Introduction to the Physical and Biological Oceanography of Shelf Seas

Book Website Software Guides

Introduction

These guides are designed to help you make the most of the software which is available at the book website (**http:www.cambridge.org/shelfseas**). The software consists of a series of MATLAB scripts which range from simple demonstrations of key aspects of ocean dynamics and mixing to numerical models of shelf sea processes. Each of the guides provides suggestions for running the programmes and for varying the input parameters to illustrate essential points. You will also find that the guides pose a number of questions, some easy, some not so easy, to expand and test your understanding of the processes involved.

On the following page, you will see a table showing the subject areas of each of the six guides along with the relevant MATLAB scripts and the book chapters to which the guide mainly relates. Remember also that, in the book, reference to the relevant software on the website is indicated by the software icon:

Accessing and running software

In order to utilise the the software suite, you will need a computer equipped with MATLAB (version 7 or later). To install the Matlab Menu, proceed as follows:

1) From the Website, download the folder OCEANDYN2 and install in an appropriate directory on your computer.

2) Open MATLAB and select OCEANDYN2 as your current folder

3) Click on “file” and then “ Set Path”

4) In dialogue box, select “Add with Subfolders”

5) Select OCEANDYN2 in browser window and click OK

6) In the MATLAB command window type OCEANDYN and the main menu will appear. Thereafter follow the instructions in the individual guides below.

Note that all the MATLAB programme scripts are available for you to copy, develop and use to explore your own ideas and enhance your understanding

Software Guides

Guide	MENU	Sub-Menu	MATLAB script
G1	BASIC DYNAMICS (Chapter 3)	(a) Inertial Occillations	Spindown2
		(b) Wind-driven flow	Ekman3
		(c) Tide-driven flow	Tidyn
G2	HEATING versus STIRRING (Chapter 6)	(a)Seasonal Cycle	HS3
		(b) Front generation	TMF1
G3	DENSITY-DRIVEN FLOW (Chapter 9)	(a) Estuarine Circulation	EstCirc
		(b)Heaps Solution	Heaps Theory
		(c) ROFI Circulation	ROFI2
G4	TURBULENCE CLOSURE MODEL (Chapter4 & 7)	(a) Run with real wind data	TC22aDem
		(b)Run with seasonal Met data	TC22aY
G5	TIDAL ANALYSIS (Chapter 2)	(a)Least Squares Demo	LST2
		(b)Analysis of heights & currents	LSPrac
G6	FICKIAN DISPERSION (Chapter 4)	(a) 1-d channel dispersion	Gauss1
		(b) Gaussian patches merging	Gauss2
		(c) Dispersion by Random Walk	RWalk2
		(d) 1-d Advection +Dispersion	GPAD2
		(e) Steady Shear Dispersion	ShDis
		(f) Tidal Shear Dispersion	TidShDis

G1: **BASIC DYNAMICS (Chapter 3)**

Introduction

This suite of 3 programs is designed to enable the student to study the dynamical response of the water column to the principal forcing mechanisms which operate in the shelf seas. The set of dynamical equations are solved under simplifying assumptions to give profiles of velocity at a single point for different types of forcing. In the exercises which follow the student is introduced to fundamental motions, starting with inertial oscillations and proceeding to classical examples of motion forced by windstress and tidal forces. The concept of a boundary layer and its development over time will be illustrated for both rotating and oscillating systems. The mathematical requirement is limited to appreciating the terms in the primitive equations (which are set out in Chapter 3 of the book) and some simple algebraic and arithmetic manipulations. Most important results are presented graphically to assist in development of physical intuitions. In addition to the final steady state solutions, which can be obtained in many cases by analytical methods, you will also see the transient time-dependent response which occurs as the system adjusts to the imposed forcing from an initial state at rest.

These exercises allow you to vary the main controlling variables to see how they affect the ocean's response. You should do this in a systematic way, based on the suggested sequence of runs, to educate your own intuitions about the way the dynamical processes involved.

As you work through the exercises below, try to answer the questions identified in the text (*Q*).

PART I :Inertial Oscillations (Spindown2)

First read through the account of inertial oscillations in Section 3.4.1 of the book to see the balance of forces involved in this type of motion. In the simple scenario to be modelled here, we assume that a whole water column is impulsively accelerated (i.e. given a push) in the y direction to a speed V_0 at t=0; It then proceeds to move under the influence of the Coriolis force which, in the northern hemisphere, means it will be deflected further and further to the right so that it follows a circular path. The radius of the circle is controlled by the Coriolis parameter, which depends on latitude, and the initial speed V_0 . If the motion is frictionless, as in the analysis in the book, the whole water column would continue indefinitely in the same circular orbit. In the numerical model, however, we introduce friction at the seabed which acts to slow the near-bed current and extracts energy from the flow so that the motion slowly spins down. The frictional stress at the bottom boundary is set by a quadratic drag law for the magnitude of the stress where k_b =0.0025 is a drag coefficient and u_b is the current speed at the bottom boundary. The vertical extent of the influence friction is controlled by the eddy viscosity N_z which is an adjustable parameter.

Procedure

1) Open MATLAB and select "OCEANDYN2" as your current folder. Set Path to this folder and include all subfolders.

2) In the command window, type: OCEANDYN then press enter

3) From the main menu select "Basic Dynamics", then choose "Inertial Oscillations" from the sub-menu

4) For the first run, use the default option for the selectable parameters; the program pauses after 3 and 6 hours to show the progressive deflection of the current (press enter to continue)

You will see in fig.1 the slowly decreasing circular motion at the surface with sinusoidal components (shown in fig.2) and much weaker circular motion near the bed (fig.1 right panel).

5) Now try experimenting with different values of latitude from tropics (+/- 10°) to the poles (+/- 90°). For the moment, keep a fixed N_z = 0.01 m ^{2}s^{-1} and V_0=0.5 observe how the period of oscillations ("the inertial period") and the size of the inertial circle changes with latitude. Note the change in the sense of rotation between northern and southern hemispheres.

6) Next, try varying N_z to see how it changes the velocity profile and controls the rate of loss of kinetic energy from the flow.

