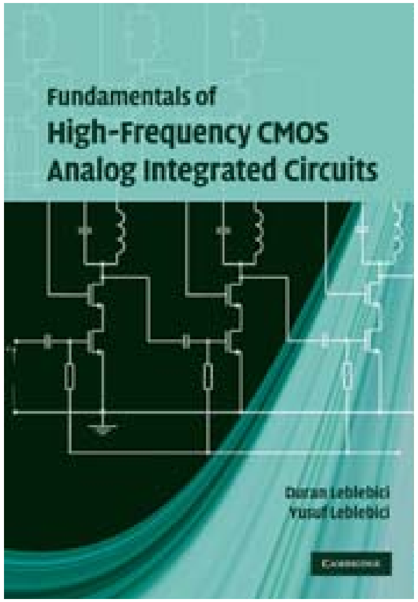


Fundamentals of High-Frequency CMOS Analog Integrated Circuits

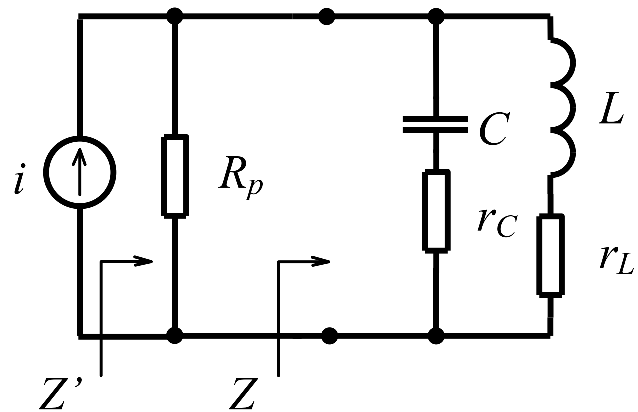
Duran Leblebici
Yusuf Leblebici

Chapter 4

Frequency-Selective RF Circuits (Resonance Circuits)

Resonance Circuits

a) Parallel resonance circuit:



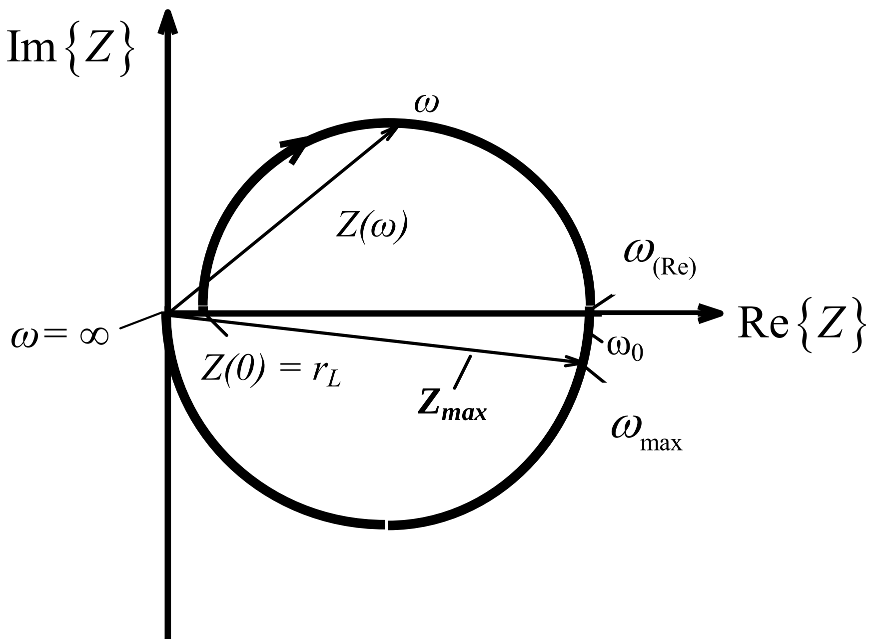
$$Z(\omega)=\frac{(r_L-\omega^2 LCr_C)+j\omega(L+Cr_C r_L)}{(1-\omega^2 LC)+j\omega C(r_C+r_L)}$$

$$\mathrm{Re}\{Z\}=\frac{(r_L-\omega^2 LCr_C)(1-\omega^2 LC)+\omega^2 C(r_L+r_C)(L+Cr_L r_C)}{(1-\omega^2 LC)^2+\omega^2 C^2 (r_L+r_C)^2}$$

$$\mathrm{Im}\{Z\}=\frac{\omega(L+Cr_L r_C)(1-\omega^2 LC)-\omega C(r_L-\omega^2 LCr_C)(r_L+r_C)}{(1-\omega^2 LC)+\omega^2 C^2 (r_L+r_C)^2}$$

Locus of the impedance phasor:

(For $r_C \ll r_L$)



