

### 3

## Single-interface modes in the microwave regime

### Tables

| Item | Topic                    | Chapter 3 | Chapter 4          | Chapter 5          | Chapter 6 | Chapter 7 |
|------|--------------------------|-----------|--------------------|--------------------|-----------|-----------|
| 1    | Interfaces               | 1         | 1                  | 2                  | 1         | 2         |
| 2    | Types                    | ENG, DNG  | DPS, ENG, DNG, MNG | DPS, ENG, DNG, MNG | ENG       | ENG       |
| 3    | $\epsilon_r, \mu_r$      | complex   | real               | real               | complex   | complex   |
| 4    | Dispersion               | yes       | no                 | no                 | no        | no        |
| 5    | Free $\beta$             | yes       | yes                | yes                | yes       | yes       |
| 6    | Loaded $\beta$           | yes       | no                 | no                 | yes       | yes       |
| 7    | Configuration            | 0, K      | free               | free               | 0, K      | G         |
| 8    | $\mathcal{R}$            | yes       | no                 | no                 | yes       | yes       |
| 9    | G-H                      | yes       | no                 | no                 | no        | no        |
| 10   | $E$ and $H$              | no        | yes                | yes                | yes       | yes       |
| 11   | $s_z$                    | no        | yes                | yes                | yes       | yes       |
| 12   | $s_x$                    | no        | no                 | no                 | yes       | yes       |
| 13   | $\eta$                   | yes       | no                 | no                 | yes       | yes       |
| 14   | $v_{ph}$ and $v_{group}$ | yes       | no                 | no                 | no        | no        |
| 15   | Charge density           | no        | no                 | no                 | yes       | yes       |

Table 2.10. List of topics investigated in chapters 3-7.

| Item | Topic                    | Chapter 3 |
|------|--------------------------|-----------|
| 1    | Interfaces               | 1         |
| 2    | Types                    | ENG, DNG  |
| 3    | $\epsilon_r, \mu_r$      | complex   |
| 4    | Dispersion               | yes       |
| 5    | Free $\beta$             | yes       |
| 6    | Loaded $\beta$           | yes       |
| 7    | Configuration            | 0, K      |
| 8    | $\mathcal{R}$            | yes       |
| 9    | G-H                      | yes       |
| 10   | $E$ and $H$              | no        |
| 11   | $S_z$                    | no        |
| 12   | $S_x$                    | no        |
| 13   | $\eta$                   | yes       |
| 14   | $v_{ph}$ and $v_{group}$ | yes       |
| 15   | Charge density           | no        |

Table 3.1. List of topics investigated in this chapter.

| $F$  | $f_0$ (GHz) | $f_\mu$ (GHz) | $f_p$ (GHz) |
|------|-------------|---------------|-------------|
| 0.56 | 4.          | 6.03023       | 10.         |

Table 3.2. The parameters  $F$ ,  $f_0$  and  $f_p$  appearing in Eqs. (2.61) and (2.62), where  $f_\mu$  is the frequency at which  $\mu_r$  vanishes.

| case | Loss             | $\gamma/f_p$ | $\Gamma/f_0$ |
|------|------------------|--------------|--------------|
| (a)  | Lossless         | 0            | 0            |
| (b)  | Moderately lossy | 0.01         | 0.01         |
| (c)  | Highly lossy     | 0.03         | 0.03         |

Table 3.3. Relative loss (damping) parameters,  $\gamma/f_p$  and  $\Gamma/f_0$ , associated with  $\epsilon_m$  and  $\mu_m$ , respectively, for (a) lossless, (b) moderately lossy and (c) highly lossy metamaterials.

| $f$ (GHz) | $\epsilon_m$ | $\mu_m$  | $n_m$    | $r_m$  | $\phi_m$ (deg) |
|-----------|--------------|----------|----------|--------|----------------|
| 4.4       | -4.16529     | -2.22667 | -3.04544 | 4.7231 | 208.128        |

Table 3.4. Values of  $\epsilon_m$ ,  $\mu_m$  and  $n_m$  at a frequency  $f = 4.4$  GHz, and their related radius,  $r_m$ , and angle,  $\phi_m$ , in an  $\epsilon_r$ - $\mu_r$  parameter space.

|     | $f$ [GHz] | Limits                              |
|-----|-----------|-------------------------------------|
| (b) | 4.        | $\mu_m \rightarrow -\infty$         |
| (c) | 4.71405   | $\mu_m \rightarrow -1$              |
| (d) | 5.20102   | $\beta_{TE} \rightarrow \beta_{TM}$ |
| (e) | 7.07107   | $\epsilon_m \rightarrow -1$         |

