_1}{2Q_1} = \frac{\omega_2}{2Q_2} = \frac{\Delta\omega}{\sqrt{2}}$$

# One of the stages




$L = 10$ nH, $Q_L = 10$

$Q_C = 40$

$R_L = 100$ k ohm

Technology: UMC018

$P_{tot} \leq 10$ mW

$$Q_{1(eff)} = \frac{\sqrt{2}\cdot\omega_1}{2\Delta\omega} = \frac{1}{\sqrt{2}}\frac{f_1}{\Delta f}$$

$$Q_{1(eff)} = \frac{1}{\sqrt{2}}\frac{857.6}{60} = 10.1$$

$$Q_{2(eff)} = \frac{1}{\sqrt{2}}\frac{942.4}{60} = 11.1$$

$Q$ enhancement is necessary for both!

$$\frac{1}{Q_{1(eff)}} = \frac{1}{Q_L} + \frac{1}{Q_C} + \frac{1}{Q_{N1}} \quad \rightarrow \quad \frac{1}{Q_{N1}} = \frac{1}{10.1} - \frac{1}{10} - \frac{1}{40} \quad \rightarrow \quad Q_{N1} = -38.5$$

$$R_{N1} = Q_{N1}\cdot\omega_0 L = -2154.5 \text{ ohm}$$

Similarly; $R_{N2} = -1617.3$ ohm

Voltage gain per stage: $\left|A_v\right| = 10$

$$A_{v1} = -g_{m1} \cdot R_{L1(eff)}$$

$$R_{L1(eff)} = Q_{1(eff)} \cdot L\omega_0 = 10.1 \times (10 \times 10^{-9}) \times (2\pi \times 900 \times 10^9) = 571 \text{ ohm}$$

$$g_{m1} = \frac{10}{571} = 17.5 \text{ mS}$$

Similarly;

$$R_{L2(eff)} = 627.7 \text{ ohm} \quad \rightarrow \quad g_{m2} = 16 \text{ mS}$$

# 1st Stage:

$$g_{m1} = 17.5 \text{ mS}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}(W/L)I_D} \quad \rightarrow \quad W \cdot I_D = \frac{g_{m1}^2 L}{2\mu_n C_{ox}}$$

$$C_{ox} = 8.2 \times 10^{-7} \text{ F/cm}^2$$

$$\mu_{n0} = 326 \text{ cm}^2/\text{V.s} \quad \rightarrow \quad \mu_n = 196 \text{ cm}^2/\text{V.s} \;\; \text{for } V_{GS} = 1 \text{ V}$$

$\downarrow$

$$W \cdot I_D = 1.71 \times 10^{-5}: \quad I_D = 1 \text{ mA}; \quad W = 171\ \mu\text{m} \text{ (excessive parasitics)}$$

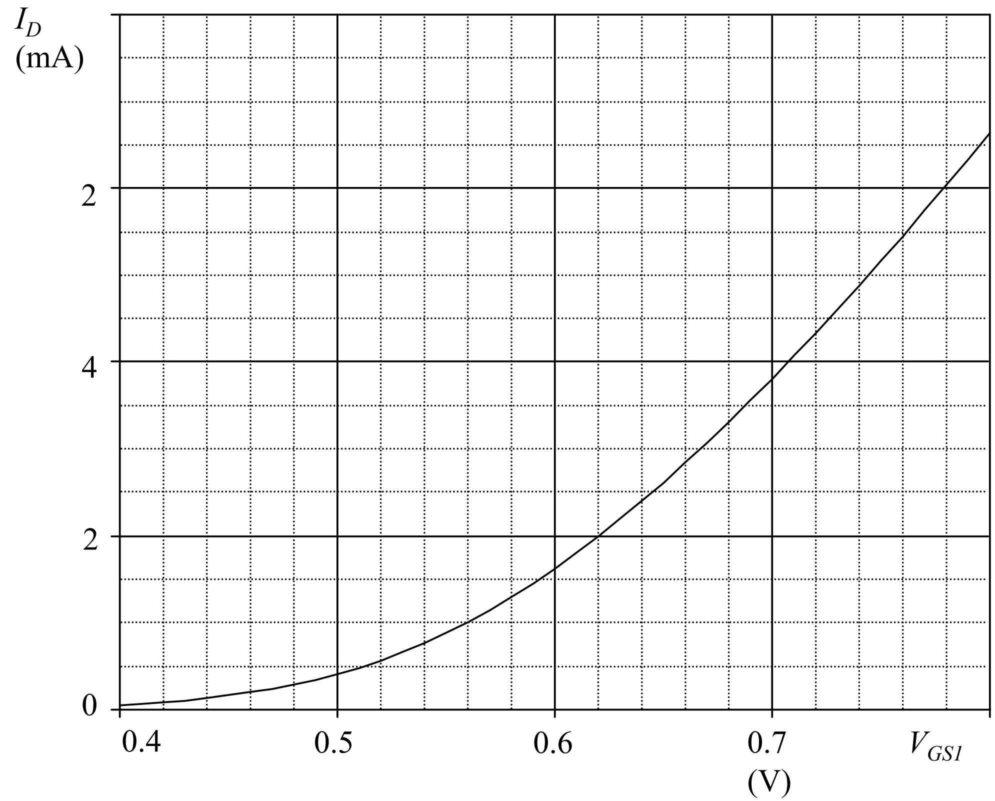
$$I_D = 2 \text{ mA}; \quad W = 85.7\ \mu\text{m} \text{ (preferred)}$$

