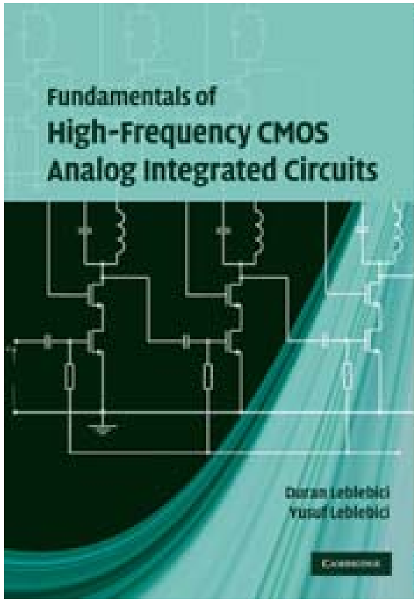

Fundamentals of High-Frequency CMOS Analog Integrated Circuits

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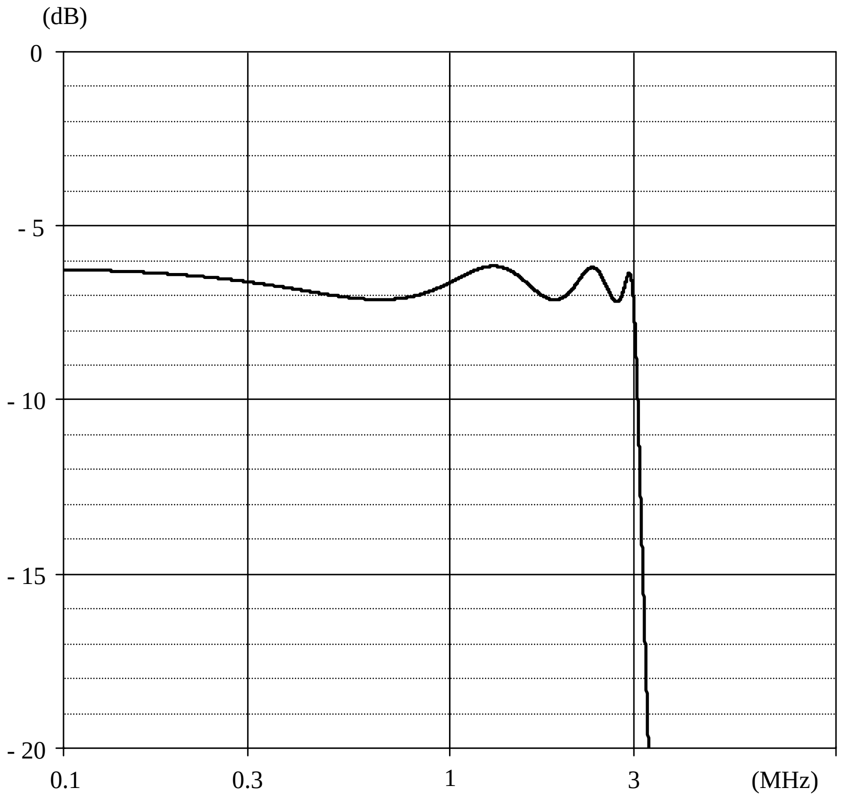
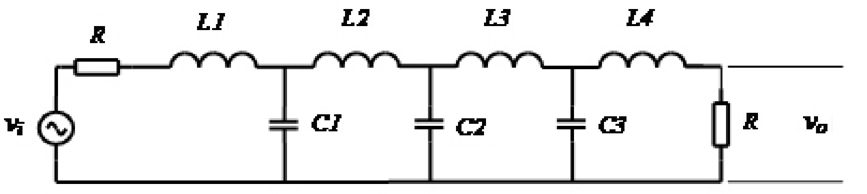
Chapter 4

Frequency-Selective RF Circuits (Gyrator)

An example to the need of high-value inductances: A 7th order low-pass filter for a GPS receiver





R = 100 ohm

L1 = 11.49 µH

L 2 = 16.41 µH

L3 = 16.41 µH

L4 = 11.49 µH

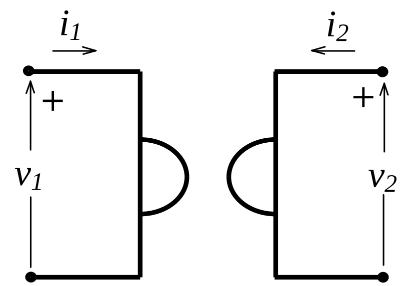
C1 = 589.7 pF

C2 = 622.6 pF

C3 = 589.7 pF

Inductors are not realisable as on-chip inductors !

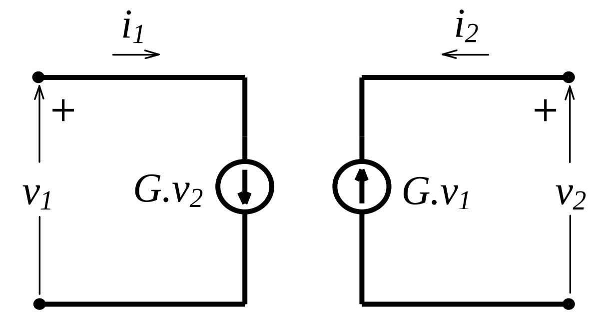
Gyrator



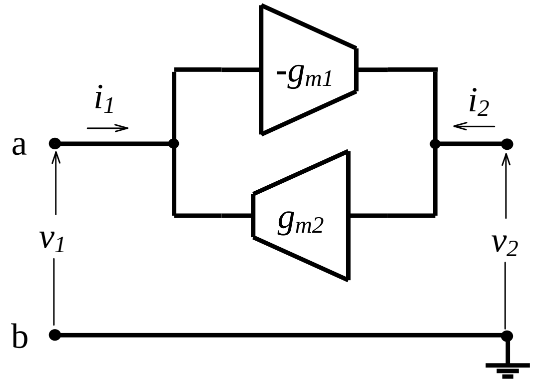
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

(Theoretically described -or invented- by Prof. Tellegen, 1948)

The gyrator in terms of the controlled current sources:

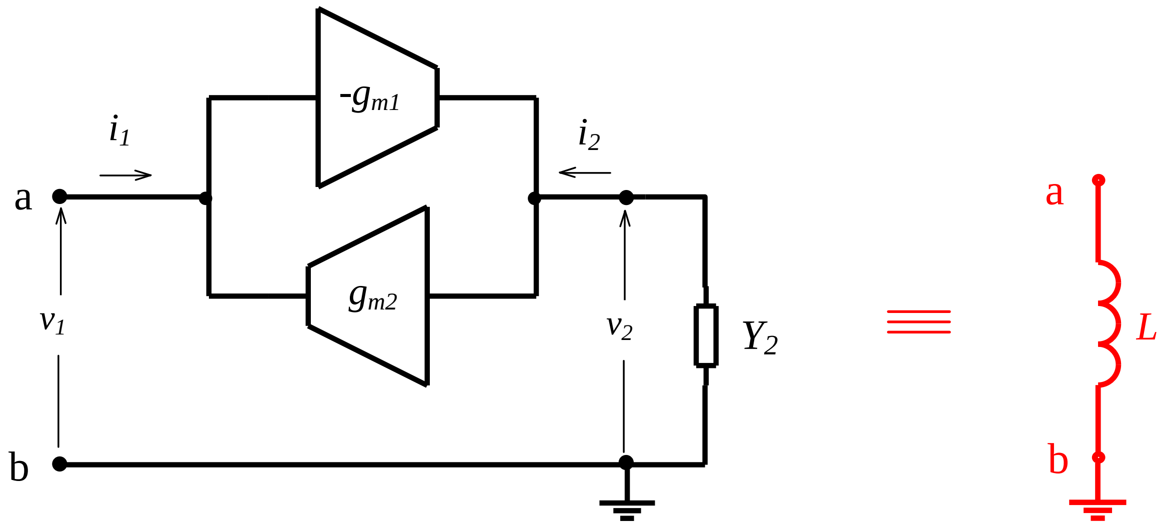


Realisation: The gyrator composed of OTAs.



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g_{m2} \\ -g_{m1} & 0 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Input impedance of a gyrator:



$$i_2 = -Y_2 v_2 = -g_{m1} v_1 \ , \qquad i_1 = g_{m2} v_2 \qquad \Rightarrow \quad Z_1 = \frac{v_1}{i_1} = \frac{Y_2}{g_{m1} g_{m2}}$$

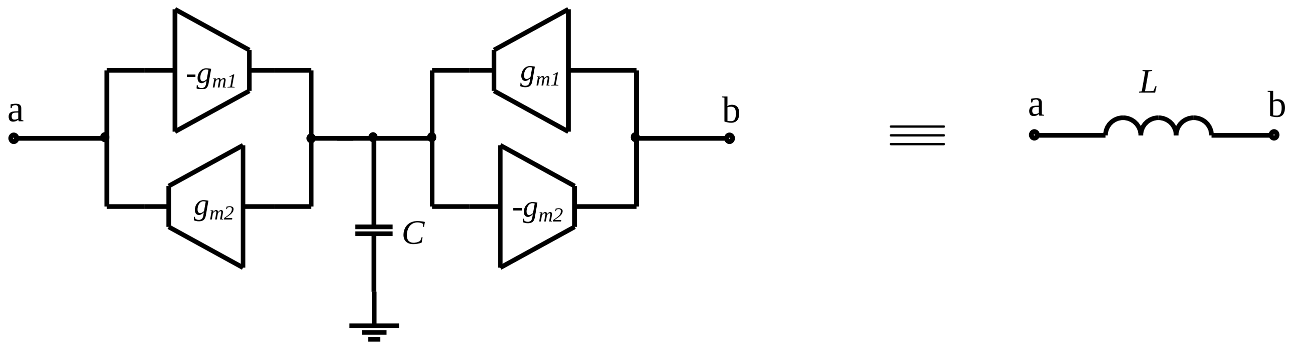
Input impedance of a capacitance loaded gyrator:

$$Z_1 = s\frac{C}{g_{m1} g_{m2}} \quad \Rightarrow \quad L = \frac{C}{g_{m1} g_{m2}}$$

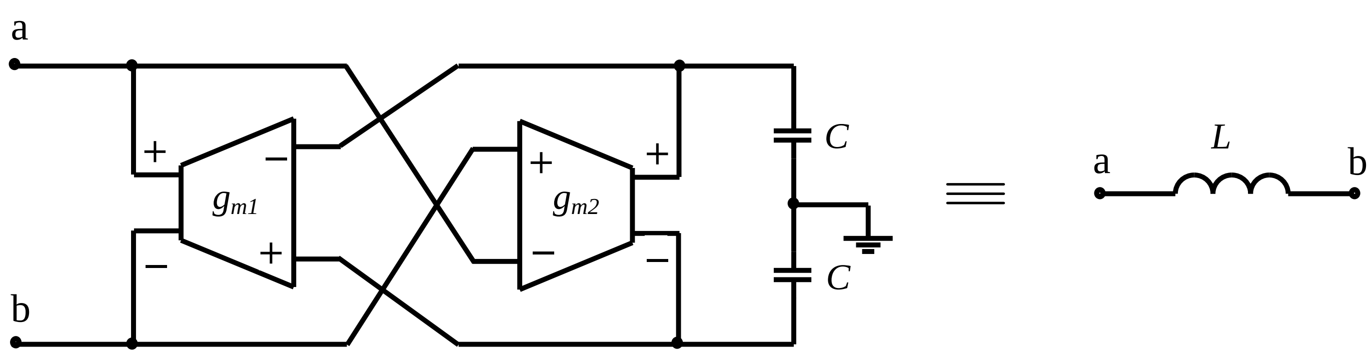
Leblebici/Leblebici, Fundamentals of High-Frequency CMOS Analog Integrated Circuits, Cambridge University Press, 2009

Gyrator based floating inductances:

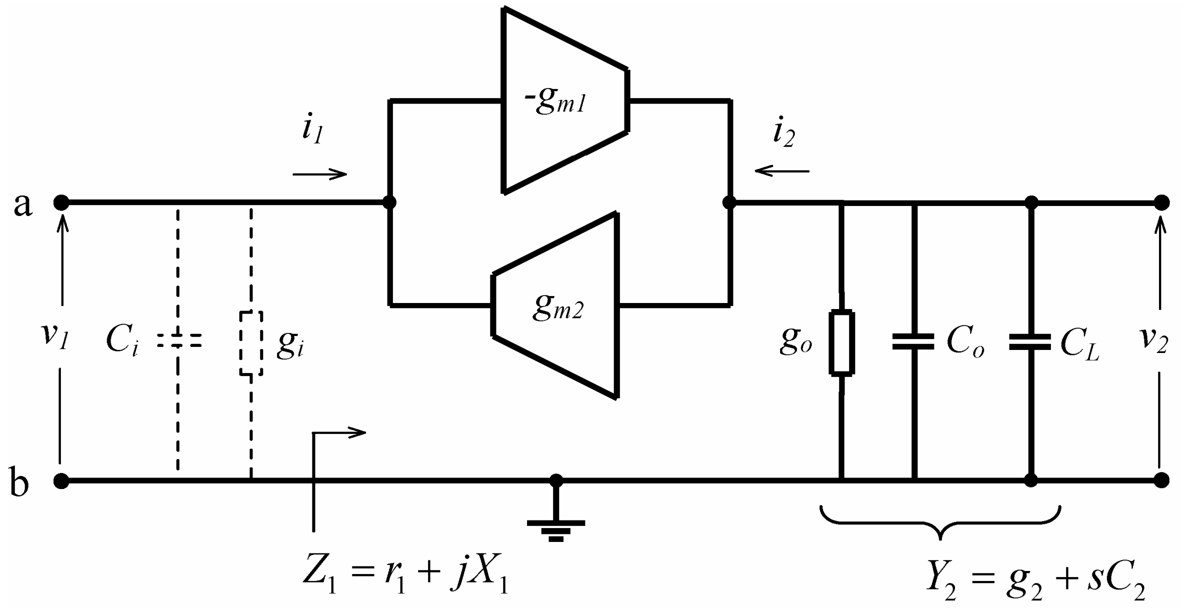
a) Floating inductance with single-ended input / single-ended output OTAs



b) Floating inductance with differential-input / differential-output OTAs



Parasitics of a non-ideal gyrator:



Transfer admittance of a typical OTA with its dominant pole:

$$y_m = g_m s_p \frac{1}{(s - s_p)}$$

$$Z_1 = \frac{Y_2}{y_{m1} y_{m2}}$$

$$Z_1 = \frac{1}{g_{m1} g_{m2} s_{p1} s_{p2}} \left(s - s_{p1} \right) \left(s - s_{p2} \right) (g_2 + sC_2)$$

$$Z_1(\omega) = \frac{1}{g_{m1} g_{m2} \omega_{p1} \omega_{p2}} (j\omega + \omega_{p1})(j\omega + \omega_{p2})(g_2 + j\omega C_2)$$

For $\omega_{p1} = \omega_{p2};$

$$Z_1(\omega) = \frac{1}{g_{m1} g_{m2} \omega_p^2} (j\omega + \omega_p)^2 (g_2 + j\omega C_2)$$

$$\operatorname{Im}\{Z_1\} = \omega\left(\frac{C_2}{g_{m1}g_{m2}} + 2\frac{g_2}{\omega_p g_{m1}g_{m2}}\right)$$

For $\omega \ll \omega_p$;

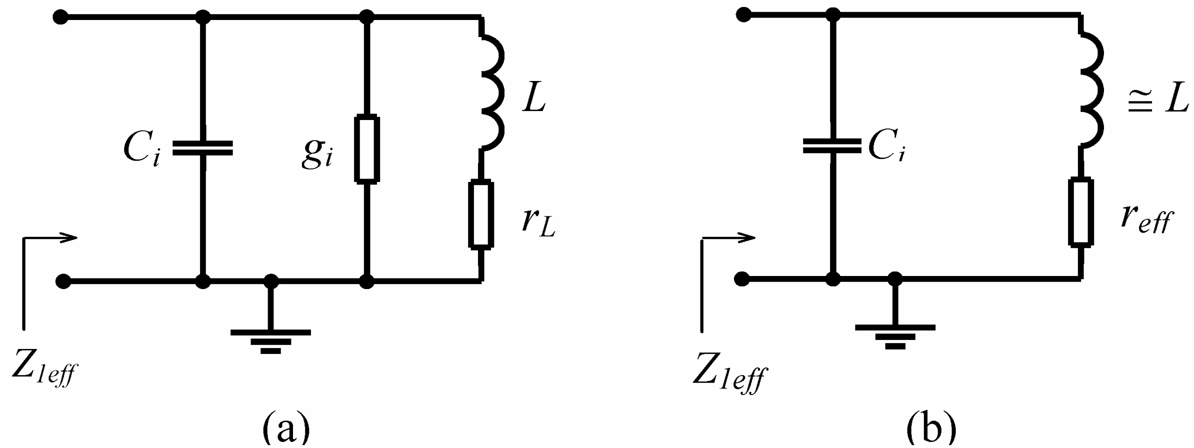
$$\operatorname{Im}\{Z_1\} \cong \omega\left(\frac{C_2}{g_{m1}g_{m2}}\right) = \omega L$$

$$\operatorname{Re}\{Z_1\} = \frac{g_2}{g_{m1}g_{m2}} - \frac{\omega^2}{\omega_p^2}\frac{1}{g_{m1}g_{m2}}(g_2 + 2\omega_p C_2) = r_L$$

