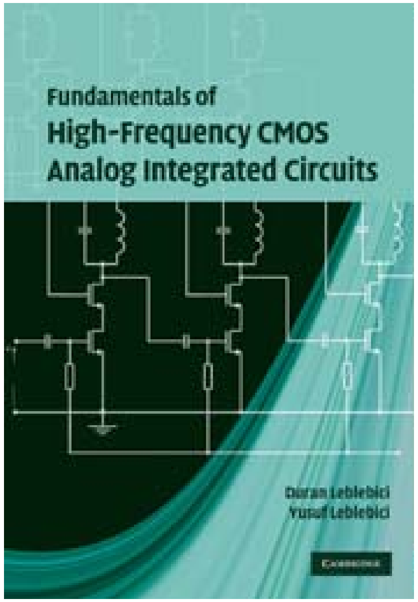


Fundamentals of High-Frequency CMOS Analog Integrated Circuits

Duran Leblebici
Yusuf Leblebici

Chapter 4

Frequency-Selective RF Circuits (Low-Noise Amplifiers: LNAs)

Types of LNAs:

- Tuned LNAs (Conventional)

  (The selectivity is provided by the internal tuned load of the amplifier)

- Wide-band LNAs

  (The selectivity is provided by an external input filter)

Requirements:

- Signal transfer from the incoming electromagnetic wave with maximum efficiency,

- Minimum interactions of different incoming signals in the amplifier,

- Minimum noise, generated in the amplifier.

- Signal transfer from the incoming electromagnetic wave to the output port of the antenna:

$$v_A = \alpha_{(antenna)} \times E$$

($\alpha_{(antenna)}$ depends on the type and the dimensions of the antenna and the direction of the incoming wave)

- Signal transfer from the output port of the antenna to the input of the LNA:

$$v_{in} = v_A \times \beta_{(coupling)}$$

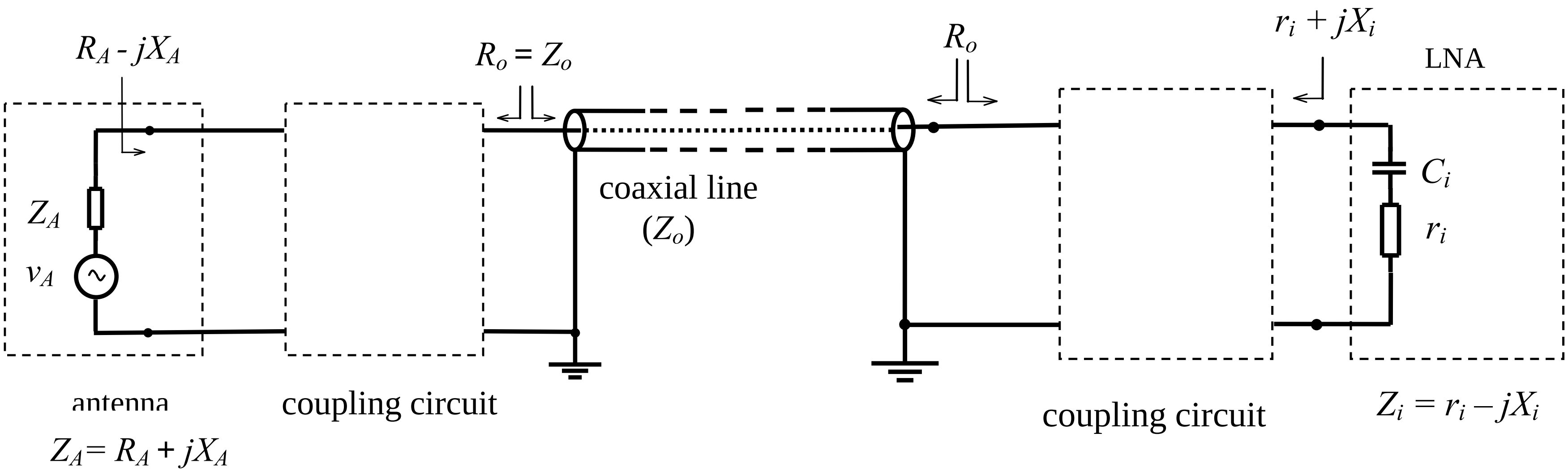
($\beta_{(coupling)}$ depends on the properties of the connection and the method of matching)

- Signal transfer from the incoming electromagnetic wave to the input of the LNA:

$$v_{in} = \alpha_{(antenna)} \times \beta_{(coupling)} \times E$$

Maximum signal transfer from antenna to the LNA

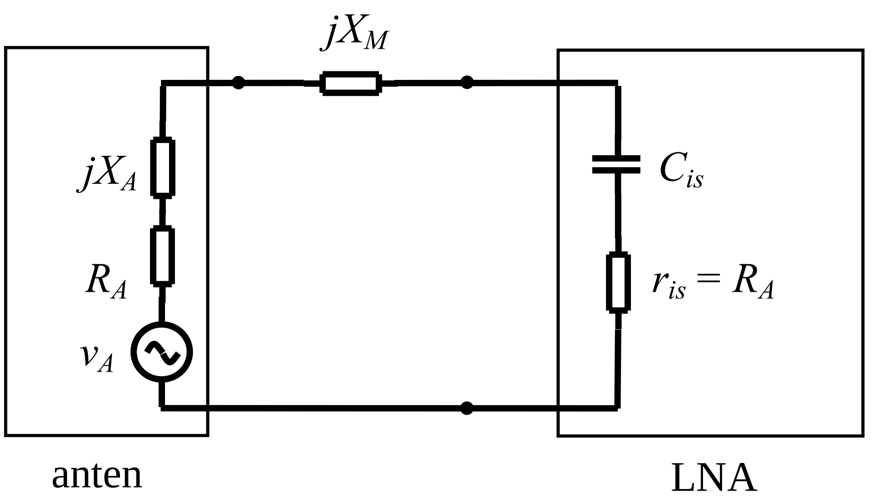
a) Antenna is connected to the LNA with a transmission line:



b) The output port of the antenna is very close to the input of the amplifier:

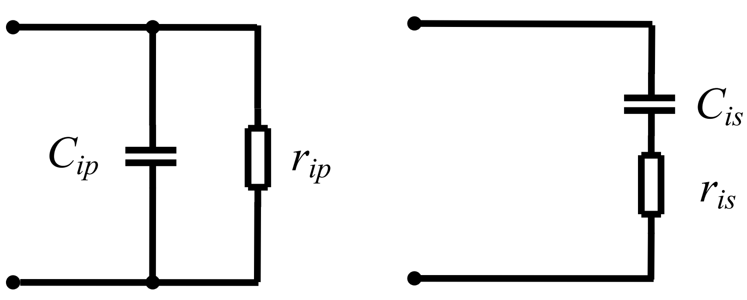
(No transmission line is needed)

b1) With maximum power transfer condition:



$$jX_A + jX_M + \frac{1}{j\omega_0 C_i} = 0$$

Conversion of the the parallel C_{ip} and r_{ip} input components to the parallel C_{is} and r_{is}