$V_{GS}$ for $I_D = 2$ mA:

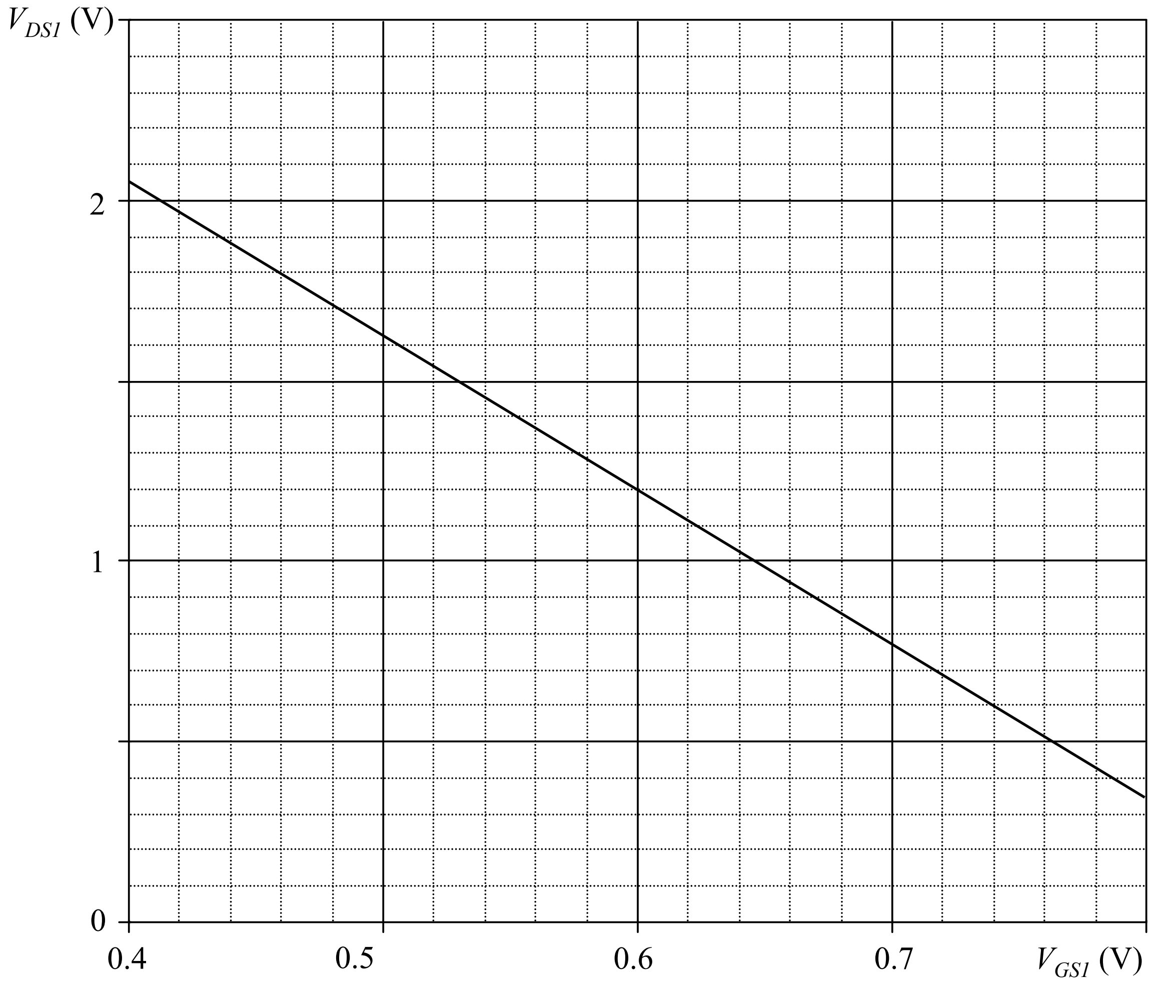
$$\left(V_{GS} - V_T\right)^2 = \frac{2\times(2\times10^{-3})}{196\times(8.2\times10^{-7})\times(85.7/0.18)} = 0.052 \;\rightarrow\; V_{GS} = 0.55 \text{ V}$$

