11

Agent based modelling

11.1 Excitable agents as cells in a tissue

11.1.1 *Simulate the excitable media cellular automata with* $\sigma = 10$ *and* $\sigma = 20$*, for an* $N = 100$ *system with site number 50 always being excited.*

Answer The cellular automaton is defined in terms of a site variable $s(x)$, which takes value $0, 1, 2, ... \quad \sigma + 1$ where the last state is the excited one. At each time step all sites $x = 1, 2, ...L$ are updated synchronously:

- For all sites x where $s(x) < \sigma$ then $s(x) \to s(x) + 1$
- For all sites where $s(x) = f$ then neighboring sites are set $= f$ if they already have a value $s = \sigma$. Subsequently the firing site is reset to zero, $s(x) = f \to s(x) = 0$.
- The source is maintaining by always setting $s(50) = \sigma + 1$.

Figure 11.1 shows the results of simulations for $\sigma = 10$, and $\sigma = 20$.

11.1.2 *Simulate the excitable media cellular automaton with* $\sigma = 10$*, and size* $N = 100$ *where a random site becoming excited for every system update. Also consider a random excitation for every 10 system updates.*

Answer The algorithm from the previous question is repeated, except that now the excited state is set to a random point with probability α after all sites are updated. Figure 11.2 shows the results of simulations for $\alpha = 1$ and $\alpha = 0.1$.

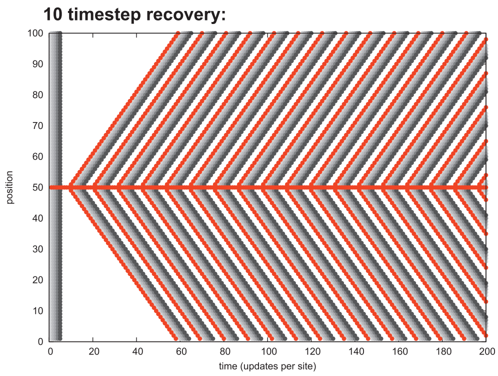

Figure 11.1 Spreading of excitation from a source at x=50, using a refractory period of respectively 10 and 20 time steps.

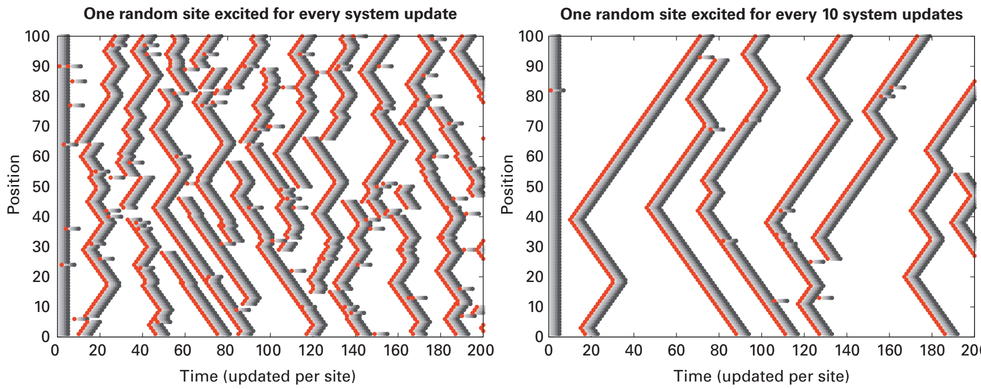

Figure 11.2 Spreading of excitation waves from random sources, using a refractory period of 10 time steps.

11.1.3 *Simulate an excitable medium in two-dimensions using stochastic updating with $p = 0.5$ and $\sigma = 5$.*

Answer As for Question 11.1.1, except that one now allows spreading to the four nearest neighbors. A snapshot is shown in Fig. 11.3.

