ADDENDUM ET CORRIGENDUM – CHAPTER 7

In my chapter, I devote attention to Leibniz's *lex homogeneorum*. I underline that Leibniz, and his correspondents, such as Johann Bernoulli and Pierre Varignon, paid great attention to the dimensional homogeneity of the terms occurring in the algebraic and differential equations. This reading allows me to claim that geometrical interpretation was important for Leibniz and the early practitioners of the differential and integral calculi, even for Euler in his early works.

On pp. 215-216, I consider Leibniz's handling of differentials. I refer to a manuscript penned in the late 1690s, and I exemplify what Leibniz has in mind there by considering a formula of my own in which differentials occur. I explain that all differentials, according to the *lex homogeneorum,* must have the same geometrical dimension in order to be compared one to another. Thus, *addx* can be summed to *dxdx*, for example, since these two terms are bidimensional: the former is a rectangle with sides *a* and *ddx*, the latter is a square with side *dx*.

What I should have added is that Leibniz refers to another law, the *lex homogeneorum transcendentalis*. In this second law, Leibniz considers the differential *d* and how it operates on magnitudes, so that only magnitudes of the same order of infinity can occur in an equation. As Bos explains:

> The geometric interpretation of the quantities entering the analysis requires the equations to be homogeneous in dimension. In addition, there is a second kind of homogeneity, which requires that all the terms of an equation should be of the same order of infinity. A quantity which is infinitely small with respect to another quantity can be neglected if compared with that quantity. (Bos, Henk J.M. (1974). Differentials, higher-order differentials and the derivative in the Leibnizian calculus. *Archive for History of Exact Sciences* **14**, 1-90 (on p. 33)).

Thus, in my example: $adx+addx+dxdx=adx$. Leibniz bases his transcendental law on a sum of the differential exponents. He writes:

> [A] transcendental law of homogeneity appears, which is not equally obvious in the usual way of notation for differentials. For instance, if we use this new kind of *Characteristica*, it appears that a*ddx* and *dxdx* are not only algebraically homogeneous (as in both cases two quantities are multiplied), but that they are also transcendentally homogeneous and comparable. For the former can be written as d^0ad^2x, and the latter as d^1xd^1x, and in both cases the differential exponents have the same sum, for $0 + 2 = 1 + 1$. The transcendental law of homogeneity presupposes the algebraical law. (Leibniz, Gottfried W. (1710). Symbolismus memorabilis calculi algebraici et infinitesimalis, in comparatione potentiarum et differentiarum; et de lege homogeneorum transcendentali. *Misc. Berol.,* 160-165 (on p. 165). *Math. Schr*. V, 377-382 (on pp. 381-382)).

It is interesting to note that if we focus on the first law, we distance Leibniz into a mathematical culture that still gives great importance to geometrical interpretation (indeed, only terms equal in geometrical dimension can be added and subtracted). If we take into consideration the second law, we realize that Leibniz is thinking about *d* as an operator, whose exponents determine the order of infinity (only terms of the same order of infinity can be compared).

It is possible to propose different historical narratives of Leibniz's calculus: one – to use Venuti's terminology – "foregnizing" its equations by reading them as subject to a Viètian principle of dimensional homogeneity, the other "domesticating" them as precursors of the Lagrangian calculus of operators. This dialectic between recognition of familiarity and wonder for diversity is a theme that is discussed in many chapters of this book.

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