HISTORICAL NOTES

for

A First Course in Mathematical Analysis

David A. Brannan

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This booklet contains short biographical information on the thirty or so mathematicians mentioned in the book **A First Course in Mathematical Analysis** by *David Alexander Brannan*, first published in 2006 by *Cambridge University Press* as ISBN: 0-521-68424-2, and reprinted with corrections in 2012.

The author believes that it is interesting and indeed positively encouraging while studying any mathematics to learn a little about the mathematicians who developed the subject and their historical environment. He hopes that this short booklet will whet readers' appetites to learn more about these fascinating historical figures, after their own efforts reading the mathematics in his book!

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was created and is maintained by John J. O'Connor and Edmund F. Robertson at the Mathematical Institute, University of St Andrews, Scotland. The archive is an impressive and invaluable resource to students and active mathematicians alike, and is warmly recommended to readers.

The archive includes:
- the biographies of more than 1600 mathematicians;
- chronologies showing the overlapping lives of the mathematicians in the archive;
- a collection of articles on Famous Curves which have been extensively studied by mathematicians, giving their history as well as pictures of the curves and various curves (evolutes, inverses, caustics etc.) associated with them;
- anniversaries for the year containing details for all dates;
- maps showing the birthplaces of those Western European or North American mathematicians in the list of biographies.

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ACKNOWLEDGEMENTS

The author gratefully acknowledges information received from June Barrow-Green and Ian Tweddle.

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ix **Johann Wolfgang von Goethe** (1749-1832), the distinguished German writer, poet, artist and statesman, was born in Frankfurt. He studied law in Leipzig (1765-68); but spent most of his life in Weimar where he held a succession of offices under the Duke of Saxe-Weimar. He was ennobled by the Duke in 1782 (hence the "von" in his name) following the success of his first novel. Because of Goethe's reputation, Weimar was chosen after WW1 (in 1919) as the location for the assembly that drafted a new constitution for Germany, which become known as the *Weimar Republic*.

A major literary figure, he wrote epic and lyric poetry, novels, dramas, memoirs and literary criticism; and made over 2,000 drawings. Germany's international cultural institution *The Goethe-Institut* is named after him.

As a polymath he is said to have studied all areas of science of his day except mathematics, for which he apparently had no aptitude. In particular, he wrote on morphology (he was said to have 17,800 rock samples, the largest private collection of minerals in Europe; 'goethite' [iron oxide] is named after him), botany, and the theory of colour.

Goethe is widely quoted - for example, his epigram '*Divide and rule, a sound motto; unite and lead, a better one'.* Other well-known quotations are often incorrectly attributed to him; for example, '*Art is long, life is short!*' [from Hippocrates] or of uncertain provenance, such as *'You really do not see a plant until you draw it'*.

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ix Calculus was invented independently by **Sir Isaac Newton** and **Gottfried Wilhelm Leibniz**.

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ix **Sir Isaac Newton** (1643-1727) was born in the manor house of Woolsthorpe, near Grantham in Lincolnshire, three months after his father's death. In 1661 Newton went to Trinity College, Cambridge University, to study law but became interested in mathematics. When the university closed in 1665 because of the Great Plague, Newton returned to Lincolnshire and in the next two years made revolutionary advances in mathematics, physics, optics, and astronomy.

Newton contributed to the study of power series, generalised the *Binomial Theorem* to non-integer exponents, and developed a method for approximating the roots of a function. His major *method of fluxions*, or *differential and integral calculus* as we would call it, was based on his realisation that integration is precisely the inverse procedure to differentiation. However from 1699 he was involved in a bitter controversy with Leibniz over priority for the discovery of calculus; this soured relations between English and Continental mathematicians for a very long time.

His greatest achievement was his work in physics and celestial mechanics, which culminated in the *theory of gravitation* and the *inverse-square law*, published in his book *Philosophiae naturalis principia mathematica* (1687); this explained the eccentric orbits of comets, the tides and their variations, the precession of the Earth's axis, and motion of the Moon as perturbed by the Sun's gravity. Newton himself often recounted that he was inspired to formulate his theory of gravitation by watching an apple fall from a tree at Woolsthorpe.

Newton built the first practical reflecting telescope in 1668, and developed a theory of colour based on the observation that a prism decomposes white light into the many colours of the visible spectrum. His book *Opticks* (1704) addressed the theory of light and colour, 'Newton's rings' and diffraction of light; and used a wave theory of light as well as a corpuscular theory.

The University of Cambridge elected Newton as one of their members in the Convention Parliament that offered the crown to **William and Mary** (1689) after the flight to France of **King James II & VII** (1633-1701). He became Warden of the Royal Mint in 1696 and Master in 1699. He served as President of the Royal Society from 1703 till his death, and was knighted by Queen Anne in 1705. He is buried in Westminster Abbey.

The poet **Alexander Pope** (1688-1744) wrote the famous epitaph '*God said "Let Newton be" and all was light*'; Newton himself wrote '*If I have seen further it is by standing on the shoulders of giants'*.

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ix **Gottfried Wilhelm Leibniz** (1646-1716) was born in Leipzig, and entered the University of Leipzig in 1661 where he studied philosophy, mathematics and law.

In Paris Leibniz studied mathematics and physics under **Christiaan Huygens** (1629-1695) beginning in the autumn of 1672. He then began to study the geometry of infinitesimals, developing the basic features of his version of the calculus. He developed the modern notation dy/dx using a 'd' from the Latin *differentia* (rather than Newton's $\dot{y}$ notation) for differentiation, and the integral sign $\int$ representing an elongated S from the Latin *summa* for integration. In 1684 Leibniz published details of his approach to calculus, including the rules for computing the derivatives of powers, products and quotients. Much of the mathematical activity of Leibniz's last years involved the dispute with Newton over priority for invention of the calculus.

He also carried out novel work on dynamics, studying kinetic energy, potential energy and momentum; refined the binary system of arithmetic, which plays a crucial role in digital computing; and worked on Gaussian elimination and determinants. As a logician, Leibniz enunciated the principal properties of conjunction, disjunction, negation, identity, set inclusion, and the empty set.

He was a prolific inventor of mechanical calculators, and in 1673 demonstrated to the Royal Society a calculating machine that he had designed that could execute the four basic operations of addition, subtraction, multiplication and division.

In 1676 Leibniz became Librarian and Court Councillor for the Duke of Hanover. There he undertook many duties and projects in diplomacy, librarianship, public health and administration; meanwhile he carried out much original work in history, theology, philosophy, metaphysics, psychology, and geology, with a huge European-wide correspondence. He invented the idea of '*monads*', elementary particles with blurred perceptions of one another; and believed that much of human reasoning could be reduced to calculations of a sort, and that such calculations could resolve many differences of opinion.

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x **Karl Theodor Wilhelm Weierstrass** (1815-1897) is often called '*the father of Modern Analysis'*.

Weierstrass entered the University of Bonn in 1834 to study law, finance and economics, but did little work (instead studying mathematics privately) and left without graduating. He then trained in Münster as a school mathematics teacher, and followed this career from 1841 to 1856; he was required to also teach physics, botany, geography, history, German, calligraphy and gymnastics; he was very unhappy with his lot, having no colleague for mathematical discussions nor access to a mathematical library. In 1854 he published a paper describing his theory of inversion of hyperelliptic integrals, which launched his academic career. In 1856 he accepted a chair at the Industry Institute in Berlin, moving a few years later to the University of Berlin.

He published little until his complete works (including his ground-breaking lecture notes) appeared between 1894 and 1927. However his rigorous lectures covered leading edge topics including analytic functions (including the concept of 'function', power series, continuity and differentiability, analytic continuation, points of singularity, analytic functions of several variables, and contour integrals), elliptic functions, Abelian functions and the calculus of variations. He also worked on irrational numbers as limits of convergent series, the formal definitions of continuity and uniform continuity, entire functions, uniform convergence, and infinite products.

He discovered (1872) a function that is continuous on $\mathbb{R}$ but differentiable nowhere on $\mathbb{R}$, and proved the *Intermediate Value Theorem*, the *Bolzano–Weierstrass Theorem* and the *Heine–Borel Theorem*.

He had many distinguished students, including his particular protégé **Sofia Kovalevskaya** (1850-1891) – the first major Russian female mathematician and the first woman appointed to a chair in Northern Europe.

The lunar *Weierstrass Crater* is named in his honour.