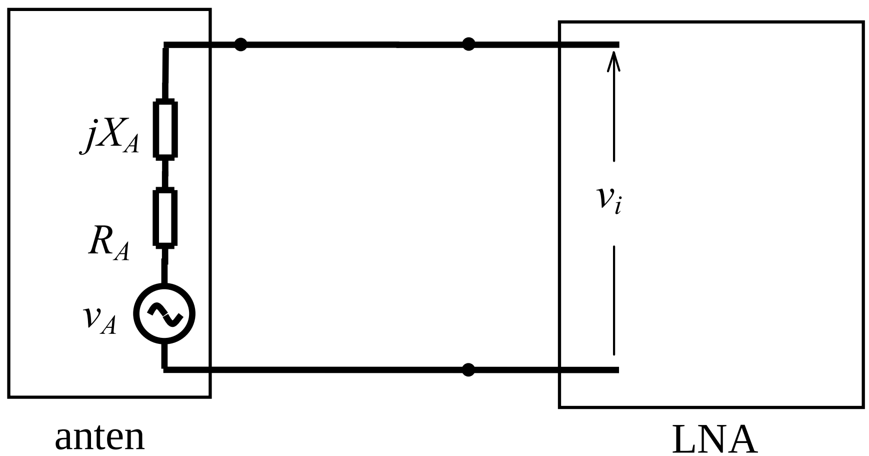


$$r_{ip} = \frac{1+\omega^2 C_{is}^2 r_{is}^2}{\omega^2 C_{is}^2 r_{is}}$$

$$C_{ip} = \frac{C_{is}}{1+\omega^2 C_{is}^2 r_{is}^2}$$

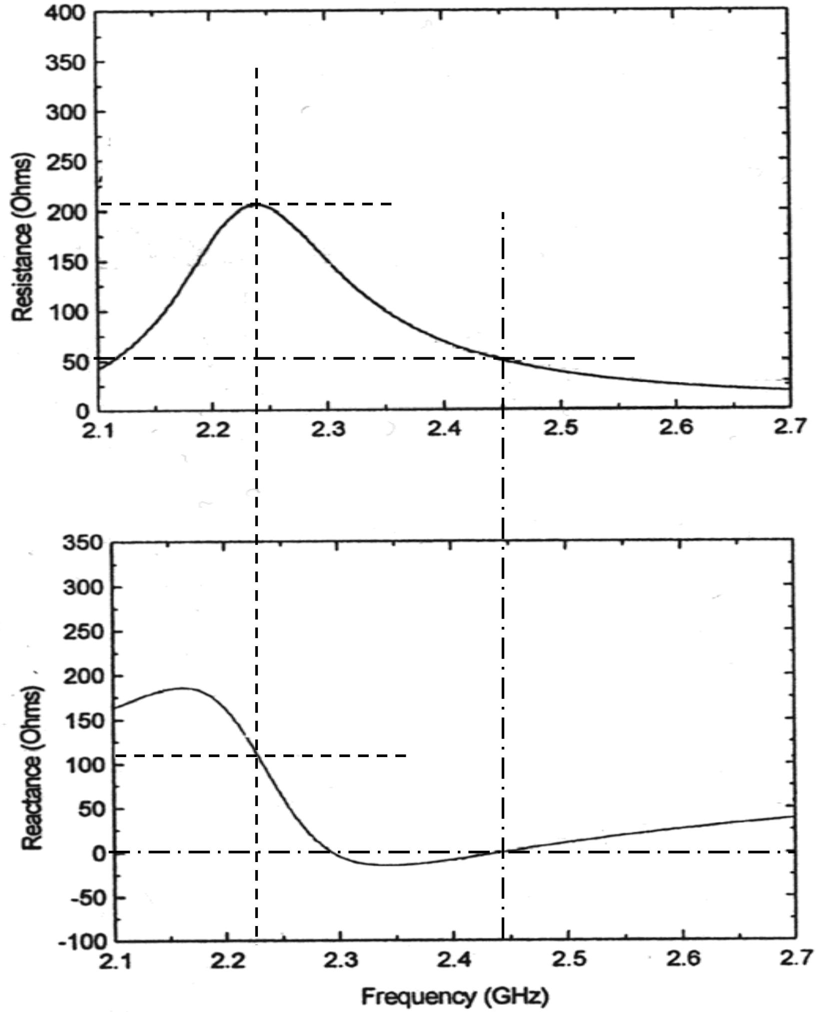
@ ω

b2) With maximum voltage transfer condition:



$$Z_i \gg Z_A \Rightarrow v_i \cong v_A$$

Example: PIFA (printed inverted-F antenna)
(Z. Feng et al.)



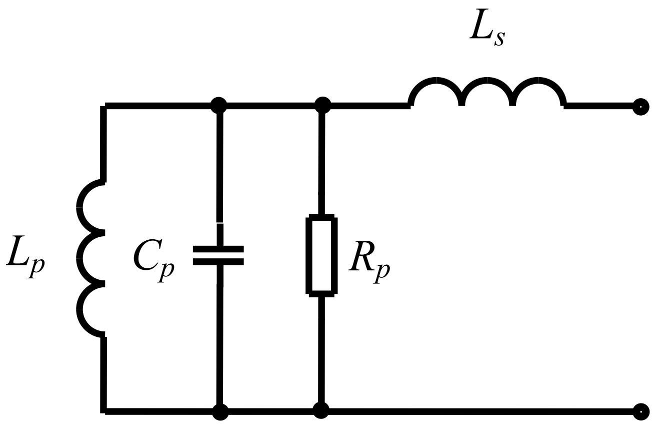
Frequency for which the impedance is 50 ohm (real):

f(50 ohm) = 2440 MHz

At resonance ($f_0 = 2225$ MHz)

