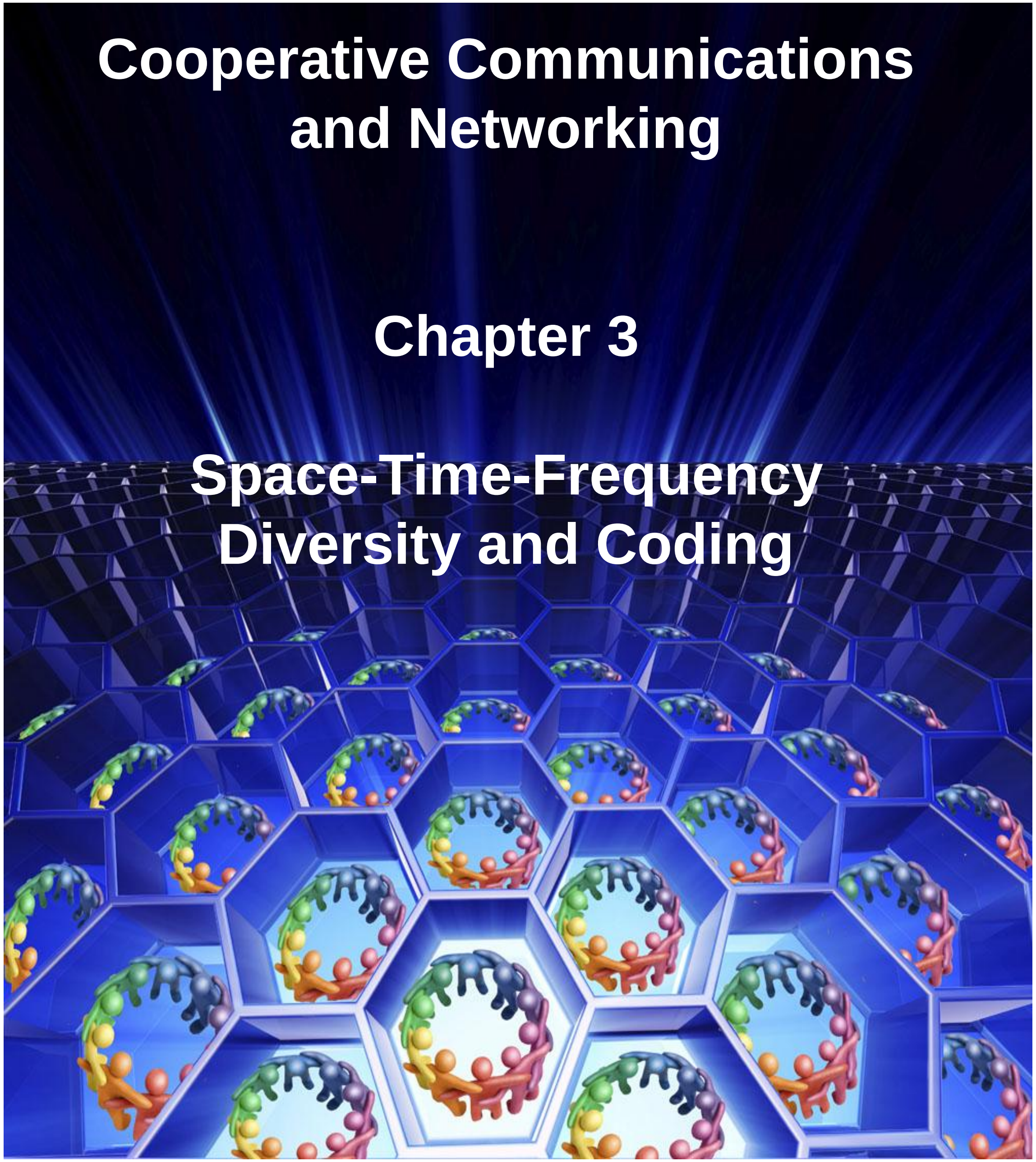

Cooperative Communications
and Networking
Chapter 3
Space-Time-Frequency
Diversity and Coding

Power Delay Profiles

- Suppose that the frequency-selective fading channels between each pair of Tx. and Rx. have L independent delay paths and the same delay profile.
- Then, the channel impulse response from Tx. i to Rx. j can be modeled as

$$h_{i,j}(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}(l)\delta(\tau - \tau_l),$$

where τ_l is the delay of the l-th path, and $\alpha_{i,j}(l)$ is the amplitude of the l-th path between Tx. i and Rx. j.
The $\alpha_{i,j}(l)$`s are modeled as zero-mean complex Gaussian random variables with variances $E|\alpha_{i,j}(l)|^2 = \delta_l^2$, and with energy normalization $\sum_{l=0}^{L-1} \delta_l^2 = 1$.

[2]

- Then, the frequency response of the channel is given by

$$H_{i,j}(f) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-j2\pi f \tau_l}$$

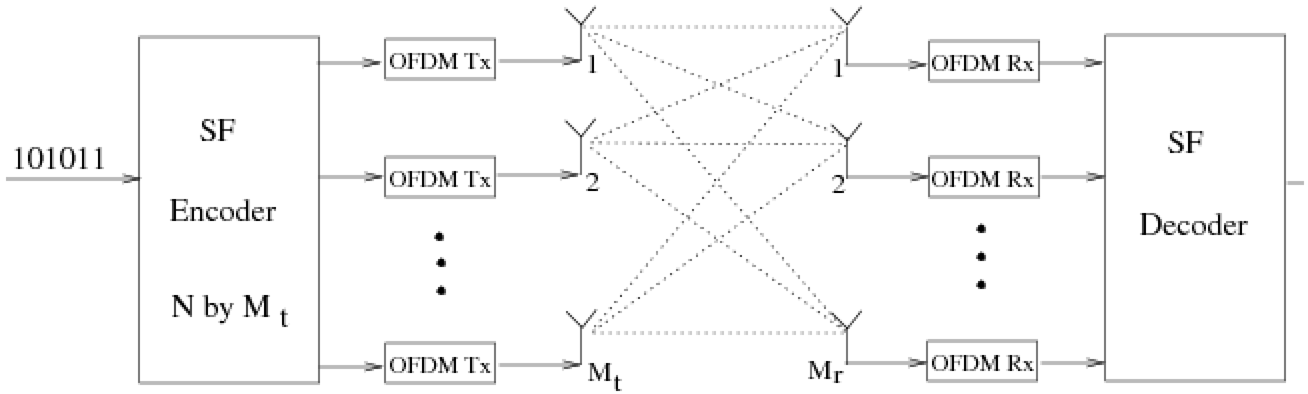
- The channel frequency response at the *n*-th subcarrier is

$$H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-j2\pi n \Delta f \tau_l}$$

where $\Delta f = 1/T$ is the tone space, and T is the OFDM block duration.

SF-Coded MIMO-OFDM System Model

- Consider a system with M_t Tx. and M_r Rx. antennas, and N subcarriers.



- Each SF codeword can be expressed as an $N \times M_t$ matrix

$$C = \begin{bmatrix} c_1(0) & c_2(0) & \cdots & c_{M_t}(0) \\ c_1(1) & c_2(1) & \cdots & c_{M_t}(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_2(N-1) & \cdots & c_{M_t}(N-1) \end{bmatrix}_{N \times M_t},$$

where $c_i(n)$ is the symbol transmitted over the n-th subcarrier by transmit antenna i. The energy normalization is $E\|C\|_F^2 = NM_t$.