Q1: At which latitudes would you expect resonant forcing of inertial oscillations by (a) diurnal (sea breeze) winds which have a period of 24 hours and (b) the M_2 tidal constituent which has a period of 12.42 hours ? *(Hint: Resonance occurs when the forcing frequency coincides with the natural period of oscillations)*

PART II : Wind-driven flow (Ekman3)

This is the classic problem of an unbounded ocean forced only by a steady windstress which you will find in Section 3.4 of the book. The effects of the surface stress are communicated down through the water column by friction between horizontal layers. There are no pressure gradients due to surface slopes or density changes so the steady state has to be a balance between the frictional stresses and the Coriolis forces. The motion, however, starts from rest at t = 0 when the wind is switched on, so that, initially, there are strong transient motions which decay as the steady state velocity profile (the Ekman Spiral) emerges. You will see in the contrasting runs below that it is the effect of the earth's rotation which limits the penetration of the boundary stress into the interior and the net transport forced by the wind.

1) Start by considering the non-rotating case in which a steady wind stress is applied at the surface in the x direction. Decline default values in Ekman2 so that you can set latitude=0° and hence f=0. Select the following values for parameters: h=200m, dz=1, windstress T=0.1 Pa, direction =0°, latitude = 0°, N_z = 0.05 m^{2}s^{-1},run time fin = 200 hours. Note the direction of the flow, the acceleration of the current and its increasing penetration in depth until it reaches the seabed.

*Q2: How long does it take for the stress to reach the bottom ?

What will the steady state be like ?

Change the parameter values to obtain a steady state in a run time of 300 hours. * (*Hint: Vertical mixing time for momentum is ~h^2/N_z*)

2) Now examine the response when rotation is involved. Choose a steady windstress of 0.2 Pa at latitude 33°N with N_z = 0.005 m^2s^{-1} which is the default case.

Note the limited depth penetration of the effects of the surface stress indicated by the exponential decay of the current speed with depth. In the bottom right hand panel you see the projection of the tips of the velocity vectors into the horizontal plane, forming a pattern which evolves into an increasingly steady Ekman spiral. Notice that the velocity vector rotates to the right with increasing depth (for the northern hemisphere).

*Q3: - What is the nature of the transient motion ?

- What is the direction of the net transport ?

- Compare the transport with the Ekman estimates: $Qx = 0$ and $Qy = \tau_x/f\rho$

where τ_x is the applied stress. What are the units of the transport ? *

3) Try varying the latitude and the eddy viscosity independently and then together.

*Q4: -How does the velocity profile vary with latitude ? (other parameters fixed)

- If both f and N_z are doubled at the same time, how does the solution change:

in amplitude ? in shape ?

- Compare the thickness of the boundary layer (BL) you estimate from the velocity profiles generated by the model with the depth of frictional influence $D=\pi(2N_z/f)^{0.5}$ given by Ekman theory ?*

Part III: **Tidal Dynamics without rotation (Tidyn)**

Next we consider the formation of another boundary layer in the response to tidal forcing by an imposed periodic surface slope in the presence of friction at the bottom boundary. We start with the non-rotating case in which the latitude is set to zero so that there is no Coriolis force and we can examine the development of a unidirectional, oscillating boundary layer in which the boundary layer thickness is limited by the tidal frequency. The graphics display includes the stress profile so that we can see the extent of frictional influence directly. At the end of the run the amplitude and phase of the current at each depth is computed and displayed in separate figure.

1) Run Tidyn2 from the sub-menu with the default values with the eddy viscosity N_z set to 0.01 $m^2\ s^{-1}$. Note the stress profile (green curve in the top RH plot) decreasing with height above the bed to zero at the surface. In figure 2 you will see a phase *lead* in the current increasing towards the boundary and a weak maximum in the current at ~30mab.

*Q5: - What is the amplitude of the oscillation at the surface? at the bottom ?
- Compare the surface value with the undamped (frictionless) response to forcing by the surface slope term.

- What is the thickness of the region of boundary influence on the stress and velocity profiles ? Compare with the theoretical value of

$\delta = (2N_z/\omega)^{0.5}$ for decline by a factor of 1/e.*

2) Investigate the influence of N_z on the velocity profile for values of N_z=0.0002 -0.1 $m^2 s^{-1}$.

*Q6: - For what values of N_z is there a maximum in the velocity amplitude ?

- Explain how the response can exceed the undamped oscillation ?

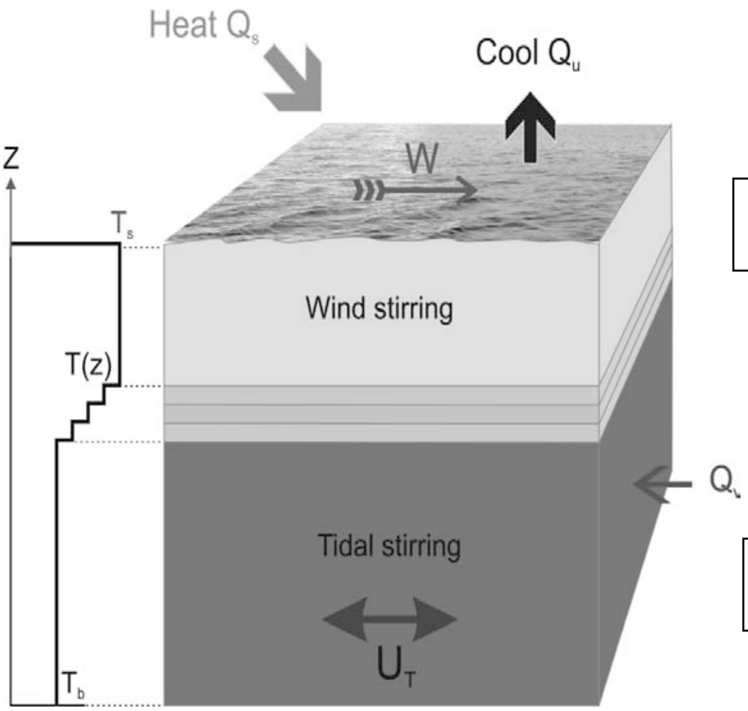
*(Hint: examine the stress profile for change in sign of the stress gradient which determines the frictional force at each level)**