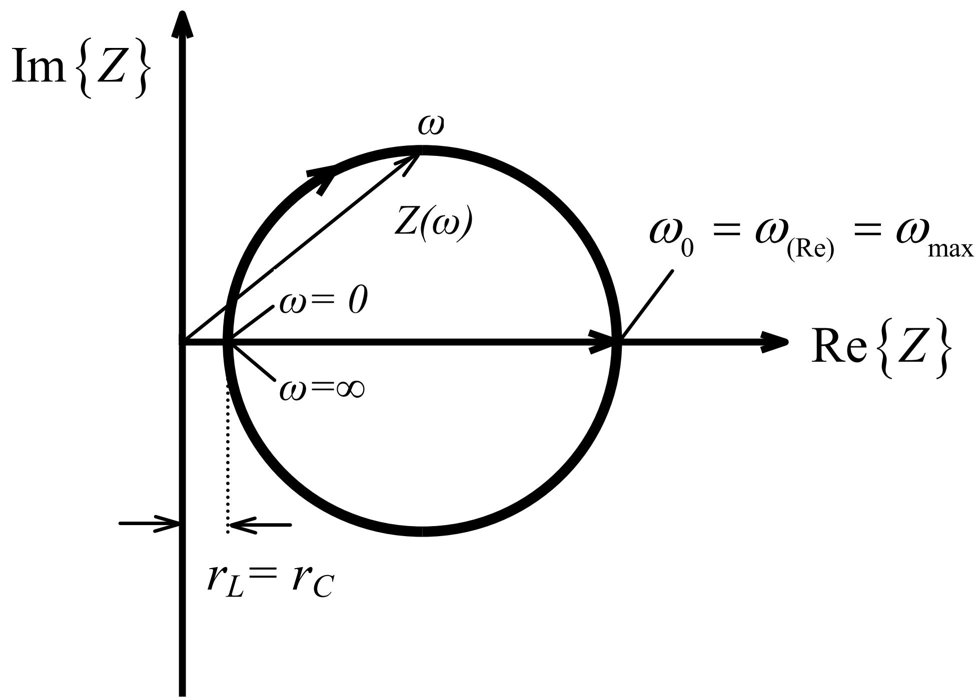
$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_{(\mathrm{Re})}^2 = \frac{1}{LC} - \frac{r_L^2}{L^2}$$

$$\omega_{\max}^2 = \frac{1}{LC}\sqrt{1 + 2r_L^2\frac{C}{L} - \frac{r_L^2}{L^2}}$$

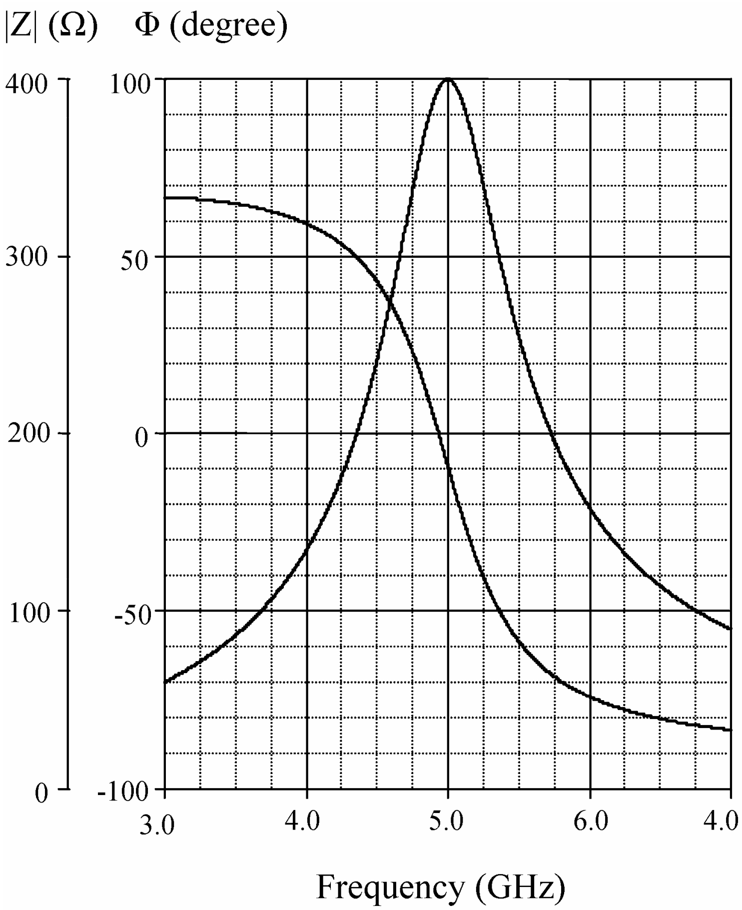
Locus of the impedance phasor:

(For $r_C = r_L$)

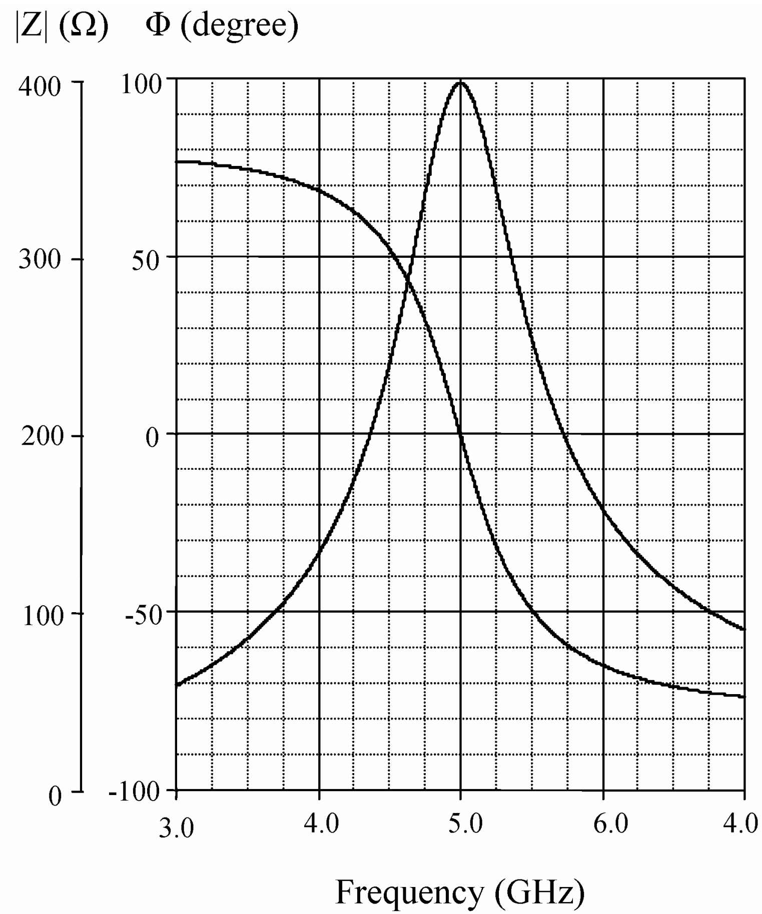


Variations of the magnitude and the phase of the impedance:

