Errata and Suggested Updates for

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Joseph M. Powers and Mihir Sen

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- 1. In many places "non-linear" appears that should be recast as "nonlinear." This is found on pages 248, 559, 559, 566, 566, 566.
- 2. p. 98: Sec. 2.6.3: In the analysis of Eqs. (2.346-2.348), several factors of 1/2 are missing; e.g.  $dx_1$  should be replaced by  $(dx_1)/2$  in Eq. (2.346).
- 3. p. 89, 91, 112, 113: We would be better to employ an opposite sign convention for torsion  $\tau$ . This is more common in the literature. For example, the Frenet-Serret equations, Eqs. (2.259-2.261), p. 89, are better stated with the more common sign convection as

$$\begin{aligned} \frac{d\mathbf{t}}{ds} &= \kappa \mathbf{n}, \\ \frac{d\mathbf{n}}{ds} &= -\kappa \mathbf{t} + \tau \mathbf{b} \\ \frac{d\mathbf{b}}{ds} &= -\tau \mathbf{n}. \end{aligned}$$

4. p. 98: Note: some sources define

div 
$$\mathbf{T} = \nabla \cdot \mathbf{T}^T = \frac{\partial T_{ji}}{\partial x_i}$$

As long as the analysis is internally consistent, as it is here, it is correct.

- 5. p. 111: Problem 4g is a repeat of Problem 1.
- 6. p. 113: Problem 22–The first equation in the problem may need to be

$$\mathbf{t}^T \cdot \frac{d^2 \mathbf{t}}{ds^2} \times \frac{d \mathbf{t}}{ds} = \kappa^2 \tau$$

This would change if we change the sign convention for  $\tau$ .

- 7. p. 113: Problem 30 should have corners at (0, 0, 0), (0, 1, 0), (1, 1, 0), and (1, 0, 0).
- 8. p. 113: Problem 30 should have "For  $\mathbf{f}(x, y, z) = \dots$ "
- 9. p. 114: Problem 38 should read "...one finds that the three principal..."

- 10. p. 114: Problem 38 should have  $q_i = -k_{ij}\partial T/\partial x_j....$
- 11. p. 145: Problem 29: The problem is better stated as "...are possible when b = 2.2617, and find the corresponding frequency."
- 12. p. 156: Eq. (4.102) has a small error. The argument of the second term should be  $-i\sqrt{14}\ln x$ . So we should have

$$y(x) = \frac{1}{x} \left( C_1 (\exp(\ln x))^{i\sqrt{14}} + C_2 (\exp(\ln x))^{-i\sqrt{14}} \right),$$
  
$$= \frac{1}{x} \left( C_1 \exp(i\sqrt{14}\ln x) + C_2 \exp(-i\sqrt{14}\ln x) \right),$$
  
$$= \frac{1}{x} \left( \hat{C}_1 \cos(\sqrt{14}\ln x) + \hat{C}_2 \sin(\sqrt{14}\ln x) \right).$$

13. p. 214: Problem 29: We should have

$$P(u,v) = \sum_{k=1}^{n} \sum_{j=1}^{k} (-1)^{j-1} \frac{d^{k-j}u}{dx^{k-j}} \frac{d^{j-1}}{dx^{j-1}} (a_{n-k}v).$$

- 14. p. 216: Problem 4.52, should read "...to being connected as well as identical to mass 1..." and "... of the two masses and the potential energy of the three springs."
- 15. p. 217: Problem 4.53 needs an additional factor of "2" in the error function, so as to be consistent with the correct Eq. (A.103) on p. 594.
- 16. p. 230: Fig. 5.5's caption should be  $dy/dx = -\sqrt{xy}$
- 17. p. 236: The second term in Eq. (5.146) be

$$2k\left(1+x+\frac{1}{4}x^2+\ldots\right)$$

instead of

$$2k\left(1+x+\frac{1}{2}x^2+\ldots\right)$$

18. p. 239: Eq. (5.179) should be

$$y = x + \frac{x^2}{2} + \frac{x^4}{12} + \dots$$

19. p. 257: Eq. (5.319) should have

$$y(x) = y_0(x) + \epsilon y_1(x) + \epsilon^2 y_2(x)...$$

20. p. 269: Just before Eq. (5.429) we should find  $\xi^2 + \eta^2 = r^2$  instead of  $x^2 + y^2 = r^2$ .

