Chapter 5

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
> with(plots):
Warning, the name changecoords has been redefined
> with(DEtools):
Warning, the name adjoint has been redefined
```

Question 1

The two equations are:

$$x(t+1) = -8 - x(t) + y(t)$$
$$y(t+1) = 4 - .3\,x(t) + .9\,y(t)$$

with difference equations:

$$\Delta\, x(t) = -8 - 2\,x(t) + y(t)$$
$$\Delta\, y(t) = 4 - .3\,x(t) - .1\,y(t)$$

```
> solve({-8-2*xstar+ystar=0,4-0.3*xstar-0.1*ystar},{xstar,ystar
});
```

$$\{xstar = 6.400000000,\ ystar = 20.80000000\}$$

```
> A:=matrix([[-1, 1], [-0.3, 0.9]]);
```

$$A := \begin{bmatrix} -1 & 1 \\ -.3 & .9 \end{bmatrix}$$

```
> eigenvalues(A);
```

$$-.8262087348,\ .7262087348$$

```
> eigenvectors(A);
```

$$[-.826208735, 1, \{[-.9852320015, -.1712247161]\}],$$
$$[.7262087348, 1, \{[-.6538121239, -1.128616198]\}]$$

It will be noted that although the eigenvalues are those given in the text, it appears that the eigenvectors are different. But this is not the case. Consider the eigenvector v^r associated with the eigenvalue $r = 0.7262087348$. This is given in the programme by,

$$v^r = \begin{bmatrix} -.6538121239 \\ -1.128616198 \end{bmatrix}$$

But in the text we arbitrarily set v_2
by -1.128616198

```
> -.6538121239/(-1.128616198);
```

$$.5793042179$$

then the eigenvector is,

$$v^r = \begin{bmatrix} .579304 \\ 1 \end{bmatrix}$$

which is the eigenvector derived in the text.

```
> V:=transpose(matrix([[-.9852320015, -.1712247161],
```

```
[-.6538121239, -1.128616198]]));
```

$$V := \begin{bmatrix} -.9852320015 & -.6538121239 \\ -.1712247161 & -1.128616198 \end{bmatrix}$$

```
> V_1:=inverse(V);
```

$$V\_1 := \begin{bmatrix} -1.128616197 & .6538121232 \\ .1712247159 & -.9852320005 \end{bmatrix}$$

```
> u0:=matrix([[-4.4], [-12.8]]);
```

$$u0 := \begin{bmatrix} -4.4 \\ -12.8 \end{bmatrix}$$

```
> DD:=matrix([[(-.8262087348)^t, 0], [0, (.7262087348)^t]]);
```

$$DD := \begin{bmatrix} (-.8262087348)^t & 0 \\ 0 & .7262087348^t \end{bmatrix}$$

```
> u:=multiply(V,DD,V_1,u0);
```

$$u := \begin{bmatrix} 3.352630124\,(-.8262087348)^t - 7.752630124\ .7262087348^t \\ .5826578322\,(-.8262087348)^t - 13.38265783\ .7262087348^t \end{bmatrix}$$

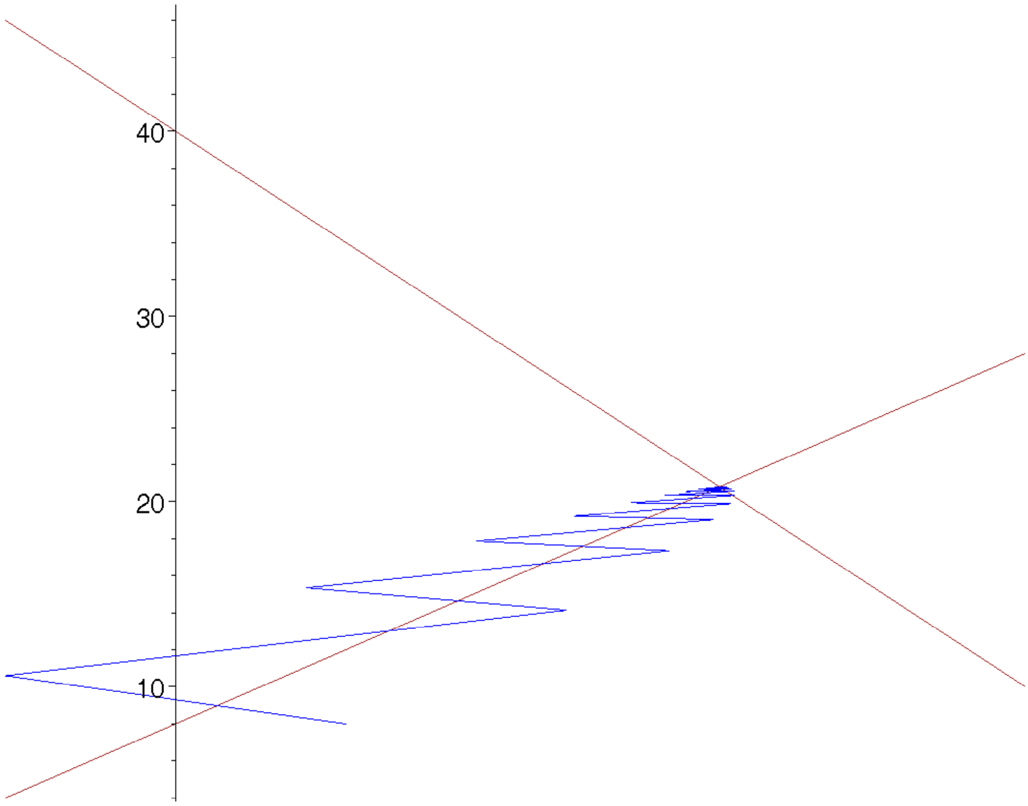
```
> xt:=6.4+3.352630124*(-.8262087348)^t-7.752630124*.7262087348^
  t;
```

$$xt := 6.4 + 3.352630124\,(-.8262087348)^t - 7.752630124\ .7262087348^t$$

```
> yt:=20.8+.5826578322*(-.8262087348)^t-13.38265783*.7262087348
  ^t;
```

$$yt := 20.8 + .5826578322\,(-.8262087348)^t - 13.38265783\ .7262087348^t$$

```
> points:=[seq([xt,yt],t=0..20)]:
> trajectory:=plot(points,colour=blue):
> phaselines:=plot({8+2*x,40-3*x},x=-2..10, colour=brown):
> display({trajectory,phaselines});
```



This figure clearly shows the trajectory oscillating across the phase line, but converging on the fixed point, which confirms the stability established in Section 5.3.

Question 2

Example 5.10 is given by the following equations

$x(t+1) = -.85078\ x(t) - y(t)$

$y(t+1) = x(t) + 2.35078\ y(t)$

which has a fixed point at the origin. Subtracting $x(t)$ from both sides of the first equation and $y(t)$ from both sides of the second equation we obtain,

$\Delta\ x(t) = -x(t+1) - x(t) = -1.85078\ x(t) - y(t)$

$\Delta\ y(t) = y(t+1) - y(t) = x(t) + 1.35078\ y(t)$

The two phase lines are found by equating $\Delta\ x(t) = 0$ and $\Delta\ y(t) = 0$ respectively, i.e.,

```
> solve(-1.85078*x-y=0,y);
```

$$-1.850780000\ x$$

```
> solve(x+1.35078*y=0,y);
```

$$-.7403130043\ x$$

It should be noted that both the phase lines have a negative slope.

The vector forces can be established by noting:

If $\Delta\ x(t) > 0$ then $y(t) < -1.5078\ x(t)$, i.e., below $\Delta\ x(t) = 0$, x is rising

If $\Delta\ x(t) < 0$ then $y(t) > -1.85078\ x(t)$, i.e., above $\Delta\ x(t) = 0$, x is falling

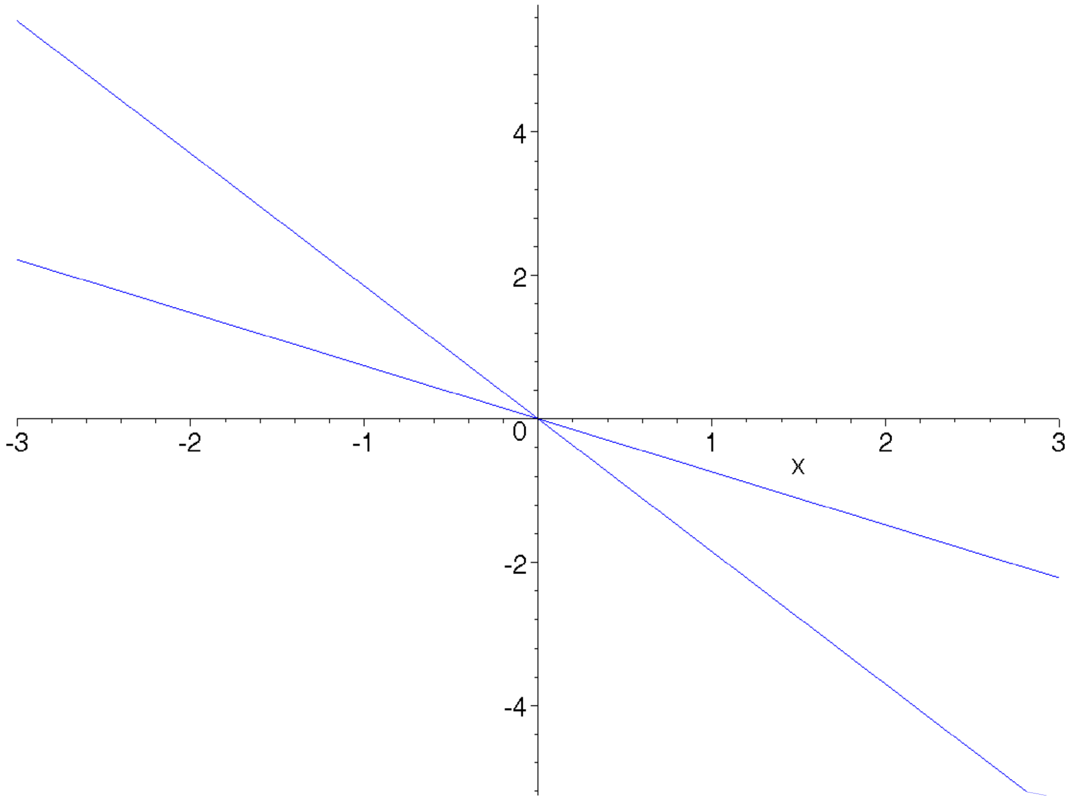
and

If $\Delta\ y(t) > 0$ then $y(t) < -.740313\ x(t)$, i.e., above $\Delta\ x(t) = 0$, y is rising

If $\Delta\ y(t) < 0$ then $y(t) < -.740313\ x(t)$, i.e., below $\Delta\ x(t) = 0$, y is falling

```
> isoclines:=plot({-1.85078*x,-0.740313*x},x=-3..3,colour=blue)
  :
```

```
> display(isoclines);
```




```
> A:=matrix([[-0.85078, -1], [1, 2.35078]]);
```

$$A := \begin{bmatrix} -.85078 & -1 \\ 1 & 2.35078 \end{bmatrix}$$

```
> eigenvalues(A);
```

-.4999986434, 1.999998643

```
> eigenvectors(A);
```

[-.499998643, 1, {[-.9436282315, .3310071912]}],

[1.999998643, 1, {[.4238962133, -1.208434272]}]

To find the saddle-path equations

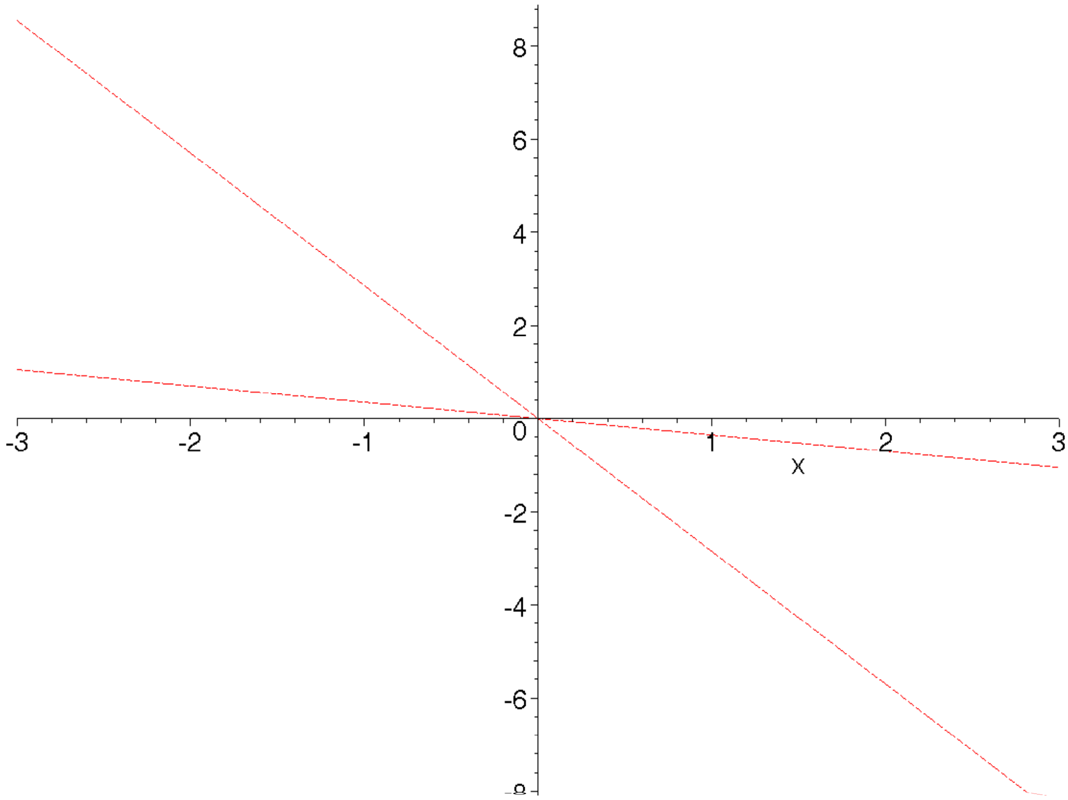
```
> solve(0.3310071912*x=-0.9436282315*y,y);
```

$-.3507813566\, x$

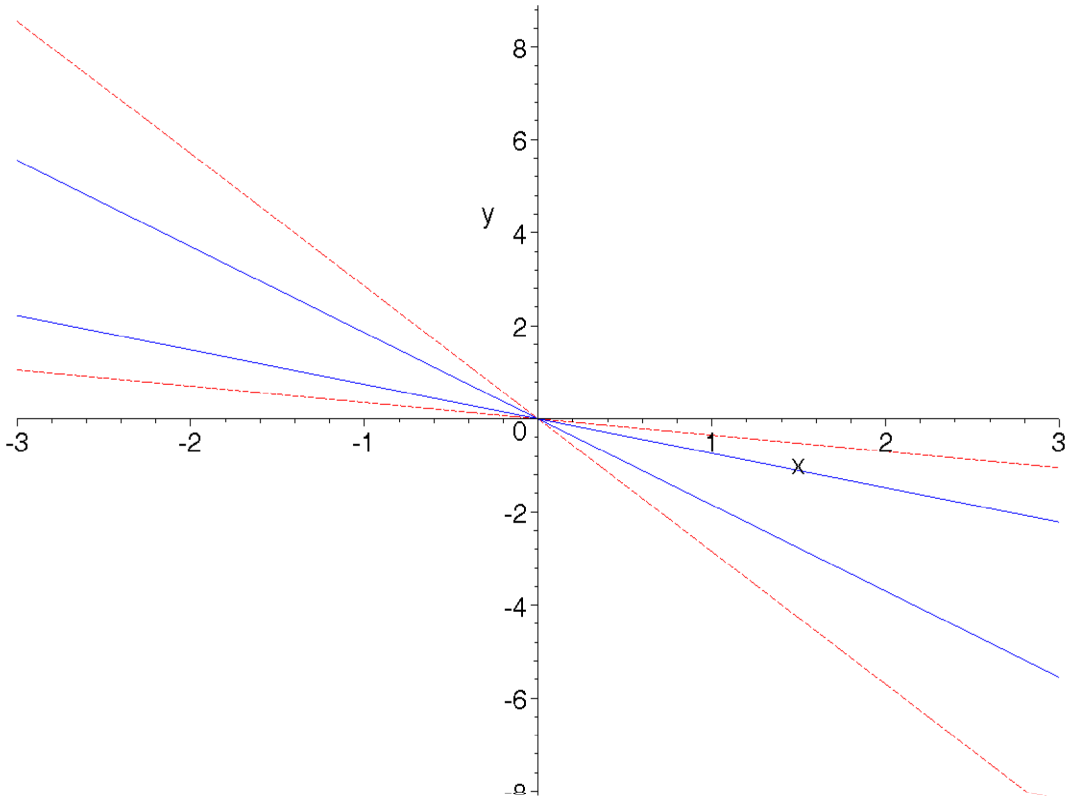
```
> solve(-.9436282315*x=.3310071912*y,y);
```

$-2.850778643\, x$