(a) For r_C □ r_L , (b) For $r_C = r_L$



(a)



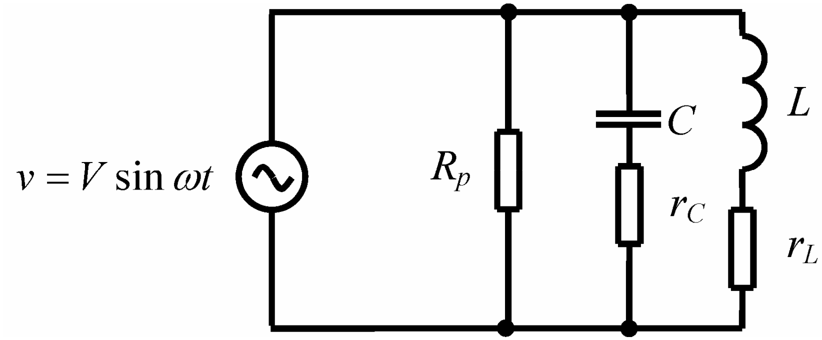
(b)

$f_0 = 5$ GHz, $L = 2$ nH, $r_L = 10$ ohm, $r_C = 0$ $f_0 = 5$ GHz, $L = 2$ nH, $r_L = 5$ ohm, $r_C = 5$ ohm

The quality factor of an oscillatory system

$$Q = 2\pi \frac{\text{The total energy of the system}}{\text{The energy lost in one period}}$$

Quality factor of a parallel resonance circuit:



Total (maximum) energy in the circuit: $E = \frac{1}{2} C V^2$

Energy consumed in one period in R_p : $E_p = T \times \frac{1}{2} \frac{V^2}{R_p} = \frac{1}{2 f_0} \frac{V^2}{R_p}$

Energy consumed in one period in r_L : $E_L = \frac{1}{2 f_0} V^2 \frac{r_L}{r_L^2 + \omega_0^2 L^2}$

Energy consumed in one period in r_C : $E_C = \frac{1}{2 f_0} V^2 \frac{r_C}{r_C^2 + \frac{1}{\omega_0^2 C^2}}$

$$Q = 2\pi \frac{\frac{1}{2}V^2 C}{\frac{1}{2}V^2 \left[\frac{1}{f_0 R_p} + \frac{1}{f_0}\frac{r_L}{(r_L^2 + \omega_0^2 L^2)} + \frac{1}{f_0}\frac{r_C}{(r_C^2 + \frac{1}{\omega_0^2 C^2})} \right]}$$

This expression can be arranged as follows:

$$\frac{1}{Q_{eff}} = \frac{1}{Q_p} + \frac{1}{Q_L} + \frac{1}{Q_C}$$

where;

Q_{eff} is called as the "effective quality factor" of the circuit

$$Q_p = \omega_0 C R_p$$

(quality factor if R_p were the only lossy component)

$$Q_L = \frac{r_L^2 + \omega_0^2 L^2}{\omega_0 L r_L} \cong \frac{\omega_0 L}{r_L}$$

(quality factor if r_L were the only lossy component)

$$Q_C = \frac{\omega_0^2 C^2 r_C^2 + 1}{\omega_0 C r_C} \cong \frac{1}{\omega_0 C r_C}$$

(quality factor if r_C were the only lossy component)

Important Equivalencies

Parallel resistance R_{LP} equivalent to r_L :

$$Q_L = \frac{r_L^2 + \omega_0^2 L^2}{\omega_0 L r_L} \equiv \frac{R_{LP}}{\omega_0 L} \quad \Rightarrow \quad R_{Lp} = r_L + \frac{\omega_0^2 L^2}{r_L} \cong r_L(1+Q_L^2) \cong \frac{L}{r_L C}$$

Similarly, equivalent parallel resistance to r_C :

$$R_{Cp} = r_C + \frac{1}{\omega_0^2 C^2 r_C} \cong r_C(1+Q_C^2) \cong \frac{L}{r_C C}$$

Note: These equivalencies are not valid for phase relations

The total parallel equivalent (effective) resistance:

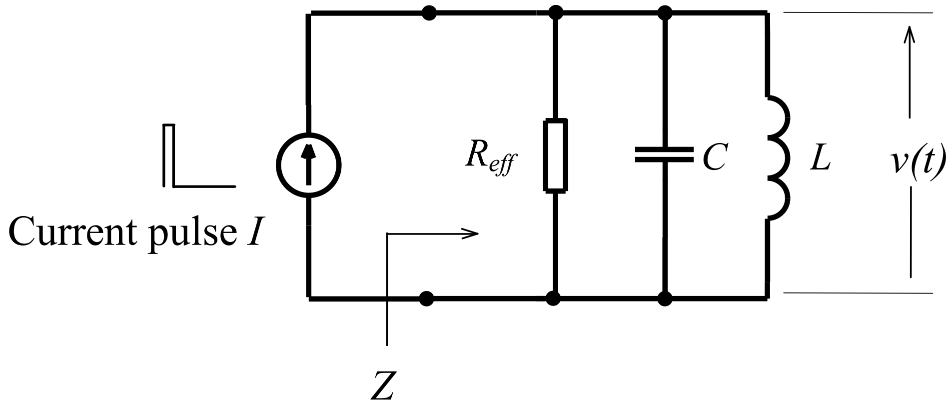
$$R_{eff} = (R_{Lp} // R_{Cp} // R_p) \ , \ G_{eff} = G_{Lp} + G_{Cp} + G_p$$