[4]

- At the receiver, after matched filtering, removing the cyclic prefix and applying FFT, the received signal at the *n*-th subcarrier at receive antenna *j* is given by:

$$y_j(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=0}^{M_t} c_i(n) H_{i,j}(n) + z_j(n),$$

for $n = 0, 1, \ldots, N\text{-}1$, in which

$$H_{i,j}(n) = \sum_{l=0}^{L-1} \alpha_{i,j}(l) e^{-j2\pi\Delta f \tau_l}.$$

- Denote $H_{i,j} = [H_{i,j}(0) \quad H_{i,j}(1) \quad \cdots \quad H_{i,j}(N-1)]^T$, then

$\boxed{H_{i,j} = W \cdot A_{i,j},}$ where

$$A_{i,j} = [\alpha_{i,j}(0) \quad \alpha_{i,j}(1) \quad \cdots \quad \alpha_{i,j}(L-1)]^T,$$

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix}_{N\times L},$$

$$w = e^{-j2\pi\Delta f}.$$

- The correlation matrix of the channel frequency response from Tx. i to Rx. j can be calculated as

$$\begin{aligned} R_{i,j} &= E\{H_{i,j} H_{i,j}^{\mathrm{H}}\} \\ &= W\ E\{A_{i,j} A_{i,j}^{\mathrm{H}}\}\ W^{\mathrm{H}} \\ &= W\ diag(\delta_0^2, \quad \delta_1^2, \quad \cdots, \quad \delta_{L-1}^2)\ W^{\mathrm{H}} \\ &=: R. \end{aligned}$$

SF Code Design Criteria

- The pair-wise error probability of two distinct SF signals C and $\tilde{C}$ can be upper bounded as

$$P(C \to \tilde{C}) \le \binom{2KM_r - 1}{KM_r} \left(\prod_{i=1}^{K} \lambda_i \right)^{-M_r} \left(\frac{\rho}{M_t} \right)^{-KM_r},$$

where K is the rank of $\Delta \circ R$, and $\lambda_1, \lambda_2, \cdots, \lambda_K$ are the non-zero eigenvalues of $\Delta \circ R$, in which

$$\Delta = (C - \tilde{C})(C - \tilde{C})^{\mathrm{H}}.$$

- If $A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & \dots & b_{1n} \\ \cdots & \cdots & \cdots \\ b_{m1} & \cdots & b_{mn} \end{pmatrix}$, then the Hadamard product of A and B is

$$A \circ B = \begin{pmatrix} a_{11}b_{11} & \dots & a_{1n}b_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1}b_{m1} & \cdots & a_{mn}b_{mn} \end{pmatrix}.$$

- Based on the upper bound, two design criteria can be proposed as:
  - *Diversity criterion*: The minimum rank of $\Delta \circ R$ over all pairs of distinct SF signals C and $\tilde{C}$ should be as large as possible;
  - *Product criterion*: The minimum value of the product $\prod_{i=1}^{K} \lambda_i$ over all pairs distinct SF signals C and $\tilde{C}$ should be as large as possible.
- According to a rank inequality of Hadamard product, we have

$$rank(\Delta \circ R) \leq rank(\Delta)\ rank(R).$$

So $$rank(\Delta \circ R) \leq \min\{LM_t,\ N\}.$$

Thus, the *maximum achievable diversity* is at most

$$\min\{LM_t M_r,\ NM_r\}.$$

A Systematic Design of Full-Diversity SF Codes

- Here we will design full-diversity SF codes from ST codes via mappings. The resulting full-diversity SF code
  - Having larger coding rates than the existing SF codes;
  - Taking into account arbitrary power delay profiles;
  - All ST codes (block and trellis) with full diversity in quasi-static flat fading environment can be used.
- For any $l = 1, 2, \dots L,$ define a repetition mapping

$$\mathrm{M}_l : \mathrm{ST} \rightarrow \mathrm{SF}$$

$$\mathrm{M}_l(G) = [I_{M_t} \otimes 1_{l\times 1}]\, G.$$

- For example,

$$\mathrm{M}_2 : \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \rightarrow \begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \\ -x_2^* & x_1^* \\ -x_2^* & x_1^* \end{bmatrix}.$$

Theorem 1: *If a space-time (block or trellis) code designed for* M_t *transmit antennas achieves full diversity for quasi-static flat fading channels, then the SF code obtained from this ST code via the mapping* $\mathrm{M}_l \ (1 \le l \le L)$ *will achieve a diversity order of at least* $\min\{lM_tM_r,\ NM_r\}$.

- In case of the full diversity SF codes, the *coding advantage* or *diversity product* is defined as

$$\varsigma_{\mathrm{SF},R} = \frac{1}{2\sqrt{M_t}} \min_{C \neq \tilde{C}} \left(\prod_{i=1}^{LM_t} \lambda_i \right)^{\frac{1}{2LM_t}},$$

where $\lambda_1, \quad \lambda_2, \quad \cdots \quad \lambda_{LM_t}$ are non-zero eigenvalues of $\Delta \circ R$.

- Recall that the *diversity product* of a full diversity ST code designed for flat fading channels is

$$\varsigma_{\mathrm{ST}} = \frac{1}{2\sqrt{M_t}} \min_{G \neq \tilde{G}} \left(\prod_{i=1}^{M_t} \beta_i \right)^{\frac{1}{2M_t}},$$

where $\beta_1, \quad \beta_2, \quad \cdots \quad \beta_{M_t}$ are non-zero eigenvalues of $(G-\tilde{G})(G-\tilde{G})^{\mathrm{H}}$.

Theorem 2: *The diversity product of the SF code is bounded by that of the corresponding ST code as follows:*

$$\sqrt{\eta_L}\,\Phi\,\varsigma_{\mathrm{ST}} \le \varsigma_{\mathrm{SF},R} \le \sqrt{\eta_1}\,\Phi\,\varsigma_{\mathrm{ST}},$$

in which $\Phi = \left(\prod_{l=0}^{L-1}\delta_l\right)^{1/L}$, η_1 *and* η_L *are the largest and smallest eigenvalues of the following matrix*

$$H = \begin{bmatrix} H(0) & H(1)^* & \cdots & H(L-1)^* \\ H(1) & H(0) & \cdots & H(L-2)^* \\ \vdots & \vdots & \ddots & \vdots \\ H(L-1) & H(L-2) & \cdots & H(0) \end{bmatrix},$$