```
> saddles:=plot({-.3507813566*x,-2.850778643*x},x=-3..3,linesty
le=3,colour=red):
> display(saddles);
```




```
> display({isoclines,saddles},labels=["x","y"]);
```



Question 3

The two equations of Example 5.4 are:

$$x(t+1) = -8 - x(t) + y(t)$$

$$y(t+1) = 4 - .3\,x(t) + .9\,y(t)$$

and although the question refers to setting up the problem on a spreadsheet, we can also use

Maple's spreadsheet facility to show a table of values. We do this by solving the system of equations for each of the initial conditions and set the results out as a table.

For (3.10)

```
> x:='x': y:='y':
> sol1:=evalf(rsolve({x(t+1)=-8-x(t)+y(t),y(t+1)=4-0.3*x(t)+0.9
*y(t),x(0)=3,y(0)=10},{x(t),y(t)}));
```

$sol1 := \{$

$$y(t) = 20.80000000 - 11.35200722\ .7262087346^t + .5520072287\ (-.8262087346)^t,$$

$$x(t) = 6.399999996 - 6.576265660\ .7262087346^t + 3.176265662\ (-.8262087346)^t\}$$

```
>
```

t	x(t)	y(t)
0	3.0000	10.0000
1	-.9999	12.1001
2	5.1000	15.1901
3	2.0901	16.1411
4	6.0510	17.9000
5	3.8490	18.2947
6	6.4457	19.3105
7	4.8648	19.4458
8	6.5809	20.0417
9	5.4608	20.0633
10	6.6024	20.4187
11	5.8162	20.3961
12	6.5798	20.6116
13	6.0317	20.5765
14	6.5447	20.7093
15	6.1646	20.6749
16	6.5103	20.7581
17	6.2477	20.7291
18	6.4814	20.7819
19	6.3004	20.7593
20	6.4588	20.7932

```
> x:='x': y:='y':
```

For (3,30)

```
> sol2:=evalf(rsolve({x(t+1)=-8-x(t)+y(t),y(t+1)=4-0.3*x(t)+0.9
  *y(t),x(0)=3,y(0)=30},{x(t),y(t)}));
```

sol2 := {

$y(t) = 20.80000000 + 10.88696867\ .7262087346^t - 1.686968675\ (-.8262087346)^t,$

$x(t) = 6.399999996 + 6.306866863\ .7262087346^t - 9.706866861\ (-.8262087346)^t$ }

t	x(t)	y(t)
0	3.0000	30.0000
1	18.9999	30.0999
2	3.1000	25.3898
3	14.2897	25.9208
4	3.6311	23.0417
5	11.4106	23.6482
6	4.2376	21.8602
7	9.6225	22.4028
8	4.7803	21.2758
9	8.4954	21.7141
10	5.2186	20.9940
11	7.7753	21.3290
12	5.5536	20.8635
13	7.3098	21.1110
14	5.8012	20.8070
15	7.0057	20.9859
16	5.9801	20.7856
17	6.8054	20.9130
18	6.1075	20.7800
19	6.6724	20.8697
20	6.1972	20.7810

```
> x:='x': y:='y':
```

For (10,10)

```
> sol3:=evalf(rsolve({x(t+1)=-8-x(t)+y(t),y(t+1)=4-0.3*x(t)+0.9
  *y(t),x(0)=10,y(0)=10},{x(t),y(t)}));
```

$sol3 := \{$

$x(t) = 6.399999996 - 7.359907225\ .7262087346^t + 10.95990722\ (-.8262087346)^t,$

$y(t) = 20.80000000 - 12.70473614\ .7262087346^t + 1.904736144\ (-.8262087346)^t \}$

t	x(t)	y(t)
0	10.0000	10.0000
1	-7.9998	10.0001
2	9.9999	15.4001
3	-2.5996	14.8602
4	9.4598	18.1541
5	.6943	17.5008
6	8.8064	19.5424
7	2.7360	18.9462
8	8.2102	20.2308
9	4.0206	19.7447
10	7.7240	20.5640
11	4.8400	20.1904
12	7.3504	20.7193
13	5.3689	20.4423
14	7.0733	20.7873
15	5.7140	20.5866
16	6.8725	20.8137
17	5.9411	20.6706
18	6.7294	20.8212
19	6.0917	20.7202
20	6.6284	20.8206

```
> x:='x': y:='y':
```

For (10,30)

```
> sol4:=evalf(rsolve({x(t+1)=-8-x(t)+y(t),y(t+1)=4-0.3*x(t)+0.9
  *y(t),x(0)=10,y(0)=30},{x(t),y(t)}));
```

$sol4 := \{$

$x(t) = 6.399999996 + 5.523225298\ .7262087346^t - 1.923225300\ (-.8262087346)^t,$

$y(t) = 20.80000000 + 9.534239754\ .7262087346^t - .3342397592\ (-.8262087346)^t \}$

t	$x(t)$	$y(t)$
0	10.0000	30.0000
1	11.9998	27.9998
2	7.9999	25.5998
3	9.5998	24.6398
4	7.0399	23.2958
5	8.2558	22.8542
6	6.5983	22.0920
7	7.4936	21.9033
8	6.4096	21.4648
9	7.0552	21.3954
10	6.3402	21.1393
11	6.7990	21.1233
12	6.3242	20.9712
13	6.6470	20.9768
14	6.3298	20.8850
15	6.5552	20.8976
16	6.3423	20.8412
17	6.4988	20.8544
18	6.3555	20.8193
19	6.4637	20.8307
20	6.3669	20.8085

Question 4

The system for example 5.5 is

$$x(t) = x(t-1) + 2\,y(t-1) + z(t-1)$$
$$y(t) = -x(t-1) + y(t-1)$$
$$z(t) = 3\,x(t-1) - 6\,y(t-1) - z(t-1)$$

(a)

We can first use *Maple* to check the solution in the text for this set of equations for the initial conditions $x(0) = 3$, $y(0) = -4$, $z(0) = 3$.

```
> x:='x': y:='y': z:='z':
```

```
> sol:=rsolve({x(t)=x(t-1)+2*y(t-1)+z(t-1),y(t)=-x(t-1)+y(t-1),
  z(t)=3*x(t-1)-6*y(t-1)-z(t-1), x(0)=3,
  y(0)=-4,z(0)=3},{x(t),y(t),z(t)});
```

$$sol := \{y(t) = 3\,(-1)^t - 2\,2^t,\ z(t) = -18\,(-1)^t + 6\,2^t,\ x(t) = 6\,(-1)^t + 2\,2^t\}$$

which is the solution given in the text.

(b)

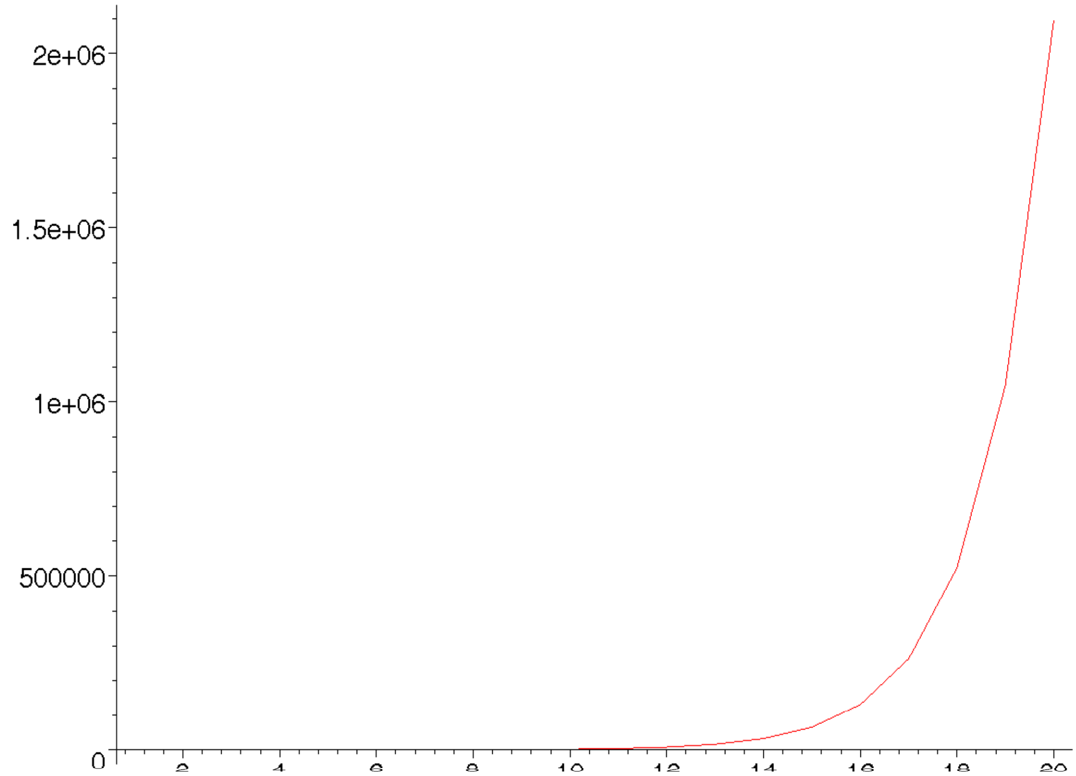
t	x(t)	y(t)	z(t)
0	8	1	-12
1	-2	-7	30
2	14	-5	6
3	10	-19	66
4	38	-29	78
5	58	-67	210
6	134	-125	366
7	250	-259	786
8	518	-509	1518
9	1018	-1027	3090
10	2054	-2045	6126
11	4090	-4099	12306
12	8198	-8189	24558
13	16378	-16387	49170
14	32774	-32765	98286
15	65530	-65539	196626
16	131078	-131069	393198
17	262138	-262147	786450
18	524294	-524285	1572846
19	1048570	-1048579	3145746
20	2097158	-2097149	6291438

It is important to note that Maple does *not* give the correct initial conditions. The general formula only applies for $1 \le t$

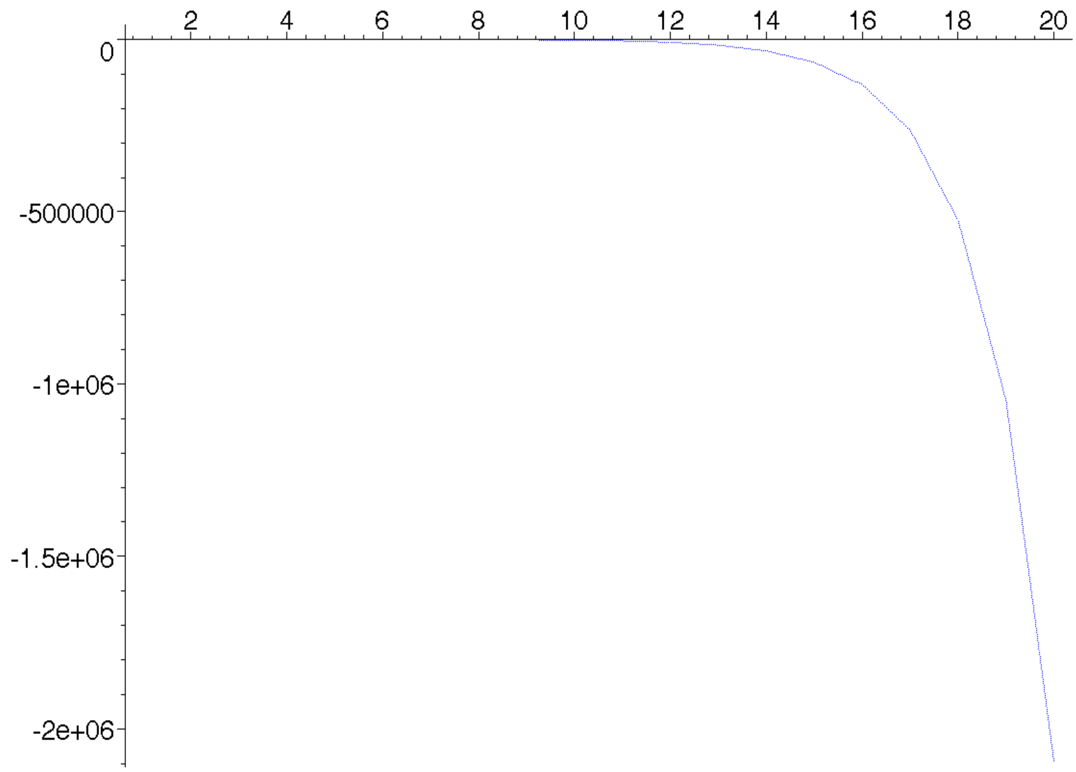
explosive. Plotting each variable against time ($1 \le t$), we have:

```
> xt:=seq([t,6*(-1)^t+2*2^t],t=1..20):
> linex:=plot([xt]):
```

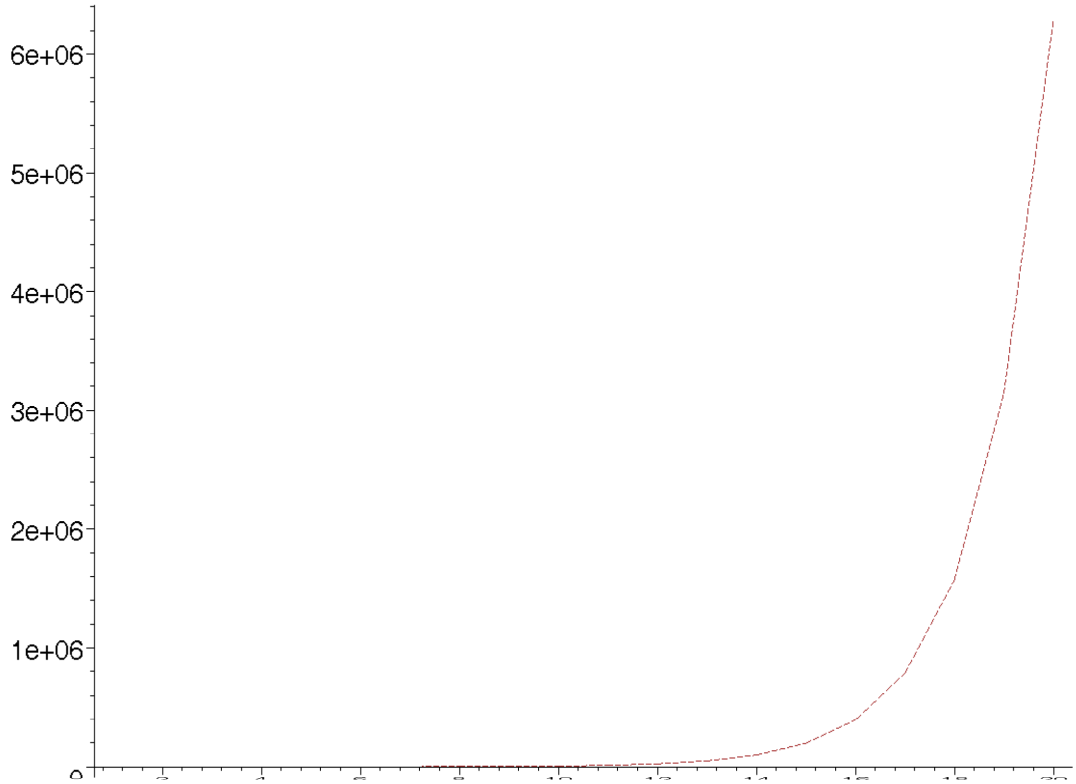
```
> display(linex);
```



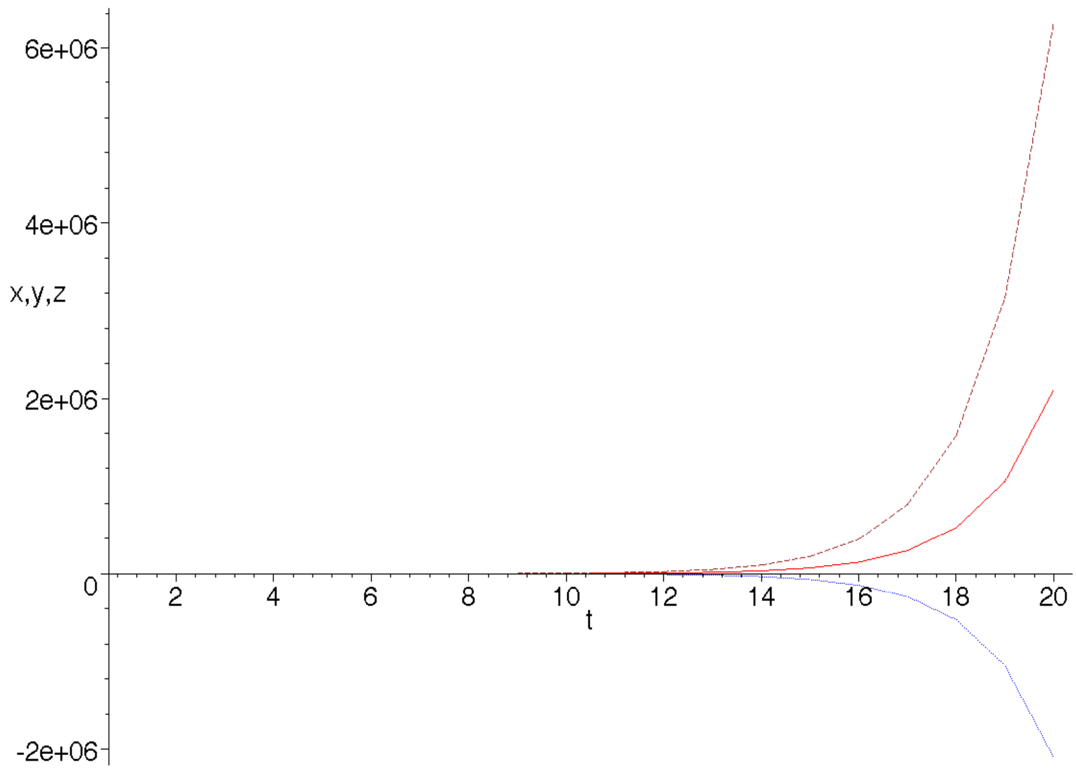

```
> yt:=seq([t,3*(-1)^t-2*2^t],t=1..20):
> liney:=plot([yt],colour=blue,linestyle=2):
> display(liney);
```




```
> zt:=seq([t,-18*(-1)^t+6*2^t],t=1..20):
> linez:=plot([zt],colour=brown,linestyle=3):
> display(linez);
```

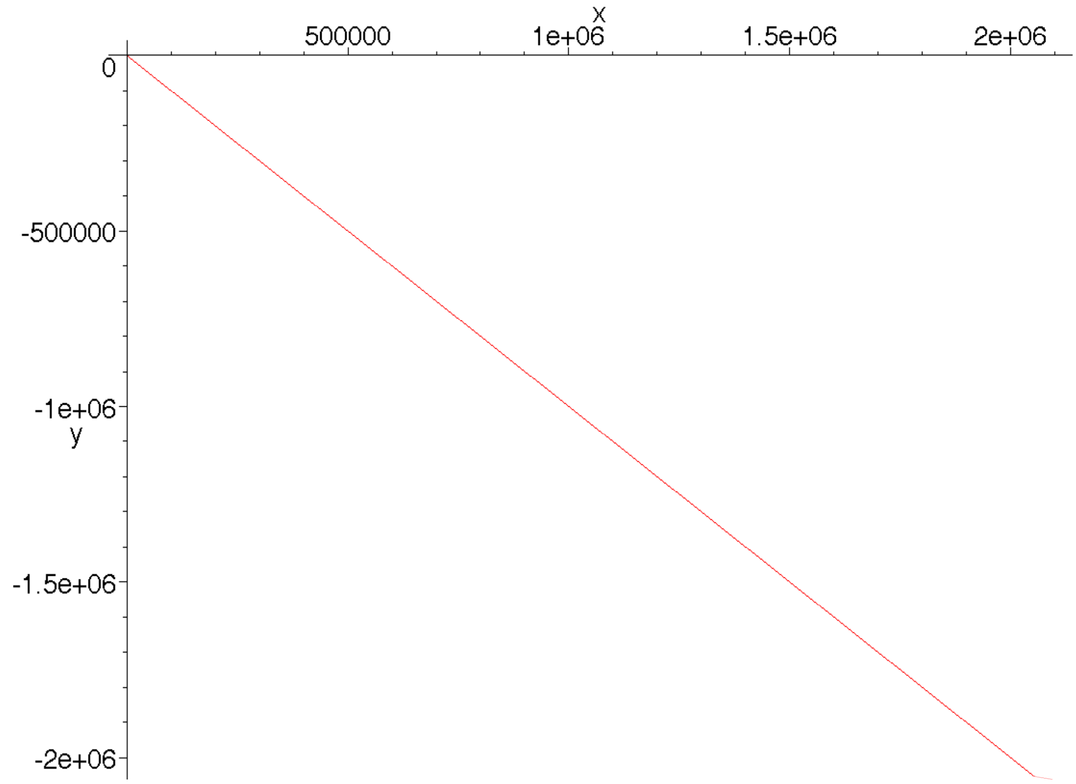



```
> display({linex,liney,linez},labels=["t","x,y,z"]);
```



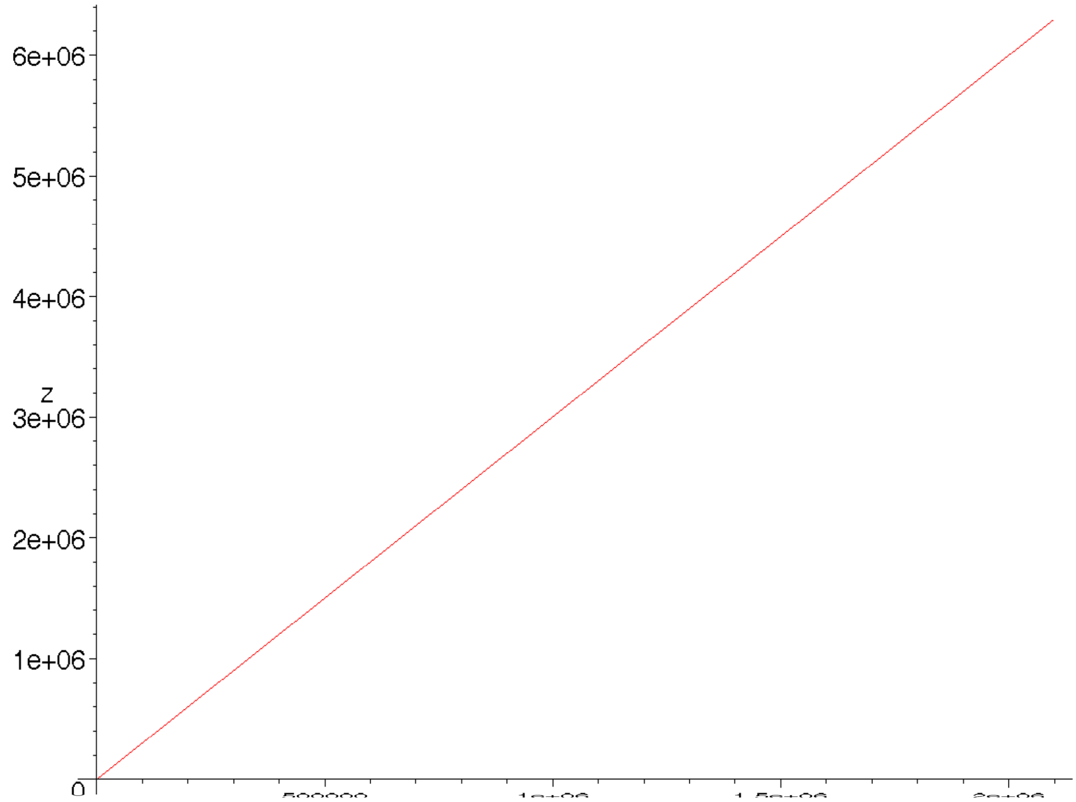
(c)

```
> xypoints:=seq([6*(-1)^t+2*2^t,3*(-1)^t-2*2^t],t=1..20):
> linexy:=plot([xypoints]):
> display(linexy,labels=["x","y"]);
```



(d)

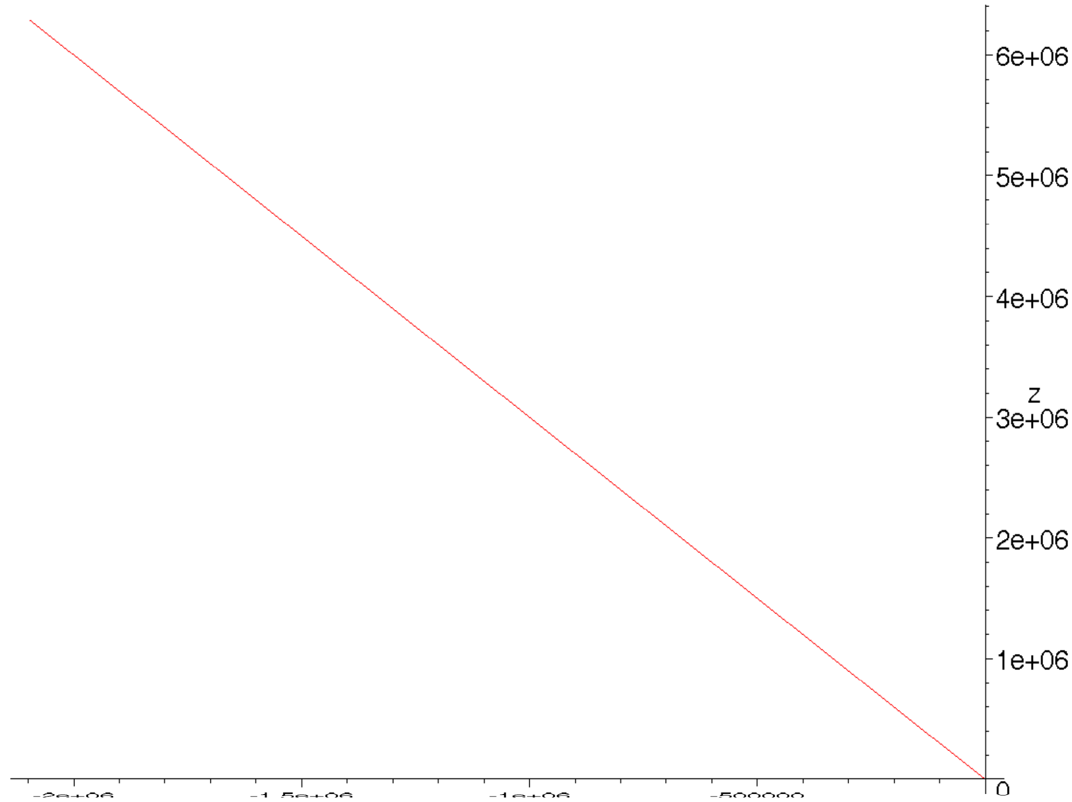
```
> xzpoints:=seq([6*(-1)^t+2*2^t,-18*(-1)^t+6*2^t],t=1..20):
> linexz:=plot([xzpoints]):
> display(linexz,labels=["x","z"]);
```



(e)

```
> yzpoints:=seq([3*(-1)^t-2*2^t,-18*(-1)^t+6*2^t],t=1..20):
> lineyz:=plot([yzpoints]):
```

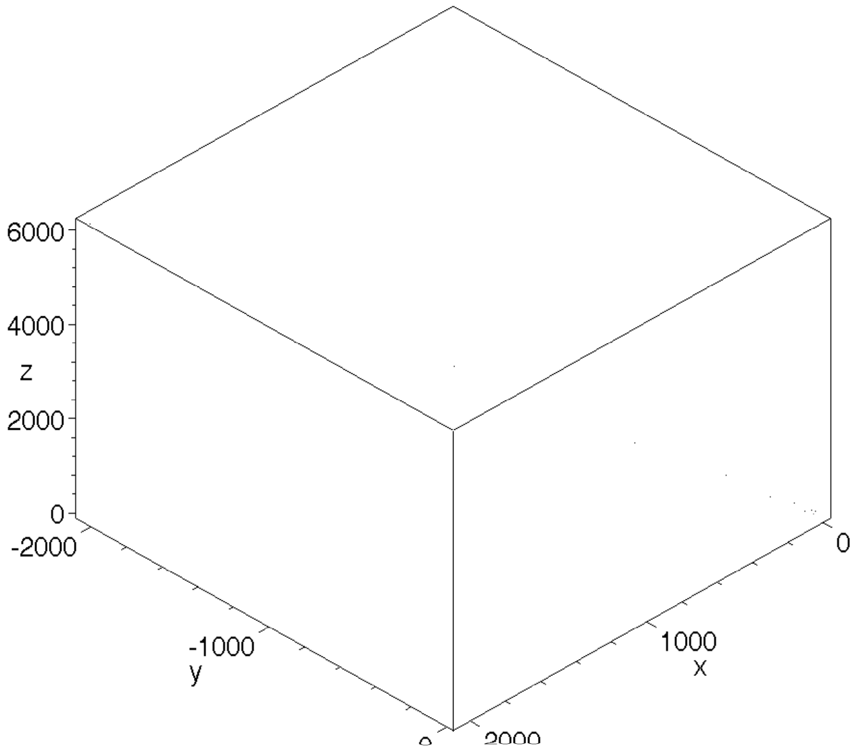
```
> display(lineyz,labels=["y","z"]);
```



Question 5

We already have the values for x(*t*), y(*t*) and z(*t*) in Question 4. However, to plot such a set of point in 3-dimensional space, we need to utilise the **pointplot3d** command. Since the figures become large (both negatively and positively) we plot only the first 10 points.