Effective quality factor in terms of R_{eff} and G_{eff} :

$$Q_{eff} = \frac{R_{eff}}{\omega_0 L} = \omega_0 C R_{eff} \ , \ Q_{eff} = \frac{1}{\omega_0 L G_{eff}} = \frac{\omega_0 C}{G_{eff}}$$

Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

Oscillation capability of a resonance circuit



$$Z = \frac{1}{C}\frac{s}{s^2 + s\frac{1}{R_{eff}C} + \frac{1}{LC}} = \frac{1}{C}\frac{s}{(s - s_{p1})(s - s_{p2})}$$

$$V = \left(\frac{I}{s}\right)\frac{1}{C}\frac{s}{(s - s_{p1})(s - s_{p2})}$$

$$v(t) = I\frac{1}{C}\frac{1}{(s_{p1} - s_{p2})}\left(e^{s_{p1}t} - e^{s_{p2}t}\right)$$

$$s_{p1,p2} = -\frac{1}{2R_{eff}C} \mp \sqrt{\left(\frac{1}{2R_{eff}C}\right)^2 - \frac{1}{LC}}$$

$$s_{p1,p2} = \sigma \mp j\sqrt{\omega_0^2 - \sigma^2} = \sigma \mp j\bar{\omega}_0$$

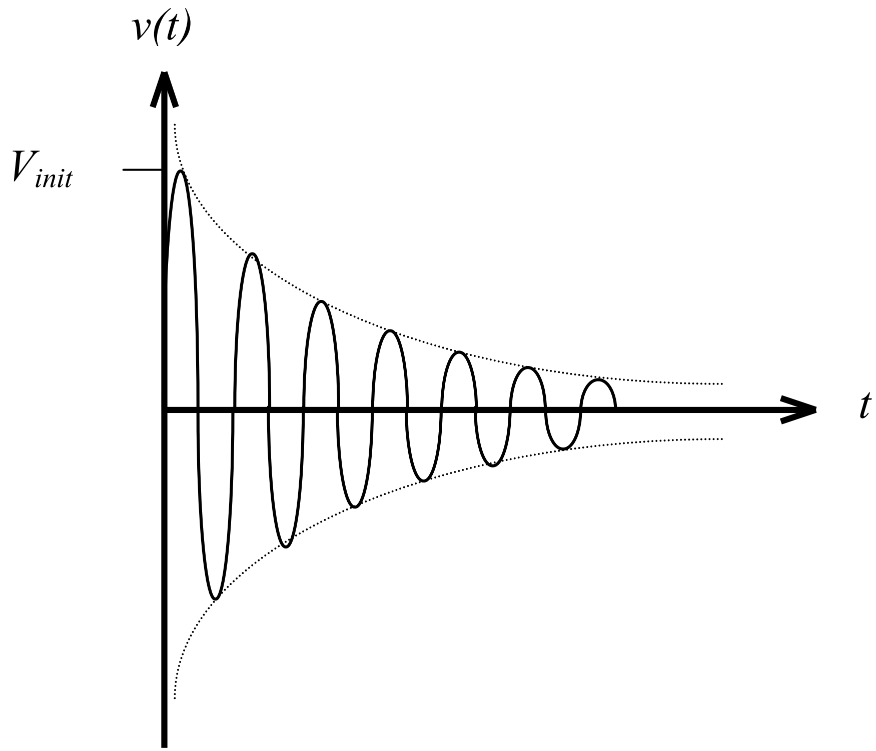
where $\sigma = -\frac{1}{2R_{eff}C}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ and $\bar{\omega}_0 = \sqrt{\omega_0^2 - \sigma^2}$

$$v(t) = I\frac{1}{C}\frac{1}{2j\bar{\omega}_0}e^{\sigma t}(e^{j\bar{\omega}_0 t} - e^{-j\bar{\omega}_0 t})$$

$$v(t) = I\frac{1}{C}\frac{1}{\bar{\omega}_0}e^{\sigma t}\sin\bar{\omega}_0 t$$

with $\sigma = -(\omega_0 / 2Q_{eff})$ and $\omega_0 \cong \overline{\omega}_0$;

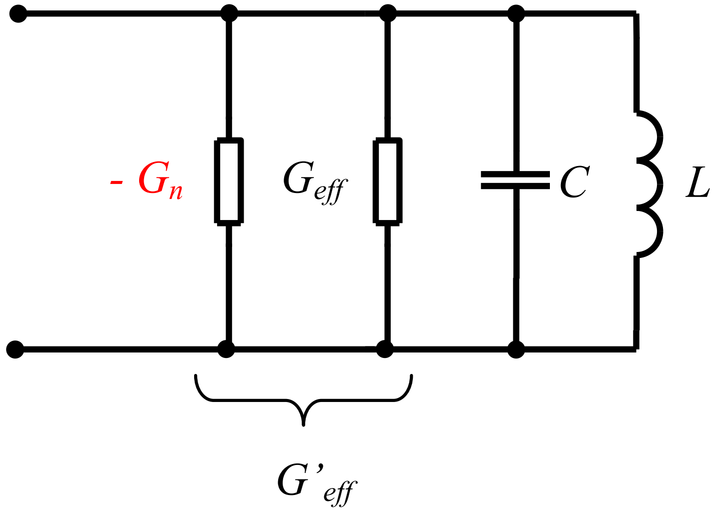
$$v(t) = V_{(init)} e^{-(\overline{\omega}_0 / 2Q_{eff})t} \sin \overline{\omega}_0 t$$



- At the end of $n = Q_{eff}$ periods, the amplitude drops to $V_{init} e^{-\pi}$.
- The number of periods corresponding to $\hat{v} = (1/e) V_{init}$ is (Q_{eff} / π).

