

INSTRUCTOR'S
MANUAL
FOR
*COMPUTABILITY
AND LOGIC*
FIFTH EDITION

PART A.
FOR ALL READERS

JOHN P. BURGESS

Professor of Philosophy
Princeton University
jburgess@princeton.edu

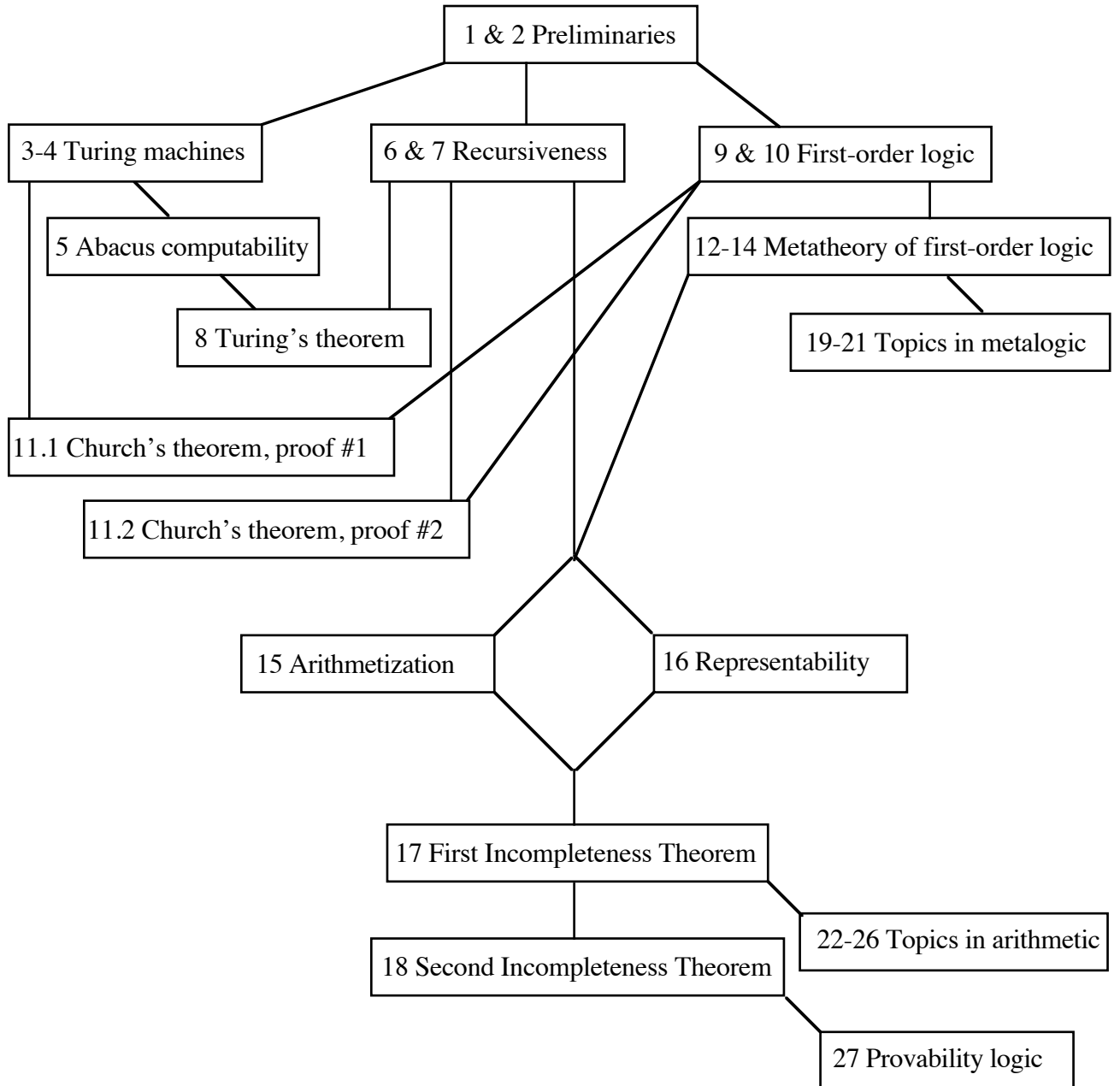
Note

*This work is subject to copyright,
but instructors who adopt Computability & Logic as a textbook
are hereby authorized to copy and distribute the present Part A.
This permission does not extend to Part B.*

Contents

Dependence of Chapters (<i>Leitfaden</i>)	2
General Remarks on Problems (for Students)	3
Hints for Odd-Numbered Problems Computability Theory	4
Hints for Odd-Numbered Problems Basic Metalogic	11
Errata	20

Dependence of Chapters



General Remarks on Problems (for Students)

- The problems for each chapter should be read as part of that chapter, even those that are not assigned. They often add important information not covered in the text.
- The results of earlier problems, whether or not assigned, may be used in later problems. Many problems are parts of sequences.
- Before working the problems for any chapter, check to see whether there are any errata listed for that chapter, and especially for its problems.
- Hints are provided for odd-numbered problems in chapters 1-18. The hints for some problems are inevitably more substantial than those for others.