```
> pointsxyz:=seq([6*(-1)^t+2*2^t,3*(-1)^t-2*2^t,-18*(-1)^t+6*2^
  t],t=1..10):
> pointplot3d({pointsxyz},axes=BOXED,thickness=3,colour=black,l
  abels=["x","y","z"],tickmarks=[3,3,3]);
```



Question 6

Example 5.5 can be written first as a set of difference equations:

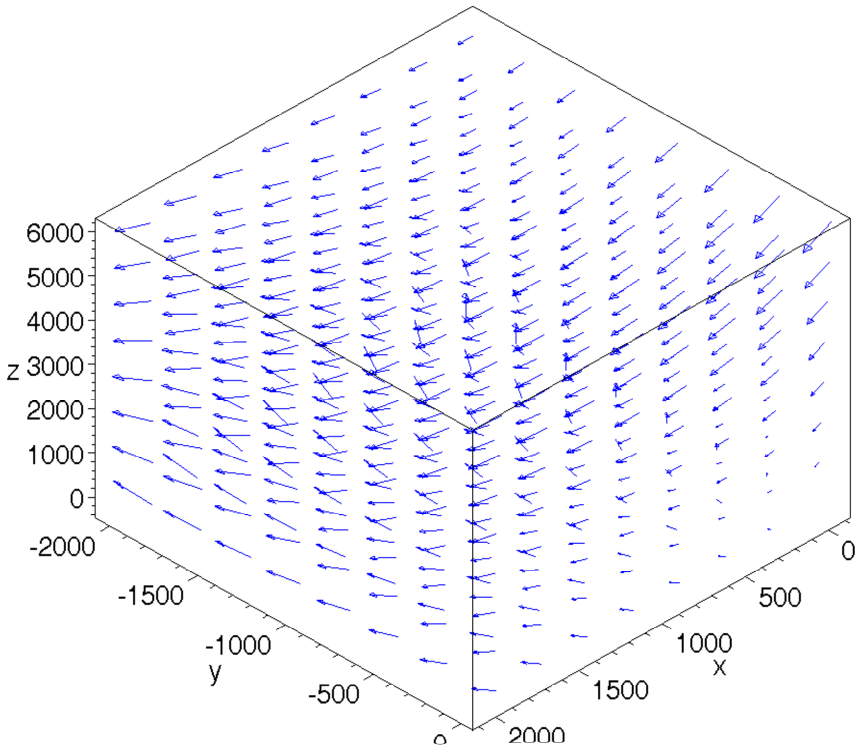
$\Delta\, x(t) = 2\, y(t) + z(t)$

$\Delta\, y(t) = -x(t)$

$\Delta\, z(t) = 3\, x(t) - 6\, y(t) - 2\, z(t)$

The 3-dimensional direction field can be obtained using the following instructions.

```
> fieldplot3d([2*x+z,-x,3*x-6*y-2*z],x=0..2000,y=-2000..0,z=0..
  6000,axes=boxed,arrows=SLIM,colour=blue);
```



Question 7

The matrix for each system is respectively:

$$\mathbf{A}1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \mathbf{A}2 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \quad \mathbf{A}3 = \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$$

If r and s **D**, is given by:

$$\mathbf{D} = \begin{bmatrix} r & 0 \\ 0 & s \end{bmatrix}$$

(i) System 1

```
> A1:=matrix([[0, 1], [-1, 0]]);
```

$$A1 := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

(a)

```
> eigenvalues(A1);
```

$$I, -I$$

(b)

```
> eigenvectors(A1);
```

$$[I, 1, \{[-I, 1]\}], [-I, 1, \{[I, 1]\}]$$

(c)

The diagonal matrix for this system is

$$\mathrm{D}1 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

(ii) System 2

```
> A2:=matrix([[-2, 1], [1, -2]]);
```

$$A2 := \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

(a)

```
> eigenvalues(A2);
```

-1, -3

(b)

```
> eigenvectors(A2);
```

[-1, 1, {[1, 1]}], [-3, 1, {[-1, 1]}]

(c)

The diagonal matrix for this system is

$$\mathbf{D}2 = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$$

(iii) System 3

```
> A3:=matrix([[3, -4], [1, -2]]);
```

$$A3 := \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$$

(a)

```
> eigenvalues(A3);
```

2, -1

(b)

```
> eigenvectors(A3);
```

[2, 1, {[4, 1]}], [-1, 1, {[1, 1]}]

(c)

The diagonal matrix for this system is

$$\mathbf{D}3 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

Question 8

Maple's spreadsheet. First we solve for the equilibrium values of the instrumental variables, r and g.

(a) Initial point $(g_0, r_0) = (20,12)$

```
> solve({r=-3.925+0.5*g,r=7.958+0.186*g},{r,g});
```

$\{r = 14.99697452, g = 37.84394904\}$

First we shall simply list the computations for g(t) and r(t) for each of the initial conditions. This requires us to solve the system of difference equations for each initial condition. We do this for ten periods.

```
> sol81:=evalf(rsolve({g(t+1)=g(t)-0.5*(g(t)-37.84394904),r(
  t+1)=r(t)-0.75*(r(t)-14.99697452),g(0)=20,r(0)=12},{g(t),r
  (t)}));
```

$$sol81 := \{ g(t) = -17.84394904\ .5000000000^t + 37.84394904,$$
$$r(t) = -2.996974520\ .2500000000^t + 14.99697452 \}$$

t	g(t)	r(t)
0	20.0000	12.0000
1	28.9219	14.2477
2	33.3829	14.8096
3	35.6134	14.9501
4	36.7286	14.9852
5	37.2862	14.9940
6	37.5650	14.9962
7	37.7044	14.9968
8	37.7741	14.9969
9	37.8090	14.9969
10	37.8264	14.9969

(b) Initial point $(g_0,r_0) = (20,20)$

```
> sol82:=evalf(rsolve({g(t+1)=g(t)-0.5*(g(t)-37.84394904),r(
  t+1)=r(t)-0.75*(r(t)-14.99697452),g(0)=20,r(0)=20},{g(t),r
  (t)}));
```

$$sol82 := \{ g(t) = -17.84394904\ .5000000000^t + 37.84394904,$$
$$r(t) = 5.003025480\ .2500000000^t + 14.99697452 \}$$

t	g(t)	r(t)
0	20.0000	20.0000
1	28.9219	16.2477
2	33.3829	15.3096
3	35.6134	15.0751
4	36.7286	15.0165
5	37.2862	15.0018
6	37.5650	14.9982
7	37.7044	14.9973
8	37.7741	14.9970
9	37.8090	14.9970
10	37.8264	14.9970

(c) Initial point $(g_0,r_0) = (50,10)$

```
> sol83:=evalf(rsolve({g(t+1)=g(t)-0.5*(g(t)-37.84394904),r(t+1)=r(t)-0.75*(r(t)-14.99697452),g(0)=50,r(0)=10},{g(t),r(t)}));
```

$sol83 := \{ r(t) = -4.996974520\ .2500000000^t + 14.99697452,$

$g(t) = 12.15605096\ .5000000000^t + 37.84394904 \}$

```
>
```

t	$g(t)$	$r(t)$
0	50.0000	10.0000
1	43.9219	13.7477
2	40.8829	14.6846
3	39.3634	14.9189
4	38.6036	14.9774
5	38.2237	14.9921
6	38.0338	14.9957
7	37.9388	14.9966
8	37.8913	14.9969
9	37.8676	14.9969
10	37.8557	14.9969

(d) Initial point $(g_0,r_0) = (50,20)$

```
> sol84:=evalf(rsolve({g(t+1)=g(t)-0.5*(g(t)-37.84394904),r(t+1)=r(t)-0.75*(r(t)-14.99697452),g(0)=50,r(0)=20},{g(t),r(t)}));
```

$sol84 := \{ g(t) = 12.15605096\ .5000000000^t + 37.84394904,$

$r(t) = 5.003025480\ .2500000000^t + 14.99697452 \}$

t	g(t)	r(t)
0	50.0000	20.0000
1	43.9219	16.2477
2	40.8829	15.3096
3	39.3634	15.0751
4	38.6036	15.0165
5	38.2237	15.0018
6	38.0338	14.9982
7	37.9388	14.9973
8	37.8913	14.9970
9	37.8676	14.9970
10	37.8557	14.9970

It is already apparent from each of these tables that the trajectories converge on the equilibrium values. To show this more clearly we place each of the trajectories in the (g,r)-space along with the "target lines".

(a)

```
> points81:=seq([-17.84394904*.5000000000^t+37.84394904,-2.996974520*.2500000000^t+14.99697452],t=0..10):
> traj81:=plot([points81]):
```

(b)

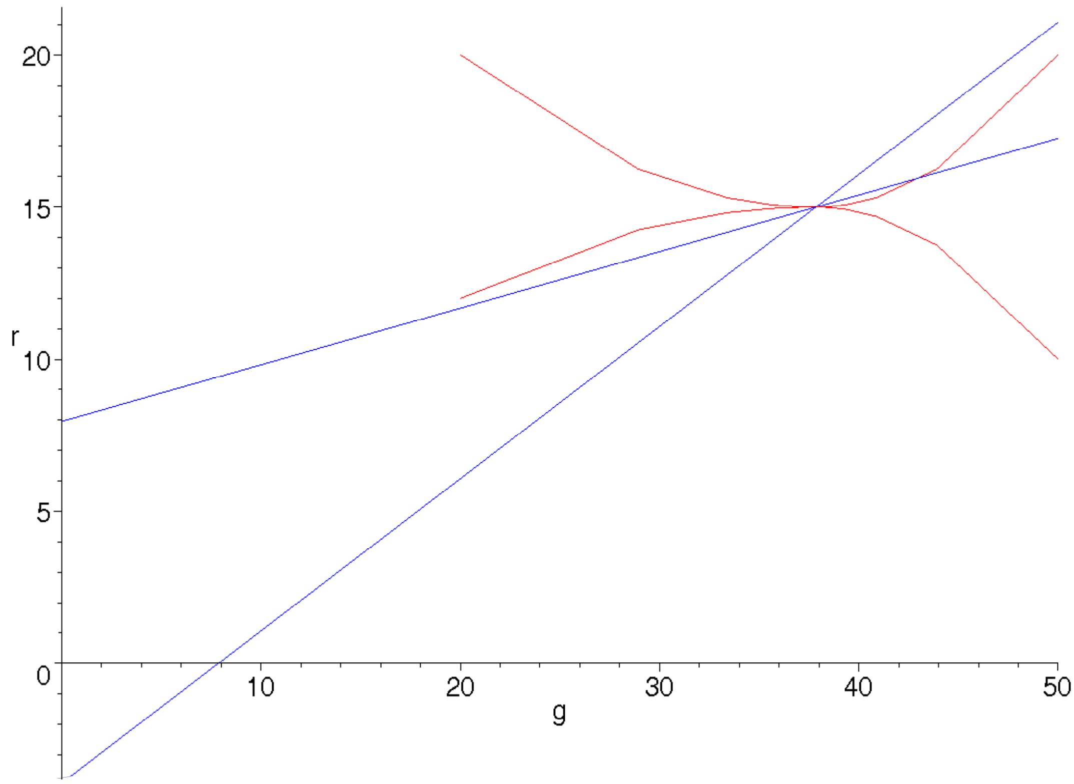
```
> points82:=seq([-17.84394904*.5000000000^t+37.84394904,5.003025480*.2500000000^t+14.99697452],t=0..10):
> traj82:=plot([points82]):
```

(c)

```
> points83:=seq([12.15605096*.5000000000^t+37.84394904,-4.996974520*.2500000000^t+14.99697452],t=0..10):
> traj83:=plot([points83]):
```

(d)

```
> points84:=seq([12.15605096*.5000000000^t+37.84394904,5.003025480*.2500000000^t+14.99697452],t=0..10):
> traj84:=plot([points84]):
> targetlines:=plot({-3.925+0.5*g,7.958+0.186*g},g=0..50,colour=blue):
> display({traj81,traj82,traj83,traj84,targetlines},labels=["g","r"]);
```



Question 9

If interest rates are set to achieve internal balance and government spending to achieve external balance, then the set of equations are:

$$g(t+1) = -21.3925 + .5\, g(t) + 2.688\, r(t)$$
$$r(t+1) = -2.94375 + .375\, g(t) + .25\, r(t)$$

(a) Four intial points, one in each quadrant

We consider four inital points as follows:

$(g0, r0) = (20, 20)$ Quadrant I
$(g0, r0) = (50, 20)$ Quadrant II
$(g0, r0) = (20, 40)$ Quadrant III
$(g0, r0) = (20, 10)$ Quadrant IV

which are determined by the following two target lines:

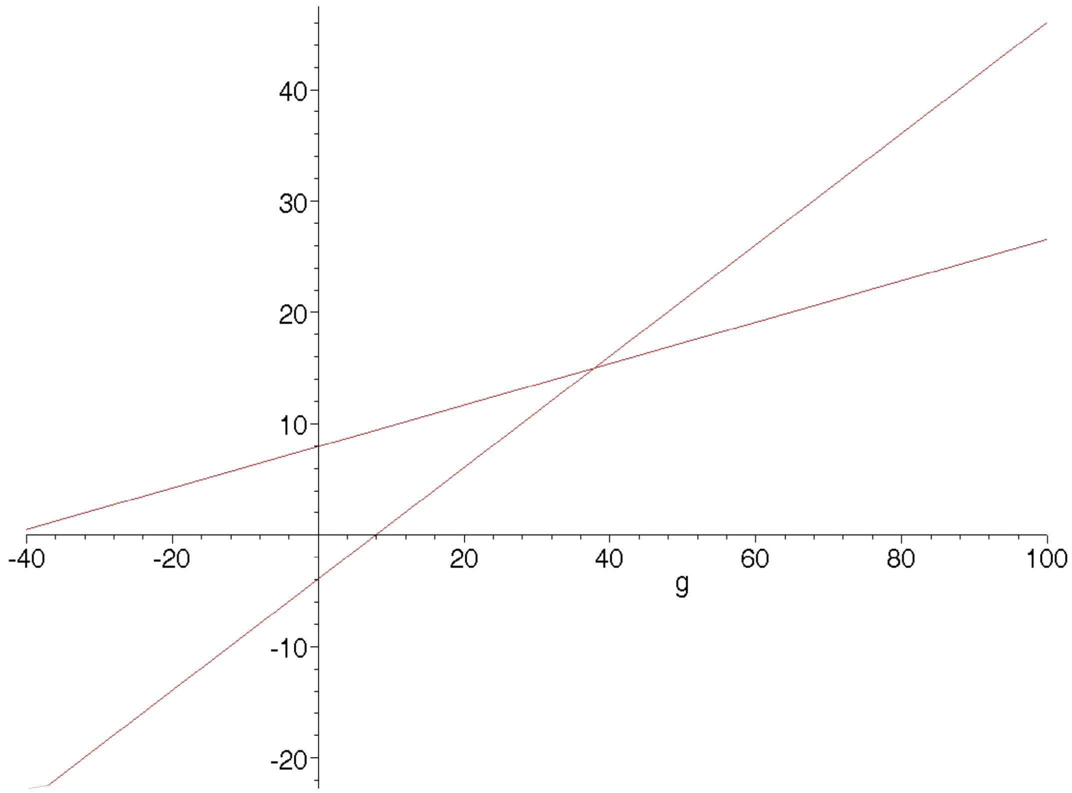
$$r = -3.925 + .5\, g$$
$$g = -42.785 + 5.376\, r$$

and equilibrium points:

```
> solve({r=-3.925+0.5*g,g=-42.785+5.376*r},{g,r});
```

$$\{r = 14.99851896, g = 37.84703791\}$$

```
> targetlines:=plot({-3.925+0.5*g,
  (42.785/5.376)+(1/5.376)*g},g=-40..100,colour=brown):
> display(targetlines);
```



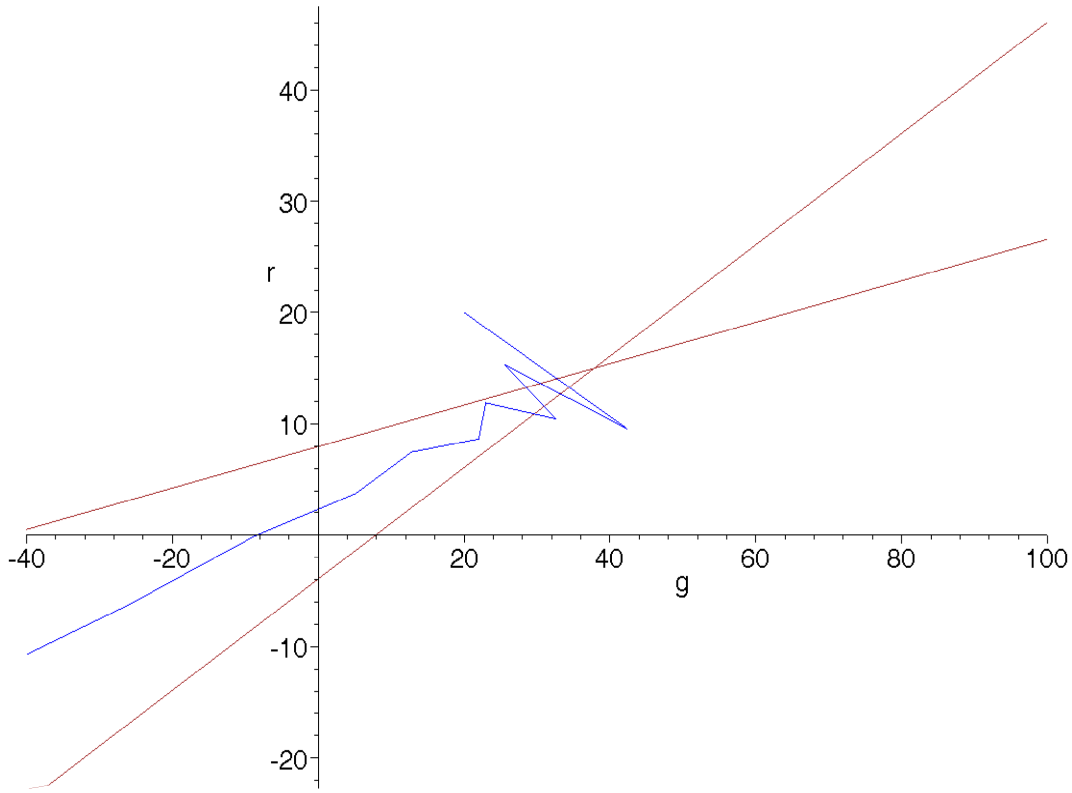

```
> result1:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
  ,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=20,r(0)=20},{g(
  t),r(t)}));
```

result1 := {

$$g(t) = 37.84703792 - 3.382044855\ 1.386743545^t - 14.46499307\ (-.6367435446)^t,$$

$$r(t) = 14.99851897 - 1.115701802\ 1.386743545^t + 6.117182846\ (-.6367435446)^t\}$$

```
> points91:=seq([37.84703792-3.382044855*1.386743545^t-14.46
  499307*(-.6367435446)^t,14.99851897-1.115701802*1.38674354
  5^t+6.117182846*(-.6367435446)^t],t=0..10):
> traj91:=plot([points91],colour=blue):
> display({targetlines,traj91},labels=["g","r"]);
```



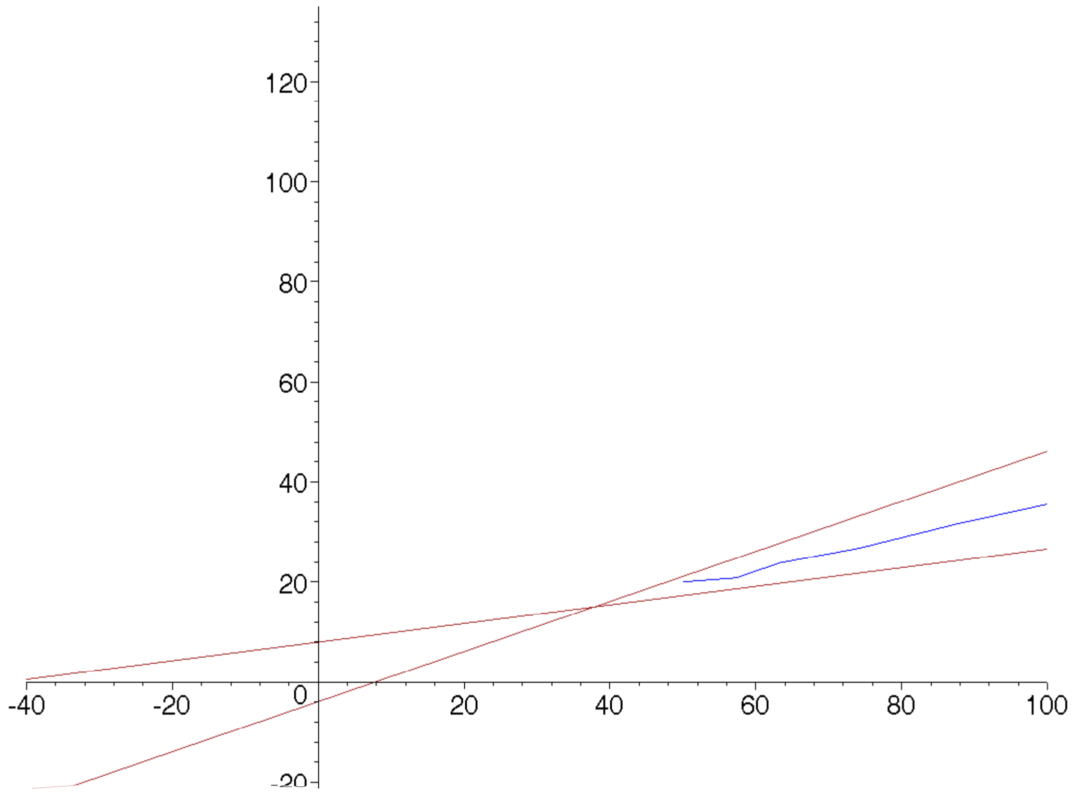

```
> result2:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=50,r(0)=20},{g(
t),r(t)}));
```

result2 := {

$$r(t) = 14.99851897 + 4.444007506\ 1.386743545^t + .5574735378\ (-.6367435446)^t,$$

$$g(t) = 37.84703792 + 13.47119159\ 1.386743545^t - 1.318229496\ (-.6367435446)^t\}$$

```
> points92:=seq([37.84703792+13.47119159*1.386743545^t-1.318
229496*(-.6367435446)^t,14.99851897+4.444007506*1.38674354
5^t+.5574735378*(-.6367435446)^t],t=0..10):
> traj92:=plot([points92],colour=blue):
> display({targetlines,traj92});
```



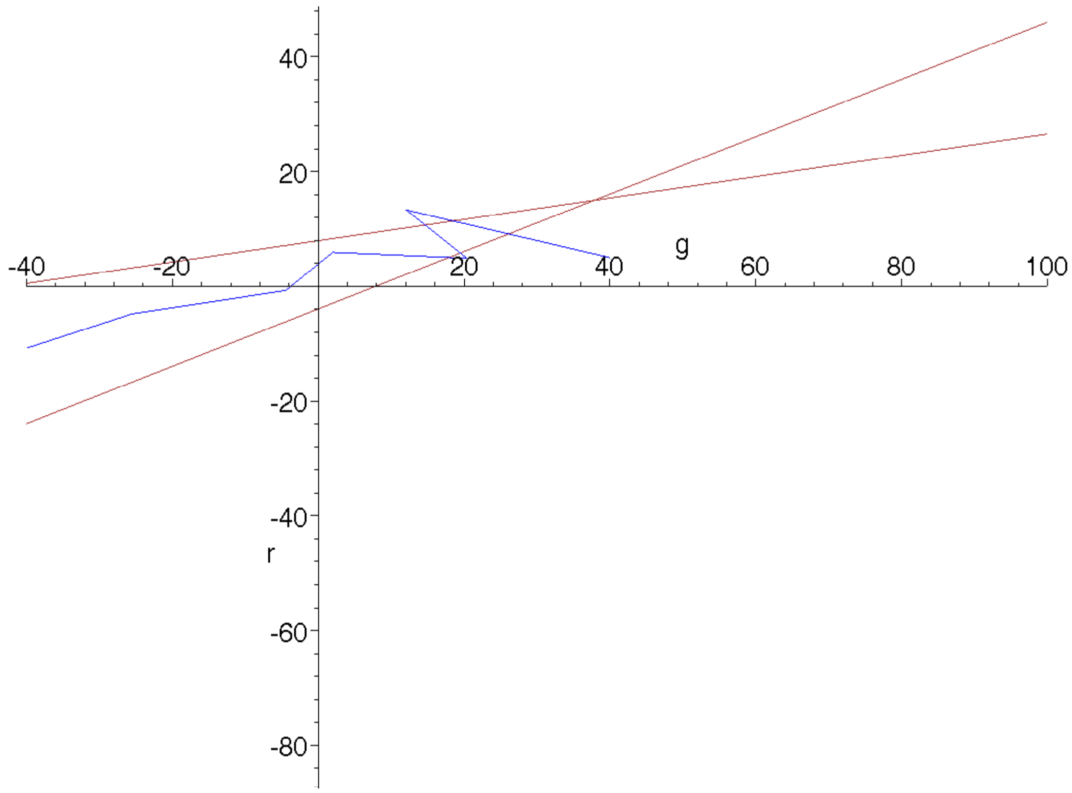

```
> result3:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=40,r(0)=5},{g(t
),r(t)}));
```

result3 := {

$r(t) = 14.99851897 - 3.982610714\ 1.386743545^t - 6.015908247\ (-.6367435446)^t$,

$g(t) = 37.84703792 - 12.07255205\ 1.386743545^t + 14.22551414\ (-.6367435446)^t$ }

```
> points93:=seq([37.84703792-12.07255205*1.386743545^t+14.22
551414*(-.6367435446)^t,14.99851897-3.982610714*1.38674354
5^t-6.015908247*(-.6367435446)^t],t=0..10):
> traj93:=plot([points93],colour=blue):
> display({targetlines,traj93},labels=["g","r"]);
```



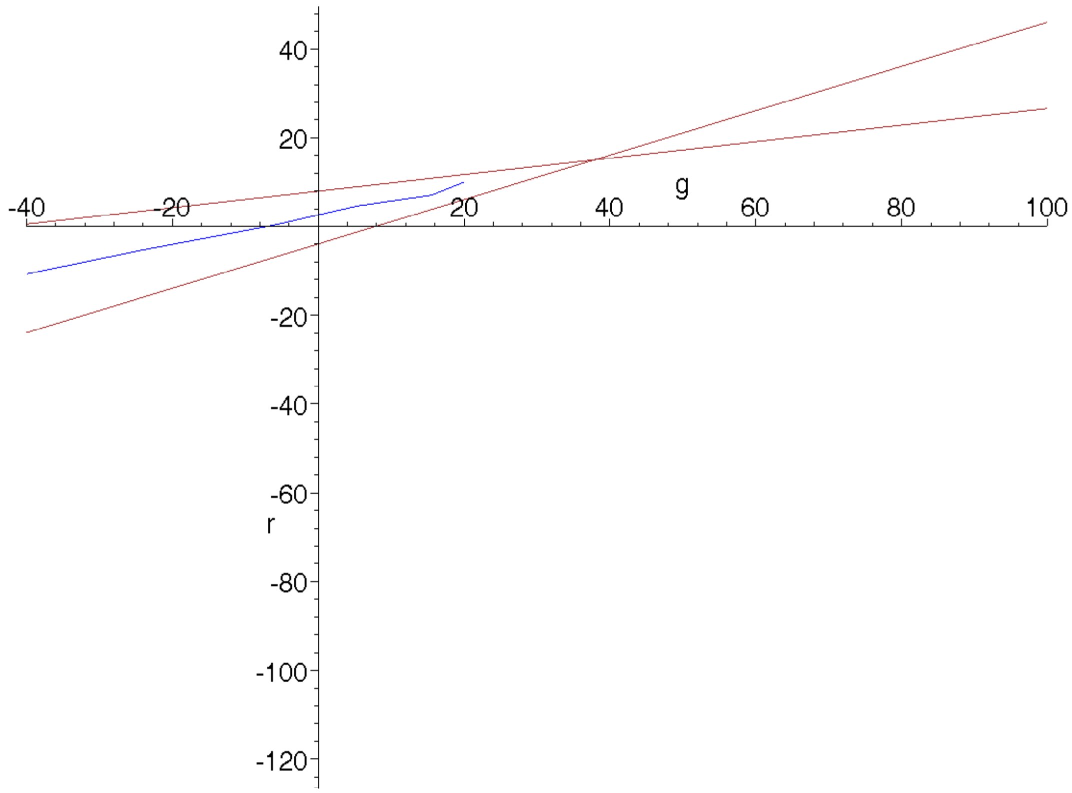

```
> result4:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=20,r(0)=10},{g(
t),r(t)}));
```

result4 := {

$r(t) = 14.99851897 - 5.497956324\ 1.386743545^t + .4994373649\ (-.6367435446)^t,$

$g(t) = 37.84703792 - 16.66604363\ 1.386743545^t - 1.180994293\ (-.6367435446)^t$ }