Summary points to remember and think about

1) Inertial oscillations (IOs) are circular motions at the natural period of horizontal motion which is set by the Earth's rotation.
2) IOs involve clockwise/anticlockwise circular motion in the northern/southern hemisphere; both the radius of the motion and its period decrease with latitude.
3) An unbounded ocean responds to the onset of a steady wind with a transient response involving inertial oscillations which decay to leave a steady spiral velocity profile. which has a limited downward extent from the surface.
4) The downward penetration of the motion is limited to a surface boundary layer (the Ekman layer) whose thickness is controlled by the Coriolis parameter f and the eddy viscosity N_z .
5) An analogous bottom boundary layer develops in unidirectional tidal flow with the tidal frequency ω_2 replacing f in the control of boundary layer thickness, i.e. the time for which the boundary can grow is now limited to the tidal period rather than the inertial period.
6) A surprising feature of the tidal motion is that, in some cases, it involves a maximum velocity amplitude which exceeds the frictionless response.
7) A question to ponder is: Why doesn't a steady boundary layer develop in the damped inertial oscillations of Part I ?

If you wish, you can make a copy of the Matlab code and modify it to tackle more involved the problems like that of an oscillating boundary layer with rotation which combines tidal and Ekman dynamics.

G2: THE HEATING-STIRRING COMPETITION (Chapter 6)



$P_W = k_s \rho_a W^3$

$P_T = k_b \rho U_T^3$

Introduction

This programme is designed to let the student explore the mechanisms responsible for the seasonal cycle of stratification in the shelf seas. The ocean is considered as a two layer system with local vertical mixing as the dominant exchange process driven by stirring due to tidal flow and windstress which act to produce separate bottom and surface mixed layers in the manner described in Chapter 6. The model assumes that horizontal heat transfer due to the mean flow is negligible and that there is no heat exchange with the seabed. The available stirring power of the tide and the wind are proportional to the cube of the tidal current and wind speed respectively; a small fraction of this power is used in working against the buoyancy forces and bringing about vertical mixing. The surface heating and cooling is described by an air-sea interaction formulation which uses long term average cycles of surface winds, solar insolation and dew point temperature to determine surface fluxes.

The exercise allows you to vary the stirring variables (wind and tide) to simulate the annual cycle for a full range of conditions on the shelf. You should do this in a systematic way, based on the suggested sequence of runs, to educate your own intuitions about the way the system works. This should enable you to appreciate the basics of the interaction between the heating/cooling and stirring processes and also understand more subtle features like the role of tidal stirring in controlling heat storage in the shelf seas. When you understand the operation of the program, you should proceed to a sequence of model runs which will enable you to construct a cross-frontal section in Part III.

As you work through the exercises below, try to answer the questions identified in the text (*Q*).

PART I (the seasonal cycle: HS3)

1) Read through the account of the model in Section 6.2.1 of the book and make sure you understand the essence of the TML model

2) Open MATLAB and select “OCEANDYN2” as your current folder.

3) In the command window, type: OCEANDYN then press enter

4) From the main menu select “HEATING vs STIRRING ”

5) From sub-menu select “Seasonal Cycle HS3”

6) For the first run (A), accept default values for selectable parameters and note scales on plot; press enter; default values are: h=90m; dz=1m; ki=χ=1500 $m^{-2}s^{3}$; WF=1; fin=730days;

(note that SH=$\log_{10}\chi$)

7) RUN A

Observe the development starting from an initially mixed condition **at January 1st** . Watch the evolution of the temperature profile (top left panel) over the seasonal cycle. The corresponding surface and bottom temperature plots (bottom left panel) illustrate the development and decay of stratification which is also depicted as Φ (blue, bottom right) with a peak value of ~ 150 Jm^{-3} . Note also in the bottom right panel the cycle of the total heat stored in the water column H_T (green plot in units of 10^8 Jm^{-2}). The rate of heat transfer across air-sea interface (top right panel) shows a marked response to the onset and breakdown of stratification. At the end of the run, press enter to display the amplitude (in units of 10^8 Jm^{-3}) and phase (in days from Jan 1st) of the heat storage cycle. Then answer yes to "another run?"

Q1: Determine the approximate times and values of the maximum bottom and surface temperatures; explain why they differ so greatly.

8) RUN B This time, decline default values and increase the level of tidal stirring by setting ki=50, but keep all other values as in run A. Note the absence of stratification and increased heat storage and heat exchange.

* Q2: Explain differences in heat storage and surface heat exchange between runs A & B *
(repeat runs to check details)

Part II (exploring the parameter ranges)

You should now feel free to change the input parameters to HS3 at will to answer your own "what would happen if ?" questions. Here are a few suggested avenues of exploration together with some limitations:

9) RUN C. How long does the system take to adjust ? Try varying the initial temperature e.g. 5° C (too low) and 11°C (too high)) and see how long the system takes to settle to a stable cycle. You might want to increase the run time to 3 years (1095 days) or more.

*Q3: Explain why lower winter temperatures result in stronger summer stratification and vice versa *.

10) RUN D What is the effect of wind stirring ?

Try progressively reducing the wind factor from WF=1 (climatic average winds) to WF=0.02 (but not zero). Then try increased winds (WF=2). Note the changes in the surface mixed layer and the affect of low winds on adjustment times. You might like to progressively remove tidal stirring as well by putting in a large values of χ (say up to 10,000)

* Q4: Can you explain how greatly decreased wind stress may result in a *reduction* of the maximum stratification ?*

11)RUN E What happens as the water column gets shallower ?

Compare these four cases with χ=200 $m^{-2}s^{3}$

(i) h=100m (ii) h=60m (iii) h=30m (iv) h=10m

* Q5: Explain the differences in stratification and the change in phase of the heat storage?*

Part III (Constructing a frontal section: TMF1)

12) You might now like to try assembling the results from a series of runs at different values of χ to produce a section across a front. To set up the data base for this you will need to make a series of runs using the front generation program TMF1 in the Heating-Stirring sub-menu.