Since usually $2\omega_p C_2 \gg g_2$;

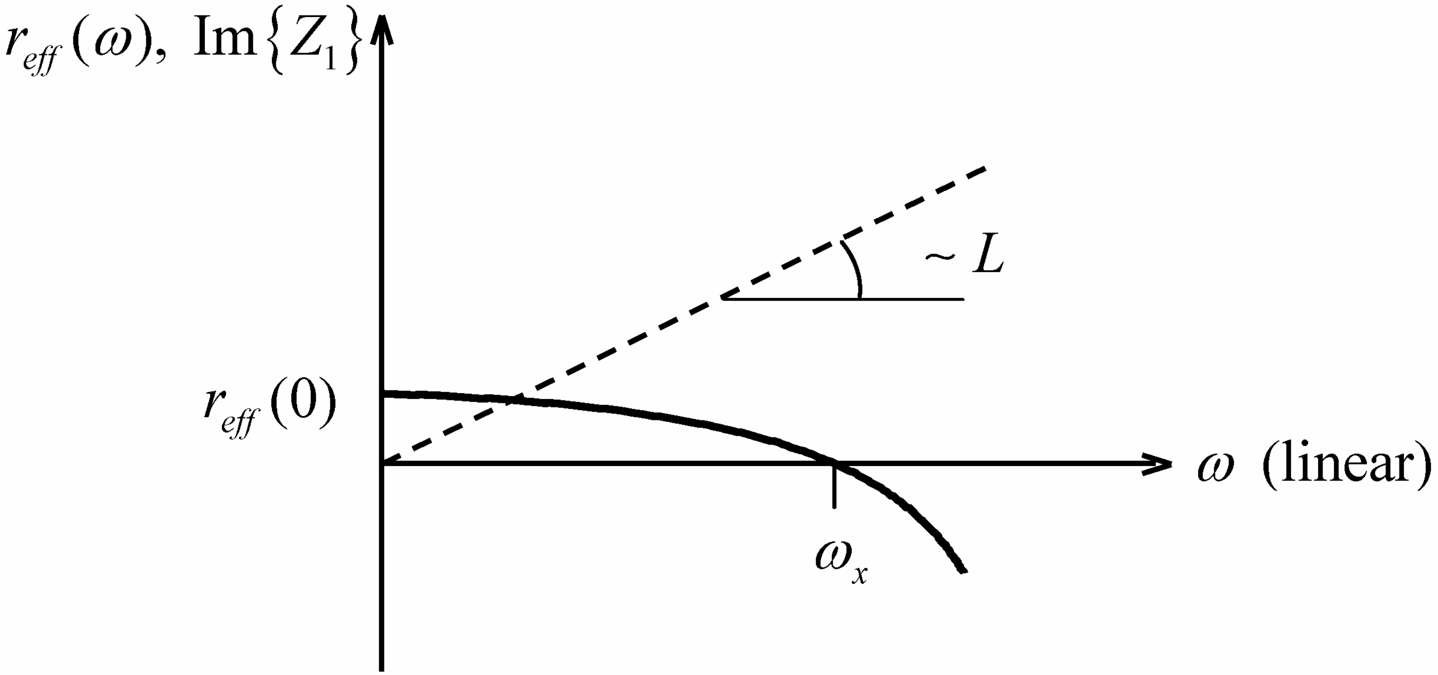
$$r_L \cong \frac{g_2}{g_{m1}g_{m2}} - \frac{\omega^2}{\omega_p}\frac{2C_2}{g_{m1}g_{m2}}$$

The effective series resistance of the emulated inductance:



$$r_{eff} \cong r_L + \omega^2 L^2 g_i$$

$$r_{eff} = \frac{g_2}{g_{m1} g_{m2}} - \omega^2 L^2 \left(\frac{2}{\omega_p L} - g_i \right)$$



$r_{eff}(\omega)$, $\mathrm{Im}\{Z_1\}$
$\sim L$
$r_{eff}(0)$
ω (linear)
ω_x

Interpretation of results:

- The slope of $\mathrm{Im}\{Z_1\}$ is constant and equal to L.