Table 3.5. The frequencies describing the horizontal lines (b) - (d) in Fig. 3.6, associated with the asymptotic value of  $\epsilon_m$  and  $\mu_m$ .

| Mode         | TM (1)    | TE        | TM (2)      |
|--------------|-----------|-----------|-------------|
| Type         | ENG       | DNG       | ENG         |
| $f$ (GHz)    | 3.57136   | 5.07339   | 6.0456      |
| $\epsilon_m$ | -6.84029  | -2.88511  | -1.73603    |
| $\mu_m$      | 3.20086   | -0.479984 | 0.00397536  |
| $n_m$        | 4.67919 i | -1.17678  | 0.0830743 i |
| $r_m$        | 7.55216   | 2.92476   | 1.73604     |
| $\phi_m$     | 154.923   | 189.446   | 179.869     |

Table 3.6. The TE, TM(1) and TM(2) modes: Media type, frequency of excitation, value of  $\epsilon_m$ ,  $\mu_m$ ,  $n_m$ ,  $r_m$  and  $\phi_m$ .

| $f$ (GHz) | Mode   | Type | $v_{\text{phase}}/c$ | $v_{\text{group}}/c$ |
|-----------|--------|------|----------------------|----------------------|
| 3.57136   | TM (1) | ENG  | 0.806222             | 0.300724             |
| 5.07339   | TE     | DNG  | 0.998469             | -0.146865            |
| 6.0456    | TM (2) | ENG  | 0.828664             | 0.286238             |

Table 3.7. Relative phase and group velocities,  $v_{\text{phase}}/c$  and  $v_{\text{group}}/c$ , respectively, for the TE, TM(1) and TM(2) modes, where  $c$  is the speed of light in free space.

| $f$ (GHz) | Mode   | Type | $\eta_c/\eta_0$ | $\eta_m/\eta_0$ |
|-----------|--------|------|-----------------|-----------------|
| 3.57136   | TM (1) | ENG  | 1.24035         | -0.184685       |
| 5.07339   | TE     | DNG  | 0.998469        | -0.371025       |
| 6.0456    | TM (2) | ENG  | 1.20676         | -0.678803       |

Table 3.8. Frequency, mode, media type, and relative cover and metamaterial impedance,  $\eta_c/\eta_0$  and  $\eta_m/\eta_0$ , respectively, where  $\eta_c$  and  $\eta_0$  are the impedance of the cover and free space, respectively.

## Figures

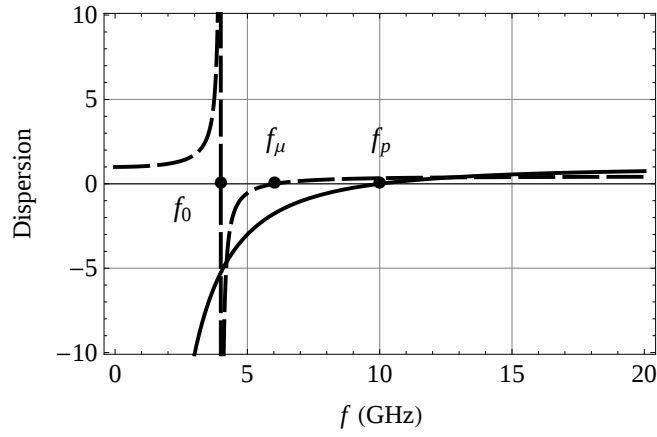


Fig. 3.1. Dispersion of  $\epsilon_m$  (solid line) and  $\mu_m$  (dashed line) as a function of frequency,  $f$ . The dots denote the frequencies at which  $\epsilon_m$  and  $\mu_m$  switch sign.

