Chapter 7 --The Scattering Potential

Changes in red.

Change on p 159. mismatched parentheses in eq.(7.22)

$$\begin{aligned}\mathcal{V}_a(\mathbf{x},\omega) &\to \boldsymbol{k}_i\cdot\boldsymbol{k}_r\left(\frac{a_\rho}{\rho_0}\right) - \frac{\omega^2}{\rho_0\,{c_0}^2}(a_\rho + a_c)\\ &= \frac{\omega^2}{\rho_0\,{c_0}^2}\left(\hat{\boldsymbol{k}}_i\cdot\hat{\boldsymbol{k}}_r\,a_\rho - (a_\rho + a_c)\right).\end{aligned}\tag{7.22}$$

Change on p 160, last paragraph. “bulk modulus” should read “P-modulus”.

In these expressions, the subscripts i and j have been used to indicate x, y, and z components. The quantity γ stands for the P-modulus $\lambda+2\mu=\rho\alpha^2$, and is not to be confused with half-opening angle. The quantities ρ_0, α_0, and β_0 are the unperturbed or background values of density, P-velocity, and S-velocity, respectively. The parameters a_ρ, a_γ, and a_μ are the perturbations

Changes on p 161 to second and fourth paragraphs in section 7.4.1

7.4.1 Diagonalization operators

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The background wave operator $\mathcal{L}_{\mathrm{E}0}$ can be diagonalized into P- and S- components with the transformation matrix $\Pi\,\mathcal{L}_{\mathrm{E}0}\,\mathbf{\Pi}^{\mathrm{T}}\,\mathcal{J}^{-1}$ as defined in section 5.2.3.

Application of the same transformation to the scattering potential will separate it into compressional and shear components. The upper left diagonal term will correspond to an incoming and a reflected P-wave. Off-diagonal terms will correspond to converted waves of one sort or another.

We next apply the transformation (5.29) to $\mathcal{V}_{\mathrm{E}}$, forming $\mathbf{\Pi}\,\mathcal{V}_{\mathrm{E}}\,\mathbf{\Pi}^{\mathrm{T}}\,\mathcal{J}^{-1}$. With P- and S- components effectively separated, we can, using the logic of the last section, replace derivatives with their corresponding wavenumbers. In the matrix $\mathbf{\Pi}$ on the left, we replace derivatives with $\imath$ times a local reflected-wave wavenumber. For $\mathbf{\Pi}^T$ on the right, we replace derivatives with $\imath$ times a local incident-wave wavenumber. The operator $\mathcal{J}^{-1}$ evaluates to the unit operator inside the Born approximation, and can be ignored. We have to be a little careful in making these replacements, because P- and S-waves have different wavenumbers and velocities. The first row in $\mathbf{\Pi}$ and the first column in $\mathbf{\Pi}^T$ will have P-wave wavenumbers. All remaining elements will have S-wave wavenumbers. Denoting $\boldsymbol{k}_{\mathrm{P}r}$ and $\boldsymbol{k}_{\mathrm{P}i}$ to be the P-wave wavenumbers, and $\boldsymbol{k}_{\mathrm{S}r}$ and $\boldsymbol{k}_{\mathrm{S}i}$ to be the S-wave wavenumbers, with respective magnitudes

Changes on p 162:

then the matrices $\mathbf{\Pi}$ and $\mathbf{\Pi}^T$ on the left and right of $\mathcal{V}_{\mathrm{E}}$ become

$$\boldsymbol{\Pi} \to \boldsymbol{\Pi}_r = \frac{-1}{\omega}\begin{pmatrix} \alpha_0\, k_{\mathrm{Pr}\,x} & \alpha_0\, k_{\mathrm{Pr}\,y} & \alpha_0\, k_{\mathrm{Pr}\,z} \\ 0 & -\beta_0\, k_{\mathrm{Sr}\,z} & \beta_0\, k_{\mathrm{Sr}\,y} \\ \beta_0\, k_{\mathrm{Sr}\,z} & 0 & -\beta_0\, k_{\mathrm{Sr}\,x} \\ -\beta_0\, k_{\mathrm{Sr}\,y} & \beta_0\, k_{\mathrm{Sr}\,x} & 0 \end{pmatrix} = i \begin{pmatrix} \alpha_0\, \boldsymbol{k}_{\mathrm{Pr}}\,\cdot \\ \beta_0\, \boldsymbol{k}_{\mathrm{Sr}}\, \boldsymbol{\times} \end{pmatrix}, \tag{7.33}$$

