Chapter 8

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> with(plots):
Warning, the name changecoords has been redefined
```

Question 1

(i)

This is a purely theoretical question. Given

$q_d(t) = a - b\,\mathrm{p}(t)$

$q_s(t) = c + d\,\mathrm{p}(t+1)$

and $q_d(t) = q_s(t)$, in equilibrium, which we set equal to $\mathrm{q}(t)$, then

```
> solve(a-b*pt=c+d*pt_1,pt);
```

$$\frac{a - c - d\,pt\_1}{b}$$

or

$$\mathrm{p}(t) = \frac{a-c}{b} + \left(-\frac{d}{b}\right)\mathrm{p}(t-1)$$

Since $b < 1$, then $1 < -b$ and with $0 < d$, then $0 < -\frac{d}{b}$. If $0 < -\frac{d}{b} < 1$, then the system converges on the equilibrium value. On the other hand, if $1 < -\frac{d}{b}$, then the system diverges from equilibrium.

(ii)

Since $0 < -\frac{d}{b}$, then the systems does not oscillate.

An example of this type of situation is given in Question 2(ii) in which $-\frac{d}{b} = \frac{1}{2}$, which we shall consider next.

Question 2

(i)

The system is:

$q_d(t) = 100 - 2\,\mathrm{p}(t)$

$q_s(t) = -20 + 3\,\mathrm{p}(t-1)$

with initial condition $p_0 = 10$

```
> solve(100-2*pt=-20+3*pt_1,pt);
```

$$60 - \frac{3}{2}\,pt\_1$$

with equilibrium price:

```
> solve(p=60-(3/2)*p,p);
```

$$24$$

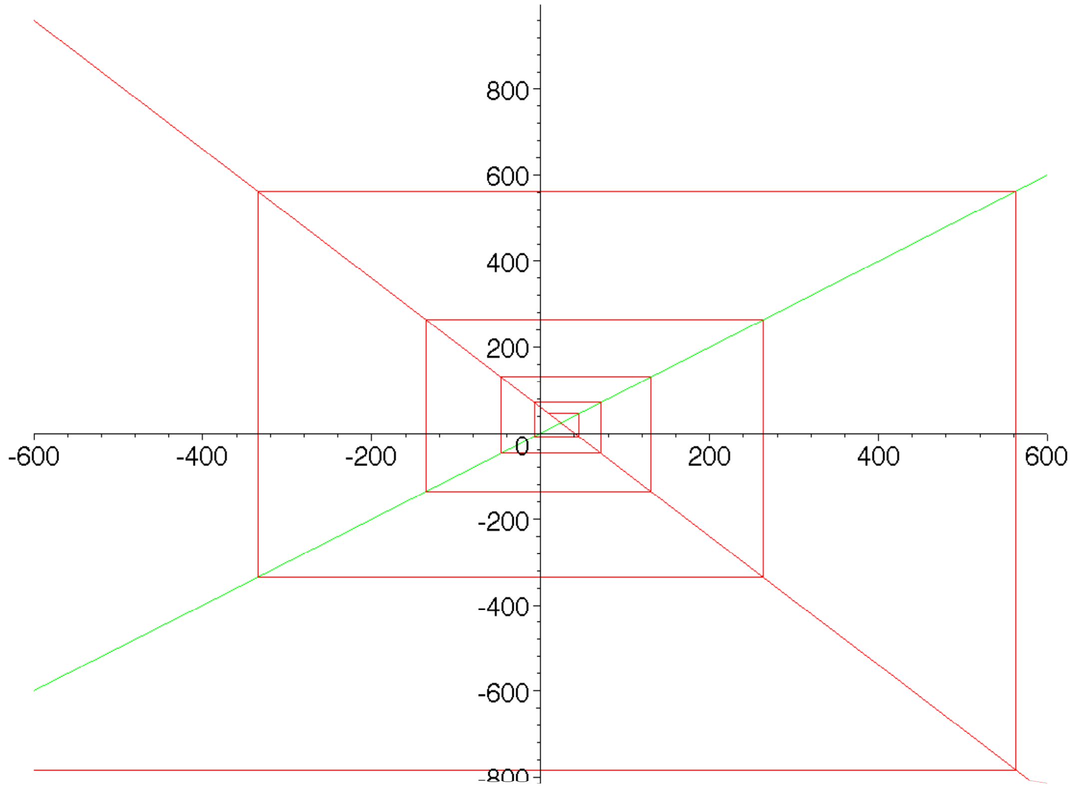
```
> f:=p->60-1.5*p;
```

$$f := p \to 60 - 1.5\,p$$

```
> fn:=(p,n)->simplify((f@@n)(p));
```

$$fn := (p, n) \to \text{simplify}((f^{(n)})(p))$$

```
> list1:=transpose(array([[seq(fn(10,n),n=0..10)],
       [seq(fn(10,n),n=0..10) ] ] ) ):
> list2:=convert(convert(list1,vector),list):
> list3:=convert(transpose(array([
  list2[1..nops(list2)-1],list2[2..nops(list2)] ])
  ),listlist):
> list4:=[list3[2..nops(list3)]]:
> web:=plot(list4):
> lines:=plot({f(p),p},p=-600..600):
> display(web,lines);
```



Since we shall be using cobwebs frequently in this chapter, it is useful to provide a small programme, using *Maple's* **proc** command.

```
> cobweb:=proc(f,p0,n,minp,maxp)
    local fk,list1,list2,list3,list4,web,lines;
       fk:=(p,k)->simplify((f@@k)(p));
       list1:=transpose(array([[seq(fk(p0,k),k=0..n)],
           [seq(fk(p0,k),k=0..n) ] ] ) ):
       list2:=convert(convert(list1,vector),list):
       list3:=convert(transpose(array([
  list2[1..nops(list2)-1],
```

```
            list2[2..nops(list2)] ]) ),listlist):
        list4:=[list3[2..nops(list3)]]:
        web:=plot(list4):
        lines:=plot({f(p),p},p=minp..maxp,colour=blue):
        display(web,lines);
    end;
```

cobweb := **proc**(*f*, *p0*, *n*, *minp*, *maxp*)

local *fk*, *list1*, *list2*, *list3*, *list4*, *web*, *lines*;

$fk := (p, k) \to \mathrm{simplify}(\,(f@@k)(p)\,)$;

$list1 :=$

$\mathrm{transpose}(\,\mathrm{array}(\,[\,[\,\mathrm{seq}(fk(p0, k), k = 0 .. n\,)\,], [\,\mathrm{seq}(fk(p0, k), k = 0 .. n\,)\,]\,]\,)\,)$

;

$list2 := \mathrm{convert}(\,\mathrm{convert}(\,list1, vector\,), list\,)$;

$list3 := \mathrm{convert}($

$\mathrm{transpose}(\,\mathrm{array}(\,[\,list2[\,1 .. \mathrm{nops}(\,list2\,) - 1\,], list2[\,2 .. \mathrm{nops}(\,list2\,)\,]\,]\,)\,)$,

$listlist\,)$;

$list4 := [\,list3[\,2 .. \mathrm{nops}(\,list3\,)\,]\,]$;

$web := \mathrm{plot}(\,list4\,)$;

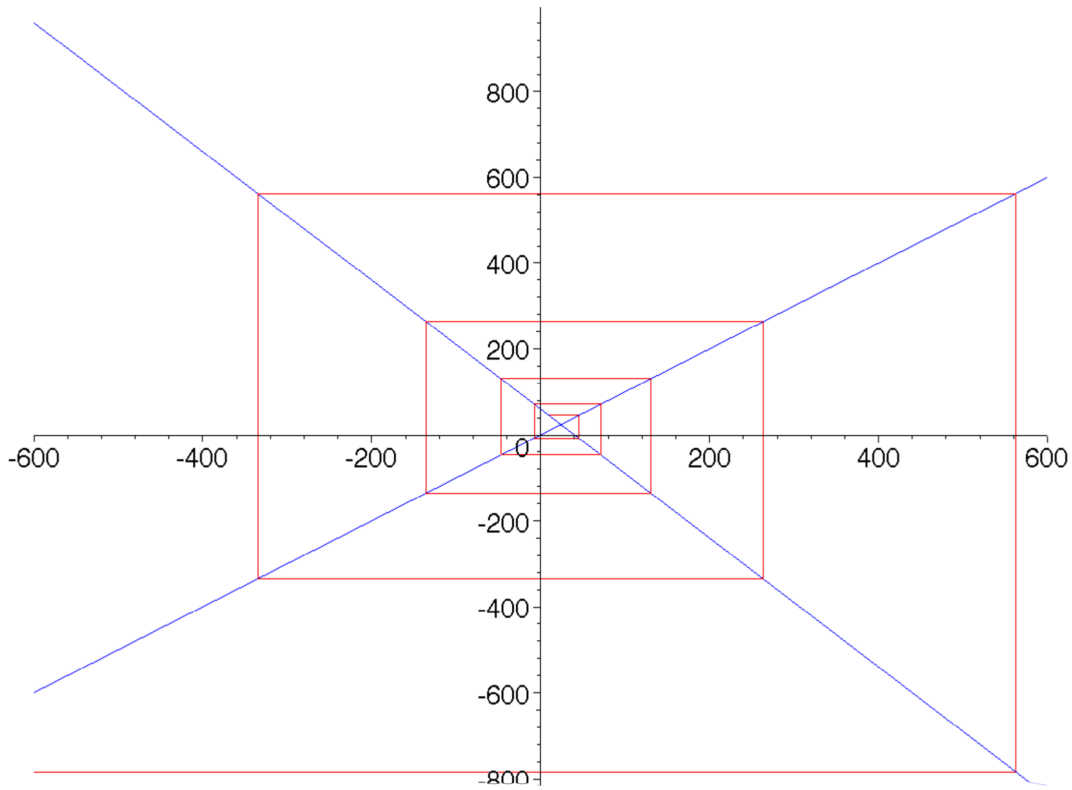
$lines := \mathrm{plot}(\,\{p, f(p)\,\}, p = minp .. maxp, colour = blue\,)$;

$\mathrm{display}(\,web, lines\,)$

end proc

