X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 9. EUV & Soft X-ray Lasers Homework Problems

9.1 (a) Describe the process of light amplification by stimulated emission of radiation. (b) Why is a population inversion required? (c) Why does such radiation often propagate in a well defined direction? (d) Is this always the case? (e) Give two examples in which amplified spontaneous emission is not directional, one laboratory and one interstellar. (f) How does the use of a radiation cavity improve the characteristics of a laboratory laser? (g) What roles do the various cavity components play in a visible light laser? (h) What sets the directional properties of laboratory lasers? (i) In what circumstances might these components be successfully employed in an EUV, soft x-ray, or x-ray laser? For part (e), consult the paper by C. H. Townes, "Astronomical Masers and Lasers", *Kvant. Elektr. (Moscow),* 24, 1063 (1997), or *Quant. Electr. (London),* 27, 1031 (1997), and references therein.

9.2 (a) Describe the advantage of a single ionization stage in a hot-dense plasma for the generation of spectrally intense short wavelength laser radiation? (b) What role do ionization bottlenecks, as discussed in Section 8.7.2, play in this? (c) What electron configurations are most conducive to the generation of spectrally intense laser lines? (d) What ionization energies would be required to form these electron configurations in ions of carbon, aluminum, argon, titanium, and molybdenum? (e) Express the strength of the ionization energy jump for each case in terms of $\Delta E_i / E_i$, that is, the increase in ionization energy for the closed shell over that with one additional electron. (f) What range of electron temperature would be required in each case for the formation of the requisite closed shells in the atoms considered? (g) Assuming appropriate ion density, plasma length, and population inversion, would the resultant ions lase in the EUV, soft x-ray, or x-ray spectral regions? Organize your answers in tabular form for each element considered.

9.3 (a) Calculate the ionization energy to form hydrogen-like aluminum, and concomitant photon energies for n = 3 to 2, 3 to 1, and 2 to 1 transitions. (b) For the 3d to 2p transition, what is the transition wavelength, oscillator strength, and lifetime? (c) Describe important features of such a plasma for successful lasing. (d) What problems might be encountered?

9.4 (a)What processes are represented by the Einstein A and B coefficients? (b) What are the degeneracy factors g_l and g_u?

9.5 You are investigating the scaling of short wavelength lasing in plasmas of high concentration hydrogen-like ions. You consider two potential candidates, H-like carbon, and H-like aluminum.
(a) For each case what electron impact energy is required to achieve the desired ionization stage? What is the total energy that must be provided by successive electron impacts to proceed from the neutral atom to the H-like ion in each case? (b) What is the wavelength and photon energy, in each case, for 3d to 2p transitions in H-like carbon and H-like aluminum? (c) What would the inverse relative spectral bandwidth ($\lambda/\otimes\lambda$) be for these lines? (d) Calculate the stimulated scattering cross-section, in each case assuming ion temperature of 20eV and 80eV, respectively. (e) For an electron density of 3×10^{19} e/cm^3 what would the total ion density be in each case? (f) If 20% of the ions were in a H-like configuration, and 1% of those (0.2% of all ions) in a 3d state,

what is the expected gain (G) assuming a transient inversion factor F = 0.8? (g) What plasma length would be required to achieve a gain-length product $GL = 12$?

9.6 (a) What is the relative spectral bandwidth of the 18.22 nm Doppler broadened line of H-like carbon ions with an ion temperature of 20 eV? (b) How might this line be narrowed by laser amplification? (c) Repeat the calculation for the 13.17 nm lasing line of nickel-like Cd at an ion temperature of 40 eV.

9.7 A discharge pumped Ne-like argon laser operating at a wavelength of 46.86 nm has been demonstrated at Colorado State University. Lasing is between the 3p ($^1S_0$) and 3s ($^1P_1$) states. The effective gain has been measured to reach a value of 1.2/cm. It is estimated that refraction has reduced the gain by 0.4/cm from what it might otherwise have achieved, 1.6/cm. (a) Assuming an ion temperature κT_i = 100 eV, an upper state radiative life time of 10 ps, and statistical weights g_u=1 and g_l =3, estimate the population inversion $n_u F = n_u - n_l\,(g_u / g_l)$. (b) With what atomic element might a similar laser be constructed which would lase near 13 nm wavelength, in the high reflectivity region of a Mo/Si multilayer mirror? (c) Describe requisite plasma parameters, special challenges, and special opportunities that would accrue in this shorter wavelength case.

9.8 Consider a nickel-like Nd laser as discussed in Section 9.4, operating at Osaka University's Institute of Laser Engineering at a wavelength of 7.905 nm (photon energy of 165.8 eV). (a) If the output energy is 40 μJ per 130 ps FWHM pulse, what is the peak power in watts? (b) If the radiation emerges from a 40 μm diameter circular region into an angular cone of 3 mr (2θ) FWHM, what is the spatially coherent peak power per pulse? (c) If pulses are generated every 400 ps, what is the average coherent power? If the laser linewidth is Doppler broadened by an ion temperature of 220 eV, what is the (longitudinal) coherence length?

9.9 The Ne-like argon laser discussed in Section 9.5, operates at a wavelength of 46.86 nm (26.46 eV photon energy) in experiments at Colorado State University. It generates an average power of about 1 mW in a relative spectral bandwidth $\Delta\lambda/\lambda = 10^{-4}$. Its measured source size and divergence, which depend on gas pressure in the discharge tube, are approximately 200 μm FWHM and 5 mr (2θ) FWHM at a background argon pressure of 650 mtorr. (a) What is the spatially coherent power $P_{coh,\,\Delta\lambda/\lambda}$, and (b) what is the (longitudinal) coherence length? In place of a monochromator, assume that a multilayer mirror of 30% reflectivity is available to suppress emission at other wavelengths from entering your experimental chamber.

9.10 Calculate the average and peak coherent power of a Ni-like Cd laser at 13.2 nm with an average power of 1 μW. Treat the laser output as originating from a source of 10 μm (FWHM) in size and the divergence angle of the beam is 10 mrad (FWHM). Assume the laser operates at 5 Hz and EUV pulse duration is 8 ps. Furthermore, calculate the average and peak spectral brightness of the laser. Express the answer in the typical unit of photons/s·mm^2·mrad^2·(0.1% bandwidth) and include your understanding of the laser's spectral bandwidth.

9.11 Explain how HHG seeding imparts a high degree of spatial coherence to a compact EUV atomic laser. What dominates the degree of temporal coherence and the concomitant longitudinal coherence length? Describe the resultant spatial and temporal coherence properties of the HHG seeded atomic laser.

9.12 (a) Based on the achievement of saturated EUV lasing at 13.2 nm in a plasma of nickle-like Cd, what scaling would be required to achieve lasing at a wavelength of 3 nm? (b) How much hotter must the plasma be? (c) How much faster must the energy be deposited? (d) Does the ion density need to be higher? (e) Qualitatively, what would be required to maintain the same degree of spatial coherence and temporal coherence length?