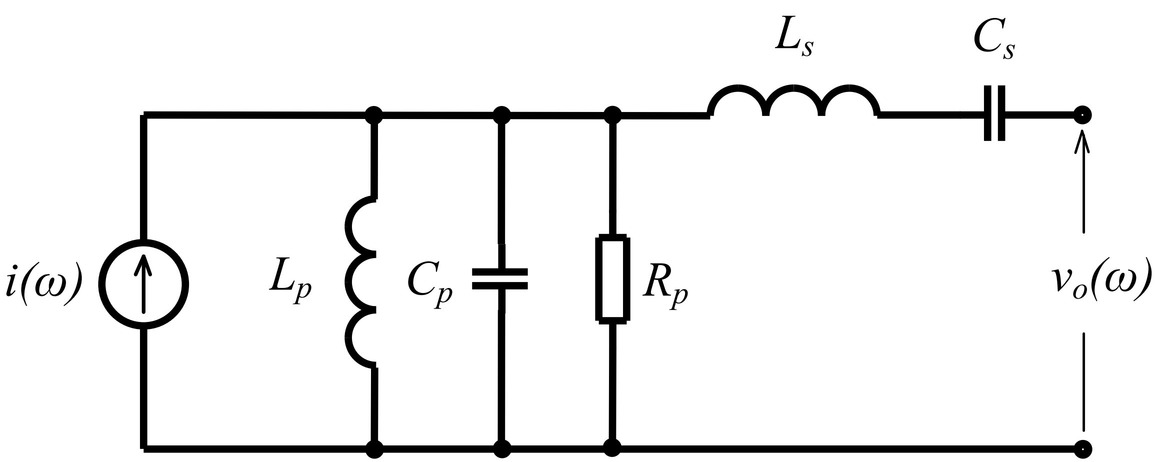
$Z_A = (210 + j\ 110)$ ohm

Derived equivalent circuit

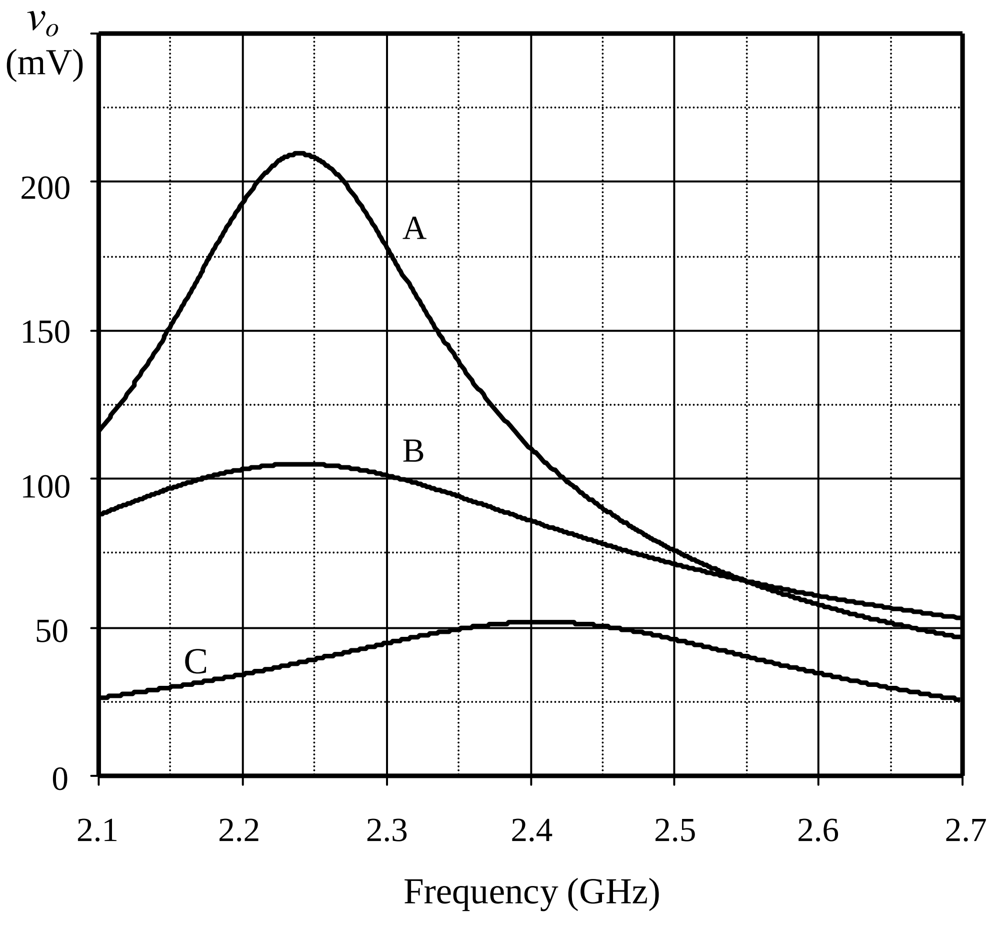


Rp=210 ohm
Cp=3.96 pF
Lp=1.27 nH
Ls=6.39 nH

With a current source representing the illuminating wave:



Output voltages for various loading conditions:

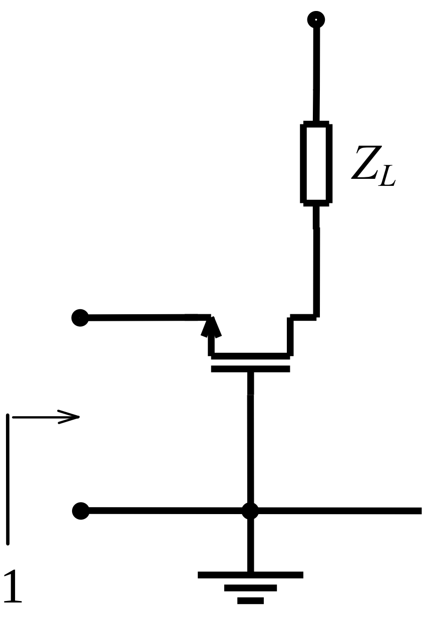


(A) High impedance load
(B) 210 ohm resistive load
(C) 50 ohm resistive load

(For 1 mA drive current)

Suitable circuits as antenna amplifier

a) Common gate amplifier

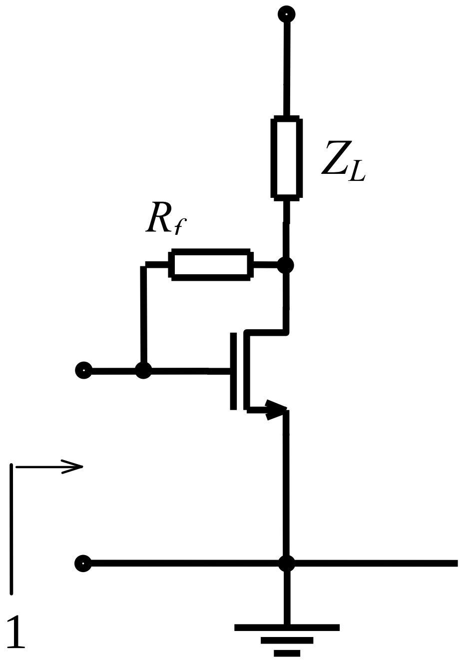


$$Z_i = \frac{1}{g_m + j\omega C_{gs}}$$

(a)

- Low input impedance.
- High output impedance.
- Suitable for tuned load.
- Suitable for wide-band applications with a resistive load.

b) Trans-impedance amplifier with parallel current feedback:

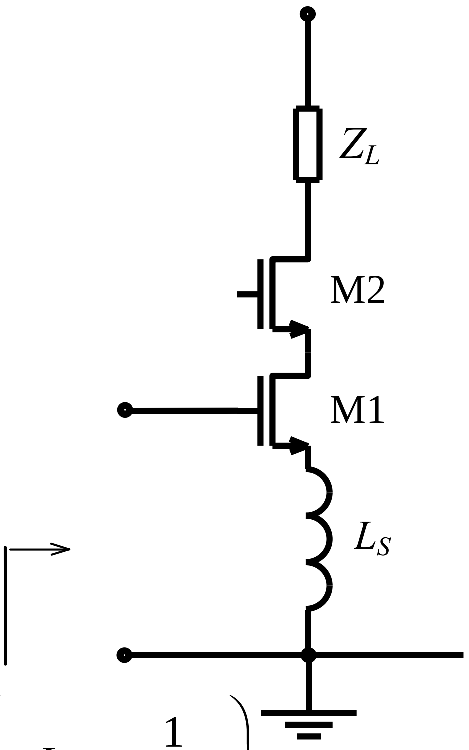


$$Z_i \approx \frac{1}{g_m + j\omega C_{gs}}$$

(b)

- Low input impedance.
- Low output impedance.
- Not suitable for tuned load.
- Suitable for wide-band applications with a resistive load.

c) Amplifier with inductive source degeneration:

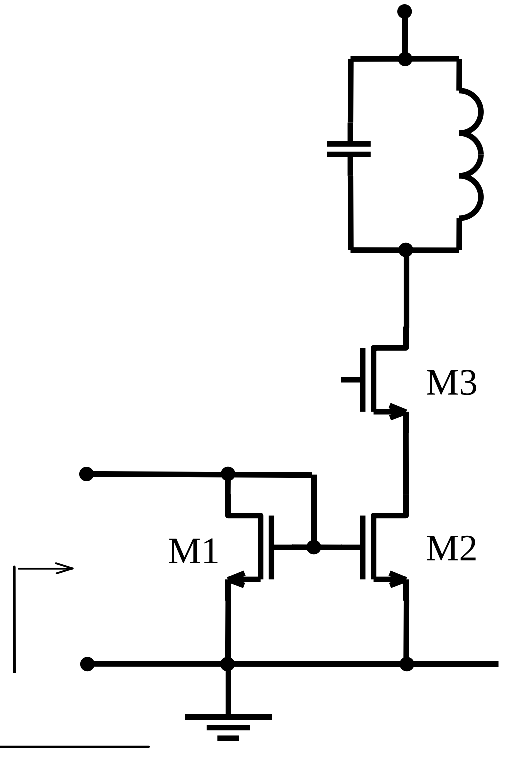


$$Z_i = \left(\frac{g_{m1} L_S}{C_{gs1}} \right) + j \left(\omega L_S - \frac{1}{\omega C_{gs1}} \right)$$

(c)

- Low input impedance for a certain frequency.
- High output impedance.
- Suitable for tuned load.
- Not suitable for wide-band applications.

d) Current mirror input amplifier



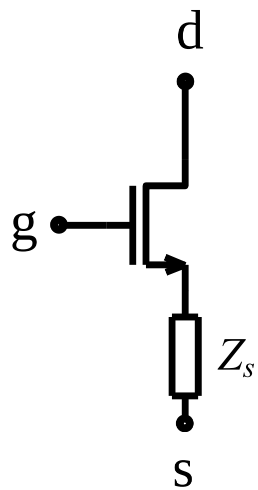
$$Z_i = \frac{1}{g_{m1} + j\omega(C_{gs1} + C_{gs2})}$$

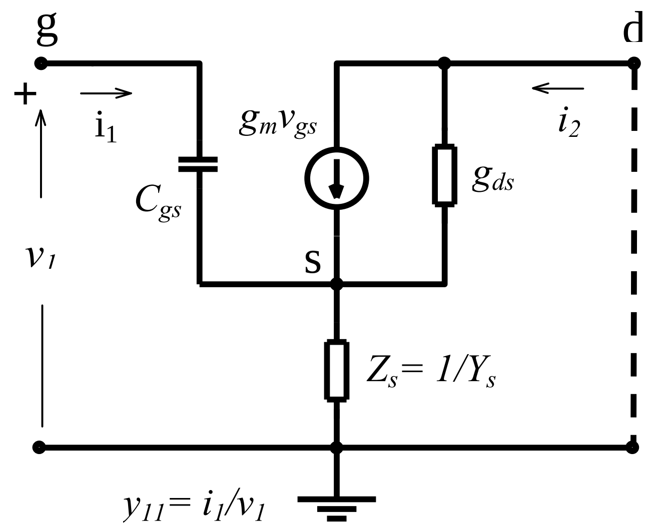
- Low input impedance.
- High output impedance.
- Suitable for tuned load.
- Suitable for wide-band applications with a resistive load.

Thanks to its better noise performance,

the source degenerated circuit is extensively used for tuned applications.

The input impedance of a source degenerated transistor:

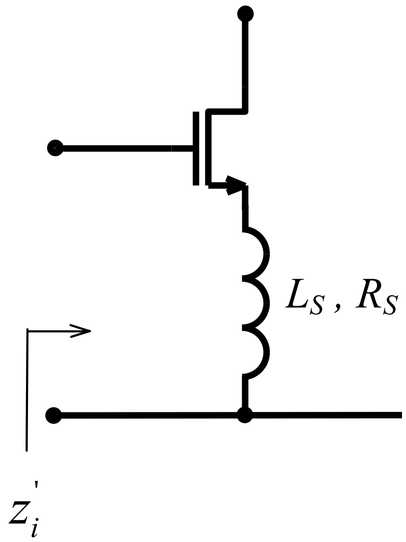


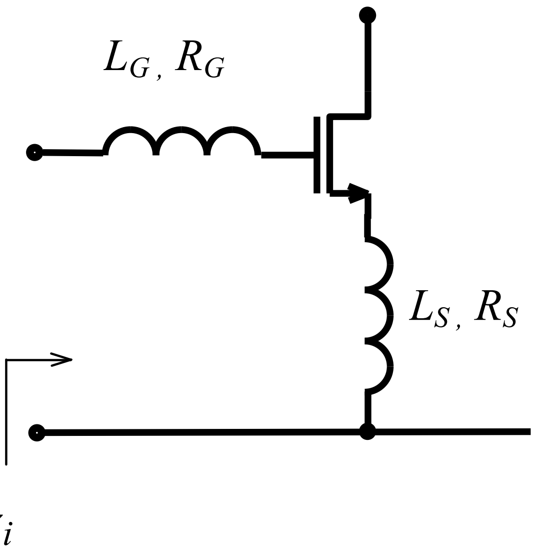


$$y_{11} \cong \frac{sC_{gs}}{1+(g_m + sC_{gs})Z_s}$$

Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

Input impedance of a inductive source degenerated transistor

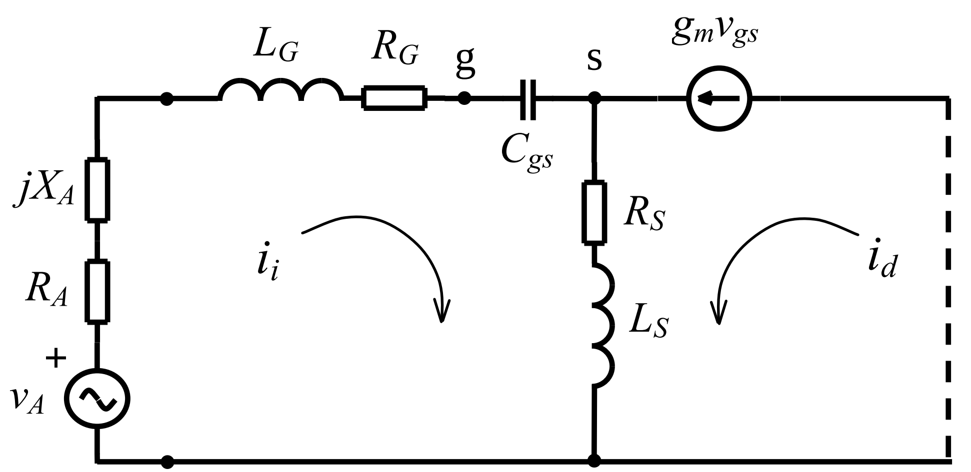




$$z_i' \cong \frac{1}{y_{in}} = \left(\frac{g_m L_S}{C_{gs}} + R_S \right) + sL_S + \frac{1 + g_m R_S}{sC_{gs}} = r_i' + sL_S + \frac{1}{sC_i'}$$

$$z_i = (r_i' + R_G) + s(L_S + L_G) + \frac{1}{sC_i'} \ , \ z_i = (r_i' + R_G) \ @ \ \omega_0 = \sqrt{1/(L_S + L_G)C_i'}$$

The trans-admittance of an inductive source degenerated amplifier



$$Y_{mG}(\omega_0) = \frac{i_d}{v_A} = \frac{g_m}{g_m R_S + j\omega_0 \left[C_{gs}(R_A + R_G + R_S) + g_m L_S \right]}$$

If the input impedance is matced to the antenna impedance;

$$Y_{mG}(\omega_0) = \frac{g_m}{g_m R_S + j\omega_0 C_{gs}(2R_A)}$$

$$\left|Y_{mG}(\omega_0)\right| = \frac{g_m}{\sqrt{(g_m R_S)^2 + (2\omega_0 C_{gs} R_A)^2}}$$

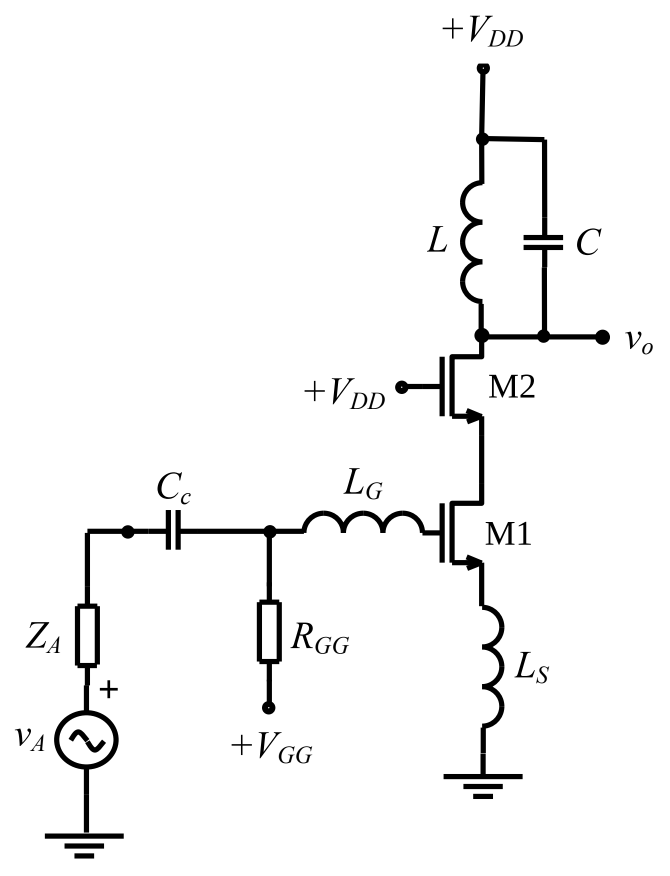
For $g_m R_S \ll 2\omega_0 C_{gs} R_A$

$$\left|Y_{mG}(\omega_0)\right| \cong \frac{g_m}{2\omega_0 C_{gs} R_A}$$

Voltage gain for the resonance frequency of the load impedance:

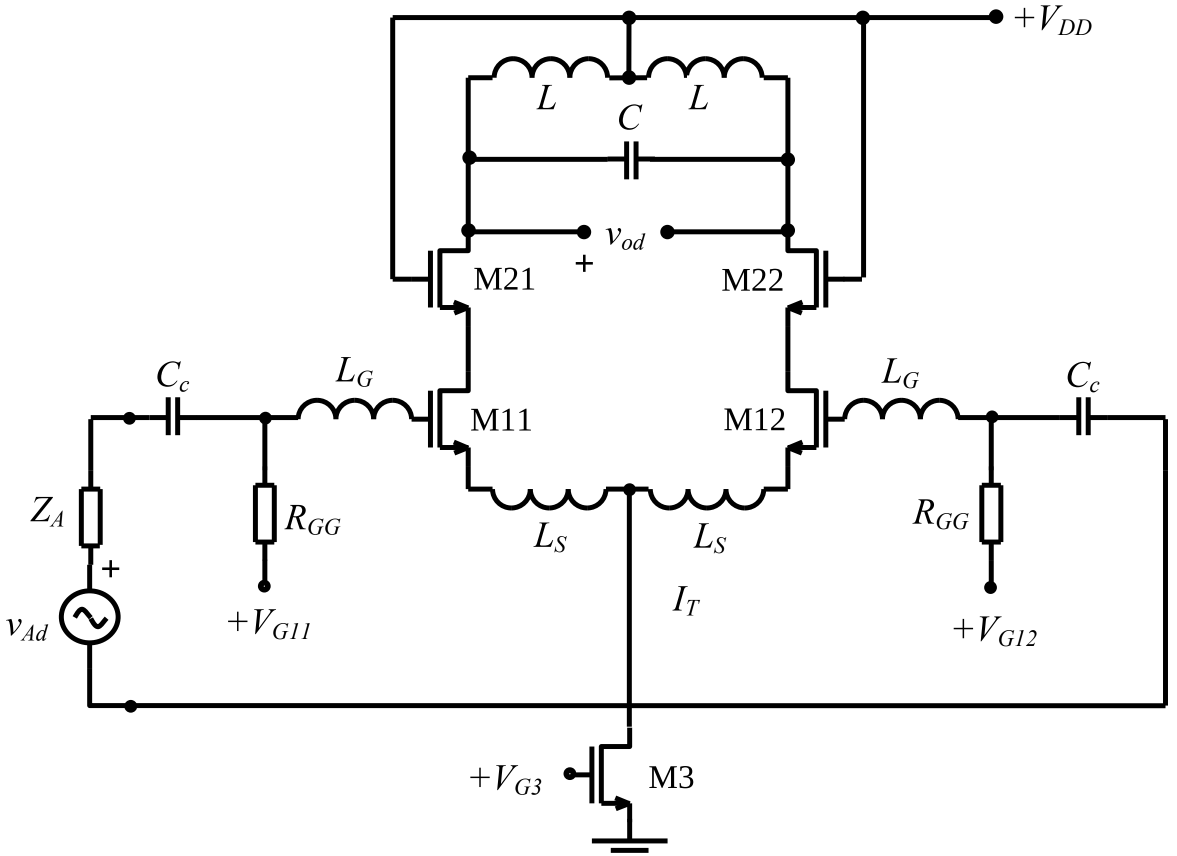
$$\left|A_v(\omega_0)\right| = \left|Y_{mG}(\omega_0)\right| \times \left|Z_L(\omega_0)\right| = \left|Y_{mG}(\omega_0)\right| \times Q_{eff} L \omega_0$$