Hints for Odd-Numbered Problems: Computability Theory (Chapters 1-8)

Chapter 1

1.1 The converse assertion then follows from the first assertion by applying it to f^{-1} and its inverse f^{-1-1} .

1.3 For (a) consider the *identity* function $i(a) = a$ for all a in A . For (b) and (c) use the preceding two problems, as *per* the general hint above.

1.5 Show both sets are denumerable.

1.7 If we can fix for each i an enumeration of A_i

$$A_i = \{a_{i1}, a_{i2}, a_{i3}, \dots\}$$

Then we can enumerate $\bigcup A$, which is the set of all a_{ij} for all i and j in the same way we enumerated pairs (i, j) in Example 1.2. However, when we assume that for each A_i there exists an enumeration of it, it follows that there exist many different such enumerations for each A_i ; and when set theory is developed rigorously, in order to conclude that there is a way of fixing *simultaneously* for *each* i some one, specific enumeration out of all the many different enumerations that exist, we need a principle known as the *axiom of choice*. As this is not a textbook of set theory, we are not going to go into such subtleties.

Chapter 2

2.1 Imitate the proof for the set of positive integers.

2.3 You do *not* need to use trigonometry or give an analytical formula for the correspondence to do this problem; a simple geometric description of a correspondence will be enough.

2.5 While this can be done using the preceding two problems, as *per* the general hint, for students who remember trigonometry, a correspondence can also be defined directly using the tangent function.

2.7 Note that rational numbers whose denominator (when written in lowest terms) is a power of two have two binary representations, one ending in all 0's and the other in all 1's from some point on (as in $1/2 = .1000000\dots = .0111111\dots$), while *in every other case the binary representation is unique and does not involve all 0's or all 1's from any point on*.

2.9 In addition to the immediately preceding problems, Problem 1.6 may be useful.

2.11 Read carefully through the sequence of preceding problems.

2.13 This is a philosophical rather than a mathematical question, and as such does not have a universally agreed answer, though there is a consensus that somehow *defining a set in terms of the notion of definability itself* is somehow to blame for the paradox.

Chapters 3 & 4

3.1 One state will be required in (a), two in (b).

3.3 Proceed as in Problem 3.1(b) but when reaching a blank in state 2 print a stroke and go into state 3. At this stage you will have a block of n strokes followed by a blank followed by a block of $m + 1 + k$ strokes. In state 3 on a stroke move right and go into state 4. In state 4 on a stroke erase it. You will now have blocks of n , $m + 1$, and $k - 1$ strokes. Take it from there.

3.5 Proceed in cycles, during each of which you erase the leftmost stroke of the first block and the rightmost stroke of the second block, and add a stroke to a third block to the right of them both. When one of the two original blocks has been completely erased, erase also the other. The trick is to keep track of when this happens.

4.1 It is certainly not possible just *exploring* without *marking* the tape.

4.3 It is not possible to preserve the original block unaltered while making a copy.

4.5 A description of a function of the kind a universal machine would have to compute is implicit in the discussion of the diagonal function in the text.

Chapter 5

5.1 Subtraction is to the predecessor function as addition is to the successor function.

5.3 Use problem 5.1.

5.5 Keep subtracting y from x , while checking each time you do so that what is left is still $\geq y$.

5.7 Manœuvres of just this kind take place the simulation of abacus machines by Turing machines.

5.9 See preceding problems.

5.11 See the proof of Theorem 4.1.

Chapter 6

6.1 For instance, in (a), $g(x, y) = f(\text{id}_2^2(x, y), \text{id}_1^2(x, y))$.

6.3 These can be done ‘from scratch’ or, generally more easily, by showing the indicated functions are compositions of functions already known to be primitive recursive.

6.5 Proposition 6.5 may be useful.

6.7 Each recursive function is denoted by some expression built up using Cn, Pr, and Mn from names for the zero, successor, and identity functions.

6.9 Use the following fact: There is a recursive function f such that $f(0) = 0$ but $f(x)$ is undefined for $x > 0$. (For instance, $f(x) =$ the least y such that $|x - y| + y = 0$.)

Chapter 7

7.1 Compare with Problem 6.1.

7.3 Use Corollary 7.8.

7.5 Consider the auxiliary function $g(n) =$ the least element of A that is $> n$.

7.7 Apply the preceding two problems to obtain a recursive function a and use it and the original f to define a suitable g .

7.9 First show that the auxiliary function $g(n) = J(f(n), f(n + 1))$ is primitive recursive, where J is as in Problems 6.2 and 6.5.

7.11 First introduce a suitable auxiliary function, as in Example 7.20.

7.13 Suppose that c_i and d are the numbers associated with g_i and f respectively, so that

$$g_i(x_1, \dots, x_n) < c_i \max(x_1, \dots, x_n) + c_i,$$

$$f(y_1, \dots, y_m) < d \max(y_1, \dots, y_m) + d.$$

Show that $d(c + 1)$ will do as a number associated with h .