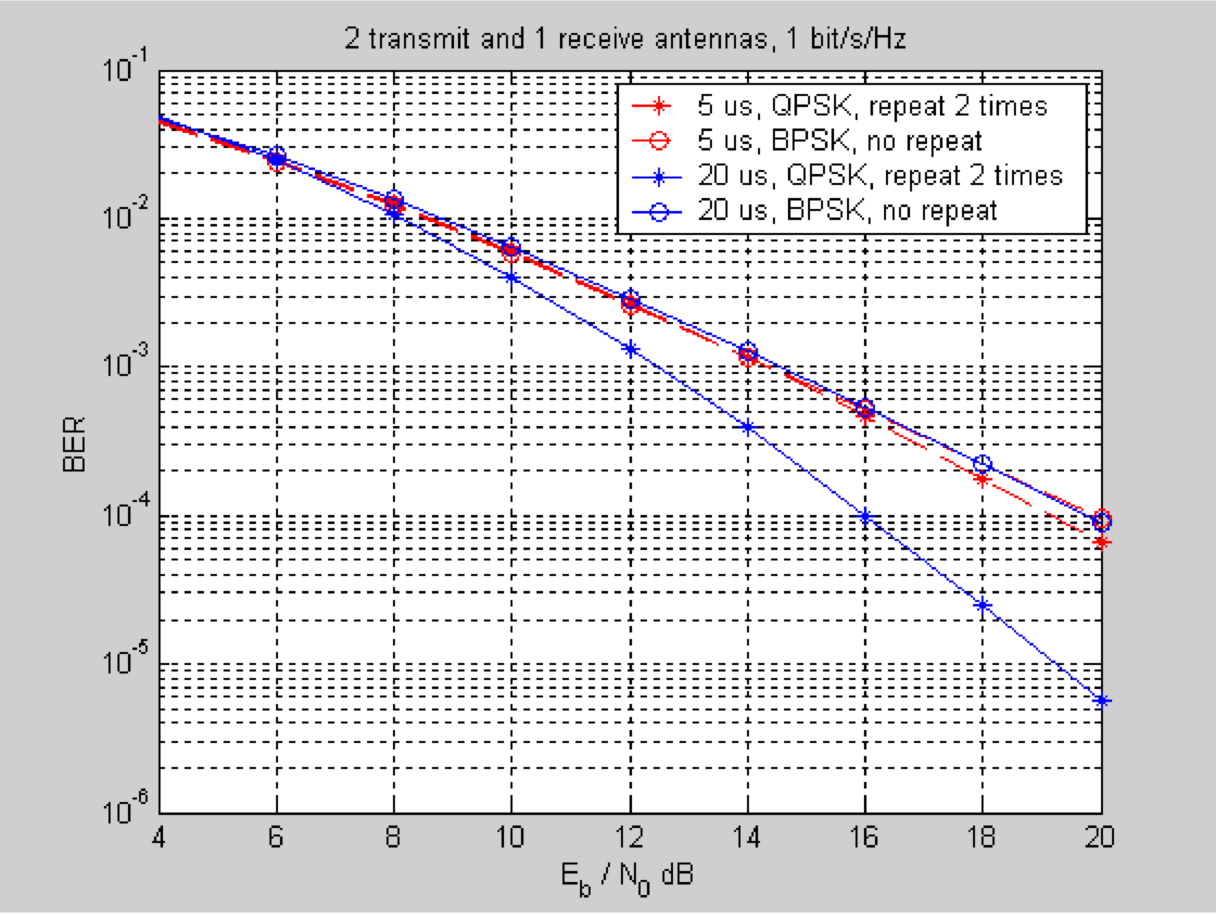
$$H(n) = \sum_{l=0}^{L-1} e^{-j2\pi\Delta f\tau_l}, \quad n = 0,1,\cdots,L-1.$$

[14]

Simulation Results

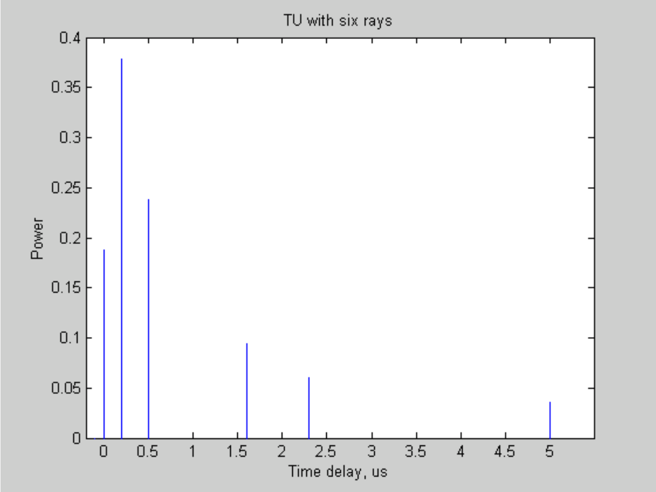
- The simulated system has $M_t = 2$ Tx. and $M_r = 1$ Rx.
- Bandwidth *1* MHz
- $N = 128$ subcarriers (tones)
- 128 us data symbol duration
- Full-diversity SF codes are obtained from *orthogonal* ST block codes.

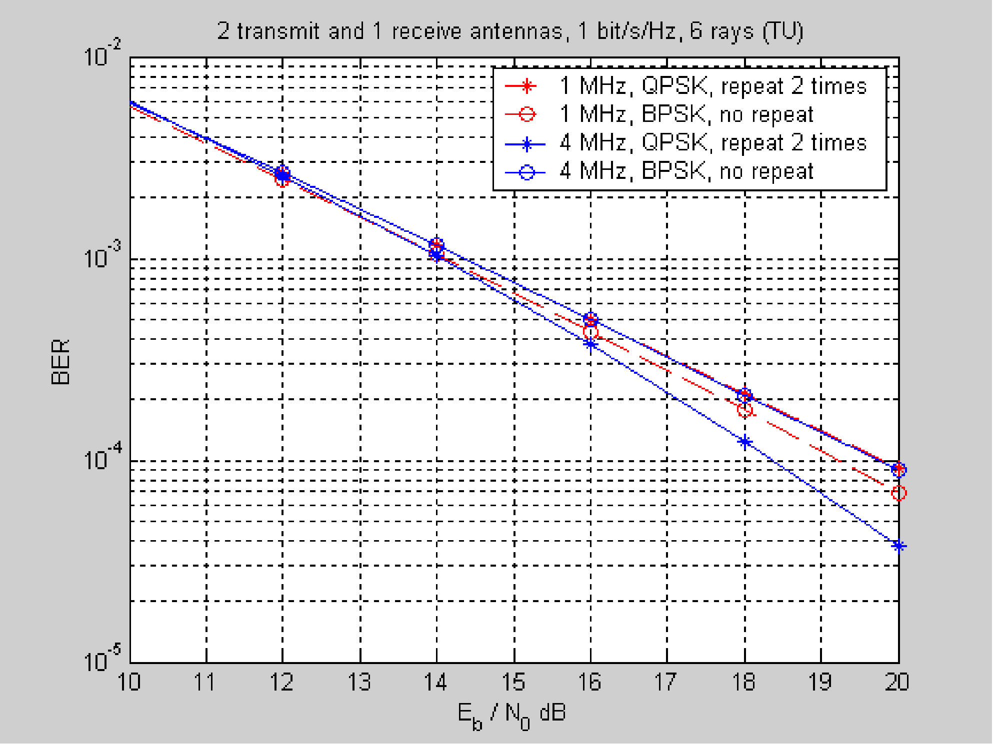
- 2 transmit and 1 receive antennas, two delay paths at: (i) 0 and 5 us (5us/128us); and (ii) 0 and 20 us (20us/128us).



- Two Tx. and one Rx., 6-ray *Typical Urban* (TU) model, with two different bandwidths: (a) 1 MHz (5.0us / 128us); (b) 4 MHz (5.0us / 32us).

Time delay:	$[0.0 \quad 0.2 \quad 0.5 \quad 1.6 \quad 2.3 \quad 5.0]$
Power profile:	$[0.189 \quad 0.379 \quad 0.239 \quad 0.095 \quad 0.061 \quad 0.037]$





3.8. A Design of Full-Rate Full-Diversity SF Codes

- More recently, we propose a systematic design method to construct SF codes with full rate and full diversity.
- Moreover, we further permute the rows of the proposed SF codes in order to achieve larger coding advantage. The permutations have been optimized for any specific power delay profile.