- For $g_i \prec \dfrac{2}{\omega_p L}$, $r_{eff}(\omega)$ decreases and crosses zero at

$$\omega_x = \sqrt{\frac{g_2}{g_{m1} g_{m2}} \frac{1}{L^2 \left(\dfrac{2}{\omega_p L} - g_i \right)}}$$

(Risk of oscillation for $\omega \succ \omega_x$!)

- For $g_i \succ \dfrac{2}{\omega_p L}$, $r_{eff}(\omega)$ is always positive (no risk of instability!)

- The quality factor of the emulated inductor:

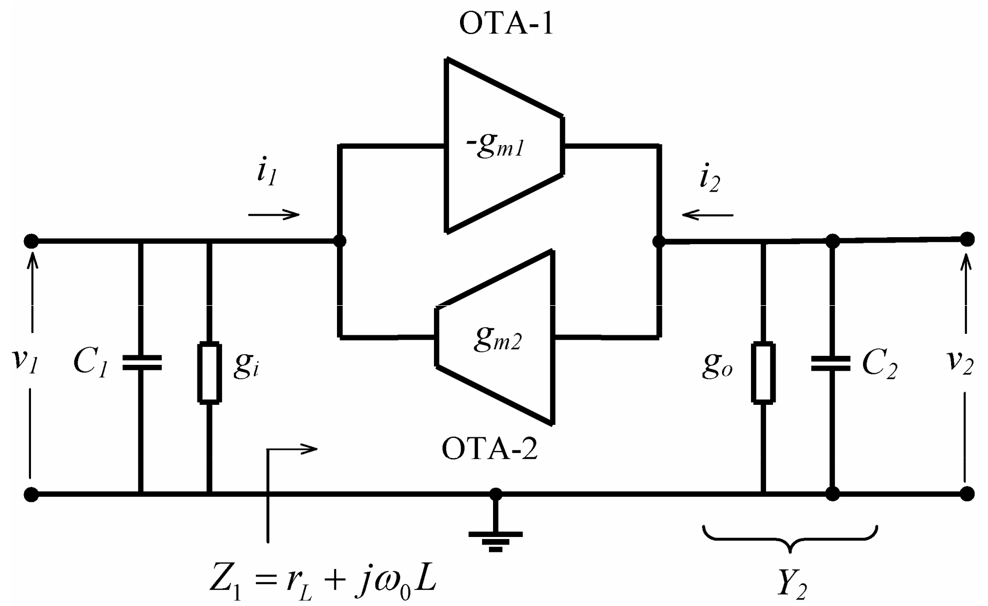
$$Q_{eff}(\omega) = \frac{L\omega}{r_{eff}(\omega)} = \frac{L\omega}{\dfrac{g_2}{g_{m1}g_{m2}} - \omega^2 L^2 \left(\dfrac{2}{\omega_p L} - g_i \right)}$$

  - The quality factor increases with the transconductances of OTAs.
  - The quality factor decreases with the input and output conductances of OTAs

- For $g_i \prec \dfrac{2}{\omega_p L}$, the circuit has an inherent Q enhancement feature.

(Note the increased sensitivity and excessively high Q for $\omega \rightarrow \omega_x$!)

Dynamic range of a gyrator-based inductor:



(C_1 is the capacitor resonating the emulated L at ω_0)

- The dynamic range is limited with the permitted input voltage swings of both of OTAs.

The voltage at the output port:

$$v_2 = -\frac{i_2}{Y_2} = -i_2 \frac{1}{sC_2 + g_2} = v_1 g_{m1} \frac{1}{sC_2 + g_2}$$

The magnitude of v_2 in the ω domain:

$$V_2 = V_1 \frac{g_{m1}}{\sqrt{\omega^2 C_2^2 + g_2^2}} \cong V_1 \frac{g_{m1}}{\omega C_2}$$