7.15 This is the problem that requires most familiarity with mathematical induction, according to which, in order to prove that all x and all y have some property it is enough to show that

(1) 0 and 0 have the property

(2) if 0 and j have the property, then 0 and $j + 1$ have the property

and that if i is such that i and j have the property for all j , then

(3) $i + 1$ and 0 have the property

(4) if $i + 1$ and k have the property, then $i + 1$ and $k + 1$ have the property.

7.17 First show that the auxiliary function

$$f(p, q) = \text{the least } s \text{ that covers } (p, q)$$

is a recursive total function.

Chapter 8

- 8.1** Remember that the right numeral is obtained by reading *backwards*, so that if $x_1 = 2$ and $x_2 = 3$, say, then the right numeral is 11110111.
- 8.3** Use the graph theorems.
- 8.5** Use the fact, noted just before the statement of Theorem 8.5 that the graph relation of the universal function F constructed in the proof of that theorem has the form $F(m, x) = y \Leftrightarrow \exists t Qmxyt$ where Q is *primitive recursive*.
- 8.7** See the problems for chapter 7.
- 8.9** Let A be as in the proof of Corollary 8.8.
- 8.11** Show that if this claim failed for some f , then A would be recursive.

Hints for Odd-Numbered Problems: Basic Metalogic (Chapters 9-18)

Chapter 9

9.1 For readers who have not previously studied logic, or whose memories of their previous study of logic are rusty, there will be one subtlety here, over how to represent ‘All Ms are Ss’. For an indication of the manner in which this construction is treated in modern logic, see displayed formulas (9) and (10) in section 9.1.

9.3 Here ‘in colloquial terms’ would mean, for instance, saying ‘grandparent’ rather than ‘parent of a parent’.

9.5 Use induction on complexity. In connection with the conjunction case, for instance, you will need the fact that the only subformulas of $(F \& G)$ are itself and the subformulas of F and G .

9.7 We do (c) as an example. If $(F \& B)$ is to be anything less than the whole of $(F \& G)$, then B must be a left part of G , and hence by the Lemma 9.4(c) must have an excess of left over right parentheses. But this is impossible, since B , being a formula, has equally many parentheses of each kind, and therefore B has one more right parenthesis than it has left parentheses.

Chapter 10

10.1 First show that substituting t for c in a closed term does not change the denotation of the term.

10.3 You will need to describe an interpretation, specifying its domain and the two-place relation on it that is to serve as the denotation of R . Reading R as ‘greater than’ may help suggest one.

10.5 In mathematics, ‘All A s are B s’ counts as ‘vacuously’ true if there are no A s.

10.7 Compare with Example 10.3(d).

10.9 Compare with Example 10.5.

10.11 For (c), think of replacing A by B as a two-step process: introduce a new atomic C , and first replace A by C , then C by B .

10.13 For (a), the result for multiple variables is immediate from the result for a single replacement, on repeated application of the latter. To prove the result for a single variable, define a transformation $*$ on formulas, eliminating bound occurrences of the variable y , by induction on complexity as follows. For an atomic formula G , let $G^* = G$. If $G = \sim F$, let $G^* = \sim F^*$, and if $G = (F_1 \ \& \ F_2)$, let $G^* = (F_1^* \ \& \ F_2^*)$, and similarly for \vee . If $G = \forall x F(x)$, where x is a variable other than y , let $G^* = \forall x F^*(x)$, while if $G = \forall y F(y)$, let $G^* = \forall z F^*(z)$, where z is the alphabetically first variable not already occurring, and similarly for \exists . It remains to prove G and G^* are equivalent for any sentence G .

Chapter 11

11.1 Describe how to obtain a solution to the decision problem for implication from a solution to the decision problem for validity.

11.3 Show that the second premiss implies

$$\forall u \forall v \forall w ((\exists y (Rwy \ \& \ Syv) \ \& \ Suv) \rightarrow Rwu)$$

11.5 There is no help for it but to reread the proof carefully.

11.7 Re-examine the proof in section 11.1, then modify the definition of *standard interpretation* so that the domain consists only of the operating interval for the computation as defined in Problem 11.6.

11.9 Try proving it first for $n = 0$, then for $n = 1$, then for $n = 2$, and so on.

11.11 First show that the function *stdh* in the proof of Theorem 8.2 is a *three-place* function such that the set of pairs (x, y) for which there exists a z with $\text{stdh}(x, y, z)$ is non-recursive.

Chapter 12

12.1 What does A tell us about the relative numbers of elements in the domain satisfying Px and satisfying $\sim Px$?

12.3 This can be done with a language having a one-place predicate Px and two one-place function symbols f and g . The trick is to find a sentence saying that there are as many elements in the domain altogether as there are *pairs* of elements satisfying Px .

12.5 Label the vertices in clockwise order A, B, C, D , and label the sides suggestively as $a = AB, b = BC, c = CD, d = DA$.

12.7 In the days before modern computers and calculators, a shortcut used with multiplication problems was to turn them into addition problems. How was this done?

