**Problems for Chapter 20 of 'Ultra Low Power Bioelectronics'**

Problem 20.1

Figure P20.1 shows a simple model of the interface between the human body and signal-acquisition electronics. The differential signal $v_d$ is sensed through electrodes with impedance $Z_1$and $Z_2$ and safety resistors $R_{prot}$. The common-mode signal $i_{cm}$ is unwanted, and is often created by 60 Hz pickup.

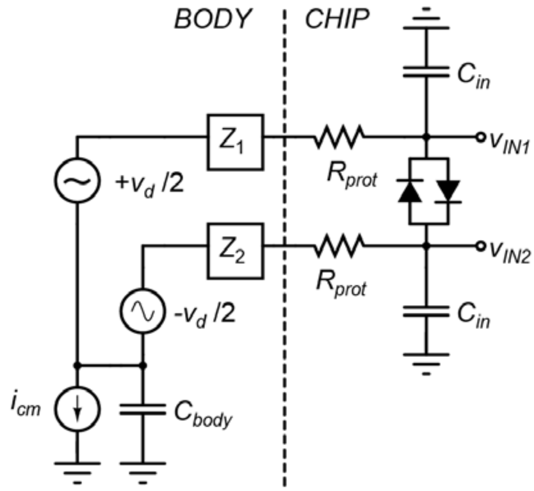


**Figure P20.1**: A model of the body/electronics interface.

a) Explain the function of the diodes, and also the resistors labeled $R_{prot}$.
b) Find the transfer function between differential input voltage $v_d$ and output differential voltage $v_{in} = v_{in1} - v_{in2}$.
c) Find the transfer impedance between common-mode input current $i_{cm}$ and output differential voltage $v_{in}$. Justify any assumptions that you make.
d) Explain why non-zero common-mode transfer impedance is undesirable. Use your analysis in part c) to suggest ways of reducing this quantity.
e) We want to sense an EKG signal, with $v_d = 1$ $mV_{pp}$. Assume that $R_{prot} = 10$ k$\Omega$, $C_{body} = 200$ pF, and $i_{cm}$ is a sinusoid of amplitude 0.5 $\mu$A and frequency 60 Hz. At this frequency, $Z_1$ and $Z_2$ are well-modeled as resistors of value 40 k$\Omega$ and 80 k$\Omega$, respectively. Under what conditions will $v_{in}$ not be dominated by 60 Hz pickup?

Problem 20.2

For the purposes of estimating 60 Hz pickup, assume that the body is well-modeled as a capacitor of value $C_{body}$ in parallel with a current source of amplitude $I_{60}$.

a) Find the amplitude of the 60 Hz voltage waveform displayed on an oscilloscope when your finger touches one of its input terminals. Assume that $C_{body} = 200$ pF, $I_{60} = 0.4$ $\mu$A, and that the input impedance of the oscilloscope consists of a 1 M$\Omega$ resistor in parallel with a 20 pF capacitor.
b) In order to reduce 60 Hz pickup you decide to connect the ground terminal in your circuit to the body through an electrode. The impedance of the electrode

at 60 Hz is approximately 50 kΩ. What is the new 60 Hz amplitude measured on the oscilloscope?

c) You are unsatisfied with the results obtained in part b), and decide to use a feedback loop to sense and actively drive the common-mode potential of the body. You use the same electrode as before, and design the loop gain of the feedback loop at 60 Hz to be 100. What is the new 60 Hz amplitude measured on the oscilloscope?

Problem 20.3

Use the block diagram shown in Figure 20.8 to derive Equations (20.5) and (20.6), i.e., the differential gain and common-mode rejection ratio (CMRR) of the EKG amplifier shown in Figure 20.7.

Problem 20.4

Consider the EKG amplifier shown in Figure 20.7. Assume that the output conductance of the super-buffer is much larger than that of the electrode, i.e., $G_{BUF} \gg G_{elec}$.

a) We want the common-mode feedback loop to have no overshoot or ringing in its transient response. We also want the magnitude of the loop gain to be at least 40 dB at 60 Hz. Complete the design of the loop, i.e., find the values of $G_M^{M4}$ and $C_{11}$, given that $\kappa = 0.7$, $C_{body} \approx 200$ pF, and $G_{elec} \approx 40\ \mu$S. Justify any assumptions that you make.

b) What is the bias current required by transconductor $M_4$, given that it is a 5-transistor OTA operated in subthreshold?

Problem 20.5

Use the block diagram shown in Figure 20.8 to derive Equation (20.9). In other words, calculate the output noise current of each OTA, and also the input-referred noise voltage of the entire EKG amplifier shown in Figure 20.7.