- Specifically, for any integer $\Gamma(1 \le \Gamma \le L)$, we are able to design SF codes with a diversity order of $\Gamma M_t M_r$.
- We consider a coding strategy that each SF codeword C is a concatenation of some matrices as follows:

$$C = \begin{bmatrix} G_1^T & G_2^T & \cdots & G_P^T & 0_{N-P\Gamma M_t}^T \end{bmatrix}^T,$$

where each matrix $G_p, p = 1, 2, \cdots, P$ of size $\Gamma M_t \times M_t$ has the same structure which is specified as

$$G = \sqrt{M_t} diag\left(X_1, \quad X_2, \quad \cdots, \quad X_{M_t} \right),$$

where $X_i = \begin{bmatrix} x_{(i-1)\Gamma+1} & x_{(i-1)\Gamma+2} & \cdots & x_{i\Gamma} \end{bmatrix}^T$, $i = 1, 2, \cdots, M_t$, and all x_k 's are complex variables and will be specified later.

- For example, $M_t = 2$ and $\Gamma = 2$,

$$G = \sqrt{2} \begin{pmatrix} x_1 & 0 \\ x_2 & 0 \\ 0 & x_3 \\ 0 & x_4 \end{pmatrix}.$$

Theorem 3: *For any SF code described above, if* $\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \neq 0$ *for any pair of distinct variables* $X = \begin{bmatrix} x_1 & x_2 & \cdots & x_{\Gamma M_t} \end{bmatrix}$ *and* $\tilde{X} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \cdots & \tilde{x}_{\Gamma M_t} \end{bmatrix}$, *then the SF code achieves a diversity order of* $\Gamma M_t M_r$, *and the diversity product is*

$$\zeta = \zeta_{in} \,|\det(Q_0)|^{1/2\Gamma},$$

in which ζ_{in} *is the "intrinsic" diversity product defined as*

$$\zeta_{in} = \frac{1}{2} \min_{X \neq \tilde{X}} \left(\prod_{k=1}^{\Gamma M_t} |x_k - \tilde{x}_k| \right)^{1/\Gamma M_t},$$

and Q_0 *is related to the power delay profile as follows:*

$$Q_0 = W_0 diag\left(\delta_0^2, \quad \delta_1^2, \quad \cdots, \quad \delta_{L-1}^2\right),$$

$$W_0 = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(\Gamma-1)\tau_0} & w^{(\Gamma-1)\tau_1} & \cdots & w^{(\Gamma-1)\tau_{L-1}} \end{pmatrix}.$$

- How to maximize the "intrinsic" diversity product

$$\zeta_{in} = \frac{1}{2}\min_{X \neq \tilde{X}} \left(\prod_{k=1}^{\Gamma M_t} | x_k - \tilde{x}_k | \right)^{1/\Gamma M_t} ?$$

- The problem of signal design is related to the problem of constructing signals for Rayleigh fading SISO systems. The optimum solution has been obtained via the algebraically rotated signal constellations (Belfiore *et al* 1997, Viterbo *et al* 1998).

- For example, $M_t = 2$ and $\Gamma = 2$,

$$G = \sqrt{2}\begin{pmatrix} x_1 & 0 \\ x_2 & 0 \\ 0 & x_3 \\ 0 & x_4 \end{pmatrix},$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \end{bmatrix} \cdot \frac{1}{2} V(\theta, -\theta, j\theta, -j\theta),$$

where s_1, s_2, s_3, s_4 are chosen from BPSK or QAM constellations, $V(\square)$ is the Vandermonde matrix, and $\theta = e^{j\pi/8}$.

Maximization of the Coding Advantage by Permutations

- Furthermore, we permute the rows of the proposed Full-rate full-diversity SF codes in order to achieve larger coding advantage.

Theorem 4: *For any permutation, the diversity product of the resulting SF code is*

$$\zeta = \zeta_{in}\zeta_{ex},$$

where ζ_{in} *is the "intrinsic" diversity product, and* ζ_{ex} *is the "extrinsic" diversity product defined as*

$$\zeta_{ex} = \left(\prod_{m=1}^{M_t} |\det(W_m \Lambda W_m^{\mathrm{H}})| \right)^{1/(2\Gamma M_t)}.$$

Moreover,

(i) $\zeta_{ex} \le 1$; *and*

(ii) *If we sort the power profile in a non-increasing order as:* $\delta_{l_1} \ge \delta_{l_2} \ge \cdots \ge \delta_{l_L}$, *then*

$$\zeta_{ex} \le \sqrt{L} \left(\prod_{i=1}^{\Gamma} \delta_{l_i} \right)^{1/\Gamma}.$$

- We consider here a specific permutation strategy. Define a one-to-one mapping σ over set {1,2,…,N} as follows:

$$\sigma(n) = v_1 \mu \Gamma + e_0 \mu + v_0 + 1,$$

where

$$e_1 = \left\lfloor \frac{n-1}{\Gamma} \right\rfloor, \quad e_0 = n - e_1 \Gamma - 1,$$

$$v_1 = \left\lfloor \frac{e_1}{\mu} \right\rfloor, \quad v_0 = e_1 - v_1 \mu.$$

We call the integer μ as a *separation factor*.

- For example, if $\Gamma = 2$, and $\mu = 3$,

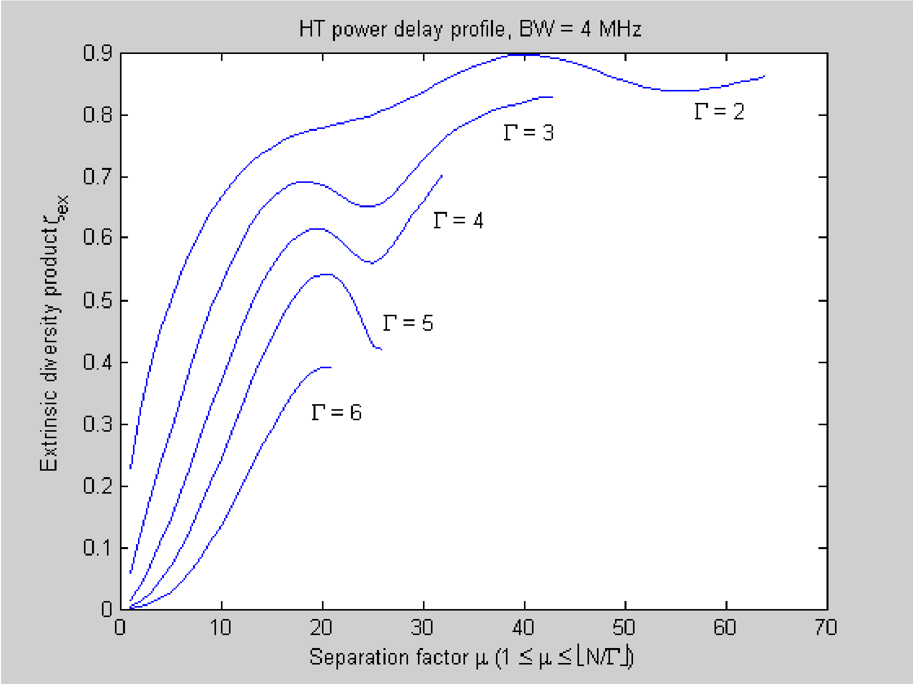
$$n \;:\; 1,\; 2,\; 3,\; 4,\; 5,\; 6,\; \cdots$$