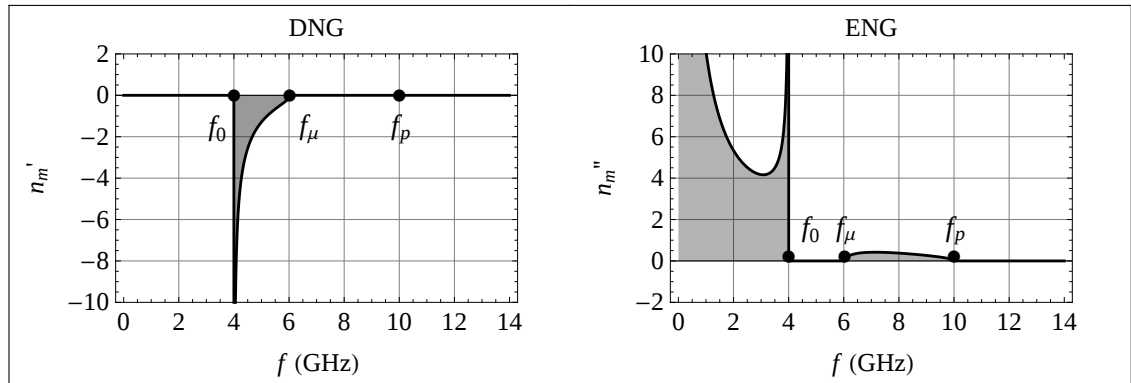


Fig. 3.2. Dispersion of  $n_m'$  and  $n_m''$  as a function of frequency,  $f$ , where the dots are as in Fig. 3.1.

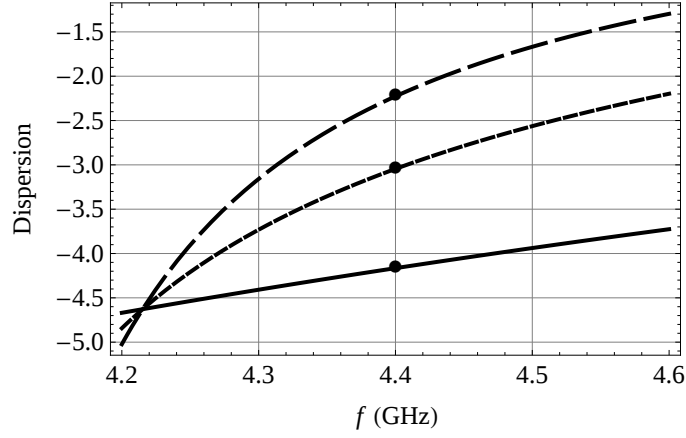


Fig. 3.3. Dispersion of  $\epsilon_m$ ,  $\mu_m$  and  $n_m$  in a narrow frequency range where all three are negative, and their value at  $f = 4.4$  GHz are depicted by the dots.

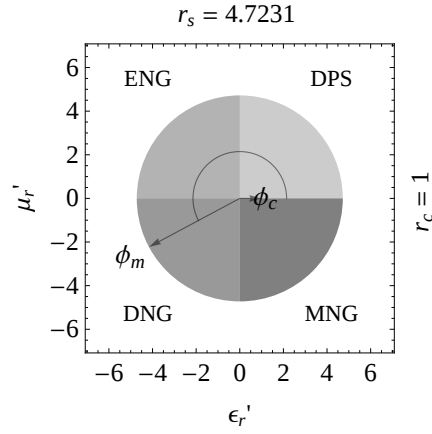


Fig. 3.4. Metamaterial type at a frequency  $f = 4.4$  GHz, described in an  $\epsilon_r'$ - $\mu_r'$  parameter space by the vector  $r_m$  and angle  $\phi_m$ , pointing in the DNG quadrant.

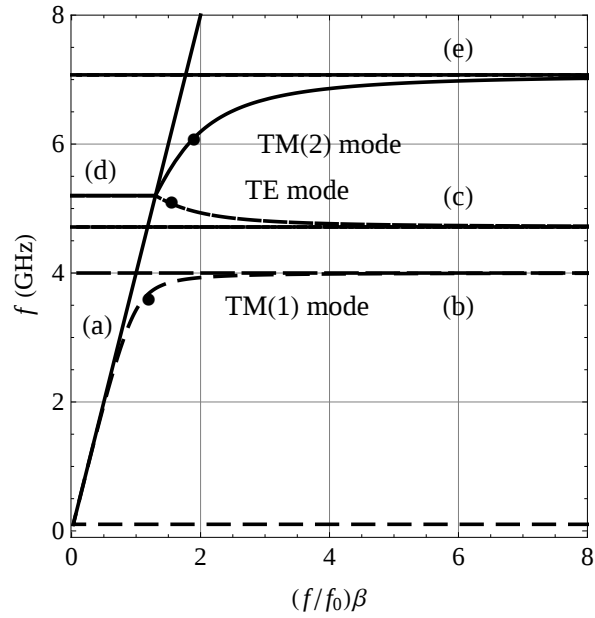


Fig. 3.6. The dispersion of the propagation constant of a single-interface mode,  $\beta$ , propagating along the interface separating a metamaterial and DPS-type cover. The dots represent the frequencies associated with the TE, TM(1) and TM(2) modes that could be excited by a prism at an angle  $\theta_p = 45$  deg. The lines denoted by (a) - (e) are discussed in the text.