Problem 20.6

a) Derive an expression for $SpO_2$, the oxygen saturation of blood, by combining Equations (20.15) and (20.16), and show that it is equal to Equation (20.17).

b) Measured extinction coefficients for oxy-hemoglobin ($HbO_2$) and hemoglobin ($Hb$) at various wavelengths are shown in the following table:

| Wavelength $\lambda$ (nm) | $\epsilon_{HbO_2}$ (cm$^{-1}$/(moles/liter)) | $\epsilon_{Hb}$ (cm$^{-1}$/(moles/liter)) |
|---|---|---|
| 660 | 319.6 | 3226.56 |
| 940 | 1214 | 693.44 |

Use the data provided in the table to derive an expression for $SpO_2$ that is only a function of $R$.

Problem 20.7

Microphones are typically sold in two- or three-terminal packages that include an internal JFET buffer. A circuit for interfacing with a two-terminal electret microphone is shown in Figure P20.7. Sound waves generate a voltage source $v_{elec}$ in series with the electret capacitance $C_{elec}$.

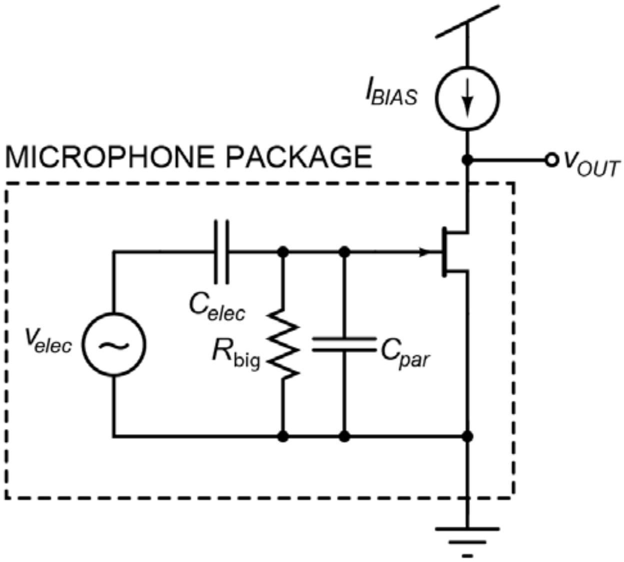


**Figure P20.7**: A circuit for interfacing with a two-terminal electret microphone.

a) Explain the purpose of $R_{big}$. What should its value be? Write down the transfer function from $v_{elec}$ to the gate of the JFET and describe its characteristics.

b) Assume that $I_{BIAS}$ is an ideal current source and that the I-V characteristics of the JFET are described by a square-law equation given by,

$$i_{DS} = \beta[(v_{GS} - V_T)v_{DS} - v_{DS}^2/2], \quad v_{DS} \le (v_{GS} - V_T)$$
$$= \beta(v_{GS} - V_T)^2(1 + \lambda v_{DS})/2, \quad v_{DS} > (v_{GS} - V_T)$$

where $\beta$, $V_T$, and $\lambda$ are constants. Is $V_T$ positive or negative? Explain.

c) Find the small-signal voltage gain of the circuit from $v_{elec}$ to $v_{out}$ when the JFET is saturated.

d) Now assume that the JFET is *not* saturated. Show that the small-signal voltage gain at frequencies of interest is given by

$$\frac{v_{out}}{v_{elec}} = -\left(\frac{C_{elec}}{C_{elec} + C_{par}}\right)\left(\sqrt{\frac{1}{1 - I_{BIAS}/I_{SAT}}} - 1\right)$$

where $I_{SAT} = \beta(V_{GS} - V_T)^2/2$ is the saturated bias current of the JFET. Is this equation accurate as $I_{BIAS}$ approaches $I_{SAT}$? Explain.

e) Sketch the small-signal voltage gain as a function of $I_{BIAS}/I_{SAT}$ both when $I_{BIAS} < I_{SAT}$, and when $I_{BIAS} \ge I_{SAT}$. As a designer, do you have control over $I_{SAT}$? Explain.

f) Explain how the voltage gain and power consumption of this circuit are coupled. Draw a simple circuit that allows the voltage gain to be varied by the user.