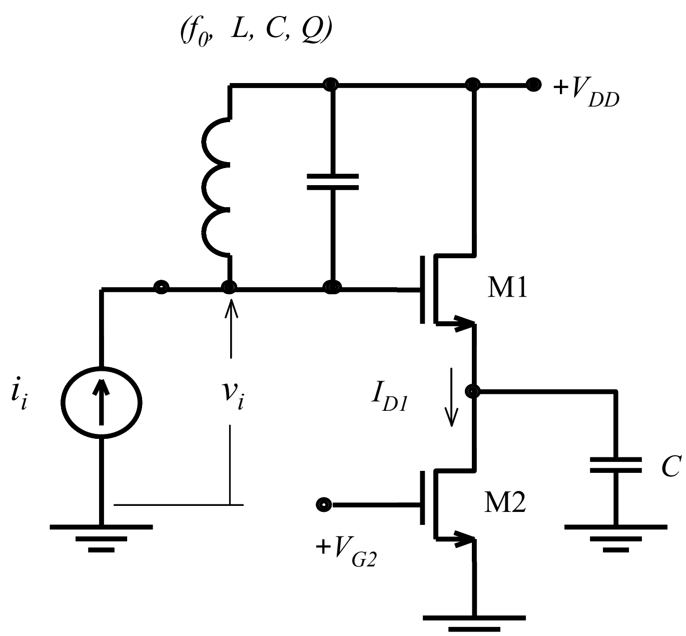
Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