Run TMF1 and respond to the prompts for parameter values:

i) Depth h = ? (50-120m)
ii) bin size dz = ? (1 or 2m); program runs faster with larger bin size.
iii) Initial value of χ ? (choose in range 25-75, 50 is a good starting value). χ value will be doubled for each successive run.
iv) NR = number of runs you require ? To make a convincing frontal section, you will need at least 5 runs which should range from mixed (χ< 100) to fully stratified (χ>1500). If you have time, choose ~10 runs and start with a low value

of χ. For example if you choose NR=8 with an initial χ=25, you will get runs at χ=25,50,100,200,400,800,1600,3200 $m^{-2}s^3$.

v) dn =day number (in days from Jan.1st of second year) at which the profile is sampled ?
This sampling time is shown as a vertical line in each run plot. Choose dn in the range 120-300; maximum stratification is typically at dn~200.

13) Once you have completed the full sequence of runs, press enter to assemble all the plots and compare them. Press enter again to assemble the frontal section which shows the temperature contoured in the vertical plane with logχ as the x coordinate. Compare with the observed temperature section across the Tidal Mixing Front in the western Irish Sea shown below.

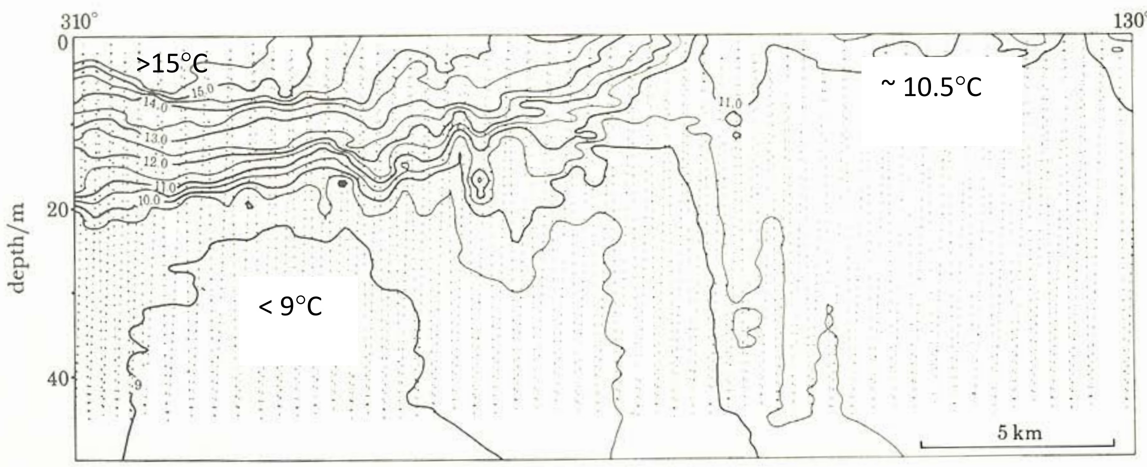


FIGURE 4. High resolution temperature section of the western Irish Sea front. Dots represent data points from the undulating CTD system which described a vertical cycle in a horizontal distance of *ca.* 500 m. Salinity variations measured at the same time were small, not exceeding ± 0.1 ‰.

* Q7: Try and explain these features of these frontal sections:

(i) The intensified horizontal gradients at surface and bottom in the front

(ii) The upward slope of the pycnocline as the front is approached from the stratified side*

15) Feel free to try out other thought experiments and let us know if you find any interesting results or bugs in this developing package.

G3: DENSITY-DRIVEN FLOW (Chapter 9)

This group of exercises is based on the theory discussed in Sections 9.1-9.3 of the book. The aim is to help the student to further understand the circulation in estuaries and ROFIs[1] forced by horizontal density gradients which result from the inflow of freshwater from the land.

As you work through the exercises below, try to answer the questions identified in the text (*Q*).

Part I: Estuarine Circulation (without rotation)

In this first simulation, we examine the flow that results from a depth-uniform density gradient along the axis of an estuary starting from rest ( ). The pressure gradient component due to the density gradient is opposed by a surface slope which can be adjusted by the parameter GM so that the depth integrated flow is close to zero or set to match the rate of river run off. As well as the developing velocity profile, you will see the evolution of stable stratification of the water column.

Procedure

1) **Select "Density-driven Flow" from the main menu; then choose "Estuarine Circulation"**

2) Run with default options depth h=40; kgm^{-4}; $N_z = 0.02$ m^{2}s^{-1}.
The surface slope adjustment parameter is found from theory GM=0.441.
At pause, note scales in the four panel display before continuing.
You will then see the development of a steady state solution with inflow in the lower layer and outflow from the estuary in the upper half of the water column. This shear flow moves lighter, fresher water over heavier more saline water creating significant stratification shown in the bottom left panel.

3) Press return for contour summary of velocity. *Q1 Estimate the time for a steady state to develop and compare with the time scale for the vertical mixing of momentum *.

4) Press return to compare model profile with the Hansen-Rattray analytical theory (NB Theory here is based on quadratic drag law for bottom friction corresponding to the model; derivation in the book (Section 9.1.1) assumes a no slip (u=0) condition at the bottom and so gives slightly different results).

5) Now experiment with different values of the parameters: e.g. try decreasing the depth by a factor of 2. Note the greatly reduced circulation ($\propto h^3$); then increase by a factor of 2^3 to compensate. *Q2 How do the circulation and stratification now compare with the default case ? What is the change in the time required to achieve a steady state ?*

[1] ROFI = Region Of Freshwater Influence

Part II Circulation with rotation in a ROFI (with rotation)

The problem of the circulation in a ROFI is considerably more difficult than the estuary case because the Earth's rotation has a strong influence on the flow and so we have to include the Coriolis forces in the equations of motion. Nevertheless, an analytical solution for the steady state in a an idealised ROFI, corresponding to the Hansen-Rattray theory, was obtained by Norman Heaps (see book Section 9.2). The solution gives the form of the steady state flow induced by a constant horizontal gradient of density in the presence of bottom friction which in order to make the maths simpler, is specified in terms of a linear drag law for the bottom stress: . As well as determining the form of the velocity profile, the Heaps solution also specifies a surface slope adjustment parameter, equivalent to GM in the estuarine circulation, which controls the net flow normal to the coast.