12.9 If \mathcal{M} is a model of Δ , and if j were an isomorphism from \mathcal{M} to the standard model \mathcal{N} , what would be $j(c^{\mathcal{M}})$?

12.11 Combine the methods of the appropriate parts of the preceding problem.

12.13 Given a correspondence f from N to X_1 , call one element a of X_1 less than another element b of X_1 if $f^{-1}(a)$ is less than $f^{-1}(b)$ in the usual order on natural numbers. Let $a_{0,0}$ be $f^{-1}(0)$, the least element of X_1 . For each k let $a_{k+1,0}$ be the least element of X_1 not E_1 -equivalent to any $a_{i,0}$ for $i \leq k$. For each m let $a_{k,m+1}$ be the least element of X_1 that is equivalent to $a_{k,0}$ and not identical to any $a_{k,i}$ for any $i \leq m$.

12.15 See Problem 10.6

12.17 Use the preceding problem and the observation that for any one, given denumerable nonstandard model or arithmetic, the set of sets of primes encrypted in that model is enumerable, since the set of elements of the domain available to encrypt sets of primes is.

12.19 Look at the problems to follow.

12.21 List the elements of the domain of j in increasing $<_A$ order as a_0, a_1, \dots, a_n , and let $b_i = j(a_i)$, so that $b_0 < b_1 < \dots < b_n$ in the usual order on natural numbers. What the problem asks you to show is that, given any new a in A there will be a rational number b such that b is related to the b_i in the usual order on rational numbers in the same way a is related to the a_i .

12.23 It will suffice to build a sequence of finite partial isomorphisms j_i as in Problem 12.22. Problem 12.21 can be used to get from j_i to j_{i+1} , but some care will be needed to arrange that every element of A gets into the domain of some j_i eventually.

12.25 Proceed as in Problem 12.23, but this time also take care to arrange that every rational number gets into the range of some j_i .

12.27 The preceding problems do not yet cover all the possibilities.

Chapter 13

13.1, 13.3, 13.5, 13.7 Hints are given in the text of section 13.5.

13.9 Imitate the proof of the isomorphism lemma, Proposition 12.5.

13.11 For (a) use the preceding problem; for (b) first note that if $B(c)$ implies A and c does not appear in A , then $\exists x B(x)$ implies A . (For if $\exists x B(x)$ does not imply A , then $\{\exists x B(x), \sim A\}$ is satisfiable, and then by Example 10.5(b) so is $\{B(c), \sim A\}$, and $B(c)$ does not imply A .)

13.13 See Problem 13.12.

13.15 See Problem 12.18.

Chapter 14

- 14.1** The compactness theorem is relevant.
- 14.3** Look how we got from (2) to (7) in Example 14.4.
- 14.5** Look how we got from (2) to (6) in Example 14.12.
- 14.7** Remember that you may use the results of earlier problems.
- 14.9** As in Example 14.13, all rides on making a suitable choice of formula $A(x)$ to apply (R8) to.
- 14.11** Imitate the proof of the inversion lemma for negation.
- 14.13** To show the effect of (R11) can be obtained using (R12), use the relevant inversion lemmas.

Chapter 15

15.1 The length is the number of digits in the decimal expansion of e that are < 8 .

15.3 Apply Corollaries 12.17 and 15.7 to $T \cup \{A\}$ and $T \cup \{\sim A\}$ where neither A nor $\sim A$ is a theorem of T .

15.5 How many of the A_i would it take to deduce all the B_j ?

15.7 The idea is just to 'check for each n through all possible models of size n ', or more precisely, through a set of possible models containing at least one representative of each isomorphism type of models of size n . Generalize the preceding problem appropriately to show the set of isomorphism type representatives for a fixed n can be taken to be finite.

15.9 Let R be a recursive relation such that a is the code number of theorem of T if and only if $\exists n Ran$. Consider the set of sentences B such that for some A and n , B is the conjunction of n copies of A , and Ran holds, where a is the code number of A .

Chapter 16

16.1 Use Lemma 16.6.

16.3 Use Theorem 16.13.

16.5 See the proof of Corollary 15.6(a).

16.7 Use Proposition 7.17 and the remark following.

16.9 Again use Proposition 7.17.

16.11 Recall that we have proved $0 + y = y$ and $1 + y = y'$ have been proved in Examples 16.18 and 16.19.

16.13 For (c) first note that there is a least n with the property 'there is a sequence of length n with property P .'

16.15 To make (Q1)-(Q2) and (Q7)-(Q10) true, the denotation of $\mathbf{0}$ should be taken to be the least pair (in the $<_2$ -order) and the denotation of $'$ the function that given any pair as argument yields as value the least pair (in the $<_2$ -order) among the pairs greater (in the $<_2$ -order). It remains to devise a suitable addition function.

16.17 Use 'induction in the metalanguage,' proving the result first for $m = 0$, then for $m = n'$ assuming it holds for n .

16.19 For (a) again use 'induction in the metalanguage,' proving the result first for $b = 0$, then for $b = c'$.

16.21 The first half of the problem is to show how, using induction and the axioms of \mathbf{Q} , to obtain the two axioms of \mathbf{R} that are not axioms of \mathbf{Q} . But one of these, (Q0), has already been done as Example 16.17, so it only remains to do (Q11). The other half of the problem is to show how, using induction and the axioms of \mathbf{R} , to obtain the four axioms of \mathbf{Q} that are not axioms of \mathbf{R} . But two of these, (Q7) and (Q9), have already been done in section 16.4, so it only remains to do (Q8) and (Q10). For the first half of the problem note that according to Problems 16.10 and 16.11, we can get the commutative law for addition using induction and axioms common to \mathbf{Q} and \mathbf{R} .

Chapter 17

17.1 Use Theorem 16.16 and Problem 16.9.

17.3 Imitate the proof of the diagonal lemma, beginning as follows: For formulas $E_1(x, y)$ and $E_2(x, y)$ with code numbers e_1 and e_2 , let the first and second double diagonals be

$\exists x \exists y (x = \mathbf{e}_1 \ \& \ y = \mathbf{e}_2 \ \& \ E_1(x, y))$, logically equivalent to $E_1(\mathbf{e}_1, \mathbf{e}_2)$ and

$\exists x \exists y (x = \mathbf{e}_1 \ \& \ y = \mathbf{e}_2 \ \& \ E_2(x, y))$, logically equivalent to $E_2(\mathbf{e}_1, \mathbf{e}_2)$.

17.5 The logical equivalence of $A(t)$ and $\exists y (y = t \ \& \ A(y))$ will be useful.

17.7 To begin with R be the Rosser sentence of T , $T_0 = T + \{\sim R\}$, $T_1 = T + \{R\}$. Let R_e for $e = 0$ or 1 be the Rosser sentence of T_e . Let $T_{00} = T_0 + \{\sim R_0\}$, $T_{01} = T_0 + \{R_0\}$, $T_{10} = T_1 + \{\sim R_1\}$, $T_{11} = T_1 + \{R_1\}$, and continue in this way.

17.9 Use the Craig reaxiomatization lemma, Problem 15.9.

17.11 To obtain \mathcal{N} , take the set of elements satisfying $N(x)$ as the domain $|\mathcal{N}|$. Let $R^{\mathcal{N}}$ hold for elements of the domain $|\mathcal{N}|$ if and only if $R^{\mathcal{M}}$ does, let $c^{\mathcal{N}} = c^{\mathcal{M}}$ for each constant c (as we may since it is given that $c^{\mathcal{M}}$ satisfies $N(x)$, that is, belongs to the domain $|\mathcal{N}|$), and let the value of $f^{\mathcal{N}}$ for elements of the domain $|\mathcal{N}|$ be the same as the value of $f^{\mathcal{M}}$ (as we may since it is given that if elements satisfy $N(x)$ so does the value of f for those elements).

17.13 It is enough to find a formula $N(x)$ that is satisfied in \mathcal{Z} by an integer if and only if that integer is non-negative, for then by Problem 17.12, relativization will be a translation, and by Problem 17.10 the set T of sentences true in \mathcal{Z} will be undecidable since the set S of sentences true in \mathcal{N} is undecidable. If we have \prec , we can simply use $x = \mathbf{0} \vee \mathbf{0} \prec x$ for $N(x)$. To find an $N(x)$ that does *not* involve \prec requires a major result of number theory, but one that has been mentioned more than once in this book.

Chapter 18

No problems.

Errata

Please report any discovered to jburgess@princeton.edu

Thanks to Mauro Allegranza, E. Biedermann, Nan Chen, Sean Duggan, Michael Forbes, David Furcy, David Keyt, Ran Lanzet, Yuval Masory, Cory Nichols, Christopher Pincock, Peter Smith, Bram van Heuveln.

p. xii, line 10, read "first incompleteness" for "first completeness"

p. xiii, line 4, read "Saul Kripke" (as in all earlier editions) for "Paul Kripke"

p. 6, two lines after displayed definition of $s(n)$: read "of other sorts" for "of others sorts"

p. 15, problem 1.5(a), first line: read "positive rational numbers less than one" for "rational numbers"; last line: add "and $m < n$ " before final period.

p. 24, line 7: read "write smaller and smaller" for "writer smaller and smaller"

p. 24, line 15 from bottom: read "for most of us" for "for most us"

p. 26, line 13: read "of the box" for "of box"

p. 27, line 4 from bottom: reader "shortened" for "shorted"

p. 31, two lines above specification (a): read "values are represented" for "values represented"

p. 37, three lines below **Theorem 4.1**: read "we were able to compute" for "we were able compute"

p. 38, line 11 from bottom: read "standard" for "stardard"

p. 39, line 14: delete the remark in parentheses. [Hints are not longer at the back of the book, but are now in the Instructor's Manual.]

p. 42, Example 4.6: read " $\geq 2n$ " for " $\geq 2p(n)$ "

p. 42, statement of Example 4.6, read " $\geq 2n$ " for " $\geq 2p(n)$ "

p. 43 Figure 4-3: replace each of the three occurrences of " k " by " j "
[alternatively, leave the figure as is, and change the text of the last paragraph on p. 43 through the end of the proof on p. 44, replacing each " j " by " k " and the three occurrences of " k " in the last three lines of the proof by " j "]

p. 45, Abstract, line 3 from bottom: read "an abacus" for "a abacus"

p.51, middle, paragraph (b), first line: read "stones in some" for "stones is some"

p. 52, two lines below displayed equation: 1 should be the subscript, not superscript; r should be the superscript, not subscript.

p. 58, four lines from bottom: read "function f " for "functions f "

p. 60, lines 5 and 6 from bottom: read i for j both places

p. 60, line 2 from bottom: read " $f(x, 0) = 0$ " for " $f(x, 1) = 0$ "

p.62, problem 5.8, first line: read "given" for "give"

p. 64, line 11: read "any" for "any any"

p.77, end of proof of Theorem 7.4: replace the last words "only replace y by $y-1$ " by "to make some slight changes"

p. 78, Example 7.7, 2nd line after displayed formula:
add "or ($y = 0$ & $z = x$)" after " $y \cdot z < x$ "

p.78, Corollary 7.8, first line read "regular primitive recursive function" for "regular primitive function"

p. 79, line 10 from bottom: read "argue" for "agruue"

p.86, Problem 7.3, line 6:

add " $z > 0$ and " before $x \mid z$ "

and line 7:

add before period at end of next-to-last sentence:

"[except that, by convention, we let $\text{lcm}(x, 0) = \text{lcm}(0, y) = 0$]"

p.87, problem 7.15, three displayed lines (definition of β -code):

first line: read \leq for $<$

second line: add $+ 1$ at the end

p.93 displayed equation defining stdh : the last term should be:

$\text{nstd}(\text{left}(\text{conf}(m, x, t)), \text{right}(\text{conf}(m, x, t)))$

p. 92, displayed definition of newstat : add at end before period: " $\cdot \text{sg}(q)$ "

p.97, proof of Corollary 8.8: Replace the third sentence by "If it were recursive, its characteristic function c would be a recursive function." and delete the words "the complement of" from the fifth sentence.

p.113, problem 9.8: read "How could we change the definition of formula" for "How could we change the definition".

p. 126, line 4 from end of italicized chapter summary: Read "not making any avoidable use of Turing's or Church's thesis" for "not depending on Turing's or Church's thesis"

p. 128, displayed formula (5): read $m > 0$ for $m \neq 0$

p. 129, equation (12): read \mathbf{Q}_1 for \mathbf{Q}_0

p. 132, line 5 of §11.2: read "contains the constant" for "contains the constants"

p. 132, line 2 from bottom: read "from 0 to r " for "from 1 to r "

p. 133, line 7: read " ≥ 0 " for " > 0 ".

p. 133, line 7 from bottom: read "where all sentences" for "where also sentences"

p. 134, displayed formula (6a): read " $\mathbf{f}_j(\mathbf{a})$ " for " $\mathbf{f}_j(\mathbf{a}, \mathbf{0})$ "

- p. 134, displayed formula (7b): read " $f_i(\mathbf{a}, \mathbf{p})$ " for " $f_i(\mathbf{a}, \mathbf{p})$ "
- p. 134, line 14 from bottom: read " $\exists y$ " for " $\forall y$ "
- p. 136, problem **11.10** read "Problem 8.12" for "Problem 8.13"
- p. 136, problem **11.16** read "Theorem 11.14" for "Theorem 11.16"
- p. 152, problem **12.21**, line 4: read " $j(a_1) <_B j(a_2)$ " for " $j(a_1) <_A j(a_1)$ "
- p. 152, problem **12.22**, first line: insert "show that" after "Continuing the preceding problem,"
- p. 153, displayed formula (**S5**): delete brackets from " $\{\exists x B(x)\}$ " and delete "or in $\exists x B(x)$ "
- p. 154, statement of Lemma **13.2**, second line: read "all sets of sentences" for "all sets of formulas"
- p. 154, proof of Lemma **13.2**, line 9: read " $\Gamma_0 \cup \{B\}$ " for " $\Gamma \cup \{B\}$ "
- p. 154, proof of Lemma **13.2**, line 10: read " $\Gamma \cup \{B\}$ " for " $\Gamma_0 \cup \{B\}$ "
- p. 160, paragraph 2 of §13.4, last line: read (S1) for (S0)
- p. 162, line 6 from bottom: read "for every sentence" for "for every formula"
- p. 169, last paragraph, line 4: read "one or another" for "one of another"
- p. 172, Example 14.5, annotation to line (2): read (R2a) for (R2b)
- p. 172, Example 14.6, annotation to line (2): read (R2a) for (R2b)
- p. 172, Example 14.6, annotation to line (4): read (R2b) for (R2a)
- p. 172, Example 14.8, annotation to line (2): read (R2a) for (R2b)
- p. 172, Example 14.8, annotation to line (5): read (R2b) for (R2a)
- p. 173, Example 14.10, annotation to line (5): read (R2a) for (R2b)
- p. 178, verification of (S2): exchange annotations (R2a), (R2b)
- p. 178, verification of (S4), last annotation: read (R2b) for (R2a)
- p. 179, verification of (S6), last annotation: read (R2a) for (R2b)
- p. 179, verification of (S8), second annotation: read (R2a) for (R2b)