```
> f:p->60-1.5*p; cobweb(f,10,10,-600,600);
```

$$p \to 60 - 1.5\,p$$



(ii)

The system is:

$q_d(t) = 5 + 2\,\mathrm{p}(t)$

$q_s(t) = 35 + \mathrm{p}(t-1)$

with initial condition $p_0 = 5$.

First we need to derive the difference equation.

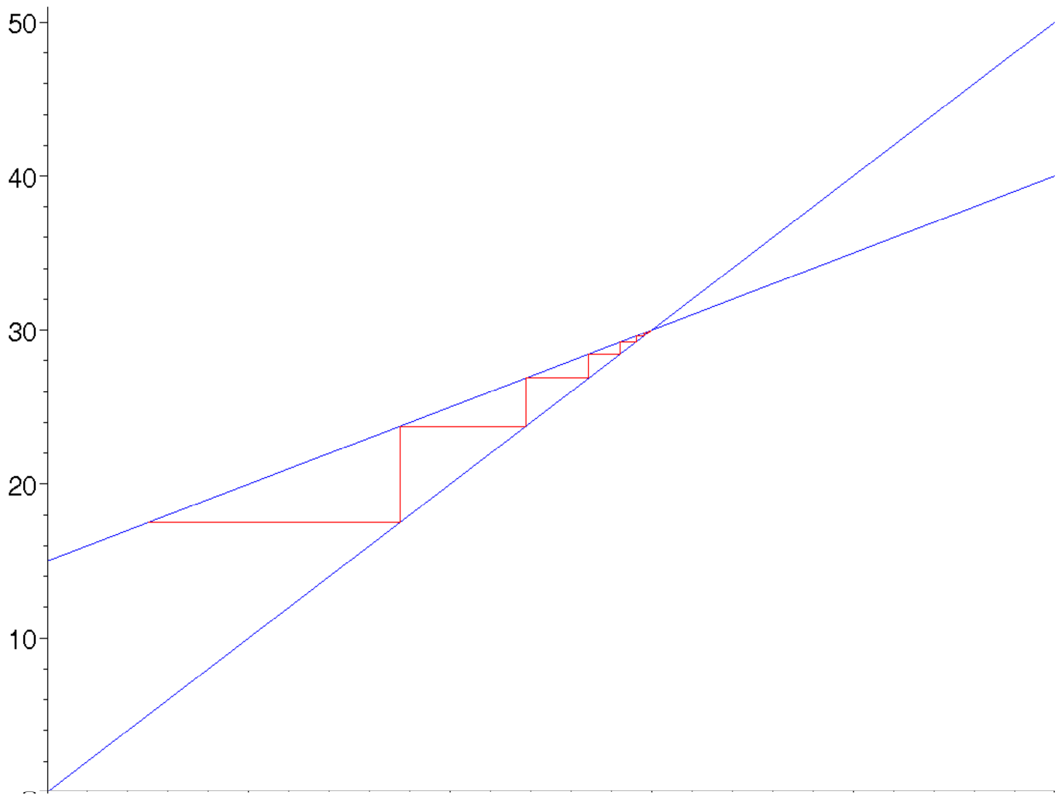
```
> solve(5+2*pt=35+pt_1,pt);
```

$$15 + \frac{1}{2}pt\_1$$

So our difference equation is $\mathrm{p}(t) = 15 + .5\,\mathrm{p}(t-1)$. Using the procedure above for the cobweb, we have:

```
> f:=p->15+0.5*p; cobweb(f,5,10,0,50);
```

$$f := p \to 15 + .5\,p$$



(iii)

The system is

$q_d(t) = 100 - 2\,\mathrm{p}(t)$

$q_s(t) = -20 + 2\,\mathrm{p}(t-1)$

with initial condition $p_0 = 24$.

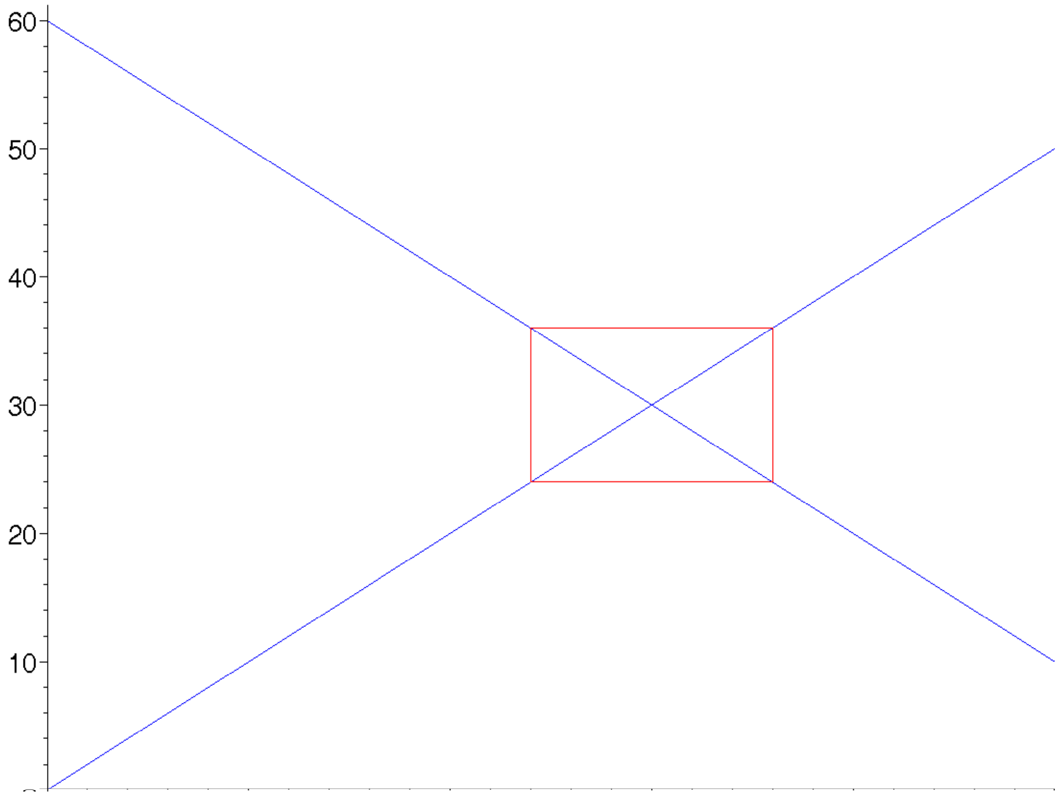
First we need to derive the difference equation.

```
> solve(100-2*pt=-20+2*pt_1,pt);
```

$$60 - pt\_1$$

```
> f:=p->60-p;cobweb(f,24,10,0,50);
```

$$f := p \to 60 - p$$



This illustrates an oscillatory cobweb which is cyclical.

(iv)

The system is

$q_d(t) = 18 - 3\,\mathrm{p}(t)$

$q_s(t) = -10 + 4\,\mathrm{p}(t-1)$

with initial condition $p_0 = 1$.

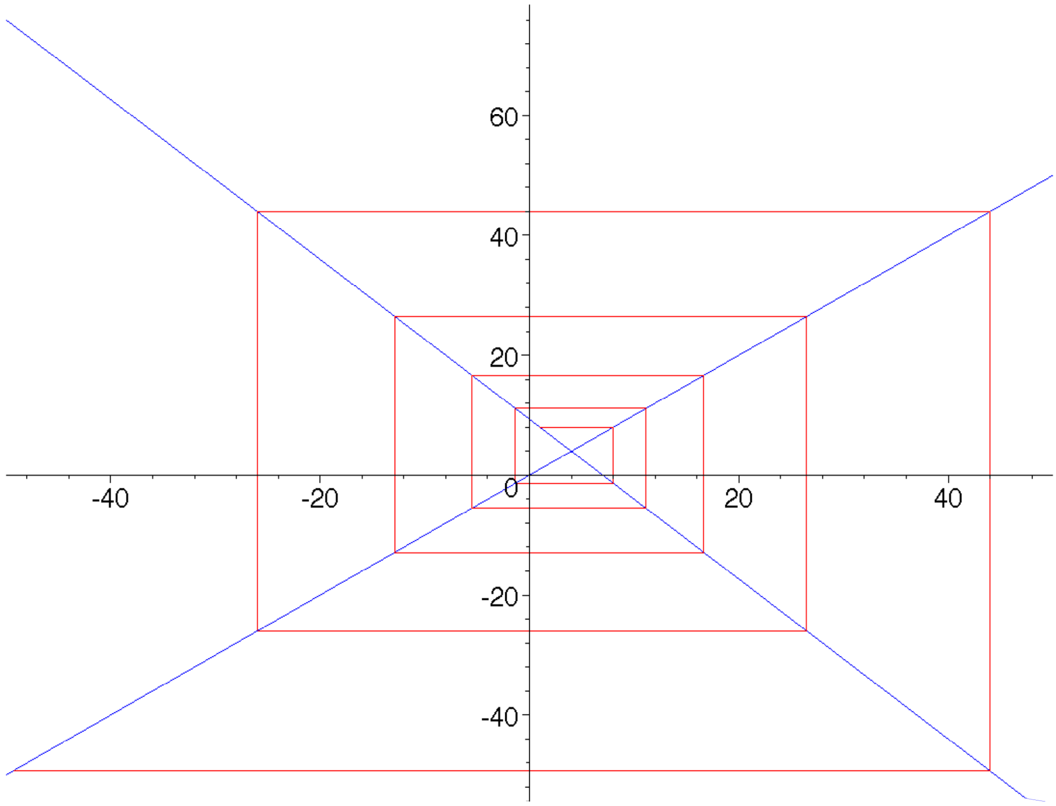
First we need to derive the difference equation.

```
> solve(18-3*pt=-10+4*pt_1,pt);
```

$$\frac{28}{3} - \frac{4}{3}\,pt\_1$$

```
> f:=p->(28/3)-(4/3)*p; cobweb(f,1,10,-50,50);
```

$$f := p \to \frac{28}{3} - \frac{4}{3}\,p$$



This illustrates an oscillatory cobweb which is divergent.

Question 3

For the general model:

$$q_d(t) = a - b\,p(t) \qquad 0 < b$$

$$q_s(t) = c + d\,(p^e)_t \qquad 0 < d$$

$$q_d(t) = q_s(t) = q$$

$$(p^e)_t = (p^e)_{t-1} - \lambda\,((p^e)_{t-1} - p_{t-1}) \qquad 0 \le (\lambda \le 1)$$

$$p_t = \frac{\lambda\,(a-c)}{b} + \left[1 - \lambda - \frac{\lambda\,d}{b}\right] p_{t-1}$$

We can deal with each porblem by substituting the parameter values and then solving the difference equation.

(i)

```
> const:=lambda*((a-c)/b);
```

$$const := \frac{\lambda\,(a-c)}{b}$$

```
> coef:=(1-lambda-(lambda*d/b));
```

$$coef := 1 - \lambda - \frac{\lambda\,d}{b}$$

```
> const1:=subs({a=100,b=2,c=-20,d=3,lambda=0.5},const);
```

$$const1 := 30.00000000$$

```
> coef1:=subs({a=100,b=2,c=-20,d=3,lambda=0.5},coef);
```

$$coef1 := -.2500000000$$

```
> rsolve({p(t)=30-0.25*p(t-1),p(0)=10},p(t));
```

$$-14\left(\frac{-1}{4}\right)^t + 24$$

In other words,

$$p(t) = 24 - 14(-.25)^t$$

(ii)

```
> const2:=subs({a=5,b=-2,c=35,d=1,lambda=0.2},const);
```

$$const2 := 3.000000000$$

```
> coef2:=subs({a=5,b=-2,c=35,d=1,lambda=0.2},coef);
```

$$coef2 := .9000000000$$

```
> rsolve({p(t)=3+0.9*p(t-1),p(0)=10},p(t));
```

$$-20\left(\frac{9}{10}\right)^t + 30$$

In other words,

$$p(t) = 30 - 20(.9)^t$$

Question 4

The stability of the system in this question can be obtained immediately by considering the ratio $-\frac{d}{b}$, where d is the slope of the supply curve and b is the slope of the demand curve.

Thus,

(i) $-\frac{d}{b} = -\frac{25}{50}$ or $-\frac{d}{b} = -\frac{1}{2}$ so the system is oscillatory and convergent.

(ii) $-\frac{d}{b} = \frac{.1}{.5}$ or $-\frac{d}{b} = .2$ so the system is non-oscillatory and convergent.

We can, however, investigate these systems in more detail.

(i)

Given

$$q_d(t) = 250 - 50\,p(t)$$

$$q_s(t) = 25 + 25\,p(t-1)$$

$$q_d(t) = q_s(t) = q(t)$$

then

```
> solve(250-50*pt=25+25*pt_1,pt);
```

$$\frac{9}{2} - \frac{1}{2}pt\_1$$

So we can investigate the stability by deriving the cobweb for the initial price $p_0 = 1$.

```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$

$$p := p$$

$$p0 := p0$$

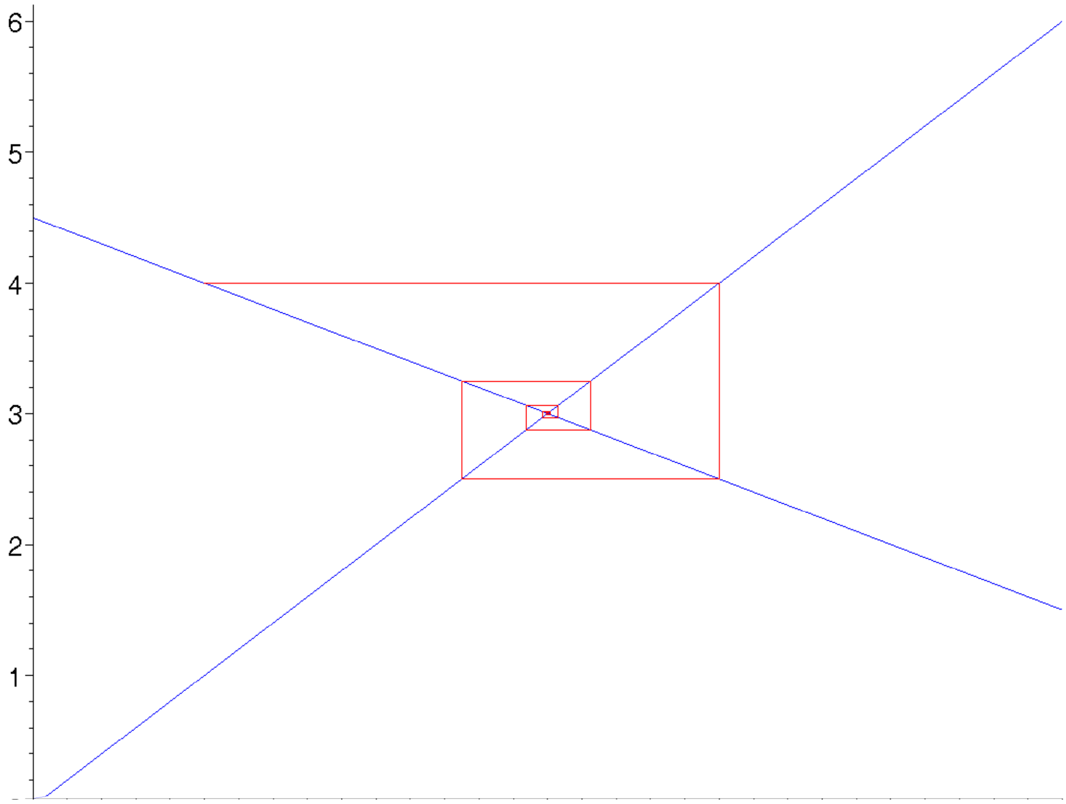
```
> f:=p->4.5-0.5*p;
```

$$f := p \rightarrow 4.5 - .5\,p$$

```
> solve(p=f(p),p);
```

$$3.$$

```
> cobweb(f,1,10,0,6);
```



(ii)

Given

$$q_d(t) = 100 - .5\,\mathrm{p}(t)$$

$$q_s(t) = 50 - .1\,\mathrm{p}(t-1)$$

$$q_d(t) = q_s(t) = \mathrm{q}(t)$$

then

```
> solve(100-0.5*pt=50-0.1*pt_1,pt);
```

$$100. + .2000000000\,pt\_1$$

```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$

$$p := p$$

$$p0 := p0$$

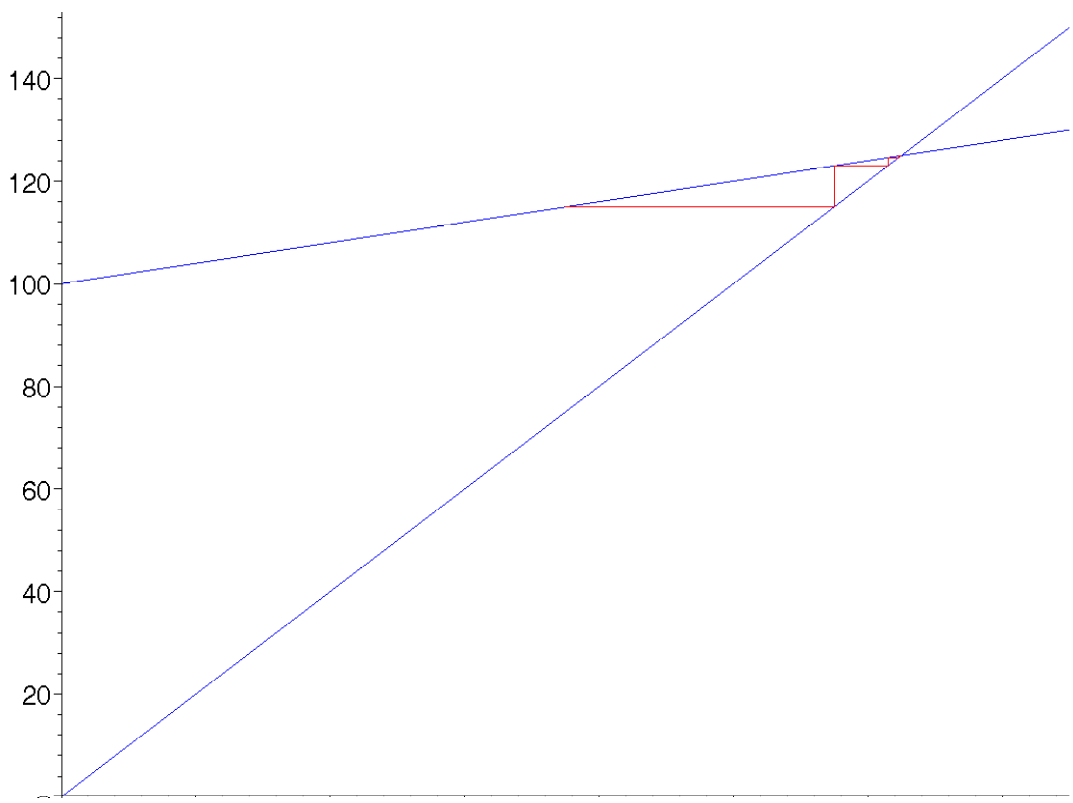
```
> f:=p->100+0.2*p;
```

$$f := p \rightarrow 100 + .2\,p$$

```
> solve(p=f(p),p);
```

$$125.$$

```
> cobweb(f,75,10,0,150);
```



As pointed out above, this system is non-oscillatory and convergent.

Question 5

(i)

Given the equations

$q_d(t) = 50 - 4\,p(t)$

$q_s(t) = 10 + 10\,p(t-1) - {p_{t-1}}^2$

$q_d(t) = q_s(t) = q(t)$

then

```
> solve(50-4*pt=10+10*pt_1-pt_2^2,pt);
```

$$10 - \frac{5}{2}pt\_1 + \frac{1}{4}pt\_2^2$$

Maple cannot solve this difference equation with the **rsolve** command.

First establish the fixed points.

```
> solve(p=10-(5/2)*p+(1/4)*p^2,p);
```

$$4, 10$$

The difference equation can be linearized in the neighbourhood of the fixed point by using

$p_t = f(pe) + \left(\frac{\partial}{\partial p} pe\right)(p - pe)$

where *pe* denotes the fixed point and $f(p) = 10 - \frac{5\,p}{2} + \frac{1\,p^2}{4}$.

```
> f:='f'; f:=p->10-(5/2)*p+(1/4)*p^2;
```

$$f := f$$

$$f := p \to 10 - \frac{5}{2}p + \frac{1}{4}p^2$$

```
> subs(p=4,diff(f(p),p));
```

$$\frac{-1}{2}$$

```
> subs(p=10,diff(f(p),p));
```

$$\frac{5}{2}$$

Thus,

$p_t = 4 - \frac{1\,(p-4)}{2}$ which is stable since $\left|\frac{\partial}{\partial p} pe\right| < 1$

$p_t = 10 + \frac{5\,(p-10)}{2}$ which is unstable since $1 < \left|\frac{\partial}{\partial p} pe\right|$

Although the linear approximations can give us some insight about stability and instability, there is no guarantee that the non-linear system may lead to a cyclical solution. To see what might be taking place around each equilibrium, we shall utilise the procedure for constructing non-linear cobwebs.

Consider first the fixed point $pe = 4$.

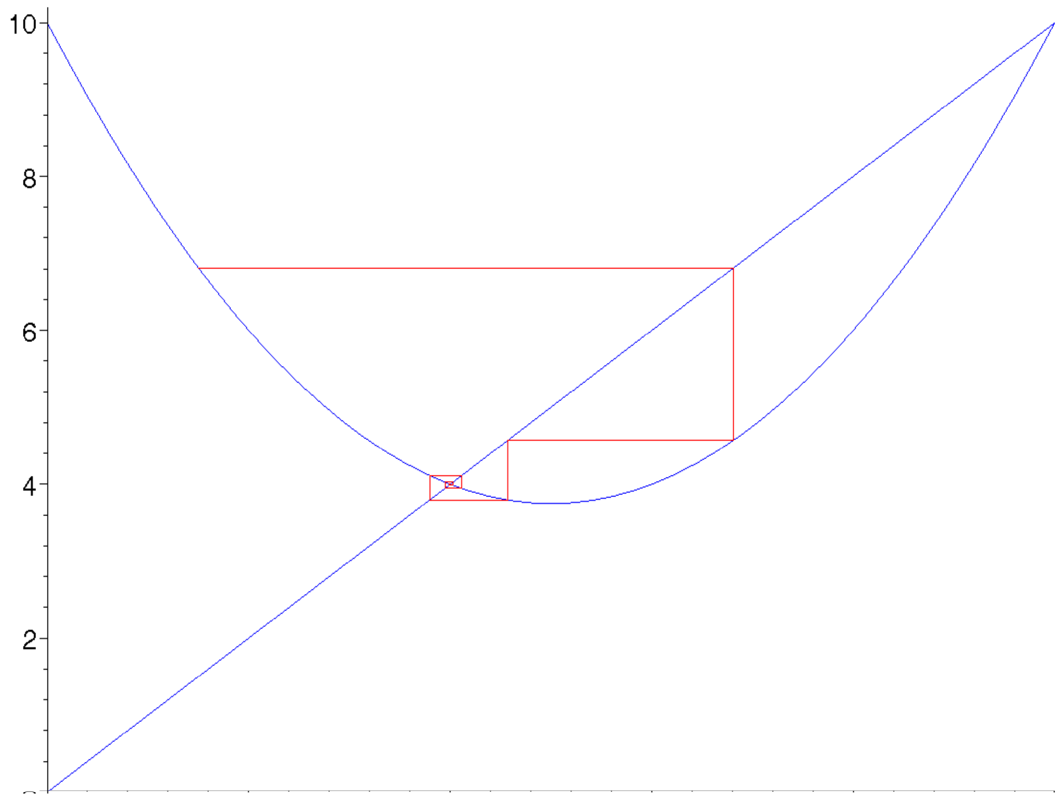
```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$
$$p := p$$
$$p0 := p0$$

```
> f:=p->10-(5/2)*p+(1/4)*p^2;
```

$$f := p \to 10 - \frac{5}{2}p + \frac{1}{4}p^2$$

```
> cobweb(f,1.5,8,0,10);
```



Consider next the fixed point $pe = 10$.

```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$

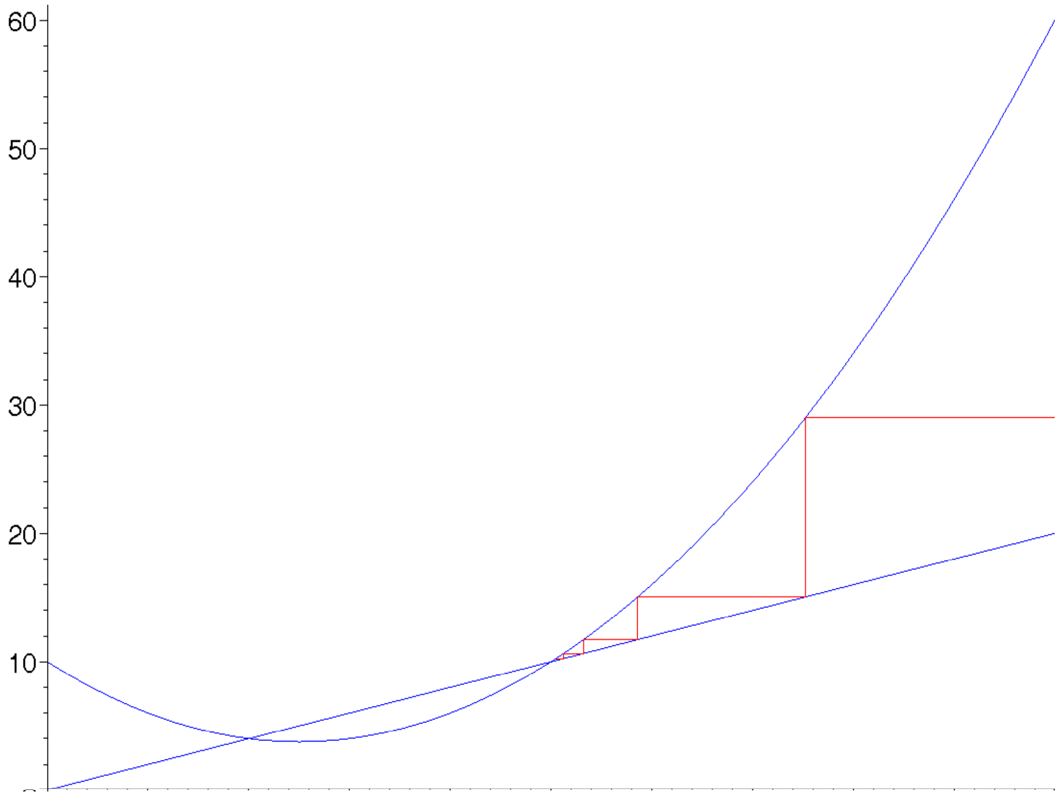
$$p := p$$

$$p0 := p0$$

```
> f:=p->10-(5/2)*p+(1/4)*p^2;
```

$$f := p \to 10 - \frac{5}{2} p + \frac{1}{4} p^2$$

```
> cobweb(f,10.1,5,0,20);
```



(ii)

Given the equations

$$q_d(t) = 50 - 4\,\mathrm{p}(t)$$

$$q_s(t) = 2 + 10\,\mathrm{p}(t-1) - {p_{t-1}}^2$$

$$q_d(t) = q_s(t) = \mathrm{q}(t)$$

then

```
> solve(50-4*pt=2+10*pt_1-pt_2^2,pt);
```

$$12 - \frac{5}{2}pt\_1 + \frac{1}{4}pt\_2^2$$

Maple cannot solve this difference equation with the **rsolve** command.

First establish the fixed points.

```
> solve(p=12-(5/2)*p+(1/4)*p^2,p);
```

$$6,\ 8$$

```
> f:='f'; f:=p->12-(5/2)*p+(1/4)*p^2;
```

$$f := f$$

$$f := p \to 12 - \frac{5}{2}p + \frac{1}{4}p^2$$

```
> subs(p=6,diff(f(p),p));
```

$$\frac{1}{2}$$

```
>
> subs(p=8,diff(f(p),p));
```

$$\frac{3}{2}$$

Thus

$$p_t = 6 + \frac{1\,(p-6)}{2} \quad \text{which is stable since} \left|\frac{\partial}{\partial p} pe\right| < 1$$

$$p_t = 8 + \frac{3\,(p-8)}{2} \quad \text{which is unstable since } 1 < \left|\frac{\partial}{\partial p} pe\right|$$

Again, let us investigate the non-linear system in the neighbourhood of each equilibrium value. Take first $pe = 6$.

```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$

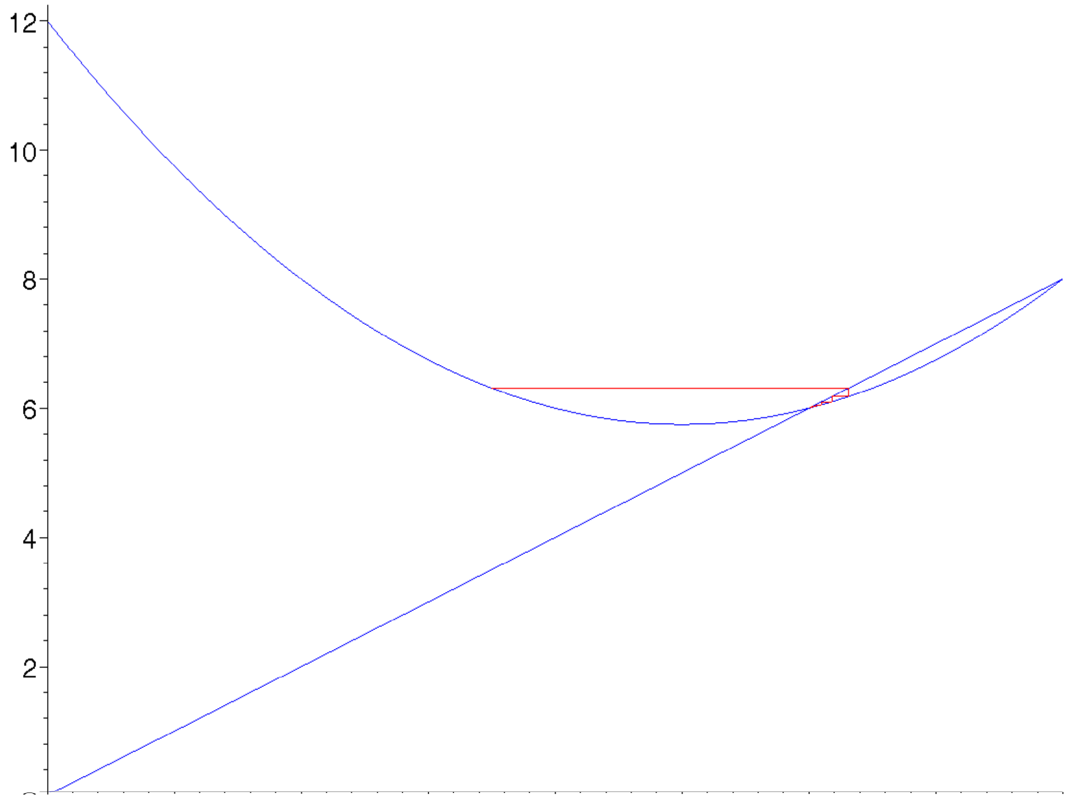
$$p := p$$

$$p0 := p0$$

```
> f:=p->12-(5/2)*p+(1/4)*p^2;
```

$$f := p \rightarrow 12 - \frac{5}{2}p + \frac{1}{4}p^2$$

```
> cobweb(f,3.5,20,0,8);
```



Take next the fixed point $pe = 8$.

```
> f:='f'; p:='p'; p0:='p0';
```

$$f := f$$

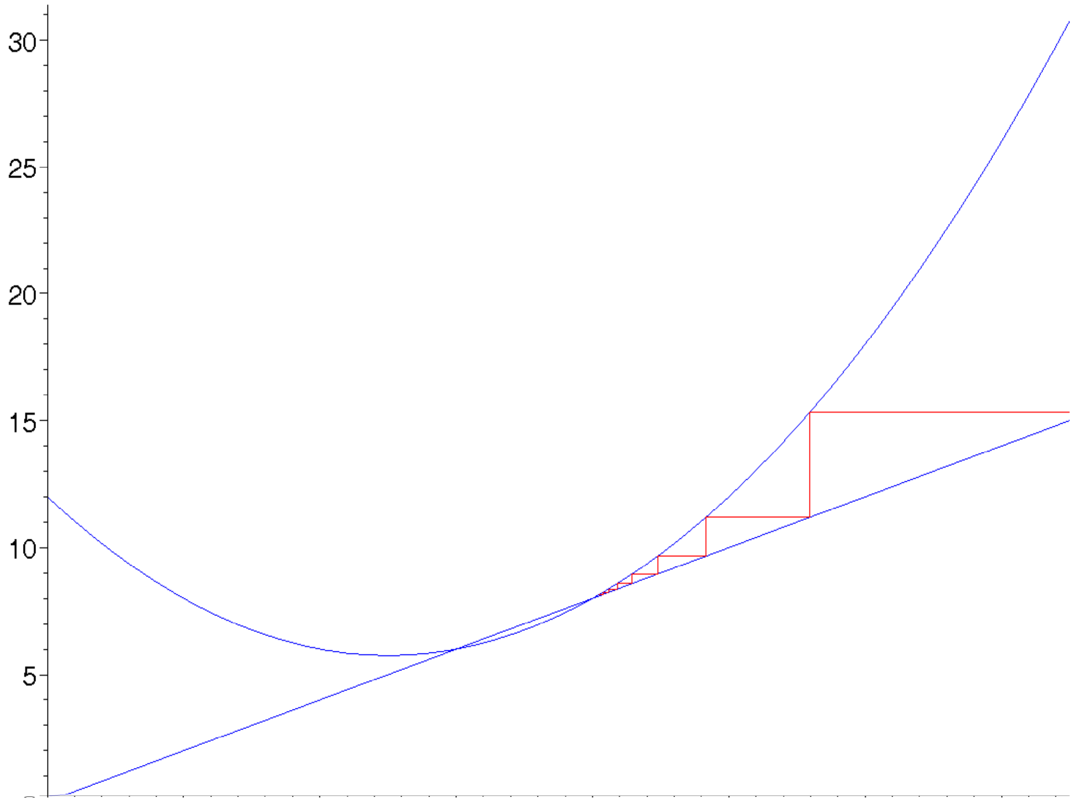
$$p := p$$

$$p0 := p0$$

```
> f:=p->12-(5/2)*p+(1/4)*p^2;
```

$$f := p \to 12 - \frac{5}{2}p + \frac{1}{4}p^2$$

```
> cobweb(f,8.1,8,0,15);
```



Question 6

(i)

We have the following system of equations:

$q_d(t) = 52 - 9\,\mathrm{p}(t)$

$q_s(t) = 3 + 5\,\mathrm{p}(t-1)$

$q_d(t) = q_x(t) = \mathrm{q}(t)$

First we need to solve the system for equilibrium price and quantity. Solving we obtain

```
> solve(52-9*pt=3+5*pt_1,pt);
```

$$\frac{49}{9} - \frac{5}{9}pt\_1$$

```
> solve(p=(49/9)-(5/9)*p,p);
```

$$\frac{7}{2}$$

We solve for equilibrium q

```
> subs(p=7/2,52-9*p);
```

$$\frac{41}{2}$$

The equilibrium price and quantity are: $pe = 3.5$ and $qe = 20.5$.

(ii)

The initial price is,

```
> p0:=3.5-0.1*3.5;
```

$$p0 := 3.15$$

t	p(t)	%dev
0	3.1500	10.0000
1	3.6944	-5.5555
2	3.3919	3.0864
3	3.5600	-1.7146
4	3.4666	.9525
5	3.5185	-.5292
6	3.4897	.2940
7	3.5057	-.1633
8	3.4968	.0907
9	3.5017	-.0504
10	3.4990	.0280

It is readily seen that by period 4 the price is within 1% of the of the equilibrium price.

Question 7

Equation 8.16 of the text, p.336 gave the general result

$$p_t = \frac{\lambda(a-c)}{b} + \left[1 - \lambda - \frac{\lambda d}{b}\right] p_{t-1}$$

substituting the values $a = 52$, $b = 9$, $c = 3$ and $d = 5$, then

$$p_t = \frac{\lambda(52-3)}{9} + \left[1 - \lambda - \frac{5\lambda}{9}\right] p_{t-1}$$

$$p_t = \frac{49\lambda}{9} + \left[1 - \lambda - \frac{5\lambda}{9}\right] p_{t-1}$$

We can first check this result by obtaining the equilibrium and showing that it is independent of λ.

```
> solve(p=(49/9)*lambda+(1-lambda-(5/9)*lambda)*p,p);
```

$$\frac{7}{2}$$

(i)-(iv)

Using the equation for p_t and the initial value $p_0 = 3.15$ we obtain the following results.

t	$p_1(t)$	$p_2(t)$	$p_3(t)$	$p_4(t)$	$\%dev_1$	$\%dev_2$
0	3.150	3.150	3.150	3.150	10.000	10.000
1	3.694	3.558	3.422	3.286	-5.555	-1.666
2	3.391	3.490	3.482	3.369	3.086	.277
3	3.560	3.501	3.496	3.420	-1.714	-.046
4	3.466	3.499	3.499	3.451	.952	.007
5	3.518	3.500	3.499	3.470	-.529	-.001
6	3.489	3.499	3.499	3.481	.294	.000
7	3.505	3.500	3.499	3.488	-.163	-.000
8	3.496	3.499	3.499	3.493	.090	$.600\ 10^{-5}$
9	3.501	3.500	3.499	3.495	-.050	$-.971\ 10^{-6}$
10	3.499	3.499	3.499	3.497	.028	$.200\ 10^{-6}$

From the spreadsheet we have the following periods when the price series comes within 1% of the equilibrium price:

For $\lambda = 1$, period 4
For $\lambda = .75$, period 2
For $\lambda = .5$, period 2
For $\lambda = .25$, period 5

Question 8

(i)

Given

$$q_d(t) = 250 - 50\,\mathrm{p}(t) - 2\left(\frac{\partial}{\partial t}\mathrm{p}(t)\right)$$

$$q_s(t) = 25 + 25\,\mathrm{p}(t)$$

then we can solve for $\frac{\partial}{\partial t}p$ as follows:

```
> solve(250-50*p-2*dp=25+25*p,dp);
```

$$\frac{225}{2} - \frac{75}{2}p$$

```
> dsolve(diff(p(t),t)=(225/2)-(75/2)*p(t),p(t));
```

$$p(t) = 3 + e^{(-75/2\,t)}\,\_C1$$

(ii)

We can establish the stability by considering the limit of this equation.

```
> limit(3+c1*exp(-75*t/2),t=infinity);
```

$$3$$

Hence the market is stable.