We shall first examine the Heaps solution and then look at the time dependent problem as the ROFI system adjusts to a depth-uniform density gradient starting from rest.

Procedure

1) **Select "Density-driven Flow" from the main menu;**
   then choose "Heaps ROFI" and run with default values

   h=40m; Nz=0.02m^{2}s^{-1}; lat=55°N; = kgm^{-4}.

2) Note the GM value is larger than for the estuary case reflecting the influence of rotation.

3) Press enter to see the steady velocity profiles. Note an estuarine type exchange flow in the cross-shore direction and a unidirectional flow along the coast increasing from *v*=0 at the bed to a maximum at the surface

4) Now, investigate the effects of changing the parameters. E.G. (a) try the effect of reducing the N_z to lower the influence of friction. *Q3 Compare the alongshore surface current with a geostrophically balanced flow as $N_z \rightarrow 0$ and (b) try going close to the equator (lat =1°N but not zero) where the flow should be predominantly estuarine in character*.

Now we turn to a numerical simulation of the time dependent problem in which the flow is developed from rest at *t*=0. The forces in the equations of motion are the same as in the Heaps problem except that the bottom boundary condition is now the quadratic drag law

5) **Select "Density-driven Flow" from the main menu;**
   then choose "ROFI Circulation"

6) Run ROFI2 with default values to observe the adjustment to a steady state.
   Q4 Identify the transient oscillatory motions which decay as the system settles down. What is their period ? and how are they losing energy ?

7) Now try other parameter values. Generally you will not know the appropriate value of GM but you can find a good first approximation by running the HeapsTheory programme with the parameter set you intend to use. (Some iterative adjustment of this initial value of GM may be necessary to obtain an exact zero cross-shore transport)

8) Then run ROFI2 for a few representative cases. For example, you might look at:
   (a) the low latitude case where rotation has little influence;*Q5 how do the results compare to estuarine flow ?*
   (b) the near frictionless case (low N_z) in which the flow should be nearly geostrophic; *Q6 compare to purely geostrophic flow ?*

9) *Q7 Finally choose one example to compare the steady state profiles generated by the ROFI2 model with the steady state flow from the Heaps solution. If your Matlab skills are up to it, you should be able to include both sets of profiles in the same plot*. (In comparing the theory and the model remember that they use different boundary conditions at the bottom. Heaps analytical theory uses a linear drag law to make the maths tractable whereas the numerical model has the, rather more realistic, quadratic drag law.)

G4: **TURBULENCE CLOSURE MODEL (Chapter 7)**

Introduction

This guide aims to introduce you to the 1-d TC model, discussed in Section 7.1 of the book,

The operation of the model is illustrated by extensive run-time graphics which show you the evolution of the density and velocity fields in response to wind and tide forcing. Two rather different applications of the model are presented. In the first, the guide takes you through the detailed model hindcasting of the dynamics during a period when forcing data is available from observations. In the second application, climatic average data for surface forcing is used in a simulation of the seasonal cycle in shelf seas which provides a much fuller picture of the cycle and the processes involved than the TML model which we looked at in Chapter 6 and in the Heating vs Stirring exercise.

This type of 1-d model is proving valuable in understanding the physical processes in shelf seas and similar models are forming the basis of Biophysical models like that used in the book.

Procedure

Before running the software, study the account of the model (Section 7.1) and the concept of turbulence closure (Section 4.4.4) given in the book.

1) From the main menu, select "TC MODEL" and then "Run with real wind data"
2) For the first run, accept the defaults which correspond to observational data from a strongly stratified site in the Celtic Sea in 2003. Press return successively to see:
   i) the initial density profile showing strong stratification.
   ii) the length scale used in the turbulence closure scheme
   iii) a plot of the wind speed and direction for the 12 day observation period
3) Select a run time of 150 hours (maximum is 288 hours)
   You will now see a full screen display showing the following parameters to be displayed during the run:
   i) u and v velocity profiles in top two panels
   ii) surface velocity v versus u (second row, left)
   iii) eddy viscosity N_z profile (second row, right)
   iv) density profile $\rho(z)$ (3rd row left)
   v) dissipation $\varepsilon(\mathrm{Wm^{-3}})$ (3rd row right)
   vi) Wind speed and direction versus time (bottom panels)
4) Press return to start the run. Observe the behaviour of parameters, especially ε and N_z over the tidal cycle and in response to the stronger wind forcing which occurs on days 215-216.
5) At the end of the run, a contoured plot of dissipation will appear. Set to full screen and then use the zoom facility to select a period of ~2 days to expand. Note the regular M_4 cycle of dissipation in the bottom layer with dissipation decreasing rapidly from large values at the bed with a lag in the time of maximum dissipation increasing with height above the bed. This lag is not the result of the diffusion of turbulence from the boundary but is mainly due to an increasing delay in the production of turbulence with height above the bed. (Remember that production $\approx$dissipation for local equilibrium). Notice the low level of dissipation($< 1\text{e-}6\mathrm{Wm^{-3}}$) in the pycnocline.

6) The programme stores data for profiles of u and v velocity, density and dissipation stored as Ust, Vst, rhost, Epst respectively. From these files, you can plot the time evolution of each parameter using the Matlab contour routine. You can also derive the velocity shear (and the stability frequency using the diff operator and then contour these parameters on a time –depth plot. Try this, commencing 2 days after the start of the run, to allow for spin up of the motion. Useg the Matlab routine **pcolor** followed by **shading interp**. You can go one stage further and obtain a contour plot of the Richardson number , from which you can deduce where mixing could be occurring (*Ri* <0.25). The high Ri values in the pycnocline are consistent with the low levels of dissipation noted above.
7) This short run did not include surface heat exchange so you can see how much mixing was done by the wind and tide over the period . Use the function fi to compute the potential energy anomaly Φ from the density profiles before and after the run.

