Portfolio Theory and Risk Management
Errata

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Theorem 3.7 (Taylor formula)
Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ *is a twice continuously differentiable function. Then for any* $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ *there exists a point* $\boldsymbol{\xi}$ *contained in the line segment joining* $\mathbf{v}$ *and* $\mathbf{v} + \mathbf{w}$*,*

$$\boldsymbol{\xi} \in \{\mathbf{v}+\alpha\mathbf{w} : \alpha \in [0, 1]\},$$

such that

$$f(\mathbf{v} + \mathbf{w}) = f(\mathbf{v}) + \nabla f(\mathbf{v}) \cdot \mathbf{w} + \frac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w},$$

where the dot stands for the scalar product.

by

Theorem 3.7 (Taylor formula)
Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ *is a twice continuously differentiable function. Then for any* $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$

$$f(\mathbf{v} + \mathbf{w}) = f(\mathbf{v}) + \nabla f(\mathbf{v}) \cdot \mathbf{w} + \mathbf{w}^{\mathrm{T}} \left(\frac{1}{2} \int_0^1 (1 - t)\, H(f, \mathbf{v} + t\mathbf{w}) dt \right) \mathbf{w},$$

where the dot stands for the scalar product.

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$$f(\mathbf{v}^* + \mathbf{w}) = f(\mathbf{v}^*) + \nabla f(\mathbf{v}^*) \cdot \mathbf{w} + \frac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w}, \qquad (3.16)$$

for some point $\boldsymbol{\xi}$ on the line segment in $\mathbb{R}^n$ between $\mathbf{v}^*$ and $\mathbf{v}^* + \mathbf{w}$.

by

$$f(\mathbf{v}^* + \mathbf{w}) = f(\mathbf{v}^*) + \nabla f(\mathbf{v}^*) \cdot \mathbf{w} + \mathbf{w}^{\mathrm{T}} B\mathbf{w}, \qquad (3.16)$$

where $B := \frac{1}{2}\int_0^1 (1-t)\, H(f, \mathbf{v} + t\mathbf{w})dt$. Observe that

$$\begin{aligned}
\mathbf{w}^{\mathrm{T}} B\mathbf{w} &= \mathbf{w}^{\mathrm{T}}\left(\frac{1}{2}\int_0^1 (1-t)\, H(f, \mathbf{v} + t\mathbf{w})dt\right)\mathbf{w} \\
&= \frac{1}{2}\int_0^1 (1-t)\, \mathbf{w}^{\mathrm{T}} H(f, \mathbf{v} + t\mathbf{w})\mathbf{w} dt \\
&\geq 0.
\end{aligned}$$

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$$\begin{aligned}
f(\mathbf{v}) &= f(\mathbf{v}^* + \mathbf{w}) \\
&= f(\mathbf{v}^*) + \nabla f(\mathbf{v}^*) \cdot \mathbf{w} + \tfrac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w} && \text{(from (3.16))} \\
&= f(\mathbf{v}^*) + A^{\mathrm{T}}\boldsymbol{\lambda} \cdot \mathbf{w} + \tfrac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w} && \text{(from (3.9) and (3.14))} \\
&= f(\mathbf{v}^*) + \left(A^{\mathrm{T}}\boldsymbol{\lambda}\right)^{\mathrm{T}} \mathbf{w} + \tfrac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w} \\
&= f(\mathbf{v}^*) + \boldsymbol{\lambda}^{\mathrm{T}} A\mathbf{w} + \tfrac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w} \\
&= f(\mathbf{v}^*) + \tfrac{1}{2}\mathbf{w}^{\mathrm{T}} H(f, \boldsymbol{\xi})\mathbf{w} && \text{(from (3.15))} \\
&\geq f(\mathbf{v}^*). && \text{(from (3.10))}
\end{aligned}$$

by

$$\begin{aligned}
f(\mathbf{v}) &= f(\mathbf{v}^* + \mathbf{w}) \\
&= f(\mathbf{v}^*) + \nabla f(\mathbf{v}^*) \cdot \mathbf{w} + \mathbf{w}^{\mathrm{T}} B\mathbf{w} && \text{(from (3.16))} \\
&\geq f(\mathbf{v}^*) + \nabla f(\mathbf{v}^*) \cdot \mathbf{w} \\
&= f(\mathbf{v}^*) + A^{\mathrm{T}}\boldsymbol{\lambda} \cdot \mathbf{w} && \text{(from (3.9) and (3.14))} \\
&= f(\mathbf{v}^*) + \left(A^{\mathrm{T}}\boldsymbol{\lambda}\right)^{\mathrm{T}} \mathbf{w} \\
&= f(\mathbf{v}^*) + \boldsymbol{\lambda}^{\mathrm{T}} A\mathbf{w} \\
&= f(\mathbf{v}^*) && \text{(from (3.15))}
\end{aligned}$$

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Lemma 7.6
Let X be a random variable. If $f : \mathbb{R} \to \mathbb{R}$ is right-continuous and non-decreasing then

$$q^{\alpha}(f(X)) = f(q^{\alpha}(X)).$$

Proof Since

$$\begin{aligned}
F_{f(X)}(f(q^{\alpha}(X))) &= P(f(X) \leq f(q^{\alpha}(X))) \\
&\geq P(X \leq q^{\alpha}(X)) \\
&= F_X(q^{\alpha}(X)) \\
&\geq \alpha,
\end{aligned}$$

we see that

$$f(q^\alpha(X)) \geq q_\alpha(f(X)).$$

If we can show that $y \geq q^\alpha(f(X))$ whenever $y > f(q^\alpha(X))$, then $f(q^\alpha(X))$ is the largest α-quantile for $f(X)$.

