#### Problems for Chapter 4 of 'Ultra Low Power Bioelectronics'

Problem 4.1

This problem investigates the source-follower shown in Figure P4.1. We shall use body-referenced transistor models throughout the problem. Assume that the two transistors are perfectly matched with identical parameters and that the bulk is connected to ground;  $\kappa_0 = 0.7$ ,  $V_{TO} = 1.0$  V,  $\phi_0 = 1.0$  V,  $V_{DD} = 5$  V,  $V_1 = 0.7$  V, and  $V_2 = 2$  V.

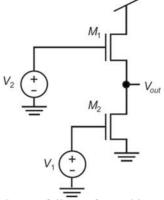


Figure P4.1 Source follower for Problems 4.1 and 4.2.

- a) Assuming that all the transistors are in saturation, what is the value of  $V_{OUT}$ ? Prove that all transistors are in saturation.
- b) Using the definition of  $\kappa$ , estimate  $\Psi_S$ , the surface potential of  $M_1$ .
- c) Plot the mobile charge per unit area,  $Q_1(x)$ , in  $M_1$ , as we go from the source, x = 0, to the drain, x = L. You will need to get the shape correct, but you do not need to evaluate  $Q_1(0)$  quantitatively.

#### Problem 4.2

In this problem, we will use the circuit shown in Figure P4.1 and the parameters listed in Problem 4.1; however, for this problem we shall set  $V_1 = 1.3$  V. Use body-referenced transistor models throughout the problem.

- a) What is the new value for  $V_{OUT}$ ? Prove that all transistors are still in saturation.
- b) Draw a qualitative plot of transistor  $M_1$ , showing the widths of all depletion regions, source, drain, and gate. You do not need to get quantitative accuracy but your plot should have the right qualitative features.
- c) What is the value of the surface potential,  $\Psi_S(0)$ , at the source of transistor  $M_1$ ?
- d) What is the value of the surface potential,  $\Psi_S(L)$ , at the drain of transistor  $M_1$ ?

## Problem 4.3

Figure P4.3 shows a transistor circuit. With respect to this circuit, you are given the following parameters:  $V_{DD} = 3 \text{ V}$ ,  $I_{BIAS} = 50 \text{ nA}$ ,  $\phi_t = 25 \text{ mV}$ ,  $\kappa_p = \kappa_n = 0.75$ ,  $|V_{Tp}| = V_{Tn} = 0.9 \text{ V}$ ,  $|\phi_0| = 1.0 \text{ V}$ ,  $\mu_p C_{ox} = \mu_n C_{ox} = 40 \text{ }\mu\text{A}/\text{V}^2$ , W/L=2 for all NMOS transistors,

while the W/L of PMOS transistors is 1 unless otherwise stated.  $C_{BIG}$  is large enough such that it may be considered a short circuit at the ac frequencies of interest. Use body-referenced transistor models throughout the problem.

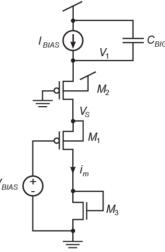


Figure P4.3 Circuit schematic for Problems 4.3 and 4.4

- a) Given that  $I_0 = 10^{-18}$  A, that  $M_1$  operates in subthreshold, and that  $V_{BLAS}$  is set such that  $M_1$  is saturated, determine  $V_{SG}$  for  $M_1$ .
- b) Suppose that we desire  $V_S = 2.7$  V. Using your answer from part a), determine  $V_{BLAS}$ .
- c) Determine  $V_{DS}$  for  $M_1$ .

### Problem 4.4

In this problem we will use the circuit shown in Figure P4.3 again with the parameters given in problem 4.3. Suppose  $V_{SD} = 100 \text{ mV}$  for  $M_2$  and that the body effect is negligible. Assume that  $V_S = 2.7 \text{ V}$  and that body-referenced transistor models are accurate.

- a) In what region of inversion does  $M_2$  operate? Justify.
- b) Is it saturated? Justify.
- c) Determine the W/L for  $M_2$ .
- d) Sketch the surface potential of  $M_2$  along the entire length of its channel. You should specify the values of the surface potential at the drain and at the source.

### Problem 4.5

From Figure 3.4 and the intuition of the diode-clamp approximation, provide a *physical* argument for why the second-line of Equation (4.42) must follow from the first line of Equation (4.42).

# Problem 4.6

- a) From the Einstein relation, the fundamental equations of drift and diffusion (Equation (3.19)), and the charge profile of transistors in saturation, show that Equation (4.28) has the following physical interpretation: When the inversion charge is large enough to cause the surface potential at the source to change from its subthreshold value by  $2\phi_t$ , thus establishing a significant lateral electric field from drain to source, the drift current just dominates over the diffusion current.
- b) Provide an intuitive explanation for why the change is  $2\phi_t$  rather than  $\phi_t$  as might naively be expected from the relationship between drift and diffusion in Equation (3.19).
- c) Relate this threshold amount of inversion charge at the border between weak inversion and strong inversion to the pre-constant of the empirical EKV model of Equations (4.47) and (4.48).

## Problem 4.7

Design an experiment that will allow you to use subthreshold and above-threshold measurements to estimate  $\gamma$  and  $\phi_0$  for a transistor.

## Problem 4.8

- a) If the depletion capacitance in a MOSFET were completely linear (no square-root dependence for any set of terminal voltages) would the MOSFET still exhibit a body effect? If so, what quantitative equation in the text would model this effect?
- b) In this linear-depletion-capacitance case, would there be any advantage to having separate source-referenced and body-referenced models?
- c) In a certain fully-depleted silicon-on-insulator transistor, the depletionregion depth is at a maximum value and this depth is invariant to changes in surface potential in all regimes of operation. Would such a transistor exhibit a body effect? What would the value of  $\kappa$  be in such a transistor?

## Problem 4.9

This problem requires the use of a circuit simulator such as SPICE. In this problem we will use SPICE to generate plots of  $I_D$  vs.  $V_{DS}$  of an NMOS transistor with  $W = 4.95 \ \mu m$  and  $L = 2.55 \ \mu m$  for different  $V_{GS}$  and  $V_{SB}$  voltages.

- a) Set  $V_{SB} = 0$  V and  $V_{GS} = 1$  V. Vary  $V_{DS}$  from 0 V to 5 V in increments of 0.1 V and measure the drain current. Repeat the measurement for  $V_{GS} = 1$  V, 2 V, 3 V, 4 V, and 5 V. Plot all curves in the same graph.
- b) Repeat the measurement from part a), but this time set  $V_{SB} = 2.5$  V. How does the result differ from the one in part a)?

## Problem 4.10

Prove that, under appropriate limiting conditions, the body-referenced equation and source-referenced equation of the empirical EKV model (Equations (4.47) and (4.48) respectively) lead to the corresponding body-referenced and source-referenced strong-inversion equations listed in Table 4.1.