11.2 Information spreading on social scales

11.2.1 *Simulate a Schelling-like model with three colors, where all agents want four nearest neighbors to be the same color as themselves. At each step,*

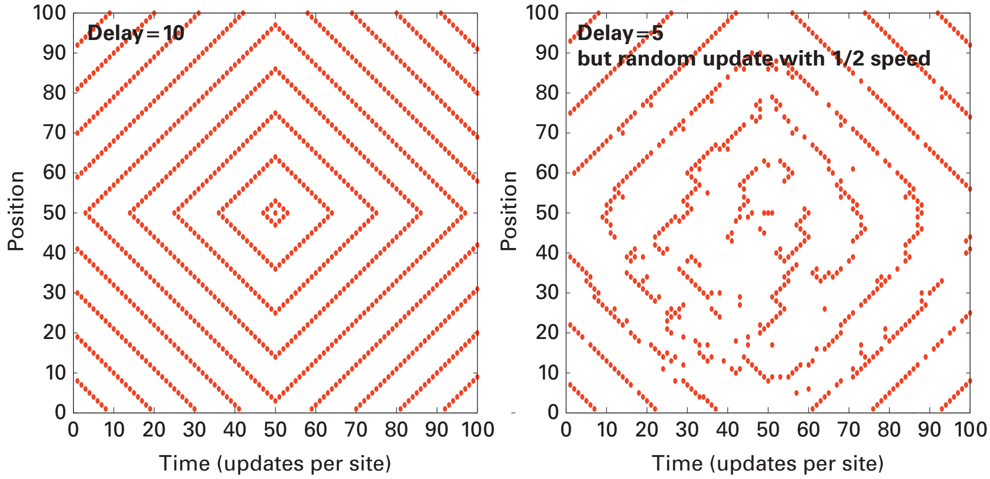

Figure 11.3 Two-dimensional spreading of excitation waves from a source in the middle. In the right-hand panel we only update each site with probability 0.5, giving the same average ageing as in the left-hand panel.

select two agents. If both agents either win or at least do not lose in the exchange, then switch positions. Simulate an $N = 100$ *system for 2000 updates per agent in the system.*

Answer At each step, select two agents i and j. Count the number of agents at $i-2$, $i-1$, $i+1$, $i+2$ that have the same color as agent number i, $c(i)$ and also the number of agents that have the same color as the agent at position j, $c(j)$. Similarly, count the colors of agents at positions $j-2$, $j-1$, $j+1$, $j+2$. If none of the agents lower their number of similar color neighbors, then switch their positions, i.e. update $c(i), c(j) = c(j), c(i)$. Coarsening dynamics are illustrated in Fig. 11.4.

11.2.2 *Simulate and visualize the spread of signals along a one-dimensional line, with new words appearing at position* $x = n/2$ *with high frequency (for example, each time each agent has been involved in one word exchange). At each step, select two neighbors, and let the youngest word spread to replace the oldest word.*

Answer There is one parameter in the problem, the word-initiation frequency. Define a state vector $a(i)$ that specifies the birth age of a word at position $i \in [1, n]$, where $n = 100$. At each time t select a random number $r \in [0, 1]$. If $r < \alpha$ then set $a(n/2) = t$. At the same time step, select

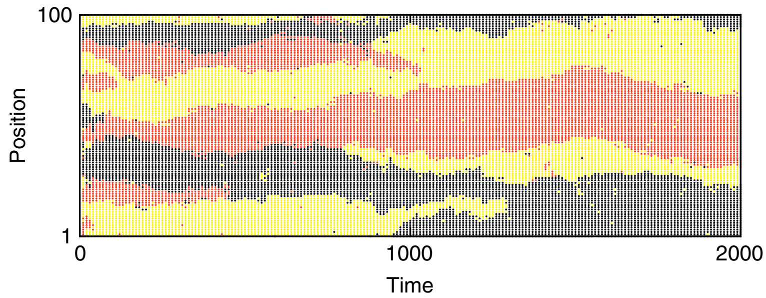

Figure 11.4 Simulation of the three-state Schelling model with time counted as the number of attempted updates per agent.