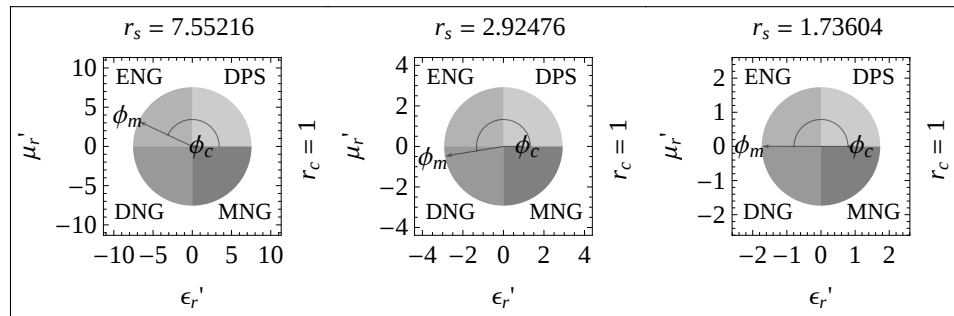


Fig. 3.7. Type of the TM(1), TE and TM(2) modes and their associated radii  $r_m$  and  $r_c$ , and angles  $\phi_m$  and  $\phi_c$ , in an  $\epsilon_r'$ - $\mu_r'$  parameter space. Here, (left) the TM(1) mode belongs to the ENG-type metamaterial, (center) the TE mode belongs to the MNG-type metamaterial and (right) the TM(2) mode belongs to the ENG-type metamaterial.

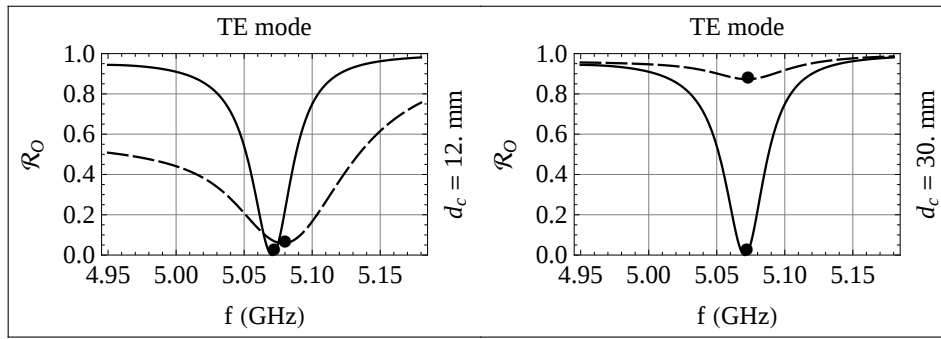


Fig. 3.10(b). The  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_O$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 12$  mm and (right) 30 mm.

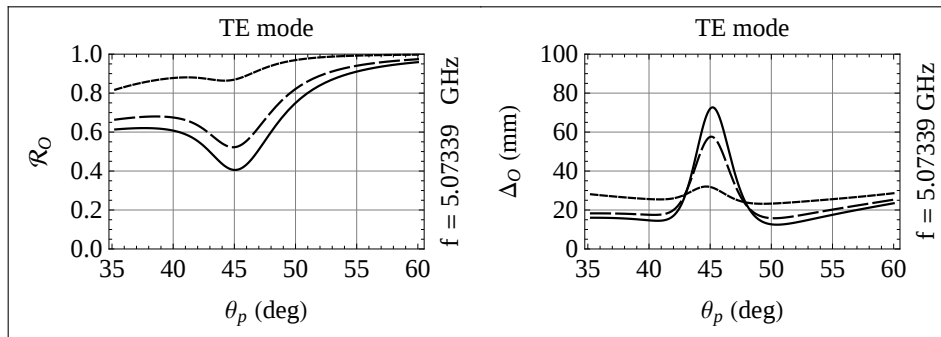


Fig. 3.10(c). (left) The  $\mathcal{R}_O$ -spectrum and (right)  $\Delta_O$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 18$  mm (solid line), 20 mm (dashed line) and 30 mm (dotted line).