- p. 188, line 2 after table: read "7, '" for "7', " (i.e. transpose accent and comma)
- p. 189, 1st full ¶, line 5 from bottom: read Γ for Γ_0
- p. 191, last ¶, 2nd line: read T for T
- p. 193, lines 12-13 below table: replace all four occurrences of 36 by 8
- p. 193, lines 13 below table: replace 89 by "over 50"
- p. 195, second paragraph, lines 9-10: read "less than" for "less that"
- p. 201, line 2: the first subscript in the displayed formula should be 2, not 3, and the last subscript should be 3, not 2.
- p. 201, line 4: the first subscript should be 3, not 2
- p. 203, lines 9 and 8 from bottom: read $N + 1$ for N both places
- p. 204, example **16.7**, displayed formula: replace all four instances of $<$ by \leq
- p. 208, 1st ¶ of proof of Theorem 16.13, line 3: read "successor" for "sucursal"
- p. 212, proof of Theorem 16.16: the boldface **0** on the last line should be **1**.
- p. 212, Problem 16.9: omit the words "extending **Q**".
- p. 215, last line: read $\mathbf{m}' \neq \mathbf{n}$ for $\mathbf{m} \neq \mathbf{n}$
- p. 219, Problem 16.18. The italicized sentence at the end is not part of the problem and should be on a separate line
- p. 223, line 2 from bottom, delete "as it is in the axioms of **Q**,"
- p. 225, first line of proof of **17.8**. Read "then there is some n " for "then there is some a "

p. 226, paragraph after proof of **17.9**: corners around G_Q are missing in all three instances of " $\text{Prf}_Q(G_Q, \dots)$ "

p. 229, problem 17.2, line 3: read "in T " for "in P ".

p. 234, line 9 from bottom: read "Theorem 18.3" for "Theorem 18.1"

p. 234, line 7 from bottom: read Prv_T for Prv

p. 244, statement of **Proposition 19.2** It should read: Any formula built up by \sim and $\&$ and \vee from given formulas is logically equivalent to one in disjunctive normal form.

p. 245, line 1, the first sentence should read: First move negations inside junctions as in the proof of Proposition 19.1.

p. 245, first line after proof of Proposition 19.2: read "each disjunct" for "each disjunction"

p. 246, proof of Theorem 19.5, lines 8-9: read "so no variable appearing bound in one conjunct or disjunct appears in the other" for "so the conjuncts or disjuncts have no variables in common"

p. 249, displayed formula (S1): Read b_1 for (b_1) and b_n for (b_n)

p. 249, displayed formula (S2): delete parentheses

p. 249, next-to-last line: read $|A|$ for A

p. 250, Example 19.8, line 4 from bottom (= two lines below second displayed formula): read $(a + b)/2$ for $(a - b)/2$

p. 251, proof of Theorem 19.9, line 6: read "there will be an enumerable" for "there will be enumerable"

p. 252, displayed material, last line: delete initial $\forall z$

p. 252, line 9 from bottom: read "if there is no set" for "if there is set"

p. 254, second paragraph before **19.11**, first line: read "Proposition 19.7" for "Proposition 12.7" β

- p. 262 (S5): read "then" for "and then", delete final "or $\exists xF(x)$ "
- p. 262, 1st paragraph after (S6), line 2: read "S(0)-S(6)" for "S(0)-S(1)"
- p. 262, 1st paragraph after (S6), line 10: read "an unbarred" for "and unbarred"
- p. 263, line 8: subscript on 2nd D in line should be 1, not 2.
- p. 263, line 12: read "both" for "both both"
- p. 263, *Proof, Part II*, line 4: read "equivalence" for "equality"
- p. 265, Example 20.8, next-to-last line: delete ", while L is".
- p. 266, 3rd line after 2nd displayed formula: read "left" for "right"
- p. 270, line 3 from bottom: read "problem for full" for "problem full"
- p. 274 "*Proof of Lemma 21.7*" should be "*Proof of Lemma 21.8*"
- p. 274, Proof of Lemma 21.12, line 3: read "appears in B " for "appears in F "
- p. 275, 1st line after 1st displayed formula: read "occur free" for "occur"
- p. 275, 3rd line after 1st displayed formula: read "atomic or negated atomic" for "atomic"
- p. 275, both displayed formulas: subscripts r should be t
- p. 276, Proof of Lemma 21.3: read " v_1, \dots, v_k " for " u_1, \dots, u_k "
- p. 276, Proof of Lemma 21.4, line 4: delete subscript i on P
- p. 278, paragraph beginning "Also", line 3: read " $4n + 1$ " for " $4m + 1$ "
- p. 278, last paragraph of proof, 1st line: read Sxb for Swb
- p. 283, next-to-last paragraph of proof, next-to-last line: read "**m** be m " for "**m** by m "
- p. 285, problem 22.2, displayed formula: add a left parenthesis immediately to the right of the left bracket, and a right parenthesis immediately to the left of the final arrow
- p. 288, 2nd paragraph, line 2: read "either a is" for "either k is"
- p. 289, displayed item (5): last S should be B .