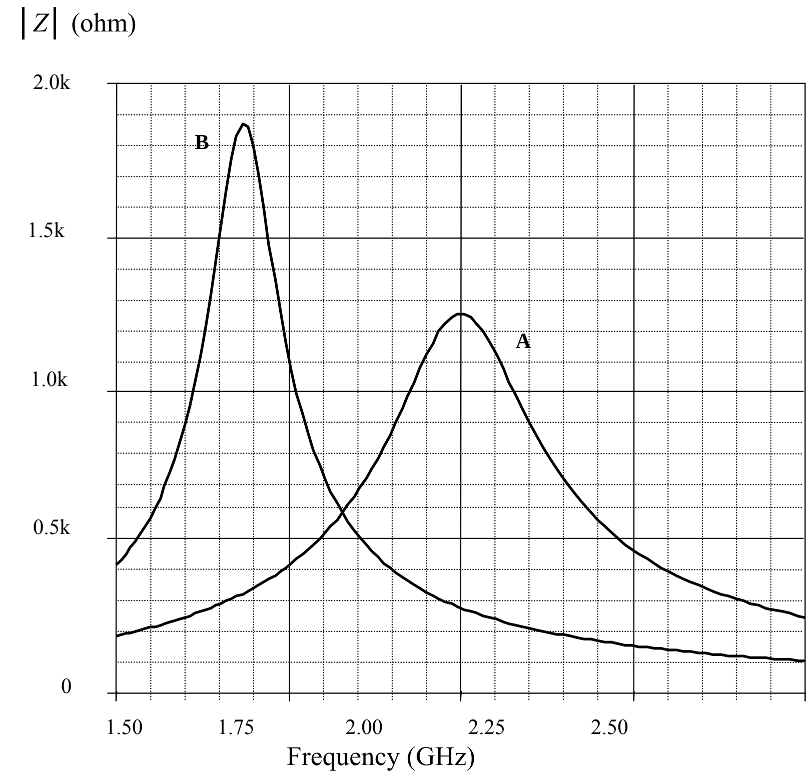
Q Enhancement



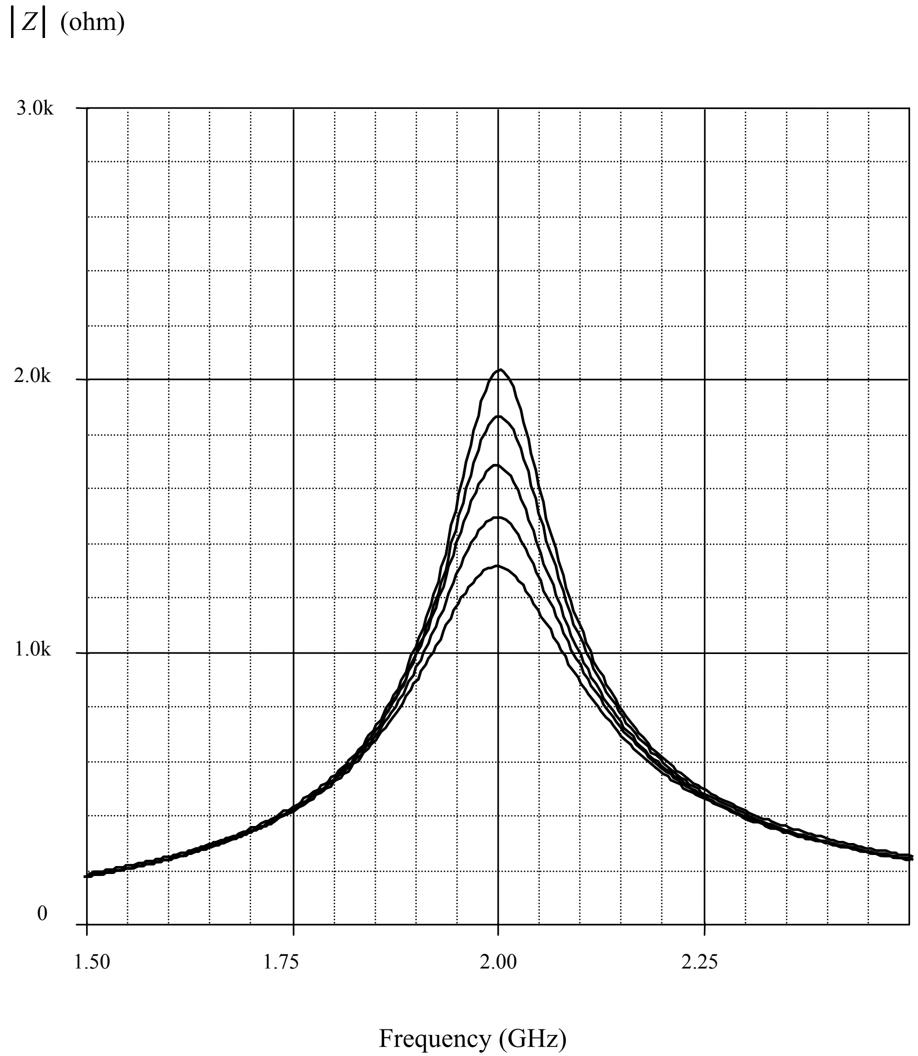
$$Q'_{eff} = \frac{1}{L\omega_0 (G_{eff} - G_n)}$$

Example: Q enhancement with a capacitive loaded source follower





(Shift of the resonance frequency is due to the input capacitance of M1)



Parameter: The gate voltage of the current source transistor, T2
(Shift of the resonance frequency is compensated in the graph!)

The bandwidth of a parallel resonance circuit

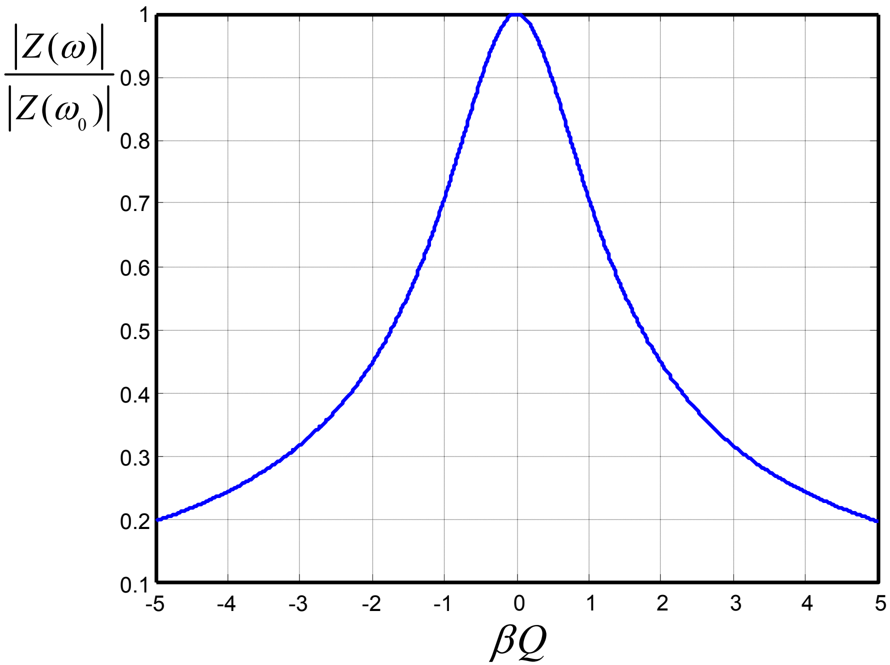
$$Z = \frac{1}{C}\frac{s}{s^2 + s\frac{1}{R_{eff}C} + \frac{1}{LC}} \qquad \rightarrow \quad Z(\omega) = R_{eff}\frac{1}{1 + j\frac{R_{eff}}{L\omega}(\omega^2 LC - 1)}$$

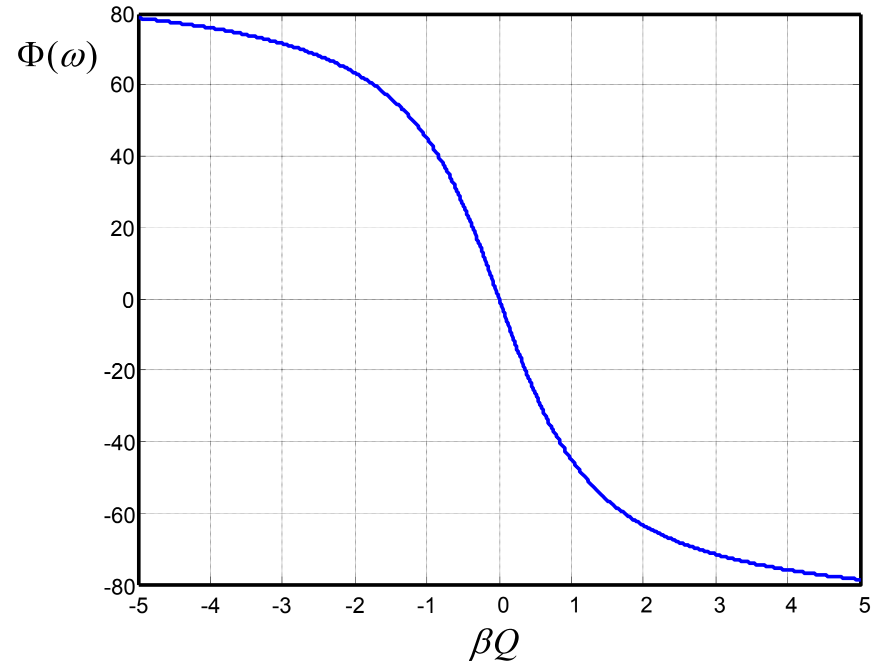
with $LC = \dfrac{1}{\omega_0^2}$, $Q_{eff} = \dfrac{R_{eff}}{L\omega_0}$;