$$\sigma(n) \;:\; 1,\; 2,\; 3,\; 4,\; 5,\; 6,\; \cdots$$

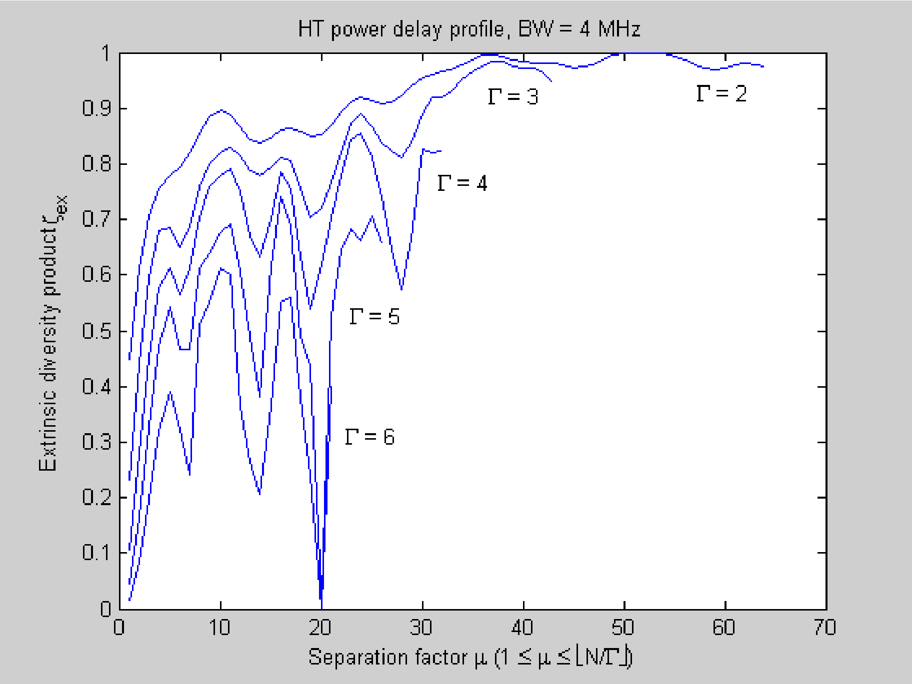
- We need to optimize the separation factor μ in order to maximize the "extrinsic" diversity product ζ_{ex}.
- For example, we consider a 2-ray delay profile with a delay τ us. Suppose that the system has N = 128 subcarriers, and the bandwidth is 1 MHz.
  - If $\tau = 5us$, then $\mu_{op} = 64$ and $\zeta_{ex} = 1$.
  - If $\tau = 20us$, then $\mu_{op} = 16$ and $\zeta_{ex} = 1$.

  In both cases, the resulting "extrinsic" diversity product achieves the upper bound 1 which is stated in Theorem 4.

- In case of the 6-ray TU channel model, we plot the curves of the "extrinsic" diversity product vs. separation factor for different Γ $(2 \le \Gamma \le L)$.



(a) BW = 1 MHz

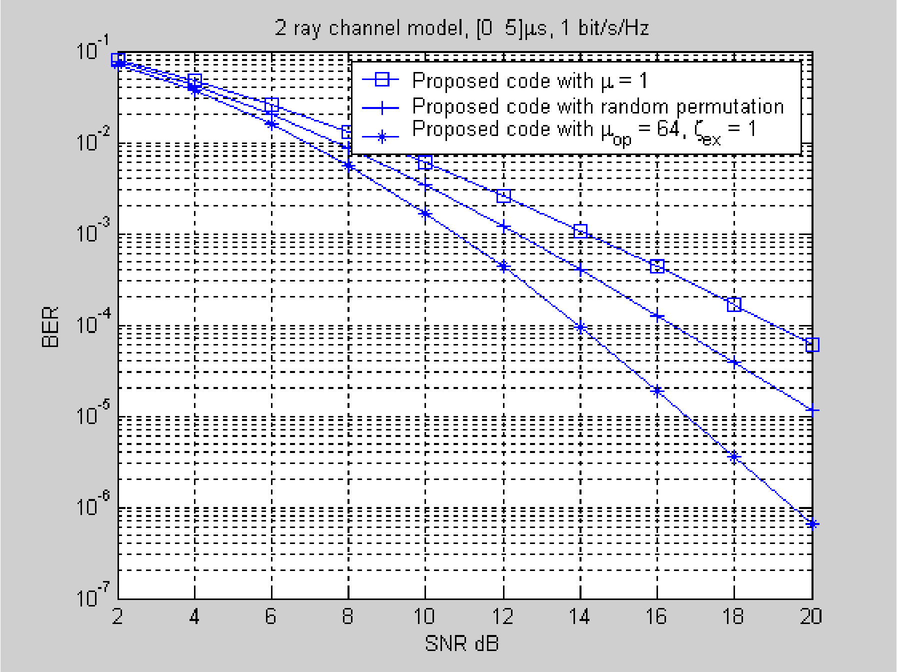


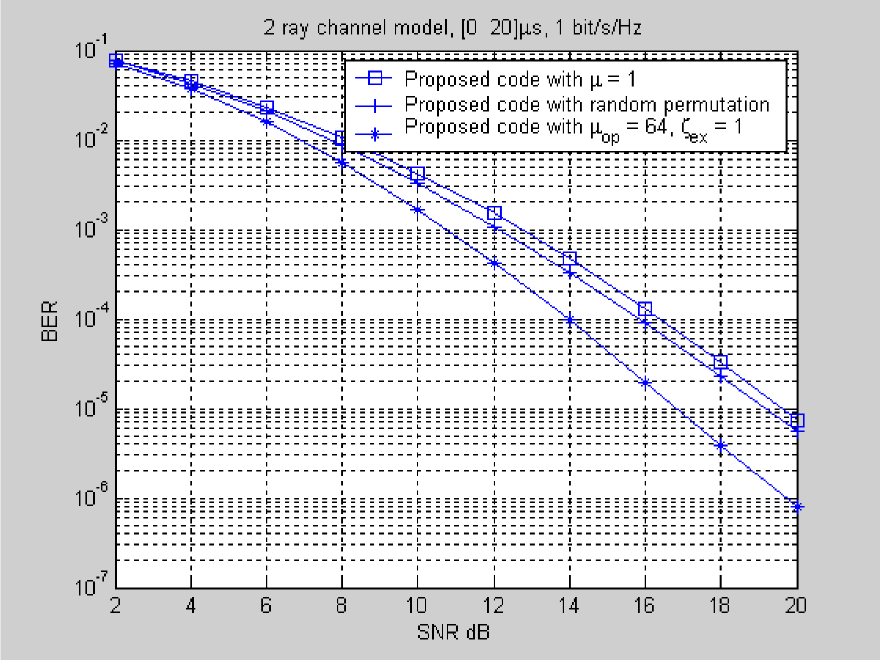
(b) BW = 4 MHz

[25]

Simulation Results

- Performance of the proposed SF code with different permutations in two-ray channel model.