The voltage at the input port:

$$v_1 = i_1 Z_1 = i_1 (sL + r_L) = v_2 g_{m2} (sL + r_L)$$

The magnitude of v_1 in the ω domain:

$$V_1 = V_2 g_{m2} \sqrt{\omega^2 L^2 + r_L^2} \cong V_2 g_{m2} \omega L$$

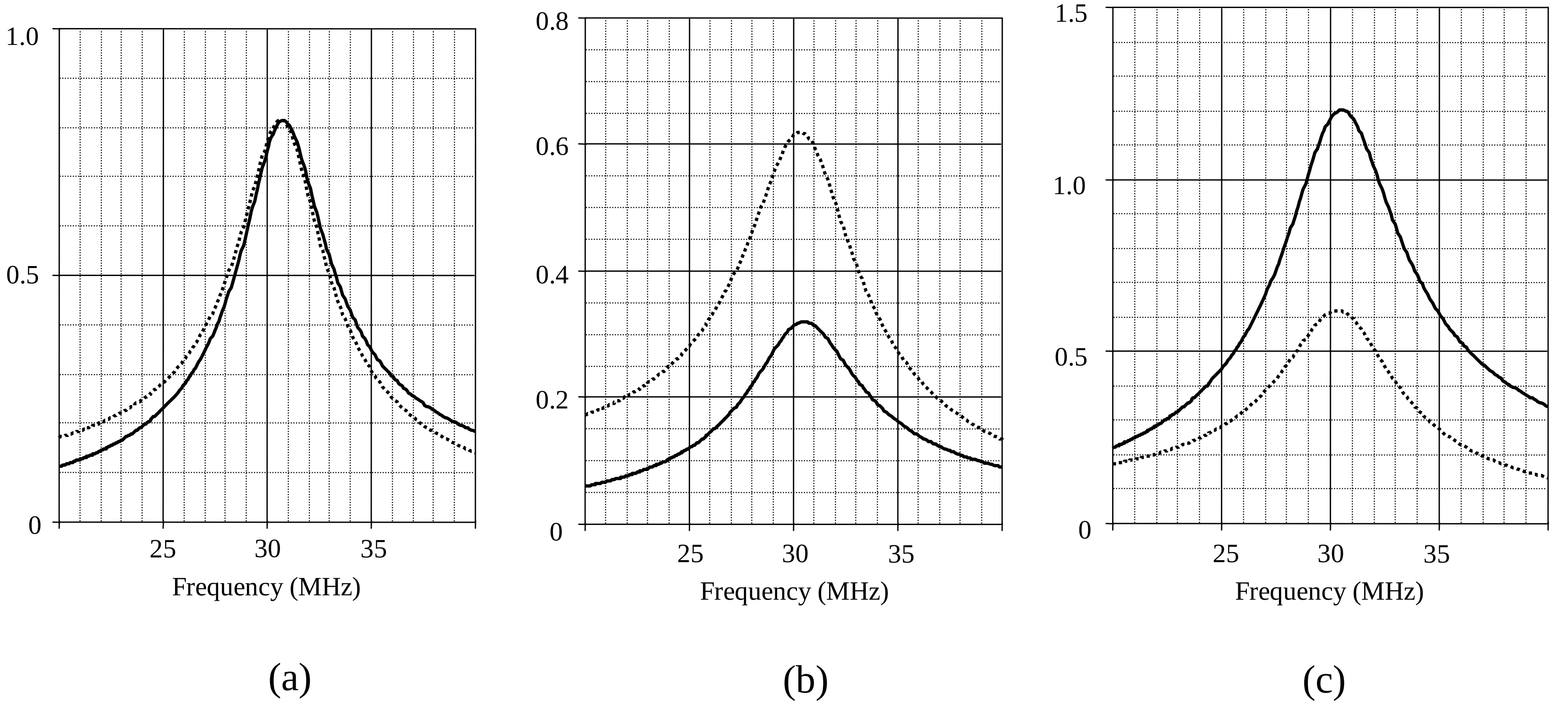
In terms of ω_0, the resonance frequency of L and C_1 :

$$V_2 \cong V_1 \frac{g_{m1}}{\omega_0 C_2} \quad \text{and} \quad V_1 \cong V_2 \omega_0 L = V_2 \frac{g_{m2}}{\omega_0 C_1}$$

The condition to operate OTA1 and OTA2 with their permitted maximum input voltages;

$$V_1 = V_2 \quad \rightarrow \quad \frac{g_{m1}}{g_{m2}} = \frac{C_2}{C_1}$$

Simulation results for $g_{m1} = g_{m2}$



The input (solid line) and output port (dashed line) voltages (in volts) of the gyrator for different ratios: (a) $C_1 = C_2 = 5$ pF, (b) $C_1 = 10$ pF, $C_2 = 2.5$ pF, (c) $C_1 = 2.5$ pF, $C_2 = 10$ pF