   [fi]=PHIrho(n,dz,Rho); (Rho is the density profile at n= levels with depth intervals dz) . Density profile at the start of the run is “rhostart” and after is “rho2”
8) You can run with data from other sources providing you put the files in the formats specified in the TC22aDem programme comments. You will need:

   i) the initial density profile
   ii) wind speed and direction time series
   iii) tidal ellipse information: ellipticity, orientation, major axis, difference in phase between M_2 and S_2 at the start of run

To make longer runs over all or part of the seasonal cycle, you can run a version of the TC model which is forced by average climate data derived from fits to observed seasonal cycle of the meteorological parameters for a mid latitude station in the western Irish Sea (lat=54°N). The runs start from a vertically uniform profile corresponding to conditions in winter. The cpu time required for runs extending over the seasonal cycle are rather long (depending on your computer) so it is generally advisable to start with relatively short runs covering the period of the onset of stratification.

9) Select “Run with seasonal Met data” from the sub-menu and accept default values. Choose a run time of 20 days or less and a start date before the vernal equinox (day 80) when the water column is vertically uniform.
10) Lower plot shows development of stratification as surface and bottom temperatures diverge. Notice the drop in mid-water dissipation and N_z as stratification becomes established At the end of the run press enter for a summary of the temperature profiles, in which you will see a rapid switch to a a run away stratification. Next you can make a plot of dissipation to see how the top and bottom layers become separated by a region in which turbulence is effectively shut down.
11) Finally try a long run for one or two seasonal cycles (e.g.365 or 730 days duration; start day 70) the time required depends on the speed of your computer but will be of the order of an hour or more.

Note the limited impact of springs neaps stirring cycle
Compare the temperature seasonal cycle summary plot with the equivalent result from the simpler representation of the heating-stirring competition in the TML model (G2).

G5: **LEAST SQUARES TIDAL ANALYSIS (Chapter 2)**

Introduction

This exercise is designed to introduce you to the basic ideas of the least squares fitting procedures which are used in tidal analysis and other branches of geophysical data analysis. The particular aim here is to show you the essential steps in the analysis of tidal data to determine the harmonic constants which are used to characterise and predict the tides as explained in section 2.5.1 of the book.

The problem is to determine the constants H_n and κ_n in the harmonic expansion:

$$\eta(t) = a_0 + \sum_{1}^{N} H_n \cos(\omega_n t + \alpha_n + \kappa_n)$$

from a set of observations of $\eta(t)$. Ideally the data would cover a time span of a year or more but frequently we have to work with shorter data sets, sometimes of only a few days duration and perhaps containing significant gaps. In these circumstances, least squares analysis offers a significantly better solution than simple Fourier analysis.

Theory

We compare the observed tide $\eta(t)$ with the "predictor":

$$\hat{\eta}(t) = a_0 + \sum_{1}^{N} \left(a_n \sin \omega_n t + b_n \cos \omega_n t\right) = a_0 + \sum_{1}^{N} H_n \cos(\omega_n t + \alpha_n + \kappa_n)\ .$$

via the sum of squares of differences for the m observations :

$$\boxed{S^2 = \sum_{1}^{m} (\eta_i - \hat{\eta}_i)^2}$$

The desired set of a_n and b_n are those which MINIMISE the value of S^2. At the required minimum, the derivatives of S^2 with respect to each a and b will be zero, i.e.

$$\frac{\partial S^2}{\partial a_0} = 0$$

$$\frac{\partial S^2}{\partial a_1} = \frac{\partial S^2}{\partial b_1} = 0$$

$$\frac{\partial S^2}{\partial a_2} = \frac{\partial S^2}{\partial b_2} = 0$$

....

....

and so on for 2N+1 equations for 2N+1 unknowns

Implementation for M_2

We start with the simplest case where we are fitting only one constituent

in the harmonic expansion:

$$\hat{\eta}(t) = a_0 + a_1 \sin \omega t + b_1 \cos \omega t$$

to a set of observations of η(t) where ω=0.50587 rad/hour. In this case, the sum of squares is just:

$$S^2 = \sum_1^m (\eta_i - a_0 - a_1 \sin \omega t_i - b_1 \cos \omega t_i)^2$$

If we differentiate this expression w.r.t. a_0,a_1 and b_1, and set the result equal to zero,

we get 3 equations which can be written in the matrix form (with $\sum \ = \sum_1^m$)

$$\begin{pmatrix} m & \sum \sin \omega t & \sum \cos \omega t \\ \sum \sin \omega t & \sum \sin^2 \omega t & \sum \sin \omega t \cos \omega t \\ \sum \cos \omega t & \sum \sin \omega t \cos \omega t & \sum \cos^2 \omega t \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} \sum \eta_i \\ \sum \eta_i \sin \omega t \\ \sum \eta_i \cos \omega t \end{pmatrix}$$

M x **A** = **N**

which can be solved for A by standard inversion procedures.

Note that for a long record

$$\sum_1^m \sin \omega t = \sum_1^m \cos \omega t = \sum_1^m \sin \omega t \cos \omega t = 0$$

$$\sum_1^m \sin^2 \omega t = \sum_1^m \cos^2 \omega t = \frac{m}{2}$$

so that only the diagonal terms are non-zero and that least squares is then equivalent to direct Fourier analysis.