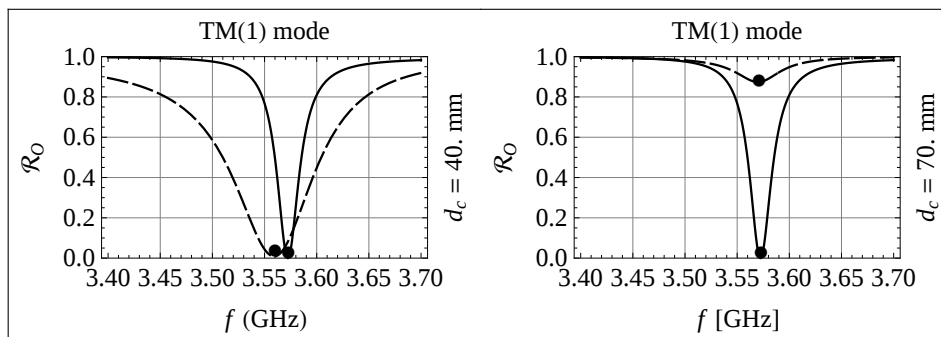


Fig. 3.11(b). The  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_O$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 40$  mm and (right) 70 mm.

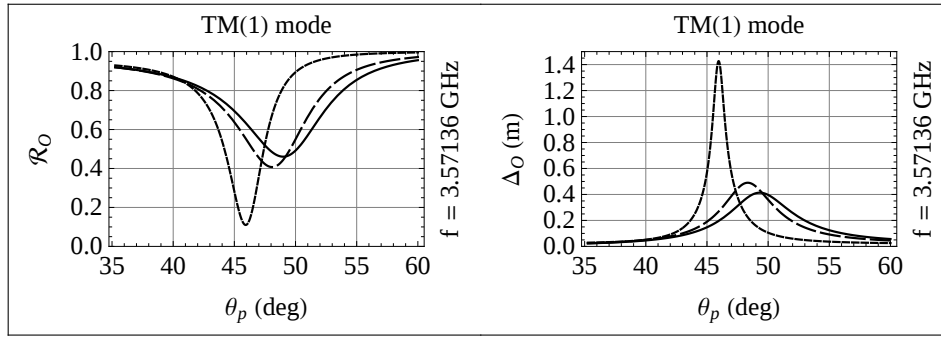


Fig. 3.11(c). The (left)  $\mathcal{R}_O$ -spectrum and (right)  $\Delta_O$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 18$  mm (solid line), 20 mm (dashed line) and 30 mm (dotted line).

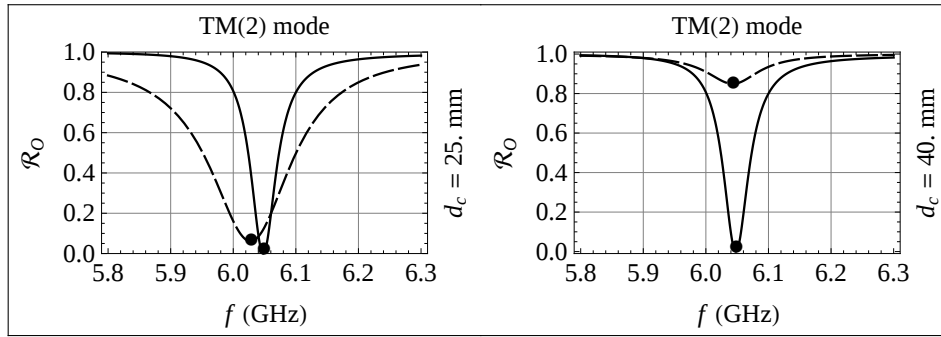


Fig. 3.12(b). The  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_O$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 15$  mm and (right) 25 mm.

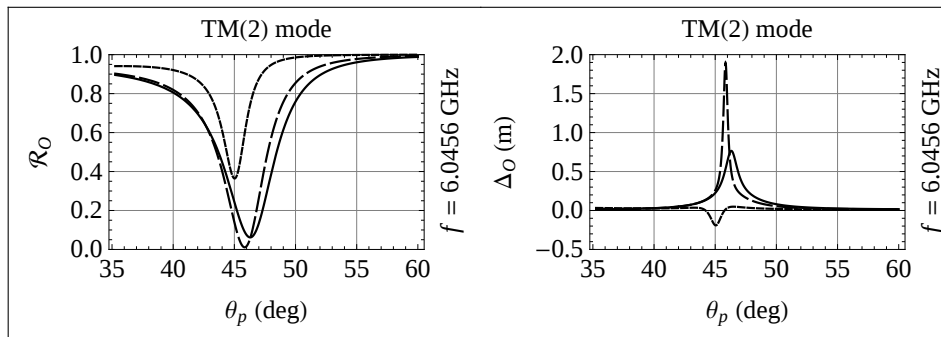


Fig. 3.12(c). (left) The  $\mathcal{R}_O$ -spectrum and (right) the  $\Delta_O$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 18$  mm (solid line), 20 mm (dashed line) and 30 mm (dotted line).