With DC sweep:

$V_{GS} = 0.62$ V



Check the DC operating point (the saturation control) for M1 with $W_2 = W_1$



## Capacitors:

- Resonance capacitance of the first stage:

(Parasitics and the load cap. included)

$$C_{1T} = \frac{1}{\omega_1^2 L} = \frac{1}{(2\pi \times 857.6 \times 10^6)^2 \times (10 \times 10^{-9})} = 3.44\ \text{pF}$$

- Resonance capacitance of the second stage:

(Parasitics and the load cap. included)

$$C_{2T} = \frac{1}{\omega_2^2 L} = \frac{1}{(2\pi \times 942.4 \times 10^6)^2 \times (10 \times 10^{-9})} = 2.85\ \text{pF}$$

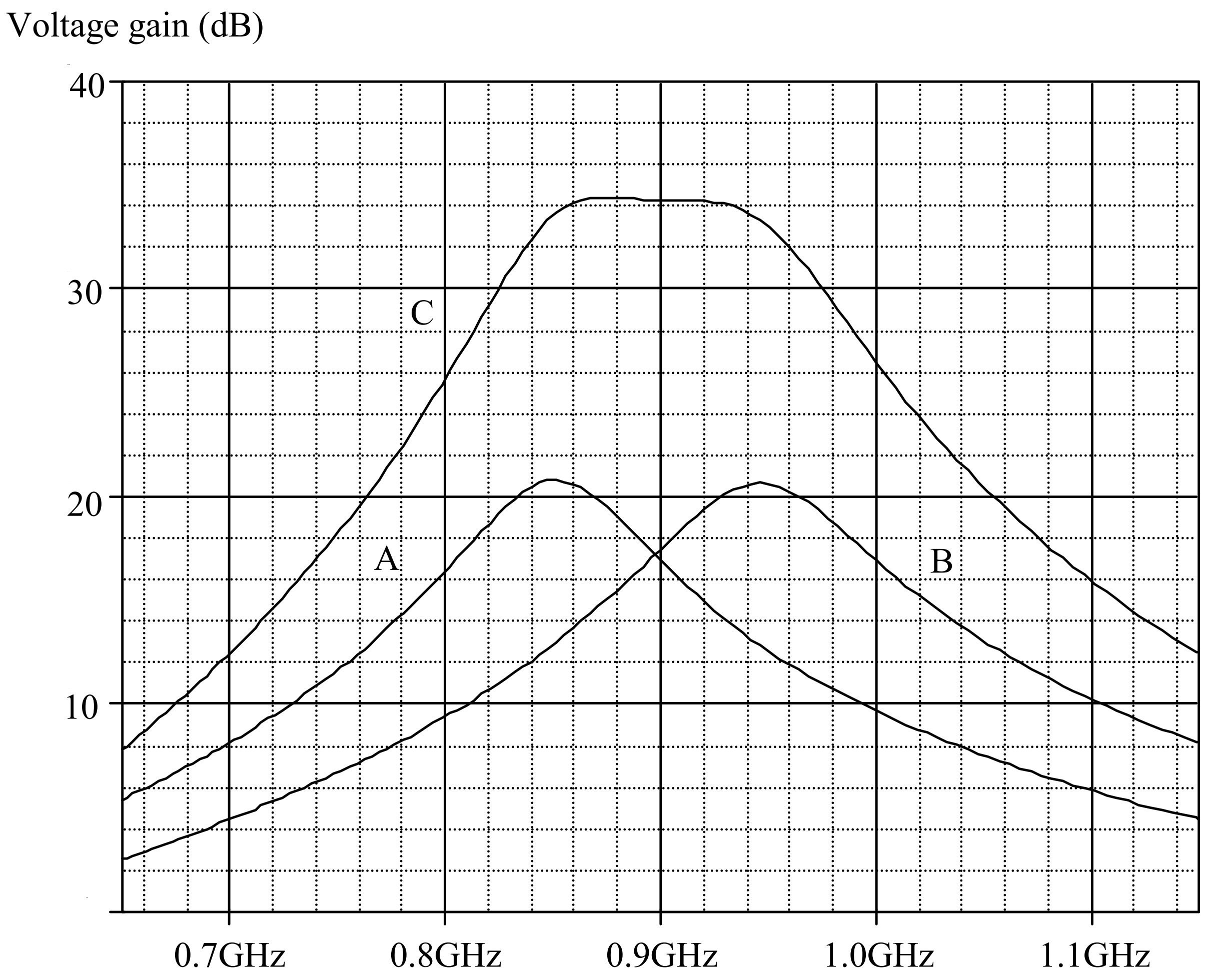
- Coupling capacitor for $\leq 1\%$ signal loss:

$$C_c \geq 100 \times C_{in1}$$

$$C_c \geq 100 \times \left[ (W_1 \times L \times C_{ox}) + \text{CGSO} \times W_1 \right] = 2.66\ \text{pF}$$

# pSpice Simulation results

## Frequency characteristics: (A) First stage, (B) Second stage, (C) Total



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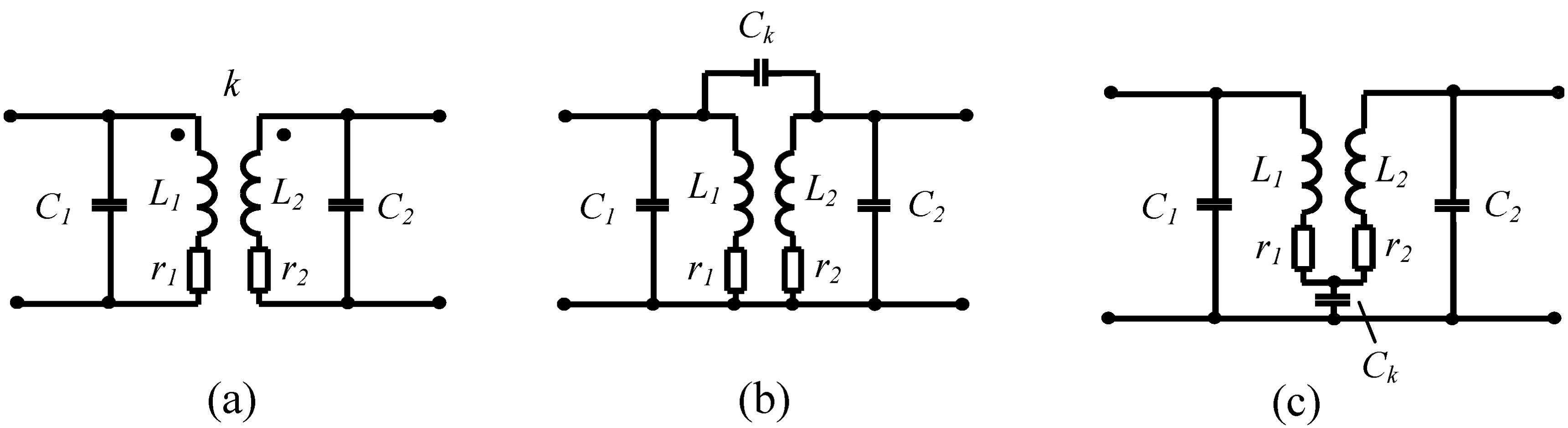
# The classical solution for 2nd order Butterworth or Chebyshev characteristics: Use of coupled, double tuned circuits.