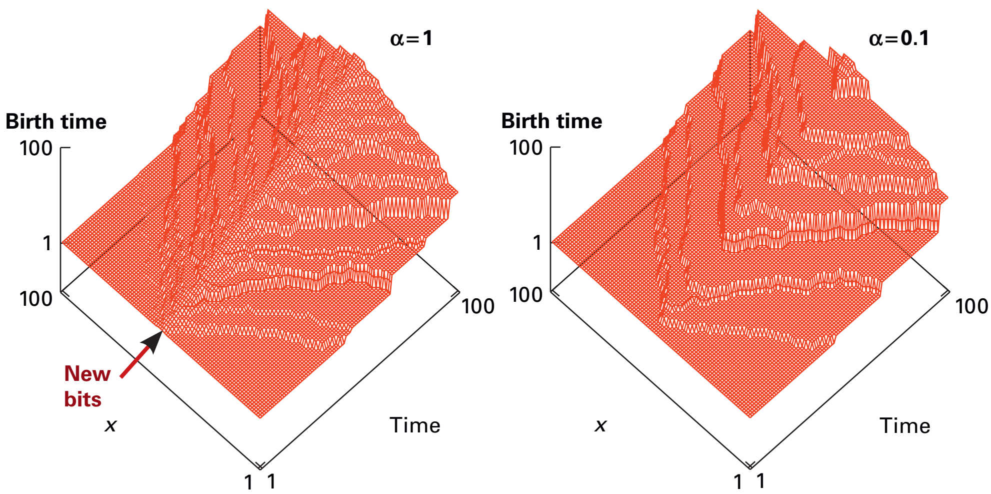

Figure 11.5 Minimal model of word spreading, with new words initiated at position x = 50 at various rates. Rate $\alpha = 1$ is the same rate as pairs of agents communicate with each other. In the right-hand panel we explore a lower word-initiation rate.

subsequently n pairs of neighboring nodes i, $i+1$. For each pair compare $a(i)$ with $a(i+1)$ and assign both positions the maximum value of their a values: $a(i) = a(i+1) = \max(a(i), a(i+1))$. Results for $\alpha = 1$ and $\alpha = 0.1$ are shown in Fig. 11.5. Notice that the dynamics far from initiation become independent of the initial "innovation rate."

11.2.3 *A classical way to obtain scale-free behavior is "the rich get richer dynamics" [588, 589, 473]. For networks this is formulated in terms of a* *growth model where each node links to already present nodes by linking up to the nodes at the end of random selected links in the network [589, 473]. Start with one node at time* $t = 0$ *and let* $n(k,t)$ *be the number of nodes with connectivity* k *at time* t*. Show that [590]:*

$$n(k,t+1) - n(k,t) \; = \; \frac{(k-1)\cdot n(k-1,t) - k \cdot n(k,t)}{\sum kn(k)} \quad for \;\; k > 1$$

Each added node is associated with one link, which means two edge ends, and thus $\sum_k kn(k,t) = 2 \cdot t$*.*

Argue that:

$$\frac{\mathrm{d}n}{\mathrm{d}t} \; = \; -\frac{1}{2t} \cdot \frac{\mathrm{d}(k \cdot n)}{\mathrm{d}k}$$

Use the "ansatz" $n(k,t) = f(t) \cdot N(k)$ *to prove that* $N(k) \propto 1/k^3$*.*

Answer Notice that the presented growth model is based on minimal information in the sense that each new node is attached to the end of a randomly selected old link. Thus one connects new nodes to old nodes, with a probability of connecting that is proportional to the degree of the older nodes. Highly connected nodes therefore grow faster; it pays to be "popular." After t steps, t nodes are added and, for the simplest version, also t edges.

Let $n(k,t)$ be the number of nodes with connectivity k at time t. Adding one link to previous nodes means that the number of nodes with connectivity k increases by one with probability $n(k-1) \times (k-1)/\sum kn(k)$. Similarly, the number of nodes of connectivity k is decreased by one with probability $n(k)k/\sum kn(k)$. As a result:

$$n(k,t+1) - n(k,t) \; = \; \frac{(k-1)\cdot n(k-1,t) - k \cdot n(k,t)}{\sum kn(k)} \quad for \;\; k > 1 \quad (11.1)$$

In this equation the first term represents the addition of a link to a node of connectivity $k-1$, thereby adding to the number of nodes of connectivity k. The second term represents the addition of a link to a node of connectivity k, thereby reducing the number of nodes with connectivity k by moving one of them to the next connectivity value.

Each added node is associated by one link, which means two edge ends, and thus $\sum_k kn(k,t) = 2 \cdot t$. Accordingly, the continuous limit:

$$\frac{\mathrm{d}n}{\mathrm{d}t} \; = \; -\frac{1}{2t}\frac{\mathrm{d}(k \cdot n)}{\mathrm{d}k} \qquad (11.2)$$

To solve this equation we make the "ansatz":

$$n(k,t) = f(t) \cdot N(k) \tag{11.3}$$

which gives:

$$2t\frac{f'(t)}{f(t)} \;=\; -\frac{1}{N}\frac{\mathrm{d}(kN)}{\mathrm{d}k} \tag{11.4}$$

where $f(t) \propto t$ since $f(t)\sum kN(k) = 2 \cdot t$. Thus, our factorization implies that the addition of nodes at constant rate implies that nodes of low and intermediate connectivity are increased at a constant rate. This implies that:

$$-1 - \frac{\mathrm{d}\ln(N)}{\mathrm{d}\ln(k)} = 2 \tag{11.5}$$

Using this, one obtains $N \propto k^{-3}$, and subsequently:

$$n(k) \propto \frac{1}{k^3} \tag{11.6}$$

Notice that if we instead added two links for each new node, each attached to the new node, then the nominator and the denominator in Eq. (11.1) would both double and the scaling word therefore be exactly maintained.

Notice also, that if one instead added links preferentially without adding new nodes, each such link would contribute with two times the nominator (one for each link end), and with $2t$ in the denominator, changing $1/(2t) \rightarrow (1{+}2)/((2{+}2)t)$. Thus the factor 2 in $2t\frac{f'(t)}{f(t)} \;=\; -\frac{1}{N}\frac{\mathrm{d}(kN)}{\mathrm{d}k}$. changes to $4/3$ and the scaling becomes $N(k) \propto 1/k^{2.33}$. In general, adding links preferentially between older nodes changes the scaling law $1/k^3 \rightarrow 1/k^\gamma$ with $\gamma \in [2;3]$.

11.2.4 *Consider the distribution [591, 592, 588]*

$$p(s) \propto 1/s^\tau$$

as a distribution for wealth in human society. Ague that $\tau \leq 2$ is fundamentally different society than $\tau > 2$. Notice that $\tau = 2$ is the famous Zipf distribution observed, for example, for word frequencies in books [593].

Answer The distribution $p(s) \propto 1/s^2$ is marginal in the sense that the average

$$\langle s \rangle \;=\; \int_{\min}^{\max} \frac{s\mathrm{d}s}{s^\tau} \tag{11.7}$$