$$Z = R_{eff}\frac{1}{1 + jQ_{eff}\frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right)} = R_{eff}\frac{1}{1 + jQ_{eff}\frac{\omega_0}{\omega}\frac{(\omega + \omega_0)(\omega - \omega_0)}{\omega_0^2}}$$

$Z \cong R_{eff}\dfrac{1}{1 + j\beta Q_{eff}}$, where $\beta = \dfrac{2\Delta\omega}{\omega_0} = \dfrac{2\Delta f}{f_0}$

$$|Z| = Z(\omega_0)\frac{1}{\sqrt{1+(\beta Q_{eff})^2}} \qquad \Phi = -\arctan(\beta Q_{eff})$$





- At resonance; $Z(\omega_0) = R_{eff} = Q_{eff} L\omega_0$

- For $\beta Q_{eff} = \mp 1$; $|Z| = Z(\omega_0)/\sqrt{2}$

- $\Delta f = \dfrac{f_0}{2Q_{eff}} \quad \Rightarrow \quad f(-3\text{ dB}) = f_0 \mp \dfrac{f_0}{2Q_{eff}} \;, \quad B = \dfrac{f_0}{Q_{eff}}$

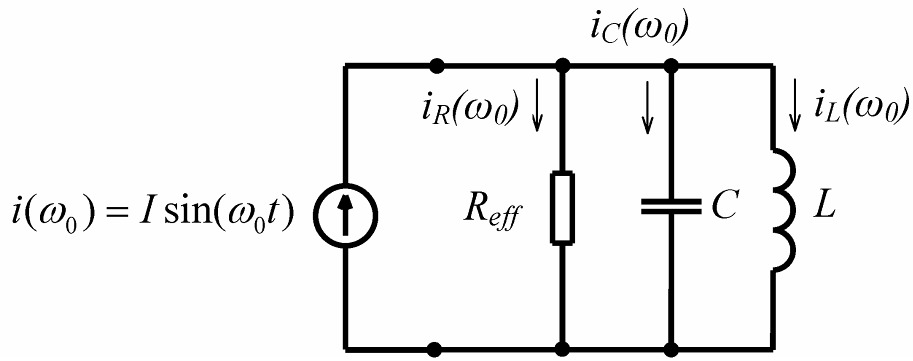
- $\Phi = 0$ at resonance.

- Impedance is

  - Inductive below the resonance frequency,
  - Capacitive above the resonance frequency.

- $\Phi = \mp \dfrac{\pi}{4}$ at -3 dB frequencies.

Branch currents at resonance:



$$i_C(\omega_0) = v(\omega_0)(jC\omega_0) = i(\omega_0)R_{eff}(jC\omega_0)$$

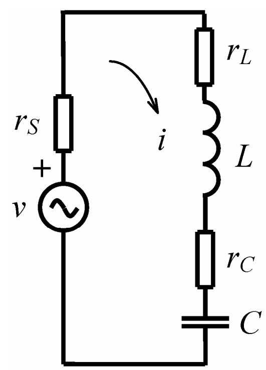
$$i_C(\omega_0) = j\ [i(\omega_0)\times Q_{eff}]$$

$$i_L(\omega_0) = \frac{v(\omega_0)}{jL\omega_0} = \frac{i(\omega_0)R_{eff}}{jL\omega_0}$$

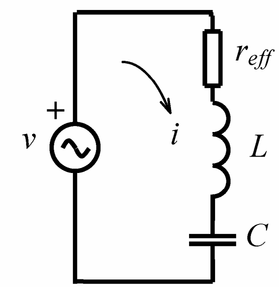
$$i_L(\omega_0) = -j\ [i(\omega_0)\times Q_{eff}]$$

(Possible adverse effect: Electromigration!)

b) The series resonance circuit:



(a)



(b)

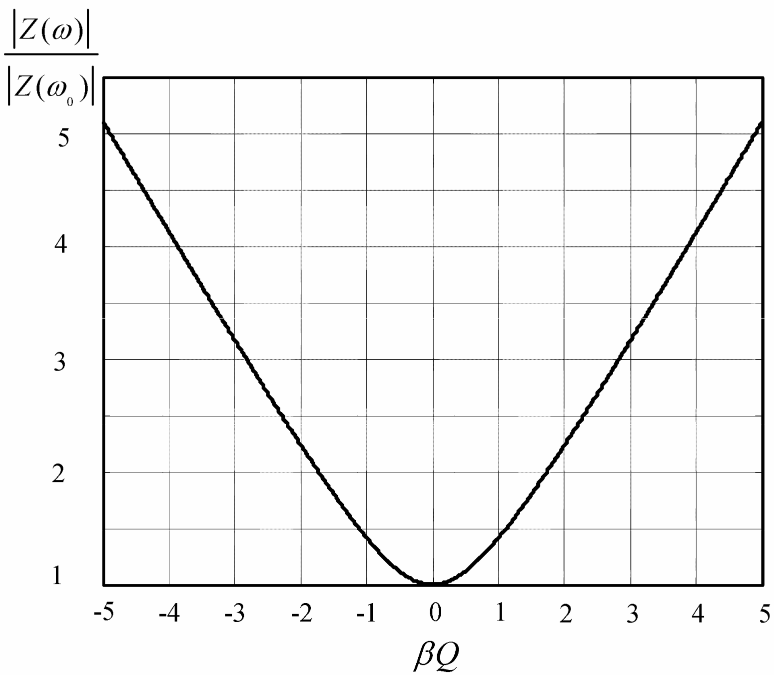
$$Z = (r_{eff} + sL + \frac{1}{sC}) \quad \rightarrow \quad Z = L\frac{(s - s_{01})(s - s_{02})}{s}$$

