and

$$\vec{m}_l(t) \equiv \int_0^t \gamma\, \vec{G}(\tau) \frac{\tau^l}{l!} \mathrm{d}\tau \quad l = 0, 1, 2, \ldots . \tag{4.89}$$

$\vec{m}_l$ is the l^{th} *order gradient moment.*

Proof of Eq. 4.87. The exact path $\vec{r}(t)$ followed by the moving spin is unknown. However, any physical motion can be expanded in a Taylor series around $t = 0$. Hence,

$$\vec{r}(t) = \vec{r}(0) + \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}(0)t + \cdots + \frac{\mathrm{d}^l\vec{r}}{\mathrm{d}t^l}(0)\frac{t^l}{l!} + \cdots . \tag{4.90}$$

The position $\vec{r}$, the velocity $\vec{v}(\vec{r})$, and the acceleration $\vec{a}(\vec{r})$ of the spin at time $t = 0$ can be introduced in this equation:

$$\vec{r}(t) = \vec{r} + \vec{v}(\vec{r})\; t + \vec{a}(\vec{r})\frac{t^2}{2} + \cdots . \tag{4.91}$$

Substituting Eq. 4.91 in Eq. 4.86 yields:

$$\Phi(\vec{r}, t) = \vec{r} \cdot \int_0^t \gamma\, \vec{G}(\tau)\, \mathrm{d}\tau + \vec{v}(\vec{r}) \cdot \int_0^t \gamma\, \vec{G}(\tau)\tau\, \mathrm{d}\tau + \vec{a}(\vec{r}) \cdot \int_0^t \gamma\, \vec{G}(\tau)\frac{\tau^2}{2}\mathrm{d}\tau + \cdots$$

or, using the gradient moments as defined in Eq. 4.89

$$\Phi(\vec{r}, t) = \vec{r} \cdot \vec{m}_0(t) + \vec{v}(\vec{r}) \cdot \vec{m}_1(t) + \vec{a}(\vec{r}) \cdot \vec{m}_2(t) + \cdots . \tag{4.92}$$

Rewriting Eq. 4.47 as

$$s(t) = \int_{\vec{r}} \rho^*(\vec{r})\; \mathrm{e}^{-\mathrm{i}\Phi(\vec{r},t)}\; \mathrm{d}\vec{r}, \tag{4.93}$$

and substituting Eq. 4.92 into Eq. 4.93, yields Eq. 4.87.

Without motion, only the zeroth-order moment $\vec{m}_0(t)$ in Eq. 4.87 causes a phase shift. This phase shift is needed for position encoding when using the $\vec{k}$-theorem. Motion introduces additional dephasing of the signal $s(t)$, yielding distortion and contrast loss. However, motion-induced dephasing can be reduced by *back-to-back symmetric bipolar pulses of opposite polarity*. They are able to restore hyperintense vessel signals for blood flowing at a *constant* velocity. In case of constant velocity Eq. 4.87 becomes

$$s(t) = \int_{\vec{r}} \rho^*(\vec{r})\; \mathrm{e}^{-\mathrm{i}\vec{v}(\vec{r})\cdot\vec{m}_1(t)}\; \mathrm{e}^{-\mathrm{i}\vec{r}\cdot\vec{m}_0(t)}\; \mathrm{d}\vec{r} \tag{4.94}$$

and contains only two dephasing factors, one necessary for position encoding and the other introduced by the blood velocity $\vec{v}(\vec{r})$.

Eq. 4.55 shows that for *stationary* spins ($\vec{v}(\vec{r}) = 0$) the net phase shift due to simple bipolar gradient pulses (Figure 4.27(a)) is zero. This is the case at t = TE in the frequency-encoding and slice-selection directions. For *moving* spins ($\vec{v}(\vec{r}) \neq 0$), however, a simple bipolar pulse sequence as in Figure 4.27(a) introduces a phase shift because its first gradient moment $\vec{m}_1$ at t = TE is nonzero:

$$m_1(\mathrm{TE}) = -\gamma\; \vec{G}\,(\Delta t)^2 \neq 0. \tag{4.95}$$

Back-to-back symmetric bipolar pulses of opposite polarity on the other hand (Figure 4.27(b)) remove the velocity-induced phase shift at t = TE while they have no net effect on static spins. Both their zeroth and first-order gradient moments m_0(TE) and m_1(TE) are zero. Higher order motion components are *not* rephased, however, and will still cause dephasing.

The rephasing gradients can be applied only in the frequency-encoding and slice-selection directions. This technique is known as *gradient moment nulling*, *gradient moment rephasing*, or *flow compensation*. It is built-in in sequences to remove velocity-induced artifacts. A diagram of a 3D spoiled GE sequence with first-order motion compensation is shown in Figure 4.28. Technical considerations limit the motion compensation to the first-order or at most the second-order gradient moments. Very complex motion patterns, such as the turbulence in the aortic arch and behind stenotic plaques, continue to produce signal dephasing.

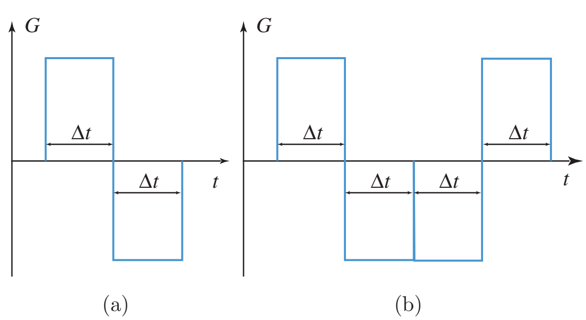

Figure 4.27 **(a)** Simple bipolar pulses cannot provide a phase-coherent signal for moving spins. **(b)** Back-to-back symmetric bipolar pulses of opposite polarity on the other hand restore the phase coherence completely for spins moving at a constant velocity.