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xi **Georg Friedrich Bernhard Riemann** (1826-1866) was born in Breselenz, Hanover, and enrolled at the University of Göttingen in 1846 to study theology, but transferred to the Faculty of Philosophy so that he could study mathematics, taking courses under **Johann Carl Friedrich Gauss** (1777-1855; prolific German mathematician and astronomer, who proved the *Fundamental Theorem of Algebra* and made major contributions to number theory, geometry, algebra, analysis, astronomy, statistics and mathematical physics). Riemann moved from Göttingen to Berlin University in the spring of 1847 to study under **Jakob Steiner** (1796-1863; Swiss geometer), **Carl Gustav Jacob Jacobi** (1804-1851; German mathematician, who made fundamental contributions to elliptic functions, dynamics, differential equations, and number theory) and Dirichlet (who had a great influence on him). He completed his PhD thesis under Gauss in Göttingen in 1851, with ground-breaking discoveries in the geometric properties of analytic functions, abelian and theta functions, conformal mappings and the connectivity of surfaces. He was appointed to a chair at Göttingen in 1857.

He made lasting contributions to analysis, number theory, and differential geometry; and established a geometric foundation for complex analysis via Riemann surfaces. His famous *Riemann Mapping Theorem* says that a simply-connected domain in the complex plane $\mathbb{C}$ (that is not $\mathbb{C}$ itself) can be mapped one-one by an analytic function onto the unit disc. In his proof he used a minimality condition, called the *Dirichlet Principle*, that Weierstrass found to be incomplete: Riemann had not noticed that his working assumption that a certain minimum existed might not be valid. However in 1901 **David Hilbert** (1862-1943; highly influential German mathematician who worked on the foundations of geometry and mathematics in general; best known for his *Hilbert's Programme* [which lead to the development of *Computability Theory*], *Hilbert's Problems* [major influences on 20th century mathematics], *Hilbert spaces* and *Hilbert's Basis Theorem*) gave a rigorous statement of Dirichlet's Principle that made Riemann's proof valid.

He studied the relationship of geometry (in n-dimensions) to the real world, discovering in *Riemannian geometry* extensions to the differential geometry of surfaces using the *Riemann curvature tensor* that played a crucial role in the later development of relativity.

Riemann examined the properties of the (Riemann) Zeta function $\zeta(s)=\sum_{n\in\mathbb{N}}\frac{1}{n^s}=\prod_{p\text{ prime}}\left(1-p^{-s}\right)^{-1}$, $s\in\mathbb{C}$; and his *Riemann hypothesis* (1859) that the complex zeros of $\zeta(s)$ all lie on the line $\operatorname{Re}s=\frac{1}{2}$ is still a major unsolved problem in mathematics.

In real analysis, he discovered the *Riemann integral*. He showed that every piecewise continuous function is integrable; that integrable functions are representable by Fourier series; gave an example of a Fourier series representing a continuous, almost nowhere-differentiable function; and proved the *Riemann-Lebesgue Lemma* that if a function is representable by a Fourier series, then its Fourier coefficients tend to zero.

The lunar *Riemann Crater* is named after him.

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5 **Johann Heinrich Lambert** (1728-1777) was born in Mulhouse, then an exclave of the Swiss Confederation. He left a job in an ironworks at 17 to become secretary to a newspaper editor, studying mathematics, astronomy, and philosophy in his free time. In 1748, he became tutor for **Count Peter von Salis** (1738-1807) in Chur, and continued his studies; while in Chur, he made his own astronomical instruments. Later (1764) he was appointed to the Prussian Academy of Sciences in Berlin, where he worked immensely productively.

Lambert is best known for proving that π is irrational (1768). He showed that, if x is a non-zero rational number, then neither e^x nor $\tan x$ can be rational; so, since $\tan(\pi/4)=1$, it follows that π must be irrational. He also conjectured that e and π are transcendental (i.e. are not the real roots of any polynomial equation with integer coefficients). (**Charles Hermite** (1822-1901) proved that e is transcendental (1873) and Carl Louis Ferdinand von Lindemann proved that π is transcendental (1882).)

In a study of the *parallel postulate* (1766) he managed to deduce a large number of non-Euclidean results by assuming that the parallel postulate was false, and noticed that in this new *hyperbolic geometry* the sum of the angles of a triangle increases as its area decreases.He also made the first systematic development of hyperbolic functions.

Lambert was the first mathematician to address the general properties of map projections, looking particularly at the properties of conformality and equal area preservation - pointing out that they are mutually exclusive. He published seven new map projections in 1772.

He discovered *Lambert's Law of Absorption* of the exponential decrease of the light in a beam passing through an absorbing medium of uniform transparency, and *Lambert's Cosine Law* that the brightness of a diffusely radiating plane surface is proportional to the cosine of the angle formed by the line of sight and the normal to the surface. He also developed the notion that the Universe is composed of galaxies of stars; worked on photometry; and invented the first practical hygrometer.

The asteroid *187 Lamberta* and the photometric unit *lambert* are named in his honour.

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7 **Archimedes** (c. 287 BC – 212 BC) was a Greek geometer, physicist and engineer, who lived in Syracuse, Sicily**.**

He invented the *Archimedes screw* to transfer water from a low-lying body of water into irrigation canals. He gave an explanation of the *principle of the lever* (reputedly saying '*Give me a place to stand on, and I will move the Earth*'); designed block-and-tackle pulley systems; and invented many engines of war for the defence of Syracuse when it was attacked by the Romans, including the *Claw of Archimedes* to lift and shake an enemy ship and *parabolic reflecting mirrors* to set fire to besieging ships.

In Mechanics he discovered fundamental theorems concerning the centre of gravity of plane figures and solids. He also laid down the basic principles of Hydrostatics, including *Archimedes Principle* which he used to determine whether the gold crown of **King Hiero II** (c. 308 BC – 215 BC) of Syracuse contained pure gold or had been adulterated. It is said that when he discovered the Principle in his bath he was so excited that he ran naked into town shouting '*Eureka!*' ('*I have found [it]*') - now the state motto of California, '*the Golden State*'!

In Mathematics Archimedes used infinitesimals via his *method of exhaustion* in a way similar to modern integral calculus, enabling him to to find areas, volumes and surface areas of many bodies. Through his method he was also able to approximate the value of π by 96-sided polygons inscribed inside and circumscribed outside a circle; he found that $3\tfrac{10}{71}(\cong 3.1408)<\pi(\cong 3.1416)<3\tfrac{1}{7}(\cong 3.1429)$; and he proved that the area of a circle of radius r is πr^2. He stated the *Archimedean Property* of real numbers; devised a method for calculating square roots accurately; determined the area of a segment of a parabola cut off by any chord; and studied the volumes of various solids of revolution cut by planes.

The *Fields Medal* for outstanding achievement in mathematics carries a portrait of Archimedes; and a lunar *Archimedes Crater*, a lunar mountain range *Montes Archimedes* and an asteroid *3600 Archimedes* are named after him.

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20 **Jacob Bernoulli** (1654-1705) was born in Basel, Switzerland; his father ran a family spice business in Basel. He graduated from the University of Basel with a master's degree in philosophy in 1671 and a licentiate in theology in 1676, meantime studying mathematics and astronomy against the wishes of his parents. He then moved to Geneva to work as a tutor, to France to study with the followers of the French philosopher and mathematician **René Descartes** (1596-1650), then to the Netherlands, and on to England where, among others, he met **Robert Boyle** (1627-1691; Anglo-Irish physicist and chemist) and **Robert Hooke** (1635-1703; English physicist, architect and polymath).

He returned to the University of Basel in 1683, to teach mechanics; there he read Descartes' *Géométrie* and the work of the English mathematicians John Wallis and **Isaac Barrow** (1630-1677; English mathematician and theologian, with an important early role in the development of infinitesimal calculus; he discovered the *Fundamental Theorem of Calculus*), and became interested in infinitesimal geometry. He was appointed professor in 1687, and began to study Leibniz's differential calculus with his brother Johann; unfortunately their collaborations turned to bitter rivalry and a complete breakdown of relations by 1697.

He made important contributions to the parallels of logic and algebra (1685), probability (1685) and geometry (1687) (e.g. he gave a construction to divide any triangle into four equal parts with two perpendicular lines), infinite series and the law of large numbers in probability theory (1689), solved the isochrones problem (1690), solved *Bernoulli's equation* [$y' = p(x)y + q(x)y^n$], obtained a general method to determine evolutes of a curve as the envelope of its circles of curvature, and studied caustic curves. While studying compound interest he discovered the constant e as the limit $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. His influential book on probability *Ars Conjectandi* was published posthumously in 1713.

The lunar *Bernoulli Crater* is named after him, jointly with his brother Johann.