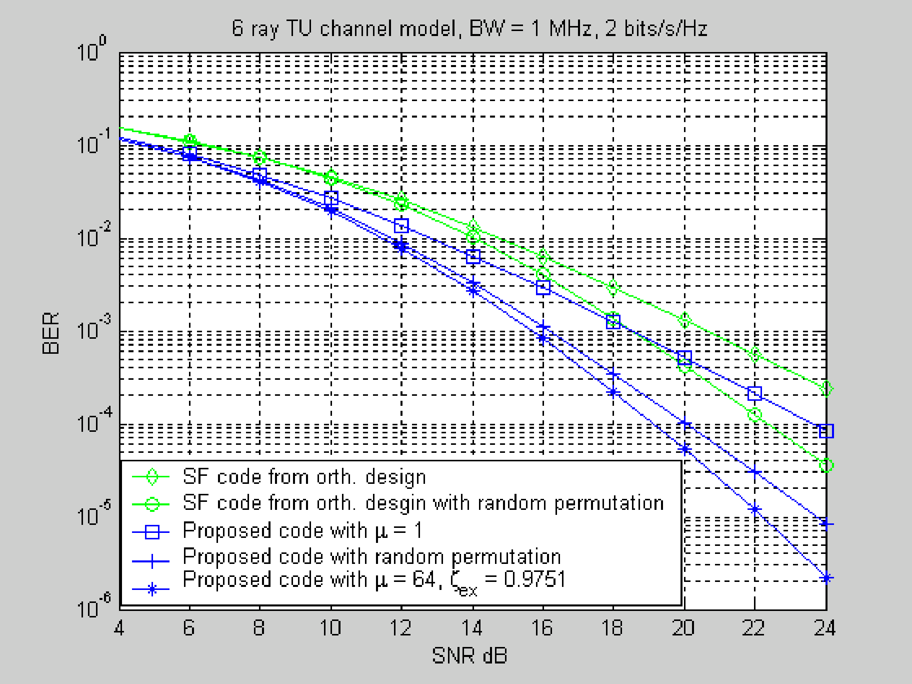




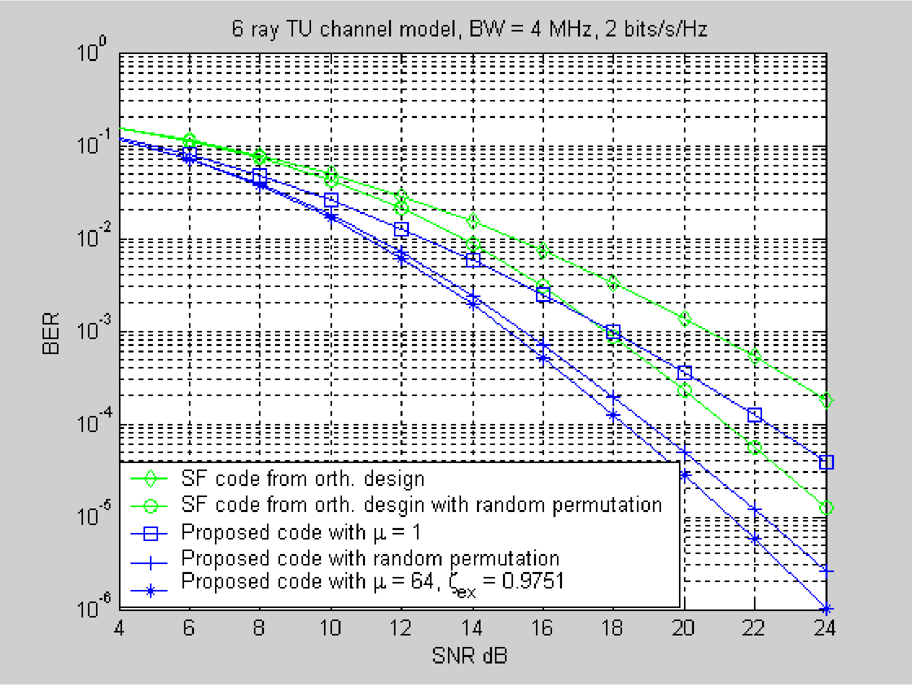
(a) two rays at 0 and 5us

(b) two rays at 0 and 20us

- Comparison of the proposed SF code and the code from orthogonal design in TU channel model.



(a) BW = 1 MHz

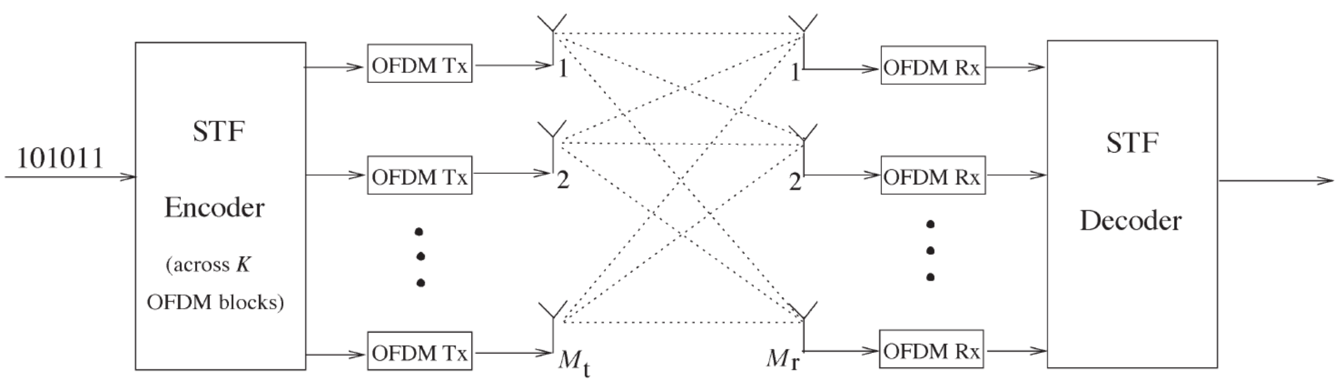


(b) BW = 4 MHz

Space-Time-Frequency (STF) Codes

- Now consider codes that exploit all spatial, temporal, and frequency diversity obtainable
- Same Goal: Obtain full diversity and full rate codes
- Similar channel model and modulation/demodulation system as SF OFDM-MIMO codes, however, our codes now span *k* OFDM blocks to add a temporal domain
- Channel model from the Tx *i*, and *R*x *j* for the k^{th} OFDM block (frequency response):

$$H_{i,j}^{k}(f)=\sum_{l=0}^{L-1}\alpha_{i,j}^{k}(l)e^{-j2\pi f\tau_l}$$



Space-Time-Frequency (STF) Codes

- Each STF codeword represented by an $KN \times M_t$ matrix:

$$C = \left[C_1^T \; C_2^T \cdots C_K^T\right], \qquad E\|C\|_F^2 = K\,N\,M_t, \qquad C_k = \begin{bmatrix} c_1^k(0) & c_1^k(0) & \cdots & c_1^k(0) \\ c_1^k(0) & c_1^k(0) & \cdots & c_1^k(0) \\ \vdots & \vdots & \ddots & c_1^k(0) \\ c_1^k(0) & c_1^k(0) & \cdots & c_1^k(0) \end{bmatrix}$$

- Received signal at j^{th} antenna for k^{th} block:

$$y_j^k(n) = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} c_i^k(n) H_{i,j}^k(n) + z_j^k(n), \qquad H_{i,j}^k(n) = \sum_{l=o}^{L-1} \alpha_{i.j}^k(l) e^{-j2\pi n \Delta f \tau_l}$$