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Part I : Fitting a single constituent

1. Make sure you understand the basics outlined above.

2. Use the short data set given below in Matlab or in a spreadsheet to construct the 3 x 3 matrix **M** and the vector **N**

3. Invert the matrix to find the M^{-1} either manually or by writing your own programme or using Matlab.

4. Determine a_0, a_1, b_1 and hence the amplitude and phase of the M_2 constituent.

5. Plot out the data and your fitted curve. Determine the root mean square deviation from your fitted curve.

6. Estimate the value of η at times t=3, 7, 8 hours

Menai Bridge tidal heights 27/4/88

Time (hours)	Height(m)
0000	2.17
0100	1.71
0200	1.56
0300	-
0400	2.36
0500	3.14
0600	3.90
0700	-
0800	-
0900	4.35
1000	3.81
1100	3.07
1200	2.30
1300	1.65

Part II : Setting up and Testing the Least Squares method

As you will now appreciate, composing the matrix and vectors is tedious using the above methods especially when we want to determine a large number of constituents. We can speed things up by constructing an intermediate matrix of the form :

Q =

$$\begin{array}{cl} \begin{pmatrix} 1 & \sin\omega_1 t_1 & \cos\omega_1 t_1 & \sin\omega_2 t_1 & \cos\omega_2 t_1 \\ 1 & \sin\omega_1 t_2 & \cos\omega_1 t_2 & \sin\omega_2 t_2 & \cos\omega_2 t_2 \\ 1 & \sin\omega_1 t_3 & \cos\omega_1 t_3 & \sin\omega_2 t_3 & \cos\omega_2 t_3 \\ 1 & \sin\omega_1 t_4 & \cos\omega_1 t_4 & \sin\omega_2 t_4 & \cos\omega_2 t_4 \\ 1 & \sin\omega_1 t_5 & \cos\omega_1 t_5 & \sin\omega_2 t_5 & \cos\omega_2 t_5 \\ \vdots & & & & \\ \vdots & & & & \\ 1 & \sin\omega_1 t_m & \cos\omega_1 t_m & \sin\omega_2 t_m & \cos\omega_2 t_m \end{pmatrix} \cdots & \rightarrow \text{ N constituents} \end{array}$$

$\downarrow$

m data points

and a column vector of the data:

$$y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{pmatrix}$$

The matrix M is then found simply by forming

$$\boxed{M = Q'Q}$$

where Q' is the transpose of Q . We also use Q to get N from:

$$\boxed{N = Q'y}$$

The solution is found as before, by using the inverse of matrix M in $MA = N$

to obtain

$$\boxed{A = M^{-1}N}$$

This inversion is readily performed using Matlab for a large data set as you can demonstrate for yourself using the Book Menu Software.

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From the MATLAB Menu, select "Tidal Analysis" and then the "Least Squares Demo" option in the sub-menu.

This programme will guide you through the following steps :

i) You construct an artificial time series for three tidal constituents for which you specify amplitude(in metres) and phase led (in radians). You also need to choose a tidal datum level from which the tidal height are measured and the length of the time series. (You can choose a wide range of parameter inputs within guide limits shown in square brackets)

ii) You can add some noise to make the time series more realistic and to check the influence of random noise on the results

iii) The time series is analysed by least squares to estimate the constituent amplitude and phases along with the height datum

iv) You are then invited to compare the analysis results with the inputs specified.

v) Make repeated runs to determine how the accuracy of the method varies as you change the length of the time series and increase the noise input.

Part III: Analysis of tidal elevations in Maputo Bay

Now that you have checked the validity of the Least Squares analysis, we shall look at the use of the method to analyse large sets of elevation and current meter data using the LSprac programme. The specimen set is a record of elevation in Maputo Bay, Mozambique of more than one month's duration in a file named **Inhacamonth.txt**. Data was recorded at 10 min intervals; there are 4520 data points in the series which consists of two columns representing elevation in metres and time in year days. You can use any tidal data series providing you present the data in this format.

Proceed as follows for elevation analysis:

Select "Practical Analysis of data" from the Tidal Analysis sub-menu in order to start the LSPrac programme.

Respond to requests for input. The data series Inhacamonth is loaded by the programme and called simply IM thereafter.

1. Compare prediction and analysis which are plotted together. Note rms deviation. Zoom in on the plots to see how good the fit is.
2. Now re-run and shorten the series to 10 days data (m =1440)
3. Consider the differences in the results and, in particular the reduction in s.d. of the fit. How much of the variance between data and prediction of the full length record is due to non-tidal, low frequency changes in sea level ?

Part IV: Current meter data

You can use the same approach to analyse tidal currents. The file Rhinevel.txt contains data for velocities at heights of 6m and 16m above the seabed from the ROFI on the Dutch coast. The data, which was recorded at 30min intervals, is arranged in the following form

Time(Hours) depth(m) U_{16} (m/s) V_{16} (m/s) U_6 (m/s) V_6 (m/s)

Proceed to analyse as follows:

1. Load the file Rhinevel.txt
2. Covert time column to time in days
3. Create separate filesfor each velocity component in LSPrac format
   i.e. 2 columns (Velocity, time)

4. Do the same for the depth data which includes a tidal signal
5. Run LSPrac for each of velocity components and depth. Deduce amplitude and phase of constituents for currents and elevation along with residuals.
6. Plot the residual currents and the tidal ellipses for M_2 at 6 and 16m above the bed.
7. Identify the sense of rotation of the current vector near the surface (16mab) and near the bed (6mab).

8. Compare the phase of the elevation and the current in order to determine the nature of the tidal regime here: is itmainly a progressive or standing wave ?

Further Applications

Once you are familiar with the least squares method, you can go on to develop the LSPrac code to undertake a wide range of tidal analyses of currents and sea surface elevation. More tidal constituents can be added and the phases related to a standard time origin. Alternatively, and with much less labour, you can utilise the excellent Least Squares harmonic analysis system known as T tide which is available on the internet:

http://www.eos.ubc.ca/~rich/t_tide/t_tide_v1.1.zip

R. Pawlowicz, B. Beardsley, and S. Lentz, "Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE", Computers and Geosciences 28 (2002), 929-937.

G6: **FICKIAN DIFFUSION DEMONSTRATIONS (Chapter 4)**

This set of demonstrations is designed to illustrate the process of diffusion in the simple idealised case of the dispersion of a passive tracer in a long narrow channel (Book Section 4.3.5). A mass of dye (a conservative, passive tracer[2]) is released into the channel at a location x=0 and at time t=0. The subsequent spreading is due to " Fickian diffusion", i.e. diffusion in which the diffusivity K is constant. The concentration of dye is assumed to be uniform across the channel section so that the concentration varies only in x and t. With no mean flow in the channel, the advection-diffusion equation reduces to:

$$\frac{\partial c}{\partial t} = K \frac{\partial^2 c}{\partial x^2}$$

The solution of this equation is the well-known Gaussian function :

$$c\left(x,t\right) = \frac{M}{\sqrt{4\pi K t}} e^{-\frac{x^2}{4Kt}}$$

which you may already have met as the "normal distribution" in statistics with standard deviation $\sigma=\sqrt{(2Kt)}$.