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20 **Augustin-Louis Cauchy** (1879-1857) was born in Paris, France during the French Revolution. He entered the École Polytechnique in 1805, graduating in 1807; then entered the École des Ponts et Chaussées, where he was assigned to the Ourcq Canal project. In 1810 Cauchy was appointed to work on port facilities in Cherbourg for **Napoleon**'s English invasion fleet, and studied mathematics in his own time; in 1813 he returned as an engineer on the Ourcq Canal project. In 1815 he was appointed to the École Polytechnique.

He was rather contentious! He inherited his father's staunch pro-Bourbon royalism and hence refused to take oaths to any government after the overthrow of French **King Charles X** (1757-1836) in 1830, resulting in him not receiving several academic appointments. He went to Switzerland (1830), Turin (1831-1833), Prague (1833-1838), and returned to Paris in 1838. In 1839 he was appointed to the Paris Bureau des Longitudes, and directed his research to celestial mechanics; on account of not taking the loyal oath, his appointment was not approved by **King Louis Philippe I** (1773-1850), and he was eventually replaced there in 1843. After the 1848 overthrow of the king, France became a Republic, and Cauchy was reinstated at the Faculté des Sciences, as professor of mathematical astronomy, but continued to be difficult with his colleagues. He was a staunch Catholic, and pleaded on behalf of the Irish during the disastrous Irish Potato Famine (1845-1852).

He proved in 1811 that the angles of a convex polyhedron are determined by its faces, and in 1814 he published the memoir on definite integrals that later became the basis of his theory of complex functions.

He made a rigorous study of the convergence of infinite series, and discovered the *Cauchy Condensation Test* and many of the basic formulas for q-series. He was the first to define complex numbers as pairs of real numbers, and founded the field of complex analysis making fundamental discoveries in the calculus of residues, *Cauchy's Theorem* and *Cauchy's Integral Formula*. He worked on permutation groups, symmetric functions, the symmetric group, the theory of higher-order algebraic equations, differential equations and determinants; and proved the *Fermat Polygonal Number Theorem*. He also worked in mathematical physics and astronomy, making crucial discoveries in the theory of wave propagation, Fresnel's wave theory and the dispersion and polarization of light, mechanics, elasticity and stress theory (introducing the *Cauchy stress tensor).* He produced 789 mathematics papers; his collected works were published in 27 volumes!

In his book *Cours d'Analyse* (1821) Cauchy stressed the importance of rigour in Mathematical Analysis, and defined continuity in modern terms: '*The function* $f(x)$ *is continuous with respect to* x *between the given limits if, between these limits, an infinitely small increment in the variable always produces an infinitely small increment in the function itself.*'

His name is one of the 72 names inscribed on the Eiffel Tower; and the lunar *Cauchy Crater* and escarpment *Rupes Cauchy* are named in his honour.

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20 **Karl Hermann Amandus Schwarz** (1843-1921) was born in Hermsdorf, Silesia (now Poland). He entered the Gewerbeinstitut, now the Technical University of Berlin, to study chemistry, but switched to mathematics under the influence of **Ernst Eduard Kummer** (1810-1893; German analyst, geometer and number theorist) and Karl Weierstrass. He became interested in geometry and analysis, gaining his doctorate in 1864 under Weierstrass. In 1867 he was appointed to the University of Halle, then to a chair at the Eidgenössische Technische

Hochschule in Zurich (1869), a chair at Göttingen University (1875), and finally a chair in Berlin (1892).

Schwarz worked first on the calculus of variations, discovering the *Schwarz minimal surface*. He then produced a rigorous proof of the *Riemann Mapping Theorem* (1870), avoiding the hole in the current version of the *Dirichlet Principle*, and solved the *Dirichlet Problem*. He also answered the question of whether a given minimal surface really yields a minimal area; in doing so he used a special case of the *Cauchy-Schwarz Inequality* (also called the *Cauchy-Schwarz-Bunyakovsky Inequality*; **Viktor Yakovlevich Bunyakovsky** (1804-1889) was a Russian student of Cauchy who worked on number theory, geometry and applied mathematics). He defined a conformal mapping of a triangle with arcs of circles as sides onto the unit disc, the *Schwarz function*; this is an example of an automorphic function, and Schwarz's ideas led to the development of the theory of automorphic functions.

Among his doctoral students were **Leopold Fejer** (1880-1959; Hungarian harmonic analyst), **Paul Koebe** (1882-1945; German complex analyst), **Robert Erich Remak** (1888-1942; German group theorist and number theorist) and **Ernst Friedrich Ferdinand Zermelo** (1871–1953; German logician and mathematician, whose work has major implications for the foundations of mathematics).

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37 **Georg Ferdinand Ludwig Philipp Cantor** (1845-1918) was born in St Petersburg, Russia, moving to Germany in 1856. He entered the Polytechnic of Zurich in 1862, moving to the University of Berlin in 1863. There he became friends with a fellow student Hermann Amandus Schwarz and attended lectures by Weierstrass, Kummer and **Leopold Kronecker** (1823-1891; German algebraist and number theorist), graduating with a thesis on number theory in 1867; he received his habilitation in 1869. He was then appointed to the University of Halle, where his research turned towards analysis; he solved the problem of the uniqueness of representation of a function as a trigonometric series in 1870.

Between 1879 and 1884 Cantor published a series of six papers providing a basic introduction to set theory. He proved that there is a 1-1 correspondence between points in the interval $[0,1]$ and points in $\mathbb{R}^p$; that the rational numbers and the algebraic numbers (i.e. those that are roots of polynomial equations with integer coefficients) are countable (i.e. may be put in 1-1 correspondence with the natural numbers); and discovered the *Cantor diagonalisation method* and the *Cantor set* (important in modern dynamical systems). He is best known as the inventor of set theory; established the importance of 1-1 correspondences between the members of sets, and defined the cardinal and ordinal numbers and their arithmetic.

Cantor's work on set theory had many virulent critics: e.g. Kronecker described Cantor as a '*scientific charlatan* and *corrupter of youth*'; and **Ludwig Josef Johann Wittgenstein** (1889-1951; Austrian-British philosopher, who contributed to the philosophy of mathematics) lamented that mathematics is '*ridden through and through with the pernicious idioms of set theory*' which he dismissed as '*utter nonsense*'.

On the other hand, in 1926 David Hilbert declared '*No one shall expel us from the Paradise that Cantor has created'*, echoing the unanimous modern opinion.

The lunar *Cantor Crater* is named in his honour.

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37 **Julius Wilhelm Richard Dedekind** (1831-1916) was born in Braunschweig, Germany. He attended school in Brunswick, and entered the University of Göttingen in 1850. There he attended a course on least squares given by Gauss that greatly inspired him; he did his doctorate on the theory of Eulerian integrals under Gauss in 1852, the last of Gauss's pupils.

in 1854 he was awarded his habilitation, and began teaching at Göttingen giving courses on probability and geometry. Gauss died in 1855 and Dirichlet was appointed to fill his chair at Göttingen; Dedekind attended courses by Dirichlet on the theory of numbers, on potential theory, on definite integrals, and on partial differential equations, plus courses by Riemann on abelian functions and elliptic functions; he studied the work of Galois, and became one of the first to understand the fundamental importance of the notion of groups for algebra. He was appointed to a chair at the Zürich Polytechnikum (now "ETH") in 1858, and to the Brunswick Polytechnikum in 1862. In that year the idea of a *Dedekind cut* came to him: viz. every real number r divides the rational numbers into two subsets, those greater than r and those less than r; so that the real numbers can be represented by such divisions of the rationals.

In 1879 Dedekind introduced the notion of an *ideal* which is fundamental to ring theory, later applying this to the theory of Riemann surfaces, giving powerful results such as a purely algebraic proof of the *Riemann-Roch Theorem*. His work on Mathematical Induction, including the definition of finite and infinite sets, and his work in number theory, particularly in algebraic number fields, is also of major importance.

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70 **Bernard Placidus Johann Nepomuk Bolzano** (1781-1848) was born in Prague, Bohemia (now Czech Republic); he was influential in logic, philosophy and theology as well as mathematics. He entered Charles University, Prague in 1796 to study mathematics, philosophy, physics and theology, and in 1804 wrote a doctorate thesis on geometry (particularly his view of what constitutes a correct mathematical proof) and became a Catholic priest. He was appointed to a chair in philosophy of religion in 1805; but was dismissed in 1819 for his social/economic/political views, and exiled to the countryside (returning to Prague only in 1842).

He continued to develop his ideas across his whole range of interests, publishing them in minor journals.

