X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 4. Coherence Homework Problems

4.1 (a) Use Heisenberg's uncertainty principle for transverse position and momentum to derive the limiting phase space condition for radiation of wavelength λ

$$d \cdot \theta \geq \lambda/2\pi$$

where $d = 2\Delta x$ is the uncertainty in emitted photon position, $\theta = \Delta\theta$ is the detection cone half-angle, and where Δx and $\Delta\theta$ are single-sided (half width) root-mean-square ($1/\sqrt{e}$) measures of Gaussian spatial and angular distributions. (b) Explain under what circumstances this is a condition for spatial coherence. (c) Convert this to an expression involving full-width at half maximum (FWHM) measures of source size and divergence.

4.2 Consider the lowest order laser cavity mode (TEM_{00}), described by a Gaussian intensity distribution

$$\frac{I}{I_0} = e^{-r^2/r_0^2}$$

where $r(z) = r_0\sqrt{1+(\lambda z/4\pi r_0^2)^2}$, z is measured along the direction of propagation (symmetry axis), and λ is the wavelength. (a) Show that in the far-field, where $z >> 4\pi r_0^2/\lambda$, that

$$d \cdot \theta = \lambda/2\pi$$

where $\theta = r(z)/z$ is the far field half-angle and $d = 2r_0$ is the "waist" diameter, i.e., the smallest diameter as a function of z. (b) Why is this condition associated with spatial coherence? (c) How is this achieved in a laser? (d) What happens in a multimode laser, and how does this affect spatial coherence of the resultant radiation field?

4.3 (a) Describe the effects of a monochromator and a pinhole spatial filter as they are used to improve the coherence properties of partially coherent radiation. (b) Relate the improved spatial and temporal coherence to reductions in photon flux (or power). (c) For the monochromator, separate the effects of narrowed spectral width from the "insertion loss" of the monochromator due to component reflectivities and grating (or crystal) efficiency. (d) For the pinhole spatial filter draw a diagram to illustrate how this works as a "phase-space filter", reducing some combination of acceptance angle and size. (e) Describe three types of experiments where these techniques would be utilized.

4.4 Derive an expression relating coherent power, $P_{coh,\ \lambda/\Delta\lambda}$ to spectral brightness $B_{\Delta\omega/\omega}$ for an arbitrary source of radiation. Start with an expression such as Eq. (5.58), as given in Chapter 5, for spectral brightness, and an expression to convert photon flux F to power, such as P=($\hbar\omega$/photon)F = (hc/λ·photon)F, and a suitable definition of coherent power in terms of $d \cdot \theta$ and $\lambda/\Delta\lambda$ (or $\lambda^2/2\Delta\lambda$). Your expression should be equally valid for undulator and bending magnet radiation, laser radiation, and emissions from extended sources such as plasma discharges and laser produced plasmas.

4.5 Consider the diffraction of radiation by a pinhole illuminated with varying degrees of partial coherence, as first studied in detail by R.A. Shore, B.J. Thompson, and R.E. Whitney [*JOSA* 56, 733-738 (1966)]. In that paper the authors report both theoretical modeling involving the van Cittert-Zernike theorm, and experiments involving mercury arc lamp emission in the visible at 546.1 nm. In their experiment emission from the arc lamp uniformly illuminates a small pinhole of diameter d which serves as the first element in a pinhole-angular-aperture spatial filter. A lens of focal length f is then used to collimate radiation from this first pinhole. A second pinhole of diameter $D = 2a$ then intercepts the quasi-collimated radiation, setting the angular acceptance of the spatial filter at half-angle $\theta = a/f$. The second pinhole also serves as the "diffractive aperture" for these studies, with the intensity distribution of its diffracted radiation recorded in the far-field as a function of the partial coherence parameter $C = \pi d\theta/\lambda$. The figure below shows the far-field intensity distribution as a function of the normalized diffraction angle $ka\psi$, for various values of the parameter C. As reported in the cited article, the measured diffraction pattern closely matches the theoretical predictions. (a) Discuss the theoretical limits of the diffraction pattern for small and large C. Which corresponds to essentially coherent illumination, and which to largely incoherent illumination? (b) What is the value of C for which the normalized degree of spatial coherence μ (see Figure 4.15b of the text) goes to zero just along the circumference of the second pinhole? (c) How does the corresponding curve in the figure below compare to your intuitive expectation for pinhole diffraction with this degree of coherence across the area of the pinhole? (d) To observe the sharp null features associated with an Airy pattern, what correlation among fields would one expect across the aperture? (e) How does this relate to values of C or C^2? (f) The authors refer to C as the "number of correlation intervals contained in the aperture radius"; relate this quantitatively to the cases where $C = 2$, 1 and 1/4.

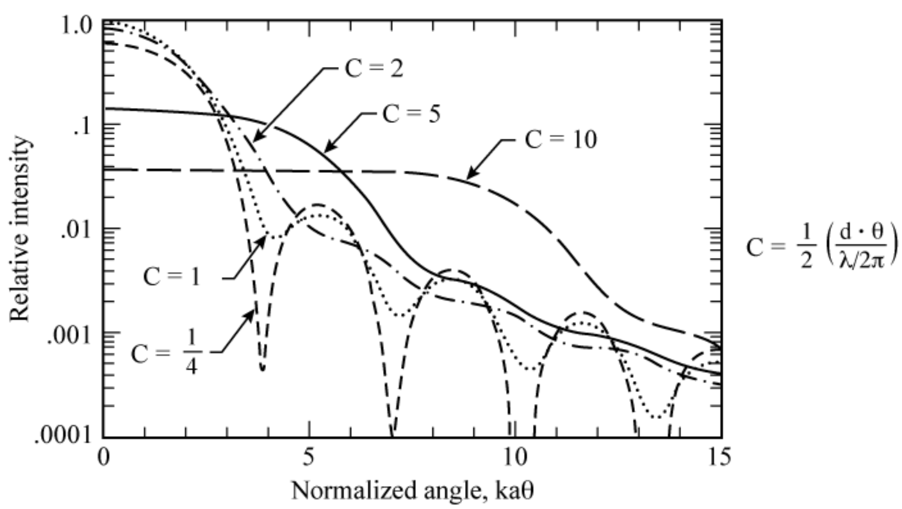

Figure for problem 4.5 [Fig.4.21 in the text]. Far-field intensity distribution of a circular aperture (pinhole) illuminated uniformly, but with varying degrees of spatial coherence. (Courtesy of B.J. Thompson, University of Rochester.)

Further homework problems involving coherence are found towards the end of homework problems for Chapters 5 through 9, and also in Chapter 11.