```
> points94:=seq([37.84703792-16.66604363*1.386743545^t-1.180
994293*(-.6367435446)^t,14.99851897-5.497956324*1.38674354
5^t+.4994373649*(-.6367435446)^t],t=0..10):
> traj94:=plot([points94],colour=blue):
> display({targetlines,traj94},labels=["g","r"]);
```



(b) Sectors I and III

We shall consider just two points in each of the two sectors as follows:

$(g0, r0) = (20, 30)$ $\quad (g0, r0) = (40, 40)$ $\quad$ for Sector I

$(g0, r0) = (40, 5)$ $\quad (g0, r0) = (80, 5)$ $\quad$ for Sector III

```
> result5:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=20,r(0)=30},{g(
t),r(t)}));
```

result5 := {

$$r(t) = 14.99851897 + 3.266552723\ 1.386743545^t + 11.73492832\ (-.6367435446)^t,$$

$$g(t) = 37.84703792 + 9.901953915\ 1.386743545^t - 27.74899184\ (-.6367435446)^t\}$$

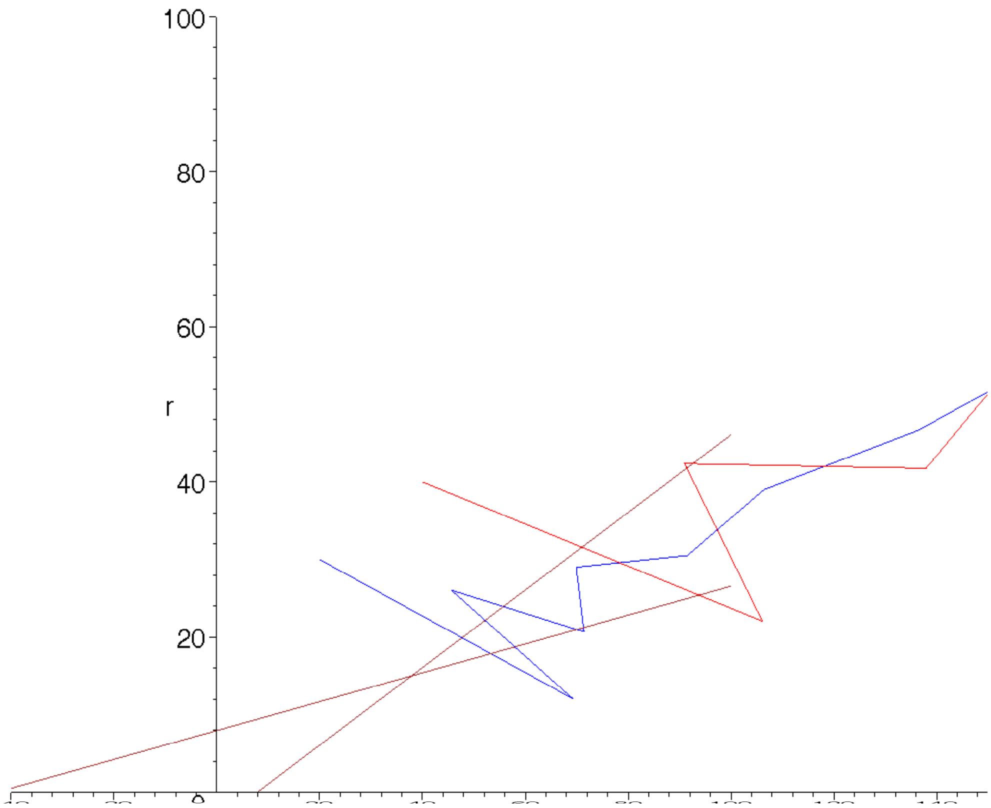
```
> points95:=seq([37.84703792+9.901953915*1.386743545^t-27.74
899184*(-.6367435446)^t,14.99851897+3.266552723*1.38674354
5^t+11.73492832*(-.6367435446)^t],t=0..10):
> traj95:=plot([points95],0..100,0..100,colour=blue):
> result6:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=40,r(0)=40},{g(
t),r(t)}));
```

result6 := {

$$g(t) = 37.84703792 + 34.42144365\ 1.386743545^t - 32.26848156\ (-.6367435446)^t,$$

$$r(t) = 14.99851897 + 11.35528012\ 1.386743545^t + 13.64620093\ (-.6367435446)^t\}$$

```
> points96:=seq([37.84703792+34.42144365*1.386743545^t-32.26
848156*(-.6367435446)^t,14.99851897+11.35528012*1.38674354
5^t+13.64620093*(-.6367435446)^t],t=0..10):
> traj96:=plot([points96],0..150,0..100,colour=red):
> display({targetlines,traj95,traj96},labels=["g","r"]);
```




```
> result7:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=40,r(0)=5},{g(t
),r(t)}));
```

result7 := {

$r(t) = 14.99851897 - 3.982610714\ 1.386743545^t - 6.015908247\ (-.6367435446)^t,$

$g(t) = 37.84703792 - 12.07255205\ 1.386743545^t + 14.22551414\ (-.6367435446)^t$ }

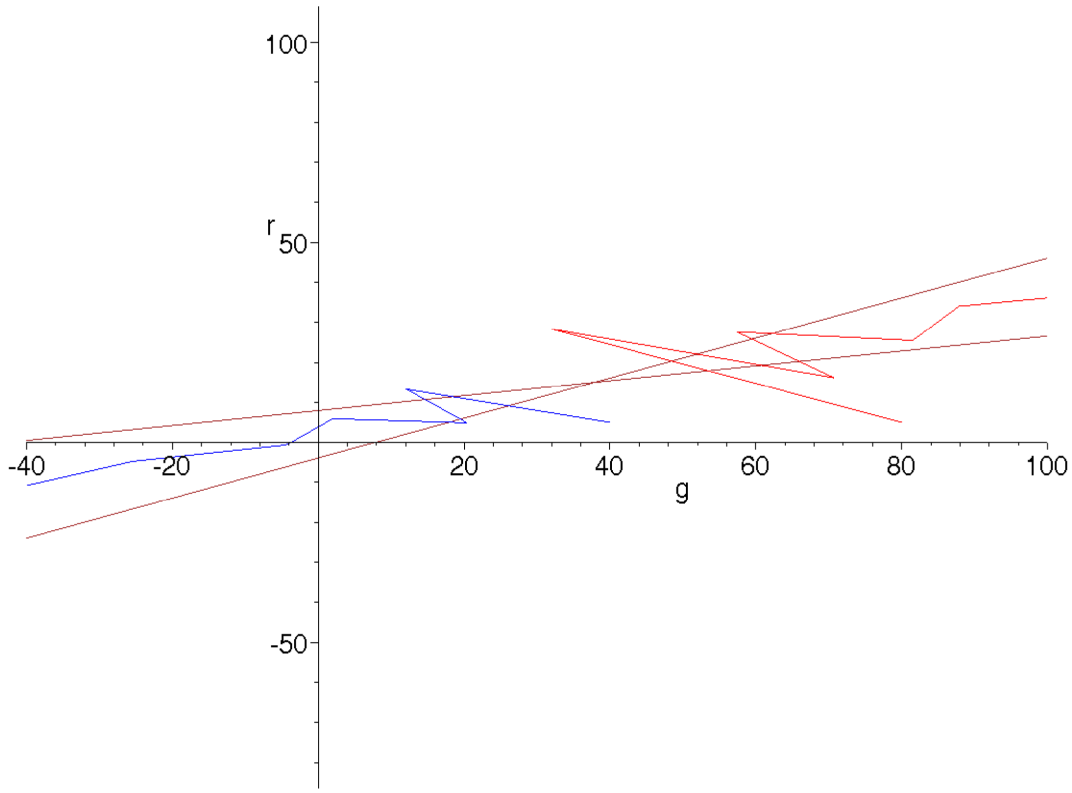
```
> points97:=seq([37.84703792-12.07255205*1.386743545^t+14.22
551414*(-.6367435446)^t,14.99851897-3.982610714*1.38674354
5^t-6.015908247*(-.6367435446)^t],t=0..10):
> traj97:=plot([points97],colour=blue):
> result8:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=80,r(0)=5},{g(t
),r(t)}));
```

result8 := {

$g(t) = 37.84703792 + 10.39842987\ 1.386743545^t + 31.75453224\ (-.6367435446)^t,$

$r(t) = 14.99851897 + 3.430335029\ 1.386743545^t - 13.42885399\ (-.6367435446)^t$ }

```
> points98:=seq([37.84703792+10.39842987*1.386743545^t+31.75
453224*(-.6367435446)^t,14.99851897+3.430335029*1.38674354
5^t-13.42885399*(-.6367435446)^t],t=0..10):
> traj98:=plot([points98],colour=red):
> display({targetlines,traj97,traj98},labels=["g","r"]);
```



(c) A point on the stable arm

The stable and unstale arms of the saddle point can be obtained from the eigenvectors of the system defined by the following matrix:

$$\mathbf{A9} = \begin{bmatrix} .5 & 2.688 \\ .375 & .25 \end{bmatrix}$$

```
> A9:=matrix([[0.5, 2.688], [0.375, 0.25]]);
>
```

$$A9 := \begin{bmatrix} .5 & 2.688 \\ .375 & .25 \end{bmatrix}$$

```
> eigenvalues(A9);
```

1.386743545, -.6367435446

```
> eigenvectors(A9);
```

[-.6367435443, 1, {[-1.398816565, .5915534594]}],
[1.386743545, 1, {[.9496598132, .3132830017]}]

Taking a unit value in the *g*-direction, we obtain the following values for *r*. These values represent points on the stable and unstable arms, where the positive value is on the unstable arm and the negative value is on the stable arm, as can be seen in terms of Figure 5.17 (p.242) of the text.

```
> [[1,.3132830017/.9496598132],[1,.5915534594/(-1.398816565)
  ]];
```

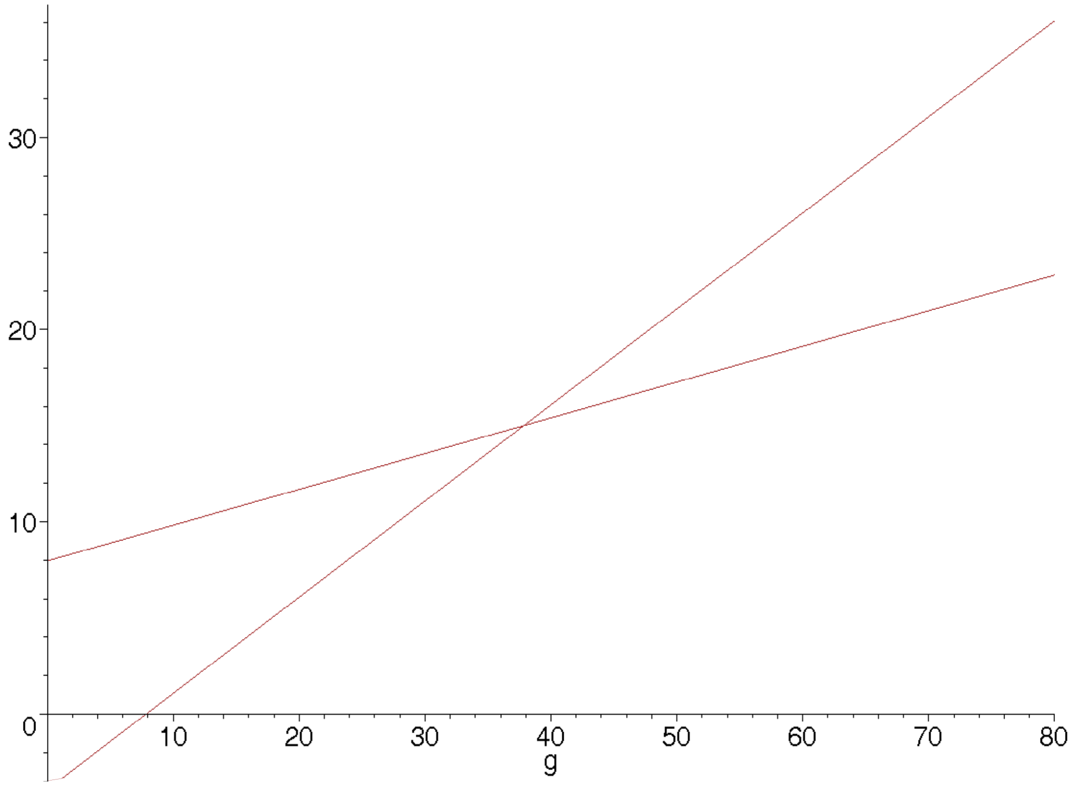
[[1, .3298897114], [1, -.4228956635]]

The equation for the stable arm is then,

$$14.9985 - .422896\,(g - 37.847)$$

We shall consider two points, one in quadrant I and a second in quadrant III. The first is determined by $g = 20$ and the second by $g = 50$.

```
> targetlines2:=plot({-3.925+0.5*g,(42.785/5.376)+(1/5.376)*
  g},g=0..80,colour=brown):
> display(targetlines2);
```




```
> [[20,14.9985-.422896*(20-37.847)],[50,14.9985-.422896*(50-
  37.847)]];
```

$$[[20, 22.54592491], [50, 9.859044912]]$$

```
> result9:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t)
  ,r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=20,r(0)=22.5459
  },{g(t),r(t)}));
```

$result9 := \{ g(t) =$

$37.84703794 - .00007160918098\ 1.386743545^t - 17.84696631\ (-.6367435446)^t$, $r(t)$

$= 14.99851897 - .00002362260322\ 1.386743545^t + 7.547404667\ (-.6367435446)^t \}$