Bolzano introduced the modern $\varepsilon-\delta$ definition of a limit; he was one of the earliest mathematicians to begin instilling rigour into Mathematical Analysis (thus distinguishing it from the Newton/Leibniz Calculus) via books published in 1810, 1816 and 1817, though his ideas were not widely accepted until the work of Weierstrass. He also defined the definition of *Cauchy sequence* (4 years before Cauchy), and gave the first purely analytic proofs of the *Fundamental Theorem of Algebra* (originally proved geometrically by Gauss) and the *Intermediate Value Theorem*; and is mainly remembered for the *Bolzano-Weierstrass Theorem* (which he discovered before Weierstrass). He also attempted to put the whole of mathematics on a logical foundation with his work on paradoxes (1851); the word *set* appears here for the first time, and he gives examples of 1-1 correspondences between the elements of an infinite set and the elements of a proper subset.

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78 **Euclid of Alexandria** (c. 325 BC – c. 265 BC): So little is reliably known of him that there are those who believe that he is simply the *nom de plume* of a team of mathematicians in Alexandria! (After all, in the 20th century **Henri Paul Cartan** (1904-2008), **Claude Chevalley** (1909-1984), **Jean Alexandre Eugène Dieudonné** (1906-1992), **Samuel Eilenberg** (1913-1998), **Alexander Grothendieck** (1928-2014), **Serge Lang** (1927-2005), **Jean-Pierre Serre** (1926-), **André Weil** (1906-1998) et al. wrote collectively more than 30 volumes of *Eléments de mathématiques* under the name **Nicolas Bourbaki** – even with their own office at the École Normale Supérieure in Paris!)

Euclid is the most prominent mathematician of antiquity, best known for his treatise on mathematics *The Elements*, in 13 volumes. He starts with definitions, axioms, and five construction postulates (e.g. *it is possible to draw a straight line between any two points* [Postulate 1] and *a unique line can be drawn through a point parallel to a given line* [Postulate 5]). *The Elements* covers plane geometry, number theory (including the *Euclidean algorithm* for finding the greatest common divisor of two numbers), irrational numbers and three-dimensional geometry.

Probably no results in *The Elements* were first proved by Euclid (e.g. many were due to **Eudoxus of Snidus** (c. 408 BC – c. 355 BC; Greek astronomer and mathematician) and **Theaetetus** (c. 417 BC – 369 BC; Greek geometer)) but the organisation of the material and its clear exposition and logical rigour are certainly due to him. More than one thousand editions of *The Elements* have been published since it was first printed in 1482, and it was still widely used a a school textbook in the middle of the 20th century. It is said that, next to the Bible, '*the "Elements" may be the most translated, published, and studied of all the books produced in the Western world'*. **Proclus Lycaeus** (412 – 485; Greek Neoplatonist philosopher) tells an anecdote that, when **King Ptolemy I** (303 BC – 285 BC) asked if there was a shorter path to learning geometry than Euclid's *Elements* '*Euclid replied there is no royal road to geometry'*. **Abraham Lincoln** (1809-1865) was highly motivated to read Euclid: '*In the course of my law reading I constantly came upon the word "demonstrate". I thought at first that I understood its meaning, but soon became satisfied that I did not. … At last I said - Lincoln, you never can make a lawyer if you do not understand what "demonstrate" means; and I left my situation in Springfield, went home to my father's house, and stayed there till I could give any proposition in the six books of Euclid at sight. I then found out what demonstrate means, and went back to my law studies.'*

Euclid wrote other books that have survived: *Data* (on what properties of figures can be deduced when other properties are given), *On Divisions* (on constructions to divide a figure into two parts with areas of given ratio) *Optics* (the first Greek work on perspective); and *Phaenomena* (an elementary introduction to mathematical astronomy). Some of Euclid's books have been lost: *Surface Loci* (two books), *Porisms* (three books), *Conics* (four books), *Book of Fallacies* and *Elements of Music.*

The lunar *Euclides Crater* is named in his honour.

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83 **Zeno of Elea** (c. 490 BC – c. 425 BC) was born and died in Elea, Lucania (now southern Italy). Little is known about him, other than some information from Plato's dialogue *Parmenides* and a mention in Aristotle's *Physics*. He was a philosopher in the Eleatic School, one of the leading pre-Socratic schools of Greek philosophy. [**Plato** (427 BC – 327 BC; Greek philosopher and mathematician); **Aristotle** (384 BC – 322 BC; Greek philosopher and scientist); **Socrates** (469 BC – 399 BC; Greek philosopher)]

He is said to have written a book containing 40 paradoxes concerning the continuum, several of which have had great influence on the development of mathematics. His paradoxes have influenced and inspired philosophers, mathematicians and physicists for over 2,000 years; the most famous are the so-called *arguments against motion* described by Aristotle:

Achilles and the tortoise paradox: Achilles races a tortoise, giving it a head start of 100 yards, for example. By the time Achilles has run 100 yards, to the tortoise's starting point, the tortoise has walked on a short distance, say 10 yards. By the time Achilles has run another 10 yards, to the tortoise's recent position, the tortoise has walked on another short distance, say 1 yard. And so on. Every time Achilles reaches a position of the tortoise, the tortoise has already moved on. Therefore, because there are an infinite number of points Achilles must reach where the tortoise has already been, he can never catch the tortoise!

Arrow paradox: At any instant of time an arrow in flight is motionless. Then, since time is entirely composed of instants, motion is impossible!

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197 **Johann Peter Gustav Lejeune Dirichlet** (1805-1859) was born in Düren, France (now Germany). By the age of 12 when he entered the Gymnasium in Bonn, he had developed a passion for mathematics and spent his pocket-money on buying mathematics books. He then moved to the Jesuit College in Cologne, where he was taught by **Georg Simon Ohm** (1789-1854; German physicist and mathematician, best known for *Ohm's Law*). He entered University of Paris in 1822, where he came into contact with the distinguished French analysts **Joseph Fourier** (1768–1830), **Pierre-Simon Laplace** (1749-1827), **Adrien-Marie Legendre** (1752-1833) and **Siméon Denis Poisson** (1781-1840). In 1823-25 he was employed by one of Napoleon's generals, **General Maximilien Sébastien Foy** (1775-1825), to teach German to his wife and children.

He moved to the University of Breslau in 1827, then to the University of Berlin (1828-1855) – indeed it is said that '*the golden age of mathematics in Berlin began with Dirichlet'* - and moved to Gauss's chair at Göttingen in 1855. In 1831 he married a sister Rebecca Mendelssohn (1811-1858) of the composer Jakob Ludwig Felix Mendelssohn Bartholdy (1809-1847).

In his first paper he proved the case $n = 5$ of the famous *Fermat's Last Theorem* (viz. that for $n > 2$ there are no non-zero positive integers x, y, z such that $x^n + y^n = z^n$); the general case was only proved by **Sir Andrew John Wiles** (1953-) in 1995. In 1837 he proved a conjecture of Gauss that in any arithmetic progression with first term coprime to the difference there are infinitely many primes. He introduced Dirichlet series and the *Pigeonhole Principle*, determined the formula for the class number for quadratic forms, created the field of analytic number theory, did important work on ideals, and introduced (1837) the modern definition of function: '*If a variable y is so related to a variable x that whenever a numerical value is assigned to x, there is a rule according to which a unique value of y is determined, then y is said to be a function of the independent variable x.*'

In mechanics he investigated the equilibrium of systems, potential theory, the gravitational attraction of an ellipsoid, the stability of the solar system, the *Dirichlet Problem* concerning harmonic functions with given boundary conditions, a sphere in an incompressible fluid, the exact integration of the hydrodynamic equations, the convergence of trigonometric series and the use of the Fourier series to represent arbitrary functions.

Dirichlet's brain is preserved in the Department of Physiology at the University of Göttingen, along with the brain of Gauss. The lunar *Dirichlet Crater* and the asteroid *11665 Dirichlet* are named in his honour.

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231 **Michel Rolle** (1652-1719) was born in Ambert, Basse-Auvergne, France. After some elementary schooling, he was largely self-educated. He went to Paris in 1675 where he worked as a scribe and arithmetical expert.

In 1682 he solved a problem posed by **Jacques Ozanam** (1640-1717; French mathematician and scientist): *Find four numbers the difference of any two being a perfect square, in addition the sum of the first three numbers being a perfect square.* Ozanam stated that the smallest of the four numbers with these properties would have at least 50 digits, but Rolle found four numbers satisfying the conditions with each number having seven digits – viz. 2399057, 2288168, 1873432 and 6560657. For this achievement **Jean-Baptiste Colbert** (1619-1683), the controller general of finance and secretary of state for the navy under **King Louis XIV** (1638-1715; known as '*the Sun King*'), arranged a pension that gave him financial security; and he came under the patronage of **François Michel le Tellier** (1641-1691), Marquis de Louvois, the French Secretary of State for War. He was admitted to the Académie Royale des Sciences in 1685, and promoted to a salaried position in the Academy, a *pensionnaire géometre,* in 1699 (only 20 of the 70 members of the Academy were paid).