The following demonstrations show how, after release, the dye spreads as a "Gaussian patch", how such patches merge to form larger Gaussian patches and how this type of diffusion can be simulated by an equivalent random walk procedure. You will also see in the final demonstration how Fickian dispersion combines with advection in the mean flow.

Procedure

1) From the main menu select " **FICKIAN DIFFUSION**" , then choose "**1-d dispersion in a channe**l"
2) Accept the default value of K for first run. The programme pauses to illustrate the dye distribution after 1 hour. With the default K = 139 m^2s^{-1}, the rms deviation of the dye from the origin is 1.00 km so the x axis can also be interpreted as the number of standard deviations from x=0. For the normal distribution, 90% of the area under the curve lies within $\pm 2\sigma$ of the origin which for our diffusion scenario means that 90% of the dye is to be found in this range of x.
   Now press enter to see how the distribution spread in time at hourly intervals.
   Re-run with large (300 m^2s^{-1}) and small (5m^2s^{-1})values of K
3) Return to the sub-menu and select "**Merging of two Gaussian patches**" and run with K=40 m^2s^{-1}. As the two patches diffuse, you will see that they merge to form a single large patch which eventually becomes indistinguishable from a single patch spreading from a single dye release. Solutions for more complicated inputs of dye inputs can be found in this way by combining the results of a series of single "dumps". For example, a continuous spatial pattern of inputs along the channel at a specified time can be simulated by a number of discrete inputs along the x axis. Similarly time-dependent inputs can be represented by a number of discrete inputs at different times. It is also possible to extend this approach of combining discrete inputs to simulate Fickian dispersion in 2 or 3 dimensions.

4) In Section 4.3.5 we demonstrated how 1-d Fickian spreading of scalar properties can be implemented by a random walk procedure. In this approach, the dye is represented by a large number of dye particles which are moved a small distance dx at

[2] a passive tracer is one which does not influence the motion but simply acts as a marker

intervals of dt with the direction of movement (positive or negative) being decided by the toss of a coin. To see an experimental test of such random movement, select : "**Dispersion by RandomWalk**" from the sub-menu and run the programme accepting the suggested default values for the number of particles and the length of the run. The time and space steps are non-dimensional variables set to dt=1; dx=1; so that the equivalent Fickian diffusivity $K = (\mathrm{d}x)^2/(2\mathrm{d}t) = ½$. At the end of the run, press enter to compare the Random walk result and its standard deviation σ with the equivalent Gaussian results.

5) See next how advection can be combined with Fickian diffusion. Select: "**Diffusion + Advection**" Which simulates 1-d advection and diffusion of a series of dye patches injected at the origin every 20 hours. Choose values of K and the advective velocity U in the suggested ranges.
The results will demonstrate that, in coordinates moving with the flow each dye patch spreads as in the stationary solution for U=0, which is what you might have anticipated for this simple case.

Generally diffusion in the ocean is not Fickian, i.e. the effective values of K increases with the length scale of the motion as discussed in the book Section 4.3.6. Nevertheless Fickian diffusion serves as a very useful idealised model and the superposition of solutions to small discrete inputs allows the construction of models of more complicated scenarios with inputs varying with time and spatial location. Moreover, in the shelf seas, diffusion is often dominated by Tidal Shear Dispersion a process which is essentially Fickian.

6) To see demonstrations of how Shear Dispersion works, return to the main menu and select "**Steady Shear Dispersion**". In this simulation, a large number (150) of dyed water particles are tracked as they move in response to a steady horizontal sheared flow combined with vertical diffusion which is represented by random walk, upward or downward, displacements of the particles. The water particles are released at x=0 into water of depth h=30m with a uniform distribution in the vertical. The steady horizontal flow is in the x direction with a uniform velocity gradient from surface to bottom and zero depth-mean. The random walk has a step length adjusted to match the selected vertical diffusivity.

7) Select the suggested run time, the vertical diffusivity K_z and the surface to bottom velocity difference U_0 ; press enter to begin run. You will see the development, from the initial vertical distribution of a dispersing cloud of particles whose variance increases in time(lower plot). Compare the shear diffusion K, determined from the average rate of increase of the variance of the particle cloud, with the theoretical value . Account for any significant difference ? (*Hint: Check the Vertical mixing time ?*)

8) Now try experimenting with higher and lower values of K_z while keeping U_0 fixed. You should see a, perhaps surprising, inverse relation between the shear diffusion K and K_z. Try and explain this relation in terms of the development of the cloud.

Frequently the largest velocity shears in the shelf seas are those due to oscillatory motion of the tide so that we need to consider the dispersion which occurs in such periodic flow. While the theory of Tidal Shear Dispersion (TSD) is complicated, the particle dispersion model can be readily adapted to simulate the process and estimate the resulting diffusivity.

9) Select **"Tidal Shear Dispersion"** from the sub menu. Run with suggested values. You will again see the development of a diffusing cloud in which, after some time has elapsed, it is difficult to identify the tidal shear in the motion.

10) Make a series of runs in which you vary K_z to make the mixing time ( ) vary over the range $T_2/4$ to $4T_2$ where T_2 is the tidal period. Plot the resulting K values versus K_z to identify the value of T_m which gives the maximum K. Can you offer an explanation of this maximum from your observation of the development of the particle cloud ?

TSD is a major contributor to horizontal mixing in shelf seas and dominates in many areas. Estimating realistic values of the shear dispersion K requires knowledge of the vertical eddy diffusivity. When this is not available, the maximum in K, which occurs for an optimum T_m, sets a useful upper bound to the contribution of this form of mixing.