(a) Magnetic coupling.

(b) Capacitive voltage coupling.

(c) Capacitive current coupling.

# Transfer impedance of a magnetic coupled double tuned circuit:

$$\frac{V_2}{I_1} = \frac{sM}{(1+sr_1C_1+s^2L_1C_1)(1+sr_2C_2+s^2L_2C_2)-s^4M^2C_1C_2}$$

$$\omega_0^2 = \frac{1}{L_1C_1} = \frac{1}{L_2C_2}\ , \quad Q_1 = \frac{1}{\omega_0 C_1 r_1}\ , \quad Q_2 = \frac{1}{\omega_0 C_2 r_2}$$

$$\frac{V_2}{I_1} = \frac{sM\omega_0^4 Q_1 Q_2}{(s^2Q_1+s\omega_0+\omega_0^2Q_1)(s^2Q_2+s\omega_0+\omega_0^2Q_2)-s^4k^2Q_1Q_2}$$

Denominator polynome with $s^4 Q_1 Q_2 (1-k^2) = s^4 Q_1 Q_2 (1-k)(1+k)$ :

$$\left[(s^2 Q_1 (1-k) + s\omega_0 + \omega_0^2 Q_1)\right]\left[(s^2 Q_2 (1+k) + s\omega_0 + \omega_0^2 Q_2)\right]$$

Poles:

$$s_1, s_1^{'} = -\frac{\omega_0}{2Q_1(1-k)} \mp j \frac{\omega_0}{Q_1(1-k)} \sqrt{4Q_1^2(1-k) - 1}$$

$$s_2, s_2^{'} = -\frac{\omega_0}{2Q_2(1+k)} \mp j \frac{\omega_0}{Q_2(1+k)} \sqrt{4Q_2^2(1+k) - 1}$$

For small $k$ values; $4Q_1^2(1-k) \gg 1,\ \ 4Q_2^2(1+k) \gg 1$

$$\Downarrow$$

$$s_1, s_1^{'} \cong -\frac{\omega_0}{2Q_1(1-k)} \mp j\frac{\omega_0}{\sqrt{1-k}}$$

$$s_2, s_2^{'} \cong -\frac{\omega_0}{2Q_2(1+k)} \mp j\frac{\omega_0}{\sqrt{1+k}}$$

With $(1-k) \cong 1$ , $(1+k) \cong 1$ and $1/\sqrt{1-k} \cong 1+(k/2)$ , $1/\sqrt{1+k} \cong 1-(k/2)$ ;

$$s_1, s_1^{'} \cong -\frac{\omega_0}{2Q_1(1-k)} \mp j\omega_0\left(1+\frac{k}{2}\right)$$

$$s_2, s_2^{'} \cong -\frac{\omega_0}{2Q_2(1+k)} \mp j\omega_0\left(1-\frac{k}{2}\right)$$

For a Butterworth or Chebyshev response

the real parts of the poles must be equal:

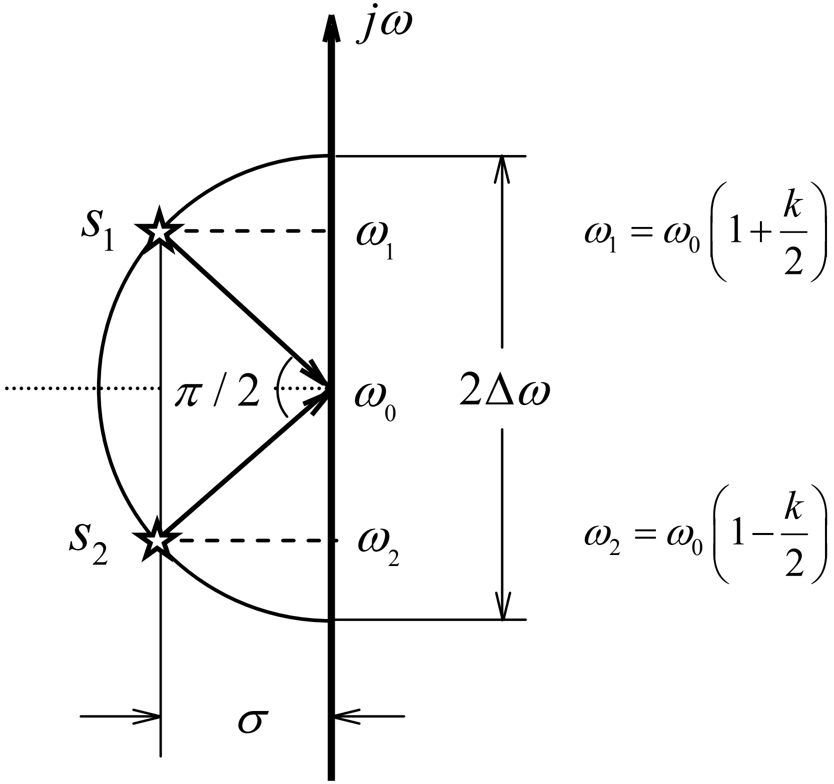
$$\sigma = -\frac{\omega_0}{2Q_1(1-k)} = -\frac{\omega_0}{2Q_2(1+k)}$$

$$k \ll 1$$

$$\Downarrow$$

$$\sigma \cong -\frac{\omega_0}{2Q_1} \cong \frac{\omega_0}{2Q_2} \cong \frac{\omega_0}{2Q}$$

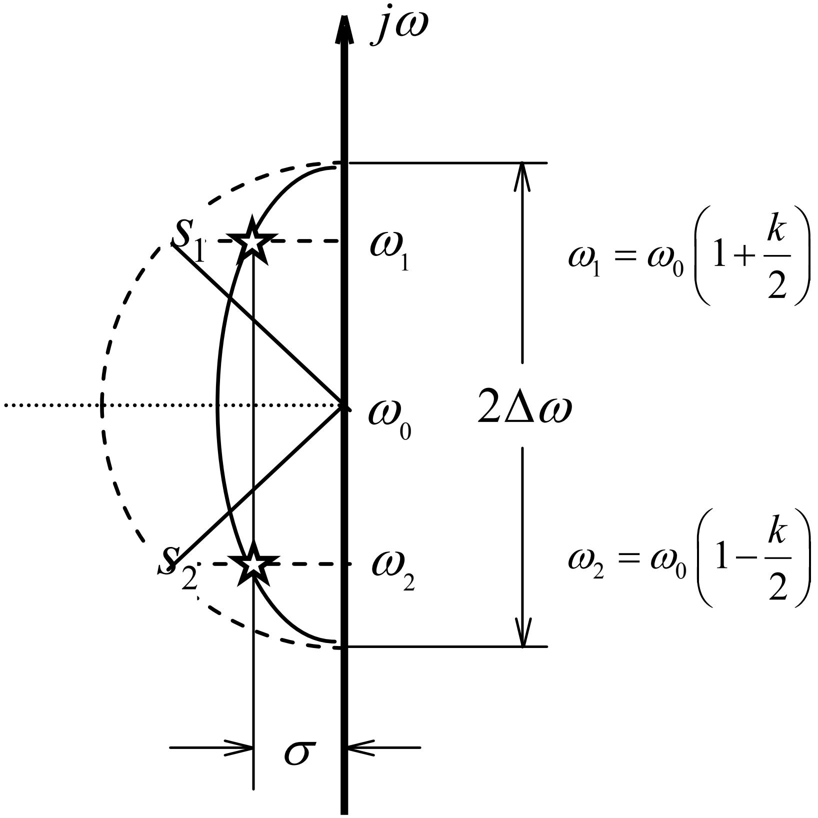
# Positions of the poles for a Butterworth characteristic:




$$|\sigma| = \frac{\omega_0}{2Q} = \omega_0 \frac{k}{2} \quad \Rightarrow \quad k = \frac{1}{Q}$$

$$2\Delta\omega = 2 \times \left(\sqrt{2}\,|\sigma|\right) = \sqrt{2}\,\frac{\omega_0}{Q} \quad \Rightarrow \quad B = 2\Delta f = \sqrt{2}\,\frac{f_0}{Q}$$