In his *Traité d'algèbre* (1690) on the theory of equations he invented the notation $\sqrt[n]{x}$ for the nth root of x, used the *Euclidean algorithm* to find the greatest common divisor of two polynomials and solve Diophantine linear equations, gave the first *published* description in Europe of (what is now known as) the *Gaussian elimination algorithm*, and introduced the notion of *cascades* [essentially derivatives] of a polynomial. He also published another important work on solutions of indeterminate equations in 1699. However Rolle is best remembered for his *Rolle's Theorem* (1691) – the name was given to it by the Italian mathematician **Giusto Bellavitis** (1803-1880) in 1846.

Remark In 1691 Rolle used - and popularised - the equals sign '=', not then in wide use. This had been introduced earlier by **Robert Recorde** (c. 1512 - 1558), a Tenby, Wales physician and mathematician, in his 1557 book *The Whetstone of Witte*. Recorde was physician to King Edward VI and to Queen Mary, became Comptroller of the Royal Mint, and died in debtors' prison after being sued for defamation!

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241 **Guillaume François Antoine Marquis de l'Hôpital** (1661-1704) was born in Paris, France, into a military family - his father being a Lieutenant-General of the French King's army. He developed a real passion for mathematics as a child. Though he followed a military career (as captain in a cavalry regiment) he maintained his interests in mathematics and often studied geometry in his tent; he resigned from the army because of short sight, and directed his principal attention to mathematics. Among his accomplishments were the determination of the arc length of the logarithmic graph, one of the solutions to the brachistochrone problem, and the discovery of a turning point singularity on the involute of a plane curve near an inflection point.

l'Hôpital (already one of the best mathematicians in France) attended lectures by Johann Bernoulli in 1691 on Leibniz's differential calculus, and in 1692 employed Bernoulli to give him private lessons in mathematics. But after l'Hôpital had published a result of Bernoulli anonymously, relations between them cooled. Yet in 1694 l'Hôpital made the following proposal to Bernoulli: in exchange for an annual payment of 300 francs, Bernoulli would inform l'Hôpital of his latest mathematical discoveries, withholding them from others. Bernoulli accepted the proposition!

In 1696 l'Hôpital's famous book *Analyse des infiniment petits pour l'intelligence des lignes courbes* was published; it was the first textbook to be written on the differential calculus. In it, l'Hôpital determined tangents to a curve; and considered maximum and minimum problems, points of inflection, cusps, curvature, evolutes, involutes, higher order derivatives, and *l'Hôpital's Rule* for rational functions; he acknowledged his indebtedness to Leibniz, Newton, Jacob Bernoulli and Johann Bernoulli but regarded the foundations provided by him as his own ideas. Johann Bernoulli grew increasingly unhappy with the accolades bestowed on l'Hôpital's work and complained privately about being sidelined; after l'Hôpital's death Bernoulli stated that the book was essentially his, and this view is supported by a manuscript copy of Bernoulli's notes. However, l'Hôpital's pedagogical expository talent remains undimmed!

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241 **Johann Bernoulli** (1667-1748) was born in Basel, Switzerland, the younger brother of **Jacob Bernoulli**. His parents wanted him to take over the family spice business, but Johann disliked this greatly. In 1683 he entered the University of Basel to study medicine; there he also studied mathematics under his brother Jacob.

In 1691 he met the Marquis de l'Hôpital in Paris; l'Hôpital asked Bernoulli to explain Leibniz's new calculus methods to him, as a (paid) private tutor in France and later by correspondence - see above for the rest of this particular episode! He accepted a chair at Groningen in 1695, moving to Basel in 1705 when Jacob died.

Johann Bernoulli made many contribution to the Calculus: he solved the problem of the catenary, worked with Jacob on caustic curves (although the two formed a strong rivalry and never published any joint work), investigated series using the method of integration by parts, had great success in integrating differential equations, discovered addition theorems for trigonometric and hyperbolic functions, solved the brachristochrone problem (1696), solved the isoperimetric problem (1718) with an elegant solution which was to form a foundation for the calculus of variations, and made important contributions to mechanics with his work on kinetic energy (1732).

'Archimedes of his age' is inscribed on his tombstone. His sons Nicolaus (II) Bernoulli, Daniel Bernoulli, and Johann (II) Bernoulli all became mathematicians; Nicolaus (I) Bernoulli was his nephew. Leonhard Euler was one of his students in Basel.

The lunar *Bernoulli Crater* is named after him and Jacob (jointly), and the Paris street *Rue Bernoulli* is named after the Bernoulli family.

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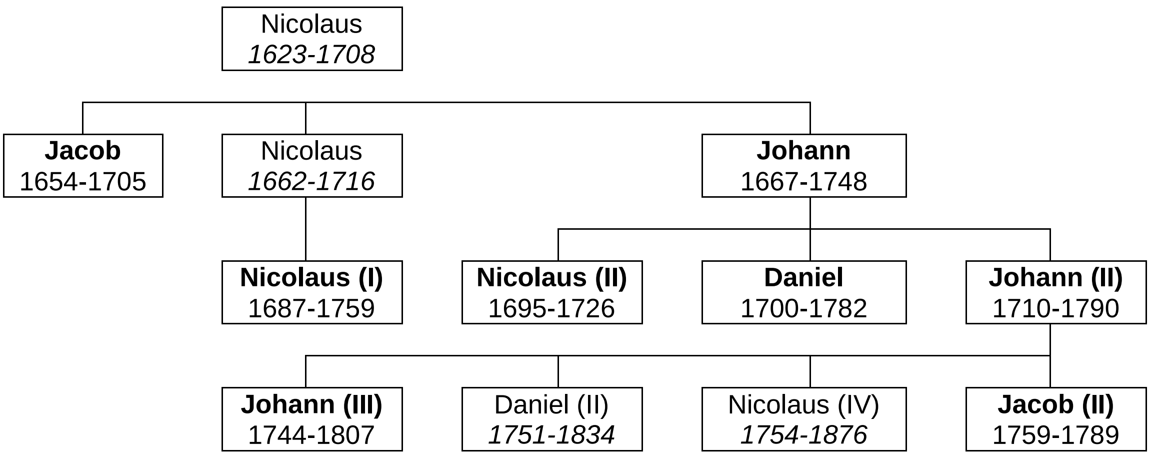
241 **Bernoulli family mathematical dynasty**

The Bernoulli family contains so many mathematicians that it is useful to see their various relationships, in the table below. (Those in bold are the mathematicians.)



Hans Benno Bernoulli (1876-1959), the Swiss and city planner, was a great grandson of Nicolaus (IV).

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244 The function that Weierstrass used in 1872 was $f(x)=\sum_{n=0}^{\infty} a^n \cos\left(b^n \pi x\right)$, where $0<a<1$, b is a positive odd integer, and $ab>1+3\pi/2$.

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244 The 'blancmange function' was first studied by the Japanese mathematician Teiji Takagi in 1903.

Teiji Takagi (1875-1960) was born in Gifu Prefecture, Japan. At that time there were no mathematics texts written in Japanese so the pupils studying mathematics had to use English texts; Takagi studied *Algebra for beginners* by **Isaac Todhunter** (1820-1884; English mathematician and expositor) and *Elementary Geometry* by **James Maurice Wilson** (1836-1931; English theologian, mathematics teacher and astronomer). He studied mathematics at Tokyo University, the only university in Japan at that time, in 1894-97; in 1897-1901 he went to Berlin and Göttingen universities, returning to Tokyo University where in 1903 he presented a thesis based on work he had undertaken in Göttingen; and worked on algebra till he retired in 1936.

He wrote 13 textbooks (in 20 volumes) between 1904 and 1911, valuable for the development of Japanese mathematics at school and university level. After WW1 he started research in class field theory. He wrote his most important paper in 1920 introducing *Takagi class field theory* generalising Hilbert's class field theory; this became the framework of algebraic number theory.

Later he was involved in the development of the Japanese Foreign Office's diplomatic cryptographic machine used just before and during WW2, code-named *Purple* by United States cryptographers.

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244 According to **David Orme Tall** (1941- ; Warwick University, England), the name *blancmange curve* was given to the graph in 1982 by **John Trevor Stanton Mills** (1932-2008) (Warwick University) on the grounds that the function is not differentiable because its graph '*wobbles so much'*.

