2
Electromagnetics of planar surface waves
Figures and tables

2.2 Topics in electromagnetic theory

2.2.7 Group velocity and phase velocity

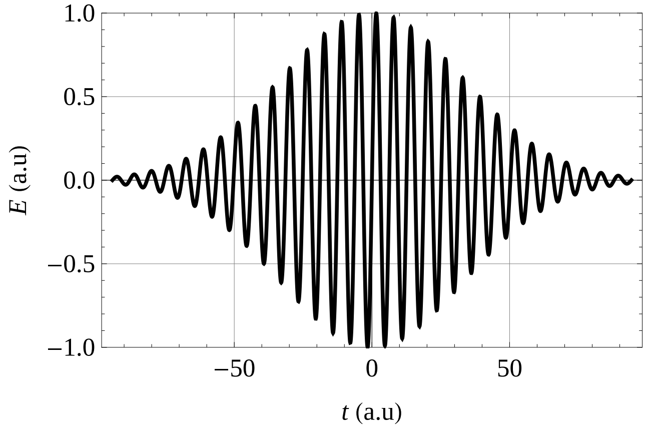

Fig. 2.1. A wave packet, E(t,z), consisting of a superposition of scalar plane-parallel harmonic waves propagating in the z-direction that has a Gaussian envelope.

2.3 Media type notation

2.3.1 Material- and geometry-dependent $\boldsymbol{\epsilon}$ *and* $\boldsymbol{\mu}$

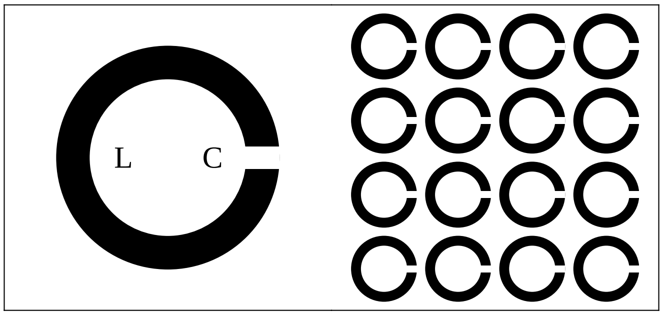

Fig. 2.2. (left) A conducting unit with the shape of a miniature split ring has an electromagnetic response at high enough frequencies that exhibits both inductive and capacitive characteristics. The curved part of this split ring acts as an inductor (L) while the gap acts as a capacitor (C). (right) A two-dimensional array of 16 split-rings.

Type	Term	sign(ϵ_r')	sign(μ_r')
Double positive	DPS	+	+
ϵ_r' negative	ENG	−	+
Double negative	DNG	−	−
μ_r' negative	MNG	+	−

Table 2.1. A medium can be regarded as an entity composed of a combination of moiety and form that can be specified by its unique set (ϵ_r',μ_r').

2.3.3 Characterization of DPS-, ENG-, DNG- and MNG-type media

Type	sign(ϵ_r')	sign(μ_r')	sign(n')
DPS	+	+	+
ENG	−	+	−
DNG	−	−	−
MNG	+	−	−

Table 2.2. The four media types, DPS, ENG, DNG and MNG, together with their respective sign of n'.

2.3.4 DPS-, ENG-, DNG- and MNG-type media presented in an $\boldsymbol{\epsilon_r}$'-$\boldsymbol{\mu_r}$' parameter space

Type	ϕ_{DPS}	ϕ_{ENG}	ϕ_{DNG}	ϕ_{MNG}
Angle	ϕ	$\pi-\phi$	$\pi+\phi$	$2\pi-\phi$

Table 2.3. To form ENG-, DNG- and MNG-type media based on a DPS-type medium, we arbitrarily choose the corresponding angles ϕ_{ENG}, ϕ_{DNG} and ϕ_{MNG}.