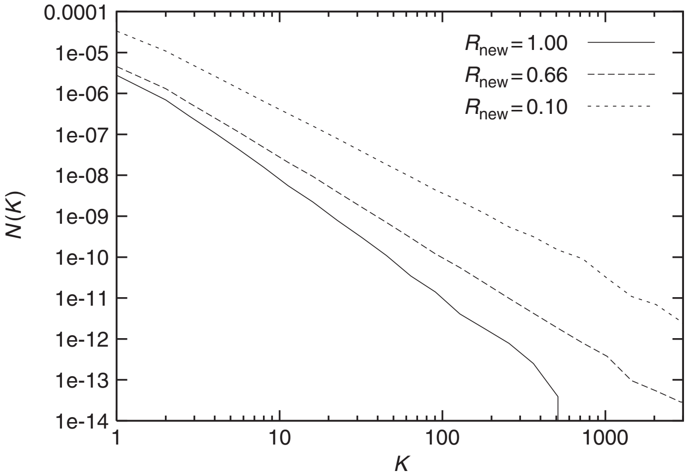

Figure 11.6 Connectivity distribution in a preferential attachment model for three different values of R_{new}, a parameter that specifies the fraction of times one adds a node with a link, and $1-R_{new}$ the fraction of times one add links to the growing network, each end being linked to a node with a probability proportional to the connectivity of that node. For all values of R_{new} one obtains a power law, dependent on R_{new} and approaching $1/K^3$ when $R_{new} = 1$.

has a large contribution from the upper cut-off of the integral, i.e. with power laws that are wider than $1/s^2$, like $1/s^{1.5}$, a huge fraction of the assets is bound relatively close to the upper cut-off. On the other hand, a narrower scaling, like $1/s^{2.5}$, will have an average that is independent of the upper cut-off.

If s denotes the resources/money, social systems should become unstable when the exponent τ becomes less than 2: popularly speaking, the rich then becomes so rich that by taxing their fortunes, the majority could increase their living standard substantially.

11.3 Tragedy of the Commons

11.3.1 *The deterministic counterpart of the non-spatial model can be given in terms of the coupling matrix* Γ *through the population dynamics of any species* i*,*

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = \sum_i \Gamma(i,j) \cdot p_i \cdot p_j - \sum_j \Gamma(j,i) \cdot p_i \cdot p_j \tag{11.8}$$

where the first sum overruns over all prey species of i, i.e. where $\Gamma(i,j) = 1$, and the second sum is over all predators of j (species where $\Gamma(j,i) = 1$). Simulate a three cycle using this equation, and subsequently add an additional species that only prey on one of the cycle species, but do not give anything back (a parasite).

Answer The simulation of this system is complicated by the numerical instability of the cyclic movement, i.e. that the simple first-order integration is inherently wrong. To avoid the technicalities of a higher-order integration, here we, for the limited time period of $t = 10$, use a very small $\mathrm{d}t = 0.00005$. Initially we set $p(1) = 1$, $p(2) = 2$ and $p(3)$, and simple integration of:

$$\frac{\mathrm{d}p_1}{\mathrm{d}t} = p_1 p_2 - p_3 p_1$$
$$\frac{\mathrm{d}p_2}{\mathrm{d}t} = p_2 p_3 - p_2 p_1$$
$$\frac{\mathrm{d}p_3}{\mathrm{d}t} = p_3 p_1 - p_2 p_3$$

gives the results in the upper left-hand panel of Fig. 11.7. Adding a parasite, the modified equations reas:

$$\frac{\mathrm{d}p_1}{\mathrm{d}t} = p_1 p_2 - p_3 p_1$$
$$\frac{\mathrm{d}p_2}{\mathrm{d}t} = p_2 p_3 - p_2 p_1$$
$$\frac{\mathrm{d}p_3}{\mathrm{d}t} = p_3 p_1 - p_2 p_3 - q p_3$$
$$\frac{\mathrm{d}q}{\mathrm{d}t} = p_3 q - q$$

and initiating $q = 0.1$, leads to overall collapse of the ecosystem, as shown in the upper right-hand panel of Fig. 11.7.

11.3.2) *Repeat the above simulation for a five-cycle system.*

Answer Start the simulation with $p_1 = p_2 = p_3 = p_4 = 1$ and $p_5 = 3$, and use same basic integration with a very small time step, as used in the previous question. Results we shown in the lower panels of Fig. 11.7.