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254 **Jean Gaston Darboux** (1842-1917) was born in Nimes, France. He attended Lycée at Nimes and then at Montpellier. In 1861 he entered the École Normale Supérieure (where he published his first paper, on orthogonal surfaces). He was awarded his doctorate in 1866 with a thesis on orthogonal surfaces, and held posts at the Collège de France (1866-1867), the Lycée Louis le Grand (1867-1872), the École Normale Supérieure (1872-1881), and the Sorbonne (1873-1917). In 1900 he was appointed permanent secretary of the Mathematics section of the Académie des Sciences de Paris.

He made important contributions to differential geometry and analysis. He is best known for the *Darboux integral* which he introduced in 1870; in 1875 he introduced the definition of the integral that *a function is integrable if the difference between its upper and lower sums tends to zero as the mesh size gets smaller*. In 1887-1896 he produced four volumes on the differential geometry of surfaces which included most of his earlier work on geometry. He was said to possess '*a rare combination of geometrical fancy and analytical power*'.

He was also renowned as an exceptional teacher, writer and administrator. His doctoral students included **Félix Édouard Justin Émile Borel** (1871-1956; French mathematician and politician; Minister for the Marine in 1925), **Élie Joseph Cartan** (1869-1951; French mathematician who worked on Lie groups, differential geometry and mathematical physics), **Édouard Jean-Baptiste Goursat** (1858-1936; French mathematician best known for his book *Cours d'analyse mathématique*), **Charles Émile Picard** (1856-1941; French analyst and applied mathematician) and **Thomas Joannes Stieltjes** (1856-1894; Dutch analyst who also worked on continued fractions and number theory).

The Paris street *Rue Gaston Darboux* is named in his honour.

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295 **John Wallis** (1616-1703) was born in Ashford, Kent, England. He went to school in Ashford, but moved to Tenterden after an outbreak of the plague, and then to Felstead, Essex.

He went to Cambridge University in 1632, finishing in 1640; he was then ordained, and served as chaplain in Yorkshire, Essex and London. He became skilled at deciphering coded messages for the Parliamentarians during the Civil War. After a Fellowship at Cambridge (1644-1645), on his marriage he moved to London where he began to meet weekly with a group of enthusiastic scientists that would eventually become the *Royal Society of London*. In 1647 he found a love of mathematics via reading *Clavis Mathematicae* by **William Oughtred** (1574-1660; best known for his invention of an early form of the slide rule; and for many new symbols, including $\times$ for multiplication); and soon began to publish research of his own. He was appointed to the Savilian Chair of Geometry at Oxford University in 1649 by **Oliver Cromwell** (1599-1658) mainly because of his support for the Parliamentarians. However he spoke out against the execution of **King Charles I** (1600-1649) and on the Restoration (1660) had his appointment in the Savilian Chair confirmed by **King Charles II** (1630-1685), who even appointed Wallis as a royal chaplain and nominated him as a member of a committee to revise the *Prayer Book*. He became involved in a bitter 20-year dispute with **Thomas Hobbes** (1588-1679; English philosopher) who claimed in 1655 to have discovered a method of *squaring the circle*.

Wallis contributed substantially to the development of the Calculus and was the most influential English mathematician before Newton. His most famous work was *Arithmetica infinitorum* (1656), where he did much work on integration ['*quadrature*'], established *Wallis's Formula* $\frac{1}{2}=\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot...$ (cf. page 295) while trying to evaluate $\int_0^1 \sqrt{1-x^2}\,dx$, and introduced the term *interpolation*. In his *Tract on Conic Sections* (1655) he approached conic sections algebraically rather than geometrically, and introduced the symbol ∞, the notation for rational powers (viz. $x^{p/q} = \sqrt[q]{x^p}$), and the term *continued fraction* - that notion being introduced by **William Brouncker** (1620-1684) (first President of the Royal Society).

He possessed a great ability to do mental calculations, once calculating the square root of a 53 digit number during the night and dictating the 27 digit answer in the morning.

The asteroid *31982 Johnwallis* was named in his honour.

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297 **Colin Maclaurin** (1698-1746) was born in Kilmodan, Scotland. After attending school in Dumbarton he entered Glasgow University in 1709, became interested in geometry via Euclid's *Elements*, and was greatly influenced by the professor **Robert Simson** (1687-1768), a geometer. He graduated in 1713, writing a dissertation concerning gravity that developed Newton's theories. After a year studying divinity at Glasgow he returned to live with relatives at Kilfinnan while continuing his study of mathematics, before being appointed professor of mathematics at Marischal College, Aberdeen University. He moved to Edinburgh University in 1725, following a controversial 2-year period of absence from Aberdeen without its leave. There he served as a Secretary of the *Medical Society of Edinburgh*, which later became the *Royal Society of Edinburgh*.

In 1740 he was awarded a prize (jointly awarded to Maclaurin, Euler and **Daniel Bernoulli** (1700-1782; he worked on the basic properties of fluid flow, pressure, density and velocity, and discovered the *Bernoulli Principle* for fluid flow) from the Académie des Sciences de Paris, for a study of the tides. In 1742 he published his 2 volume *Treatise of fluxions* on Newton's methods, written as a reply to an attack by **Bishop George Berkeley** (1685-1753; Bishop of Cloyne, Church of Ireland, and Anglo-Irish philosopher) on the Calculus for its lack of rigorous foundations. This treatise covered the *Fundamental Theorem of Calculus*, work on maxima and minima, the attraction of ellipsoids, elliptic integrals, the *Euler-Maclaurin summation formula*, the special case of *Taylor series* known as *Maclaurin series*, and the *Integral Test* for series. Other topics on which Maclaurin wrote were the annular eclipse of the sun in 1737 and the structure of bees' honeycombs.

Maclaurin superintended the defence operations for Edinburgh during the Jacobite rebellion of 1745, fled to England as guest of the Archbishop of York when Edinburgh fell, and returned fatally weakened by his exertions in late 1745. His *Treatise on algebra* was published two years after his death.

The lunar *Maclaurin Crater* is named in his honour.

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300 **Leonhard Euler** (1707-1783) was born in Basel, Switzerland. He entered the University of Basel in 1720, where Johann Bernoulli (a friend of his father) discovered his great potential for mathematics, graduating in 1726. In 1727 he was appointed to a chair at the St Petersburg Academy of Sciences two years after it had been founded by **Tsaritsa Catherine I** (1684-1727), the wife of **Tsar Peter the Great** (1672-1725). He moved to the Berlin Society/Academy of Sciences in 1741 following political turmoil in Russia. In 1766 Euler returned to St Petersburg in 1766, but became totally blind in 1771. Because of his remarkable memory he was able to continue with his work, producing almost half his total research output with the help of his two sons and other assistants. After his death the St Petersburg Academy continued to publish Euler's unpublished work for nearly 50 more years.

Euler was the most prolific writer of mathematics in history. He brought together Leibniz's differential calculus and Newton's *method of fluxions* as *Mathematical Analysis*; made decisive contributions to geometry, calculus and number theory; introduced beta and gamma functions, and integrating factors for differential equations; studied continuum mechanics, fluid mechanics, lunar theory, the three body problem, graph theory (solving the *Seven Bridges of Königsberg problem*), elasticity, acoustics, the wave theory of light, hydraulics, music, analytical mechanics, astronomy and the cartography of Russia. We also owe him the *Euler-Maclaurin summation formula*, *Euler's Formula* $e^{ix} = \cos x + i \sin x$ (1748), *Euler's Identity* $e^{i\pi} = -1$, and *Euler diagrams* for representing sets and their relationships.

In 1735 he proved that $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, $\sum_{n=1}^{\infty} 1/n^4 = \pi^4/90$, $\sum_{n=1}^{\infty} 1/n^6 = \pi^6/945$, $\sum_{n=1}^{\infty} 1/n^8 = \pi^8/9450$, $\sum_{n=1}^{\infty} 1/n^{10} = \pi^{10}/93555$ and $\sum_{n=1}^{\infty} 1/n^{12} = 691\pi^{12}/638512875$; and in 1737 he proved that $\sum_{n\in\mathbb{N}} \frac{1}{n^s} = \prod_{p \text{ prime}} \left(1-p^{-s}\right)^{-1}$, $s \in \mathbb{C}$; and identified the *Euler constant* $\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}-\log_e n\right)$.

Euler introduced much modern notation such as $f(x)$ for a function (1734), e for the base of natural logarithms (1727), i for the square root of -1 (1777), $\sum$ for summation (1755), and Δy and $\Delta^2 y$ for finite differences; and popularised the use of the symbol π for pi.

The lunar *Euler Crater* and *Rima Euler*, the asteroid *2002 Euler* and the Paris street *Rue Euler* are named in his honour.

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306 **James Stirling** (1692-1770) was born at Garden, near Stirling, Scotland into a strongly Jacobite family.