and

$$\boldsymbol{\Pi}^{\mathrm{T}} \to \boldsymbol{\Pi}^{\mathrm{T}}{}_i = \frac{-1}{\omega}\begin{pmatrix} \alpha_0\, k_{\mathrm{Pi}\,x} & 0 & \beta_0\, k_{\mathrm{Si}\,z} & -\beta_0\, k_{\mathrm{Si}\,y} \\ \alpha_0\, k_{\mathrm{Pi}\,y} & -\beta_0\, k_{\mathrm{Si}\,z} & 0 & \beta_0\, k_{\mathrm{Si}\,x} \\ \alpha_0\, k_{\mathrm{Pi}\,z} & \beta_0\, k_{\mathrm{Si}\,y} & -\beta_0\, k_{\mathrm{Si}\,x} & 0 \end{pmatrix}$$

$$= \frac{-1}{\omega}\left(\alpha_0\, \boldsymbol{k}_{\mathrm{Pi}}\,\cdot^{\mathrm{T}} \quad \beta_0\, (\boldsymbol{k}_{\mathrm{Si}}\, \boldsymbol{\times})^{\mathrm{T}} \right). \tag{7.34}$$

Delete last sentence in section 7.4.1.

▪ 7.4.2 Rotation into SV and SH components

The product $\boldsymbol{\Pi}_r\, \mathcal{V}_{\mathrm{E}}\, \boldsymbol{\Pi}^{\mathrm{T}}{}_i$ is a 4 by 4 matrix, with elements corresponding to P-waves and the *x*-, *y*-, and *z*-components of S-waves. Since the shear waves are transverse, there are really only three independent dimensions. It would be convenient to explicitly reduce the number of dimensions to three, and deal with P-waves and the two independent S-wave components. We can do this by rotating the shear-wave components to a local coordinate system in which the third (longitudinal) S-wave component is zero.

Changes on pages 163-165:

Define 4 by 3 rotation matrices for the incoming and outgoing waves

$$\boldsymbol{E}_i \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e_{\mathrm{SV}ix} & e_{\mathrm{SV}iy} & e_{\mathrm{SV}iz} \\ 0 & -e_{\mathrm{SH}x} & -e_{\mathrm{SH}\,y} & -e_{\mathrm{SH}\,z} \end{pmatrix} = \begin{pmatrix} 1 & \boldsymbol{0}^{\mathrm{T}} \\ 0 & \hat{\boldsymbol{e}}_{\mathrm{SV}i}{}^{\mathrm{T}} \\ 0 & -\hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}} \end{pmatrix} \tag{7.42}$$

and

$$\boldsymbol{E}_r \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e_{\mathrm{SV}r\,x} & e_{\mathrm{SV}r\,y} & e_{\mathrm{SV}r\,z} \\ 0 & -e_{\mathrm{SH}x} & -e_{\mathrm{SH}\,y} & -e_{\mathrm{SH}\,z} \end{pmatrix} = \begin{pmatrix} 1 & \boldsymbol{0}^{\mathrm{T}} \\ 0 & \hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}} \\ 0 & -\hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}} \end{pmatrix} \tag{7.43}$$

that transform a PS – decomposed wave $\boldsymbol{\Psi} = \boldsymbol{\Pi}\, \boldsymbol{u}$ into SV and SH components $\boldsymbol{\Phi} = \boldsymbol{E}\, \boldsymbol{\Pi}\, \boldsymbol{u}$. We have

$$\boldsymbol{E}\, \boldsymbol{E}^{\mathrm{T}} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{7.44}$$

The matrix $\boldsymbol{E}^T\, \boldsymbol{E}$ is 4 by 4 and not the unit matrix. However,

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}\,\boldsymbol{\Pi} = \boldsymbol{\Pi}\,, \tag{7.45}$$

so that effectively, the transpose of $\boldsymbol{E}$ is its inverse.

The combination of the transposes of $\boldsymbol{E}$ and $\boldsymbol{\Pi}$ is, for incident waves,

$$\begin{aligned}\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i^{\mathrm{T}} &= \frac{-1}{\omega}\begin{pmatrix} \alpha_0\,k_{\mathrm{P}i\,x} & -\beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SV}i}\right)_x & \beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_x \\ \alpha_0\,k_{\mathrm{P}i\,y} & -\beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SV}i}\right)_y & \beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_y \\ \alpha_0\,k_{\mathrm{P}i\,z} & -\beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SV}i}\right)_z & \beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_z \end{pmatrix} \\ &= \frac{-1}{\omega}\left(\alpha_0\,\boldsymbol{k}_{\mathrm{P}i} \quad -\beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SV}i}\right) \quad +\beta_0\left(\boldsymbol{k}_{\mathrm{S}i}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)\right).\end{aligned} \tag{7.46}$$

Invoking the orthogonality relations (7.40), the product matrix can be written

$$\begin{aligned}\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i^{\mathrm{T}} &= -\begin{pmatrix} \frac{\alpha_0}{\omega}k_{\mathrm{P}i\,x} & e_{\mathrm{SH}x} & e_{\mathrm{SV}i\,x} \\ \frac{\alpha_0}{\omega}k_{\mathrm{P}i\,y} & e_{\mathrm{SH}\,y} & e_{\mathrm{SV}i\,y} \\ \frac{\alpha_0}{\omega}k_{\mathrm{P}i\,z} & e_{\mathrm{SHz}} & e_{\mathrm{SV}i\,z} \end{pmatrix} \\ &= -\left(\frac{\alpha_0}{\omega}\boldsymbol{k}_{\mathrm{P}i} \quad \hat{\boldsymbol{e}}_{\mathrm{SH}} \quad \hat{\boldsymbol{e}}_{\mathrm{SV}i}\right).\end{aligned} \tag{7.47}$$

Similarly, the product of $\boldsymbol{E}$ and $\boldsymbol{\Pi}$ is, for reflected waves,

$$\begin{aligned}\boldsymbol{E}_r\,\boldsymbol{\Pi}_r &= \frac{-1}{\omega}\begin{pmatrix} \alpha_0\,k_{\mathrm{P}rx} & \alpha_0\,k_{\mathrm{P}ry} & \alpha_0\,k_{\mathrm{P}rz} \\ -\beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SV}r}\right)_x & -\beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SV}r}\right)_y & -\beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SV}r}\right)_z \\ \beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_x & \beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_y & \beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)_y \end{pmatrix} \\ &= \frac{-1}{\omega}\begin{pmatrix} \alpha_0\,\boldsymbol{k}_{\mathrm{P}r}{}^{\mathrm{T}} \\ -\beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SV}r}\right)^{\mathrm{T}} \\ \beta_0\left(\boldsymbol{k}_{\mathrm{S}r}\times\hat{\boldsymbol{e}}_{\mathrm{SH}}\right)^{\mathrm{T}} \end{pmatrix}.\end{aligned} \tag{7.48}$$

Invoking the orthogonality relations (7.41), this product matrix can be written

$$\begin{aligned}\boldsymbol{E}_r\,\boldsymbol{\Pi}_r &= -\begin{pmatrix} \frac{\alpha_0}{\omega}k_{\mathrm{P}r\,x} & \frac{\alpha_0}{\omega}k_{\mathrm{P}r\,y} & \frac{\alpha_0}{\omega}k_{\mathrm{P}r\,z} \\ e_{\mathrm{SH}x} & e_{\mathrm{SH}y} & e_{\mathrm{SHz}} \\ e_{\mathrm{SV}r\,x} & e_{\mathrm{SV}r\,y} & e_{\mathrm{SV}r\,z} \end{pmatrix} \\ &= -\begin{pmatrix} \frac{\alpha_0}{\omega}\boldsymbol{k}_{\mathrm{P}r}{}^{\mathrm{T}} \\ \hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}} \\ \hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}} \end{pmatrix}.\end{aligned} \tag{7.49}$$

Applied to a displacement vector, the top row of $\boldsymbol{E}_r\,\boldsymbol{\Pi}_r$ and first column of $\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i^{\mathrm{T}}$ separate out the

P-component. The second row of $\boldsymbol{E}_r\,\boldsymbol{\Pi}_r$ and second column of $\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i{}^{\mathrm{T}}$ separate out the SH-component, and the third row of $\boldsymbol{E}_r\,\boldsymbol{\Pi}_r$ and third column of $\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i{}^{\mathrm{T}}$ separate out the SV-component.

The operator we now want to evaluate is the scattering potential in the P-SH-SV coordinate system, or

$$\mathcal{V}_{\mathrm{W}} \equiv \boldsymbol{E}_r\,\boldsymbol{\Pi}_r\,\mathcal{V}_{\mathrm{E}}\,\boldsymbol{\Pi}^{\mathrm{T}}{}_i\,\boldsymbol{E}_i{}^{\mathrm{T}}. \tag{7.50}$$

The elements of this potential matrix can be identified with P, SH and SV-wave components as follows:

$$\mathcal{V}_{\mathrm{W}} = \begin{pmatrix} \mathcal{V}_{\mathrm{PP}} & \mathcal{V}_{\mathrm{PSH}} & \mathcal{V}_{\mathrm{PSV}} \\ \mathcal{V}_{\mathrm{SHP}} & \mathcal{V}_{\mathrm{SHSH}} & \mathcal{V}_{\mathrm{SHSV}} \\ \mathcal{V}_{\mathrm{SVP}} & \mathcal{V}_{\mathrm{SVSH}} & \mathcal{V}_{\mathrm{SVSV}} \end{pmatrix}$$

$$= \begin{pmatrix} \left(\alpha_0{}^2 / \omega^2\right) \boldsymbol{k}_{\mathrm{P}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\boldsymbol{k}_{\mathrm{P}i} & (\alpha_0/\omega)\,\boldsymbol{k}_{\mathrm{P}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SH}} & (\alpha/\omega)\,\boldsymbol{k}_{\mathrm{P}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SV}i} \\ (\alpha_0/\omega)\,\hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\boldsymbol{k}_{\mathrm{P}i} & \hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SH}} & \hat{\boldsymbol{e}}_{\mathrm{SH}}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SV}i} \\ (\alpha_0/\omega)\,\hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\boldsymbol{k}_{\mathrm{P}i} & \hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SH}} & \hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\hat{\boldsymbol{e}}_{\mathrm{SV}i} \end{pmatrix}. \tag{7.51}$$

The elements of $\mathcal{V}_{\mathrm{E}}$, given in equation (7.27), also contain derivatives. Consistent with our treatment of $\boldsymbol{\Pi}$, we replace the derivatives with wavenumbers as follows: In the first row of $\mathcal{V}_{\mathrm{W}}$, derivatives appearing to the left of the elastic parameters a_ρ, a_γ, or a_μ are replaced by $i\,\boldsymbol{k}_{Pr}$. In remaining rows, left-derivatives are replaced by $i\,\boldsymbol{k}_{Sr}$. In the first column of $\mathcal{V}_{\mathrm{W}}$, derivatives appearing on the right of the elastic parameters are replaced by $i\,\boldsymbol{k}_{Pi}$. In the remaining columns, right-derivatives are replaced by $i\,\boldsymbol{k}_{Si}$.

■ 7.4.3 The P-to-P scattering potential

page 166. " α_γ " → " a_γ " in equation 7.53 and 7.55

$$\mathcal{V}_{\mathrm{PP}} = -\frac{\rho_0\,\alpha_0{}^2}{\omega^2}\left(a_\gamma\,\alpha_0{}^2\,k_{\mathrm{P}r}{}^2\,k_{\mathrm{P}i}{}^2 - a_\rho\,\boldsymbol{k}_{\mathrm{P}r}\cdot\boldsymbol{k}_{\mathrm{P}i}\,\omega^2 - 2\,a_\mu\,\beta_0{}^2\left|\boldsymbol{k}_{\mathrm{P}r}\times\boldsymbol{k}_{\mathrm{P}i}\right|^2\right) \tag{7.53}$$

$$\mathcal{V}_{\mathrm{PP}} \to \omega^2\,\mathbb{V}_{\mathrm{PP}}(\boldsymbol{x},\,\sigma) = -\frac{\omega^2}{\alpha_0{}^2}\,C_{\mathrm{P}}{}^2\left(a_\gamma + a_\rho\cos\sigma - 2\,\frac{\beta_0{}^2}{\alpha_0{}^2}\,a_\mu\,\sin^2\sigma\right), \tag{7.55}$$

page 167. " α_μ " → " a_μ " in equation 7.60

■ 7.4.4 The SH to SH scattering potential

$$\mathcal{V}_{\mathrm{SHSH}} = \rho_0\,\omega^2 \left(a_\rho \ - a_\mu \,\frac{(\boldsymbol{k}_{\mathrm{S}r} \cdot \boldsymbol{k}_{\mathrm{S}i})}{\omega^2} \right). \tag{7.60}$$

page 170. changes to equations (7.72), (7.73), (7.76), (7.77), (7.78), (7.79). In last sentence on this page, change "proportional" to "equal".

■ 7.4.5 The P to SV and SV to P scattering potentials

The P to SV component of the scattering potential is the element in the third row and first column of $\boldsymbol{\mathcal{V}}_{\mathrm{W}}$, or

$$\mathcal{V}_{\mathrm{SVP}} = \frac{\alpha_0}{\omega}\,\hat{\boldsymbol{e}}_{\mathrm{SV}r}{}^{\mathrm{T}}\,\mathcal{V}_{\mathrm{E}}\,\boldsymbol{k}_{Pi}, \tag{7.72}$$

or

$$\mathcal{V}_{\mathrm{SVP}} = -\,\frac{\alpha_0\,\beta_0}{\omega^2}\,\rho_0 \left| \boldsymbol{k}_{\mathrm{S}r} \times \boldsymbol{k}_{\mathrm{P}i} \right| \left(a_\rho\ \omega^2 - 2\,a_\mu\,(\boldsymbol{k}_{\mathrm{S}r} \cdot \boldsymbol{k}_{\mathrm{P}i})\,\beta_0{}^2 \right). \tag{7.73}$$

Since the incoming wave is a P-wave and the outgoing wave is a shear wave, we define the opening angle σ between them with the relation

$$\boldsymbol{k}_{\mathrm{S}r} \cdot \boldsymbol{k}_{\mathrm{P}i} = -\,\frac{\omega^2}{\alpha_0\,\beta_0}\,\cos\sigma. \tag{7.74}$$

Likewise,

$$\left| \boldsymbol{k}_{\mathrm{S}r} \times \boldsymbol{k}_{\mathrm{P}i} \right| = \frac{\omega^2}{\alpha_0\,\beta_0} \left| \sin\sigma \right|. \tag{7.75}$$