```
> points99:=seq([37.84703794-.7160918098e-4*1.386743545^t-17
  .84696631*(-.6367435446)^t,14.99851897-.2362260322e-4*1.38
  6743545^t+7.547404671*(-.6367435446)^t],t=0..10):
> traj99:=plot([points99],colour=blue):
> result10:=evalf(rsolve({g(t+1)=-21.3925+0.5*g(t)+2.688*r(t
  ),r(t+1)=-2.94375+0.375*g(t)+0.25*r(t),g(0)=50,r(0)=9.859}
  ,{g(t),r(t)}));
```

$result10 := \{$

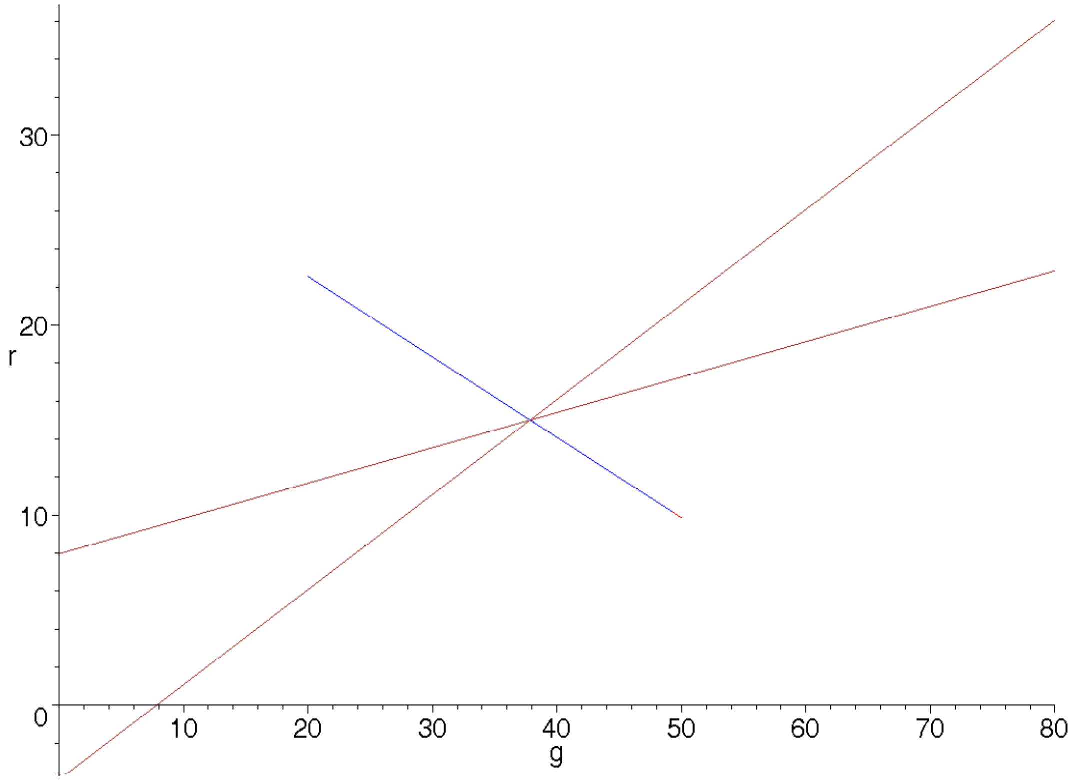
$g(t) = 37.84703792 - .0001115691551\ 1.386743545^t + 12.15307366\ (-.6367435446)^t$

$, r(t) =$

$14.99851897 - .00003680523927\ 1.386743545^t - 5.139482153\ (-.6367435446)^t \}$

```
> points910:=seq([37.84703792-.1115691551e-3*1.386743545^t+1
```

```
2.15307366*(-.6367435446)^t,14.99851897-.3680523927e-4*1.3
86743545^t-5.139482153*(-.6367435446)^t],t=0..10):
> traj910:=plot([points910],colour=red):
> display({targetlines2,traj99,traj910},labels=["g","r"]);
```



Question 10

We have the system of equations:

$g(t+1) = -1.875 + .25\, g(t) + .375\, S(t)$

$S(t+1) = 10 - 2\, g(t) + .2\, S(t)$

and we have the target equilibrium lines:

$g^*(t) = -2.5 + .5\, S(t)$

$S^*(t) = 20 - 4\, g(t)$

(a)

```
> result101:=evalf(rsolve({g(t+1)=-1.875+.25*g(t)+.375*S(t),S(t
+1)=10-2*g(t)+.2*S(t),g(0)=2.5,S(0)=12},{g(t),S(t)}));
```

$$result101 := \{S(t) = (-.0001390047262 - 0.\, I)\,(-59950. - 13189.\,(.2250000000 - .8656644845\, I)^t + 7306.208251\, I\,(.2250000000 - .8656644845\, I)^t - 7306.208251\, I\,(.2250000000 + .8656644845\, I)^t - 13189.\,(.2250000000 + .8656644845\, I)^t),\ g(t) = (.0003475118154 + 0.\, I)\,(4796. + 2319.980819\, I\,(.2250000000 - .8656644845\, I)^t + 1199.\,(.2250000000 - .8656644845\, I)^t - 2319.980819\, I\,(.2250000000 + .8656644845\, I)^t$$

$+ 1199. (.2250000000 + .8656644845\, I)^t)\}$

```
> points101:=seq([1.666666667+.4166666667*(.2250000000-.8656644845*I)^t+.8062207462*I*(.2250000000-.8656644845*I)^t-.8062207462*I*(.2250000000+.8656644845*I)^t+.4166666667*(.2250000000+.8656644845*I)^t,8.333333334-1.015597477*I*(.2250000000-.8656644845*I)^t+1.833333333*(.2250000000-.8656644845*I)^t+1.015597477*I*(.2250000000+.8656644845*I)^t+1.833333333*(.2250000000+.8656644845*I)^t],t=0..10):
> traj101:=plot([points101],colour=blue):
```

(b)

```
> result102:=evalf(rsolve({g(t+1)=-1.875+.25*g(t)+.375*S(t),S(t+1)=10-2*g(t)+.2*S(t),g(0)=3,S(0)=10},{g(t),S(t)}));
```

$result102 := \{ S(t) = (.0006950236308 + 0.\, I)\, (11990.$
$+ 1199. (.2250000000 - .8656644845\, I)^t$
$- 2250.727660\, I\, (.2250000000 - .8656644845\, I)^t$
$+ 2250.727660\, I\, (.2250000000 + .8656644845\, I)^t$
$+ 1199. (.2250000000 + .8656644845\, I)^t),\ g(t) = (-.0001390047262 - 0.\, I)\, (-11990.$
$- 2735.499772\, I\, (.2250000000 - .8656644845\, I)^t$
$- 4796. (.2250000000 - .8656644845\, I)^t$
$+ 2735.499772\, I\, (.2250000000 + .8656644845\, I)^t$
$- 4796. (.2250000000 + .8656644845\, I)^t)\}$

```
> points102:=seq([1.666666667+.6666666667*(.2250000000-.8656644845*I)^t+.3802473968*I*(.2250000000-.8656644845*I)^t-.3802473968*I*(.2250000000+.8656644845*I)^t+.6666666667*(.2250000000+.8656644845*I)^t,8.333333333-1.564308910*I*(.2250000000-.8656644845*I)^t+.8333333333*(.2250000000-.8656644845*I)^t+1.564308910*I*(.2250000000+.8656644845*I)^t+.8333333333*(.2250000000+.8656644845*I)^t],t=0..10):
> traj102:=plot([points102],colour=red):
```

(c)

```
> result103:=evalf(rsolve({g(t+1)=-1.875+.25*g(t)+.375*S(t),S(t+1)=10-2*g(t)+.2*S(t),g(0)=1,S(0)=5},{g(t),S(t)}));
```

$result103 := \{ g(t) = (-.0002780094523 - 0.\, I)\, (-5995.$
$+ 1199. (.2250000000 - .8656644845\, I)^t$
$+ 2631.620034\, I\, (.2250000000 - .8656644845\, I)^t$
$- 2631.620034\, I\, (.2250000000 + .8656644845\, I)^t$
$+ 1199. (.2250000000 + .8656644845\, I)^t),\ S(t) = (.001390047262 + 0.\, I)\, (5995.$
$+ 588.6518496\, I\, (.2250000000 - .8656644845\, I)^t$
$- 1199. (.2250000000 - .8656644845\, I)^t$
$- 588.6518496\, I\, (.2250000000 + .8656644845\, I)^t$

− 1199. (.2250000000 + .8656644845 I)^t) }

```
> points103:=seq([1.666666667-.7316152446*I*(.2250000000-.86566
44845*I)^t-.3333333334*(.2250000000-.8656644845*I)^t+.7316152
446*I*(.2250000000+.8656644845*I)^t-.3333333334*(.2250000000+
.8656644845*I)^t,8.333333334-1.666666667*(.2250000000-.865664
4845*I)^t+.8182538916*I*(.2250000000-.8656644845*I)^t-.818253
8916*I*(.2250000000+.8656644845*I)^t-1.666666667*(.2250000000
+.8656644845*I)^t],t=0..10):
> traj103:=plot([points103],colour=green):
```

(d)

```
> result104:=evalf(rsolve({g(t+1)=-1.875+.25*g(t)+.375*S(t),S(t
+1)=10-2*g(t)+.2*S(t),g(0)=1,S(0)=12},{g(t),S(t)}));
```

result104 := { S(t) = (-.0001390047262 − 0. I) (−59950.

− 13189. (.2250000000 − .8656644845 I)^t

− 5159.360329 I (.2250000000 − .8656644845 I)^t

+ 5159.360329 I (.2250000000 + .8656644845 I)^t

− 13189. (.2250000000 + .8656644845 I)^t), g(t) = (-.0001390047262 − 0. I) (−11990.

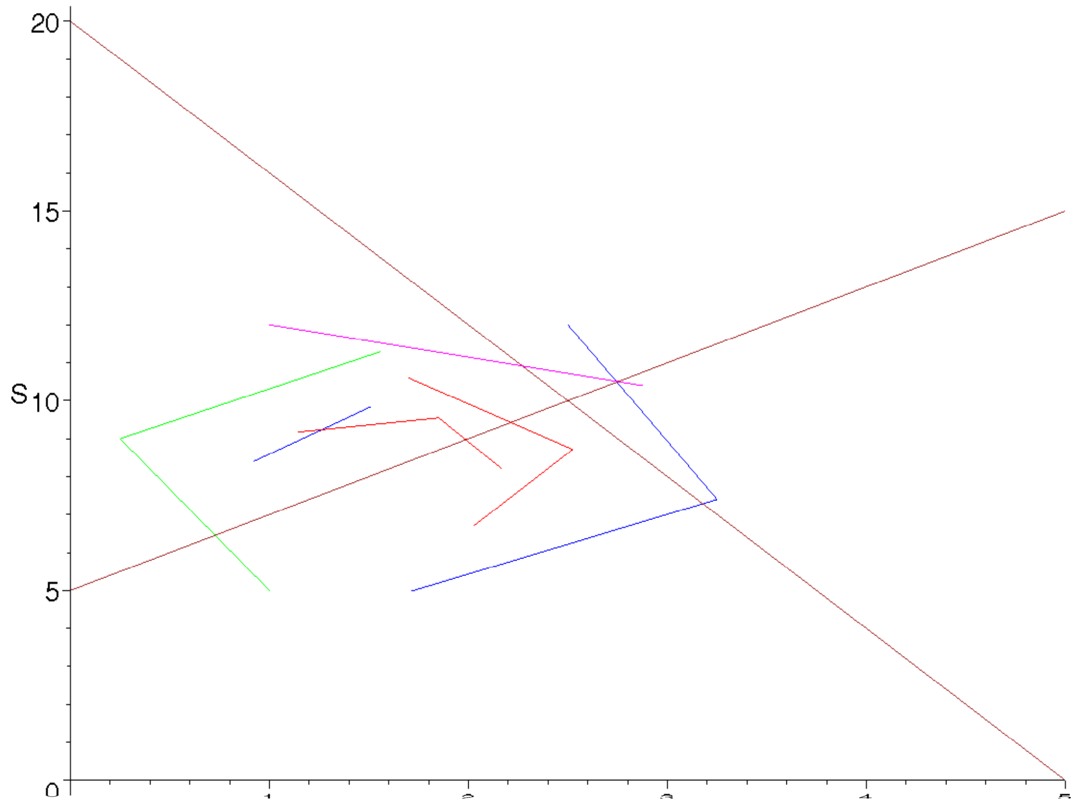
+ 2398. (.2250000000 − .8656644845 I)^t

− 5644.132441 I (.2250000000 − .8656644845 I)^t

+ 5644.132441 I (.2250000000 + .8656644845 I)^t

+ 2398. (.2250000000 + .8656644845 I)^t) }

```
> points104:=seq([1.666666667-.3333333334*(.2250000000-.8656644
845*I)^t+.7845610845*I*(.2250000000-.8656644845*I)^t-.7845610
845*I*(.2250000000+.8656644845*I)^t-.3333333334*(.2250000000+
.8656644845*I)^t,8.333333334+.7171754697*I*(.2250000000-.8656
644845*I)^t+1.833333333*(.2250000000-.8656644845*I)^t-.717175
4697*I*(.2250000000+.8656644845*I)^t+1.833333333*(.2250000000
+.8656644845*I)^t],t=0..10):
> traj104:=plot([points104],colour=magenta):
> targetlines10:=plot({5+2*g,20-4*g},g=0..5,colour=brown):
> display({targetlines10,traj101,traj102,traj103,traj104},label
s=["g","S"]);
```



This plot readily shows all trajectories converging on the equilibrium - but very slowly.

Question 11

```
> mA:=matrix([[2,3],[1,-2]]);
```

$$mA := \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$

```
> mB:=matrix([[4,-2],[1,-1]]);
```

$$mB := \begin{bmatrix} 4 & -2 \\ 1 & -1 \end{bmatrix}$$

```
> mC:=matrix([[3,2,1],[-1,0,3]]);
```

$$mC := \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 3 \end{bmatrix}$$

```
> evalm(mA&*mB);
```

$$\begin{bmatrix} 11 & -7 \\ 2 & 0 \end{bmatrix}$$

```
> trace(evalm(mA&*mB));
```

$$11$$

```
> det(evalm(mA&*mB));
```

$$14$$

```
> transpose(evalm(mA&*mC));
```

$$\begin{bmatrix} 3 & 5 \\ 4 & 2 \\ 11 & -5 \end{bmatrix}$$

```
> inverse(mB);
```

$$\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & -2 \end{bmatrix}$$

```
> eigenvals(mA);
```

$$\sqrt{7}, -\sqrt{7}$$

```
> eigenvals(mB);
```

$$\frac{3}{2}+\frac{1}{2}\sqrt{17}, \frac{3}{2}-\frac{1}{2}\sqrt{17}$$

```
> eigenvects(mA);
```

$$\left[\sqrt{7}, 1, \left\{\left[1, \frac{1}{3}\sqrt{7}-\frac{2}{3}\right]\right\}\right], \left[-\sqrt{7}, 1, \left\{\left[1, -\frac{1}{3}\sqrt{7}-\frac{2}{3}\right]\right\}\right]$$

```
> eigenvects(mB);
```

$$\left[\frac{3}{2}+\frac{1}{2}\sqrt{17}, 1, \left\{\left[1, \frac{5}{4}-\frac{1}{4}\sqrt{17}\right]\right\}\right], \left[\frac{3}{2}-\frac{1}{2}\sqrt{17}, 1, \left\{\left[1, \frac{5}{4}+\frac{1}{4}\sqrt{17}\right]\right\}\right]$$

```
> charpoly(mA,lambda);
```

$$\lambda^2 - 7$$

Question 12

Solve the following system using either *Mathematica* or *Maple*

$x_{t+1} = -5 + x_t - 2\,y_t$

$y_{t+1} = 4 + x_t - y_t$

$x_0 = 1, y_0 = 2$

```
> solve({x=-5+x-2*y,y=4+x-y},{x,y});
```

$$\left\{y = \frac{-5}{2}, x = -9\right\}$$

```
> sol=rsolve({x(t+1)=-5+x(t)-2*y(t),y(t+1)=4+x(t)-y(t),x(0)=1,y
  (0)=2},{x(t),y(t)});
```

$$\{\mathrm{y}(t) = 3\,(-1)^t - 2\,2^t, \mathrm{z}(t) = -18\,(-1)^t + 6\,2^t, \mathrm{x}(t) = 6\,(-1)^t + 2\,2^t\} = \{$$

$$\mathrm{x}(t) = -9 + 5\,(-I)^t + 5\,I^t + \frac{1}{2}I\,(-I)^t - \frac{1}{2}I\,I^t, \mathrm{y}(t) = -\frac{5}{2} + \frac{9}{4}(-I)^t + \frac{11}{4}I\,(-I)^t + \frac{9}{4}I^t - \frac{11}{4}I\,I^t$$

$$\}$$