After studying at either Glasgow or Edinburgh University, he went to Balliol College, Oxford University in 1710 on a Snell Exhibition scholarship (established by the will of **Sir John Snell** (1629-1679; secretary to the **Duke of Monmouth** (1649-1685)); the economist **Adam Smith** (1723-1790) was a later Snell Exhibitioner). He was awarded a **Bishop [John] Warner** (1581-1666; Bishop of Rochester and royalist) Exhibition Scholarship in 1711, but lost his scholarships when he refuse to take an oath of loyalty to the Hanoverian **King George I** (1660-1727) in 1715; he remained in Oxford till 1717 when he published work extending Newton's theory of plane curves of degree 3 (adding 4 new types of curves to the 72 given by Newton), on the curve of quickest descent, results on the catenary, and results on orthogonal trajectories.

In 1717 Stirling went to Venice, returning to Glasgow in 1722 fearing assassination on account of having discovered a trade secret of the Venetian glassmakers! From 1724 he spent 10 years in London, becoming friendly with Isaac Newton and a Fellow of the Royal Society in 1726. He then became a teacher in London at William Watt's Academy in Little Tower Street, Covent Garden, where he had to borrow money to pay for the mathematical instruments he needed.. In 1730 he published his most important work *Methodus Differentialis* on infinite series, summation, infinite products, interpolation, quadrature, the gamma function, the hyper-geometric function, and his asymptotic formula for $n!$. In 1735 he wrote that the Earth is an oblate spheroid.

In 1735 Stirling returned to Scotland where he was appointed manager of the *Scottish Mining Company* at Leadhills, Lanarkshire. In 1752 he received a silver tea-kettle for his work surveying the River Clyde with a view to rendering it navigable by a series of locks, thus taking the first step towards making Glasgow the commercial capital of Scotland. In 1753 he resigned from the Royal Society as he was in debt to the Society and could no longer afford the annual subscriptions.

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317 **Brook Taylor** (1685-1731) was born into a well-off family that gave him a love of music and painting. (His grandfather Nathaniel Taylor had represented Bedfordshire in Cromwell’s 1653 *Westminster Assembly of Divines*.) He entered Cambridge University in 1703 and graduated with an LL.B. in 1709 but had already written his first important mathematics paper in 1708 (published in 1714) using Newton’s differential calculus - it gives a solution to the problem of the centre of oscillation of a body, but resulted in a priority dispute with Johann Bernoulli. He was elected to the Royal Society in 1712, serving as its Secretary in 1714-1718; and was appointed in 1712 to its committee set up to adjudicate on whether Newton or Leibniz had invented the calculus.

Taylor published on experiments in capillary action, magnetism and thermometers; the law of magnetic attraction; an improved method for approximating the roots of an equation by giving a new method for computing logarithms; singular solutions to differential equations; a change of variables formula; and the shape of vibrating strings. He also set out the basic principles of perspective in his book *Linear Perspective* (1715), stressing the importance of the vanishing point and vanishing line - the main theorem in his theory of linear perspective is that *the projection of a straight line not parallel to the plane of the picture passes through its intersection and its vanishing point*.

In 1715 he invented the *calculus of finite differences* and *integration by parts*; and published *Taylor's Theorem*, which he had obtained in 1712 following a comment that John Machin had made in *Child's Coffeehouse,* London. (In fact, James Gregory, Isaac Newton, Gottfried Leibniz, Johann Bernoulli and **Abraham de Moivre** (1667-1754; French Huguenot refugee in London, mathematical analyst and probabilist; best known for his *de Moivre’s Formula* $(\cos x + i\sin x)^n = \cos(nx) + i\sin(nx)$) had all discovered variants of the series earlier.) The term *Taylor's series* was first used by **Simon Antoine Jean Lhuilier** (1750-1840; Swiss analyst and topologist, who corrected Euler’s solution of the *Königsberg bridge problem*) in 1786.

From 1715 his studies took a philosophical and religious bent.

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329 **Friedrich Wilhelm Bessel** (1784-1846) was born in Minden, Westphalia (Germany). He left school at 14 to become an apprentice in an import-export business; as a result he became interested in mathematics, astronomy, navigation, and the problem of determining longitude.

In 1804 Bessel wrote a paper on Halley's comet. In 1806 he accepted the post of assistant at the Lilienthal Observatory (a private observatory near Bremen) to observe the planets particularly Saturn, its rings and satellites; he also observed comets and continued his study of celestial mechanics.

Then in 1809 Bessel was appointed director of the new Königsberg Observatory of **King Frederick William III of Prussia** (1770-1840; ally and enemy of Napoleon) and professor of astronomy; and, on the recommendation of Gauss, he was awarded an honorary doctorate by the University of Göttingen in 1811.

Bessel was one of the first astronomers to realise that before a positional observation could be relied upon one must have quantitative knowledge of every possible error that might enter into the final result. By eliminating all sources of error (optical, mechanical and meteorological) he was able to obtain astronomical results of astonishing delicacy from which a great deal of new data could be extracted. He undertook the huge task of determining the positions and proper motions of over 50,000 stars, which led to the discovery in 1838 of the parallax of the dim star *61 Cygni*. From periodic variations in the proper motions of the star *Sirius* Bessel predicted in 1841 that it had a companion 'dark star' which had not been observed; *Sirius B* was later observed, in 1862. He was also the first astronomer to determine the distance from the sun to another star by the *method of parallax*, and determined the ellipticity of the Earth to be approximately $1/129$.

Bessel also introduced the *Bessel function* in 1817 to study the *three body problem*, though special cases of it had been used earlier by Jacob Bernoulli, Daniel Bernoulli, Euler and **Joseph-Louis Lagrange** ['**Giuseppe Lodovico Lagrangia**'] (1736-1813; born in Turin, worked in Italy, Berlin and Paris; made major contributions to the calculus of variations, and contributed also to probability, number theory, theory of equations, mechanics (introducing the *Lagrangian function*), and foundations of group theory). In 1824 he published a detailed book on *Bessel functions*, which are nowadays indispensable tools in differential equations, applied mathematics, classical and quantum physics, and engineering.

The lunar *Bessel Crater* and the asteroid *1552 Bessel* are named in his honour.

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333 **Niels Henrik Abel** (1802-1829) was born in Frindöe, near Stavanger, Norway; Norway was a very poor country following the British Napoleonic Wars blockade and an independence war with Sweden. In 1815 Abel was sent to the Cathedral School in Christiania, where under the supervision of an insightful teacher **Bernt Michael Holmboë** (1795-1850; Norwegian mathematician, in whose memory an annual *Holmboë Memorial Prize* for mathematics teachers been awarded since 2005) he read the works of Euler, Newton and **Jean Le Rond d'Alembert** (1717-1783; argumentative French mathematician, who also worked in mechanics, physics, philosophy and theory of music; best known for *d'Alembert's formula* for solutions to the *wave equation*). Abel's family became impoverished when his father died, but Holmboë helped Abel gain a scholarship to remain at school and enter Royal Frederick University, Christiania in 1821; he graduated in 1822.

Abel was given funds by university professors to remain in Christiania for two years, staying in one of their attics; he began working again on quintic equations (he had produced a false proof while at school); in 1824 he proved his most famous result *the impossibility of solving the general equation of the fifth degree in radicals* in a privately published pamphlet. (Abel sent a copy of this to Gauss, who did not read it.) In August 1825 Abel was given a scholarship from the Norwegian government to allow him to travel abroad. In Berlin he met **August Leopold Crelle** (1780-1855; German mathematician, with an extraordinary intuition for identifying and encouraging talented young mathematicians), who invited him to write a clearer version of his work on the insolubility of the quintic; this was published in 1827 in the first volume of *Journal für die reine und angewandte Mathematik* ["*Crelle's Journal*"], along with six other papers by Abel.

He returned to Christiania in May 1827 and was awarded a small amount of money by the university, and tutored schoolchildren; he also obtained a temporary teaching post at the University. Abel continued to pour out high quality mathematics, but his health deteriorated and he died of tuberculosis two days before Crelle wrote to tell Abel that he had obtained a chair for him at the University of Berlin.

Abel also wrote fundamental work on the theory of functions, elliptic functions and integrals, hyperelliptic functions, and *Abelian functions*. He gave a proof of the *Binomial Theorem* valid for all powers, extending Euler's proof for rational powers.

The lunar *Abel Crater*, the Paris street *Rue Abel* and the prestigious annual *Abel Prize* are named in his honour.

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347 **James Gregory** [or **Gregorie**] (1638-1675) was born in Drumoak, near Aberdeen, Scotland.