Criteria for a Good STF Code

- The received signal Y is modeled the same

$$\mathbf{Y} = \sqrt{\frac{\rho}{M_t}}\mathbf{D}\mathbf{H} + \mathbf{Z}, \qquad \mathbf{D} = \mathbf{I}_{M_r} \otimes \left[D_1\ D_2 \cdots D_{M_t}\right]$$

$$D_i = \mathrm{diag}\left\{\mathrm{c_i}(0), \mathrm{c_i}(1), \ldots, \mathrm{c_i}(KN-1)\right\}$$

  - Note: avg. SNR for each Rx ant. = ρ

- The pairwise error probability that two matrices $\mathbf{D}, \tilde{\mathbf{D}}$ constructed from codewords $\mathbf{C}, \tilde{\mathbf{C}}$, where $\tilde{\mathbf{D}}$ is received:

$$P(\mathbf{D} \to \tilde{\mathbf{D}}) \le \binom{2r-1}{r}\left(\prod_{i=1}^{r}\gamma_i\right)^{-1}\left(\frac{\rho}{M_t}\right)^{-r}$$

$$r = \mathrm{rank}\left((\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^H\right), \quad \mathbf{R} = E\left[\mathbf{H}\mathbf{H}^H\right]$$

- Where $\gamma_1, \gamma_2, \ldots, \gamma_r$ are the nonzero eigenvalues of $(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^H$

Design Upper Bounds

- Through some manipulation, Δ and **R** may be written:

$$a \otimes b \equiv \text{tensor product}, \quad a \circ b \equiv \text{Hadamard product}, \quad \mathbf{I}_{M_t M_r} \equiv M_t M_r \text{ identity matrix}$$

$$R = I_{M_r M_t} \otimes \left(R_t \otimes R_f\right), \quad \Delta \hat{=} (C - \tilde{C})(C - \tilde{C})^H$$

  – Where: R_t is the temporal correlation matrix
  – R_f is the frequency correlation matrix

- The pairwise error probability that two codewords $C, \tilde{C}$, where $\tilde{C}$ is received, is:

$$P(C \to \tilde{C}) \le \binom{2\nu M - 1}{\nu M} \left(\prod_{i=1}^{\nu} \lambda_i \right)^{-M_r} \left(\frac{\rho}{M_t} \right)^{-\nu M_r}$$

- Where $\Delta \circ R$ are the nonzero eigenvalues of $\lambda_1, \lambda_2, \ldots, \lambda_r$

$$\zeta_{STF} = \min_{C \ne \tilde{C}} \prod_{i=1}^{\nu} \lambda_i$$

- The criteria for the coding advantage of the STF code is

Design Upper Bounds

- The new performance criteria for good STF may be written:
  - *Diversity (rank) criterion:* Maximize the minimum rank of $\Delta \circ R$ over $\forall C \neq \tilde{C}$
  - *Product criterion:* Maximize the minimum of the product $\prod_{i=1}^{\nu} \lambda_i$ over $\forall C \neq \tilde{C}$
  - <u>Similar to SF code design criteria</u>
- The rank of Δ is at most M_t, and rank of R_F is at most L:

$$\mathrm{rank}(\Delta \circ R) \leq \mathrm{rank}(\Delta)\mathrm{rank}(R_T)\mathrm{rank}(R_F) \Rightarrow \mathrm{rank}(\Delta \circ R) \leq \min(L M_t rank(R_T), K N)$$

- Thus maximum achievable diversity for completely stationary channel, rank(Rt =1) is: $\min(L M_t M_r, K N M_r)$

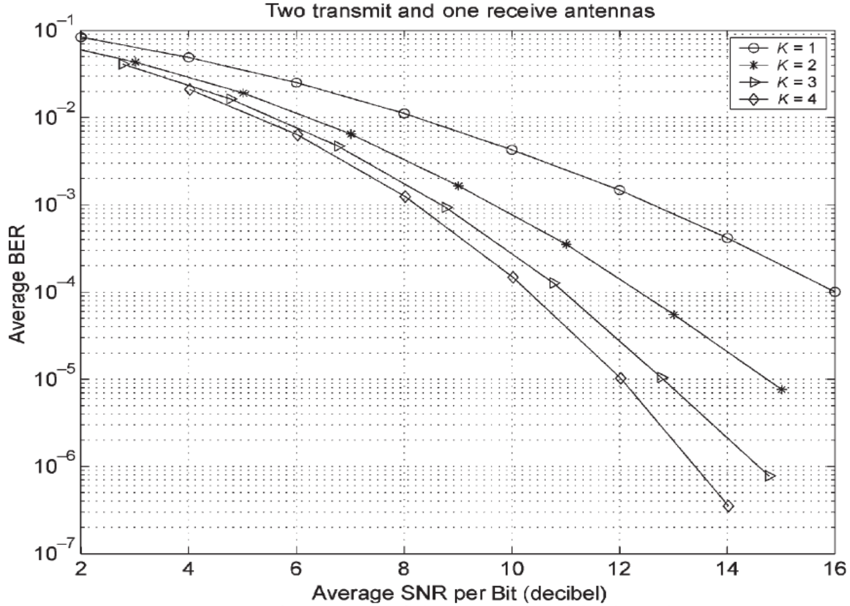
Design of Full-Diversity and Full-Rate STF Codes

- Try the simple repetition-based approach with best SF code
- The k block time-repeated code: $C_{STF} = \mathbf{1}_{k\times 1} \otimes C_{SF}$
- Results: Full diversity of K L M_t M_r,
  - at price of 1/k symbol rate

Delay-Profile Results – Repetition-based (STF)

Block codes

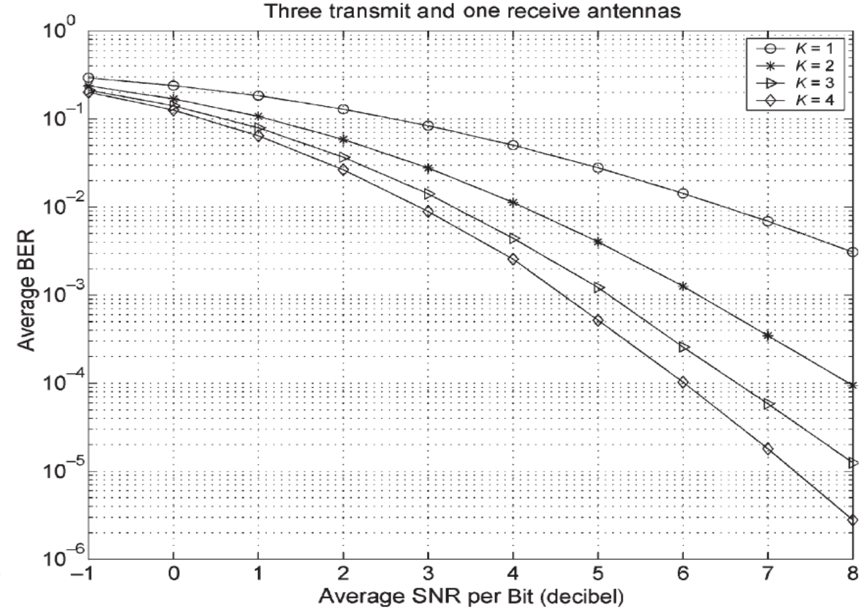
- 2-ray model, $\tau_0 = 20\mu s$, N(0,.5)
- $M_t = 2$, $M_r = 1$ antenna
- ST Alamouti codes and QPSK
- Spectral Efficiency: 1, 0.5, 0.33, 0.25 bits/s/Hz



Performance of the repetition block codes.