Problem 20.8

A non-contact method for studying respiratory and cardiac function relies on using the Doppler effect to record chest wall motion. Both ultra-sound and radio frequency (RF) sources can be used for the measurement. Consider an ultra-sound source emitting a 5 MHz tone. The source is placed in front of the subject, and the energy reflected from his/her chest back towards the source is recorded.

a) Explain why energy is reflected back towards the source. What fraction of the incident power is reflected? The acoustic impedances of air and skin at 300K are approximately 410 $\text{Ns/m}^3$ and $1.6\times10^6$ $\text{Ns/m}^3$, respectively.
b) Assume that the position of the chest wall varies sinusoidally with time, i.e., is given by $y(t) = A\sin(2\pi f_0 t)$. Derive a formula for the maximum frequency change in the reflected ultrasonic wave that is caused by this periodic motion.
c) Calculate the maximum frequency change for the following set of values, which are typical for respiration in adults: $A$ = 1 cm, $f_0$ = 15 per minute. Will the frequency change be audible? The speed of sound at sea level is approximately 340 m/s.
d) Discuss the advantages and disadvantages of this method of sensing.

Problem 20.9

An alternative approach to reducing unwanted common-mode pickup is to place a band-stop filter (usually designed to reject a narrow range of frequencies around 60 Hz) at the input of the bio-signal processing system. Such filters are known as notch filters. The circuit shown in Figure P20.9 (a) is among the most popular notch-filter topologies, and is known as the twin-T.

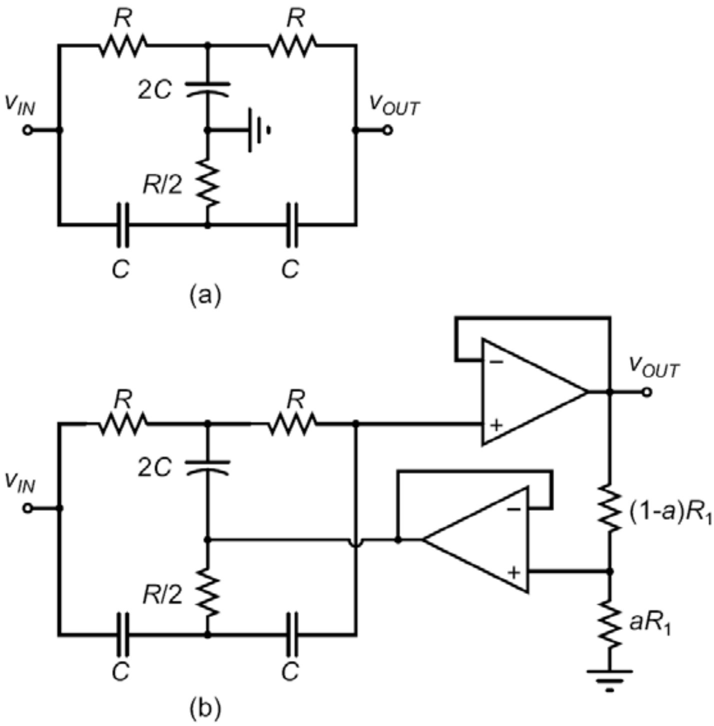


**Figure P20.9**: Notch-filter circuits.

a) Explain the basic properties of the twin-T circuit without using any mathematics.

b) Show that the twin-T circuit indeed behaves as a notch filter, and calculate the frequency and quality factor of the notch. Write down the transfer function of the circuit from $v_{IN}$ to $v_{OUT}$ in canonical form. [Hint: the circuit becomes easier to analyze if it's transformed from a T network into an equivalent $\pi$ network.]

c) Is the quality factor of the twin-T filter adjustable? Explain.

d) An improved notch filter circuit, based on the original twin-T network, is shown in Figure P20.9 (b). Assuming that the op-amps are ideal, show that the transfer function of this circuit is given by

$$H(s) = \frac{v_{out}}{v_{in}} = \frac{\tau^2 s^2 + 1}{\tau^2 s^2 + 4(1-a)\tau s + 1}$$

e) What are the advantages of the active circuit shown in Figure P20.9 (b) over the passive circuit shown in Figure P20.9 (a)? Are there any disadvantages?

f) Comment on the overall suitability of notch filtering as a method for removing unwanted common-mode signals.

Problem 20.10

**This problem requires the use of a circuit simulator such as SPICE.**

a) Implement the circuit model of the heart shown in Figure 20.2 in a circuit simulator. You may use the integrated circuit implementation shown in Figure 20.3 as a starting point for your design. However, do not neglect the compliance of heart valves or the mass of blood in the circulatory system. Use reasonable guesses for any parameter values that are unknown or research values for them from the literature. Adjust these values until the waveforms obtained from transient simulations are good approximations of actual cardiac waveforms.

b) Use your model to explore the effects of various abnormalities in the circulatory system, such as leaky heart valves and constricted arteries.