Appendix B The mixing equation for ratios

Let us write (2.6) describing the conservation of elements (or isotopes) A and B in the mixture of components $j = 1, 2, \ldots$:

$$C_0^{\mathrm{A}} = \sum_j f_j C_j^{\mathrm{A}} \tag{B.1}$$

$$C_0^{\mathrm{B}} = \sum_j f_j C_j^{\mathrm{B}} \tag{B.2}$$

and then divide one by the other:

$$\begin{aligned}\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_0 &= \frac{f_1 C_1^{\mathrm{A}} + f_2 C_2^{\mathrm{A}} + \cdots}{f_1 C_1^{\mathrm{B}} + f_2 C_2^{\mathrm{B}} + \cdots} \\ &= \frac{f_1 C_1^{\mathrm{B}}}{f_1 C_1^{\mathrm{B}} + f_2 C_2^{\mathrm{B}} + \cdots}\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_1 \\ &\quad + \frac{f_2 C_2^{\mathrm{B}}}{f_1 C_1^{\mathrm{B}} + f_2 C_2^{\mathrm{B}} + \cdots}\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_2 + \cdots\end{aligned} \tag{B.3}$$

In the fractions of the right-hand side, we recognize proportions φ_1^{B}, φ_2^{B}, ..., of the denominator element B provided by components $1, 2, \ldots$, proportions that are written, for component 1, say:

$$\varphi_1^{\mathrm{B}} = \frac{f_1 C_1^{\mathrm{B}}}{f_1 C_1^{\mathrm{B}} + f_2 C_2^{\mathrm{B}} + \cdots} \tag{B.4}$$

Summing the previous equation for all components, it can be verified that the sum of the various φ_j^{B} is equal to unity. We therefore arrive back at (2.9):

$$\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_0 = \sum_j \varphi_j^{\mathrm{B}} \left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_j \tag{B.5}$$

For a binary mixture, when allowing for closure, this equation becomes:

$$\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_0 = \left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_1 + \varphi_2^{\mathrm{B}}\left[\left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_2 - \left(\frac{C^{\mathrm{A}}}{C^{\mathrm{B}}}\right)_1\right] \tag{B.6}$$

If the element in the denominator is the same for two ratios, e.g. for A/B and X/B, or if the two denominators are proportional, φ_2^{B} can be eliminated between the two corresponding equations. We thus obtain a linear relation between the two ratios for the mixture. For two independent denominators, say for A/B and X/Y, the relation can be shown to be hyperbolic.