Trellis codes

- 2-ray model, $\tau_0 = 20\mu s$, N(0,.5)
- $M_t = 3$, $M_r = 1$ antenna
- 16-state trellis, QPSK ST code
- Spectral Efficiency: 1, 0.5, 0.33, 0.25 bits/s/Hz



Performance of the repetition trellis codes.

Design of Full-Diversity and Full-Rate STF Codes

- Try Full-Rate STF approach:

$$C = \left[C_1^T \; C_2^T \cdots C_K^T \right], \quad C_k = \left[G_{k,1}^T \; G_{k,2}^T \cdots G_{k,P}^T \; \mathbf{0}_{N-P\Gamma M_t}^T \right]$$

  - Where $\mathbf{0}_{N-P\Gamma M_t}^T$ is a zero-padding

- It may be shown for full-rank Rt, {rank(Rt) = K}, symbols

$$\zeta_{STF} = \delta M_t^{K\Gamma M_t} \left|\det(R_t)\right|^{\Gamma M_t} \left|\det(Q_0)\right|^{K M_t}, \; \delta = \min_{\mathbf{X} \neq \tilde{\mathbf{X}}} \prod_{k=1}^{K} \prod_{j=1}^{\Gamma M_t} \left| x_{k,j} - \tilde{x}_{k,j} \right|^2$$

  - Where $Q_0 = W_0 \operatorname{diag}\left(\delta_0^2, \delta_1^2, \cdots, \delta_{L-1}^2\right) W_0^H$

$$W_0 = \omega^{i\tau_j} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \omega^{\tau_0} & \omega^{\tau_1} & \cdots & \omega^{\tau_{L-1}} \\ \omega^{(\Gamma-1)\tau_0} & \omega^{2\tau_1} & \cdots & \omega^{2\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(\Gamma-1)\tau_0} & \omega^{(\Gamma-1)\tau_1} & \cdots & \omega^{(\Gamma-1)\tau_{L-1}} \end{bmatrix}_{\Gamma \times L} \qquad \omega = e^{-j2\pi\Delta f}$$

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- Also yields good codes with Vandermonde design:

$$S = \left[S_1 \, S_2 \cdots S_L \right] \in \Omega^K$$

$$\mathbf{X} = S \cdot \frac{1}{\sqrt{L}} V\left(\theta_1, \theta_2, \cdots, \theta_L\right)$$

  – Where Ω^K is the signal constellation order K

- Code Tradeoff: High-complexity ML Receiver (exponential K)

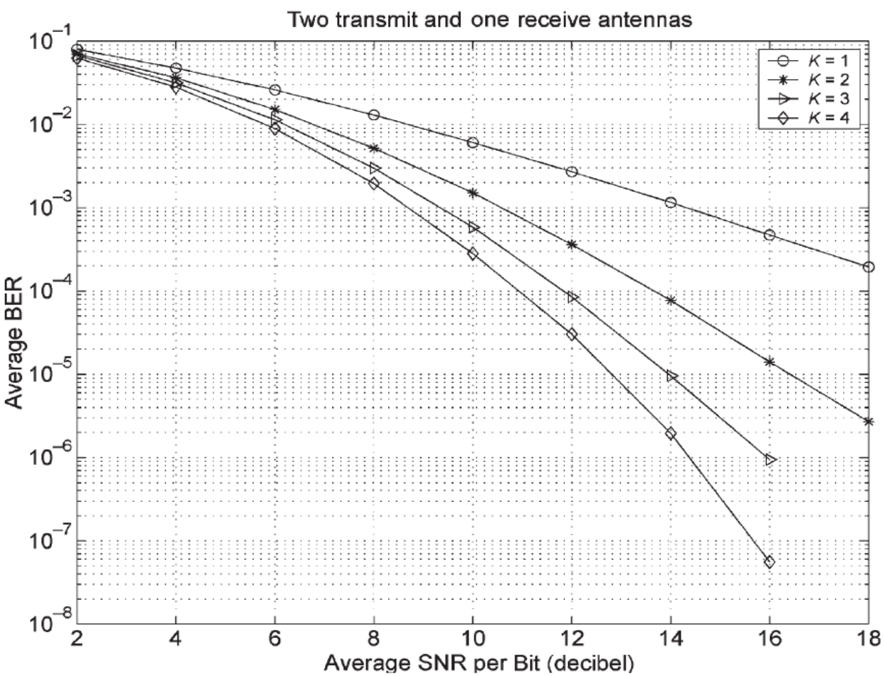
Delay-Profile Performance Simulation/Results (STF)

- Using 6-ray COTS207 typical urban (TU) power delay profile
- Simulated fading channel with different temporal correlation
- BPSK over Ω^{4K}
- Spectral Efficiency:1 bits/s/Hz

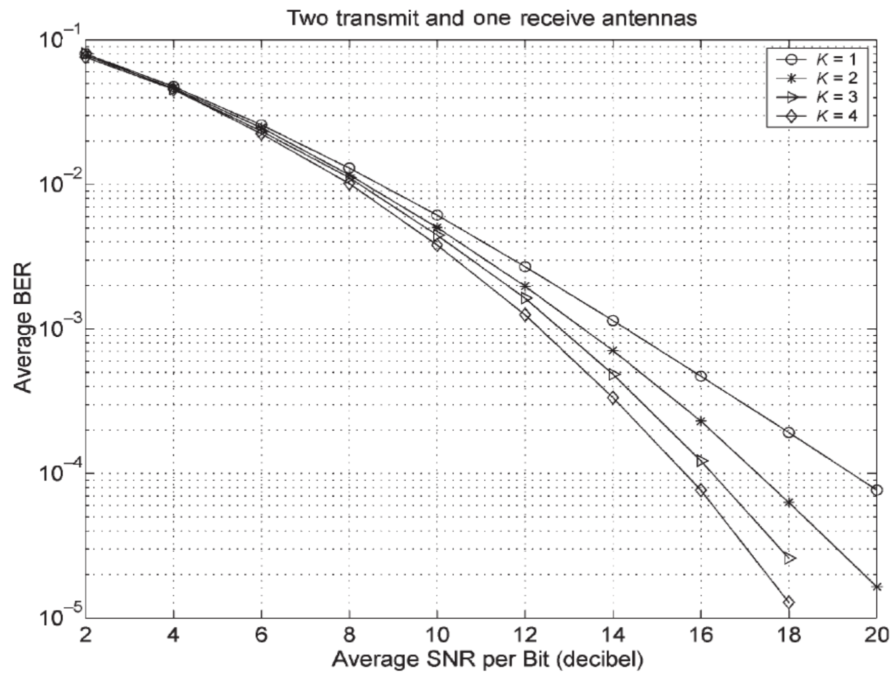
- First-order Markov Model
- ε (0 ≤ ε ≤ 1) is the amount of temporal correlation
- Model:

$$\alpha_{i,j}^{k} = \varepsilon \alpha_{i,j}^{k-1}(l) + \eta_{i,j}^{k}(l), \quad 0 \le l \le L-1$$

$$\eta_{i,j}^{k}(l) \approx N\left(0, \delta_1 \sqrt{1-\varepsilon^2}\right)$$



Performance of the full-rate STF codes, $\varepsilon = 0$.



Performance of the full-rate STF codes, $\varepsilon = 0.95$.