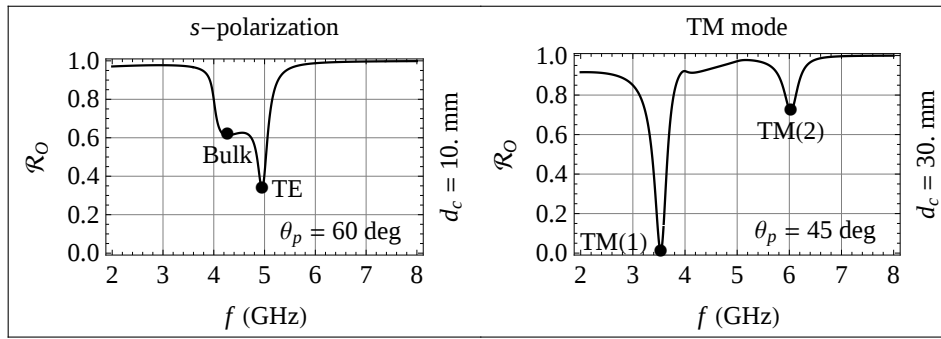


Fig. 3.13. The  $\mathcal{R}_O$ -spectrum for the Otto configuration as a function of frequency,  $f$ , for a TE mode with  $d_c = 10$  mm and  $\theta_p = 60$  deg (left), and for a TM mode with  $d_c = 30$  mm and  $\theta_p = 45$  deg (right). One observes the positions of the resonances belonging to the bulk and TE modes on the left. The resonances belonging to the TM(1) and TM(2) modes are observed on the right, but the bulk mode is barely observed.

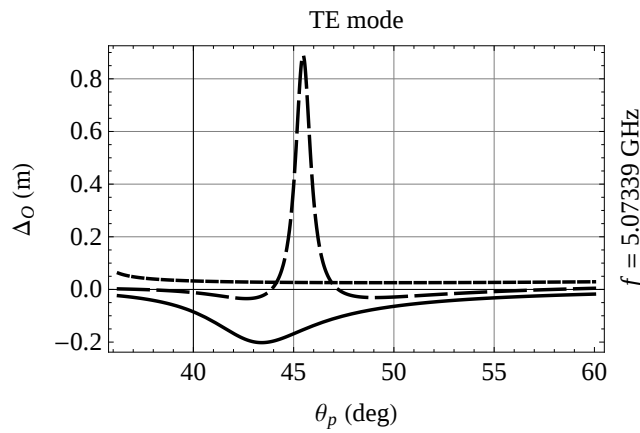


Fig. 3.14. The  $\Delta_O$ -spectrum as a function of  $\theta_p$  for  $f = f_{TE}$  and  $d_c = 5$  mm (solid line),  $d_c = 10$  mm (dashed line) and (c)  $d_c \rightarrow \infty$  (dotted line).

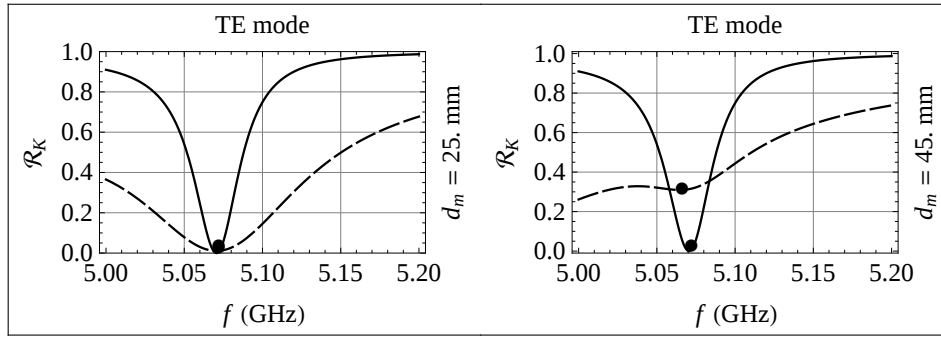


Fig. 3.16(b). The  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_K$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 25$  mm and (right) 40.5 mm.