Positions of the poles for a Chebyshev characteristic:




(b)

$$(\sigma)_C = \frac{\omega_0}{2Q} = \tanh\alpha \times \omega_0 \frac{k}{2}$$

$$\Downarrow$$

$$k = \frac{1}{\tanh\alpha}\frac{1}{Q}$$

$$2\Delta\omega = 2\times\sqrt{2}\times\omega_0\frac{k}{2}$$

$$= \sqrt{2}\times\omega_0\frac{1}{\tanh\alpha}\frac{1}{Q}$$

$$\Downarrow$$

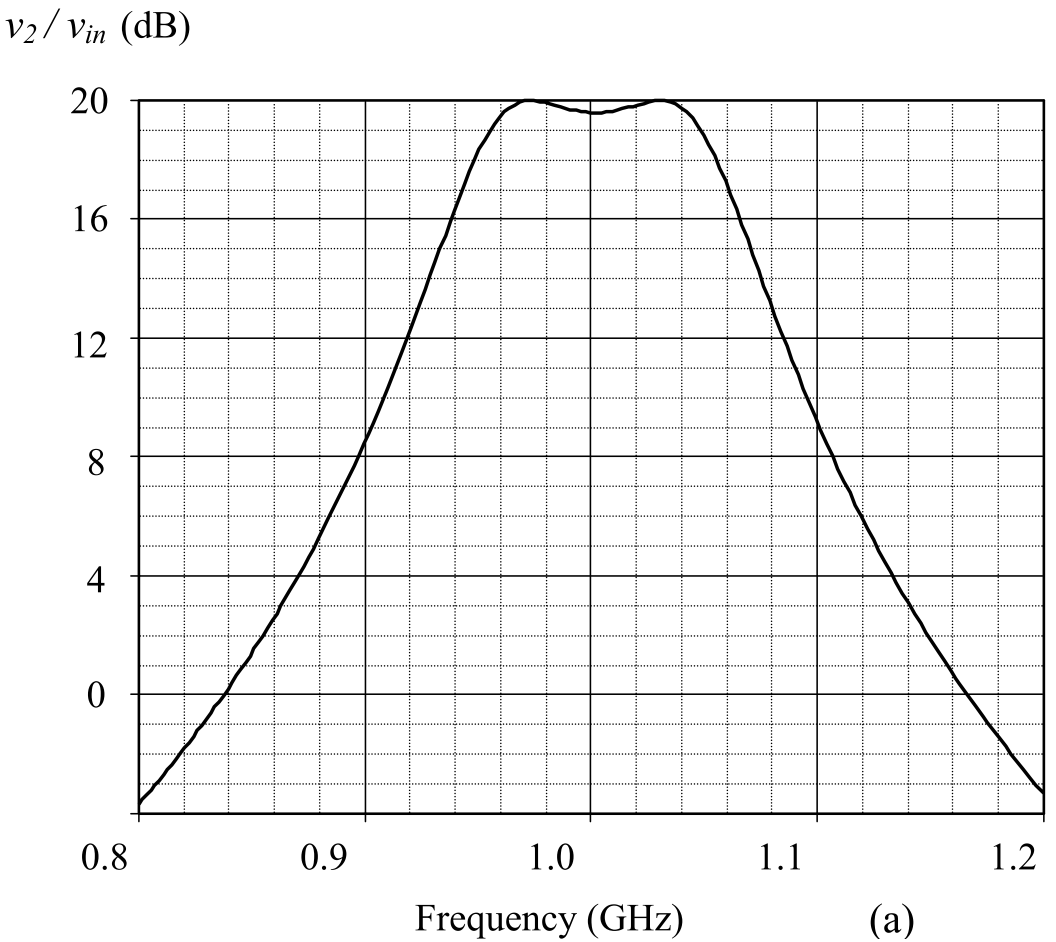
$$2\Delta f = B = \sqrt{2}\times f\frac{1}{\tanh\alpha}\frac{1}{Q}$$