His uncle **Alexander Anderson** (1582-1619) was a pupil of **François Viète** (1540-1603; French algebraist and geometer who made major contributions to geometry, trigonometry and the theory of equations – but also rejected the use of negative numbers [!]; best known for Viète's formula $\pi = 2 \times \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{2+\sqrt{2}}} \times \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \times \cdots$ (1593)) in Paris. Alexander's sister, his mother **Janet Anderson** (1600-1688), endowed him with his love of geometry by encouraging him to read Euclid's *Elements*; then he studied at Marischal College, Aberdeen. He began to study optics and the construction of telescopes, and wrote a book *Optica Promota* (published later in 1663) on reflection and refraction of light, mathematical astronomy (parallax, transits and elliptical orbits, and his invention of a *Gregorian reflecting telescope* (using a primary concave parabolic mirror and a concave ellipsoidal mirror; a design still used today in radio telescopes).

In 1663 he went to London, meeting fellow Scot **Sir Robert Moray** (1608-1703; soldier, statesman, diplomat, judge, spy and natural philosopher), one of the founders of the Royal Society; he spent 1664-1668 at the University of Padua where he wrote two books on geometry. In one he tried to prove that π and e are transcendental. The other was the first attempt to write a systematic textbook on the Calculus, which he invented independently of Newton, on *the method of tangents* (= differentiation) and its inverse *the method of quadratures* (= integration); he gave the first published statement and proof of the *Fundamental Theorem of Calculus*, the series expansions of sin, cos and tan (differentiating between convergent and divergent series), and the integrals of sec and log. By 1671 he had discovered *Taylor's Theorem* (not published by Taylor until

1715) and the expansion $\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - ...$ for $-\frac{\pi}{4} < \theta \le \frac{\pi}{4}$; this formula (cf. page 347) was used with $\theta = \frac{\pi}{4}$ to calculate digits of π.

He was elected a Fellow of the Royal Society in 1668, and became the first Regius Professor of Mathematics at the University of St Andrews - a position created for him by **King Charles II** (1630-1685), probably at the request of Robert Moray. In 1674 he moved to the first Chair of Mathematics at University of Edinburgh. He also discovered the *diffraction grating* by passing sunlight through a bird feather and observing its splitting into its component colours, a year after Newton had done the same with a prism.

The lunar features *Gregory Crater* and *Catena Gregory* are so-named in his honour.

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347 **Abraham Sharp** (1653-1742) was born in Little Horton near Bradford, Yorkshire.

He had a good early education. In 1669 he became a cloth merchant's apprentice before becoming a schoolmaster in Liverpool and subsequently a bookkeeper in London (using the *Hen and Chickens* coffee house in the Strand as a mailing address). His wide knowledge of mathematics and astronomy attracted the attention of **John Flamsteed** (1646-1719; the first Astronomer Royal, who catalogued over 3000 stars), who employed him at the Royal Observatory Greenwich as assistant and instrument maker (1684-1685 and 1688-1690). There he did notable work, producing eclipse data and tables of the motions of Jupiter's satellites, improving astronomical instruments and showing great skill as a calculator, publishing logarithmic tables and a book *Geometry Improved*.

Later he taught mathematics as the resident mathematician of William Court at the *Mariner and Anchor* on Fish Street Hill in London, and became clerk at Portsmouth shipyard; he returned to Horton Hall in 1694. There he led a fairly unstructured life and became a recluse, often forgetting to take his meals!

Sharp calculated π to 72 decimal places in 1699 using Gregory's series with $x = 1/\sqrt{3}$, viz.

$\frac{\pi}{6} = \frac{1}{\sqrt{3}}\left(1 - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^4 - \frac{1}{7}\left(\frac{1}{\sqrt{3}}\right)^6 + ...\right) = \frac{1}{\sqrt{3}}\left(1 - \frac{1}{9} + \frac{1}{45} - \frac{1}{189} + ...\right)$, briefly holding the record until John Machin calculated 100 digits in 1706.

The lunar *Sharp Crater* is named after him.

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347 **John Machin** (1680-1751) acted as a private tutor to Brook Taylor before Taylor entered Cambridge University. He was elected a Fellow of the Royal Society in 1710, becoming its Secretary in 1718-1747; he was a member of the commission which investigated the Calculus priority dispute between Leibniz and Newton in 1712. Taylor wrote in a letter in 1712 that a comment made by Machin during a coffee-house conversation had given him the idea of *Taylor's Theorem*. He was also friendly with Abraham de Moivre, another current private tutor of mathematics.

He was appointed Professor of Astronomy at Gresham College, London in 1713, and did some work in astronomy; indeed his solution to the problem of the motion of the nodes of the moon's orbit was included in the 3rd edition of Newton's *Principia*. He is best known for developing a quickly converging series $\pi/4 = 4\tan^{-1}(1/5) + \tan^{-1}(1/239)$ for π in 1706, and using it to compute π to 100 decimal places.

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348 **Carl Louis Ferdinand von Lindemann** (1852-1939) was born in Hanover, Germany. He began his university studies in Göttingen in 1870, writing a dissertation on non-Euclidean geometry at Erlangen University under **Christian Felix Klein** (1849-1925; German mathematician and expositor who tried to classify geometries via a group-theoretic approach, and made major contributions to elliptic functions); he gained his habilitation at the University of Würzburg in 1877. He was appointed to a chair at the University of Freiburg in 1877, a chair at the University of Königsberg in 1883, and a chair at the University of Munich in 1893.

Lindemann's main work was in geometry and analysis. He is best known for his proof (1882) that π is *transcendental*; that is, it is not the root of any polynomial equation with integer coefficients. This solved the classical problem of Greek mathematics whether it is possible to *square the circle* viz. construct a square with the same area as a given circle using only ruler and compasses. His proof is based on the proof that e is transcendental together with the fact that $e^{i\pi} = -1$. He also worked on the theory of the electron, and the history of mathematics.

In Königsberg he was the doctoral supervisor of David Hilbert, **Hermann Minkowski** (1864-1909; German mathematician, who used geometrical methods to solve problems in number theory and mathematical physics; best known for *Minkowski space-time*, a geometric theory of four-dimensional space–time underlying relativity theory (1907)) and **Arnold Johannes Wilhelm Sommerfeld** (1868-1951; German theoretical physicist who pioneered developments in atomic and quantum physics, and whose doctoral students earned more Nobel prizes in physics than any other supervisor).

Further sources of information on the history of Mathematical Analysis

The following list of books and articles is in no way definitive: the literature on the history of Mathematical Analysis is very large indeed!

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- **Beckmann, Petr**, *A history of π (pi),* The Golem Press, Boulder, Colo.; 1st edition (1970); 2nd edition (1971).
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- **Birkhoff, Garrett** (editor), *A source book in classical analysis*, Harvard University Press, Cambridge, Mass. (1973); ISBN: 9780674822450.
- **Bottazzini, Umberto**, *The higher calculus: a history of real and complex analysis from Euler to Weierstrass*, Springer-Verlag, New York (1986); ISBN: 0-387-96302-2.
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Many (but not all) ‘dictionaries’ of mathematics are often other useful sources of information on the history of Mathematical Analysis. These include the following:

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- **James, Glenn** & **James, Robert Clarke**, *Dictionary of mathematics* (5th edition), Van Nostrand Reinhold, New York (1992); ISBN: 0-442-00741-8.

Some academic journals regularly contain interesting articles on the history of mathematics in general, with many of these covering Mathematical Analysis in particular. These journals include the following, all of which are available electronically:

- ***American Mathematical Monthly***: http://www.maa.org/publications/periodicals/american-mathematical-monthly .
- ***Archive for History of Exact Sciences***: http://link.springer.com/journal/407
- ***Historia Mathematica***: http://www.sciencedirect.com/science/journal/03150860
- **BSHM Bulletin: Journal of the British Society for the History of Mathematics** (2004-); formerly known as **British Society for the History of Mathematics. Newsletter** (1986-2003)

Finally, the Internet contains vast amounts of information that can be found using a search engine such as Google, including information on the history of Mathematical Analysis. These sources include: the well-organised St Andrews-based MacTutor archive; the huge Wikipedia website, with its many links; and sites maintained by many groups and individual authors.

- ***MacTutor History of Mathematics Archive***, created by John J. O'Connor & Edmund F. Robertson:
  http://www-history.mcs.st-andrews.ac.uk/
- ***Wikipedia***, Wikimedia Foundation:
  http://en.wikipedia.org/wiki/Main_Page
- **Mathematics Genealogy Project**, founded by Harry Bernard Coonce (1997); Mathematics Department, North Dakota State University, ND & American Mathematical Society, Providence, RI:
  http://www.genealogy.ams.org/

Web users MUST bear in mind that, while many sites contain many wonderful articles, most are not refereed/checked and may contain minor or major errors!