- 21. p. 272: Problem 15 is identical to Example 5.11.
- 22. p. 276: Problem 55 repeats Problem 50.
- 23. p. 277: Problem 57 repeats Problem 18.
- 24. p. 296: The equation appearing just before Eq. (6.92) should receive a number.
- 25. p. 302: In Example 6.19,  $||x||_2 = 3.873$  not 3.870.
- 26. p. 311: In Fig. 6.9, need more space in needed in the term " $-0.23 \sin 4t$ ."
- 27. p. 316: Eq. (6.239) needs an approximate equals sign rather than an equals sign. So we should find

$$t^2 \approx \sum_{n=1}^N \alpha_n \varphi_n$$

28. p. 327: Eq. (6.335) has a  $\cdot$  between the matrix and the vector and should not. We should find  $\begin{pmatrix} \epsilon^1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} r^1 \\ r^1 \end{pmatrix}$ 

$$\begin{pmatrix} \xi^1\\ \xi^2 \end{pmatrix} = \begin{pmatrix} 2 & 1\\ 0 & 3 \end{pmatrix} \begin{pmatrix} x^1\\ x^2 \end{pmatrix}$$

- 29. p. 363: Eq. (6.693) should have  $\alpha \phi(t)$  instead of  $c\phi(t)$ .
- 30. pp. 364-5: In Eqs. (6.702), (6.703), one could replace  $(t^5/4)^{2/5}$  by  $t^{1/2}$ . Additional simplification is possible that is easily achieved with computer algebra.
- 31. p. 371: Eq. (6.754) should have  $\lambda t^3$  instead of  $\lambda t^2$ . So we should find

$$\int_{0}^{1} \left( -(1-2t)^{2} + \lambda t^{3}(1-t)^{2} \right) dt = 0,$$
$$-\frac{1}{3} + \frac{\lambda}{60} = 0,$$
$$\lambda = 20$$

- 32. p. 378: The y-axis of the rightmost graph of Fig. 6.28 is mis-labeled. It should be y(t = 1).
- 33. p. 379: Problem 10 is too similar to Example 6.39.
- 34. p. 381: Problem 27: the inner product should be enclosed by angle brackets instead of parentheses. So we should have

$$\langle x, y \rangle = \int_{a}^{b} w(t)x(t)y(t) dt,$$

35. p. 381: Problem 30 is a repeat of Problem 18.



Figure 1: Modified Fig. 7.5.

- 36. p. 381: Problem 31 is a repeat of Problem 8.
- 37. p. 381: Problem 33 is a repeat of Problem 4.
- 38. p. 382: Problem 37 is a repeat of Problem 11.
- 39. p. 384: Problem 57 is a repeat of Problem 14.
- 40. p. 385: Problem 61 is a repeat of Problem 56.
- 41. p. 385: Problem 66 is a repeat of Problem 64.
- 42. p. 386: Problem 72 may benefit from changing the lower limit to  $-\infty$ .
- 43. p. 424: Eq. (7.229) should actually employ  $\mathbf{u}\mathbf{u}^T$ ; it presently incorrectly uses  $\mathbf{u}^T \cdot \mathbf{u}$ , though Eq. (7.230) is correct. We should thus find

$$\begin{split} \mathbf{H} &= \mathbf{I} - 2\mathbf{u}\mathbf{u}^{T}, \\ &= \begin{pmatrix} 1 & 0\\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} -\frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix}, \\ &= \begin{pmatrix} 1 & 0\\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} \frac{1}{10} & \frac{-3}{10} \\ \frac{-3}{10} & \frac{9}{10} \end{pmatrix}, \\ &= \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{-4}{5} \end{pmatrix}. \end{split}$$

- 44. p. 426: Eq. (7.240) should read  $\mathbf{I} 2\mathbf{u}\mathbf{u}^T$ . The actual numerical values are correct.
- 45. p. 431: Figs. 7.5 and 7.6 would be more effective if it plotted  $2|c_k|/\sqrt{N}$  vs. k. When so done, it returns the actual amplitude of the mode for modes that are sufficiently resolved. It would be useful to present the analysis to show this. Figure 1 gives an example of how this could be fixed.
- 46. p. 440: Eq. (7.337) has an unneeded dot in the matrix product.

- 47. p. 441: It would be useful in Section 7.9.5 to have more discussion of algebraic and geometric multiplicity of the eigenspace.
- 48. p. 450: Columns of **Q** should be the normalized eigenvectors, not just the eigenvectors.
- 49. p. 457: More nuance may be needed for the projection matrix **P**. The matrix **P** as defined by Eq. (7.477) is guaranteed symmetric with spectral norm of 1. And it guarantees  $\mathbf{P} \cdot \mathbf{x} = \mathbf{P} \cdot \mathbf{P} \cdot \mathbf{x}$ , as required. However if

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix},$$

we find a)  $\mathbf{B} \cdot \mathbf{x} = \mathbf{B} \cdot \mathbf{B} \cdot \mathbf{x}$ , b) the eigenvalues of  $\mathbf{B}$  are 1 and 0; c)  $\mathbf{B}$  is asymmetric, d) the spectral norm of  $\mathbf{B}$  is  $||\mathbf{B}|| = \sqrt{2}$ , thus  $\mathbf{B}$  can stretch vectors of certain orientation, e.g. if  $\mathbf{x} = (1, 0)^T$ ,  $\mathbf{B} \cdot \mathbf{x} = (1, 1)^T$ . So we do not consider  $\mathbf{B}$  to be a projection because it could stretch  $\mathbf{x}$ , though some authors might allow it to be considered a projection.

50. p. 460: Eq. (7.497) needs another "dot" within its matrix multiplication. So we should find

$$\mathbf{a} = \left( \left( \mathbf{W} \cdot \mathbf{A} \right)^T \cdot \mathbf{W} \cdot \mathbf{A} \right)^{-1} \cdot \left( \mathbf{W} \cdot \mathbf{A} \right)^T \cdot \mathbf{W} \cdot \mathbf{b}.$$

- 51. p. 469: In Eq. (7.593), we should have  $q_{11} = 1/\sqrt{10}$ .
- 52. p. 475: Problem 32 has no Cholesky decomposition as the matrix is not positive definite.
- 53. p. 486: Figure 8.2 needs negative signs on some numbers on the y axis.
- 54. p. 491: Some confusion exists here. The  $\phi_i$  of Eq. (8.72) is orthonormal. The  $\phi_i$  of Eq. (8.73) is not. It is written carefully, but it is confusing. It could and should be reformulated for more clarity.
- 55. p. 518: Just after Eq. (9.159), should read "...eigenvectors  $\mathbf{e}_k, k = 1, 2, ..., K$ , as possible."
- 56. p. 553: In the caption of Fig. 9.13, one should replace dx/dx by dx/dt.
- 57. p. 562: Eq. (9.494) should have an 8 in the numerator of both terms, not  $4\sqrt{2}$ . So we should find

$$T(x,t) \approx \frac{8}{\pi^3} e^{-\pi^2 t} \sin(\pi x) + \frac{8}{27\pi^3} e^{-9\pi^2 t} \sin(3\pi x).$$

58. p. 562: Eq. (9.495) should have additional  $\sqrt{2}$  on both terms because this is part of the basis function. Thus, we should find

$$T(x,t) \approx \alpha_1(t)\sqrt{2}\sin(\pi x) + \alpha_2(t)\sqrt{2}\sin(3\pi x),$$