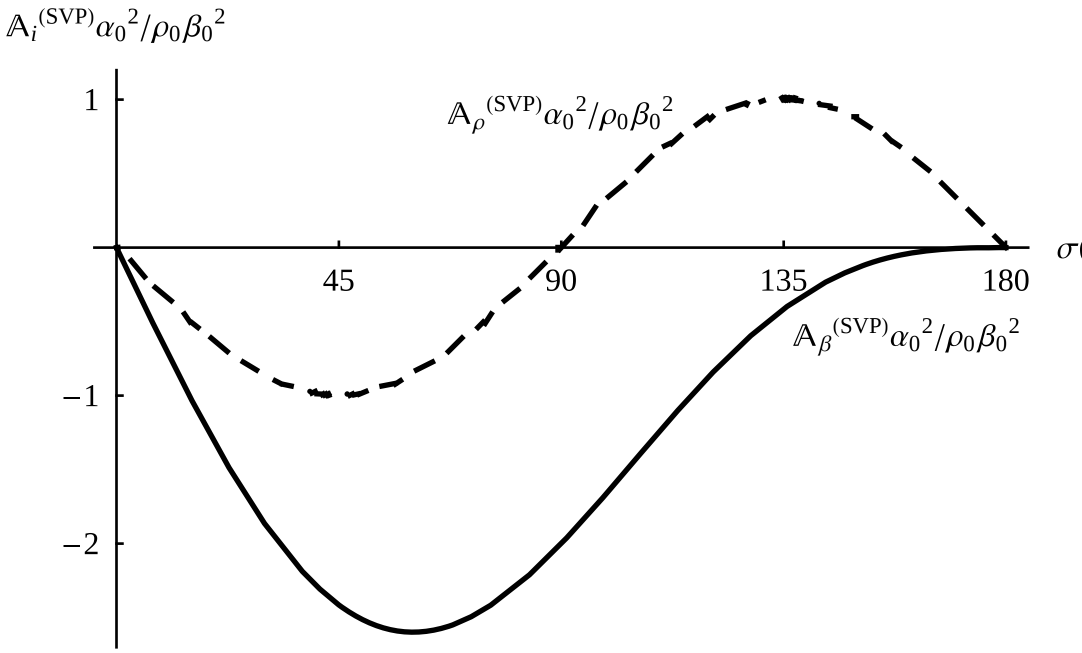
Thus,

$$\mathcal{V}_{\mathrm{SVP}} \rightarrow \omega^2\,\mathbb{V}_{\mathrm{SVP}}(\boldsymbol{x},\,\sigma) = -C_{\mathrm{P}}{}^2\,\frac{\omega^2}{\alpha_0{}^2} \left| \sin\sigma \right| \left(a_\rho\ + 2\,a_\mu\,\frac{\beta_0}{\alpha_0}\,\cos\sigma \right). \tag{7.76}$$

In terms of density and velocity perturbations,

$$\begin{aligned} \mathbb{V}_{\mathrm{SVP}}(\boldsymbol{x},\,\omega,\,\sigma) &\equiv \mathbb{A}_\beta{}^{(\mathrm{SVP})}\,a_\beta + \mathbb{A}_\rho{}^{(\mathrm{SVP})}\,a_\rho \\ &= -\rho_0 \left| \sin\sigma \right| \left(a_\rho \left(1\ + 2\,\frac{\beta_0}{\alpha_0}\,\cos\sigma \right) + 2\,a_\beta\,\frac{\beta_0}{\alpha_0}\,\cos\sigma \right). \end{aligned} \tag{7.77}$$

The components of the frequency-reduced potential are illustrated in Figure 7.6.



The components of the P to SV frequency-reduced scattering potential $\mathbb{V}_{SVP}$, as a function of opening angle σ. The ratio α_0/β_0 has been set to two in this figure. The dashed curve is the coefficient $\rho_0^{-1}\,\mathbb{A}_\beta^{(SVP)}$ of the shear velocity perturbation a_β, and the solid curve is the coefficient $\rho_0^{-1}\,\mathbb{A}_\rho^{(SVP)}$ of the density perturbation a_ρ. The SV to P to potential $\mathbb{V}_{PSV}$ is proportional to the negative of $\mathbb{V}_{SVP}$.

For the SV to P scattering potential, a similar analysis shows

$$\mathbb{V}_{PSV}(\boldsymbol{x},\,\omega,\,\sigma) = +\rho_0 \left|\sin\sigma\right| \left(a_\rho + 2\,a_\mu\,\frac{\beta_0}{\alpha_0}\cos\sigma\right), \tag{7.78}$$

or

$$\mathbb{V}_{PSV}(\boldsymbol{x},\,\omega,\,\sigma) = +\rho_0 \left|\sin\sigma\right| \left(a_\rho\left(1+2\,\frac{\beta_0}{\alpha_0}\cos\sigma\right) + 2\,a_\beta\,\frac{\beta_0}{\alpha_0}\cos\sigma\right). \tag{7.79}$$

It is apparent that $\mathbb{V}_{PSV}$ is equal to the negative of $\mathbb{V}_{SVP}$.

page 172. changes to equations (7.86), (7.87),

■ 7.5 Summary

$$\mathbb{V}_{SVP}(\boldsymbol{x},\,\sigma) = -\rho_0(\boldsymbol{x}) \left|\sin\sigma\right| \left(a_\rho(\boldsymbol{x})\left(1+2\,\frac{\beta_0(\boldsymbol{x})}{\alpha_0(\boldsymbol{x})}\cos\sigma\right) + a_\beta(\boldsymbol{x})\,2\,\frac{\beta_0(\boldsymbol{x})}{\alpha_0(\boldsymbol{x})}\cos\sigma\right) \tag{7.86}$$

for P to SV waves, and

$$\mathbb{V}_{\mathrm{PSV}}(\boldsymbol{x},\ \sigma_0) = \rho_0(\boldsymbol{x}) \left|\sin\sigma\right| \left(a_\rho(\boldsymbol{x}) \left(1 + 2\,\frac{\beta_0(\boldsymbol{x})}{\alpha_0(\boldsymbol{x})} \cos\sigma \right) + 2\, a_\beta(\boldsymbol{x})\, \frac{\beta_0(\boldsymbol{x})}{\alpha_0(\boldsymbol{x})} \cos\sigma \right). \tag{7.87}$$