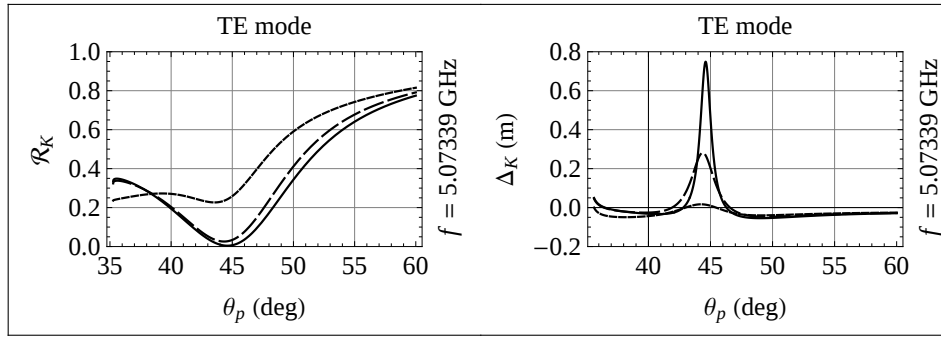


Fig. 3.16(c). (left) The  $\mathcal{R}_K$ -spectrum and (right) the  $\Delta_K$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 24.3$  mm (solid line), 27 mm (dashed line) and 45 mm (dotted line).

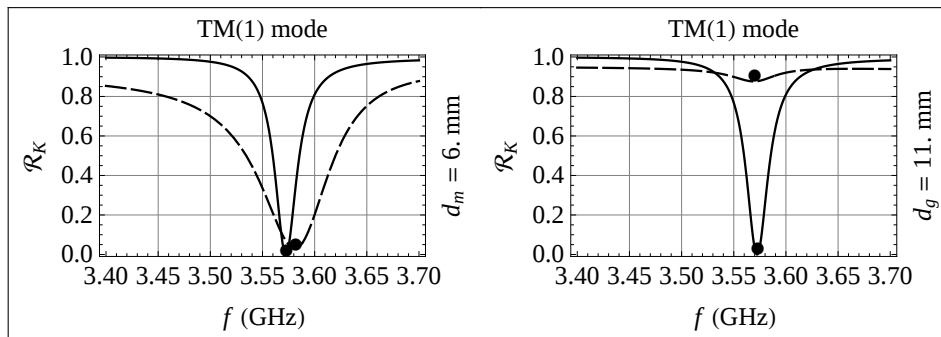


Fig. 3.17(b). The  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_K$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 6$  mm and (right) 11 mm.

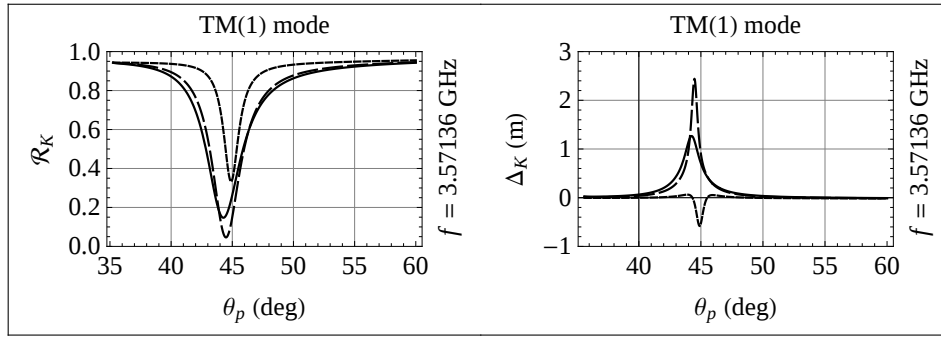


Fig. 3.17(c). (left) The  $\mathcal{R}_K$ -spectrum and (right)  $\Delta_K$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 4.5$  mm (solid line), 5 mm (dashed line) and 7.5 mm (dotted line).

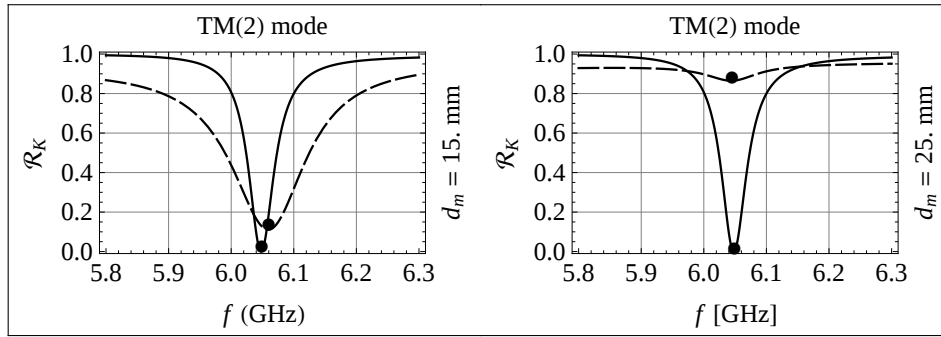


Fig. 3.18(b).  $\mathcal{L}$ -spectrum (solid line) together with the  $\mathcal{R}_K$ -spectrum (dashed line) as a function of frequency,  $f$ , for (left)  $d_c = 15$  mm and (right) 25 mm.