Question 9

(i)

Equating demand and supply and solving for w_t we get

```
> solve(42-4*wt=2+6*wt_1,wt);
```

$$10 - \frac{3}{2}\,wt\_1$$

Thus, our equation is $w_t = 10 - \frac{3\,w_{t-1}}{2}$

```
> solve(w=10-(3/2)*w,w);
```

$$4$$

(ii)

We have just established that

$$w_t = 10 - \frac{3\,w_{t-1}}{2}$$

and at the minimum wage of $w = 3$, we have

```
> solve(3=10-(3/2)*w,w);
```

$$\frac{14}{3}$$

Hence we have the equation

$$f(w) = 10 - 1.5\,w \qquad w \le \frac{14}{3}$$

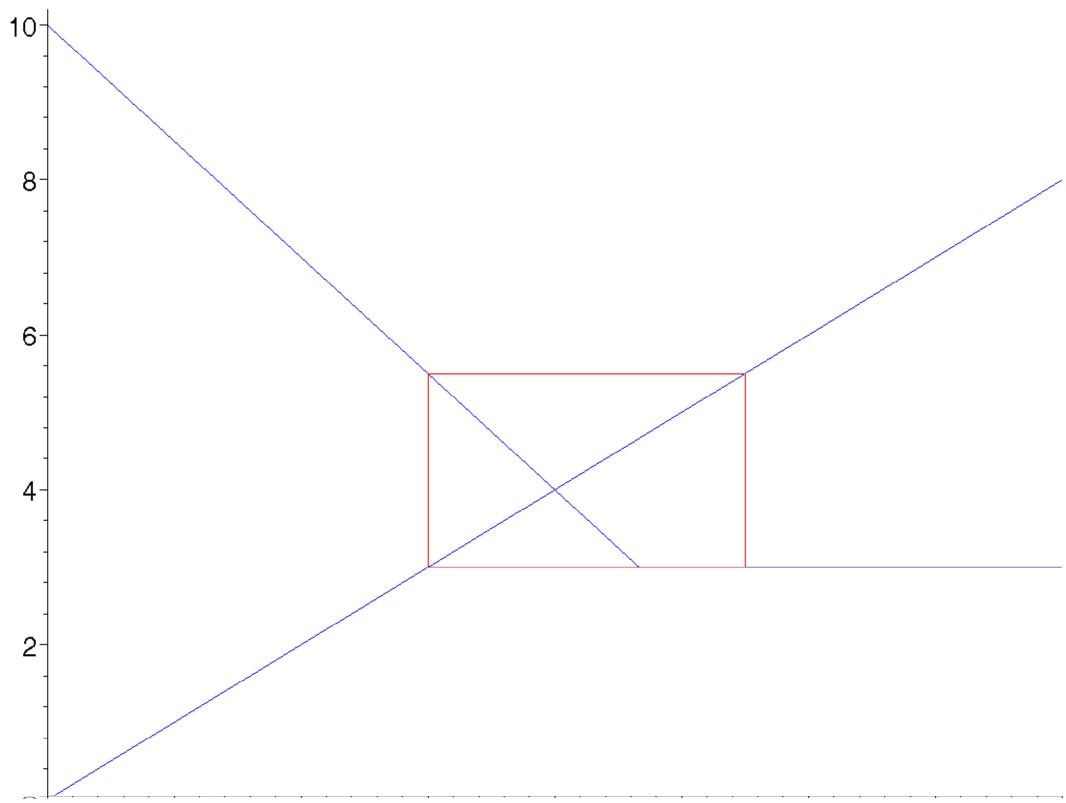
$$=3 \qquad \frac{14}{3} < w$$

```
> f:='f'; f:=w->piecewise(w<=14/3,10-1.5*w,w>14/3,3);
```

$$f := f$$

$$f := w \rightarrow \text{piecewise}\left(w \le \frac{14}{3},\, 10 - 1.5\,w,\, \frac{14}{3} < w,\, 3\right)$$

```
> cobweb(f,5,8,0,8);
```



Question 10

This is purely a theoretical question, but we can approach it in the following manner. Let the demand and supply system be given by:

$$q_t^d = a - b\,p_t$$

$$q_t^s = c + d\,p_{t-1}$$

$$q_t^d = q_t^s = q_t$$

in equilibrium

$$a - b\,p_t = c + d\,p_{t-1}$$

$$p_t = a - c + \left(-\frac{d}{b}\right)p_{t-1}$$

with solution

```
> p:='p'; p0:='p0';
```

$$p := p$$

$$p0 := p0$$

```
> rsolve({p(t)=(a-c)+(-d/b)*p(t-1),p(0)=p0},p(t));
```

$$p0\left(-\frac{d}{b}\right)^t - \frac{b\,(a-c)\left(-\frac{d}{b}\right)^t}{b+d} + \frac{b\,(a-c)}{b+d}$$

So the solution for p(t) can be written

$$p(t) = \frac{a\,b - b\,c}{b + d} + \left(-\frac{d}{b}\right)^t \left(p0 - \frac{a\,b - b\,c}{b + d}\right)$$

If $|b| > |d|$ then $|d/b| < 1$ and $0 < (-d/b$
oscillatory fashion. The greater the absolute value of the demand curve relative to the supply curve, the larger the parameter b and the smaller $(-\,d/b)$, hence the smaller the oscillations around the equilibrium and so the sooner equilibrium is reached.

Question 11

```
> p:='p'; p0:='p0';
```

$$p := p$$
$$p0 := p0$$

```
> solve(10-2*pt=4+2*pt_1,pt);
```

$$3 - pt\_1$$

```
> solve(p=3-p,p);
```

$$\frac{3}{2}$$

With difference equation,

$$p_t = 1.5 + (-1)^t\,(p - 1.5)$$

The periodicity and the periodic values are readily established using the **rsolve** command.

```
> rsolve({p(t)=3-p(t-1),p(0)=p0},p(t));
```

$$p0\,(-1)^t - \frac{3}{2}(-1)^t + \frac{3}{2}$$

with periodic values p_0 when t is even and $3 - p_0$ when t is odd.

Question 12

First let us establish the fixed points of the system. Equating demand and supply and solving for equilibrium p we find,

```
> solve(4-3*p=p^2,p);
```

$$-4,\ 1$$

Since a negative price is ruled out, we consider only $pe = 1$.

Before considering the cobweb version of this non-linear model, consider the stability of the linear approximation. First we need to obtaind the difference equation for p.

```
> solve(4-3*pt=pt_1^2,pt);
```

$$\frac{4}{3} - \frac{1}{3}pt\_1^2$$

Thus, $p_t = f(p_{t-1}) = \frac{4}{3} - \frac{1\,p_{t-1}^{\;2}}{3}$ and $\frac{\partial}{\partial p}f = -\frac{2}{3}$ with $f\,'(pe{=}1) = -2/3$. Since $|f\,'(pe)| < 1$, then the system is stable. The linear approximation is given by:

$$p_t = 1 - \frac{2\,(p_{t-1} - 1)}{3}$$

which can be expressed,

$$p_t = \frac{5}{3} - \frac{2\,p_{t-1}}{3}$$

```
> rsolve({p(t)=(5/3)-(2/3)*p(t-1),p(0)=p0},p(t));
```

$$p0\left(\frac{-2}{3}\right)^t - \left(\frac{-2}{3}\right)^t + 1$$

which can be expressed:

$$p_t = 1 + \left(-\frac{2}{3}\right)^t (p_0 - 1)$$

```
> f:='f'; p0:='p0';
```

$$f := f$$

$$p0 := p0$$

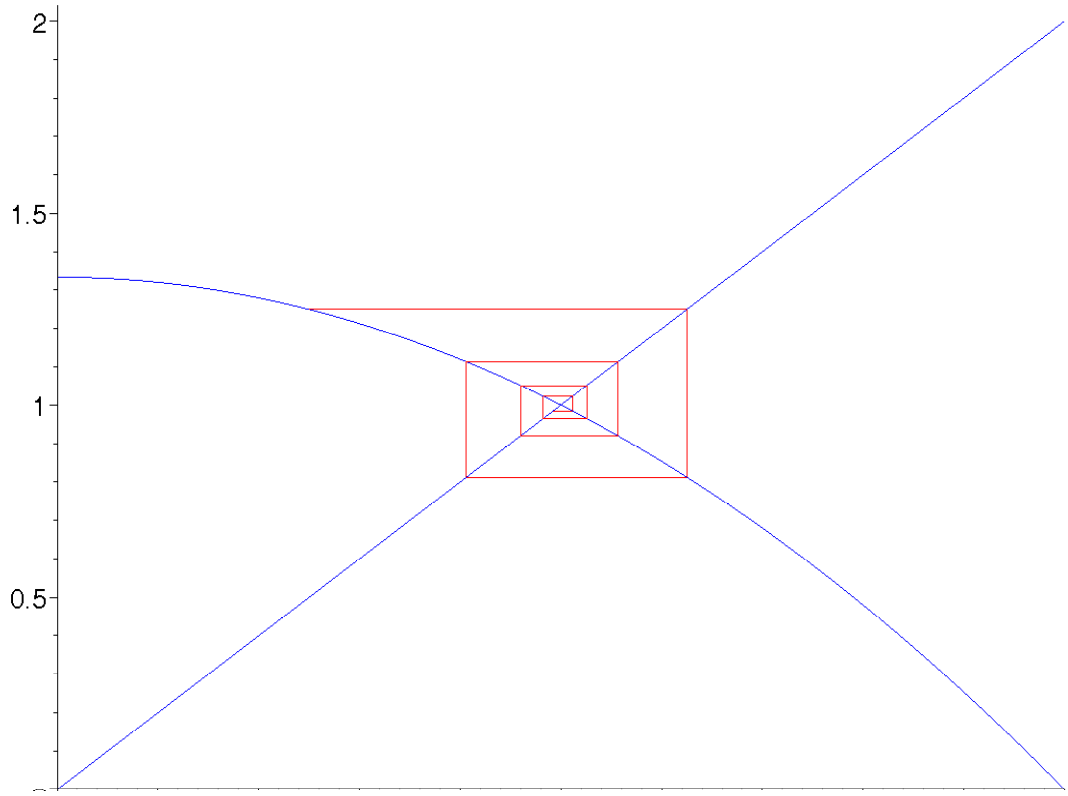
```
> f:=p->(4/3)-(1/3)*p^2;
```

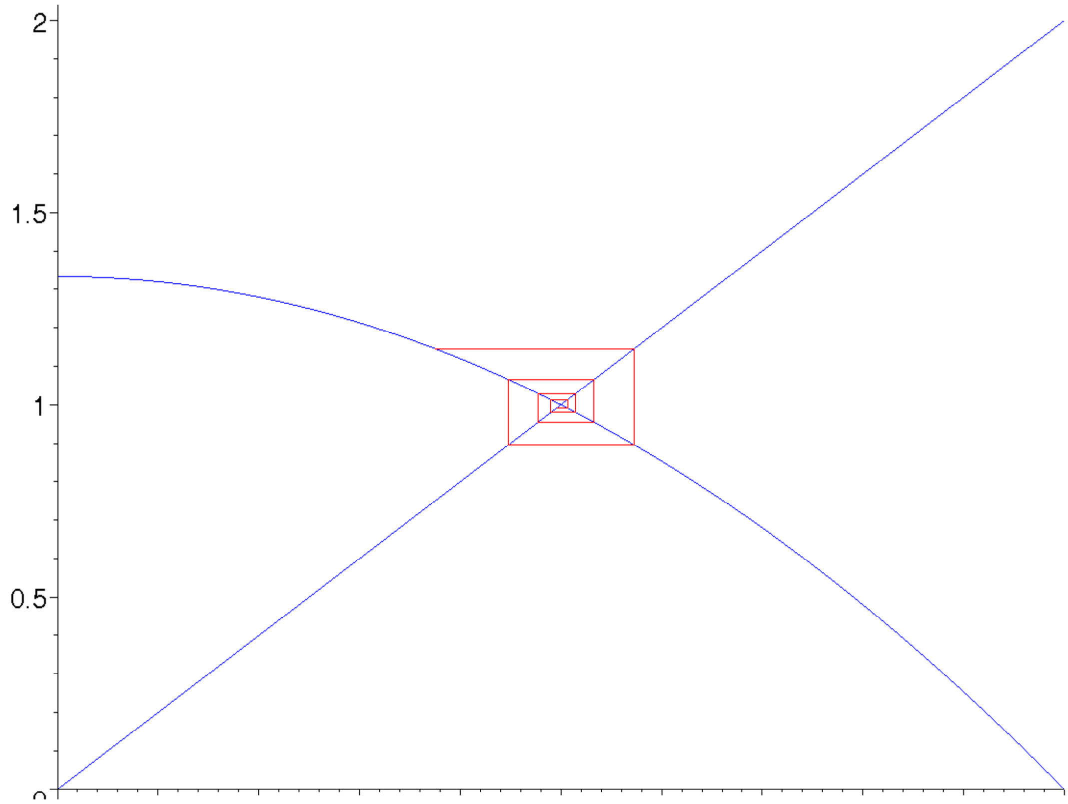
$$f := p \rightarrow \frac{4}{3} - \frac{1}{3}p^2$$

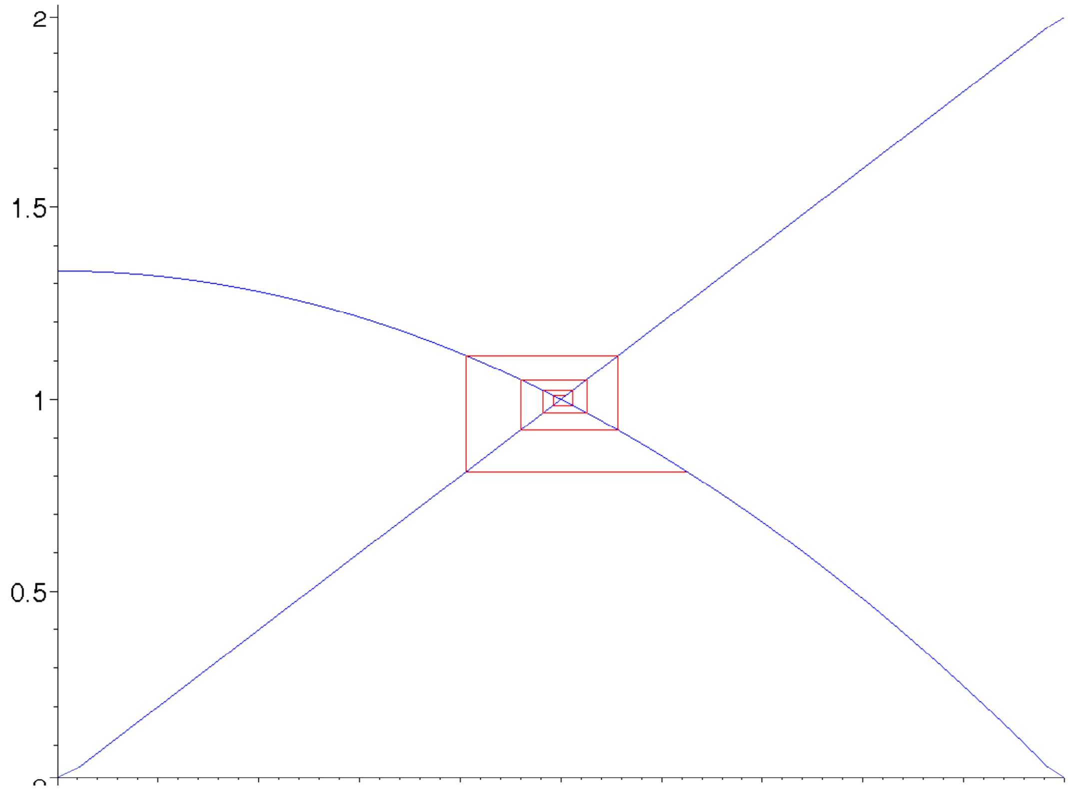
```
> cw:=array(1..2,1..2);
```

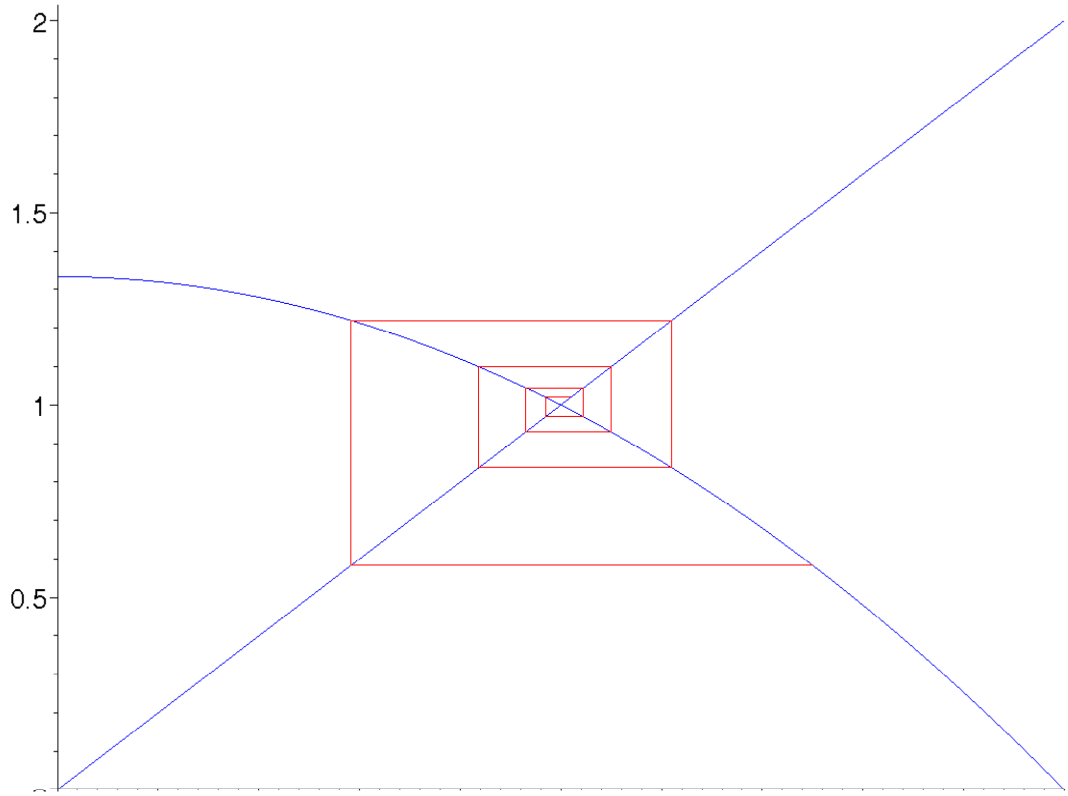
$$cw := \text{array}(1 \,..\, 2, 1 \,..\, 2, [\ ])$$

```
> cw[1,1]:=cobweb(f,0.5,8,0,2):
> cw[1,2]:=cobweb(f,0.75,8,0,2):
> cw[2,1]:=cobweb(f,1.25,8,0,2):
> cw[2,2]:=cobweb(f,1.5,8,0,2):
> display(cw[1,1]); display(cw[1,2]); display(cw[2,1]);
  display(cw[2,2]);
```