```
> solx:=-9+5*(-I)^t+5*I^t+1/2*I*(-I)^t-1/2*I*I^t;
```

$$solx := -9 + 5\,(-I)^t + 5\,I^t + \frac{1}{2}I\,(-I)^t - \frac{1}{2}I\,I^t$$

```
> soly:=-5/2+9/4*(-I)^t+11/4*I*(-I)^t+9/4*I^t-11/4*I*I^t;
```

$$soly := -\frac{5}{2} + \frac{9}{4}(-I)^t + \frac{11}{4}I\,(-I)^t + \frac{9}{4}I^t - \frac{11}{4}I\,I^t$$

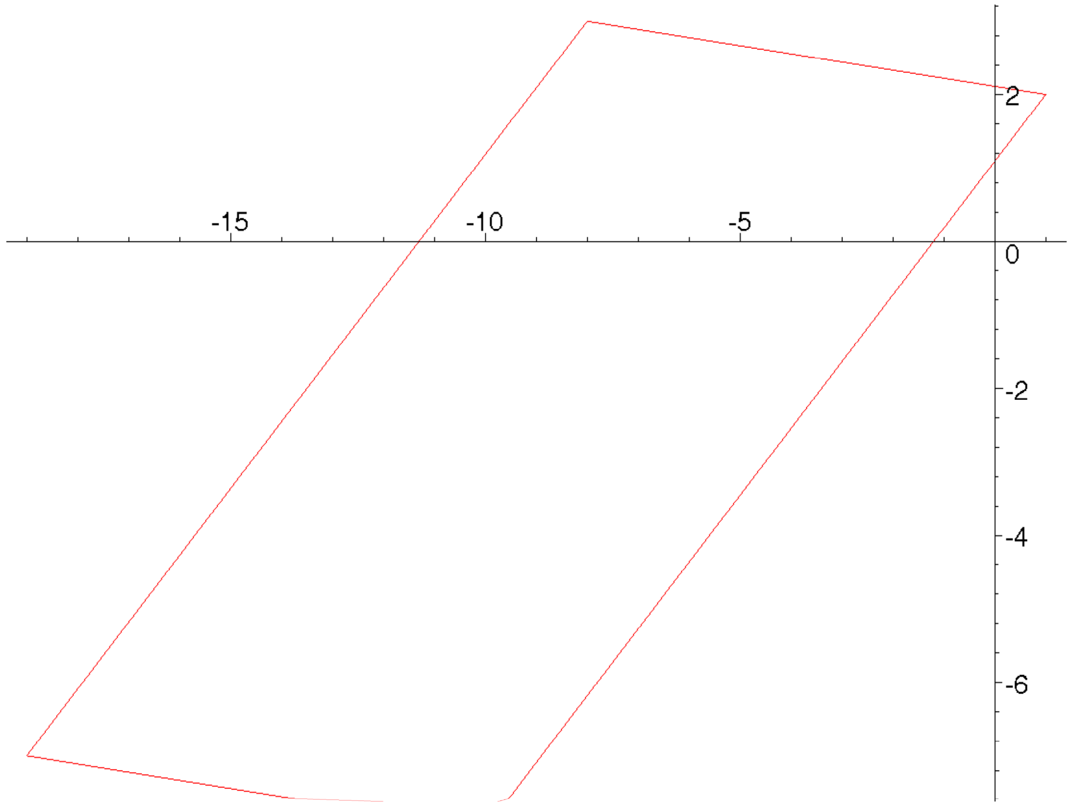
```
> seq({solx,soly},t=0..10);
```

{1, 2}, {-8, 3}, {-19, -7}, {-10, -8}, {1, 2}, {-8, 3}, {-19, -7}, {-10, -8}, {1, 2}, {-8, 3}, {-19, -7}

```
> dataset:=[seq([solx,soly],t=0..10)];
```

dataset := [[1, 2], [-8, 3], [-19, -7], [-10, -8], [1, 2], [-8, 3], [-19, -7], [-10, -8], [1, 2], [-8, 3], [-19, -7]]

```
> plot(dataset);
```



Question 13

```
> mA:=matrix([[1,-2],[1,-1]]);
```

$$mA := \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

```
> J:=jordan(mA,'V');
```

$$J := \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

```
> print(V);
```

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{2}I & \frac{1}{2}-\frac{1}{2}I \\ \frac{1}{2}I & \frac{-1}{2}I \end{bmatrix}$$

```
> evalm(V^(-1)&*mA&*V);
```

$$\begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

Question 14

We have already solved for the fixed point in question 12 and shown that for this model the system is cyclical. The question is spefically to set up the problem on a spreadsheet to show that the same cyclical pattern results.

Question 15

```
> solve({x=5.6-0.4*x,y=3.5+0.4*x-0.5*y},{x,y});
```

$$\{y = 3.400000000, x = 4.\}$$

```
> mA:=matrix([[-0.4,0],[0.4,-0.5]]);
```

$$mA := \begin{bmatrix} -.4 & 0 \\ .4 & -.5 \end{bmatrix}$$

```
> J:=jordan(mA,'V');
```

$$J := \begin{bmatrix} -.4000000000 & 0 \\ 0 & -.5000000000 \end{bmatrix}$$

```
> print(V);
```

$$\begin{bmatrix} 1.000000000 & 0. \\ 4.000000000 & -4.000000000 \end{bmatrix}$$

```
> evalm(V^(-1)&*mA&*V);
```

$$\begin{bmatrix} -.4000000000 & -0. \\ 0. & -.5000000000 \end{bmatrix}$$

```
> simplify(evalm(V^(-1)&*mA&*V));
```

$$\begin{bmatrix} -.4000000000 & 0. \\ 0. & -.5000000000 \end{bmatrix}$$

```
> u0:=matrix([[2],[1]]);
```

$$u0 := \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

```
> evalm(V^(-1)&*u0);
```

$$\begin{bmatrix} 2.000000000 \\ 1.750000000 \end{bmatrix}$$

```
> eigenvalues(mA);
```

$$-.5000000000, -.4000000000$$

```
> ustar:=matrix([[4],[3.4]]);
```

$$ustar := \begin{bmatrix} 4 \\ 3.4 \end{bmatrix}$$

```
> u0_ustar:=evalm(u0-ustar);
```

$$u0\_ustar := \begin{bmatrix} -2 \\ -2.4 \end{bmatrix}$$

```
> evalm(V^(-1)&*u0_ustar);
```

$$\begin{bmatrix} -2.000000000 \\ -1.400000000 \end{bmatrix}$$

Although the question requires the problem to be set up on a spreadsheet, we shall here use

Maple to set out the plots of the original system and its canonical form.

```
> sol15=rsolve({x(t)=5.6-0.4*x(t-1),y(t)=3.5+0.4*x(t-1)-0.5*y(t-1),x(0)=2,y(0)=1},{x(t),y(t)});
```

$$sol15 = \{y(t) = -8\left(\frac{-2}{5}\right)^t + \frac{28}{5}\left(\frac{-1}{2}\right)^t + \frac{17}{5}, x(t) = -2\left(\frac{-2}{5}\right)^t + 4\}$$

```
> solx:=-2*(-2/5)^t+4;
```

$$solx := -2\left(\frac{-2}{5}\right)^t + 4$$

```
> soly:=-8*(-2/5)^t+28/5*(-1/2)^t+17/5;
```

$$soly := -8\left(\frac{-2}{5}\right)^t + \frac{28}{5}\left(\frac{-1}{2}\right)^t + \frac{17}{5}$$

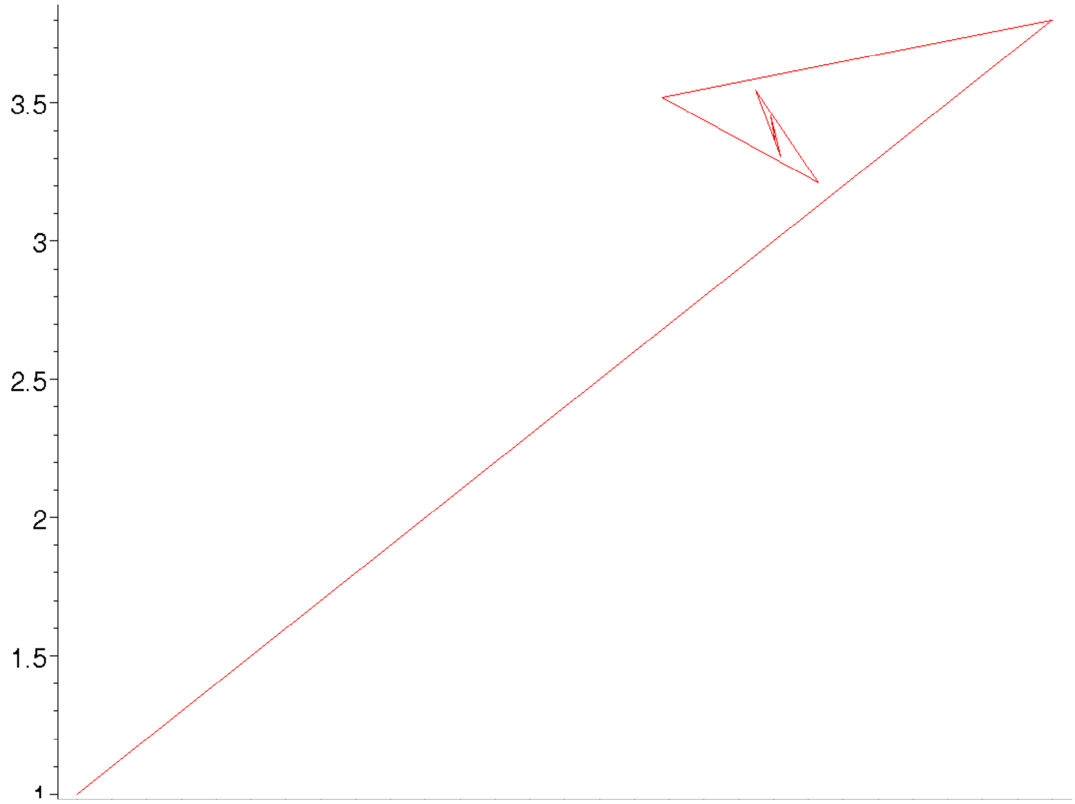
```
> seq({solx,soly},t=0..10);
```

$$\{1, 2\}, \{\frac{19}{5}, \frac{24}{5}\}, \{\frac{88}{25}, \frac{92}{25}\}, \{\frac{803}{250}, \frac{516}{125}\}, \{\frac{8863}{2500}, \frac{2468}{625}\}, \{\frac{82673}{25000}, \frac{12564}{3125}\},$$
$$\{\frac{62372}{15625}, \frac{863683}{250000}\}, \{\frac{8423393}{2500000}, \frac{312756}{78125}\}, \{\frac{85415803}{25000000}, \frac{1561988}{390625}\}, \{\frac{847789913}{250000000}, \frac{7813524}{1953125}\},$$
$$\{\frac{8511574723}{2500000000}, \frac{39060452}{9765625}\}$$

```
> dataset1:=[seq([solx,soly],t=0..10)];
```

$$dataset1 := \left[[2, 1], \left[\frac{24}{5}, \frac{19}{5}\right], \left[\frac{92}{25}, \frac{88}{25}\right], \left[\frac{516}{125}, \frac{803}{250}\right], \left[\frac{2468}{625}, \frac{8863}{2500}\right], \left[\frac{12564}{3125}, \frac{82673}{25000}\right],\right.$$
$$\left[\frac{62372}{15625}, \frac{863683}{250000}\right], \left[\frac{312756}{78125}, \frac{8423393}{2500000}\right], \left[\frac{1561988}{390625}, \frac{85415803}{25000000}\right], \left[\frac{7813524}{1953125}, \frac{847789913}{250000000}\right],$$
$$\left.\left[\frac{39060452}{9765625}, \frac{8511574723}{2500000000}\right]\right]$$

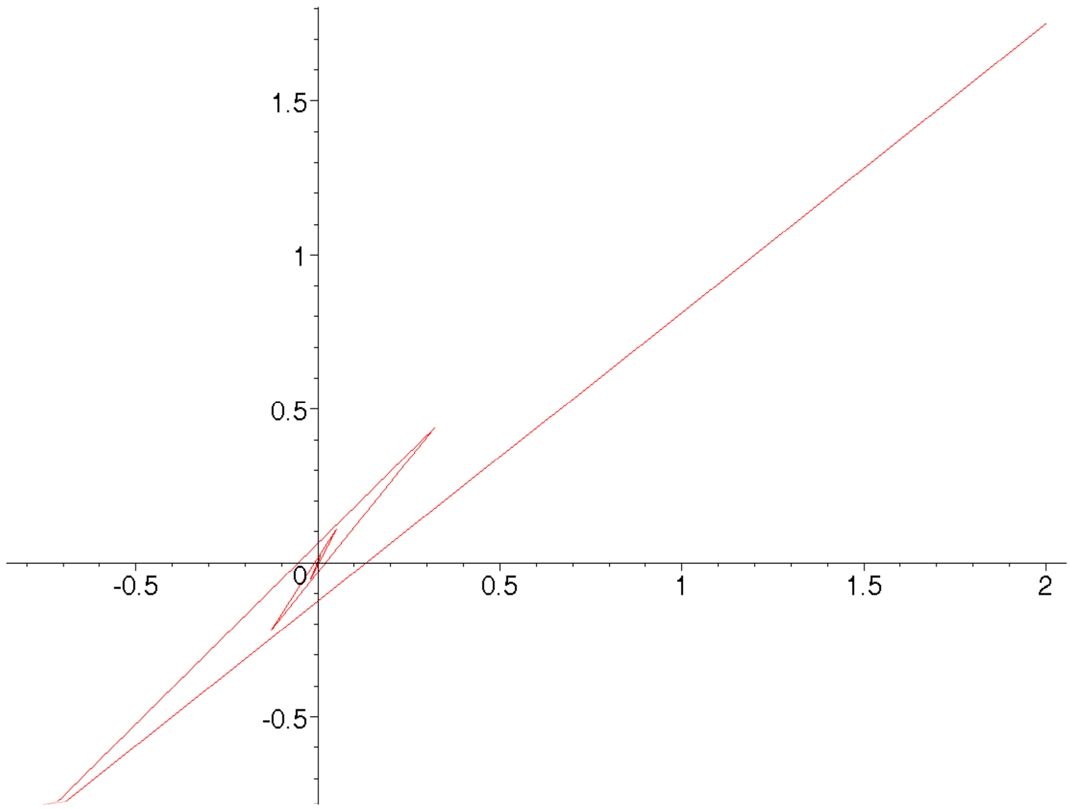
```
> plot(dataset1);
```




```
> dataset2:=[seq([((-0.4)^t)*2,((-0.5)^t)*1.75],t=0..10)];
```

dataset2 := [[2., 1.75], [-.8, -.875], [.32, .4375], [-.128, -.21875], [.0512, .109375], [-.02048, -.0546875], [.008192, .02734375], [-.0032768, -.013671875], [.00131072, .0068359375], [-.000524288, -.00341796875], [.0002097152, .001708984375]]

```
> plot(dataset2);
```



Both the original plot and itsc canonical form indicates that the system is asymptotically stable.