11.3.3 *The cyclic relationship from the previous questions includes a direct predation cost on the prey, which differs from Eigen and Schusters hypercycle*

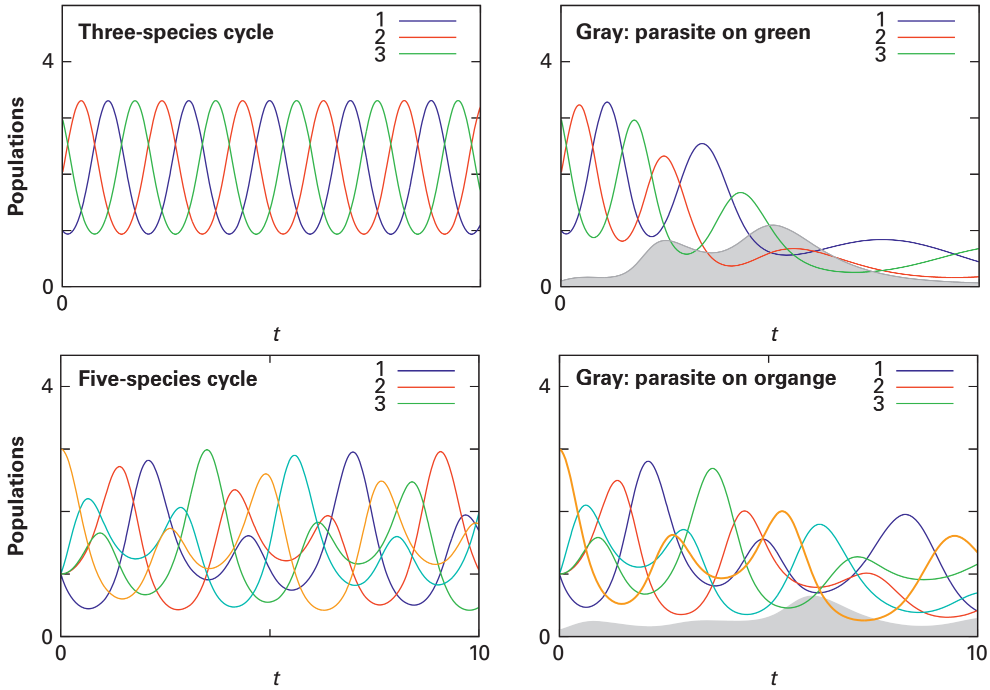

Figure 11.7 Simulation of a three-species cycle, and a five-species cycle. In the right-hand panels we show the effect of a parasite on one of the species.

[610], where the growth of one species happens at the cost of all other species. The hypercycle equation: $\frac{\mathrm{d}p_i}{\mathrm{d}t} = p_i \cdot p_{mod(i+1,n)} - \Omega \cdot p_i$ *where* $\Omega = \sum p_i \cdot p_{mod(i+1,n)}$ *describes catalytic chemical equations in a chemostat, where the sum of all species is maintained at* $\sum p_i = 1$*. Simulate a three-cycle and a five-cycle and compare with corresponding predator–prey simulations. Hint: see Fig. 11.12.*

Answer The hyper-cycle equations for the three catalytic species read:

$$\frac{\mathrm{d}p_1}{\mathrm{d}t} = p_1 p_2 - p_1 \Omega$$
$$\frac{\mathrm{d}p_2}{\mathrm{d}t} = p_2 p_3 - p_2 \Omega$$
$$\frac{\mathrm{d}p_3}{\mathrm{d}t} = p_3 p_1 - p_3 \Omega$$

where $\Omega = p_1 p_2 + p_2 p_3 + p_3 p_1$, and initially $\sum p_i = 1$. Start with $p_1 = p_2 = 0.2$ and $p_3 = 0.6$ to obtain the results shown in Fig. 11.8. The simulations are directly compared with predator–prey cyclic equations. Overall, the catalytic