- p. 295, displayed formula: insert prime ' at end of left hand side of equation
- p. 296, 4th paragraph, line 3: read "a quantifier-free" for "and quantifier-free"
- p. 296, 5th paragraph, line 2: read "no additional" for "no addition"
- p. 297, item (12), displayed formulas (ii) and (iii): read boldface " \leq " for lightface " $<$ " in all three occurrences
- p. 297, item (12), 2nd last line: read "that if (i) or (ii) holds" for "that (i) or (ii) holds"
- p. 297, item (14), line 1: read "preceding two steps" for "preceding three steps"
- p. 298, item (21), first displayed line: last k should be a subscript like the others
- p. 298, last line: read $\mathbf{D}_m(i - j)$ for $\mathbf{D}_n(i - j)$
- p. 303, next-to-last paragraph, first sentence: the first "NUMBER" (before comma) should be "number", and the last "number" (before period) should be "NUMBER".
- p. 304, line 5 from bottom: read "and a an integer" for "and a and integer".
- p. 306, line just before displayed formula: the word "theory" should be "set"
- p. 307, 1st full paragraph, last line: read "for" for "for for"
- p. 310, Proof of Theorem 25.4b, 2nd line: read "Chapter 17" for "Chapter 16"
- p. 310, Proof of Theorem 25.4b, 1st displayed line in proof: read $\sim\exists z \leq y \beta(x, z)$ for $\sim\exists z \leq y \beta(x, y)$ and $\sim\exists z \leq y \alpha(x, z)$ for $\sim\exists z \leq y \alpha(x, y)$
- p. 311, 2nd displayed formula: should end with comma (,) not period (.)
- p. 312, line 11 from bottom: read " N^{**} of L^{**} " for " N^{**} of L^* "
- p. 312, line 9 from bottom: both occurrences of L should be L^*
- p. 314, item (3), read "formula $F(x)$ of L^{**} " for "formula $F(x)$ of L^* "
- p. 315, next-to-last paragraph, line 3: read "in $F(x)$ " for "in $F(X)$ "

p. 320, statement of Ramsey's theorem, line 3: read "subset Y of size at least n " for "size- n subset Y "

p. 323, line 6: read "nine nodes" for "ten nodes"

p. 327, §27.1, first paragraph, next to last line: after "sentences are sentences" insert "(also called formulas)"

p. 327, §27.1, 2nd paragraph, line 3: read "sentences" for "formulas"

p. 328, lines 9, 10, 11 from bottom: each $\wedge C$ should be $\wedge \Gamma$

p. 330, proof of Lemma 27.3, line (2): read 27.2(1) for 25.2(1)

p. 330, line 3 from bottom: read "of whose disjuncts" for "of whose conjuncts"

p. 331, 2nd full paragraph, lines 3-4, read " B , C , and D are subsentences of A or (in the case of D) possibly \perp " for " C and D are subsentences of A "

p. 331, line 3 from bottom" read " $\Box B$ and B are both" for " $\Box B$ are both B "

p. 332, end of proof of Theorem 27.1: read " $\Box C$ is in w " for $w \models \Box C$ "

p. 332, last displayed item in proof of Proposition 27.4: a dot should precede the *first* A , and none should precede the *second* A

p. 334, Proof of Theorem 27.8, fifth paragraph: read "main other part" for "only other part"

p. 334, Proof of Theorem 27.8. End of proof read " $\Box C$ is in w " for $w \models \Box C$ "

Also, add at end "(We leave it to the reader to show that if A is not a theorem, then $\sim A$ belongs to some w in W .)"

p. 337, first line below table: read π_α for γ

p. 338, lines 10-11 read "then this is a theorem, hence so are D and $\Box D$, which may be replaced by $\sim \perp$ " for "then this is a theorem, so we may replace D by \perp "

p. 338, displayed formulas (1)-(3) the superscripts are wrong:

for (1) they should be $n + 1$ and m

for (2) and (3) they should be m and n and $n + 1$ and n

p. 339, lines 7-8: one negations should be dropped on each line, the last of the two on line 7, and the middle one of the three on line 8

p. 339, proof of Lemma 27.15, line 2: a \square is missing before the second $C_i(p)$

p. 339, proof of Lemma 27.16, line 4: the double arrow \leftrightarrow should be single \rightarrow

p. 339, proof of Lemma 27.17, line 2: read " $v > u$ " for " $v \geq u$ " and " $u < v$ " for " $u \leq v$ "

p. 339, proof of Lemma 27.17, line 4: read "and so by Lemma 27.15" for "and so by that lemma"