Circuit diagram of a source degenerated single-ended tuned LNA



Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

Circuit diagram of a source degenerated differential (symmetrical) tuned LNA



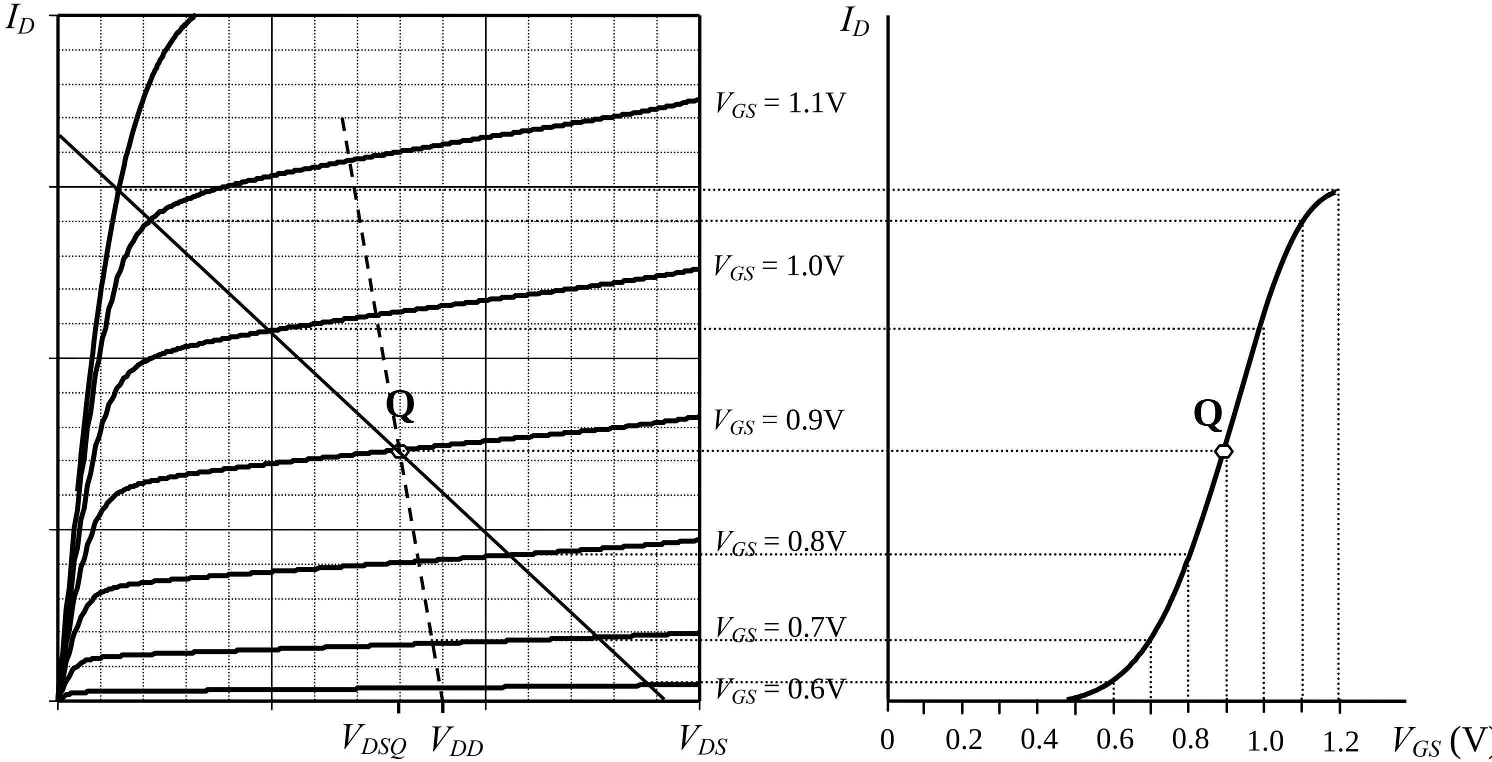
Linearity of an LNA
The immunity against intermodulation

The drain current corresponding to an input voltage, $v_i = v_{gs}$;

$$i_d = g_1 v_{gs} + g_2 v_{gs}^2 + g_3 v_{gs}^3 + g_4 v_{gs}^4 + \cdots$$

Since the $I_D = f(V_{gs})$ transfer characteristic is not linear.

The physical source of the nonlinearity :



(a) (b)

$$I_D = I_{DQ} + \left.\frac{dI_D}{dV_{GS}}\right|_Q \Delta V_{GS} + \frac{1}{2!}\left.\frac{d^2 I_D}{dV_{GS}^2}\right|_Q (\Delta V_{GS})^2 + \frac{1}{3!}\left.\frac{d^3 I_D}{dV_{GS}^3}\right|_Q (\Delta V_{GS})^3 + \cdots$$

Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

For variations of the signal around the operating point, Q :

$$i_d = g_1 v_{gs} + g_2 v_{gs}^2 + g_3 v_{gs}^3 + g_4 v_{gs}^4 + \cdots$$

The generated harmonic components of the drain current for a single sinusoidal input voltage, $v_{gs} = v_i = V_i \cos \omega t$:

$$\begin{aligned} i_d &= \left(\frac{1}{2} g_2 V_i^2 + \frac{3}{8} g_4 V_i^4 + \cdots \right) \\ &+ \left(g_1 V_i + \frac{3}{4} g_3 V_i^3 + \cdots \right) \cos \omega t \\ &+ \left(\frac{1}{2} g_2 V_i^2 + \frac{1}{2} g_4 V_i^4 + \cdots \right) \cos 2\omega t \\ &+ \left(\frac{1}{4} g_3 V_i^3 + \cdots \right) \cos 3\omega t \\ &+ \cdots \end{aligned}$$