Take any $y > f(q^\alpha(X))$. Since f is right-continuous and non-decreasing, the set $f^{-1}(-\infty, y)$ is an open interval of the form $(-\infty, a)$, for some $a \in \mathbb{R}$. This gives

$$(-\infty, q^\alpha(X)] \subset \{x : f(x) \leq f(q^\alpha(X))\} \subset \{x : f(x) < y\} = (-\infty, a),$$

which means that there exists an x^* for which $q^\alpha(X) < x^* < a$. Since $q^\alpha(X) < x^*$

$$\alpha < F_X(x^*),$$

hence, with $Y = f(X)$,

$$F_Y(y) = P(Y \leq y) \geq P(Y < y) = P(X < a) \geq P(X \leq x^*) = F_X(x^*) > \alpha,$$

which implies that $y \geq q^\alpha(Y) = q^\alpha(f(X))$. □

by

Lemma 7.6
Let X be a random variable. If $f : \mathbb{R} \to \mathbb{R}$ is right-continuous and non-decreasing then

$$q^\alpha(f(X)) = f(q^\alpha(X)).$$

Proof Since f is right continuous and non decreasing, for any $y \in \mathbb{R}$ there exists an $x \in \mathbb{R}$ such that

$$f^{-1}\left((-\infty, y)\right) = (-\infty, x)\,.$$

We need to show two facts to obtain our result:
1. for any $y > f(q^\alpha(X))$ we have

$$F_{f(X)}(y) > \alpha,$$

2. for any $y < f(q^\alpha(X))$ we have

$$F_{f(X)}(y) \leq \alpha.$$

For the proof of the first fact we take $y > f(q^\alpha(X))$. We choose any $\bar{y} \in (f(q^\alpha(X)), y)$, take $\bar{x}$ such that $f^{-1}\left((-\infty, \bar{y})\right) = (-\infty, \bar{x})$, and note that since $\bar{y} > f(q^\alpha(X))$ we have $\bar{x} > q^\alpha(X)$. Taking any $\hat{x} \in (q^\alpha(X), \bar{x})$ we obtain the following estimates

$$\begin{aligned}
F_{f(X)}(y) &= P(f(X) \le y) \\
&\ge P(f(X) < \bar{y}) \qquad (\text{Since } \bar{y} < y) \\
&= P(X < \bar{x}) \\
&\ge P(X \le \hat{x}) \qquad (\text{Since } \hat{x} < \bar{x}) \\
&= F_X(\hat{x}) \\
&> \alpha \qquad (\text{Since } q^\alpha(X) < \hat{x} \text{ and by definition of } q^\alpha(X)).
\end{aligned}$$

For the second fact we take $y < f(q^\alpha(X))$, choose any $\bar{y} \in (y, f(q^\alpha(X)))$ and an $\bar{x}$ such that $f^{-1}((-\infty, \bar{y})) = (-\infty, \bar{x})$. Note that $\bar{x} \le q^\alpha(X)$ We then have the following estimates

$$\begin{aligned}
F_{f(X)}(y) &= P(f(X) \le y) \\
&\le P(f(X) < \bar{y}) \qquad (\text{Since } y < \bar{y}) \\
&= P(X < \bar{x}) \\
&\le P(X < q^\alpha(X)) \qquad (\text{Since } \bar{x} \le q^\alpha(X)) \\
&= \lim_{x \to q^\alpha(X)^-} F_X(x) \\
&\le \alpha, \qquad (\text{By definition of } q^\alpha(X))
\end{aligned}$$

as required. $\square$

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$$\begin{aligned}
&\min \mathbf{z}^{\mathrm{T}} \mathbf{P}^\alpha, \\
&\text{subject to:} \quad \mathbf{z}^{\mathrm{T}} \mathbf{H}(0) = c, \\
&\qquad\qquad\qquad \mathbf{z}^{\mathrm{T}} \mathbf{1} \le x, \\
&\qquad\qquad\qquad z_0, \ldots, z_n \ge 0.
\end{aligned} \tag{8.29}$$

by

$$\begin{aligned}
&\min -\mathbf{z}^{\mathrm{T}} \mathbf{P}^\alpha, \\
&\text{subject to:} \quad \mathbf{z}^{\mathrm{T}} \mathbf{H}(0) = c, \\
&\qquad\qquad\qquad \mathbf{z}^{\mathrm{T}} \mathbf{1} \le x, \\
&\qquad\qquad\qquad z_0, \ldots, z_n \ge 0.
\end{aligned} \tag{8.29}$$