Example: $r = 0.5\ \text{dB} \quad \rightarrow \quad \tanh\alpha = 0.709 \cong \frac{1}{\sqrt{2}}$

$$k = \frac{1}{Q \times \tanh\alpha} \cong \frac{\sqrt{2}}{Q} \quad \rightarrow \quad kQ = \sqrt{2}$$

$$B = \sqrt{2} \times f_0 \frac{1}{\tanh\alpha}\frac{1}{Q} \cong 2\frac{f_0}{Q}$$

# Problem 4-6 (Simulation result)



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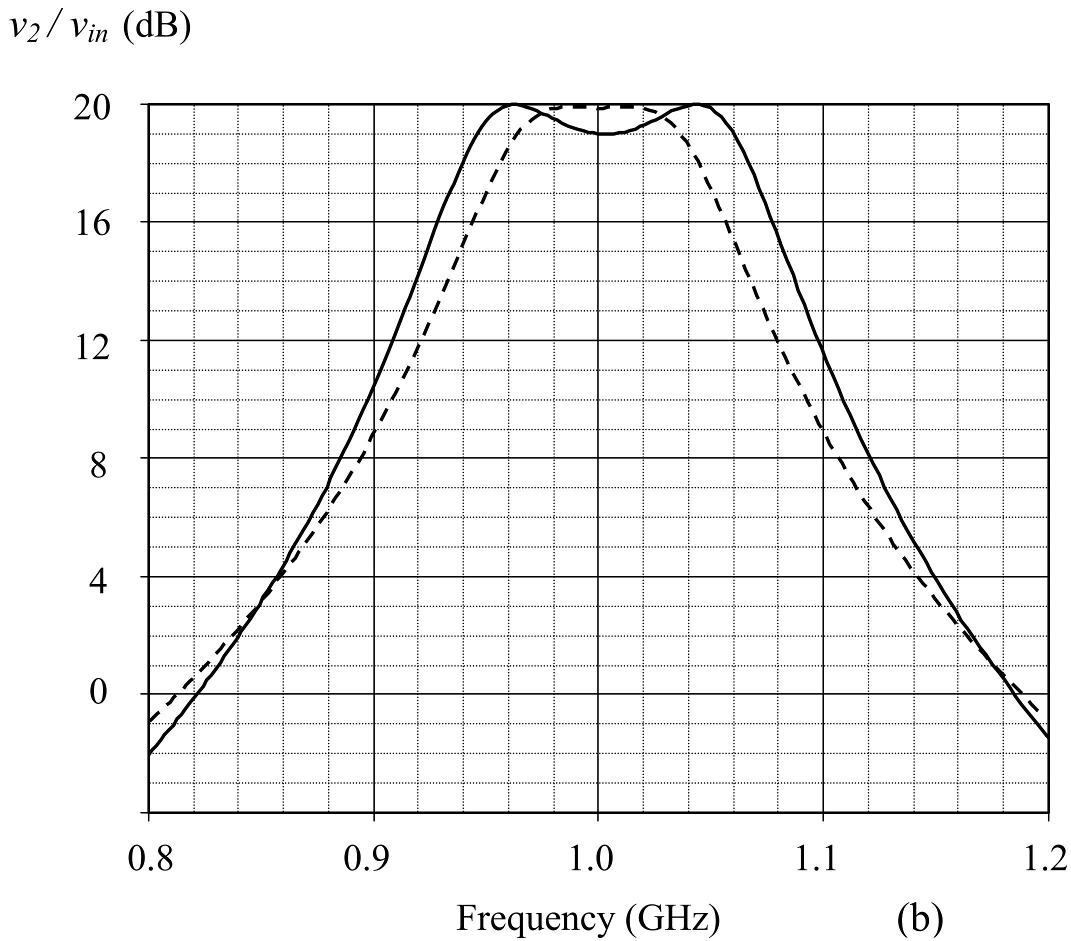
# Problem 4-6. (Simulation results for $\pm 20$ % tolerance of $k$)




(b)

Transformation formula to apply the expressions derived for magnetic coupling to capacitive coupled circuits:

a) For capacitive voltage coupling:

$$C_k = -\frac{k}{\omega_0^2 \sqrt{L_1 L_2}}$$

b) For capacitive current coupling:

$$C_k = -\frac{1}{k\omega_0^2 \sqrt{L_1 L_2}}$$