The intermodulation products for two simultaneous sinusoidal voltages,

$$v_{gs} = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t \quad :$$

$$i_d = \frac{1}{2} g_2 (V_1^2 + V_2^2)$$

$$+ \left[g_1 V_1 + \frac{3}{2} g_3 (V_1 V_2^2 + \frac{1}{2} V_1^3) \right] \cos \omega_1 t + \left[g_1 V_2 + \frac{3}{2} g_3 (V_1^2 V_1 + \frac{1}{2} V_2^3) \right] \cos \omega_2 t$$

$$+ \frac{1}{2} g_2 V_1^2 \cos 2\omega_1 t \frac{1}{2} g_2 V_2^2 \cos 2\omega_2 t$$

$$+ \frac{1}{4} g_3 V_1^3 \cos 3\omega_1 t \frac{1}{2} g_3 V_2^3 \cos 3\omega_2 t$$

$$+ \frac{1}{2} g_2 V_1 V_2 \cos(\omega_1 + \omega_2) t + \frac{1}{2} g_2 V_1 V_2 \cos(\omega_1 - \omega_2) t$$

$$+ \frac{3}{4} g_3 V_1^2 V_2 \cos(2\omega_1 + \omega_2) t + \frac{3}{4} g_3 V_1 V_2^2 \cos(2\omega_2 + \omega_1) t$$

$$+ \frac{3}{4} g_3 V_1^2 V_2 \cos(2\omega_1 - \omega_2) t + \frac{3}{4} g_3 V_1 V_2^2 \cos(2\omega_2 - \omega_1) t$$

Interpretations:

- An extra DC component appears on the drain current.

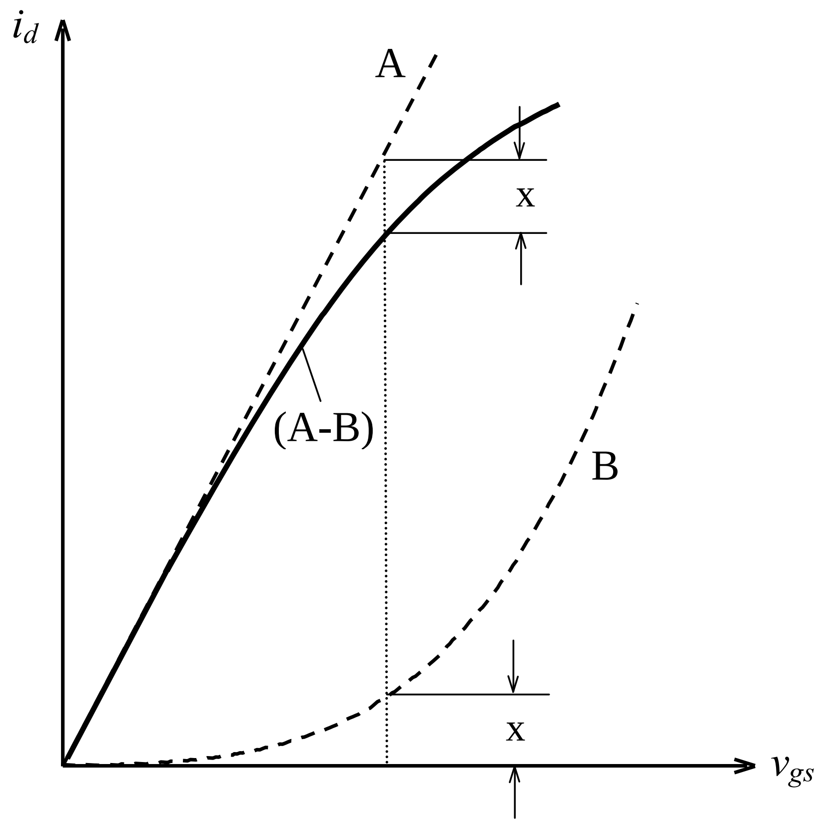
  (Shift of the operating point for high signal amplitudes)

- ω_1 and ω_2, each has two components in nature;

  - Varies linearly with the amplitude of the signal,

  - Varies with the third power of the amplitudes.

- Since for an S shaped function the coefficients of the third power components are negative, the amplitudes of the fundamental components are equal to the difference of the corresponding linear term and the cubic term.



It means that for the fundamental components, the slope of the output-versus-input amplitude curves decreases with the input amplitudes.

- In case of the amplification of a modulated signal, to maintain the amplitude relations of the carrier and side frequencies, the decrease of the amplitude of the carrier must not be more than 1 dB, that corresponds to a ratio of (1/1.122):

$$\frac{g_1 V_i}{g_1 V_i - \frac{3}{4}\left|g_3\right| V_i^3} = 1.122 \quad \rightarrow \quad V_i\Big|_{-1\mathrm{dB}} = 0.38\sqrt{\frac{g_1}{\left|g_3\right|}}$$

- The 2nd and 3rd harmonics are far from the fundamentals; they can be easily filtered out in tuned amplifiers, but important for wide-band amplifiers.

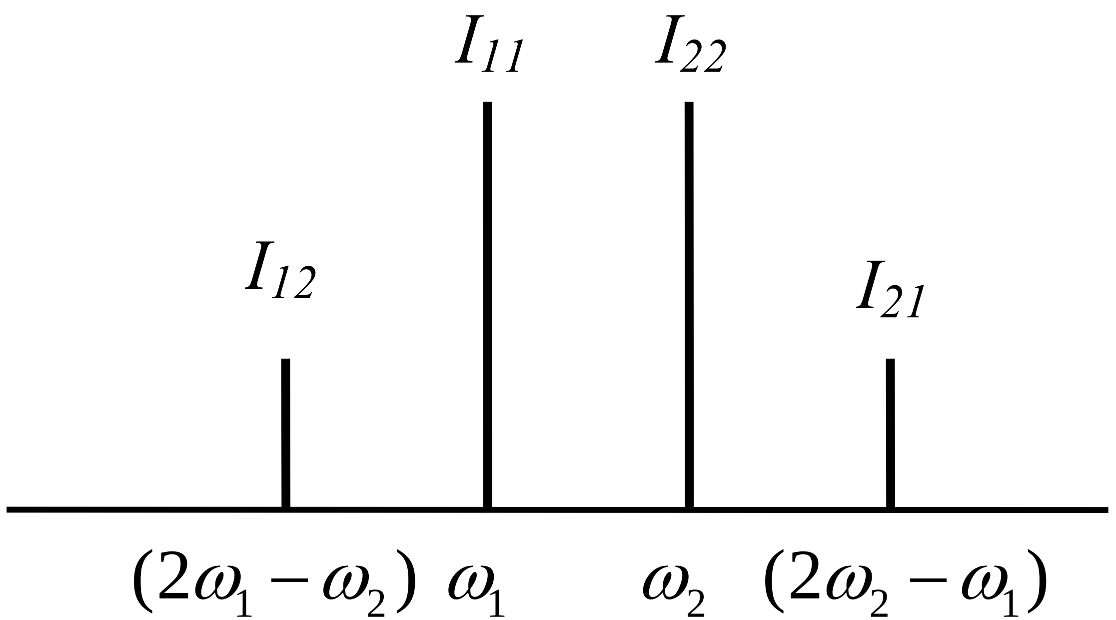
- The symmetrical circuits have an advantage; they do not contain any even harmonic at the output.

- Another advantage of the symmetrical circuits: they do not contain any extra DC component. Therefore there is no shift of the operating point.

- The most important intermodulation products are the "close" second order producs. If ω_1 and ω_2 (the neighboring channels) are close to each other, such that $\omega_2 = \omega_1 + \Delta\omega$;

$$(2\omega_1 - \omega_2) = \omega_1 - \Delta\omega$$
$$(2\omega_2 - \omega_1) = \omega_2 + \Delta\omega$$

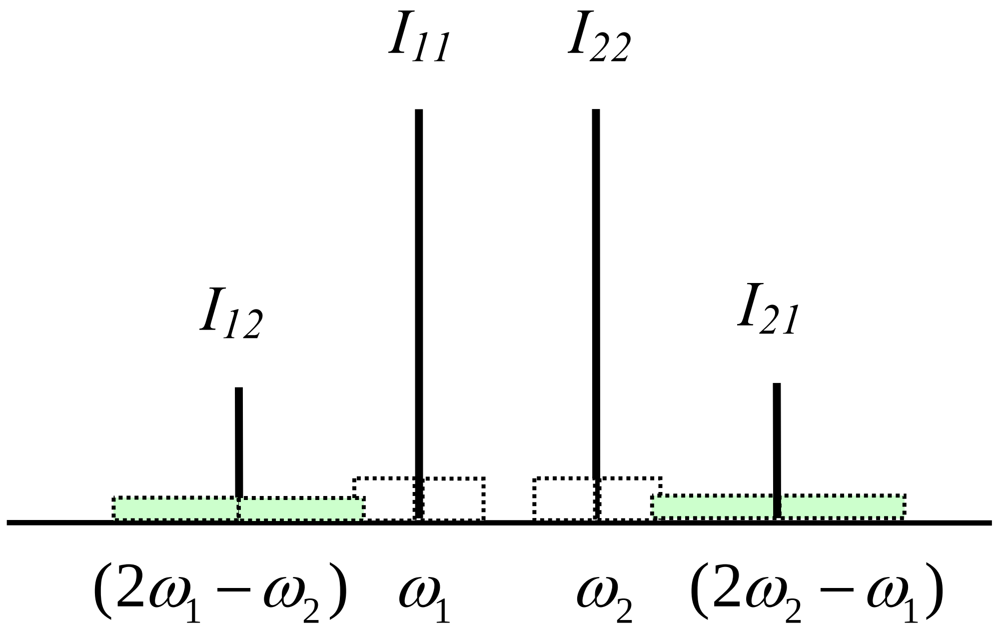
- These components are proportional to g_3 and steeply increase with the signal amplitudes.



- The carriers (ω_1 and ω_2) are always modulated and have corresponding modulation side-bands.

- Note that the width of the side-bands of the $2\omega_1$ and $2\omega_2$ components are twice wider than the side-bands of ω_1 and ω_2.

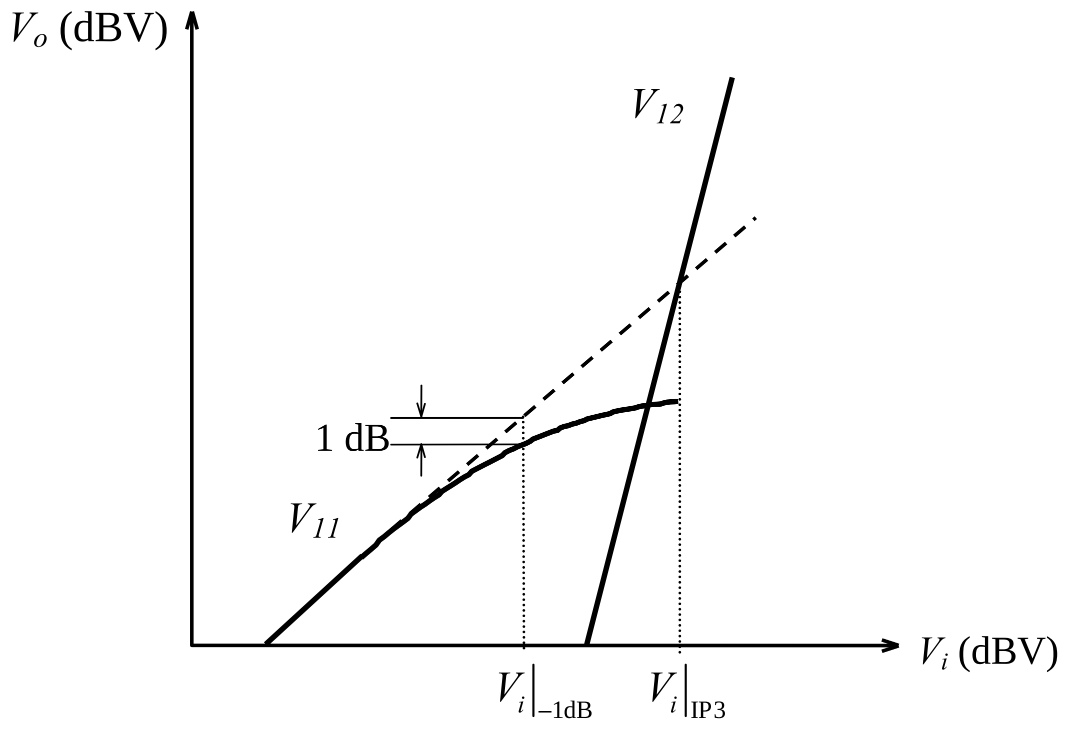
The resulting spectrum with the side-bands:



Note that the interfering effects of the close intermodulation products are more severe compared to that of the neighboring channel!

Another metric to evaluate the nonlinearity of an LNA:

The input level for which the level of a close intermodulation product becomes equal to the linear term of one of the carriers.



This is called as the "third order intercept point" (IP3).

IP3 can be calculated from its definition:

$$\frac{V_{11(lin)}}{V_{12}} = \frac{R_{eff}\, g_1 V_i}{\frac{3}{4} R_{eff} \left|g_3\right| V_i^3} = 1 \quad \rightarrow \quad V_i\big|_{IP3} = 1.154\sqrt{\frac{g_1}{\left|g_3\right|}}$$

Relation between the input levels corresponding to -1dB point and IP3:

$$\frac{V_i\big|_{\mathrm{IP3}}}{V_i\big|_{-1\mathrm{dB}}} = 3.037 \quad \rightarrow \quad 9.65\ \mathrm{dB}$$