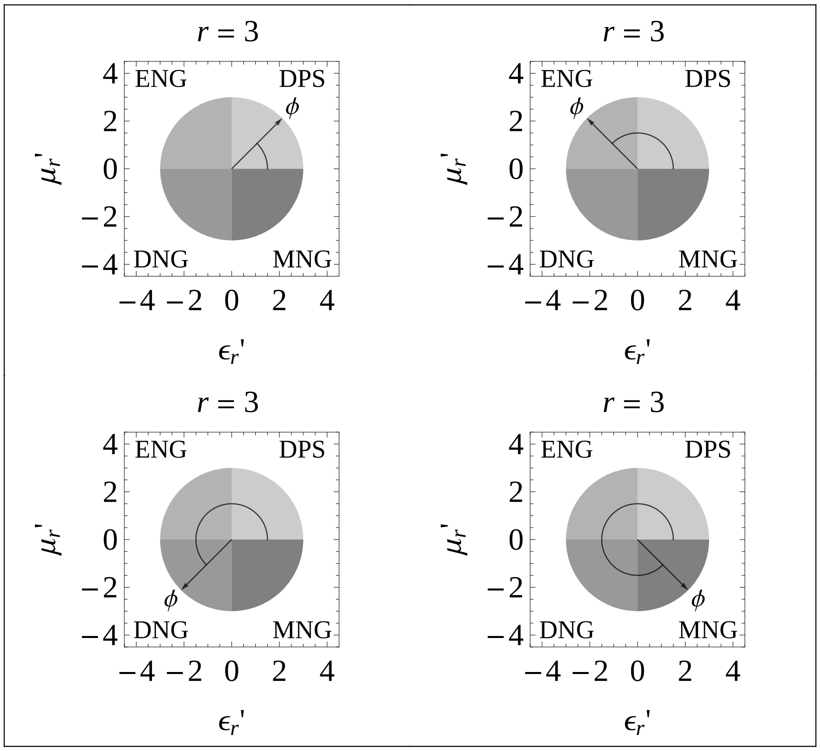

Fig. 2.3. A chosen DPS-type medium with r = 3 and ϕ = 30 deg, generates ENG-, DNG- and MNG-type media. Each medium is represented by an arrow residing in one of the four quadrants of an ϵ_r'-μ_r' parameter space.

2.4 Mode and symmetry notation

2.4.1 TE and TM modes

Property	Notation	Even TE	Odd TE	Even TM	Odd TM
Mode	m	0	0	1	1
Symmetry	s	1	-1	1	-1

Table 2.4. The parameter m denotes a TE mode if it equals 0 and a TM mode if it equals 1. The parameter s denotes an even profile of the field along the *x*-direction if it equals -1 and an odd profile if it equals +1.

2.5 Wave vector notation

2.5.1 Wave vectors in single- and double-interface structures

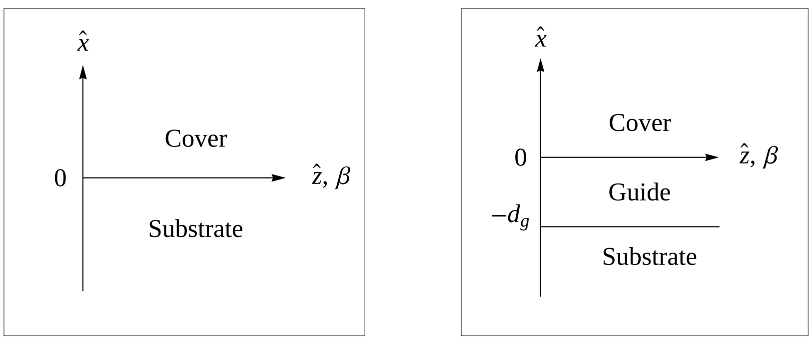

Fig. 2.4. Single-interface (left) and double-interface (right) structures showing the direction of the propagation constant, β, Note that in both cases the bottom surface of the cover is at x = 0 and that for the double-interface structure d_g denotes the thickness of the guide.

2.6 Single-interface TE mode fields

2.6.1 Schematic diagram

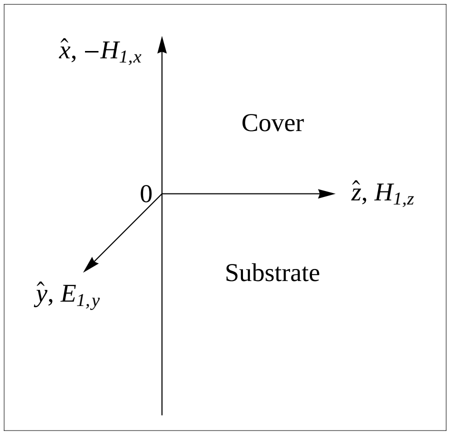

Fig. 2.5. Schematic diagram of a single-interface structure composed of a substrate and cover supporting a TE mode propagating along the z-direction with a propagation constant β. For the TE mode, the electric field has one component, $E_{1,y}$, and the magnetic field has the two components, $H_{1,x}$ and $H_{1,z}$.

2.7 Single-interface TM mode fields

2.7.1 Schematic diagram

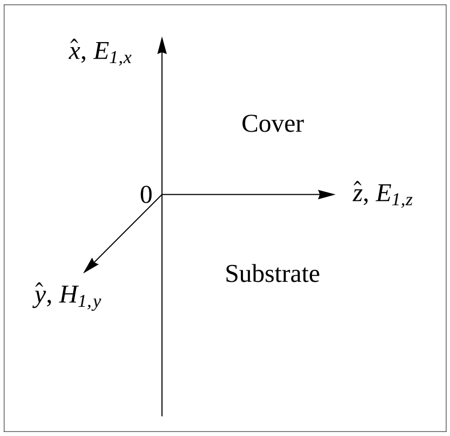

Fig. 2.6. Schematic diagrams of a single-interface structure composed of a substrate and cover supporting a TM mode propagating along the z-direction with a propagation constant β. For the TM mode, the magnetic field has one component, $H_{1,y}$, and the electric field has the two components, $E_{1,x}$ and $E_{1,z}$.

2.9 Double-interface TE mode fields

2.9.1 Schematic diagram

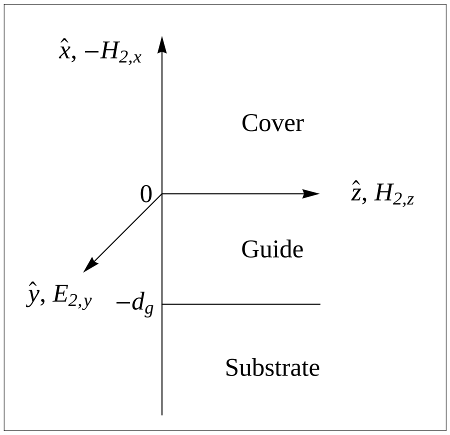

Fig. 2.7. Schematic diagrams of a double-interface structure composed of a substrate, a guide with thickness d_g and cover supporting a TE mode propagating along the z-direction with a propagation constant β. For the TE mode, the electric field has one component, $E_{2,y}$, and the magnetic field has the two components, $H_{2,x}$ and $H_{2,z}$.

2.10 Double-interface TM mode fields

2.10.1 Schematic diagram

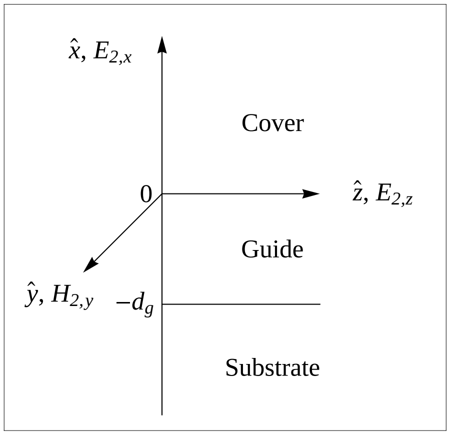

Fig. 2.8. Schematic diagrams of a double-interface structure composed of a substrate, a guide with thickness d_g and cover supporting a TM mode propagating along the z-direction with a propagation constant β. For the TM mode, the magnetic field has one component, $H_{2,y}$, and the electric field has the two components, $E_{2,x}$ and $E_{2,z}$.

2.14 Double-interface mode solution

2.14.3 Summary of mode solutions of a symmetric guide structure

ψ		Even	Odd
		$s = 1$	$s = -1$
TE	$m = 0$	$\frac{\mu_g\,\delta}{\mu_c\,\kappa}$	$-\frac{\mu_c\,\kappa}{\mu_g\,\delta}$
TM	$m = 1$	$\frac{\epsilon_g\,\delta}{\epsilon_c\,\kappa}$	$-\frac{\epsilon_c\,\kappa}{\epsilon_g\,\delta}$

Table 2.5. The four even- and odd-symmetry TE and TM mode equations of a symmetric guide structure in terms of ψ.

δ		Even	Odd
		$s = 1$	$s = -1$
TE	$m = 0$	$\frac{\mu_c\,\kappa}{\mu_g}\psi$	$-\frac{\mu_c\,\kappa}{\mu_g\,\psi}$
TM	$m = 1$	$\frac{\epsilon_c\,\kappa}{\epsilon_g}\psi$	$-\frac{\epsilon_c\,\kappa}{\epsilon_g\,\psi}$

Table 2.6. The four even- and odd-symmetry TE and TM mode equations of a symmetric guide structure in terms of δ.

2.15 Poynting vector

2.15.3 Local power flow in a single-interface mode

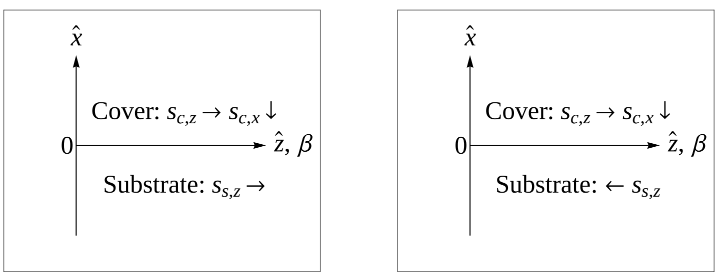

Fig. 2.9. Schematic diagram showing two possible directions of the local power flow in a single-interface mode, $s_{c,z}$ and $s_{s,x}$. (left) The case where the local power in the cover has components along the –x- and +z-directions, and in the substrate it flows along the z- direction. (right) The case where the local power flow in the cover is similar to that case on the left, but the local power in the substrate flows along the -z- direction.

2.15.4 Local power flow in a double-interface modes

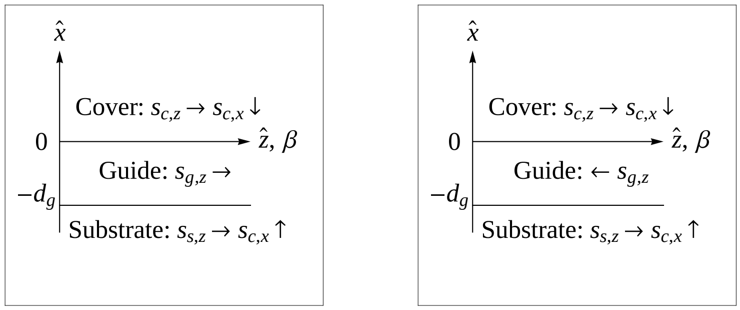

Fig. 2.10. Schematic diagram showing two possible directions of the local power flow of double interface modes, $s_{s,z}$, $s_{g,z}$ and $s_{c,x}$, in the substrate, guide and cover, respectively, along the ±x- and ±*z*-directions. (left) The case where the local power in the cover has components along the +z- and −*x*-directions, the local power in the substrate has components along the +z and +*x*-directions, and the local power in the guide flows along the +*z*-direction. (right) The case where the local power flow in the cover and substrate are similar to those shown in the figure on the left, but the local power in the guide flows along the -*z*-direction.

2.16 Prism coupling

2.16.2 Schematic diagram of the prism coupler

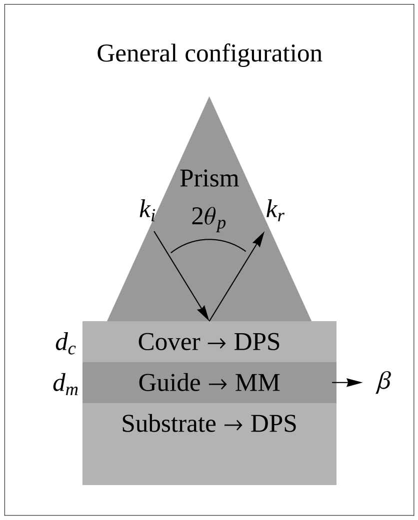

Fig. 2.11. Schematic diagram of a general configuration of a prism coupler in contact with a double-interface structure composed of a substrate, guide and cover. The prism, substrate and cover consist of DPS-type media while the guide is composed of a DPS-, ENG-, DNG-, or MNG-type medium. The arrows represent the rays of the incident and reflected beams off the prism-cover interface, k_i and k_r, respectively, with the angle in the prism between these two denoted by $2\theta_p$. The thickness of the cover and guide are denoted by d_c and d_g, respectively, and the arrow on the right depicts the direction of the propagation constant, β. Note that MM denotes metamaterial.

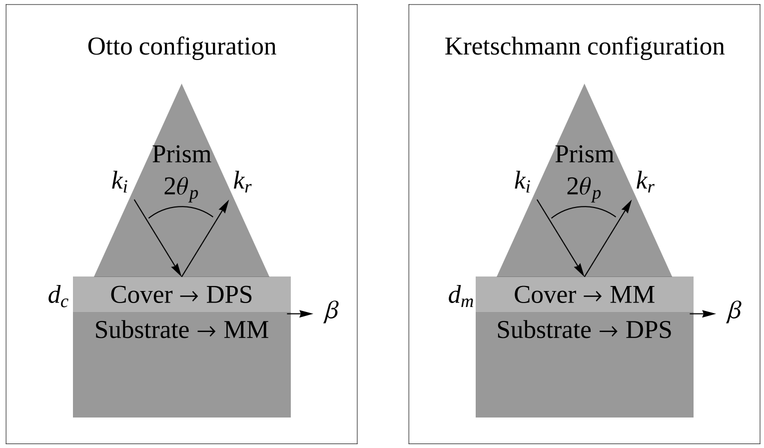

Fig. 2.12. Schematic diagrams showing prism coupling to two single-interface structured. (left) The Otto configuration where the cover is a DPS-type medium and the substrate an ENG- DNG- or MNG-type medium. (right) The Kretschmann configuration refers to the case where the cover is an ENG- DNG- or MNG-type medium and the substrate is a DPS-type medium. Note that for the Kretschmann configuration the thickness of the cover is denoted by d_m.

2.16.5 k-vectors in the cover, guide and substrate

	k−vector	Wave vector
k_c	$k_0\sqrt{(\epsilon_c\,\mu_c - \epsilon_p\,\mu_p\,\sin\theta_p{}^2)}$	$i k_0 \delta$
k_g	$k_0\sqrt{(\epsilon_g\,\mu_g - \epsilon_p\,\mu_p\,\sin\theta_p{}^2)}$	$k_0 \kappa$
k_s	$k_0\sqrt{(\epsilon_s\,\mu_s - \epsilon_p\,\mu_p\,\sin\theta_p{}^2)}$	$-i k_0 \gamma$

Table 2.7. Transverse k-vectors in the cover, guide and substrate and their respective representation as wavevectors used in the solution of the mode equation.

2.17 Reflectivity and Goos-Hänchen shift

2.17.1 Reflectivity from a single-interface structure: Fresnel equations

	TE−polarization	TM−polarization
R_{pc}	$(\mu_c\, k_p(\theta_p) - \mu_p\, k_c(\theta_p))/(\mu_c\, k_p(\theta_p) + \mu_p\, k_c(\theta_p))$	$(\epsilon_c\, k_p(\theta_p) - \epsilon_p\, k_c(\theta_p))/(\epsilon_c\, k_p(\theta_p) + \epsilon_p\, k_c(\theta_p))$
R_{cg}	$(\mu_g\, k_c(\theta_p) - \mu_c\, k_g(\theta_p))/(\mu_g\, k_c(\theta_p) + \mu_c\, k_g(\theta_p))$	$(\epsilon_g\, k_c(\theta_p) - \epsilon_c\, k_g(\theta_p))/(\epsilon_g\, k_c(\theta_p) + \epsilon_c\, k_g(\theta_p))$
R_{gs}	$(\mu_s\, k_g(\theta_p) - \mu_g\, k_s(\theta_p))/(\mu_s\, k_g(\theta_p) + \mu_g\, k_s(\theta_p))$	$(\epsilon_g\, k_c(\theta_p) - \epsilon_c\, k_g(\theta_p))/(\epsilon_g\, k_c(\theta_p) + \epsilon_c\, k_g(\theta_p))$

Table 2.8. Fresnel equations for the amplitude reflectance, R, of a single-interface structure for TE and TM polarizations as a function of the angle of incidence inside the prism, θ_p.

2.17.2 Reflectance and reflectivity from three- and a four-interface structures

	TE−polarization	TM−polarization
R_{cgs}	$\left(R_{\text{cg}}{}^{\text{TE}} + R_{\text{gs}}{}^{\text{TE}}\, e^{i\,2\,k_g\,d_g}\right)/\left(1 + R^{\text{TE}}{}_{\text{cg}}\, R^{\text{TE}}{}_{\text{gs}}\, e^{i\,2\,k_g\,d_g}\right)$	$\left(R_{\text{cg}}{}^{\text{TM}} + R_{\text{gs}}{}^{\text{TM}}\, e^{i\,2\,k_g\,d_g}\right)/\left(1 + R^{\text{TM}}{}_{\text{cg}}\, R^{\text{TM}}{}_{\text{gs}}\, e^{i\,2\,k_g\,d_g}\right)$
R_{pcgs}	$\left(R^{\text{TE}}{}_{\text{pc}} + R^{\text{TE}}{}_{\text{cgs}}\, e^{i\,2\,k_c\,d_c}\right)/\left(1 + R^{\text{TE}}{}_{\text{cgs}}\, R^{\text{TE}}{}_{\text{pc}}\, e^{i\,2\,k_c\,d_c}\right)$	$\left(R^{\text{TM}}{}_{\text{pc}} + R^{\text{TM}}{}_{\text{cgs}}\, e^{i\,2\,k_c\,d_c}\right)/\left(1 + R^{\text{TM}}{}_{\text{cgs}}\, R^{\text{TM}}{}_{\text{pc}}\, e^{i\,2\,k_c\,d_c}\right)$

Table 2.9. Cover-guide-substrate reflectance, R_{cgs}, and prism-cover-guide-substrate reflectance, R_{pcgs}, in terms of the Fresnel equations for the TE and TM polarizations.

2.18 Summary

Item	Topic	Chapter 3	Chapter 4	Chapter 5	Chapter 6	Chapter 7
1	Interfaces	1	1	2	1	2
2	Types	ENG,DNG	DPS,ENG,DNG, MNG	DPS,ENG,DNG, MNG	ENG	ENG
3	ϵ_r,μ_r	complex	real	real	complex	complex
4	Dispersion	yes	no	no	no	no
5	Free β	yes	yes	yes	yes	yes
6	Loaded β	yes	no	no	yes	yes
7	Configuration	O,K	free	free	O,K	G
8	$\mathcal{R}$	yes	no	no	yes	yes
9	G−H	yes	no	no	no	no
10	E and H	no	yes	yes	yes	yes
11	s_z	no	yes	yes	yes	yes
12	s_x	no	no	no	yes	yes
13	η	yes	no	no	yes	yes
14	v_{ph} and v_{group}	yes	no	no	no	no
15	Charge density	no	no	no	yes	yes

Table 2.10. A list of the 15 topics that will be explored in chapters 3-7.

Exercises

(1) Discuss structures that give rise to the electromagnetic response of DNG-type media.

(2) Review the meaning of the refractive index in terms of Eq. (2.63) for lossy media.

(3) Derive Eq. (2.64) that gives the critical thickness, d_{cr}, of a DNG-type waveguide and discuss its peculiar meaning.

(4) Derive Eq. (2.212) that gives the Goos-Hänchen shift and discuss its meaning in terms of the four possible media types.

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