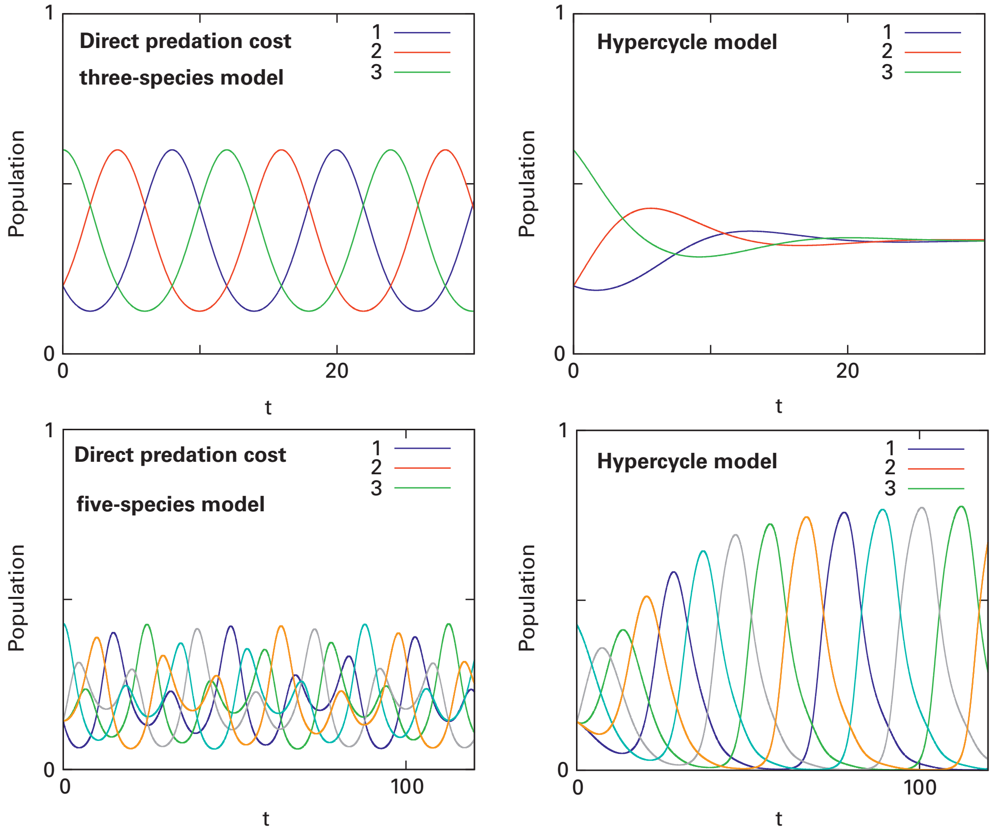

Figure 11.8 Simulation for comparison of direct cyclic predation with a hypercycle where growth of each population is taken from everybody.

reactions assumed by the hypercycle equations lead to less oscillating behavior than the direct predation cost.

11.3.4 *Social dilemmas are most often modeled in the form of the classical two-player prisoners' dilemma [611]. In this, each player decides independently whether to co-operate or defect. A given player subsequently is given a score of 1 if the opponent co-operates and 0 otherwise. Further the player is assigned an additional score* $-c$ *(i.e penalty* $c < 1$*) if he collaborate. Assume that the opponent randomly, with probability* r*, chooses to co-operate; what is your optimal strategy? Consider now an iterated game where players play multiple times with each other. Assume that the opponent always mimics your last choice; what then is your optimal long-term strategy as a function of* c*?*

Answer If you always defect, your average score is:

$$s = 1 \cdot r + 0 \cdot (1 - r) = r \tag{11.9}$$

For each time you co-operate, you lose $-c$, thus it will never be beneficial to cooperate with a non-responsive opponent. In this case, the optimal strategy is to defect persistently.

If your opponent mimics your last move, persistent defection gives an average score of $s = 0$.

For each collaboration step, the player loses c by collaborating, but gains $+1$ by subsequently cheating a collaborating opponent. Thus the average gain by alternating between collaboration and defection becomes $s = (1 - c)/2$.

If both agents always collaborate, the players average gain would be $1-c$, which is better.

However, each player would gain instantly by cheating, but it effectively forces the opponent to retaliate.

For $c > 1$ it is impossible to gain by collaboration.