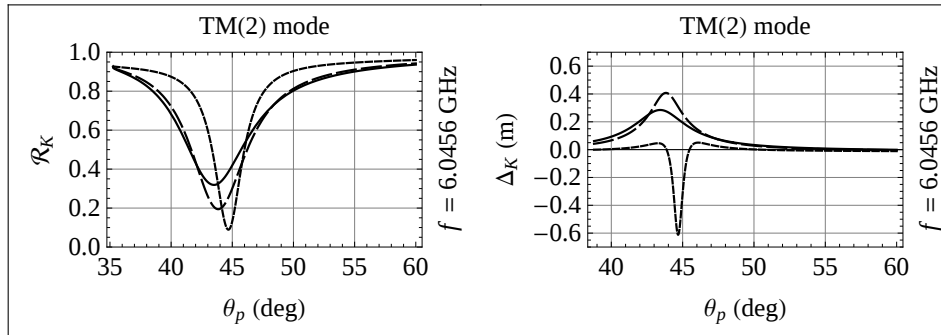


Fig. 3.18(c). (left)  $\mathcal{R}_K$ -spectrum and (right)  $\Delta_K$ -spectrum as a function of  $\theta_p$ , both for  $d_c = 9$  mm (solid line), 10 mm (dashed line) and 15 mm (dotted line).

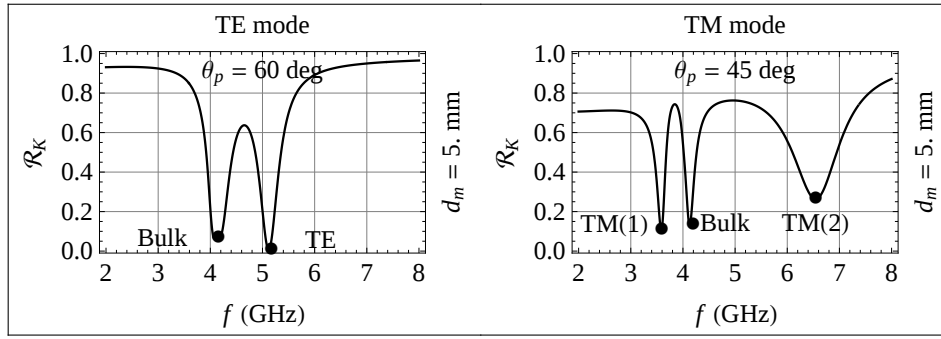


Fig. 3.19. The s-polarized  $\mathcal{R}_K$ -spectrum as a function of frequency,  $f$ , for  $d_m = 5$  mm and  $\theta_p = 60$  deg (left), and  $p$ -polarized  $\mathcal{R}_K$ -spectrum for  $d_m = 5$  mm and  $\theta_p = 45$  deg (right). One observes the positions belonging to the bulk and TE modes on the left. The TM(1), bulk and TM(2) modes are all clearly observed on the right.

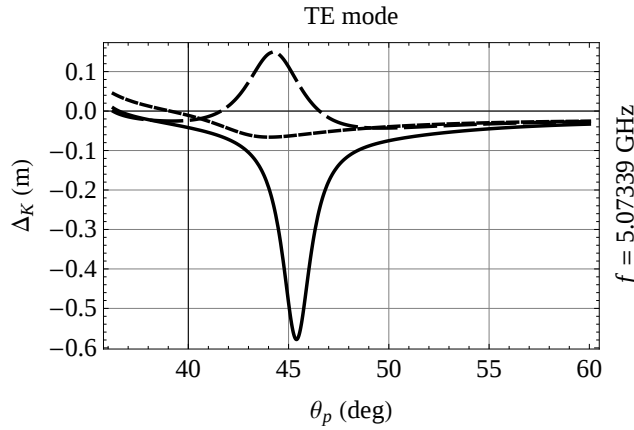


Fig. 3.20. The  $\Delta_K$ -spectrum as a function of  $\theta_p$  for  $f = f_{TE}$  and  $d_c = 20$  mm (solid line),  $d_c = 30$  mm (dashed line) and (c)  $d_c \rightarrow \infty$  (dotted line).

## Exercises

- (1) Table 3.1 includes only a partial list of the topics included in Table 2.2. Try addressing all the rest of the topics using the model metamaterial of this chapter.
- (2) Discuss the meaning of charge density waves in a DNG-type medium.
- (3) Compare the advantages and disadvantages of using the Otto and Kretschmann configurations.

## References

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