$$s_{01,02} = \sigma \mp j\sqrt{\omega_0^2 - \sigma^2}$$

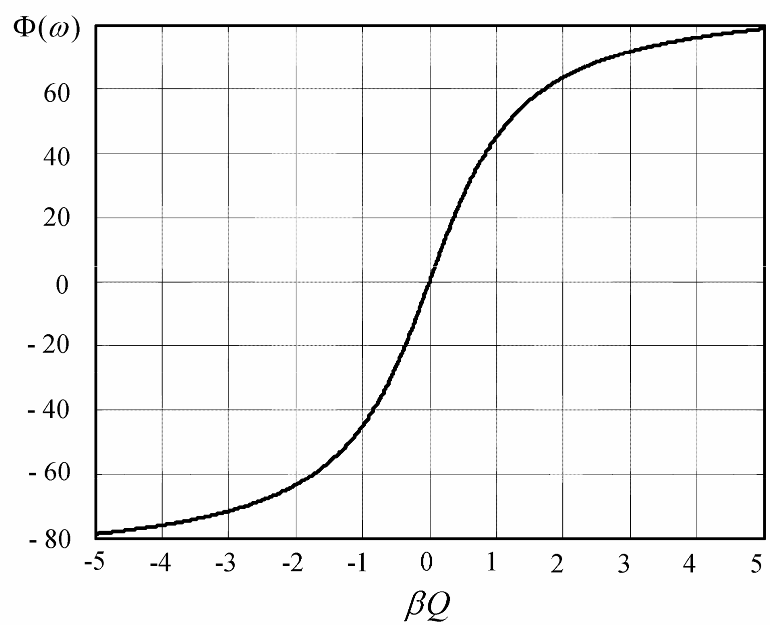
where $\omega_0^2 = \dfrac{1}{LC}$, $\sigma = -\dfrac{r_{eff}}{2L} = -\dfrac{\omega_0}{2Q_{eff}}$, $Q_{eff} = \dfrac{\omega_0}{2\sigma} = \dfrac{L\omega_0}{r_{eff}}$

$$Z(\omega) = Z(\omega_0)(1 + j\beta Q_{eff}) \ , \qquad \beta = 2\Delta\omega / \omega_0$$

$$\left|Z(\omega)\right| = Z(\omega_0)\sqrt{1 + (\beta Q_{eff})^2} \qquad \Phi(\omega) = \arctan(\beta Q_{eff})$$



(a)



(b)

- At resonance; $Z(\omega_0) = r_{eff}$

 For $\beta Q_{eff} = \mp 1$ $\quad |Z| = \sqrt{2} \times Z(\omega_0)$

 $f(+3 \text{ dB}) = f_0 \mp \dfrac{f_0}{2Q_{eff}}$, $\quad B = \dfrac{f_0}{Q_{eff}}$

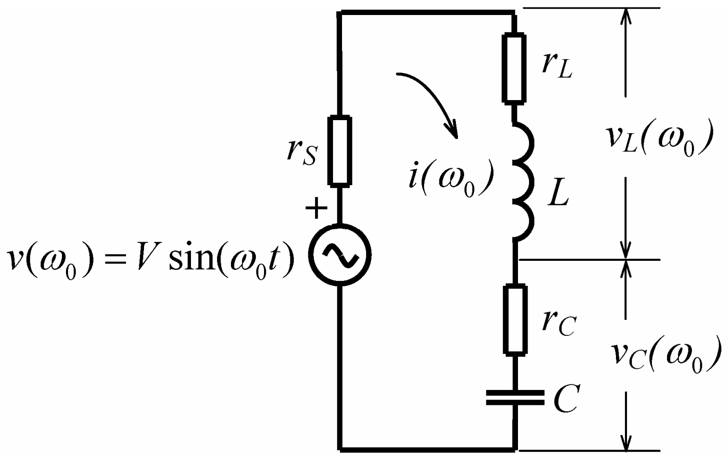
- At resonance; $\Phi = 0$

- Impedance is

  - Capacitive below the resonance frequency,
  - Inductive above the resonance frequency.

 $\Phi = \mp \dfrac{\pi}{4}$ at $+3$ dB frequencies.

Voltages at resonance:



$$v_L(\omega_0) = i(\omega_0)(r_L + jL\omega_0) = v(\omega_0)\frac{r_L + jL\omega_0}{r_{eff}}$$

$$= v(\omega_0)\left(\frac{r_L}{r_{eff}} + j\,Q_{eff}\right) \cong j\,v(\omega_0)\times Q_{eff}$$

$$v_C(\omega_0) \cong -j\,v(\omega_0)\times Q_{eff}$$

(Possible adverse effect: Breakdown of the capacitor dielectric!)