2
1.5
1
0.5


2
1.5
1
0.5



Question 13

Equation 8.16 of the text, p.336 gave the general result

$$p_t = \frac{\lambda(a-c)}{b} + \left[1 - \lambda - \frac{\lambda d}{b}\right] p_{t-1}$$

substituting the values $a = 24$, $b = 5$, $c = -4$ and $d = 2$, then

$$p_t = \frac{\lambda(24+4)}{5} + \left[1 - \lambda - \frac{2\lambda}{5}\right] p_{t-1}$$

$$p_t = \frac{28\lambda}{5} + \left[1 - \frac{7\lambda}{5}\right] p_{t-1}$$

We can first check this result by obtaining the equilibrium and showing that it is independent of λ.

```
> solve(p=(28*lambda/5)+(1-(7*lambda/5))*p,p);
```

$$4$$

```
> eq0:=(28*lambda/5)+(1-(7*lambda/5))*p;
```

$$eq0 := \frac{28}{5}\lambda + \left(1 - \frac{7}{5}\lambda\right) p$$

```
> eq1:=subs(lambda=0.25,eq0);
```

$$eq1 := 1.400000000 + .6500000000\, p$$

```
> eq2:=subs(lambda=0.5,eq0);
```

$$eq2 := 2.800000000 + .3000000000\, p$$

```
> eq3:=subs(lambda=0.75,eq0);
```

$$eq3 := 4.200000000 - .050000000\, p$$

```
> eq4:=subs(lambda=1,eq0);
```

$$eq4 := \frac{28}{5} - \frac{2}{5}p$$

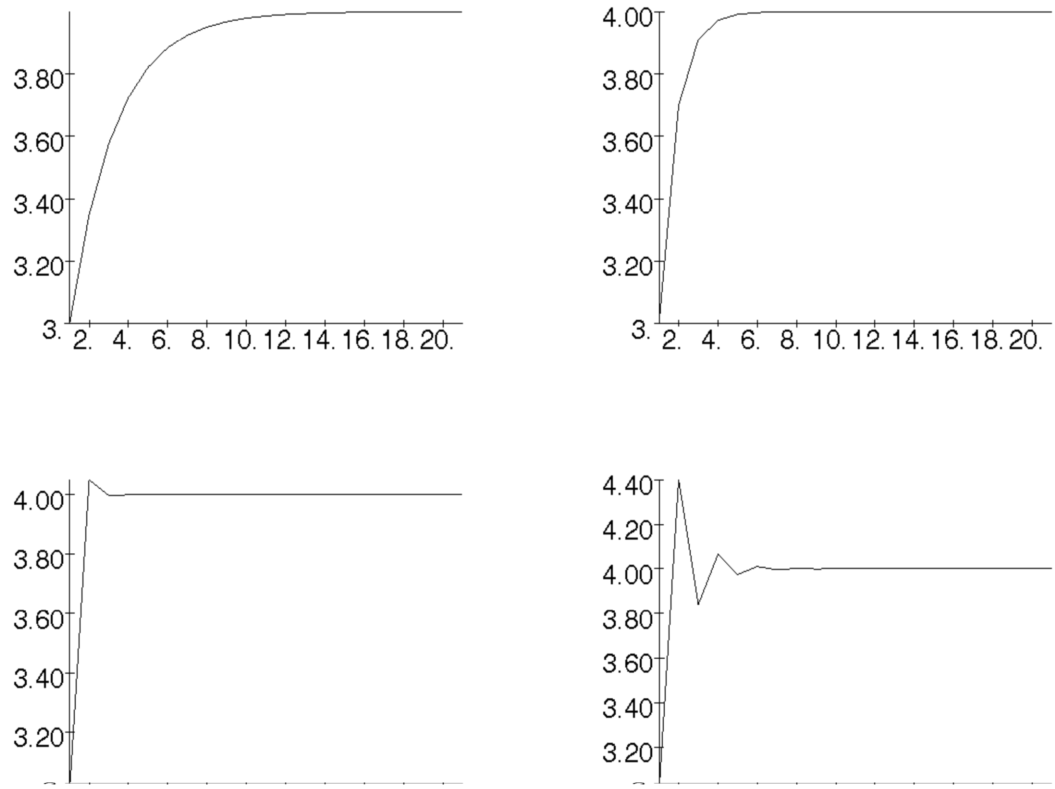
```
> f1:=p->1.4+0.65*p; f2:=p->2.8+0.3*p; f3:=p->4.2-0.05*p;
  f4:=p->5.6-0.4*p;
```

$$f1 := p \rightarrow 1.4 + .65\,p$$
$$f2 := p \rightarrow 2.8 + .3\,p$$
$$f3 := p \rightarrow 4.2 - .05\,p$$
$$f4 := p \rightarrow 5.6 - .4\,p$$

```
> graph:=array(1..2,1..2);
```

$$graph := \text{array}(1\,..\,2, 1\,..\,2, [\ ])$$

```
> graph[1,1]:=listplot([seq((f1@@k)(3),k=0..20)]):
> graph[1,2]:=listplot([seq((f2@@k)(3),k=0..20)]):
> graph[2,1]:=listplot([seq((f3@@k)(3),k=0..20)]):
> graph[2,2]:=listplot([seq((f4@@k)(3),k=0..20)]):
> display(graph);
```



Question 14

(i)

The linear approximation is given by:

$$\frac{\partial}{\partial t}P = r\,(P - Pe) - \left(\frac{\partial}{\partial h}\text{R}(he)\right)(h - he)$$

$$\frac{\partial}{\partial t}h = \left(\frac{\partial}{\partial P}\text{g}(Pe)\right)(P - Pe) - (d + n)\,(h - he)$$

(ii)

Given the assumptions, then

$$\frac{\partial}{\partial t} P = .05\,(P - Pe) + .5\,(h - he)$$

$$\frac{\partial}{\partial t} h = P - Pe - .03\,(h - he)$$

Assume $Pe = 1$ and $he = 1$, then if $\frac{\partial}{\partial t} P = 0$ and $\frac{\partial}{\partial t} h = 0$,

```
> solve(0.05*(P-1)+0.5*(h-1)=0,P);
```

$$11. - 10.\,h$$

```
> solve((P-1)-0.03*(h-1)=0,P);
```

$$.9700000000 + .03000000000\,h$$

(iii)

```
> A:=matrix([[0.05, 0.5], [1, -0.03]]);
```

$$A := \begin{bmatrix} .05 & .5 \\ 1 & -.03 \end{bmatrix}$$

```
> eigenvalues(A);
```

$$.7182372484,\ -.6982372484$$

Because eigenvalues r and s

This is also verified by the fact that the determinant is negative, thus

```
> det(A);
```

$$-.5015$$

```
> eigenvectors(A);
```

$[.7182372484, 1, \{[.5990963249, .8006769594]\}],$

$[-.6982372484, 1, \{[-.5892025114, .8817265314]\}]$

Question 15

The equation we wish to investigate is

$$p_{t+1}^{\ e} = (1-\lambda)\,p_t^{\ e} + \frac{\lambda\,a}{b} - \frac{\lambda\arctan(\mu\,p_t^{\ e})}{b}$$

with $a = .8$, $b = .25$ and $\mu = 4$.

Note

The following plot takes a long time to produce and may exhaust your computer's

c in the equation for λ.

The range for c will then need to be altered so λ ranges from 0.15 to 0.75.

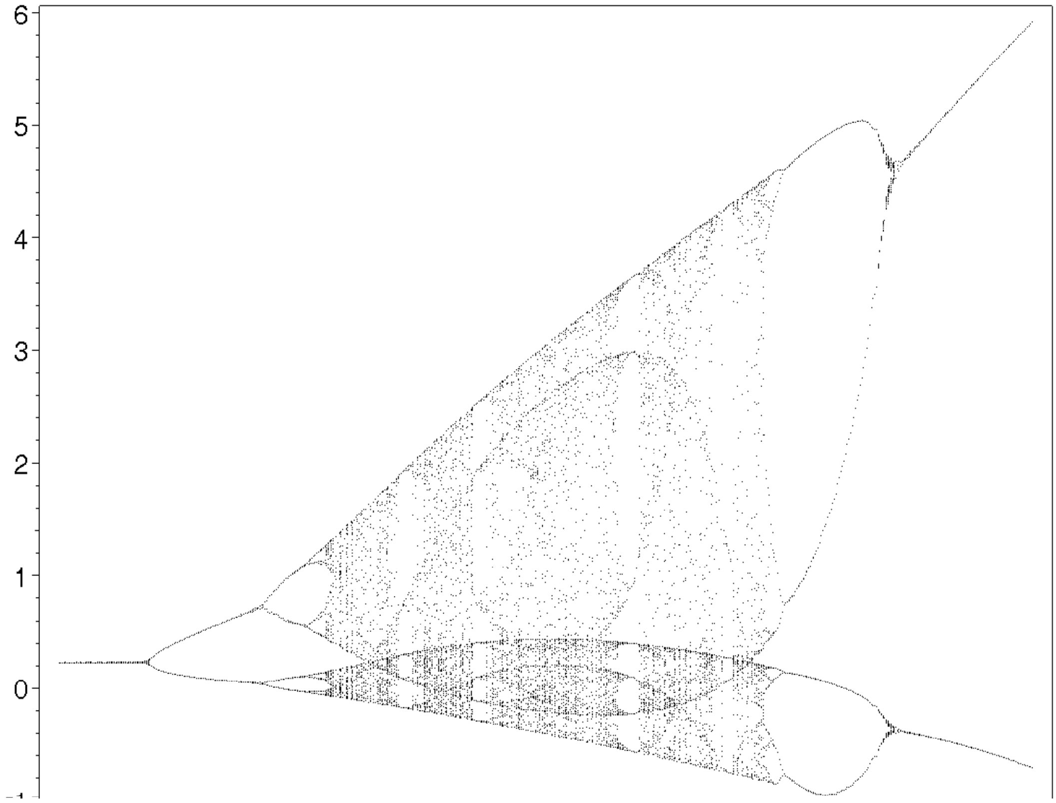
```
> lambda:='lambda': c:='c':
> f:=pe->(1-lambda)*pe+lambda*0.8/0.25-lambda*arctan(4*pe)/0
.25;
```

$$f := pe \to (1-\lambda)\,pe + 3.200000000\,\lambda - 4.000000000\,\lambda\arctan(4\,pe)$$

```
> lambda:=0.15+0.001*c;
```

$$\lambda := .15 + .001\,c$$

```
> points:=evalf(seq(seq([lambda,(f@@k)((f@@50)(0.5))],k=0..5
0),c=0..600),4):
> pointplot({points},symbol=point,axes=BOXED);
```



>

When λ has a value around 0.15 the model exhibits a stable equilibrium expected price. As λ increases infinitely many doubling bifurcations occur, eventually reaching chaotic behaviour. However, as λ continues to increase there are many period halving bifurcations arising and expected price behaviour takes on a more regular pattern once again. At λ around 0.75 the system settles down to a stable two-period cycle. Also note that as λ increases so does the amplitude of the expected price oscillations.