$$\mathbf{J}_{\text{cond}}(t) = \int_{-\infty}^{t} \sigma(t - t')\mathbf{E}(t')\mathrm{d}t', \tag{2.35}$$

where $\sigma(t)$ is the same as that in (2.34). The frequency domain relation is obtained by taking the Fourier transform on (2.35):

$$\mathbf{J}_{\text{cond}}(\omega) = \sigma(\omega)\mathbf{E}(\omega), \tag{2.36}$$

where

$$\sigma(\omega) = \int_{-\infty}^{\infty} \sigma(t)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}t = \frac{Ne^2\tau}{m^*}\frac{1}{1 - \mathrm{i}\omega\tau}. \tag{2.37}$$

This frequency-dependent optical conductivity, also known as the *AC conductivity*, can be expressed in terms of the *DC conductivity*:

$$\sigma(\omega) = \frac{\sigma(0)}{1 - \mathrm{i}\omega\tau}, \tag{2.38}$$

where $\sigma(0)$ is the DC conductivity,

$$\sigma(0) = \frac{Ne^2\tau}{m^*}. \tag{2.39}$$

As discussed in Section 1.1, there are two alternative, but equivalent, ways to describe~~d~~ the optical response of free charge carriers: (1) by treating it as part of the total susceptibility and total permittivity in the total displacement $\boldsymbol{D}$, as in (1.12); or (2) by treating it as an optical conductivity through an explicit conduction current $\boldsymbol{J}_{\text{cond}}$, as in (1.16). The discussion above follows the second alternative, which allows us to find the optical conductivity in (2.38). By equating the two alternative approaches, the *conduction susceptibility*, χ_{cond}, due to the free charge carriers can be found.

Equating (1.12) and (1.16) but expressing them in complex fields, we have

$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial \mathbf{D}_{\text{bound}}}{\partial t} + \mathbf{J}_{\text{cond}}. \tag{2.40}$$

Converting this relation to the frequency domain, we find

$$-\mathrm{i}\omega\mathbf{D}(\omega) = -\mathrm{i}\omega\mathbf{D}_{\text{bound}}(\omega) + \mathbf{J}_{\text{cond}}(\omega). \tag{2.41}$$

By using the relations $\mathbf{D}(\omega) = \epsilon(\omega)\mathbf{E}(\omega)$, $\mathbf{D}(\omega)_{\text{bound}} = \epsilon_{\text{bound}}(\omega)\mathbf{E}(\omega)$, and $\mathbf{J}_{\text{cond}}(\omega) = \sigma(\omega)\mathbf{E}(\omega)$ from (2.36), we find the total permittivity that includes all contributions from bound and free charges in a material:

$$\epsilon(\omega) = \epsilon_{\text{bound}}(\omega) + \frac{\mathrm{i}\sigma(\omega)}{\omega} = \epsilon_{\text{bound}}(\omega) - \frac{\sigma(0)}{\omega(\omega\tau + \mathrm{i})}, \tag{2.42}$$

where $\epsilon_{\text{bound}}(\omega)$ is the permittivity from bound charges discussed in Section 2.3. Therefore, we can identify the conduction susceptibility due to the free charge carriers: